UNIVERSITY OF CANTERBURY

USE OF THESIS

Full name of author   Francisco Javier Cristofulli

Full title of thesis  Seismic Behaviour of Reinforced
                    Concrete Structures with Masonry Infills

Degree              Ph.D. (Civil Eng.) Year of Presentation 1997

University Department Civil Engineering

Supervisor/or H.O.D. Dr. Athol Carr

Either (1) I agree to this thesis being consulted, for research or study purposes only, provided that due
acknowledgement of its use is made where appropriate;
and
I consent to copies of this thesis, in part or as a whole, being made, for research or study
purposes, at the discretion of the University of Canterbury Librarian.

Or (2) I wish the following special conditions to apply, for the period of time specified, to the use made
of this thesis;

Signature of author   [Signature] Date 29/7/97

If special conditions are to apply, the approval of the Supervisor and the University Librarian must be
obtained.

Conditions accepted by:

_________________________  ____________________________
(Supervisor)                  (University Librarian)

Date____________________   Date____________________
SEISMIC BEHAVIOUR OF REINFORCED
CONCRETE STRUCTURES WITH
MASONRY INFILLS

A thesis submitted in partial fulfilment
of the requirements for the degree
of
Doctor of Philosophy
in Civil Engineering
by
Francisco Javier Crisafulli

Department of Civil Engineering
University of Canterbury
Christchurch, New Zealand
July 1997
CONTENTS

ABSTRACT ........................................................................................................... ix

ACKNOWLEDGEMENTS .................................................................................... xi

NOTATION ...................................................................................................... xiii

CHAPTER 1: INTRODUCTION
1.1 Background .................................................................................................. 1
1.2 Objectives of the Research Work ................................................................. 2
1.3 Terminology .................................................................................................. 4
1.4 Organization .................................................................................................. 4

CHAPTER 2: MASONRY MATERIALS
2.1 Introduction .................................................................................................. 7
2.2 Masonry Materials ....................................................................................... 7
  2.2.1 General .................................................................................................. 7
  2.2.2 Masonry Units ....................................................................................... 7
    2.2.2.1 Clay Bricks ..................................................................................... 7
    2.2.2.2 Concrete Masonry Units ............................................................... 8
    2.2.2.3 Calcium Silicate Masonry Units .................................................... 9
  2.2.3 Mortars .................................................................................................. 9
2.3 Properties of Masonry Units ......................................................................... 10
  2.3.1 General .................................................................................................. 10
  2.3.2 Compressive Strength of Masonry Units .............................................. 10
  2.3.3 Proposed Method for the Evaluation of the Compressive Strength of Masonry Units .......................................................................................... 11
  2.3.4 Tensile Strength of Masonry Units ......................................................... 14
  2.3.5 Failure Criteria under Multi-Axial Stress for Masonry Units ............... 16
  2.3.6 Modulus of Elasticity and Poisson's Ratio of Masonry Units .............. 19
  2.3.7 Moisture Content and Absorption of Masonry Units ......................... 21
2.4 Properties of Mortar .................................................................................... 21
  2.4.1 Uniaxial Compressive Strength of Mortar .......................................... 21
  2.4.2 Compressive Strength of Confined Mortar ......................................... 23
  2.4.3 Modulus of Elasticity and Poisson's Ratio of Mortar ......................... 26
  2.4.4 Workability and Water Retentivity ...................................................... 28
2.5 Conclusions ................................................................................................ 28

CHAPTER 3: BEHAVIOUR OF MASONRY IN COMPRESSION
3.1 Mechanism of Failure under Direct Compression ......................................... 31
3.2 Compressive Strength of Masonry ................................................................ 34
  3.2.1 General .................................................................................................. 34
  3.2.2 Experimental Determination of Compressive Strength ....................... 34
  3.2.3 Factors Affecting the Compressive Strength ........................................ 37
3.3 Stress-Strain Relationship of Masonry ....................................................... 40
3.4 Modulus of Elasticity and Poisson's Ratio of Masonry ................................ 42
### 3.5 Evaluation of the Compressive Strength of Masonry

- **3.5.1 General** .......................................................... 45
- **3.5.2 Empirical Formulas** .......................................... 46
- **3.5.3 Failure Theories**
  - 3.5.3.1 Hilsdorf's Theory ........................................... 47
  - 3.5.3.2 Khoo and Hendry's Theory ................................ 47
  - 3.5.3.3 Francis, Hormann and Jerrem's Theory ................. 50
  - 3.5.3.4 Atkinson, Noland and Abrams' Theory .................. 51
  - 3.5.3.5 Older's Theory ............................................. 52
- **3.5.4 Proposed Failure Theory** ................................. 53
  - 3.5.4.1 General Formulation ....................................... 53
  - 3.5.4.2 Stress Distribution Factor $C_d$ ....................... 55
- **3.5.5 Nonlinear Analysis of Masonry Prism in Compression** 56
- **3.5.6 Comparison with Experimental Results** ............... 56

### 3.6 Discussion on the Validity of Failure Theories ............ 61

### 3.7 Conclusions ...................................................... 63

#### CHAPTER 4: MASONRY SUBJECTED TO SHEAR, TENSION AND BIAXIAL STRESSES

### 4.1 Masonry Subjected to Shear .................................... 65

- **4.1.1 Introduction** ................................................ 65
- **4.1.2 Modes of Failure**
  - 4.1.2.1 General ..................................................... 65
  - 4.1.2.2 Shear-Friction Failure .................................. 66
  - 4.1.2.3 Diagonal Tension Failure ................................. 66
  - 4.1.2.4 Compressive Failure ..................................... 66
- **4.1.3 Shear Test Procedures** ................................... 67
- **4.1.4 Shear Strength of Mortar Joints** ......................... 70
  - 4.1.4.1 Introduction .............................................. 70
  - 4.1.4.2 Response of Bed Joints Subjected to Shear .......... 71
  - 4.1.4.3 Nature of Bond .......................................... 73
  - 4.1.4.4 Factors Affecting the Bond Strength ................. 74
  - 4.1.4.5 Coefficient of Friction .................................. 75
- **4.1.5 Deformational Properties** ................................ 75

### 4.2 Shear Failure Theories .......................................... 76

- **4.2.1 General** ...................................................... 76
- **4.2.2 Mohr-Coulomb Criterion** .................................. 77
- **4.2.3 Mann and Müller's Theory**
  - 4.2.3.1 General Assumptions .................................... 77
  - 4.2.3.2 Shear-Friction Failure .................................. 78
  - 4.2.3.3 Diagonal Tension Failure ................................. 79
  - 4.2.3.4 Compressive Failure .................................... 79
  - 4.2.3.5 Failure Envelope Curve ................................. 79
- **4.2.4 Modification of Mann and Müller's Theory** ............ 80
  - 4.2.4.1 General Formulation ..................................... 80
  - 4.2.4.2 Evaluation of the Coefficients $C_s$ and $C_t$ ....... 82
  - 4.2.4.3 Total Participation of the Head Joints .............. 83
  - 4.2.4.4 Influence of the Axial Stress $f_a$ .................... 83
  - 4.2.4.5 Comments and Discussion of Results ................. 84
- **4.2.5 Masonry Subjected to Shear and Tensile Stresses** .... 86

### 4.3 Masonry Subjected to Tension .................................... 86

- **4.3.1 Introduction** ................................................ 86
- **4.3.2 Modes of Failure and Strength in Direct Tension** .... 87
- **4.3.3 Testing Procedures** ....................................... 88

### 4.4 Masonry Subjected to Biaxial Stresses ........................ 88

- **4.4.1 Introduction** ................................................ 88
- **4.4.2 Modes of Failure** .......................................... 89
5.8.9 Presence of Openings ................................................. 156
5.8.10 Three-Dimensional Configuration of the Building .......... 157
5.9 Conclusions ............................................................ 157

CHAPTER 6: REVIEW OF PROCEDURES FOR ANALYSIS OF INFILLED FRAMES
6.1 Introduction ............................................................. 161
6.2 Macro-Models for Infilled Frames ................................. 162
   6.2.1 Diagonal Strut Model ............................................ 162
      6.2.1.1 General Description ...................................... 162
      6.2.1.2 Modification of the Diagonal Strut Model .......... 163
      6.2.1.3 Properties of the Diagonal Strut ..................... 165
      6.2.1.4 Hysteretic Behaviour of the Diagonal Struts ....... 171
   6.2.2 Storey Mechanism Model ...................................... 173
   6.2.3 Other Macro-Models .......................................... 174
6.3 Micro-Models for Infilled Frames ............................... 176
   6.3.1 Finite Element Models ........................................ 176
      6.3.1.1 Introduction ............................................. 176
      6.3.1.2 Modelling of the Masonry Panel .................... 177
      6.3.1.3 Modelling of the Surrounding Frame ............... 178
      6.3.1.4 Modelling of the Interfaces ........................ 178
   6.3.2 Other Micro-Models .......................................... 183
6.4 Evaluation of the Strength of Infilled Frames ................. 183
   6.4.1 General .......................................................... 183
   6.4.2 Sliding Shear Failure of the Masonry Panel ............. 184
   6.4.3 Diagonal Tension Failure of the Masonry Panel .......... 186
   6.4.4 Compressive Failure of the Masonry Panel ............... 187
   6.4.5 Tensile Failure ................................................. 188
   6.4.6 Sliding Shear Failure of the Columns ..................... 188
   6.4.7 General Design Expressions ................................ 189
6.5 Conclusions .......................................................... 189

CHAPTER 7: PROPOSED MODEL FOR THE CYCLIC BEHAVIOUR OF MASONRY
7.1 Introduction .......................................................... 191
7.2 Analytical Model for Cyclic Axial Behaviour of Masonry .... 191
   7.2.1 General .......................................................... 191
   7.2.2 Rule 1: Envelope Curve in Compression ................... 192
   7.2.3 Unloading and Reloading ..................................... 194
      7.2.3.1 Proposed Equation ...................................... 194
      7.2.3.2 Rule 2: Unloading from the Envelope Curve .......... 196
      7.2.3.3 Rule 3: No Stress ....................................... 200
      7.2.3.4 Rule 4 and 5: Reloading after Complete Unloading . 200
   7.2.4 Small Cycle Hysteresis ....................................... 203
      7.2.4.1 General ................................................... 203
      7.2.4.2 Cyclic Tests ............................................. 204
      7.2.4.3 Unloading from Rule 4 or 5 .......................... 204
      7.2.4.4 Reloading from Rule 2 .................................. 205
      7.2.4.5 Accumulative Damage Due to Small Cycle Hysteresis 206
   7.2.5 Tensile Behaviour of Masonry - Rule 6 ..................... 206
   7.2.6 Local Contact Effects of Cracked Material on the Hysteretic Response 208
   7.2.7 Summary of the Hysteresis Rules ........................... 210
   7.2.8 Comparison with Experimental Results .................... 211
7.3 Analytical Model for Cyclic Behaviour of Masonry in Shear ... 215
   7.3.1 General .......................................................... 215
   7.3.2 Elastic Response - Rule 1 ................................... 215
   7.3.3 Sliding - Rule 2 ................................................. 216
   7.3.4 Unloading and Reloading after the Bond Failure - Rule 2 . 217
7.3.5 Effect of the Normal Stress $f_n$ ......................................................... 217
7.4 Conclusions .................................................................................. 217

CHAPTER 8: PROPOSED MODELS FOR THE ANALYSIS OF INFILLED FRAMES
8.1 Introduction ................................................................................. 219
8.2 Simple Procedure for the Design and Analysis of Infilled Frames .... 219
8.2.1 General ................................................................................ 219
8.2.2 Preliminary Study .................................................................. 219
8.2.3 Formulation of the Analytical Model ....................................... 221
8.2.3.1 General ........................................................................ 221
8.2.3.2 Shear-Friction Failure .................................................... 223
8.2.3.3 Diagonal Tension Failure ............................................... 224
8.2.3.4 Failure Envelope ............................................................ 224
8.2.4 Hysteretic Behaviour .............................................................. 225
8.2.5 Computational Implementation and Numerical Examples .......... 227
8.3 Proposed Macro-Model for Refined Analysis of Infilled Frames ...... 230
8.3.1 Introduction ........................................................................ 230
8.3.2 Diagonal Tension Failure ....................................................... 231
8.3.3 Crushing of the Corners ......................................................... 231
8.3.4 Shear Failure ....................................................................... 231
8.3.4.1 General Considerations .................................................. 231
8.3.4.2 Formulation of the Proposed Macro-Model ....................... 235
8.3.4.3 Numerical Examples ....................................................... 240
8.4 Modelling of Infilled Frames with the Program ABAQUS ............ 240
8.5 Conclusions .............................................................................. 243

CHAPTER 9: TEST PROGRAMME ON FRAMED MASONRY STRUCTURES
9.1 Introduction ............................................................................... 245
9.2 Objectives of the Test Programme ............................................... 245
9.3 Test Units .................................................................................. 246
9.3.1 Design Considerations .......................................................... 246
9.3.2 Description of the Test Units ................................................... 246
9.3.2.1 Test Unit 1 ................................................................ 246
9.3.2.2 Test Unit 2 ................................................................ 246
9.3.3 Construction of the Test Units ............................................... 249
9.4 Material Properties .................................................................. 251
9.4.1 Masonry Units ................................................................... 251
9.4.2 Mortar .............................................................................. 252
9.4.3 Masonry .......................................................................... 252
9.4.4 Concrete .......................................................................... 254
9.4.5 Reinforcing Steel ................................................................. 254
9.5 Test Arrangement ..................................................................... 255
9.5.1 General Description .............................................................. 255
9.5.2 Discussion on the Loading System ......................................... 257
9.6 Instrumentation ....................................................................... 260
9.6.1 Measurement of Loads .......................................................... 260
9.6.2 Measurements of Displacements and Deformations ............... 260
9.6.3 Measurements of Local and Average Reinforcement Strains .... 262
9.6.4 Data Acquisition System ...................................................... 265
9.7 Test Sequence ......................................................................... 265
9.8 Limitations of the Tests ............................................................. 266
9.9 Conclusions ............................................................................. 267

CHAPTER 10: TEST RESULTS
10.1 Introduction ............................................................................ 269
10.2 Unit 1 .................................................................................... 269
10.2.1 General Behaviour .................................................. 269
10.2.2 Lateral Force-Displacement Response ......................... 273
10.2.3 Beam-Column Joint Deformation .............................. 277
10.2.4 Curvature and Axial Strain in the Frame ....................... 278
10.2.5 Stresses in the Transverse Reinforcement ..................... 278
10.2.6 Evaluation of the Cracking Force and the Lateral Strength .. 279

10.3 Unit 2 ...................................................................... 281
10.3.1 General Behaviour ................................................. 281
10.3.2 Lateral Force-Displacement Response ......................... 284
10.3.3 Beam-Column Joint Deformation .............................. 286
10.3.4 Curvature and Axial Strain in the Frame ....................... 286
10.3.5 Stresses in the Transverse Reinforcement ..................... 288
10.3.6 Evaluation of the Cracking Force and the Lateral Strength .. 288

10.4 Comparison of the Behaviour of Units 1 and 2 ................. 289
10.5 Conclusions .................................................................. 292

CHAPTER 11: DESIGN CONSIDERATIONS FOR INFILLED FRAMES

11.1 Introduction .................................................................. 295
11.2 Design Considerations .................................................. 295
11.2.1 General ................................................................. 295
11.2.2 Tensile Failure of the Reinforced Concrete Frame .......... 295
11.2.3 Sliding Shear Failure of the Columns ......................... 296

11.3 Classification of Infilled Frames ..................................... 298
11.3.1 General ................................................................. 298
11.3.2 Elastic Infilled Frames ............................................. 298
11.3.3 Rocking Infilled Frames ......................................... 299
11.3.4 Ductile Infilled Frames ........................................... 299

11.4 Proposed Design of Cantilever Infilled Frames with Flexural Yielding .................................................. 300
11.4.1 General Description of the Procedure ......................... 300
11.4.2 Ductility Requirements ............................................ 304
11.4.3 Multibay Infilled Frames ......................................... 308
11.4.4 Summary of the Design Procedure ............................ 308

11.5 Proposed Design of Squat Infilled Frames with Shear Yielding .......................................................... 310
11.5.1 General Description of the Procedure ......................... 310
11.5.2 Ductility Requirements ............................................ 312
11.5.3 Summary of the Design Procedure ............................ 314

11.6 Recommendations for the Construction of Infilled Frames .......................................................... 315
11.7 Conclusions .................................................................. 315

CHAPTER 12: EFFECT OF HYSTERETIC PINCHING ON THE SEISMIC RESPONSE

12.1 Introduction ............................................................... 317
12.1.1 Objectives ............................................................. 317
12.1.2 Background .......................................................... 317

12.2 Parametric Study of Inelastic Systems Subjected to Earthquake Motions .................................................. 318
12.2.1 Earthquake Accelerograms ..................................... 318
12.2.2 Structural Model ..................................................... 324

12.3 Results ......................................................................... 326
12.3.1 Verification of the Concept of Equal Displacement ....... 326
12.3.2 Effect of the Shape of the Hysteresis Loops ................. 326
12.3.3 Ductility Demand .................................................... 331
12.3.4 Influence of the Damping Model ............................... 335

12.4 Conclusions .................................................................. 336

CHAPTER 13: CONCLUSIONS AND RECOMMENDATIONS

13.1 General ................................................................. 339
13.2 Conclusions Based on the Literature Review .................... 339
13.2.1 Behaviour of Masonry and Constitutive Materials ............................................ 339
13.2.2 Behaviour of Infilled Frames ........................................................................ 340
13.2.3 Analysis of Infilled Frames .......................................................................... 340
13.3 Conclusions Drawn from This Project .................................................................. 341
13.3.1 Masonry Materials ...................................................................................... 341
13.3.2 Evaluation of the Masonry Strength .............................................................. 341
13.3.3 Analysis of Infilled Frames ......................................................................... 342
13.3.4 Test Programme ............................................................................................ 343
13.3.5 Design of Infilled Frames ........................................................................... 344
13.4 Suggestions for Future Research ........................................................................ 345
13.4.1 Masonry ........................................................................................................ 345
13.4.2 Analysis of Infilled Frames ......................................................................... 346
13.4.3 Design of Infilled Frames ........................................................................... 346

REFERENCES .............................................................................................................. 349

APPENDIX 1: EVALUATION OF THE STRESS DISTRIBUTION FACTOR $C_d$
A1.1 Finite Element Analyses of Masonry Prisms ....................................................... 369
A1.1.1 Introduction .................................................................................................. 369
A1.1.2 Finite Element Model .................................................................................. 369
A1.1.3 Material Properties ...................................................................................... 371
A1.1.4 Procedure and Results ................................................................................ 371
A1.2 Finite Element Analyses of Bricks and Mortar Joints ........................................ 374
A1.3 Evaluation of the Stress Distribution Factors .................................................... 376
A1.4 Summary of the Mechanical and Geometrical Properties ................................ 377

APPENDIX 2: NONLINEAR ANALYSIS OF MASONRY PRISMS IN COMPRESSION
A2.1 Basis of the Procedure ....................................................................................... 379
A2.1.1 General Formulation .................................................................................... 379
A2.1.2 Analytical Model for Mortar ...................................................................... 380
A2.1.3 Analytical Model for Masonry Units ........................................................... 381
A2.2 Numerical Examples ......................................................................................... 383
A2.3 Computer Program ........................................................................................... 384

APPENDIX 3: EVALUATION OF THE COEFFICIENTS $C_q$ AND $C_d$
A3.1 Introduction ..................................................................................................... 389
A3.2 Simplified Evaluation of the Coefficients $C_q$ and $C_d$ ..................................... 389
A3.3 Finite Element Analysis of Masonry Panels Subjected to Pure Shear ............. 391

APPENDIX 4: COMPUTER PROGRAM FOR THE HYSTERETIC RESPONSE OF
MASONRY STRUTS
A4.1 General ............................................................................................................. 395
A4.2 Computer Program ............................................................................................ 395
ABSTRACT

This thesis focuses on the seismic behaviour of reinforced concrete structures with masonry infills, with particular interest in the development of rational procedures for the design and analysis of this type of structure.

The properties of masonry and its constitutive materials were reviewed, giving special emphasis to those aspects which contribute to a better understanding of the strength mechanism. Theoretical procedures were developed for the rational evaluation of the strength of masonry subjected to compressive and shear stresses.

A large amount of experimental work related to the behaviour of infilled frames was also reviewed. The main characteristics of the response under lateral loading were discussed for different types of infilled frames and a comprehensive classification of the modes of failure, for both the masonry panel and the surrounding frame, was conducted. In addition, the influence of several parameters which can affect the structural response was evaluated.

Two theoretical procedures, with different degree of refinement, are proposed in this study for the analysis of infilled frames. The first procedure is a simple approach, based on the equivalent truss mechanism, which allows the evaluation of the lateral resistance of the infilled frames, considering two different types of failure in the masonry panel, namely, shear-friction and diagonal tension failure. The compressive strength of the diagonal strut is assessed by transforming the shear failure envelope obtained from the modification of the Mann and Müller's theory. This transformation takes into account the inclination of the diagonal strut and neglects the effect of the tensile principal stresses acting on the masonry panel. The second procedure is a refined macroscopic model based on a multi-strut formulation, which is intended to represent more accurately the effect of the masonry panel on the surrounding frame. Since debonding of the mortar joints is the most common type of failure observed in the masonry panel, the formulation of the procedure is specifically developed to represent this situation. The model accounts separately for the compressive and shear behaviour of masonry using a double truss mechanism and a shear spring in each direction. Recommendations are also given for the analysis of infilled frames when a failure due to diagonal tension or crushing of the corners is expected in the panel.

A test programme was implemented to investigate the seismic response of infilled frames. The main criterion followed for the design was that the reinforced concrete columns should yield in tension in order to obtain a reasonable ductile response under lateral loading. New reinforcing details were provided in one unit, aimed at enhancing the structural response. These details consisted in tapered beam-column joints with diagonal reinforcement, and additional longitudinal reinforcement in the frame members. The additional bars placed in the columns were not anchored to the foundation in order to produce a weak region at the base of the
columns, where most of the plastic deformations were expected to occur. The most important conclusion of the experimental programme is that the response of reinforced concrete frames with masonry infills can be significantly improved by a rational design aimed at reducing the distortion of the masonry panels while plastic deformations are concentrated in selected regions of the structure.

A new design approach is proposed for infilled frames, in which two cases are considered: cantilever and squat infilled frames. In the first case, the ductile behaviour is achieved by yielding of the longitudinal reinforcement, which is limited to occur only at the base of the columns, and by avoiding large elongations of the remaining parts of the surrounding frame. A pre-cracked connection is induced between the infilled frame and the foundation, where plain round dowels can be placed to control shear sliding. In the second case, ductility is conferred to the structure by allowing controlled sliding of the infilled frame over the foundation. The applicability of this approach is limited to those cases where the total shear force exceeds the frictional strength of the pre-cracked connection.

The effect of pinching of the hysteresis loops in the response of infilled frames subjected to earthquakes was investigated. A parametric study was conducted using a one-degree-of-freedom oscillator subjected to ground accelerations recorded in five different earthquakes. Results obtained from the dynamic nonlinear analyses indicated that the effect of pinching and the damping model used can significantly influence the response of infilled frames, which normally exhibit a short to medium initial period of free vibration. Therefore, the displacement demand imposed by the earthquake can be larger than that assumed by the seismic codes if they are based on the concept of equal displacement.
ACKNOWLEDGEMENTS

The research work reported in this thesis was conducted in the Department of Civil Engineering, University of Canterbury, New Zealand, under the supervision of Dr. Athol J. Carr and Professor Robert Park, to whom I wish to express my deepest gratitude for their invaluable guidance and continuous support.

The fee paying scholarship provided by the Ministry of External Relations and Trade of New Zealand and the financial assistance given by the Faculty of Engineering, University of Cuyo, Argentina are gratefully acknowledged. I would like to sincerely thank Ings. J. M. Gómez and T. A. Montes, former and actual dean of the Faculty of Engineering, University of Cuyo, and Ings. E. Japaz and E. Villafañe for their support.

Special thanks are given to Dr. J. I. Restrepo not only for showing a continuous interest in my research and for contributing with useful advice, but also for sharing a lot of good moments of friendship; and to Emeritus Professor T. Paulay for his friendship and fruitful discussions always mixed with a good joke.

The useful advice of Dr. J. B. Berrill in the selection of the earthquake records used in the dynamic analyses, and the help of Dr. R. Meli, Professor. A. W. Page, Dr. M. Rodríguez and Professor A. R. Santhakumar, who provided copies of different research reports, are very much appreciated.

The assistance of the technical staff of the Department of Civil Engineering under the management of Mr. G. Clarke is acknowledged. Mr. G. Harvey constructed the test specimens and helped with the logistics of the tests. Messrs. R. Allen and P. Murphy collaborated in the construction of the reaction frame. The excellent job of Messrs. R. Newton and M. Weavers with the electronic equipment is very much appreciated. They helped with the data acquisition system and developed the servo valve controller used in the test programme, but also faced the enormous task of trying to explain the basic concepts of electronic to a civil engineer (even though they did not succeed). Thanks are also given to Mr. M. Roestenburg for his assistance with the photographic equipment, to Messrs. B. Hutchinson and P. J. Coursey for helping with the computer facilities and to Mrs. V. J. Grey for drawing several figures of Chapters 5 and 9. Mr. N. Dixon is thanked for his assistance with the purchase of the materials and for his ability to decipher my messages in English.

I also wish to thank my fellow post-graduate students for their constructive discussions and friendship, in particular, Drs. J. Maffei and Widodo and Messrs. C. M. Lin, A. Rahman, I. Satyarno and Y. C. Wang.

I owe a great deal to Ing. E. Villafañe, who taught me the principles of earthquake engineering and showed me with his example that research is a gratifying, even not profitable, endeavour.
Finally, I express my deepest gratitude to my wife, Vicky, for her love and comprehension, and especially for keeping always a nice smile, even when something goes wrong. She also helped with the drawing of some figures, and the correction and edition of the final draft of this thesis. Sincere thanks are given to our parents for their understanding and support during the past years. We are also grateful to Professors R. Park and T. Paulay, Drs. A. M. Remennikov, J. I. Restrepo, W. Titulaer and D. Wareham, and Mr. R. Jarquin, and their families for their friendship and for helping to make our life in New Zealand an enjoyable experience.
### NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>peak ground acceleration, expressed as a fraction of the gravity</td>
</tr>
<tr>
<td>$A_{1}, A_{2}$</td>
<td>coefficients of Sargin's stress-strain equation</td>
</tr>
<tr>
<td>$A_{b}$</td>
<td>cross-sectional area of the brick</td>
</tr>
<tr>
<td>$A_{c}$</td>
<td>area of a column section</td>
</tr>
<tr>
<td>$A_{d}$</td>
<td>cross-sectional area in the diagonal plane of square masonry panels</td>
</tr>
<tr>
<td>$A_{f}$</td>
<td>effective area of the shear friction</td>
</tr>
<tr>
<td>$A_{g}$</td>
<td>gross area of a section</td>
</tr>
<tr>
<td>$A_{m}$</td>
<td>area of a masonry panel in the horizontal plane $= t L_{m}$</td>
</tr>
<tr>
<td>$A_{ma}$</td>
<td>area of the equivalent masonry strut $= t w$</td>
</tr>
<tr>
<td>$A_{mal}, A_{ma2}$</td>
<td>initial and final area of the equivalent masonry strut</td>
</tr>
<tr>
<td>$A_{n}$</td>
<td>net cross-sectional area of the brick</td>
</tr>
<tr>
<td>$A_{re}$</td>
<td>area of anchored reinforcement</td>
</tr>
<tr>
<td>$A_{rc}$</td>
<td>area of longitudinal reinforcement crossing a pre-cracked connection</td>
</tr>
<tr>
<td>$A_{sd}$</td>
<td>total area of plain round dowels</td>
</tr>
<tr>
<td>$A_{si} f_{si}$</td>
<td>axial force in a longitudinal reinforcing bar</td>
</tr>
<tr>
<td>$A_{sl}$</td>
<td>total area of longitudinal reinforcement</td>
</tr>
<tr>
<td>$A_{s}$</td>
<td>area of the splitting section</td>
</tr>
<tr>
<td>$A_{v}$</td>
<td>vertical area of a masonry panel</td>
</tr>
<tr>
<td>$A_{v, eff}$</td>
<td>effective shear area of a masonry panel</td>
</tr>
<tr>
<td>$a$</td>
<td>empirical constant for the calculation of $e_{p}$</td>
</tr>
<tr>
<td>$a_{1}, a_{2}$</td>
<td>empirical constants</td>
</tr>
<tr>
<td>$B_{1}, B_{2}, B_{3}$</td>
<td>constants for the unloading-reloading equation</td>
</tr>
<tr>
<td>$b$</td>
<td>height of the brick</td>
</tr>
<tr>
<td>$C_{c}$</td>
<td>compressive force carried by the concrete</td>
</tr>
<tr>
<td>$C_{d}$</td>
<td>combined stress distribution factor $= C_{db} / C_{dj}$</td>
</tr>
<tr>
<td>$C_{db}$</td>
<td>stress distribution factor for brick</td>
</tr>
<tr>
<td>$C_{dj}$</td>
<td>stress distribution factor for mortar</td>
</tr>
<tr>
<td>$C_{n}$</td>
<td>normal stress distribution factor</td>
</tr>
<tr>
<td>$C_{s}$</td>
<td>shear stress distribution factor</td>
</tr>
<tr>
<td>$c$</td>
<td>empirical coefficient</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>designation of a 10 mm diameter deformed bar</td>
</tr>
<tr>
<td>$d$</td>
<td>length of the brick</td>
</tr>
<tr>
<td>$d_{b}$</td>
<td>nominal diameter of a bar</td>
</tr>
<tr>
<td>$d_{h}, d_{v}$</td>
<td>horizontal and vertical dimensions of the tapered joints</td>
</tr>
<tr>
<td>$dL$</td>
<td>differential of length</td>
</tr>
</tbody>
</table>
\( d_m \quad = \quad \text{diagonal length of the masonry panel} \)
\( d_l \quad = \quad \text{distance between the longitudinal reinforcing bars} \)
\( E_{t1}, E_{t2} \quad = \quad \text{initial and final tangent modulus for the unloading-reloading equation} \)
\( E_A \quad = \quad \text{absorbed energy} \)
\( E_b \quad = \quad \text{modulus of elasticity of the brick} \)
\( E_e \quad = \quad \text{modulus of elasticity of the concrete} \)
\( E_{eh} \quad = \quad \text{tangent modulus at changing strain } \epsilon_{eh} \)
\( E_{el} \quad = \quad \text{modulus of elasticity at strain } \epsilon_{el} \)
\( E_D \quad = \quad \text{dissipated energy} \)
\( E_i \quad = \quad \text{modulus of elasticity of the mortar} \)
\( E_{jo} \quad = \quad \text{initial modulus of elasticity of the mortar} \)
\( E_m \quad = \quad \text{modulus of elasticity of the masonry} \)
\( E_{mo} \quad = \quad \text{initial modulus of elasticity of the masonry} \)
\( E_{pl,u} \quad = \quad \text{tangent modulus at plastic strain } \epsilon_{pl} \text{ for unloading} \)
\( E_{pl,r} \quad = \quad \text{tangent modulus at plastic strain } \epsilon_{pl} \text{ for reloading} \)
\( E_{re} \quad = \quad \text{tangent modulus at reloading strain } \epsilon_{re} \)
\( E_s \quad = \quad \text{secant modulus of elasticity,} \)
\( \text{or modulus of elasticity of steel} \)
\( E_t \quad = \quad \text{tangent modulus of elasticity} \)
\( E_{un} \quad = \quad \text{tangent modulus at unloading strain } \epsilon_{un} \)
\( e_1, e_2 \quad = \quad \text{empirical exponents} \)
\( F \quad = \quad \text{force in general} \)
\( \{F\}_A, \{F\}_B \quad = \quad \text{nodal force vectors} \)
\( f_1, f_2 \quad = \quad \text{stresses at point 1 and 2 of the unloading-reloading equation,} \)
\( \text{or principal stresses} \)
\( f_a \quad = \quad \text{stress at point A, used to define the plastic strain} \)
\( f'_c \quad = \quad \text{compressive strength of concrete} \)
\( f'_e \quad = \quad \text{uniaxial compressive strength of the brick} \)
\( f_{eh} \quad = \quad \text{stress at changing strain } \epsilon_{eh} \)
\( f_{ej} \quad = \quad \text{confined strength of the mortar} \)
\( f_d \quad = \quad \text{diagonal strength of the masonry panel} \)
\( f_j \quad = \quad \text{mortar stress} \)
\( f'_j \quad = \quad \text{uniaxial compressive strength of the mortar} \)
\( f_m \quad = \quad \text{masonry stress} \)
\( f'_m \quad = \quad \text{compressive strength of the masonry} \)
\( f_m \quad = \quad \text{fictitious failure stress of the masonry} \)
\( f_{m8} \quad = \quad \text{compressive strength of the masonry for an angle } \theta \)
\( f_n \quad = \quad \text{normal stress at the bed joint} \)
\( \Delta f_n \quad = \quad \text{increment of the normal stress } f_n \)
\( \Delta f_{av} \quad = \quad \text{average value of the increment of the normal stress } f_n \)
\( f_{n1}, f_{n2} \quad = \quad \text{normal stresses} \)
\( f_p \quad = \quad \text{parallel stress to the bed joint} \)
\( f_{re} \quad = \quad \text{stress at reloading strain } \epsilon_{re} \)
\( f_{r,i} \) = stress at reloading strain \( e_{r,i} \)
\( f_u \) = ultimate (maximum) strength
\( f' \) = reduced tensile strength
\( f'_t \) = uniaxial tensile strength
\( f'_{tb} \) = uniaxial tensile strength of the brick
\( f'_{tm} \) = tensile strength of masonry
\( f'_{io} \) = tensile bond strength
\( f_{un} \) = stress at unloading strain \( e_{un} \)
\( f_x \) = lateral stress (in x direction)
\( f_{ob}, f_{sj} \) = lateral stress in x direction for brick and mortar, respectively
\( \Delta f_{ob}, \Delta f_{sj} \) = increment of the stress \( f_{ob} \) and \( f_{sj} \)
\( f_y \) = vertical stress (in y direction), or yield strength of reinforcing steel
\( \Delta f_y \) = increment of the stress \( f_y \)
\( f_{yd} \) = yield strength of a plain round dowel.
\( f_z \) = lateral stress (in z direction)
\( f'_{ob}, f_{sj} \) = lateral stress in z direction for brick and mortar, respectively
\( G_m \) = shear modulus of the masonry
\( g \) = acceleration due to gravity
\( h \) = storey height
\( h_e \) = effective column height between plastic hinges
\( h_m \) = height of masonry panel
\( h_p \) = height of masonry prism
\( h_r \) = vertical separation between diagonal struts
\( h' \) = height of the resultant of the seismic lateral forces
\( I_e \) = second moment of area (moment of inertia) of a column section
\( j \) = thickness of the mortar joint
\( K \) = stiffness in general
\( K_a \) = axial stiffness of the masonry strut
\( K_o \) = reference stiffness, or initial stiffness
\( K_s \) = secant stiffness, or shear stiffness
\( L \) = span of beam between centre lines of supporting columns
\( L_d \) = diagonal length
\( L_E \) = length of the edge of a panel element
\( L_{of} \) = total length of the infilled frame
\( L_m \) = length of masonry panel between adjacent columns
\( L_o \) = overlap length in stretcher bond masonry
\( L_{sc} \) = horizontal projection of the length of the stepped crack
\( L_{ub} \) = unbonded length
\( M_b \) = bending moment in a beam
\( M_{I_l} \) = Richter (local) magnitude
\( M_o \) = total overturning moment
\( M_s \) = surface wave magnitude
\( M_d \) = flexural strength
\( M_{dc}, M_{ut} \) = flexural strength of the compression and tension column, respectively
\( m \) = confinement factor for mortar
\( n \) = number of storeys,
or empirical coefficient
\( n_b \) = number of brick courses of a masonry panel
\( n_{xy} \) = number of inner loops
\( P \) = axial force
\( P_d \) = diagonal load in masonry tests
\([Q]\) = transformation matrix in general
\([Q]_{ed}\) = transformation matrix from external to internal nodes
\([Q]_{ol}\) = transformation matrix from global to local coordinates
\([Q]_{Id}\) = transformation matrix from internal to dummy nodes
\( R \) = compressive force in the equivalent strut,
or strength reduction factor
\( R_e \) = compressive strength of the equivalent strut
\( r \) = \( b/l \)
\( S_1, S_2 \) = numerical factors for the brick failure envelope
\( s \) = distance between a dummy node and an internal node
\( T_e \) = tensile axial force acting on a column
\( T_s \) = characteristic period of the ground motion
\( T_o \) = initial period of free vibration
\( t \) = thickness of masonry infill
\( t_p \) = thickness of masonry prism
\( U \) = nonuniformity coefficient
\( U_u \) = nonuniformity coefficient at failure
\( u \) = horizontal displacement of a node
\( \{u\}_A, \{u\}_B \) = nodal displacement vectors
\( V \) = shear force,
or peak ground velocity
\( V' \) = predicted shear strength
\( V_b \) = shear force in a beam
\( V_c \) = shear force in a column,
or cracking force
\( V_{cr} \) = shear force at which cracking occurs
\( V_d \) = shear strength due to dowel action
\( V_e \) = elastic shear force
\( V_f \) = shear strength due to friction
\( V_i \) = intercept force in the pinched model
\( V_s \) = shear force at which shear sliding occurs
\( V_t \) = shear force at which diagonal crack occurs
\( V_y \) = yield shear force
\( V_u \) = ultimate shear resistance,

or design shear force according to principles of capacity design
\( v \) = vertical displacement of a node
\( W \) = weight
\( W_C \) = weight of cement in the mortar composition
\( W_L \) = weight of lime in the mortar composition
\( w \) = effective width of the equivalent diagonal strut
\( x_{oi}, y_{oi} \) = horizontal and vertical offset of the internal node i of the panel element
\( z \) = vertical contact length between the masonry panel and the column
\( \alpha \) = \( j/(4.1 \, b) \),
or angle of kinking of the longitudinal reinforcement
\( \alpha_{ch} \) = empirical factor to define \( \varepsilon_{ch} \)
\( \alpha_m \) = \( j/(m \, b) \)
\( \alpha_{re} \) = empirical factor to define \( \varepsilon_{re} \)
\( \alpha_s \) = shear stress distribution factor
\( \beta \) = non-dimensional parameter expressing the relative stiffness of the frame to the panel,
or ratio \( E_s/E_l \)
\( \beta_s \) = empirical factor to define \( f_s \)
\( \beta_{ch} \) = empirical factor to define \( f_{ch} \)
\( \gamma \) = shear strain
\( \gamma_{pl,r} \) = empirical factor to define \( E_{pl,r} \)
\( \gamma_{pl,u} \) = empirical factor to define \( E_{pl,u} \)
\( \gamma_s \) = shear stiffness factor
\( \gamma_{un} \) = empirical factor to define \( E_{un} \)
\( \Delta \) = displacement
\( \Delta_{d}, \Delta'_{d} \) = change in length of the diagonals of beam-column joints
\( \Delta_E \) = maximum lateral displacement in an elastic system
\( \Delta_{EP} \) = maximum lateral displacement in an elasto-plastic system
\( \Delta_l \) = lateral displacement due to shear and flexural deformation of the infilled frame
\( \Delta_y \) = lateral displacement \( \Delta_l \) defined at first yield
\( \Delta_{si} \) = maximum lateral displacement in a nonlinear system with initial stiffness damping
\( \Delta_m \) = maximum displacement
\( \Delta_{pi} \) = maximum lateral displacement in a pinched system
\( \Delta_p \) = plastic lateral displacement
\( \Delta_R \) = axial displacement of the masonry strut
\( \Delta_{k1}, \Delta_{k2} \) = axial displacement of the masonry strut at which the area \( A_{mn} \) changes
\( \Delta_r \) = lateral displacement due to a rigid body rotation
\( \Delta_s \) = elongation of a longitudinal reinforcing bar
\( \Delta_{sl} \) = lateral displacement due to sliding shear
\( \Delta_{tan} \) = maximum lateral displacement in a system nonlinear with tangent stiffness damping
\( \Delta_y \) = lateral displacement at first yield
\( \Delta_{y}^{*} \) = equivalent yield displacement
\[ \Delta_{15} \] = lateral displacement corresponding to a force level of 0.75 \( V_y \)
\[ \delta \] = storey drift, equal to the ratio of the lateral displacement to the storey height
\[ \delta_y \] = storey drift at first yield
\[ \delta_u \] = storey drift corresponding to the ultimate shear resistance, \( V_u \)
\[ \varepsilon \] = axial strain
\[ \varepsilon_a \] = strain at point A, used to define the plastic strain
\[ \varepsilon_b \] = strain at point B, used to define \( E_{sh} \) and \( \varepsilon_{ch} \)
\[ \varepsilon_{ch} \] = strain for changing hysteresis rule
\[ \varepsilon_{el} \] = strain at which masonry starts to transfer compressive stresses
\[ \varepsilon_j \] = mortar strain
\[ \varepsilon_j' \] = strain corresponding to the maximum stress for confined mortar
\[ \varepsilon_{jo} \] = strain corresponding to the maximum stress for unconfined mortar
\[ \varepsilon_m \] = masonry strain
\[ \varepsilon_m' \] = masonry strain corresponding to the maximum stress
\[ \varepsilon_{max} \] = maximum strain in a reinforcing bar
\[ \varepsilon_{pl} \] = plastic strain
\[ \varepsilon_{re} \] = reloading strain at the envelope curve
\[ \Delta \varepsilon_{re} \] = increment of the reloading strain
\[ \varepsilon_{rel} \] = reloading strain for inner loops
\[ \varepsilon_\alpha, \varepsilon_\beta \] = axial strains in the longitudinal reinforcement
\[ \varepsilon_{sh} \] = strain at the beginning of strain hardening
\[ \varepsilon_u \] = ultimate (uniform) strain of reinforcing steel
\[ \varepsilon_{um} \] = ultimate strain of masonry
\[ \varepsilon_{um} \] = unloading strain at the envelope curve
\[ \varepsilon_{um} \] = unloading strain for inner loops
\[ \varepsilon_{x, y} \] = lateral strain (x direction) in brick and mortar, respectively
\[ \varepsilon_y \] = yield strain
\[ \varepsilon_{y, y} \] = vertical strain in brick and mortar, respectively
\[ \varepsilon_{z, z} \] = lateral strain (z direction) in brick and mortar, respectively
\[ \lambda_n \] = non-dimensional parameter expressing the relative stiffness of the infill to the frame
\[ \lambda_{of} \] = frictional overstrength factor
\[ \lambda_{st} \] = steel overstrength factor
\[ \phi \] = ratio of the shear resistance of the infilled frame to the shear resistance of the bare frame
\[ \phi_s \] = acceleration amplification factor
\[ \phi_o \] = flexural overstrength factor
\[ \phi_{res} \] = residual resistance of the infilled frame, at \( \delta = 2-3\% \), to the residual resistance of the bare frame
\[ \phi_v \] = velocity amplification factor
\[ \rho \] = inclination of the diagonal cracks measured from the direction of the normal stress \( f_n \)
\[ \mu \] = coefficient of friction
\[ \mu_r \] = residual coefficient of friction after bond failure
\( \mu^* \) = reduced coefficient of friction at bed joints
\( \mu_d \) = displacement ductility ratio
\( \rho \) = unit weight
\( \rho_t \) = ratio of total reinforcement = \( A_w/A_g \)
\( \sigma_l \) = lateral confinement stress
\( \tau \) = shear stress
\( \tau' \) = design shear strength
\( \tau_{av} \) = average shear stress
\( \tau_d \) = nominal shear stress along the diagonal plane
\( \tau_m \) = shear strength of the masonry
\( \tau_{max} \) = maximum shear stress
\( \tau'_{max} \) = maximum permissible shear stress for masonry
\( \tau_o \) = initial shear bond strength (cohesion)
\( \tau_{o*} \) = reduced shear bond strength (cohesion)
\( v_b \) = Poisson's ratio of the brick
\( v_j \) = Poisson's ratio of the mortar
\( v_{jo} \) = initial Poisson's ratio of the mortar
\( v_m \) = Poisson's ratio of the masonry
\( \theta \) = angle between the bed joint direction and the principal stress \( f_i \), or inclination of the diagonal of masonry panel respect to the horizontal axis, or inclination of the diagonal of a beam-column joint to the horizontal
\( \theta_E \) = inclination of the edge of a panel element
\( \xi \) = damping ratio
\( \chi \) = normalized strain
\( \psi \) = strength reduction factor
\( \omega_v \) = dynamic shear magnification factor
1. INTRODUCTION

1.1 BACKGROUND
Frame structures with masonry infills are commonly used in regions of high seismicity, especially where masonry is still an economical construction material. Adequate knowledge of the behaviour is required to design this type of structure in order to reduce the loss of life and property associated with a possible structural failure. Furthermore, infilled frame buildings designed and constructed before the development of actual seismic codes constitute an important part of the high-risk structures in different countries. The rehabilitation of these buildings to resist seismic actions implies, as a first step, the assessment of the structural behaviour.

Infilled frame structures have been used since the beginning of this century for low and medium-height buildings. Different causes originated the use of infilled frames as a construction system. In some places, the masonry panels are built mainly as architectural elements in order to profit from the advantages of masonry, such as fire resistance, aesthetic appeal, and thermal and acoustic insulation. Industrialized systems and other panel materials are also used to reduce the duration of the construction and the labour costs required by masonry walls.

In other places, infilled frames appeared due to the necessity of improving the resistance of masonry walls under lateral forces. Since the 18th century, engineers and architects tried to reduce the massive sections of masonry constructions. They intuitively used iron to cramp masonry units and later to build reinforced beams, before any work on reinforced concrete. Thereafter, reinforced masonry received little attention until this century, when it developed in some seismic areas, notably New Zealand, India and parts of USA [S27]. Infilled frames may be considered, therefore, as a special type of reinforced masonry, in which the reinforcement is concentrated at the edges of the panel to provide flexural resistance and continuity between walls and slabs and between intersecting walls.

The distinct criteria mentioned above led to two different constructive techniques, depending on the sequence followed to build the infilled frames. One option is to build the frame (steel or reinforced concrete) and later to construct the masonry panels which infill the frame. In this case, the shrinkage of the infill material or defects due to inaccurate workmanship usually results in an initial lack of fit. In the other alternative, the masonry panels are built firstly and then beams and columns are cast to form a reinforced concrete frame. Thus, it is possible to achieve an adequate bond and shear strength at the panel-frame interfaces. This technique is mainly used in seismic regions of Latin America, such as Argentina, Chile, Mexico and Peru.
Structural engineers have largely ignored the influence of the masonry panels when selecting the structural configuration, assuming that these panels are brittle elements when compared with the frame. The design practice of neglecting the infill during the formulation of the mathematical model leads to substantial inaccuracy in predicting the lateral stiffness, strength and ductility. The reluctance of numerous engineers to consider the contributions of the masonry infills has been due to the inadequate knowledge concerning to the composite behaviour of infilled frames, and to the lack of practical methods for predicting the stiffness and strength. According to the conclusions reported by the CEB Task Group III/6 in 1994 [B10] none of the computer programs commonly used by designers is provided with some rational and specific elements for modelling the behaviour of the masonry infills.

Even though seismic codes for infilled frames have been developed in some countries for some time [C9, C10, N7], numerous design codes and recommendations from all over the world do not contain rules for design of infilled frames [B10]. Fortunately, there is now a change in this attitude. New design provisions are been introduced in several countries which require that the effect of the masonry panels be taken into account in the design and analysis of infilled frames [F7, R12, R13, V1]. This has generated a need for better knowledge and for practical tools to be used in design and analysis.

The review of the literature shows that experimental and analytical research related to infilled frame structures has progressively increased in the last 40 years. Despite this effort, numerous uncertainties still remain. Infilled frames exhibit a complex composite behaviour which is affected by numerous factors, such as material properties, relative dimensions, type of loading, etc. Furthermore, some difficulties in the interpretation of the experimental results arise from unrealistic design of the specimens and the loading procedures, inadequate planning of the test programme without systematic separation of the variables or incomplete reporting of the results. As a result, it is difficult to draw general conclusions and design recommendations.

1.2 OBJECTIVES OF THE RESEARCH WORK

This research work is mainly focussed on the seismic response of reinforced concrete frames infilled with masonry panels. However, emphasis is also given to understand the basic behaviour of masonry and its component materials, which have a significant effect on the behaviour of infilled frame structures. Despite the extensive use of masonry in different types of construction, this material has not been investigated as intensely as other materials, like reinforced concrete or steel. An extensive literature review is conducted, with the aim of summarizing results from previous research work. It is worth noting that, due to practical limitations, the numerous factors affecting the structural response of infilled frames cannot be investigated in a single research programme. Therefore, general conclusions should be obtained by complementing results from different sources.

The main objectives of this study, which are grouped in four categories, can be summarised as follow:

Masonry and constitutive materials:

- To review the basic behaviour of masonry materials and the response of masonry subjected to shear and compressive loading.
• To develop reliable procedures for the evaluation of the shear and compressive strength of masonry, considering that these parameters strongly affect the response of infilled frames.

Behaviour of infilled frames:
• To investigate the seismic behaviour of infilled frame structures, modes of failure and principal factors affecting the response, based on previous research.
• To test infilled frame structures under quasi-static cyclic loading, using realistic procedures for the application of the horizontal forces.

Analysis of infilled frames:
• To review the different mathematical models used for the analysis of infilled frames and to assess their advantages and disadvantages.
• To develop a macro-model capable of representing the main characteristics of these types of structure and simple equations to be used by designers.

Design of infilled frames:
• To verify by laboratory tests the structural response of infilled frames constructed with new reinforcing details aimed at improving the seismic behaviour.
• To propose a new design approach intended to obtain a reasonable ductile response of the infilled frames.
• To investigate the effect of pinching of the hysteresis loops in the seismic response, based on nonlinear dynamic analyses.

In order to fulfill these objectives, experimental research related to masonry and infilled frames is complemented with analytical considerations. The principal premise in the development of analytical models is the rational interpretation of the physical phenomena, trying to keep the formulation as simple as possible. This criterion was clearly outlined by Thurlimann [T3]: "From a scientific standpoint an empirical approach is only tenable if the separation and control of the main variables in a test programme are assured and sufficient tests are conducted to allow a statistical treatment of the results. In testing structural components or entire structures of reinforced concrete, these conditions are practically never met. For this reason, an approach based on even a simplified model, considering only the main variables, analysed with an idealized theoretical procedure is preferable".

Design of infilled frames, according to current procedures, is mainly done by limiting the average shear stress in the masonry panel. This simple criterion, developed on the basis of an empirical approach, does not solve the problems associated with the brittle behaviour of masonry and with the uncertainties resulting from the complex interaction between the frame and the infill panel. For these reasons, a new criterion is proposed to apply the principles of capacity design to infilled frames, under the conviction that this philosophy represents a rational and deterministic approach. The designer, therefore, is able to "tell the structure what to do" [P1] by selecting specific regions of the structure to dissipate energy by plastic deformations, whereas the remaining parts are designed to resist the seismic actions in the elastic
range. This approach seems to be suitable for infilled frames in order to avoid brittle modes of failure which are inherently associated with unreinforced masonry.

1.3 TERMINOLOGY
The term "infilled frame" designates a composite structure formed by one or more infill panels surrounded by a frame. The panels are usually built with clay or concrete masonry, whereas the frame is constructed with steel or reinforced concrete.

Different terminology is normally used depending on the constructive techniques. In some cases, the term infilled frame refers to the case in which the frame is firstly built and then infilled with one or more masonry panels. Several researchers introduced the term "confined masonry" to describe the case in which the reinforced concrete frame is cast after the construction of the masonry panel. The origin of this term is probably due to the observations reported by Klingner and Bertero [K9], who pointed out that the beneficial effect produced by the frame to improve the behaviour of the brittle masonry panel can be considered as a "confining action", being conceptually similar to the way in which spirals increase the flexural ductility of a reinforced concrete member. In the author's opinion, this is an improper denomination because the masonry panel is not confined by the frame, especially under lateral forces which produce cracking at the panel-frame interfaces. Even though the composite interaction still remains after the separation, this mechanism cannot be considered as confinement. Furthermore, the apparent "confining action" of the frame is also observed in those infilled frames where the masonry panels are built after the frame. It seems that terms like "surrounded" or "framed" are more representative to describe the beneficial effect of the frame. Therefore, the denomination "framed masonry" will be used here to specifically refer to this type of structure.

Other researchers consider that frames with masonry panels can be grouped in two different types: non-integral and integral infilled frames. The former type includes the infilled frames without shear connectors or with no bond at the structural interface, whereas the latter group consider the infilled frames with shear connectors or strong bond [L1, L11]. Under these considerations, framed masonry and integral infilled frames are similar structural systems. Hence, the term "infilled frame" can be used, in a general sense, for all types of masonry infills surrounded by steel or reinforced concrete frames.

1.4 ORGANIZATION
This report is organized in 13 chapters, which are grouped in four parts.

Part 1 describes the behaviour of masonry subjected to different stress states. Chapter 2 includes a description of the materials commonly used for constructing masonry walls and their mechanical properties, based on a review of the literature. A new test procedure is proposed for the evaluation of the compressive strength of masonry units and experimental results are presented to verify its validity.

Chapter 3 relates to the behaviour of masonry subjected to compression. Different failure theories reported in the literature are discussed and two new approaches are proposed. The first approach is developed to evaluate the compressive strength of masonry based on the characteristics of the constitutive materials, where the second approach is a nonlinear procedure capable of describing the complete strain-stress path of masonry subjected to compression. Furthermore, current standard procedures for the
experimental evaluation of the compressive strength are discussed. In chapter 4, the behaviour of masonry subjected to shear, tension and biaxial stress states is examined. Failure theories are revised and a modified theory is proposed for the evaluation of the shear strength of masonry.

Part 2, presented in Chapter 5, relates to the behaviour and modes of failure of infilled frame structures. An extensive literature review is conducted to describe the structural behaviour based on previous experimental works. Some results, obtained from finite element analyses conducted by the author, are also included to complete this description. Special emphasis is given to present an adequate classification of the types of failure commonly observed in infilled frames, intended to a better understanding of the structural behaviour. Finally, the factors affecting the response of infilled frames are discussed.

Part 3 is related to the analysis of infilled frame structures. Different mathematical models developed for the analysis of this type of structure are revised in Chapter 6. Simplified procedures proposed for the evaluation of the strength are also included. Chapter 7 presents new hysteretic relationships, which are developed in this work to represent the response of masonry subjected to compression and shear. These relationships are intended to describe the cyclic behaviour of a new multi-strut model proposed for the analysis of infilled frames, whose formulation is given in Chapter 8. Furthermore, a simplified procedure is developed to evaluate the strength of the infill panel, based on the failure theory proposed for masonry in Chapter 4.

Part 4 reports the experimental research on two framed masonry structures tested under quasi-static cyclic lateral forces and the subsequently developed design recommendations for infilled frames. Details of the test units, instrumentation, test set-up and loading sequence are described in Chapter 9, whereas the test results are presented in Chapter 10. A new design approach is proposed in Chapter 11, based on rational considerations and the results obtained from the tests. Chapter 12 discusses the effect of pinched force-displacement hysteresis loops in the seismic response of infilled frames, following the results obtained from dynamic nonlinear analyses.

Finally, Chapter 13 summarizes the conclusions obtained in this research project and presents recommendations for future research work.
2. MASONRY MATERIALS

2.1 INTRODUCTION
Masonry has been used as structural material, mainly for compression members, since ancient civilizations. For a long period, masonry structures were built in accordance with empirical rules and designed only to support gravity actions using the massive dead load to stabilize the structures against lateral forces from winds and earthquakes. In this way, many buildings, towers, pillars and bridges were built since the earliest days of humanity. The Egyptian Pyramids (2500 BC), the Great Wall of China (200 BC to 220 AD), the pyramids of Yucatan, Mexico (500 AD), the stone walls of Machu Pichu, Peru (1200 to 1400 AD) and the Taj Majal, India (1650 AD) are just a few examples of these structures and testify to the durability of masonry [A2].

Masonry walls not only provide structures, but also subdivision of space, fire protection, thermal and acoustic insulation, and aesthetic appeal. Many architects value the colour, shape and texture of masonry. In addition to these desirable properties, masonry is relatively cheap and durable [H2].

The research and rationalization of masonry design began in the middle of this century. The application of the structural engineering principles has led to a better knowledge of its properties and behaviour. Consequently, masonry has been re-adopted as an engineering material.

2.2 MASONRY MATERIALS
2.2.1 General
Masonry is normally made of rectangular masonry units bonded with mortar. The construction industry offers masonry units with a large variety of shapes, materials and sizes. Constructive systems and materials also change from one country to another.

Some authors use the word "brick" referring only to solid masonry units. From now on, brick is used in a general sense as a synonym of masonry unit.

This study is mainly focussed on the materials and constructive techniques commonly used in the construction of reinforced concrete frames infilled with masonry panels. Consequently, mud bricks (adobe), grouted masonry, low-density blocks and stone masonry are not considered here.

2.2.2 Masonry Units
2.2.2.1 Clay Bricks
Clay bricks are rectangular masonry units of variable shape and size. The sizes range approximately from 120 x 90 x 45 mm to 300 x 180 x 120 mm [A2, S1]. Some countries use a modular system where the
brick thickness is 100 mm and the length is 200 or 300 mm. The volumetric weight ranges from 13 to 22 kN/m² [S1].

Bricks are made of clay, shale, fire clay or mixture of these, and shaped by moulding, pressing or extrusion. They are fired at 750 °C to 1300 °C. During the heating process the water is driven off and the clay particles become soft causing the mass to stick together (incipient fusion) due to the elevated temperatures. Then a partial vitrification occurs and the temperature is maintained for a prescribed time. The process lasts 40 to 150 hours according to the size and volume of the bricks and the type of kiln. The cooling process must be controlled because it may affect the quality. For example, a rapid cooling may cause cracking of the masonry units. Since the masonry units are usually subjected to different temperature-time regimens, bricks made with the same clay mix may have quite varied mechanical and physical properties.

During the vitrification process, alumino-silicates are formed and many of these products hydrate later in the presence of moisture, causing clay masonry units to expand with time. There is a great variation in the magnitude of the expansion depending primarily on the chemical composition of the clay mix. McNeilly [M1] presented values ranging from 0.05 to 2 mm/m collected from researchers of USA, Germany and Australia. For many types of brick, however, this phenomenon has little or no importance.

Clay brick may be classified in solid and hollow masonry units depending on the ratio of the net cross-sectional area, \( A_n \), (in any plane parallel to the bearing surface) to the total cross-sectional area, \( A_b \). It is generally accepted that solid brick must have a net area \( A_n \geq 0.75 A_b \) [N1, S3]. Usually, the compressive strength is referred to the total area \( A_b \).

2.2.2.2 Concrete Masonry Units

Concrete masonry units are basically made from Portland cement, water and mineral aggregates. They are moulded in many sizes and types under pressure and/or vibration. Other materials may be added to confer determined characteristics, such as colour, texture or reduction of the weight. Modular dimensions which are multipliers of 200 mm have been used customarily and the 400 x 200 x 200 mm block is frequently manufactured [A2]. It should be noted that the most important factors that affect the strength of the masonry units are the water-cement ratio, the unit weight, the type of aggregate and the curing process.

Concrete blocks can be classified in solid and hollow masonry units according to the same criterion applied for clay masonry units. For structural use, the hollow blocks are the most common option in order to reduce the weight.

With the inclusion of scoria, pumice, perlite or other aggregates it is possible to reduce the unit weight of the concrete. The specification ASTM C90-85 [S3] considers three weight classifications, as indicated in Table 2.1. The compressive strength decreases when the unit weight of concrete is reduced. For structural use only medium and normal weight is recommended. The volumetric weight of the masonry unit is very variable depending on the density of the concrete and the core area.
Table 2.1. Weight classification according to ASTM C90-85 [S3].

<table>
<thead>
<tr>
<th>Classification</th>
<th>Unit Weight (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light Weight</td>
<td>&lt; 16.8</td>
</tr>
<tr>
<td>Medium Weight</td>
<td>16.8 - 20.0</td>
</tr>
<tr>
<td>Normal Weight</td>
<td>&gt; 20.0</td>
</tr>
</tbody>
</table>

2.2.2.3 Calcium Silicate Masonry Units

These masonry units are made of siliceous sand and hydrated high calcium lime under pressure and elevated temperatures. Often, they are called sand-lime bricks. During autoclaving, the lime and silica react with water in calcium silicate hydrate. The volumetric weight ranges approximately from 18 to 20 kN/m³.

2.2.3 Mortars

Mortar is a mixture of cementitious materials, aggregates and water used to bind masonry units into a structural mass. Fresh mortar must be workable, and when hardened, the mortar must provide bond between the masonry units and strength to bear the loads. There are many cementitious materials for making mortars. Portland cement and lime mixed in adequate proportions are frequently used. Cement contributes to durability and strength whereas lime confers workability and water retentivity [A2].

Natural and quarried sand is commonly used as aggregate. It must be well graduated and free of dangerous substances and organic impurities. The size and shape of the aggregates influence the mortar strength as well as its workability. The maximum particle size should not be more than \( \frac{1}{3} \) of the thickness of the joint.

Numerous admixtures may be added to the mortar to modify its properties. Some of these are retarding admixtures for delaying the stiffening of the mortar or air-entraining agents for improving the workability and the resistance against freezing. Also, accelerating additives and water-repelling agents are used.

Mortar mixes are usually indicated in parts by volume of Portland cement. Different proportions are commonly used for masonry construction. For example, ACI/ASCE Specifications for Masonry [S3] consider four types of mortars, named type M, S, N and O, with increasing lime contents (see Table 2.4). The quantity of lime varies from ¼ to 2 parts for each part of cement. The volume of sand should not be less than 2.25 and not more than 3.00 times the sum of the separate volumes of the cementitious materials. The cement-water ratio (by weight) ranges from 0.5 to 1.8 [A2]. The unit weight of the mortar normally varies from 17 to 20 kN/m³. Material specifications [S3] restrict the quantity of certain materials or substances in the ingredients of the mortar because they can affect the mortar quality or induce steel corrosion.
2.3 PROPERTIES OF MASONRY UNITS

2.3.1 General
Material specifications state the minimum physical requirements that the different types of masonry units must conform. These requirements are usually compressive strength, water absorption and size tolerances. Other tests may be done in order to evaluate the properties of the masonry units, such as initial rate of absorption (suction), measurement of void area and warpage, freezing and thawing, modulus of rupture, chemical components, etc. However, structural engineers are especially interested in the strength and deformational properties.

2.3.2 Compressive Strength of Masonry Units
The compressive strength shows a wide variation, depending on the materials and type of masonry units, with typical values ranging from 5 to 100 MPa [P1]. Commonly, the strength of concrete masonry units varies from 10 to 40 MPa, and sand-lime and clay masonry units from 8 to 50 MPa. There is not much knowledge about the stress-strain relationship of masonry units under compression. Experimental results from clay bricks [S1] indicate that the stress-strain relationship is almost linear up to failure followed by rapid decrease of the resistance. For concrete masonry units, a behaviour similar to that showed by plain concrete specimens could be expected.

The compressive strength of masonry units is evaluated from a direct compression test and the strength is usually referred to the average gross area perpendicular to the direction of the load [S3]. It should be noted that the behaviour of the brick is affected by friction between the masonry unit and the platens of the testing machine. The platens restrain the lateral deformation at the ends of the masonry units and such restraint may result in an increase of the apparent strength of the test specimen. This effect increases when the aspect ratio (defined as the ratio of the height of the specimen to the thickness) decreases.

Compression test specimens are capped to diminish the effect of roughness and correct the lack of plane surfaces. Two different techniques are commonly used: hard capping (using sulphur or gypsum compound) or soft capping (using plywood or softboard). Drysdale et al. [D8] indicated that the latter technique has the advantage of reducing the test preparation time. Furthermore, it seems that soft capping decreases the confinement effect induced by friction between the platen and bearing faces.

The specifications ASTM C-67 and ASTM C-140 [S3] for testing clay and concrete masonry units indicate that the bearing surface of the masonry units must be capped using a thin uniform coat of either calcined gypsum or a mixture containing sulphur with fire clay or other suitable inert material passing a No. 100 (150-μm) sieve. The New Zealand standard NZS 3102:1983 [N5] specifies a different procedure for concrete blocks, which must be tested between two pieces of either 12 mm wood fibre softboard or 4 mm plywood. Clay bricks are capped with sulphur-pumice mixture [N6]. Atkinson et al. [A3] and McNary and Abrams [M26] found that the compressive strength measured from tests using an interface friction reduction system was almost half of the value obtained from the ASTM procedure for the same masonry units. The system consisted of a polished steel platen with a greased Teflon sheet placed between the testing machine and the specimen; the loading faces of the specimen were capped with gypsum.
The distinct testing methods not only influence the compressive strength, but also alter the mode of failure of the masonry units. Failure of capped test specimens is influenced by friction due to loading platens showing 45° shear cracks [S1], whereas vertical splitting usually occurs when the masonry units are tested using an interface friction reduction system. In the latter case, it is not clear whether the system allows or induces the splitting failure. Föppl (as reported in reference [K2]) showed that the use of a soft packing or lubricant agents between the concrete specimen and the bearing platens induce additional lateral tensile stresses with a nonuniform distribution, resulting in an apparent reduction of the compressive strength. It is expected that the same conclusion could be valid for masonry units.

For these reasons, it is difficult to indicate which is the most representative procedure to evaluate the compressive strength. The desirable methodology must avoid the frictional effect without inducing additional lateral stresses in the masonry unit. The "compressive strength" obtained according to the current standards does not represent the value of the real uniaxial compressive strength of the material due to the confinement effect induced during the test.

Page and Marshall [P15] compared the compressive strength of calcium silicate bricks, both solid and perforated, obtained from the standard test (specimens capped with 5 mm plywood sheets) and using flexible brush platens to minimize the restraint effect on the specimen. Based on these results, they derived correction factors depending on the aspect ratio of the specimen (height to thickness ratio) in order to obtain the uniaxial compressive strength from standard tests. Table 2.2 shows these correction factors.

**Table 2.2.** Correction factors for masonry units, proposed by Page and Marshall [P15].

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>0</th>
<th>0.4</th>
<th>0.7</th>
<th>1.0</th>
<th>3.0</th>
<th>&gt; 5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The effect of perforation pattern on the compressive strength has been investigated for several researchers [H2]. Schellbach (as reported in reference [H2]) tested various types of highly perforated masonry units and reported that the more convenient perforation ratio should be in the order of 38% to 43%. These kind of masonry units usually exhibit a very brittle behaviour and failure occurs suddenly. Therefore, the use of perforated masonry units is not recommended for structures subjected to seismic induced actions.

### 2.3.3 Proposed Method for the Evaluation of the Compressive Strength of Masonry Units

The main problems in the evaluation of the compressive strength arise from the confinement effect induced by friction at the loading platens and the potential influence of the material used for capping. It is proposed here that these problems could be overcome by testing a stack of three masonry units with a sheet of cork between them. This material has a Poisson's ratio of almost zero and, consequently, should not significantly affect the stress state in the masonry units. The aim of testing a stack of three bricks is to obtain a large aspect ratio of the specimen avoiding the confinement effect in the central brick.
In order to verify the validity of the proposed method, an experimental programme was conducted. Solid concrete bricks, with dimensions 230 x 90 x 75 mm, were tested in an Avery universal testing machine according to the proposed method (see Fig. 2.1). The specimens were formed by three bricks stacked between 4 mm cork sheets. These sheets were also placed between the loading faces of the specimen and the testing machine. The aspect ratio of the stack was 2.50. The cork sheets had a Poisson's ratio equal to 0.03, which was evaluated by testing the material in compression and measuring the longitudinal and transverse deformations. Additionally, standard tests were conducted using different materials for capping and changing the aspect ratio of the specimens. Different aspect ratios were obtained by testing single masonry units in the longitudinal and transverse directions and by cutting prisms, with dimensions 230 x 46 x 46 mm, from the bricks. The compressive strength was obtained as average value of five tests. Table 2.3 details the characteristics of the tests and summarises the experimental results.

![Figure 2.1](image.png)

**Figure 2.1.** Test set-up for the evaluation of the compressive strength.

The series of tests with aspect ratio of 0.83 showed the influence of different capping material on the compressive strength. The higher values were obtained from masonry units capped with gypsum, in which the frictional effect is more significant. The use of cork sheets between the specimen and the loading platens seems to reduce the frictional effects. The result in this case was similar to that obtained from bricks capped with gypsum and aspect ratio of 3.07.

The comparison of the cracks formed in the specimens capped with different materials clearly indicated that a significant confinement effect develops in the bricks capped with gypsum. In this case, the cracks were inclined and did not affect the loading faces of the brick. Contrarily, vertical cracks developed in
the bricks tested with soft capping (softboard or cork sheets). These cracks started in the region close to the edges of the brick and then additional cracks formed toward the centre.

Table 2.3. Compressive strength of solid concrete bricks tested under different conditions.

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>75/90 = 0.83</th>
<th>75/90 = 0.83</th>
<th>75/90 = 0.83</th>
<th>230/75 = 3.07</th>
<th>230/46 = 5.00</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capping</td>
<td>Gypsum</td>
<td>Softboard</td>
<td>Cork</td>
<td>Gypsum</td>
<td>Gypsum</td>
<td></td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>37.2</td>
<td>35.5</td>
<td>29.4</td>
<td>30.6</td>
<td>25.4</td>
<td>26.2</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>6.1</td>
<td>10.2</td>
<td>2.7</td>
<td>4.6</td>
<td>3.4</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Compressive tests with aspect ratio of 5.0 were also conducted, considering that the correction factors proposed by Page and Marshall [P15] (see Table 2.2) indicate that there is no effect of the loading platens in this case. It was found, however, that the tests are very sensitive to the lack of alignment between the loading faces. Due to the slenderness of the specimens, flexural stresses may be induced. Several tests had to be repeated because a flexural failure occurred. For these reasons, compressive tests of masonry units with aspect ratio of 5.0 are not practical and should be conducted with extreme care.

During the tests conducted according to the proposed method, the vertical cracks started in the central brick, close to the edges, and then propagated toward the centre. These cracks remained vertical even in the region close to the loading faces. This fact indicates the frictional effects were avoided or significantly reduced. The compressive strength evaluated according to the proposed testing technique agrees very well with that obtained from tests with aspect ratio of 5.0.

The stress state in the compressed bricks was investigated using nonlinear finite element analyses. The results show that it is possible to obtain a uniform compressive stress field in the central brick with almost no lateral stresses induced by the bed joint. In these analyses two limit conditions were considered, namely, no friction between the platen and the brick and complete restraint with no horizontal displacement at the loaded surfaces. The stress distribution in the central brick was very similar in both cases. Figs. 2.2 (a) and (b) illustrate the stress contours of the lateral and vertical stresses, respectively, for the case of complete restraint of the lateral displacement of the loaded faces. It is observed that the central brick is subjected to a uniform stress state, with some minor perturbations in the region close to the corners of the bricks. The model represents only one quarter of the three-bricks stack based on symmetry considerations. The Poisson's ratio of the joint material was equal to 0.01 and the ratio of the elastic modulus of the brick to the joint material was 3.5. The characteristics of the finite element model are described in Appendix 1.

The experimental and analytical results presented here indicate that the proposed testing technique could be used for the evaluation of the uniaxial compressive strength of masonry units. However, a more extensive research programme, considering different materials and types of bricks, should be conducted to validate the results obtained in this project.
2.3.4 Tensile Strength of Masonry Units

The tensile strength of masonry units can be evaluated from different testing techniques, namely, flexural test, splitting (or indirect tension) test and direct tension test. In the flexural test, the masonry unit is subjected to a linear bending moment produced by a concentrated load applied at the centre of the masonry unit. The tensile strength obtained from this test is commonly named modulus of rupture. In the splitting test a compressive line load is applied to opposite faces of the masonry unit causing an approximately uniform lateral tensile stress (except in the loading regions, where large compressive stresses develop). Failure of the specimen occurs along the plane which contents the line loads. Borchelt and Brown [B12] applied the compressive load with two 6 mm diameter steel rods, using gypsum capping between the steel rods and the masonry units to obtain a uniform distribution of the load and to keep the rods in position. The splitting test was originally conceived for concrete cylinders, but Davies and Bose [D28] demonstrated that this testing technique is also applicable either for rectangular sections in which the width is larger than the height, or for square sections. Therefore, the tensile strength can be calculated as $f_{tu} = 2P / (\pi A_s)$, where $P$ is the compressive load measured in the test and $A_s$ is the sectional area along the splitting plane. Fig. 2.3 (a) presents a photograph of a concrete brick ready to be tested and Fig. 2.3 (b) shows the specimen after failure.

Another method is the concentric or uniform axial tension test, in which special mechanisms must be attached to both ends of the specimen. This type of test, however, is difficult to perform. Variable results may be obtained due to alterations associated with gripping the specimen and the complications to align the test device. Atkinson et al. [A3] and McNary and Abrams [M26] tested specimens in direct tension using aluminum brushes bonded to the bricks with high strength epoxy, in order to reduce the lateral interface shear forces.
The effect of strain gradient on tensile strength was studied by Hamid and Drysdale [H1], who found that the tensile strength is highly sensitive to the test technique. The strength at maximum strain gradient (flexural test) was more than double that at zero strain gradient (uniform tension) in tests with concrete blocks. A similar phenomenon occurs for normal concrete and can be attributed to the fact that under uniform tension the entire volume of the specimen is subjected to the maximum stress. Test results obtained from concrete blocks [H1] showed that the uniaxial tensile strength, $f_{tb}$, may be related to the uniaxial compressive strength, $f_{cb}$, according to the following expression:

$$f_{tb} = c \sqrt{f_{cb}^n} \quad \text{(MPa)} \quad (2.1)$$

where:
- $c = 0.28$ for uniform tensile test
- $c = 0.34$ for splitting test
- $c = 0.69$ for flexural test.
The splitting test seems to be the most reliable measure of the tensile strength when masonry is subjected to in-plane forces. This is the case of masonry panels surrounded by steel or reinforced concrete frames subjected to gravity or lateral in-plane loads.

Several researchers have proposed that the tensile strength of masonry units can be evaluated in direct proportion to the compressive strength \( f'_{cb} \). Sahlin [S1] reported that the ratio of the modulus of rupture to the compressive strength varies from 0.11 to 0.34 for solid bricks and from 0.10 to 0.17 for hollow masonry units (materials were not specified). Mann and Müller [M2] suggested very low values for the same ratio, ranging from 0.025 to 0.045. These values are very conservative because the tests covered a wide range of materials (clay, calcium silicate, pumice) without differentiation. Francis et al. [F1] found in their tests that the splitting strength was about 9% and 5% of the compressive strength for solid and hollow masonry units, respectively. Other researchers [A3] have reported values from 4% to 7% for this parameter. For practical purposes, the tensile strength of the bricks may be assumed as 10% of the compressive strength.

The stress-strain relationship of masonry units in tension is almost linear elastic up to near the maximum stress. Then, a brittle failure occurs without a falling branch [H1].

2.3.5  Failure Criteria under Multi-Axial Stress for Masonry Units

The development of compressive failure theories for masonry requires the evaluation of the failure envelope of brick under a triaxial stress state, particularly biaxial tension and compression. However, in the author's knowledge, only two experimental works have been conducted to investigate this failure envelope. The first assumption was made by Hilsdorf [H10] in 1969, who considered a linear relationship between compressive and tensile stress (Coulomb-Mohr criterion) because no information on the behaviour of bricks under triaxial stresses was available at that moment. Thus, the failure envelope can be expressed by the following equation:

\[
\frac{f_x}{f'_{tb}} + \frac{f_y}{f'_{eb}} = 1. \quad (2.2)
\]

where \( f_x \) and \( f_y \) are the absolute value of the tensile stress in \( x \) direction and the compressive stress in \( y \) direction, as Fig. 2.4 indicates.

In 1973, Khoo and Hendry [H2, K12] conducted an experimental investigation considering a large number of specimens with compressive strengths ranging from 32 to 93 MPa. In these tests, a biaxial stress state was generated in small specimens (25 x 25 x 75 mm) cut from bricks. The ends of these specimens were capped with epoxy resin to form a "dog-bone" briquet to pull them in direct tension. Compressive stress was exerted in a perpendicular direction using two loading platens. In order to minimize the friction developed at specimen surfaces, two frictionless pads were used (consisting of a polyester film and hardened 0.025 mm thick aluminium foil, with a thin layer of grease in between). They found that the envelope of the test results followed approximately a parabolic curve and proposed:
\[
\left( \frac{f_x}{f_{th}} \right)^{0.546} + \frac{f_y}{f_{cb}} = 1
\]

(2.3)

Figure 2.4. Reference system for stresses.

Khoo and Hendry [K12] reported that "in nearly all specimens, failure occurred across a plane at the edge of the compression loaded area", which clearly indicated that the influence of the platen restraint was not eliminated. Therefore, the experimental results, especially for high compressive stresses, could be distorted showing an increased strength of the specimens.

Atkinson et al. [A3] and McNary and Abrams [M26] also investigated the biaxial constituent properties of brick. The tensile load was applied first in the horizontal plane using aluminum brushes bonded to the brick, followed by an increasing compressive load, which was exerted in the vertical plane through an interface friction reduction system (see section 2.3.2). The brick failure was generally uniform and perpendicular to the direction of the tensile load. The researchers wondered about the validity of the test device used to apply the compressive load and commented that this system could have induced additional tensile stresses, reducing the strength of the specimens. Atkinson et al. [A3] and McNary and Abrams [M26] proposed the following expression:

\[
\left( \frac{f_x}{f_{th}} \right)^n + \frac{f_y}{f_{cb}} = 1
\]

(2.4)

where \( n \) is an empirical coefficient, with a recommended value of \( n = 0.58 \) obtained by statistical analysis.

Ohler [H2] proposed a trilinear approximation to the curve given by Eq. 2.4, considering \( n = 0.58 \):

\[
S_1 \frac{f_x}{f_{th}} + \frac{f_y}{f_{cb}} = S_2
\]

(2.5)
where:
\[ S_1 = 2.218, \ S_2 = 1.000 \quad \text{if} \quad 0.0 \leq f_x/f_{tb} \leq 0.15 \]
\[ S_1 = 0.960, \ S_2 = 0.811 \quad \text{if} \quad 0.15 < f_x/f_{tb} \leq 0.50 \]
\[ S_1 = 0.662, \ S_2 = 0.622 \quad \text{if} \quad 0.50 < f_x/f_{tb} \leq 1.0 \]

The aim of this expression was to obtain an explicit solution for the compressive failure criterion of masonry that Oehler proposed (see section 3.5.3.5).

The experimental work apparently shows that the presence of the tensile stress has stronger influence in the compressive strength than that predicted by the Coulomb-Mohr criterion (Eq. 2.2). Fig. 2.5 compares the previous proposed equations and the experimental results obtained by Atkinson et al. [A3] and McNary and Abrams [M26]. It is observed an important difference between the assumed linear envelope and the curves deduced according to test results.

![Figure 2.5. Tension-compression failure envelopes proposed for bricks.](image)

The validity of the experimental data may be discussed on the ground that results were affected by the system used to apply the compressive load. It is expected, therefore, that this effect could be more important in the range of high compressive stresses. Fig. 2.5 shows that Eq. 2.3 and Eq. 2.4 are almost linear for low to medium compressive stress and the nonlinear effect occurs mainly in the range \(0.6 \leq f_x/f_{tb} \leq 1.0\).

There is not enough information available concerning the complete biaxial strength envelope for bricks. Research work about behaviour of masonry panels [A13, D4, P6], and experimental results obtained from other brittle materials [D19] indicated that a concave curve, like those represented by Eqs. 2.3 and 2.4, is not a good representation of the behaviour under tension-compression stress states. Furthermore, the consideration of this concave curve as part of a complete failure criterion under biaxial stress state introduces a slope discontinuity at the point corresponding to the uniaxial compressive strength. Fig. 2.6 shows the failure envelope for concrete [K2], which would be expected to behave in a similar way to brick, and the failure envelope represented by Eq. 2.3 (valid for the tension-compression zone). The use
of the linear envelope as failure criterion requires to define adequately the uniaxial compressive strength. It seems that the "uniaxial compressive strength", which is considered as a reference point, is not really the uniaxial compressive strength of the brick due to the perturbation effects introduced by the testing devices.

![Diagram](image)

**Figure 2.6.** Biaxial failure envelope for a brittle material.

The problems mentioned above suggest that the experimental results obtained by Khoo and Hendry [K12], and Atkinson et al. [A3] and McNary and Abrams [M26] should be revised using more realistic testing procedures. However, it must be accepted that Eqs. 2.3 and 2.4 fit adequately the experimental results obtained from standard testing procedures.

The failure criteria presented in this section have been also applied to represent the behaviour of bricks under axial compression and biaxial tension [C1, F1, H2]. In this case, \( f_x \) represents the tensile stress in two perpendicular directions to the compressive stress \( f_y \).

2.3.6 **Modulus of Elasticity and Poisson's Ratio of Masonry Units**

The strain-stress relationship of masonry units depends significantly on the constitutive material. Due to the lack of experimental information, it may be assumed that clay bricks behave nearly like a linear elastic material [A7], whereas concrete blocks exhibit a similar nonlinear behaviour to that observed for plain concrete.

The modulus of elasticity of masonry units presents a wide variation and basically depends on the type of material and on the compressive strength \( f'_{ob} \). There is no standardized method to evaluate the modulus of elasticity. Usually, this value is adopted as the secant modulus of elasticity from zero stress to one third of the material strength.
The modulus of elasticity of concrete masonry units varies from 3000 to 12000 MPa. Sahlin [S1] presented experimental data obtained at the University of Illinois from concrete blocks made of different aggregates. These results are shown in Fig. 2.7 as a function of the compressive strength $f'_{cb}$ indicating a wide scatter of the data.

![Concrete blocks](image)

2.7. Experimental values of modulus of elasticity for concrete blocks made with different aggregates [S1].

For clay bricks, the modulus of elasticity $E_b$ is usually smaller than that of concrete blocks. This parameter can be approximated by:

$$E_b = 300 \ f'_{cb}$$

(2.6)

Fig. 2.8 shows results from different tests reported by Sahlin [S1], which are compared with Eq. 2.6. It is worth noting that there is a good correlation, in a general sense, for bricks with a compressive strength ranging from 20 to 50 MPa.

Kirsch [K1] conducted an experimental investigation in Germany and proposed the following empirical expression to calculate the secant modulus of elasticity defined at 1/3 of the compressive strength:

$$E_b = 980 \ f'_{cb}^{0.77} \text{ (MPa)}$$

(2.7)

The masonry units used in these tests were perforated clay bricks, calcium silicate cellular blocks, lightweight concrete blocks and aerated concrete blocks.

There is insufficient information available about the Poisson's ratio of masonry units $v_{eb}$ since this parameter is rarely investigated by researchers. Atkinson et al. [A3] and McNary and Abrams [M26] reported values ranging from 0.13 to 0.22 for three different types of masonry units (materials were not
specified) and Ameny et al. [A1] found values ranging from 0.07 to 0.14 testing dry-pressed masonry units.

![Graph showing the relationship between modulus of elasticity and compressive strength of pressed bricks.](image)

**Figure 2.8.** Experimental values of the modulus of elasticity of clay bricks [S1].

### 2.3.7 Moisture Content and Absorption of Masonry Units

The moisture content and the water absorption are very important properties of the masonry units and have a considerable effect on the characteristics of the masonry.

The moisture content is the mass of water per unit volume, which can be expressed in absolute terms or in relative terms (to the density of the masonry unit when dry). The typical range of this parameter is 50-60 kg/m³ and 2-3% respectively.

The ability of the masonry unit to absorb water is measured by two parameters: the total absorption and the initial rate of absorption (IRA). The former parameter represents the amount of water required to saturate the masonry unit (it is a measurement of the porosity), whereas the initial rate of absorption, or suction, is the mass of water absorbed per unit area per unit time, measured in kg/(m² min). Existing standards limit the values of total absorption for different materials to minimize the potential for freeze-thaw damage and excessive volumetric change or permeability. The IRA is a measure of how quickly a masonry unit can suck water out of the mortar [S10]. Usual values of the IRA range from 0.5 to 1.5 kg/(m² min). Masonry units with a large percentage of fine pores, for example calcium silicate bricks, have a greater suction rate than others with large capillary pores [S8].

### 2.4 PROPERTIES OF MORTAR

#### 2.4.1 Uniaxial Compressive Strength of Mortar

The determination of the compressive strength is conducted using 50 mm cube specimens (ASTM C 109: Test Method for Compressive Strength of Hydraulic Cement Mortars [S3]) or cylinders of different
dimensions, usually with length-diameter ratio equal to 2.0. The compressive strength obtained from the latter specimens is usually smaller. According to the ASTM Standard Specification C 780 [S3], the ratio between 75 \times 150 \text{ mm} (3 \times 6 \text{ in}) cylinder and 50 \text{ mm} cube compressive strength may be considered equal to 0.85.

Experimental work conducted by Schmidt et al. [S30] showed that the cylinder compressive strength may be considered equal to 83\% of the cube strength, grouping all the data together. However, higher values were obtained from one series of tests. Specimens were moulded into three cylinder and two cube sizes using cement and lime-cement mortars. They also observed that the same relationship was valid for ready-mixed mortars. Matthys and Singh [M21] compared the results obtained from 50 mm cube and 75 \times 150 \text{ mm} cylinder, and found that the ratio of the compressive strength between cylinders and cubes was 0.81. The authors of this investigation recommended the use of 75 \times 150 \text{ mm} cylinders to check the compressive strength of mortars because the handling of the specimens was easier and the variation of the results was less significant.

![Figure 2.9. Variation of mortar strength with the lime content [S1].](image)

The compressive strength of the mortar usually ranges from 5 to 20 \text{ MPa} and depends on many factors, such as lime content, characteristics of the aggregates, cement-water ratio and curing process. The lime content is a very important factor and has a strong influence in the compressive strength. Fig. 2.9 shows experimental results obtained in Sweden [S1] from mortars with different lime contents. In this investigation, the mortar composition was varied by changing the proportion of cement, \( W_C \), and lime, \( W_L \). It is observed that the effect of the lime in reducing the strength of the mortar is markedly more important for the compressive strength than for the tensile strength.

Table 2.4 shows the compressive strength, \( f_{cj} \), obtained from cube and cylinder specimens. These values represent the average strength obtained at 28 days. This table also includes typical proportions of cement, lime and sand (by volume) reported by Amrhein [A2] for the different mortars considered by the ACI/ASCE Specifications for Masonry [S3].
Table 2.4. Average compressive strength, $f'_c$, for different types of mortar [A2].

<table>
<thead>
<tr>
<th>Type</th>
<th>Proportion by volume cement : lime : sand</th>
<th>$f'_c$ (MPa) cube 50 x 50 mm</th>
<th>$f'_c$ (MPa) cylinder 100 x 50 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1 : ¾ : 3½</td>
<td>17.2</td>
<td>14.5</td>
</tr>
<tr>
<td>S</td>
<td>1 : ½ : 4½</td>
<td>12.4</td>
<td>10.3</td>
</tr>
<tr>
<td>N</td>
<td>1 : 1 : 6</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>O</td>
<td>1 : 2 : 9</td>
<td>2.4</td>
<td>-</td>
</tr>
</tbody>
</table>

It has been observed that the properties of mortar in the joints of masonry walls, especially the compressive strength, could be different to those obtained from laboratory tests [A1, A11, S8]. Such differences are due to the compaction of the mortar, the water absorbed by the masonry units, and the curing conditions in situ. Schubert [S8] conducted a large research program to evaluate the compressive strength of the mortar from bed joints and to compare it with the standard strength. The investigation included clay bricks, concrete blocks and calcium silicate bricks. The results showed that, in most cases, the compressive strength in the joints exceeded the standard strength in the early age (3 and 7 days strength). This fact can be explained considering that the mortar in the joints was immediately subjected to water suction, whereas the standard specimens were kept wet during the first seven days. Notwithstanding, if too much water is initially absorbed by the masonry units the long term compressive strength of the mortar can be smaller. The tests at 28 days demonstrated a great dispersion of results. The ratio between the strength of the mortar in the bed joint and the standard strength varied from 0.5 to 1.5.

For these reasons, it has been suggested that the compressive strength of the mortar must be determined in the joints. Schubert [S8] used 10 mm cube specimens cut out from mortar joints and capped with gypsum, but this method presents some disadvantages because the cube could be damaged during the cutting process. He also proposed a new method, in which 30 x 30 mm specimens are cut out from the bed joints and then are tested using a steel plunger (diameter = 10 mm); the thickness of the specimen is equal to the thickness of the mortar joint.

2.4.2 Compressive Strength of Confined Mortar

The composite behaviour of mortar joints and masonry units causes a triaxial compressive state in the mortar (see section 3.1). Therefore, it is necessary to investigate the effect of the lateral confinement stress in the compressive strength of the mortar. Experimental work reported for several researchers indicates that the maximum stress and the ultimate strain increase as a consequence of the confinement effect. In most of these tests, the confining pressure has been exerted on the specimen by a hydraulic pressure in a triaxial test cell. The cylindrical mortar specimens were jacketed with a plastic membrane or covered with surface sealant, in this way the confining fluid was prevented from entering the pores of the material (this condition is more representative of the behaviour of mortar in masonry structures). Krahl et al. [K10] indicated that unjacketed specimens presented brittle failure, whereas jacketed specimens were ductile, showing a barrel shape before failure. They also observed that the confining
pressure acting on unjacketed cylinders increased the compressive strength. However, this effect was markedly less important than that measured for jacketed specimens.

In 1928, Richart et al. [R5, R6] proposed an equation to calculate the strength of concrete in triaxial compression, based on results from tests of plain concrete cylinders under hydraulic pressure and spirally reinforced columns. Posteriorly, Hilsdorf [H10] considered that this equation could be used for the mortar. Thus, it was assumed that:

\[ f'_{eq} = f'_j + 4.1 \sigma_1 \] (2.8)

where \( f'_{eq} \) is the compressive strength of the confined mortar, \( f'_j \) is the uniaxial compressive strength and \( \sigma_1 \) is the absolute value of the lateral confinement stress.

Other researchers investigated this effect based on experimental results considering different types of mortars. In this way, Kralh et al. [K10] tested four types of cement mortars with the uniaxial compressive strength ranging from 16.3 to 34.9 MPa. The confining pressure was varied from 0 to 123 MPa, which represents a value several times greater than the uniaxial compressive strength of the mortar. Based on the experimental results, they proposed that \( f'_{eq} = f'_j + a_1 \sigma_1^{a_2} \), where \( a_1 \) and \( a_2 \) are empirical constants calculated by statistical analysis. The results obtained in this investigation, especially for low to medium confining pressures, agree very well with those obtained by Richart et al. [R5, R6] (Eq. 2.8). However, it must be noted that the type of mortar tested correspond to medium to high strength and the specimens were completely dried (moisture content equal zero) before being tested. Sereda (discussion presented in reference [K10]) discussed these results and indicated that effect of the moisture content may affect the confined strength of mortar.

Khoo and Hendry [H2, K12] investigated the confined strength of mortar using triaxial test cell with a confining pressure ranging from 0 to 11 MPa. The composition of the tested mortars was 1:1/4:3 and 1:1:6 (proportion of cement:lime:sand) with uniaxial compressive strengths 4.9 and 20 MPa, respectively. They found a nonlinear relationship represented by:

\[ \frac{f'_{eq}}{f'_j} = 1 + 2.91 \left( \frac{\sigma_1}{f'_j} \right)^{0.805} \] (2.9)

It can be observed that this expression is similar to that proposed by Kralh et al. [K10]. However, the numerical comparison of both equations indicates that the effect of confining pressure was systematically less significant in the tests performed by Khoo and Hendry.

Atkinson et al. [A3] and McNary and Abrams [M26] investigated the behaviour of four different types of mortar using a modified triaxial cell and proposed a linear relationship similar to Eq. 2.8. Instead of the constant factor 4.1 they considered a variable factor ranging from 2.2 to 5.2 for low and high strength mortars, respectively. In these tests, the uniaxial strength ranged from 6.0 to 31.5 MPa and the confining pressure was increased up to 8 MPa. Similar results were obtained by Ohler (as reported in reference [H2]), who proposed a linear equation for the confined mortar strength:
\[ f'_{\text{cj}} = f'_j + m \sigma_l \]  

(2.10)

where \( m \) is a factor that depends on the uniaxial compressive strength, as indicates Table 2.5. Ohler's formula agrees very well with the experimental results reported by Atkinson et al. [A3].

**Table 2.5.** Confinement factor proposed by Ohler [H2].

<table>
<thead>
<tr>
<th>( f'_j ) (MPa)</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2.1</td>
</tr>
<tr>
<td>15</td>
<td>2.4</td>
</tr>
<tr>
<td>21</td>
<td>3.6</td>
</tr>
<tr>
<td>32</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Fig. 2.10 compares the confined strength of mortar obtained from Eqs. 2.8, 2.9 and 2.10. In this comparison, the uniaxial strength was equal to 10 and 20 MPa. It is observed that the confining pressure has a strong influence on the magnitude of the ultimate strength. The increment of the confined strength predicted by Eq. 2.8 is independent of \( f'_j \), whereas according to Eqs. 2.9 and 2.10 this increment is more significant for mortars with higher values of \( f'_j \).

![Graph showing confined strength of mortars](image)

**Figure 2.10.** Confined strength of mortars.

2.4.3 **Modulus of Elasticity and Poisson's Ratio of Mortar**

The stress-strain relationships obtained from mortar compressive tests are, in a general sense, similar to those for unconfined concrete [A1, A3, H2, M26]. Fig. 2.11 presents the stress-strain relationship for three mortars with different composition. It may be observed that the lime has an important effect on the
mortar behaviour, modifying the compressive strength as well as the modulus of elasticity and the ultimate strain. These experimental results indicate that the modulus of elasticity decreases when the content of lime increases.

![Graph showing stress-strain relationship for different types of mortars]  

**Figure 2.11.** Stress-strain relationship for different types of mortars [S1].

Brown and Whitlock [B2] showed that the secant modulus of elasticity determined at stress level of 50% of the compressive strength was about 15% to 25% smaller than the initial modulus. The secant modulus is usually considered in analysis and design, however, there is no general agreement in the stress level at which the secant modulus should be defined.

Several equations have been suggested to calculate the modulus of elasticity of the mortar $E_j$ based on the idea that mortar should behave like concrete. For example:

\[
E_j = 1000 \ f_j^{1.4} \quad \text{(2.11)}
\]

\[
E_j = 43 \ \rho^{1.4} \ \sqrt{f_j^{1.4}} \quad \text{(MPa)} \quad \text{(2.12)}
\]

where $\rho$ is the unit weight (kN/m$^3$) of the mortar.

Sahlin [S1] found that there is not good agreement between Eqs. 2.11 and 2.12 and the experimental results obtained by several researchers. Eq. 2.11 is close to the observed data for the range of medium strength (10 to 20 MPa). The effect of the lime is not considered in these equations, even though there is clear evidence that it has a strong influence on the response of mortar.
Another fact to take into account is the different conditions of the mortar in test specimens and in bed joints. Ameny et al. [A1] reported that the vertical strain measured in the mortar joint was higher than in compressed cylinders both loaded at the same stress. This lower stiffness, and modulus of elasticity, was explained by the poor curing conditions that exist in the masonry joint. Experimental results indicate that the modulus of elasticity for in situ mortar was between 1.2 and 5 times smaller than that of the standard mortar cylinder. This fact may influence the behaviour of masonry, which depends on the deformational properties of the mortar joint and masonry units. The results obtained from Eq. 2.11 and Eq. 2.12 could be higher than the actual values for most of the cement-lime mortars used in masonry structures.

Tests conducted by Atkinson et al. [A3] and McNary and Abrams [M26] indicated that the initial tangent modulus of elasticity obtained from the uniaxial test was very similar to that measured for mortars subjected to triaxial compression. After that, the tangent modulus decreased continuously up to the failure, showing nonlinear behaviour from the beginning of the test. The tangent modulus decreased at a lower rate when the confinement lateral pressure was increased. In these tests the uniaxial strength of the different mortars varied from 6 to 31 MPa and the confinement stress was increased up to 7 MPa.

The Poisson’s ratio \( \nu_l \) presents a wide variation. Several researchers [A1, A3, B1, B2, F1, M26] have reported values from 0.07 to 0.25, although it seems that values ranging from 0.15 to 0.20 are more representative for low to medium stress levels. This parameter remains relatively constant when the compressive stress is increased up to 70-80% of the maximum stress. Then, \( \nu_l \) increases considerably up to the theoretical maximum value \( \nu_l = 0.5 \) (incompressible material), and even more for unconfined mortars due to dilatation [A3, B2, P1, M26]. However, according to experimental results obtained by Atkinson et al. [A3], if the mortar is subjected to confinement pressure, the Poisson's ratio at failure would be less than 0.5. They tested cylindrical specimens using a triaxial cell, where the lateral pressure was exerted by hydraulic fluid acting on the specimen through a plastic membrane.

These results agree qualitatively with those reported by Krahl et al. [K10], who tested mortar specimens under combined compressive stresses. Even though the variation of the Poisson's ratio up to failure was not presented, some observations may be extracted from the measurement of volume changes. It was observed a continuous volume reduction with increasing axial stress, which indicated that the Poisson's ratio was below the value of 0.5. However, the confinement pressure range applied in these tests was very high, varying from 1 to 6 times the uniaxial compressive strength of the mortar. These values of confinement pressure are not expected to occur in masonry structures. Unconfined concrete specimens presented linear volume reduction initially, which increased roughly at medium loads; finally, the volume change reversed at high loads and the material expanded.

Ameny et al. [A1] tested two different cement-lime mortars with compressive strengths of 5.2 and 13.1 MPa. It was observed that the Poisson's ratio increases almost linearly with the applied compressive stress. The first type of mortar presented an important variation from the beginning of the test until failure, whereas the other mortar showed that Poisson's ratio varied slightly for compressive stresses ranging from 0 to 10 MPa.
Unfortunately, there is not enough experimental information available concerning the variation of modulus of elasticity and Poisson's ratio of the mortars under triaxial compression. These parameters are important to predict the compressive strength of the masonry. The failure mechanism of masonry under compressive loading depends on the Poisson's ratio and their variation with the biaxial state of stress.

2.4.4 Workability and Water Retentivity

Workability and water retentivity are important properties of the plastic mortar, which directly affect the quality of hardened mortar. There is no simple definition of workability. However, workability means the mortar will adhere well, spread easily and squeeze out of joints. In the laboratory, this property is evaluated by the flow test, which measures the increase in diameter of a cone of mortar after 25 drops on a standard flow table. Usual values range from 100% to 115%.

The water retentivity measures the ability of the mortar to retain water, preventing rapid loss of mixing water to masonry units with high suction and to the air [S1]. This parameter is measured in the laboratory based on two flow tests. The first test is conducted using the mortar in normal conditions, whereas in the second test the water is removed previously by subjecting the mortar to a vacuum pressure of 51 mm of mercury. Water retentivity is the flow test result after suction expressed as a percentage of flow test result in normal conditions. Drysdale et al. [D8] recommended water retentivity values about 90% or more for mortaring most masonry units, and about 80% when concrete blocks with low suction rates are used.

The water retentivity may be improved by adding fine particles, such as lime, or water retentive admixtures. Experimental results showed that the water retentivity increases almost linearly when the proportion of lime augments.

2.5 CONCLUSIONS

- Different types of masonry units and mortars are commonly used, with an ample range of geometric and mechanical properties. The resultant material, masonry, presents a large variation in its characteristics and distinct behaviour. For this reason, it is difficult to calibrate analytical models or empirical expressions which be valid in a general sense.

- The compressive strength of the masonry unit is an important property and other mechanical properties can be related to it. Current standards specify different test procedures, however, it seems that none of these procedures are able to evaluate adequately the real uniaxial compressive strength.

- A new testing technique, using a stack of three bricks with cork sheets in between, is proposed for the evaluation of the compressive strength of masonry units. A more complete investigation, considering different materials and types of brick, is necessary to confirm the results obtained in this study.

- The failure criterion of masonry units subjected to biaxial tension-compression stresses should be revised using realistic testing techniques. The concave failure envelope proposed
on the basis of experimental results seems to be affected by the testing techniques used to apply the compressive load.

More experimental research is needed in order to understand the complete behaviour of mortar under biaxial compression, especially the variation of its properties at high stress levels. It is also very important to investigate the real behaviour of mortar in the bed joints, considering that the water absorption of the masonry units, water retentivity of the mortar, moisture contents and curing conditions may strongly affect the mortar strength in situ.
3. BEHAVIOUR OF MASONRY IN COMPRESSION

3.1 MECHANISM OF FAILURE UNDER DIRECT COMPRESSION

Masonry structures present good behaviour when they are stressed in compression, being used for columns, arches and walls for thousands of years. However, it is only in the last forty years that a systematic investigation has been carried out in many countries.

Masonry walls concentrically loaded in the direction perpendicular to the bed joints (see Fig. 3.1) show linear behaviour at low force levels. As the compressive load is increased, the material behaves nonlinearly and vertical cracks appear at a force level smaller than the compressive capacity. The wall is divided into several "columns", as shown in Fig. 3.1, until the system is not able to remain stable and failure occurs. The splitting failure may occur in a plane parallel or perpendicular to the plane of the wall. Crushing of the mortar is rarely observed.

Figure 3.1. Typical compressive failure of masonry walls [S1].

This type of failure is related to the interaction between masonry units and mortar joints as a result of their different deformational behaviour. When masonry is subjected to compressive forces, brick and mortar expand laterally, but both materials have different properties. The mortar usually presents a lower modulus of elasticity and a higher Poisson's ratio than those of the brick. As a result, the lateral strain in the mortar tends to be greater than that in the brick, as Fig. 3.2 (a) illustrates.
Figure 3.2. Individual and composite behaviour of the brick and mortar joint in compression. Lateral deformations are constrained to be equal due to friction and bond strength at the mortar-brick interface, as depicted in Fig. 3.2 (b). This effect induces tensile and compressive horizontal stresses in the brick and mortar, respectively. Therefore, both materials are subjected to a triaxial stress state (see Fig. 3.3).

Figure 3.3. Stress state into the brick and the mortar joint.

The compressive strength of mortar is enhanced due to the beneficial confinement effect produced by the biaxial lateral compression. Thus, the mortar is able to withstand higher compressive stresses in the bed joint. In the brick, the combination of vertical compression and biaxial lateral tension decreases the crushing strength, and brick failure will occur at a lower compressive stress than would be required in the absence of lateral tension.
Fig. 3.4 shows typical stress-strain curves for the mortar, the masonry unit and the composite material considering the usual case in which the mortar is weaker than the masonry units, $f'_m < f'_{cb}$. The compressive strength of the masonry is always lower than that of the brick and, in this case, greater than the mortar compressive strength.

![Stress vs Strain Graph](image)

**Figure 3.4.** Typical stress-strain relationships for mortar, brick and masonry [P1].

McNary and Abrams [M26] pointed out that the shrinkage of mortar may alter the stress state in the constitutive materials, introducing lateral compressive stresses in the brick. This fact would produce beneficial effects in the behaviour of masonry under compression, resulting in an increase of its compressive strength. However, there are no experimental results which confirm this assumption.

The mechanism of failure described previously is valid for masonry panels subjected to compressive loading in the direction perpendicular to the bed joints. When the compressive load is applied in the direction parallel to the bed joints, failure occurs by debonding along the bed joints due to the lateral spreading of the panel [P12]. The compressive strength of the panel is significantly reduced.

Hendry [H2], on the basis of experimental results, reported that a different failure occurs for some types of concrete masonry in which the mortar is stronger than the masonry units. In this case a kind of shear failure occurs along certain lines of weakness in the masonry.

It is generally accepted that the lateral splitting failure in compression is due to the different deformational properties of both materials. Contrary to this fact, Shrive [S12, S14] argued that this cannot be the sole cause of vertical cracking. This conclusion is based on the fact that prisms of concrete, rock or even bricks stacked one on top of another (without mortar) crack in a similar pattern as masonry prisms do. According to Shrive, the experimental data in uniaxial and multiaxial stress states may be explained by a generalized theory of cracking. He applied the Griffith's concepts for fracture mechanics and analysed the stress distribution around a spherical or elliptical void in a material subjected to a compressive stress field [S12, S14]. Based on these considerations, Shrive found that tensile stresses develop around the voids with
sufficient magnitude to cause fracture, and in directions which would explain the observed cracking patterns. Even though the idea seems to be conceptually true, there is not yet a practical approach to calculate the compressive masonry strength based on this theory.

3.2 COMpressive STRENGTH OF Masonry

3.2.1 General
Masonry is a material that presents its optimal behaviour when is subjected to compression. The compressive strength is one of the most important parameters to quantify the characteristics of masonry, and has been extensively studied for researchers and engineers. This parameter is also used frequently to establish design allowable flexural and shear stresses. There are numerous possible combinations of mortars, masonry units and mortar joint thickness, which lead to an ample variation of the compressive strength. The workmanship has also an important influence on this parameter.

3.2.2 Experimental Determination of Compressive Strength
Masonry prisms are primarily used to evaluate the strength. They are small masonry "walls" built one or two bricks in length and three or more bricks in height and tested under a compressive load perpendicular to the bed joint. Failure of the masonry prism occurs due to the formation of several vertical cracks. Fig. 3.5 shows a masonry prism tested by the author and a detail of the vertical cracks. It can be observed that the damage is more important in the central masonry unit, where the confinement effect induced by the loading platens is not significant.

It has been suggested that the height of the prism, \( h_p \), should be two to five times the thickness of the prism, \( t_p \) [S3]. The New Zealand's code for masonry structures [C5] indicates that the height of a prism shall not be less than three times the thickness \( t_p \) and formed by a minimum of three courses. Drysdale and Wong [D1] recommended the use of prisms at least four bricks high.

Prisms are amply used because the test is easier and less expensive than with an entire wall and do not require special testing machines. It is worth noting that the boundary conditions in both tests are different. In a prism, bricks could deform laterally without restriction, whereas in a wall partial lateral confinement occurs in the direction parallel to the length of the wall. However, friction between bearing faces of the prism and loading platens restrains significantly the lateral deformation of the prism. As a result, compressive tests of prisms with low aspect ratios \( h_p/t_p \) result in a conical type shear-compression failure similar to that observed in concrete cylinder tests. This effect artificially increases the strength of the prism when compared with the entire wall. On the other hand, the strength of masonry prisms decreases due to slenderness effect when the ratio \( h_p/t_p \) exceeds 6 [B4].

Krefield recommended in 1938 the use of correction factors to modify the compressive strength obtained from prism tests. Even though Krefield considered in his study just one type of clay brick and mortar, these factors were adopted by most of the current standards for many years [N4, P15]. Now, ACI/ASCE specifications [S3] indicate the use of the factors presented in Table 3.1 (a) and (b), as a function of the aspect ratio \( h_p/t_p \).
Figure 3.5. Compression test of a masonry prism and detail of the vertical cracks.

Table 3.1 (a). Correction factors for clay brick prism tests, ACI/ASCE specifications [S3].

<table>
<thead>
<tr>
<th>( \frac{h}{t_b} )</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.82</td>
<td>0.85</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
<td>0.97</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3.1 (b). Correction factors for concrete block prism tests, ACI/ASCE specifications [S3].

<table>
<thead>
<tr>
<th>( \frac{h}{t_b} )</th>
<th>1.33</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.75</td>
<td>1.0</td>
<td>1.07</td>
<td>1.15</td>
<td>1.22</td>
</tr>
</tbody>
</table>

For concrete masonry, it is a common practice to compute the compressive masonry strength using two-course prisms (\( \frac{h}{t_b} = 2 \) for the usual dimensions of concrete blocks). American masonry specifications [S3] recommend correction factors in order to refer the strength of a particular geometry to that of a standard prism of \( \frac{h}{t_b} = 2 \). Thus, the correction factor is greater than one for \( \frac{h}{t_b} > 2 \), with a maximum
value of 1.22 for $h_p/t_p = 5$. This implies that a strong correlation exists between prisms with $h_p/t_p = 2$ and full scale masonry, which is obviously incorrect. Hegemier et al. [H11] recommended that this practice of evaluating the compressive strength should be reviewed. Therefore, it seems more realistic to use the correction factors of Table 3.1 (a) for both clay and concrete masonry, as Amrhein [A2] suggested, until new research confirm the validity of these factors for concrete masonry.

Page and Marshall [P15] investigated the effect of the aspect ratio, $h_p/t_p$, on the compressive strength of prisms and masonry units. Based on the results obtained from a large series of tests, they proposed the correction factors summarized in Table 3.2. These values are compared in Fig. 3.6 with those corresponding to ACI/ASCE specifications [S3]. It can be observed that there is very good agreement between the values proposed by Page and Marshall [P15] and the ACI/ASCE specifications [S3] for clay brick prisms. However, the values corresponding to concrete block prisms are consistently higher. Again, this fact suggests that the correction factors for concrete block prisms should be revised.

Table 3.2. Correction factors for masonry prisms, proposed by Page and Marshall [P15].

<table>
<thead>
<tr>
<th>$h_p/t_p$</th>
<th>0</th>
<th>0.4</th>
<th>0.7</th>
<th>1.0</th>
<th>3.0</th>
<th>&gt; 5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 3.6. Correction factors for masonry prisms.
Australian masonry specifications (as reported in reference [D8]) adopt the same correction factors for both clay and concrete masonry prisms. In this case, the factor varies linearly from 0.7 to 1.0 for prisms with $h_p/t_p$ equal to 1.0 and 5.0, respectively.

Standard procedures for testing masonry prisms present some differences. For example, the standard ASTM E-447 Test Methods for Compressive Strength of Masonry Prisms [S3] indicates that the ends of the prism must be capped in the same manner as the masonry units (using sulphur or gypsum plaster, see section 2.3.2). The standard NZS 4230 [C5] specifies that two pieces of 12 mm wood fibre softboard must be located between the prism and the upper and lower platens. In the latter case, the effect of confinement at the ends is reduced and the calculated value of the compressive strength is not affected by any correction factor. This procedure seems to be more realistic. However, if the pieces of wood are more deformable than the mortar joints, the lateral stresses induced at the ends are greater and the strength of the prism would be reduced.

The bond configuration of the prism may influence the results. Consequently, it should be similar to the bond configuration in the masonry wall. Experimental results [D8, H11] indicate that the compressive strength of prisms laid in stack bond can be greater than that of prisms laid in running bond. This effect is significant for ungrouted hollow masonry, because the cross webs may not be in vertical alignment when running bond is used.

3.2.3 Factors Affecting the Compressive Strength

The compressive strength of the masonry depends on the mechanical properties of the constitutive materials and on the way they are combined to form the brickwork. Many factors affect the strength and only a brief discussion is presented here:

- **Masonry Unit Strength**: it is generally accepted that the compressive strength of the masonry units, $f_u$, is a good indicator of the quality of the brick. Furthermore, the tensile strength of the brick, which is an important factor for the usual case that masonry fails by lateral splitting of the masonry units, depends on the compressive strength of the masonry unit. Even though the failure mechanism of bricks in the compressive test is different to that observed in masonry walls, experimental data and analytical models demonstrate that masonry strength is correlated to the brick strength. Significant scatter of results can occur, however, due to the complexity of the failure mechanism which is affected by several factors.

- **Mortar Strength**: Mortar and masonry strengths are both related. In many cases, however, the brick strength has more influence in the compressive capacity of the masonry than the mortar strength. The compressive strength of masonry obviously increases when the mortar is stronger. The influence of mortar strength is more important for low values of the ratio $b/j$ (where $j$ is the bed joint thickness and $b$ is the masonry unit height) and for large tension-to-compression brick strength ratios $f_u/f_b$. Experimental results for clay masonry reported by Drysdale et al. [D8] showed that the increase of mortar strength has more influence on the compressive strength of masonry when poor quality mortars were used.
Mortar Joint Thickness: mortar is usually the weakest part of the masonry, thus the higher masonry strength is obtained with small joint thickness. This fact was intuitively known by ancient masons who used to build masonry walls or columns with very thin bed joints and even without mortar. For example, the stone walls at Machu Pichu (Peru) have mortar joints so tight that it is difficult to insert a knife blade between the masonry units [A2]. It is usually recommended that the mortar thickness be smaller than 10 mm. Hendry [H2] reported that joint thickness about 16-19 mm may reduce the strength up to 30% when compared with normal 10 mm joints. Experimental results obtained for different joint thickness are presented in section 3.5.6. Shrive [S12] indicated that the old practice of using thin mortar joint is advantageous because it increases the confining stress in the mortar (and its strength) and decreases the lateral tensile stress in the brick. Furthermore, thinner joints reduce rain penetration and considerably increase bond strength. It must be noted that in certain cases the mortar may be less deformable than the masonry unit. In this case, the mortar tends to reduce the lateral strains in the brick and a different type of failure can occur. The influence of the bed joint thickness in this situation could be distinct [H2].

Water Retentivity and Water Absorption: masons usually prepare the mortar with more water than that required for the cement hydration to ensure good workability. The excess of water is absorbed by the masonry units, but there is the risk that too much water be taken out to result in lack of residual water necessary to develop the strength of the mortar [S8]. The absorbed water leaves cavities in the mortar and consequently the material is weaker. On the other hand, the excess of water (for example caused by saturated bricks) can result in poor adhesion between both materials, making the mortar susceptible to frost damage [H2].

Sahlin [S1], based on experimental results, reported that the masonry strength decreases when the water absorption of the bricks increases. However, very low values of water absorption can also lead to reductions in the compressive strength. Schubert [S8] pointed out that the problem is more complex. He suggested that the water retentivity and water content of the mortar, and the moisture content and water absorption of the brick should be considered to analyse this issue. Very unfavourable conditions exist when the mortar has both high water content and high retentivity and is combined with masonry units having very low suction rate or which are very wet [S8]. Environmental conditions, such as low air humidity or violent air movement, also have a negative influence.

Shrive [S10] reported that the best bond conditions occur if the suction rate (IRA) ranges from 0.8 to 1.2 kg/(m² min) and ASTM specifications [S3] indicate that the suction rate must be less than 1.55 kg/(m² min). Therefore, it is recommended that bricks with a large suction rate be wetted before laying [H2]. Specifications for masonry structures [S3] indicate that the concrete masonry units shall not be wetted before placing in the wall. Test conducted by Schubert [S8] showed that the compressive strength of masonry (made of brick with high suction rate) may be doubled by increasing the moisture content of the masonry units from 0% to 15%.
Workmanship: this factor has a strong influence in the compressive strength of the masonry, especially for low strength materials. There is not much quantitative data to investigate this effect, although some laboratory tests have been conducted [H2]. The principal defect is the incomplete filling of the joint. Tests carried out in Australia and England indicated that the incomplete filling of the bed joints is more serious, from the structural point of view, than that of the vertical joint. In those tests, defects in the bed joints reduced the compressive strength up to 33% whereas incomplete filling of the vertical joint (even unfilled joints) had no significant effect in the strength of the wall [H2]. Incomplete filling of the joints also affects the non-structural performance by reducing the noise insulation and the resistance to rain penetration. Other defects arise when the bricklayer spreads too long a bed of mortar or attempts to correct plumbing errors by hammering bricks into a right position. In the latter case the bond between mortar and masonry unit may be broken. A large experimental programme carried out in Australia and United States [H2] aimed at studying the overall effect of workmanship showed that unsupervised site brickwork had a strength ranging from 55% to 62% of that of supervised brickwork. For low strength materials, Sahlin [S1] reported that the compressive strength of the wall may be increased up to 100% by improving the workmanship.

Patterns and Method of Bonding: masonry units may be combined in many different patterns to build a wall, although the unique condition required is to overlap bricks from one course to another. Sahlin [S1] reported that such different patterns do not seem to affect the compressive strength of the masonry.

Direction of the compressive loading: the type of failure of masonry panels is different when the compressive loading is applied in the direction parallel to the bed joints (see section 3.1). The strength is significantly reduced in comparison with the uniaxial compressive strength perpendicular to the bed joints. According to experimental results reported by different researchers, this reduction was 15% [S33], 40% [P12] and 43% [P6].

Cycling Loading: the behaviour of unreinforced masonry under cyclic compressive loading has been scarcely investigated. Tests conducted by Naraine and Sinha [N2] indicated that the stress-strain curves obtained under monotonic loading coincided approximately with the envelope curve under cyclic loading. Furthermore, the observed type of failure was similar in both cases. Abrams et al. [A7] reported that the compressive strength of clay masonry was reduced as a result of repeated compressive loads. In this investigation the load was applied with respect to a prescribed sinusoidal pattern, which was centred about a constant level of sustained force. The magnitude of the strength reduction depended on the type of mortar, the amplitude of the alternating stress and the number of cycles. This fact, however, has not significant importance for masonry structures subjected to earthquakes actions, in which the occurrence of many large cycles is not expected.
3.3 STRESS-STRAIN RELATIONSHIP OF MASONRY

The complete stress-strain relationship has recently been studied, since for a long time the only parameters required for elastic design were the modulus of elasticity and the compressive strength. The characteristics of the stress-strain curves depend on the constitutive materials. Fig. 3.7 illustrates typical stress-strain relationships for masonry prisms built with four different bricks (curve A: perforated bricks, curves B, C and D: solid bricks) bonded with 1:1/4:3 mortar (proportion of cement:lime:sand). It can be observed that, in these cases, there is no relation between the stress $f_m$ and the strain $e_m$ at the maximum point of the curve.

![Graph showing stress-strain relationship for masonry](image)

**Figure 3.7.** Typical stress-strain relationships for masonry in compression [H2].

Experimental data collected by Hendry [H2] indicates that there is a considerable variation in the strain $e_m$, ranging from 0.001 to 0.0045. Tests conducted by Abrams et al. [A7] using solid clay bricks laid with different types of mortar showed that the strain $e_m$ varied from 0.0055 to 0.0088. The higher values corresponded to masonry prisms with low strength mortars. Naraine and Sinha [N8] obtained values of 0.006 and 0.0061 for $e_m$, using clay masonry specimens tested in the direction normal and parallel to the bed joints.

Tests conducted by Powell and Hodgkinson (as reported in reference [H2]) with four brick types and 1:1/4:3 mortar showed that there was some variation of results among specimens of the same material but reasonable consistency was obtained. A falling branch after maximum stress was detected in these tests using suitable load control technique. The strain $e_m$ varied from 0.002 to 0.004 and the ultimate strain from 0.003 to 0.006. They found that all curves are very similar if they are plotted on a dimensionless basis (dividing the stress and strain by the values of the maximum point $f_m$ and $e_m$). The experimental data are well represented by the parabola:

$$\frac{f_m}{f_m^*} = \frac{e_m}{e_m^*} - \left( \frac{e_m}{e_m^*} \right)^2$$ (3.1)
where $f_m$ and $e_m$ are the stress and strain of a given point of the curve, respectively.

Sinha and Pedreschi [S2] tested six different types of bricks and two grades of mortars. A mathematical representation of the experimental results was obtained using regression analysis and the following expression was proposed:

$$\frac{f_m}{f'_m} = -0.0061 + 2.265 \frac{e_m}{e'_m} - 2.092 \left( \frac{e_m}{e'_m} \right)^2 + 0.824 \left( \frac{e_m}{e'_m} \right)^3 \quad (3.2)$$

This expression is only valid if $e_m \leq e'_m$, otherwise the stress is greater than $f'_m$. In these tests, the strain measurements were taken up to 95% of the ultimate stress and the maximum strain at failure was extrapolated. It is believed that this procedure could introduce large errors in the determination of the maximum compressive stress. Naraine and Sinha [N2] have also proposed an exponential function to represent the stress-strain relationship of clay masonry. This expression contains two empirical constants, whose values are adopted to fit experimental results.

Binda et al. [B1] investigated the compressive behaviour of masonry prisms built with clay bricks and three different types of mortar. They found that the stress-strain curve may be approximated with a linear branch, up to a stress level of 75% of the compressive strength $f'_m$, connected with a parabola:

$$\frac{f_m}{f'_m} = \begin{cases} 
\frac{3}{2} \frac{e_m}{e'_m} & \text{if } 0 \leq \frac{e_m}{e'_m} \leq \frac{1}{2} \\
-\frac{1}{2} + \frac{7}{2} \frac{e_m}{e'_m} - 2 \left( \frac{e_m}{e'_m} \right)^2 & \text{if } \frac{1}{2} < \frac{e_m}{e'_m} \leq e'_m
\end{cases} \quad (3.3)$$

Fig. 3.8 presents the comparison of Eqs. 3.1, 3.2 and 3.3. It can be observed that the differences among the three expressions are small from a practical point of view. Eq. 3.3 presents a maximum stress value of 1.031 $f'_m$ for $\frac{e_m}{e'_m} = 0.875$ and the slope of the combined curve has no discontinuity.

It has been observed [A3] that the strength of the mortar and the elastic modulus influence the shape of the stress-strain curve of the masonry. Compressive tests on prisms made with solid masonry units and different mortars showed that significant softening occurs when the mortar strength decreases. This fact is not considering by neither of the expressions described above (Eqs. 3.1, 3.2 and 3.3).

Priestley and Edler (as reported in reference [P1]) indicated that the compression stress-strain curves for concrete masonry are similar to those for plain concrete, noting that due to the splitting failure, the ultimate compression strain is lower than that appropriate to concrete.

The cyclic compressive behaviour of masonry is discussed in section 7.2, where an analytical model is proposed to represent this effect.
3.4 MODULUS OF ELASTICITY AND POISSON'S RATIO OF MASONRY

Different definitions are employed to evaluate the modulus of elasticity of masonry, $E_m$. In some cases $E_m$ is defined as the secant modulus at a stress level of 30% [B1] or 75% [H2] of the compressive strength $f_m$. Amrhein [A2] used the secant method in which the slope of the line is taken from 0.05 $f_m$ to a point on the curve at 0.33 $f_m$. In many cases the method considered to define the modulus of elasticity has not been reported. Nevertheless, for low and medium stress levels the differences obtained using distinct definitions may be considered small for practical purposes.

Masonry is a composite material of bricks and mortar, each one with distinct deformational properties. It seems reasonable to develop an analytical model to calculate the elastic modulus based on the deformational properties of brick and mortar. In this way, several researchers [A1, B1, D8, S1] have derived the same expression assuming linear elastic behaviour for both materials and equating the compressive deformation of masonry to the sum of the deformation of the bricks and of the mortar joints:

$$E_m = E_b \left( \frac{b}{j} + 1 \right) \left( \frac{b}{j} + \frac{E_b}{E_j} \right)$$

This expression indicates that the modulus of elasticity of the masonry will be always smaller than that of the brick when the mortar is more deformable than the brick ($E_b < E_j$), as usually occurs. Eq. 3.4 predicts that $E_m$ increases when the mortar is stiffer and when the mortar joint thickness $j$ is smaller (see Fig. 3.9).
Figure 3.9. Modulus of elasticity of masonry, according to Eq. 3.4.

The effect of lateral stresses may be included in the analytical model, although Ameny et al. [A1] showed that the difference is only 2% and it is not worthwhile to increase the complexity of the model. From an engineering point of view, it is reasonable to reject the confinement effect in the calculation of the modulus of elasticity. Ameny et al. [A1] proposed two complex models suitable for hollow concrete blocks, considering full-shell and face-shell bedded masonry units.

A different approach has been attempted by many authors to relate the modulus of elasticity $E_m$ of masonry walls with the compressive strength of the material. These empirical equations are not based on physical reasoning, but they have practical value in some cases. Sahlin compared the modulus of elasticity of clay brick obtained from a large amount of tests conducted by Hilsdorf, Glanville and Barnett, and the Structural Clay Products Research Foundation [S1]. Most of these results vary according to $400 f_m' < E_m < 1000 f_m'$. The following approximate expression has been given by Paulay and Priestley [P1] and Sahlin [S1]:

$$E_m = 750 f_m'$$  \hspace{1cm} (3.5)

This equation should be used with caution for very low strength mortars or unusual ratios of mortar to brick strength. San Bartolome [S7] and the Colorado Building Code [A2] proposed:

$$E_m = 500 f_m'$$  \hspace{1cm} (3.6)

Sinha and Pedreschi [S2] tested masonry prisms made with different bricks and mortars. Based on the statistical analysis of these results they proposed:

$$E_m = 1180 f_m^{0.83} \text{ (MPa)}$$  \hspace{1cm} (3.7)
Schubert (as reported by Hendry [H2]) proposed a value for $E_m$ that is usually greater than the value suggested by Sinha and Pedreschi [S2]:

$$E_m = 2116 \sqrt{f_m^*} \quad \text{(MPa)}$$  \hspace{2cm} (3.8)

The modulus of elasticity of concrete blocks is usually greater than the brick masonry modulus. There is a reduce dispersion of experimental data and several researchers [P1, S1] have reported the same expression:

$$E_m = 1000 \ f_m^* \quad \text{(MPa)}$$  \hspace{2cm} (3.9)

Most of the expressions do not consider the masonry unit weight of the masonry and were computed for normal weight materials. For this reason, Holmes (as reported by Amrhein [A2]) has proposed an empirical equation for light weight masonry:

$$E_m = 28.6 \ \rho^{1.5} \ \sqrt{f_m^*} \quad \text{(MPa)}$$  \hspace{2cm} (3.10)

where $\rho$ is the unit weight of the masonry in kN/m$^3$. Ameny et al. [A4], tested light weight concrete masonry and reported that the elastic modulus could be as low as 630 $f_m^*$.

Fig. 3.10 plots the modulus of elasticity $E_m$ according to Eqs. 3.5, 3.7, 3.8, 3.9 and 3.10 as a function of the masonry compressive strength $f_m^*$ (considering two values of the unit weight $\rho$, for Eq. 3.10). It is observed that all the expressions present an acceptable agreement for $f_m^* \leq 15$ MPa, whereas large differences are found for higher values of $f_m^*$.

![Graph showing modulus of elasticity as a function of compressive strength](image)

**Figure 3.10.** Comparison of empirical formulas to calculate the modulus of elasticity.
All the empirical expressions should be used carefully because the same masonry strength may be obtained combining distinct materials, for example, medium strength mortar and bricks or low strength mortar and high strength bricks. However, the deformational properties are very different and the modulus of elasticity will not be the same in both cases [S1].

The tangent modulus decreases when the vertical compressive stress is increased up to failure. However, Hilsdorf [S1] reported that masonry prisms constructed with lime mortar presents a S-shaped stress-strain diagram. In this particular case, the modulus of elasticity first increases and then decreases when brick start to fail.

The values of modulus of elasticity discussed in this section are related to short-term measurements because creep strain has been observed in mortar and in concrete block under long-term loading. Shrive and England [S4] reported that the strain caused by the applied load at 1000 days can exceed four times the short-term strain. Experimental work conducted by Ameny et al. [A4] indicated that the initial strain for prisms made of concrete masonry units increases up to 2.2 times (at 110 days). The effect of creep increases with the stress level induced by the applied load.

The Poisson's ratio \( u_m \), as well as other mechanical properties of the masonry, present a considerable variation. According to the experimental values reported by Binda et al. [B1] and Dhanasekar et al. [D2] this parameter may vary from 0.07 to 0.24. Hilsdorf (as reported in reference [H2]) found that the Poisson's ratio increases from an initial value of 0.2 to 0.35 near failure. Baba [B3] proposed a complex expression to evaluate the Poisson's ratio of the masonry considering the properties of its components \( E_m, E_p, u_m, u_p, \) and the dimensions of the brick and mortar joint.

Most studies conducted to investigate the compressive behaviour of masonry considered prisms or walls loaded perpendicularly to the bed joints. Dhanasekar et al. [D2] carried out an extensive series of tests and evaluated the elastic properties of the masonry \( (E_m, u_m) \) applying the load in the directions normal and perpendicular to the bed joints. The mean values found in these tests indicate that masonry behaves in an approximately isotropic manner, because the values measured in both directions were similar. Despite the reasonable agreement between the mean values, the isotropic behaviour was not strictly observed for individual panels. A similar conclusion was obtained by Naraine and Sinha [N2]. The presence of perforations in hollow masonry units clearly affect the isotropy condition. Brooks and Amjad [B4] found some degree of anisotropy testing masonry prisms constructed with perforated clay bricks.

### 3.5 EVALUATION OF THE COMPRESSIVE STRENGTH OF MASONRY

#### 3.5.1 General

Many researchers have attempted to formulate the relationship between the masonry strength and the fundamental properties of the component materials. Basically, two different approaches have been used. The first develops empirical equations derived from statistical analysis of experimental data and the second assumes analytical models for the composite behaviour of mortar and bricks.

The development of failure theories constitutes a rational method to quantify the failure of masonry in compression. Masonry is not a homogeneous material and its behaviour is governed by a complex
interaction between mortar joints and masonry units, as a consequence of the distinct properties of both materials. The observed mechanism of failure was described in section 3.1 and this is the basis for all these theories.

3.5.2 Empirical Formulas

Numerous investigators have presented empirical formulas which are the best-fit equations for the compressive strength of brickwork prisms or walls. Hendry [H2] and Sahlin [S1] have collected about 25 empirical expressions considering different materials, sizes and joint thicknesses. Some formulas are very simple and consider only two parameters, such as the compressive brick and mortar strengths. Others are more complicated because they include more mechanical properties and empirical factors. It must be recognized that empirical formulas are only valid for the same conditions and materials considered when they were obtained (geographical location is probably important [S1]) and should not be used in a general sense. Consequently, only a few examples of these expressions are presented here.

Hendry and Malek (as reported in reference [H2]) conducted a comprehensive research program and tested several hundred brickwork prisms. They proposed the following expressions:

\[
\begin{align*}
    f'_m &= 1.242 \ f'_{cb}^{0.531} \ f'_j^{0.208} \quad \text{(MPa)} \quad (3.11a) \\
    f'_m &= 0.334 \ f'_{cb}^{0.778} \ f'_j^{0.234} \quad \text{(MPa)} \quad (3.11b)
\end{align*}
\]

which are valid for masonry thickness of 102 and 215 mm, respectively. The prisms were built with solid bricks and mortars with a composition of 1:1/4:3 and 1:1:6 (proportion of cement:lime:sand).

Based on the observed composite behaviour of mortar and masonry units, Kirschig [K1] proposed that the compressive strength of the masonry should be described by a function of the brick strength and the modulus of elasticity of the mortar rather than by its strengths. Thus, he found from the tests that:

\[
    f'_m = 0.161 \ f'_{cb}^{0.43} \ E_j^{0.26} \quad \text{(MPa)} \quad (3.12)
\]

This expression was derived considering several types of bricks and mortars with rather low strengths, which limit the validity of Eq. 3.12.

Mehlmann and Oppermann [M3] found a relationship between values for brick and mortar strengths and the compressive strength of the mortar, according to the following formula:

\[
    f'_m = 0.83 \ f'_{cb}^{0.66} \ f'_j^{0.18} \quad \text{(MPa)} \quad (3.13)
\]

The tests were conducted with cement-lime mortars mixed in different proportions and bricks of medium strength \((f'_{cb} \leq 20 \text{ MPa})\).
In China, a large experimental program was conducted by Shi Chuxian [S6] to evaluate the compressive strength of brickwork. On the basis of statistical results, the following polynomial expression was proposed:

\[ f_m' = 0.1 \ f_{cb}' + 0.2 \ f_j' + 4 \sqrt{f_{cb}'} + 14 \sqrt{f_j'} \] (kPa) \hfill (3.14)

In this expression, the mortar strength has a stronger influence than the brick strength, which contradicts other empirical equations. Material properties were not indicated in Shi Chuxian's report.

### 3.5.3 Failure Theories

#### 3.5.3.1 Hilsdorf's Theory

The first attempt to develop a rational expression was made by Hilsdorf [H10] in 1967, based on his experimental observations. This theory considers the following hypothesis:

- The failure envelope for bricks under vertical compression and lateral biaxial tension is represented by Coulomb-Mohr criterion (Eq. 2.2).
- The strength of mortar under triaxial compression is similar to the strength of concrete and then can be approximated with the expression proposed by Richart et al. [R5, R6] (Eq. 2.8).
- The relationship between the lateral stresses \( f_{sb} \) (tension) in the brick and \( f_{sj} \) (compression) in the mortar may be formulated on the basis of equilibrium condition. It is assumed a tributary area of brick equal to one half of the height above and below the mortar joint, equal width for both sections and uniform stress distribution, as depicted in Fig. 3.11. Thus:

\[ f_{sj} \ j + f_{xb} \ b = 0 \] \hfill (3.15)

where \( b \) and \( j \) are the thickness of the masonry unit and mortar joint, respectively.

![Diagram of brick and mortar joint](Figure 3.11. Distribution of the stresses \( f_{sb} \) and \( f_{sj} \) assumed by Hilsdorf.)
• Failure of the masonry occurs when the lateral tensile stress in the cracked brick is smaller than the stress necessary to confine the mortar. In other words, both materials reach the failure envelope simultaneously. Then, the minimum lateral confinement of the mortar joint is (substituting Eq. 3.15 into Eq. 2.8 and considering the lateral confinement stress \( \sigma_i \) is equal to the absolute value of \( f_{sb} \)):

\[
f_{sb} = \alpha (f_y - f'_j)
\]  

(3.16)

where \( \alpha = j / (4.1 \ b) \).

Based on the validity of these assumptions, Eq. 3.16 may be combined with Eq. 2.2 (considering \( f_a = f_{sb} \)) and the vertical compressive stress at failure is expressed by:

\[
f_y = f'_{cb} \frac{(f'_{th} + \alpha \ f'_y)}{(f'_{th} + \alpha \ f'_{cb})}
\]  

(3.17)

Hilsdorf introduced a nonuniformity coefficient, \( U \), which was defined as "the ratio of the maximum normal stress observed within one brick to the average normal stress acting on the masonry". Thus, the coefficient \( U \) is a measurement of the distribution of the vertical stresses \( f_y \) along the horizontal section. He found experimental values of \( U \) based on strain measurements over a gauge length of 50 mm at the surfaces of several bricks. The values of the nonuniformity coefficient \( U \) depended on the mortar type and strength, joint thickness and level of applied load. Hilsdorf considered that the compressive stress \( f_y \) given by Eq. 3.17 represents the magnitude of the maximum local stress and introduced a nonuniformity coefficient at failure, \( U_{m} \), to obtain the average masonry stress. Therefore, the compressive strength \( f'_m \) was calculated according to:

\[
f'_m = \frac{f'_{cb}}{U_u} \frac{(f'_{th} + \alpha \ f'_y)}{(f'_{th} + \alpha \ f'_{cb})}
\]  

(3.18)

The measured values of \( U_u \) varied between 1.35 and 2.18. Posteriorly, Hendry [H2] proposed \( U_u = 1.3 \) based on tests made with cement mortars in the medium strength range. Sahlin [S1] recommended two approximate values: \( U_u = 1.5 \) or \( U_u = 2 - f'_j / 35 \) (MPa), being the latter expression valid for \( f'_j < 28 \) MPa. Even though Hilsdorf and other researchers have recognized the variability of this coefficient, the constant value \( U_u = 1.5 \) is usually adopted [S1, P1]. In this case, Hilsdorf's expression predicts that the ratio between the compressive strengths of masonry and brick \( f'_m / f'_{cb} \) ranges from 0.4 to 0.65, considering the materials and dimensions commonly used in construction.

Fig. 3.12 presents a graphic interpretation of this theory. Lines A and B represent the failure criterion for brick and mortar, respectively. Lines A' and B' are the reduced envelopes. The introduction of the nonuniformity coefficient practically implicates that the three uniaxial strengths involved in Eq. 3.18, \( f'_{cb} \), \( f'_b \), and \( f'_j \) are reduced by \( U_u \). Failure occurs when the lateral tensile stress \( f_b \) in the brick (represented by line C) reaches the failure envelope of the brick (line A'). Simultaneously, the mortar reaches its failure state.
The original interpretation presented by Hilsdorf was rather different. However, the validity of the explanation indicated by Fig. 3.12 is evident when some limit conditions are analysed. For example, if the mortar joint thickness is reduced to zero \((j = 0, \alpha = 0, \text{line C is vertical})\), the masonry strength should be equal to the brick strength \(f_{cb}^\prime\). Nevertheless, Eq. 3.18 predicts that \(f_{m}^\prime = f_{cb}^\prime/U_u\). In the case that the brick height is zero \((b = 0, \alpha = \infty, \text{line C is horizontal})\) then masonry strength would be \(f_{m}^\prime = f_{j}^\prime/U_u\) according to Eq. 3.18. A similar problem occurs if it is considered that the compressive strength of the mortar is zero, \(f_{j}^\prime = 0\). For this case, the compressive strength of masonry should be also zero, however, Eq. 3.18 leads to a different value:

\[
f_{m}^\prime = \frac{f_{cb}^\prime}{U_u \left( 1 + \alpha \frac{f_{cb}^\prime}{f_{tb}^\prime} \right)}
\]

Figure 3.12. Failure criterion for masonry in compression according to Hilsdorf's theory.

Despite the fact that these limit conditions have no practical application, the results clearly indicate a conceptual mistake.

Hilsdorf fully delineated the limitations of his work, in which some assumptions were made in the lack of more complete information. The criterion of failure utilized for mortar does not truly represent its real behaviour (see section 2.4.2). The strength of confined mortar, Eq. 2.8, assumed in this theory is valid
only for high strength mortars. However, Eq. 3.18 is not very sensitive to the confinement factor, as it can be verified carrying out a numerical evaluation of this expression varying the confinement factor. The linear envelope for brick subjected to compression and biaxial tension, Eq. 2.2, would be valid only when the compressive strength is evaluated using realistic testing procedures, otherwise this criterion overestimates the real biaxial strength of the brick (see section 2.3.4). Furthermore, the hypothesis of uniform distribution of the lateral stresses is not realistic, especially for a brittle material such as brick. This assumption, expressed mathematically in Eq. 3.15, leads to underestimate the value of the lateral stress in the brick, $f_{sb}$.

It is worth noting that the coefficient $U_n$ was introduced to consider the effect of the nonuniform distribution of vertical stress $f_v$ in the direction parallel to the bed joint. However, finite element analyses (see Appendix 1) conducted by the author indicate that the vertical stress $f_v$ is almost constant (some concentration of stresses occurs at the vertical edges of the brick), whereas the distribution of the lateral stresses $f_{sb}$ and $f_{ss}$ are highly nonuniform. The lateral stress in the brick $f_{sb}$ presents a maximum at the mortar-brick interface and then decreases very rapidly towards the centre of the brick. Chai Yuk Hon and Priestley [C1] reported that "the stresses (axial and transverse stresses) vary across the section from a minimum at the edges to a maximum at the centre", however, it was not reported how this conclusion was obtained. Certainly this observation is incorrect when explaining the variation of the lateral and vertical stresses, according to the results presented in Appendix 1.

All the problems discussed above produce an overestimation of the compressive strength of the masonry (Eq. 3.17) which was compensated with the introduction of the empirical factor $U_n$ in Eq. 3.18. In the author opinion's this coefficient represents a reduction factor, rather than a nonuniformity coefficient, which was introduced to fit the experimental results. Despite the problems mentioned above, Hilsdorf reported a good agreement between theoretical and experimental results and concluded that the compressive strength of test specimens could be predicted within a range of $\pm$ 20% of the actual values. It seems that this theory represents an acceptable approximation to the compressive strength for masonry in normal cases, although it should not be considered as a general expression.

Paulay and Priestley [P1] recommended the use of Eq. 3.18 for practical purposes with a coefficient $U_n = 1.5$. It should be noted, however, that Hilsdorf reported values of $U_n$ ranging from 1.35 to 2.18 instead of a constant value of 1.5.

3.5.3.2 Khoo and Hendry's Theory
Khoo and Hendry [H1] proposed a modification of Hilsdorf's theory, based on their investigation of the behaviour of masonry units subjected to biaxial tension-compression and of mortar under triaxial compression. Eqs. 2.3 and 2.9 were adopted to represent the failure envelope of masonry units and the confined strength of mortar, respectively. In order to obtain an analytical solution, Eqs. 2.3 and 2.9 were transformed in polynomials of third order and then combined with Eq. 3.15:
\[ 0 = \left( 0.9968 \frac{f_{cb}^*}{f_{cb}^t} + 0.1620 \alpha f_{ij}^t - \left( 2.0264 \frac{f_{cb}^t}{f_{cb}^{t^2}} + 0.1126 \alpha \right) f_{m}^t \right. \\
+ \left. \left( 1.2781 \frac{f_{cb}^t}{f_{cb}^{t^2}} - 0.0529 \alpha \right) f_{m}^{t^2} - \left( 0.2487 \frac{f_{cb}^t}{f_{cb}^{t^3}} - 0.0018 \alpha \right) f_{m}^{t^3} \right) \]

(3.19)

in which \( \alpha = j / (4.1 \beta) \). The authors of this theory reported that Eq. 3.19 shows reasonable agreement with experimental results obtained by Francis et al. [F1] in Australia and the Structural Clay Products Research Foundation in United States [H2].

3.5.3.3 Francis, Horman and Jerrems' Theory

A different concept was introduced by Francis et al. [F1] who considered that the masonry failure occurs due to the brittle behaviour of bricks. Thus, they adopted a linear failure envelope (Eq. 2.2) and introduced force equilibrium and strain compatibility requirements to relate the lateral stresses induced in the brick and mortar. It was assumed perfect bond between the masonry units and the mortar joints, which results in the lateral strain of both materials being equal at the interface:

\[ \varepsilon_{xb} = \varepsilon_{xb} = \varepsilon_{m} \]

(3.20)

where the subscripts b and j indicate brick and mortar, respectively, and directions x and z are defined in Fig. 3.3. Francis et al. [F1] considered an isotropic behaviour of both materials subjected to a three-dimensional stress state and assumed that both lateral stresses were equal (\( f_{x} = f_{z} \)). Thus, the compatibility condition, Eq. 3.20, may be expressed as:

\[ \frac{1}{E_{b}} \left[ f_{xb} - \nu_{b} (f_{y} + f_{zb}) \varepsilon_{xb} = \frac{1}{E_{j}} \left[ f_{ij} - \nu_{j} (f_{y} + f_{xj}) \right] \varepsilon_{xz} \right. \\
+ \left. \frac{1}{E_{b}} \left[ f_{zb} - \nu_{b} (f_{y} + f_{zb}) \varepsilon_{xb} = \frac{1}{E_{j}} \left[ f_{jz} - \nu_{j} (f_{y} + f_{xj}) \right] \varepsilon_{xz} \right] \varepsilon_{xz} \right) \]

(3.21)

where the compressive stresses are positive. The equilibrium condition is the same as that assumed by Hilsdorf in Eq. 3.15. Combining this equation with Eq. 3.21:

\[ f_{xb} = f_{zb} = \frac{\nu_{b} - \beta \nu_{j}}{1 + \beta \nu_{b} - \beta \nu_{j}} \]

(3.22)
where $\beta = E_b/E_j$ and $r = b/j$. This expression indicates that $f_{eb}$ and $f_{sb}$ are tensile stresses (negative) when $(v_b - \beta v_j) < 0$ for a given value of a compressive stress $f_j$. Assuming elastic behaviour up to failure (linear fracture mechanism) and a failure envelope represented by Eq. 2.2, it is found that:

$$f_m = \frac{f_{eb}}{1 + \frac{f_{eb} (\beta v_j - v_b)}{f_{eb} (1 - v_b) + r \beta (1 - v_j)}}$$

Eq. 3.23 can be simplified on the basis that the term $(1 - v_b)$ in the denominator is normally smaller than $[r \beta (1 - v_j)]$. Thus:

$$f_m = \frac{f_{eb}}{1 + \frac{f_{eb} (\beta v_j - v_b)}{r \beta (1 - v_j)}}$$

(3.24)

3.5.3.4 Atkinson, Noland and Abrams' Theory

A further development of the theory proposed by Francis et al. [F1] is due to Atkinson et al. [A3] and McNary and Abrams [M26], who proposed a more general procedure to predict not only the compressive strength, but also the complete stress-strain path up to failure. This theory assumes that the stress and strain states in the masonry can be derived by considering equilibrium and compatibility conditions. The condition of strain compatibility at the mortar-brick interfaces, assuming a plane stress state, is expressed as:

$$\frac{1}{E_b} \left[ f_{eb} - v_b f_j \right] = \frac{1}{E_j} \left[ f_{ej} - v_j f_j \right]$$

(3.25)

Substituting Eq. 3.25 in the Eq. 3.15, which represents the equilibrium condition proposed by Hilsdorf, the lateral stress in the brick is:

$$f_{eb} = \frac{v_b - \beta v_j}{1 + \beta r} f_j$$

(3.26)

Results obtained from three-dimensional finite element analyses (see Appendix 1) indicate that the hypothesis of three-dimensional stress state considered by Francis et al. [F1] and the assumption of plane stress used in this procedure are both limit situations. The comparison of the lateral stress $f_{eb}$ obtained from Eqs. 3.22 and 3.26 shows that the former expression systematically leads to higher values. This result can be explained considering that when a three-dimensional stress state is assumed (Eq. 3.22) the system is "more restrained" in the lateral direction, thus the induced tensile stress is greater.
Atkinson et al. [A3] conducted an experimental investigation to determine the multi-axial constituent properties of the components of masonry. Based on the test results, they considered that the nonlinear behaviour of the mortar and their deformation parameters can be expressed as a function of the vertical and lateral stresses, thus $E_j = f(f_r, f_q)$ and $v_j = f(f_r, f_q)$. The brick was assumed to remain elastic and its failure envelope represented by Eq. 2.4. Using measured material properties, an incremental nonlinear procedure was employed to solve Eq. 3.26. The vertical stress $f_r$ was increased from zero until the failure occurs either in the brick or in the mortar. For each stress increment $\Delta f_r$, lateral stresses and strains were calculated in both materials.

The stress-strain curves obtained from the application of this theory showed a similar shape to those obtain for compressive tests of masonry prisms. Nevertheless, the predicted compressive strength was consistently smaller than the measured values. According to the results presented by Atkinson et al. [A3] and McNary and Abrams [M26], the ratio of the calculated to the measured strength ranged from 0.60 to 0.71. Atkinson et al. [A3] pointed out that "Hilsdorf's theory incorrectly assumes that the mortar is at its failure state when masonry failure occurs" and assumed that the compressive strength develops when the stress state, either in brick or mortar, reaches the corresponding failure envelope. However, the formation of one crack in the brick does not imply the failure of masonry. The observed behaviour in laboratory tests and results from nonlinear finite element analyses (see Appendix 1) indicate that several cracks may develop before the failure occur. In the author's opinion, this is the main reason to explain the large underestimation of the compressive strength obtained from the comparison with experimental results.

3.5.3.5 Ohler's Theory

Ohler [H2] proposed an expression to calculate the compressive strength of masonry. This expression was developed considering a three-linear representation of the biaxial failure curve for brick material (Eq. 2.5) and a linear relationship for the confined strength of mortar (Eq. 2.10):

$$f_m = f_j + \frac{S_2}{S_1} \frac{f_{eb} - f_j}{1 + \frac{m}{r} \frac{f_{eb}}{f_{lb}}} \quad (3.27)$$

where factors $S_1$ and $S_2$ are defined in section 2.3.5 and the coefficient $m$ is given in Table 2.5.

3.5.4 Proposed Failure Theory

3.5.4.1 General Formulation

Hilsdorf's theory is, without doubt, the most accepted criterion to calculate the compressive strength of masonry because it leads to a simple expression which agrees approximately with experimental results. However, this theory is based on some hypotheses that were assumed in the lack of experimental evidence. For this reason, some modifications are introduced to the original procedure with the aim of improving it:

- The linear approximation, given by Eq. 2.5, is adopted to represent the failure envelope of brick subjected to biaxial tension-compression. This expression allows the consideration of different failure criteria assuming the proper values of the coefficients $S_1$ and $S_2$. For the linear failure envelope (Eq. 2.2) the values are $S_1 = S_2 = 1$. Atkinson's nonlinear envelope
must be approximated for a given range; for example $S_1 = 1.70$ and $S_2 = 0.96$ represent a good approximation, with an error smaller than 10% in the range $0 < f_x / f_{xb} < 0.2$. This approximation is acceptable on the ground that the lateral stress $f_x$ is usually smaller than the uniaxial tensile strength, $f_{xb}$. The consideration of the nonlinear expressions proposed by Khoo and Hendry [H2, K12] and Atkinson et al. [A3] (Eqs. 2.3 and 2.4) significantly complicates the analytical solution of the problem.

The confined strength of mortar is represented by Eq. 2.10, considering that the confinement factor $m$ is a function of the uniaxial compressive strength of the mortar (Table 2.5).

![Figure 3.13. Nonuniform distribution of lateral stresses assumed in the proposed theory.](image)

The distribution of the lateral stresses in both brick and mortar are nonuniform along a vertical section (see Fig. 3.13). Consequently, the equilibrium condition can be expressed as:

$$C_{db} f_{xb} b + C_{dj} f_{sj} j = 0$$

(3.28)

where $C_{db}$ and $C_{dj}$ are stress distribution factors defined as the ratio of the average lateral stress to the maximum lateral stress in brick and mortar, respectively:

$$C_{db} = \frac{1}{f_{xb}} \int_0^b f_x \, dy$$

(3.29)

$$C_{dj} = \frac{1}{f_{sj}} \int_0^j f_x \, dy$$

These factors are equal to 1.0 when the stress distribution is uniform, as assumed by Hilsdorf [H10].
BEHAVIOUR OF MASONRY IN COMPRESSION

Following a similar procedure to that indicated by Hilsdorf (combining Eq. 3.28 with Eq. 2.10 and introducing into Eq. 2.5), the following expression is proposed to evaluate the compressive strength of the masonry:

\[
f'_m = f'_{cb} \frac{S_2 f'_{tb} + S_1 \frac{\alpha_m}{C_d} f_j}{f'_{tb} + S_1 \frac{\alpha_m}{C_d} f'_{cb}}
\]  
(3.30)

where \(\alpha_m\) is a factor similar to that defined by Hilsdorf, which considers the mortar confinement factor \(m\):

\[
\alpha_m = \frac{j}{m b}
\]  
(3.31)

and \(C_d\) is a combined stress distribution factor:

\[
C_d = \frac{C_{db}}{C_{dj}}
\]  
(3.32)

The determination of the stress distribution factor \(C_d\) is discussed in next section, and a comparison between analytical and experimental results is presented in section 3.5.6.

### 3.5.4.2 Stress Distribution Factor \(C_d\)

The values of the stress distribution factors \(C_{db}\) and \(C_{dj}\) were evaluated from a parametric study based on finite element models of masonry prisms subjected to compressive loads (details of this study are presented in Appendix 1). Based on these results, it was found that the stress distribution factors can be approximated by:

\[
C_{db} = \frac{0.2}{b/d}
\]  
(3.33)

\[
C_{dj} = 1 - \frac{1.1}{d/j}
\]  
(3.34)

The combined stress distribution factor \(C_d\) is obtained replacing Eqs. 3.33 and 3.34 into Eq. 3.32. Fig. 3.14 shows the variation of the factor \(C_d\) as a function of the ratio \(b/d\), for three different values of the ratio \(b/j\). It is evident that the influence of the mortar joint thickness is not very important, especially for small values of the ratio \(b/d\) (usually 0.3 < \(b/d\) < 0.5). This fact indicates that the variation of factor \(C_d\) is mainly controlled by the stress distribution factor of the brick \(C_{db}\). A simple approximation for factor \(C_d\) is:

\[
C_d = \frac{C_{db}}{C_{dj}} = \frac{0.2}{b/d} \left(1 - 0.1 \frac{b}{d}\right)
\]  
(3.35)
Eq. 3.35 was derived assuming an average value $b/j = 10$. The error introduced by considering this simplification has no practical importance.

![Graph showing stress distribution factor $C_d$ as a function of the ratio $b/d$.](image)

**Figure 3.14.** Variation of the stress distribution factor $C_d$ as a function of the ratio $b/d$.

### 3.5.5 Nonlinear Analysis of Masonry Prism in Compression

In order to complete the study of the compressive behaviour of masonry, a computer program was developed to evaluate the nonlinear behaviour of masonry in compression. The foundations of the method are similar to those proposed by Atkinson et al. [A3] (section 3.5.3.4). However, some modifications are introduced to consider a nonuniform distribution of the lateral stresses. Details of this procedure and the code of the computer program are presented in Appendix 2.

The nonlinear procedure and the proposed modification of Hilsdorf's theory (Eq. 3.30) are used to investigate the influence of the stress distribution factor $C_d$ on the compressive strength $f'_m$. Fig. 3.15 shows results obtained from a particular example in which common values for the masonry properties were assumed. Both procedures give similar results, especially for values of $C_d > 0.3$. In other cases the differences are more pronounced. The nonlinear procedure usually leads to smaller values of compressive strength. It is observed that the compressive strength $f'_m$ increases when more uniform stress distributions are considered ($C_d$ close to 1). Hilsdorf's theory assumes uniform stress distribution, hence $f'_m$ is independent of the factor $C_d$.

### 3.5.6 Comparison with Experimental Results

Experimental data obtained from compressive tests of masonry prisms are compared with Hilsdorf's equation and the analytical models developed in sections 3.5.4 and 3.5.5, in order to verify the validity of these procedures. Masonry prisms with different thickness were tested in compression using two pieces of 20 mm softboard between the prism and the loading platens. The ratio $h/f_p$ of the prism varied from...
2.61 to 2.94. Consequently, the compressive strength obtained from the tests was affected by the correction factors presented in Table 3.2.

Figure 3.15. Variation of the compressive strength as a function of the factor $C_d$.

The specimens were constructed using solid concrete bricks, with dimensions 230 x 90 x 75 mm, and mortar with a proportion of one part of cement, two parts of bricklayer lime mortar and one part of sand. The thickness of the mortar joints was 5, 7, 10, 15 and 20 mm. The mechanical properties of the masonry materials are presented in Table 3.3. The compressive strength of the bricks was obtained by testing brick stacks without mortar joints, instead of single bricks, in order to avoid the restraint induced by the loading platens, following the method proposed in section 2.3.3. Since this value can be considered as the real uniaxial compressive strength, the failure criterion for brick under tension-compression stresses assumed in the nonlinear analysis is represented by a linear envelope (Eq. 2.5, $S_1 = S_2 = 1.0$). The tensile strength was determined by an indirect tension test.

Table 3.3. Material properties of masonry prisms tested by the author.

<table>
<thead>
<tr>
<th>Concrete bricks (solid)</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 75$ mm, $d = 230$ mm</td>
<td>$f'_t = 8.0$ MPa</td>
</tr>
<tr>
<td>$f'_c = 26.2$ MPa</td>
<td>$f'_t = 8.0$ MPa</td>
</tr>
<tr>
<td>$f'_b = 2.8$ MPa</td>
<td>$m = 2.3$</td>
</tr>
<tr>
<td>$E_p = 12980$ MPa</td>
<td>$E_p = 8540$ MPa</td>
</tr>
<tr>
<td>$\nu_p = 0.15$ *</td>
<td>$\nu_p = 0.20$ *</td>
</tr>
</tbody>
</table>

*Assumed value
The comparison between the experimental values and the numerical results are presented in Fig. 3.16. In this comparison, three analytical procedures are considered, namely, Hilsdorf's equation, the modification proposed in section 3.5.4 (Eq. 3.30) and the nonlinear procedure described in section 3.5.5. The experimental results considered in Fig. 3.16 represent the average compressive strength obtained from three tests. It is observed in Fig. 3.16 that Eq. 3.30 agrees very well with the experimental results, whereas the nonlinear procedure gives similar results only for large joint thicknesses. Hilsdorf's equation leads to smaller values of the compressive strength in all the cases. For a joint thickness of 10 mm, a common value in masonry construction, the difference between the experimental value and Hilsdorf's equation is 26%. This difference decreases when the thickness of the mortar joints increases.

![Graph](image)

**Figure 3.16.** Comparison between experimental data and numerical results.

Experimental results reported by Francis et al. [F1] are also included here to verify the validity of the analytical procedures. They conducted a research program to investigate the compressive strength of masonry prisms constructed with different mortar joint thicknesses, using two types of masonry units. The measured properties of the materials are given in Table 3.4.

<table>
<thead>
<tr>
<th>Solid bricks</th>
<th>Perforated bricks</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = 74 mm, d = 225 mm</td>
<td>b = 76 mm, d = 228 mm</td>
<td></td>
</tr>
<tr>
<td>$f'_{ub} = 29.2$ MPa</td>
<td>$f'_{ub} = 39.4$ MPa</td>
<td>$f'_{i} = 6.5$ MPa</td>
</tr>
<tr>
<td>$f'_{ab} = 2.6$ MPa</td>
<td>$f'_{ab} = 2.1$ MPa</td>
<td>m = 2.1</td>
</tr>
<tr>
<td>$E_b = 26714$ MPa</td>
<td>$E_b = 20738$ MPa</td>
<td>$E_i = 1406$ MPa</td>
</tr>
<tr>
<td>$\nu_b = 0.25$</td>
<td>$\nu_b = 0.25$</td>
<td>$\nu_i = 0.25$</td>
</tr>
</tbody>
</table>

* Assumed value.

Table 3.4. Material properties for brick prisms tested by Francis et al. [F1].
The compressive strength of the bricks was obtained by testing brick stacks without mortar joints. The ratio of this value to the standard compressive strength (according to the standard AS CA47-1969 [F1]) was 0.44 and 0.69 for solid and perforated bricks, respectively. The tensile strength was evaluated according to an indirect tension test. Fig. 3.17 shows the experimental results of the compressive strength plotted for different joint thickness. Test values reported by Francis et al. [F1], and plotted in Fig. 3.17, are affected by the correction factor of Table 3.2 because they were obtained from masonry prism tests (the ratio $h_d/t_p$ varied from 2.8 to 3.5).

![Graphs showing compressive strength vs. mortar joint thickness for solid and perforated bricks](image)

Figure 3.17. Comparison between experimental data obtained by Francis et al. [F1] and numerical results.

It may be observed that the Eq. 3.30 and the nonlinear procedure presented in section 3.5.5 lead to very similar results in both cases, because the mortar is much weaker than the brick and both materials fail almost simultaneously (this is the hypothesis assumed for deriving Eq. 3.30). For the case of solid bricks the analytical results agree very well with the experimental data, whereas for perforated bricks both methods overestimate the measured strength.

Another comparison can be conducted based on the experimental results obtained by Arango [A11]. In this research program only the basic mechanical properties were measured. Therefore, it is necessary to assume
some values to complete the required data for the analytical procedures. Table 3.5 presents the material properties considered in the analysis. In these tests, the compressive strength of the block was determined according to ASTM specifications. It was mentioned in section 3.2.2 that ACI/ASCE specifications [S3] indicate that the compressive strength must be multiplied by a correction factor, which is greater than one when the prism aspect ratio $h_p/t_p$ is greater than two, as shown in Table 3.1 (b). This procedure leads to results which are not conceptually valid. The prisms tested in this investigation had average aspect ratios of 2.7 and 4.1, thus the correction factors varied from 1.05 to 1.16 for the two series of tests that are considered in this comparison. Therefore, the values reported by Arango (which were modified by the correction factor) are divided by these factors to remove this bias. Both results are plotted in Fig. 3.18, the small circles indicate the compressive strength according to the ACI/ASCE specifications [S3], and the triangles represent the modified values.

**Table 3.5. Material properties of concrete block prisms tested by Arango [A11].**

<table>
<thead>
<tr>
<th>Concrete blocks A</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_{cb} = 12.62$ MPa</td>
<td>$f'_u = 10.23$ MPa</td>
</tr>
<tr>
<td>$f'_{lb} = 1.26$ MPa *</td>
<td>$m = 2.3$</td>
</tr>
<tr>
<td>$E_s = 8834$ MPa *</td>
<td>$E_u = 5115$ MPa *</td>
</tr>
<tr>
<td>$\nu_s = 0.15$ *</td>
<td>$\nu_u = 0.20$ *</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Concrete blocks B</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_{cb} = 21.23$ MPa</td>
<td>$f'_u = 7.37$ MPa</td>
</tr>
<tr>
<td>$f'_{lb} = 2.12$ MPa *</td>
<td>$m = 2.15$</td>
</tr>
<tr>
<td>$E_s = 14861$ MPa *</td>
<td>$E_u = 3685$ MPa *</td>
</tr>
<tr>
<td>$\nu_s = 0.15$ *</td>
<td>$\nu_u = 0.20$ *</td>
</tr>
</tbody>
</table>

*Assumed value $f'_{lb} = 0.1 f'_{cb}$, $E_o = 700 f'_u$, $E_p = 500 f'_u$

The failure criterion for brick proposed by Atkinson et al. [A3] and McNary and Abrams [M26] (Eq. 2.4) was assumed in this case because the compressive strength of the brick was calculated according to standard procedures, in which the loading platens strongly affect the behaviour of the squat prisms (see section 2.3.4). For the application of Eq. 3.30, this criterion was approximated using Eq. 2.5 with $S_1 = 1.70$ and $S_2 = 0.96$. The comparison of the results indicates that Eq. 3.30 again may predict adequately the experimental data for both series of tests, whereas Eq. 3.18 behaves poorly for the first case, concrete blocks A. Results obtained from the nonlinear procedure are just an approximation in this case, because most of the required properties were crudely assumed.

Based on the comparisons described in this section, it is believed that both methods proposed in this work give a good approximation to calculate the compressive strength of unreinforced masonry walls. For the cases considered here, the Eq. 3.30 gives better agreement with experimental results than Hilsdorf's theory. The application of the nonlinear procedure also leads to good results. It is observed that, generally, Hilsdorf's theory (Eq. 3.18) does not predict adequately the compressive strength for low values of the joint
thicknss, as it was anticipated in section 3.5.3.1. Furthermore, Hilsdorf's expression is not as sensitive to the variation of the joint thickness as the experimental results indicate. It is worth noting that the splitting failure of the masonry units can occur in either the parallel or perpendicular plane to the plane of the wall. However, the analytical results presented in this section correspond to a value of the coefficient $C_d$ obtained using a ratio $b/d$ (see Eq. 3.35) measured in the plane of the wall.

![Graph](image)

**Figure 3.18.** Comparison of numerical and experimental results based on compression tests conducted by Arango [A11].

### 3.6 DISCUSSION ON THE VALIDITY OF FAILURE THEORIES

It is worth noting that the failure theories presented in this study are based on the mechanism of failure described in section 3.1. The splitting failure of brick, observed in masonry under compressive loading, is basically explained assuming that the mortar is more deformable than the brick. This condition may be expressed mathematically [B3] by:

$$\frac{E_b}{E_j} \frac{v_j}{v_b} > 1$$  \hspace{1cm} (3.36)
The measured parameters reported by different researchers indicate that Eq. 3.36 is satisfied in most of the cases, especially for the usual case in which the mortar is weaker than the brick. This condition assures that the lateral stress $f_m$ calculated with Eqs. 3.22 and 3.26 will be negative (tension) for a given value of the compressive stress $f_c$. Hilsdorf's theory does not consider the deformational properties of the materials. Therefore, it is implicitly assumed that these properties are proportional to the strength of the material. In this case the necessary condition is $f_m > f_c$.

Failure theories do not consider neither the effect of the head joints nor lateral stresses acting on the vertical faces of the brick. Therefore, these theories are strictly valid only for stack-bonded masonry prisms and not for masonry walls with running bond (the commonest bond pattern used in masonry construction). Conceptual considerations suggest that interaction between two adjacent bricks would restrain the lateral expansion reducing the lateral tensile stress induced in the masonry units. However, the quality of the head joints is usually poorer than that of the bed joints (head joints are often not properly filled with mortar and bond strength may be affected due to shrinkage) and of the masonry units. This fact could lead to a reduction of the compressive strength in masonry wall with running bond

A plane finite element model of a masonry wall was analysed to investigate the effect of the partial restraint produced by head joints. The panel was five bricks high with three bricks in each layer. Due to the symmetric properties of the structure and load distribution only one quarter of the panel was considered by using a mesh of 17 x 29 four-node elements (details of the finite element model are described in Appendix 1). Numerical results obtained from this model showed that the panel expands in the direction perpendicular to the applied load, which induced lateral stresses in both mortar joints and bricks. The head joints were subjected to a vertical compression and lateral tension stress state, as occurs in the bricks, however the vertical stresses were much smaller in the head joints due to the different mechanical properties. The lateral stress distribution in the bricks located close to the free edge of the panel was practically equal to that obtained for the masonry prisms. However, the stress state in the central bricks was more uniform. In the bed joints, a more uniform stress state was also observed, except in the zone close to the vertical edge of the panel. This model, however, did not consider the poorer quality of the head joints in comparison to the bed joints.

It is worth noting that the splitting failure may also occur in a plane parallel to the plane of the panel, which represents a condition similar to that of a stack-bonded prism. For the usual case of masonry constructed with stretcher bond (bricks laid with their long side along the face of the wall) this situation would be expected, according to Eqs. 3.30 and 3.35.

In the case of ungrouted masonry structures constructed with hollow masonry units, the presence of the hollows can affect the stress state and modified the mechanism of failure. Different factors shall be carefully analysed in this case, such as geometry of the masonry units, shell taper, vertical continuity of the webs, alignment of cores, type of bond and mortar bedding [G7, K13]. Hollow masonry units are usually mortared following two distinct techniques: face-shell and full mortaring.

Face-shell bedded masonry (mortar is applied only on the face-shells but not on the webs) is a common practice in some countries to facilitate construction and to reduce the consumption of mortar. However,
the structural behaviour of masonry deteriorates. The failure mechanism is rather different to that observed for solid brick masonry because tensile stresses develop at the centre of the webs by a mechanism similar to deep-beam bending. Shrive [S5] proposed a failure theory which was checked with experimental results and finite element analysis. Self (as reported in reference [S5]) indicated that full-bedded masonry prisms were about 50% stronger than face-shell bedded masonry prisms constructed with the same materials. For this reason, Ganesan and Ramamurthy [G7] discouraged the use of the latter constructive technique. For hollow masonry with full mortar bedding, Drysdale et al. [D8] indicated that the mechanism of failure is similar to that of solid masonry. Analytical studies conducted by Khalil et al. [K13] indicated that grouting results in improved performance of masonry, especially when the grout has similar deformational characteristics to those of the masonry units.

The theories presented in the sections 3.5.3 and 3.5.4 are not applicable to grouted masonry. Chai Yuk Hon and Priestley [C1, P1] developed a modified expression, based on Hilsdorf's theory, to calculate the compressive strength of grouted masonry. Even though the mechanism is similar, the grout strength and the net area ratio are important parameters that influence the total strength. Experimental data on grouted masonry have shown that the superposition of the capacities of the ungrotted hollow masonry and the columns of grout formed in the cells can overestimate the real compressive strength [D8].

3.7 CONCLUSIONS

Experimental work conducted in many countries have provided valuable information used to understand the masonry behaviour and to determine the strength parameters. Numerous empirical equations have been proposed according to test results, although these expressions are valid only for the same conditions and materials used when the equations were obtained. Consequently, they must be used with caution.

It is generally accepted that the failure of masonry in direct compression is due to the different deformational properties of the constitutive materials: brick and mortar. This difference induces tensile stresses in the brick which cause a splitting failure. A different type of failure may occur when the mortar is more rigid than the masonry unit.

In the last three decades, several researchers have proposed different theories and expressions to evaluate the compressive strength of masonry walls, based on the properties of the mortar and brick. These theories assume distinct hypothesis and criteria, and occasionally lead to completely different results.

Hilsdorf's theory is widely accepted for the evaluation of the compressive strength, although it presents some conceptual mistakes and should not be regarded as a general procedure. For these reasons, the original theory is modified in this work to consider a rational distribution of the lateral stresses and more realistic material models. The comparison between experimental data and theoretical results obtained from the proposed procedure indicates a good agreement.
An approximate procedure to evaluate the variation of mortar properties under biaxial compression is also proposed in order to apply a nonlinear procedure conceptually similar to that developed by Atkinson et al. [A3]. However, some modifications are introduced to take into account the variation of the lateral stresses, which were originally assumed to be uniform.

Numerous parameters are required to define the characteristics of masonry. For this reason, the experimental reports should provide a complete description of the materials and its mechanical properties, giving the adequate information required for the application of analytical models.
4. MASONRY SUBJECTED TO SHEAR, TENSION AND BIAXIAL STRESSES

4.1 MASONRY SUBJECTED TO SHEAR
4.1.1 Introduction
The adequate evaluation of the shear strength is required for the design of masonry panels when they are subjected to lateral loads induced by wind or earthquakes. Shear stresses are usually combined with compressive stresses produced by gravity loads or other actions. Consequently, the case of pure shear has no practical application and the shear strength of masonry is investigated considering the effect of compressive stresses in the direction normal to the bed joints (see Fig. 4.1). In other cases, the stress state in the masonry panel is more complex and general failure criteria are required to evaluate the strength of masonry. These general criteria are discussed in section 4.4.

![Diagram of masonry panel showing bed joint and head joint](a)

![Diagram of assumed stress state](b)

Figure 4.1. Denomination of mortar joints in the masonry panel and assumed stress state.

4.1.2 Modes of Failure
4.1.2.1 General
The behaviour of masonry is characterized by two important effects: the fragile response of the masonry units in tension and the weakness introduced by the mortar joints. Therefore, the modes of failure result from the combination of diagonal tension cracks crossing the bricks and debonding along the mortar-brick interfaces. Experimental results of specimens tested under combined stresses show that different patterns of failure may occur according to the relative magnitude of the normal stress, $f_n$ (in the direction...
perpendicular to the bed joints) and the shear stress \( \tau \) \([G1, H3, M2, P3, R1, W1]\). Three types of failure can be considered:

\subsection{4.1.2.2 Shear-Friction Failure}

The first type of failure occurs for low normal stresses and is usually called shear or shear-friction failure. In this case, the failure is caused by debonding of the mortar-brick interfaces and sliding shear occurs along the bed joints. Cracks often develop following a stepped pattern, as Fig. 4.2 (a) indicates, because the bond strength of the head joints is usually reduced by shrinkage and uncomplete filling, and the increase of the shear strength due to friction is not significant. Consequently, these zones constitute planes of weakness in the masonry where the cracks usually begin. Various researchers \([E1, P6, P9, W1]\) have indicated that the failure may also occur by debonding along only one bed joint, for very low values of the normal stress \( f_n \). Page \([P6]\), using the test technique indicated in Fig. 4.5 (a), observed this type of failure for values of the inclination \( \theta \) smaller than 23\(^\circ\), when the normal stress \( f_n \) was very low.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Modes of failure of masonry panels subjected to shear and compressive stresses.}
\end{figure}

\subsection{4.1.2.3 Diagonal Tension Failure}

For medium to high values of the normal stress, tensile failure of the bricks commonly occurs. This mode of failure is termed "diagonal tension failure". The shear strength of the mortar joints increases due to the effect of the compressive normal stresses. Hence, the cracks appear in the bricks instead of in the bed joints, as a result of the tensile stresses induced by the shear-compressive stress state. As observed in Fig. 4.2 (b), the cracks follow the head joints and pass through the bricks with an inclination which depends on the orientation of the principal stresses in the brick.

\subsection{4.1.2.4 Compressive Failure}

This type of failure occurs for very high values of the normal stress in comparison with the shear stress \((f_n > 8 \tau\) according to experimental results \([M2]\)). In this case, the failure is similar to that corresponding to masonry under direct compression (see section 3.1), although the effect of shear stresses causes a reduction in the compressive strength of the masonry.

Hamid and Drysdale \([H3]\) conducted an experimental programme with ungrouted and grouted prisms made of perforated concrete blocks following the procedure indicated in Fig. 4.5 (a). They found that the
type of failure was similar, in a general sense, for both ungrouted and grouted prisms, even though some differences were observed due to the presence of grouted cores. The contribution of the grout increases the shear strength of the masonry, especially when the shear failure of the bed joint (crossing the grouted cores) is the governing mode.

4.1.3 Shear Test Procedures

The shear behaviour of masonry has been investigated for many researchers. In 1873, Bauschinger (as reported by Dialer [D25]) apparently conducted the first tests to investigate the strength of masonry subjected to shear loading. Nowadays, different testing methodologies are used, from simple prisms made with two bricks to full-scale masonry walls subjected to biaxial load.

The simplest test procedure, usually named direct shear test, consists of testing prisms built with two (couplet), three (triplet) or four masonry units (see Fig. 4.3). The compressive load, \( P \), is applied first and then the specimen is subjected to a second load, \( V \), which induces increasing shear stresses up to failure. Even though this is an easy and inexpensive methodology, the conditions in the test are different from those in the real structure. The specimens have only one or two layers of mortar and there is no mortar joint parallel to the compressive load direction. Furthermore, the deformation of the bricks is restrained by the devices which apply the loads and the stress distributions are almost constant in most of the cases. The parameters measured in these specimens represent the behaviour of the bed joints rather than an entire masonry wall.

![Diagram](https://example.com/diagram.png)

**Figure 4.3.** Testing procedures for evaluating the shear strength of bed joints.
It is important to note that the test should be properly designed trying to induce a pure shear state in the mortar joints. In the case illustrated in Fig. 4.3 (c), for example, the mortar joints are subjected to a combined state of shear and flexural stresses, which leads to a reduction of the shear strength. The testing procedure used in the experimental programme conducted by the author for the evaluation of the shear strength of the mortar joints is described in section 9.4.3.

The test of masonry panels is a more realistic procedure to investigate the shear strength, and different methods are employed for the application of the loads. Fig. 4.4 shows two types of test conducted on small panels, which are less frequently used than the direct shear tests. Experimental testing of a wall as a cantilever (see Fig. 4.4 (a)) is difficult because the base has to be prevented from sliding and overturning. For this reason some researchers [R1] have preferred to test the masonry panels as deep beams using three loading points, as illustrated in Fig. 4.4 (b). In these cases, flexural effects induce additional normal stresses in the bed joints and this effect should be considered in the analysis of the results.

![Figure 4.4. Methods for testing the shear strength of masonry panels.](image)

Another method often used in experimental work is represented in Fig. 4.5 (a). In this case, small masonry panels are tested under compressive loading but the angle $\theta$, between the bed joint direction and the applied load, is different from $90^\circ$. Consequently, shear and axial stresses are induced in the mortar joints. Gallegos [G8] used 1.0 m diameter masonry octagons that permitted, through testing in different positions, the application of the compressive load with angles equal to $0^\circ$, $45^\circ$ and $90^\circ$. The resultant stresses can be expressed as:

\[
\begin{align*}
    f_n &= \frac{P}{A} \sin^2 \theta \\
    f_p &= \frac{P}{A} \cos^2 \theta \\
    \tau &= -\frac{P}{A} \sin \theta \cos \theta
\end{align*}
\] (4.1)
where $f_p$ and $f_n$ are the axial stresses in the direction parallel and normal to the bed joints, $\tau$ is the shear stress in the bed joints and $P/A$ is the applied compressive stress (negative). The stresses are theoretically uniform, but in practice the stress state can be affected by the friction between the specimen and the testing device, and due to the poor conditions in the head joints. The ratio between the shear and normal stresses ($f_n/\tau = -\tan \theta$) can be changed considering different orientations of the bed joint. Therefore, it is possible to investigate the masonry behaviour for distinct situations. This method has been used to study the general mechanism of failure of masonry, and it represents a good procedure to evaluate the strength of masonry panels in infilled frames [H3, S10, W1].

![Diagram](image)

**Figure 4.5.** Alternative methods of testing the strength in masonry panels.

Fig. 4.5 (b) shows a testing procedure in which the masonry panel is subjected to a diagonal compressive force. In this case, the diagonal force $P_d$ induces a complex biaxial stress state into the panel and cannot be regarded as a realistic shear test. Several researchers [A6, B5, D6] have called this method "tensile splitting test". In the author's opinion, it provides a measure of the resistance of masonry to combine in-plane loading, rather than an indication of the tensile strength. The conditions in this test are similar to those that occur in the masonry panels of infilled frame structures subjected to lateral loads (see Chapter 5). San Bartolomé [S7] compared results from direct shear test, diagonal compressive tests and full-scaled infilled frame tests, and concluded that the former method did not reflect the type of failure observed in real structures whereas the diagonal compressive test was more realistic.

The elastic stress distribution for a square wall diagonally loaded was calculated by Frocht (as reported by Yokel and Fattal [Y1]), assuming isotropic behaviour and Poisson's ratio equal to zero. He found that the principal stresses $f_1$ (tension) and $f_2$ (compression) at the centre of the wall are expressed by the following relationships:

\[
\begin{align*}
f_1 &= 0.7336 \ \tau_d \\
f_2 &= -2.380 \ \tau_d
\end{align*}
\]  

(4.2)
where $\tau_d$ is the nominal shear stress along the diagonal cross area $A_d$ (defined by the diagonal length and the thickness of the specimen):

$$\tau_d = \frac{P_d}{A_d} \quad (4.3)$$

Theoretical results obtained from finite element analysis [Y1] confirmed the validity of Frocht's solution, and indicated that the influence of Poisson's ratio is not important from the practical point of view. The diagonal strength of the panel $f_d$ has been related to the compressive strength $f_m'$ by the formula:

$$f_d = c \sqrt{f_m'} \quad (4.4)$$

where $c$ is an empirical factor with usual values ranging from 0.17 to 0.38 [D6], although test results indicate that there is a great scatter of the data.

It is worth noting that the different methods described here are not completely able to represent the real behaviour of masonry structures. The selection of the testing method depends on several factors, such as available facilities at the laboratory, cost of the tests, number of specimens to be tested, objective of the investigation, types of load to apply, etc. The stress state induced in the masonry panel can be markedly influenced by the geometry of the panel, the support conditions and the procedure of load application. Consequently, these factors must be carefully considered when comparing results from different types of shear test.

### 4.1.4 Shear Strength of Mortar Joints

#### 4.1.4.1 Introduction

Numerous researchers have shown experimentally that the shear strength of mortar joints increases in the presence of normal compressive stresses applied to the masonry. This fact is explained considering that the shear strength results from the combination of two different mechanisms, namely, bond strength and friction resistance between the mortar joints and bricks. Therefore, the shear strength can be expressed as:

$$\tau_m = \tau_o + \mu f_n \quad (4.5)$$

where $\tau_o$ is the initial shear bond strength, $\mu$ is the coefficient of internal friction and $f_n$ is the absolute value of the normal compressive stress in the direction perpendicular to the bed joints. These parameters can be experimentally evaluated using the direct shear test (see Fig. 4.3). Design codes usually adopt conservative values for $\tau_o$ and $\mu$. This criterion remains justified considering the great variation in material properties and the numerous factors which can affect the shear strength.

It is worth noting that Eq. 4.5 represents the shear strength of mortar-brick interfaces. Therefore, it is strictly valid only in those cases in which debonding of the mortar joints occurs. Even though this situation is very common, Eq. 4.5 has been adopted for most of the design codes as a general expression,
independently of the mechanism of failure. This criterion can lead to an unsafe evaluation of the strength of the masonry panel (see section 4.2).

The bond-friction mechanism is generally accepted for explaining the observed behaviour of mortar joints. However, Stafford Smith and Carter [S41] proposed that the shear failure may be due to a tensile failure of the mortar joints and, therefore, the tensile strength of the mortar would be the parameter which controls the shear strength. Their conclusion was based on elastic finite element analyses conducted to investigate the stress distributions of masonry triplets subjected to shear and compressive loading. Posteriorly, Hamid and Drysdale [H4] showed that the concept of relating shear bond strength to the tensile strength of the mortar is not valid, resulting in great differences with experimental results.

4.1.4.2 Response of Bed Joints Subjected to Shear
The shear strength of mortar joints and the mechanism of failure can be investigated using the direct shear test (see Fig. 4.3). This is a simple test procedure which has been widely applied for different researchers to evaluate only the shear parameters, \( \tau_c \) and \( \mu \).

Stöckl and Hofmann [S13] have reported results from monotonic loading tests obtained from the apparatus illustrated in Fig. 4.3 (a). The shear force-relative displacement relationships measured during the tests showed a linear response in the initial stage. After the shear strength was achieved, the specimens were able to maintain large displacements without significant reduction of the shear stress. Fig. 4.6 (a) shows the response obtained from two specimens built with clay brick and different mortars. Similar relationships were measured by Hamid and Drysdale [H4] (see Fig. 4.6 (b)), but in this study, the post-peak response was not investigated and the tests were finished after reaching the maximum shear stress. In both experimental programmes, it was observed that the shear displacements, for equal stress levels, were larger for increasing values of the applied precompression.

Fig. 4.7 presents the cyclic response of specimens tested by Atkinson et al. [A10] using a direct shear apparatus (see section 4.1.3). The measured force-displacement curves indicate a very stiff response up to the peak load. In the post-peak region, the shear resistance decreases to a residual value and remains constant for several cycles. This can be attributed to the reduction of friction due to smoothening of the surface, the lubrication effect of the pulverized particles and the failure of mechanical bond between mortar and brick [H12, P4]. The specimens failed at the mortar-brick interface with no evidence of shearing of the mortar joints. Atkinson et al. [A10] reported that the ratio of the peak to the residual shear strength varied according to the applied normal stress and the thickness of the mortar joint. Results from these tests showed an ample scatter of data. However, it was clearly observed that the difference between the peak and residual stress is equal to the shear bond strength. They also tested three specimens obtained from very old masonry walls damaged during the 1987 Whittier, California, earthquake. These specimens did not show a well defined peak stress as occurred in the other cases (see Fig. 4.7). It is possible that the bond mechanisms had been affected before testing the specimens. The coefficient of friction remained approximately constant during the tests. Based on the experimental results, these researchers proposed a hyperbolic model to represent the nonlinear pre-peak response.
Figure 4.6. Response of mortar joints obtained from direct shear tests ($f'_n$ and $f'_{cb}$ are the compressive strength of the mortar and the brick, respectively).

Figure 4.7. Typical cyclic response for shear test on masonry specimens (adapted from test results by Atkinson et al. [A10]).
The comparison of Figs. 4.6 and 4.7 shows apparently a very different behaviour. However, the results presented in these figures must be analysed carefully. The maximum displacement imposed in the tests conducted by Stöckl and Hoffmann [S13] was about three times larger than the displacement corresponding to the shear strength, whereas in the specimens tested by Atkinson et al. [A10] was about 80 times larger. Furthermore, in the latter tests the cyclic shear reversals were applied with a frequency of 0.01 Hz, which implies a high displacement rate (in the other tests the displacement rate was not reported). It seems that the breakdown of the bond strength along the mortar-brick interfaces occurs gradually, therefore, the phenomena would be quite sensitive to the displacement rate used in the test.

Even though additional experimental research is needed to draw definitive conclusions, it can be considered that the shear resistance of the mortar joints decreases after the bond is broken. Then, the shear stress remains constant, the value depending on the applied normal stress, \( f_m \), and the coefficient of friction, \( \mu \).

The relative displacements measured in different direct tests are not comparable, because different dimensions and arrangements are used in these tests. A pseudo strain could be calculated by dividing the measured displacement by the mortar joint thickness. However, the shear stress distribution and, consequently, the relative displacement also depend on the length of the mortar joints. After the breakdown of bond, the displacement has no significance because the specimen behaves as a pure frictional system.

4.1.4.3 Nature of Bond

The mechanism of bond is incompletely understood even though many investigations have been carried out. Research work conducted by Grandet, Lawrence and Cao, and Binda and Baronio (as reported in reference [H2]) indicated that the bond between mortar and clay bricks is mechanical in nature, but there could be chemical bonding as well. The mechanical bond is due to the formation of ettringite crystals, which are products of the hydration process. These crystals have a minimum diameter of 0.05 mm and grow in the mortar-brick interface. In order to provide a good bond, the crystal must penetrate through the pores of the brick.

An ample range of values of shear bond strengths, \( \tau_o \), has been measured by different researchers. Hendry [H2] presented experimental results obtained from tests using diverse materials and this parameter varied from 0.3 to 0.6 MPa. Paulay and Priestley [P1] and Shrive [S10] indicated that typical values range from 0.1 to 1.5 MPa and from 0.1 to 0.7 MPa respectively. Similar values were reported by Stöckl and Hofmann [S13] for clay and sand-lime masonry units and by Atkinson et al. [A10] for a wide range of materials. For approximate calculation the following relationship may be considered [P1]:

\[
\tau_o = 0.03 f_m^m
\] (4.6)

Other empirical expressions have been proposed to evaluate \( \tau_o \) on the basis of different parameters [N3, R7]. Considering the numerous variables that may affect the shear bond strength, these expressions must be used carefully.
4.1.4.4 Factors Affecting the Bond Strength

Numerous parameters may affect the bond strength between brick and mortar, and their influences and interrelationships are not clearly understood. The most important factors, related to the material characteristics, are:

- **Brick**: porosity, initial rate of water absorption, roughness of the surface, moisture content and chemical reactivity.
- **Mortar**: characteristic of the sand, cement-lime ratio, water retentivity, water content and presence of additives.

There is insufficient quantitative information about the effect of each one of the above mentioned factors, however, some comments are presented here. Rahman and Anand [R7] reported that the compressive strength of the mortar is directly related to the shear bond strength. They observed that an increase in the compressive strength of the bricks was generally accompanied by an increase in the shear strength of masonry. On the contrary, Hamid and Drysdale [H4] concluded that the shear strength was related to neither the strength of the mortar nor the compressive strength of the masonry prisms.

The hydration conditions at the mortar-brick interface are very important. For this reason, those properties related to moisture content and the movement of water between the brick and the mortar affect considerably the bond strength. Sinha [S11] reported that bond strength increases when the moisture content of the bricks also increases, because dry bricks absorb water from the mortar and there may be insufficient water for the hydration of the cement. But bond strength decreases drastically when bricks are saturated. This researcher indicated that the optimum bond strength was associated with a moisture content of the brick approximately equal to 2/3 of the value obtained from the 24 hours water absorption test. The characteristics of the three different types of brick tested were not indicated. Tests carried out in Germany [S13] showed that clay bricks present substantially larger values of the bond strength than sand-lime bricks, considering the same conditions. In this investigation, it was observed no influence of moisture content for sand-lime bricks. Nuss et al. [N3] tested a large series of specimens constructed with clay bricks. Results from multiple regression analysis indicated that the compressive strength of the brick, the initial rate of absorption and the water content of the mortar were the most important parameters, which strongly influenced the shear bond strength.

The presence of lime appears to have a beneficial effect according to an investigation reported by Hendry [H2]. This material reduces the microcracking at the interface and produces more continuous structure of the hydration products. However, there is not information about the most convenient cement-lime ratio for mortars. Sand grading has marked influence on the brick-mortar bond. Experimental work conducted by Sinha [S11] indicated that the sand defined as well graded coarse-medium presented a stronger bond strength. He also investigated the effect of load applied to the specimens during the curing period. According to the test results, it seems that there is no specific relationship between shear bond and the applied compression during curing, and a wide scatter of experimental values was observed.

The effect of cyclic loads was investigated by Pook et al. [P4] using specimens fabricated with three cellular masonry blocks in stacked formation. They found that the failure load, at a given precompression
level, decreased slightly after the initial cycle and then remained almost constant. Tests conducted by Wan and Yi [W1] indicated that shear strength decreases about 10% for cyclic reversal loads. Atkinson et al. [A10] proposed strength values obtained from results of linear regression analysis on direct shear test data. Terčelj et al. [T1] reported results using dynamic shear loads, in which the shear strength of small masonry walls increased up to 12% when compared with the shear strength of the specimen tested under static loads. They also proposed a linear expression, a function of the frequency of the cyclic load, to calculate the shear strength in the range of 1 to 5 Hz.

4.1.4.5 Coefficient of Friction
There is no clear knowledge about the factors that affect the coefficient of internal friction and contradictory results have been reported. Stöckl and Hofmann [S13] concluded that the values of \( \mu \) were not dependent on the type of brick, the mortar strength or the brick condition when laying (wet or dry). However, experimental results obtained by different researchers and presented by Hendry [H2] indicate that \( \mu \) varies according to the type of brick. The measured values of this coefficient range from 0.10 to 1.2 [H2, P1, S1, S13]. Atkinson et al. [A10] surveyed different results of the shear strength reported in the literature and observed that the coefficient of friction ranged from 0.7 to 0.85 for a wide variety of masonry units and materials. Consequently, they recommended \( \mu = 0.7 \) as a reliable lower bound for estimating the coefficient of friction. For design purposes, Paulay and Priestley [P1] recommended a value of \( \mu = 0.3 \).

Experimental results indicate that the coefficient of internal friction may be affected by cyclic loads. The slight decrement of the failure load observed in shear tests [A10, P4] can be attributed to the reduction of friction due to smoothening of the surfaces. Ghazali and Riddington [G1] indicated that the coefficient of friction decreased at higher precompression levels and recommended the use of average values for this parameter. Similar results were obtained by Wan and Yi [W1] who reported that the coefficient of friction decreased by about 5% to 10% when subjected to cyclic loads.

Hamid and Drysdale [H4] investigated the effect of the friction in the shear strength. Specimens previously tested were reloaded after failure using increased levels of precompression. It is clear from these results that the coefficient of friction decreased substantially as the level of precompression was increased. They concluded that the relationship between shear strength and precompression is nonlinear and recommended the adoption of more refined procedures to represent the shear strength of the masonry walls.

4.1.5 Deformational Properties
Tests conducted on couplet specimens made of different types of brick [A10, H12, S13] indicated that unreinforced masonry can sustain small plastic deformation under shear stresses. Atkinson et al. [A10] proposed a model to represent the pre-peak response of masonry bed joints, in which the nominal shear stress and the relative shear displacement are related according to a nonlinear expression. The coefficients of this expression are a function of the compressive stress \( f_c \). It is worth noting that the deformational properties measured from small specimens made of two or three masonry units cannot be considered valid for entire masonry walls because they merely represent the behaviour of bed joints.
Abrams [A5] tested unreinforced masonry walls under cyclic loading and found that the walls had a large deformation capacity after cracking. In these tests, three complete cycles of low ductility were imposed first and then the lateral deflection was increased up to failure. These tests indicated clearly that masonry walls are able to sustain large deformations. However, additional experimental information is necessary to understand completely the response of masonry, especially under cyclic loading and in the nonlinear range.

The shear modulus of masonry, $G_m$, can be calculated from deflection measurements made on masonry panels. It has been observed that the shear modulus depend on the type of brick, mortar class and the moisture content of the bricks when laying [S13]. For approximate calculations, it can be assumed that the masonry behaves as an isotropic material (in terms of deformational properties), thus:

$$G_m = \frac{E_m}{2 (1 + v_m)}$$  \hspace{1cm} (4.7)

This expression indicates that the ratio $G_m/E_m$ varies from 0.40 to 0.45 for the usual values of the Poisson's ratio, ranging from 0.10 to 0.25. Note that the modulus of elasticity, $E_m$, and the Poisson's ratio, $v_m$, depend on the stress level (see section 3.4). Experimental work [D2, H2] confirmed that Eq. 4.7 agrees reasonably with measured values. However, Alcocer and Klingner [A15] reported that the ratio $G_m/E_m$ may vary from 0.10 (for high strength bricks) to 0.30 (for weaker bricks). The shear modulus, $G_m$, was measured from diagonal compressive tests and the modulus of elasticity, $E_m$, was obtained from prism tests.

Different researchers reported contradictory conclusions about the effect of precompression on the shear modulus. Hendry [H2] and Hamid and Drysdale [H3] indicated that this parameter increases appreciably with the precompression, reflecting the nonlinear characteristics of the material, whereas Dhanasekar et al. [D2] found that there is no significant variation of $G_m$ with the normal stress. The discrepancy probably arises from the fact that different ratios $\tau/f_n$ and test conditions were considered in those investigations. Experimental work carried out in Germany [S13] indicated that the precompression increased the shear modulus for sand-lime bricks, whereas it had no influence when clay bricks were tested.

Atkinson et al. [A10] reported that the bed joints subjected to shear loading dilated under increasing shear stresses and then contracted when the load was reduced. They found that dilatancy during shear loading took place only at low normal loads and was not significant at medium and high normal load levels. This effect could produce a variation of the normal stresses affecting the shear strength under certain boundary conditions. Lotfi and Shing [L12], who developed a constitutive model for mortar interfaces, pointed out that the dilatancy may have an important effect on the response of the confined interface.

### 4.2 SHEAR FAILURE THEORIES

#### 4.2.1 General

The increase in the use of masonry as a structural material, especially to resist lateral loads, led a number of researchers to propose failure theories aimed at assessing the shear strength of this material. The usual
failure hypotheses for homogeneous materials cannot be unrestrictedly applied to masonry, because this is a composite material, made up of numerous elements which can fail individually.

In the formulation of the shear failure theories presented in sections 4.2.2, 4.2.3 and 4.2.4, the compressive normal stress, \( f_n \), and the shear stress, \( \tau \), are considered in absolute value in order to maintain the original convention and to simplify the final expressions.

### 4.2.2 Mohr-Coulomb Criterion

The Mohr-Coulomb criterion assumes that yielding, or fracture of a brittle material, occurs when a critical combination of shear and normal stresses act on a given plane. According to this criteria, the critical value of the shear stress for masonry, \( \tau_m \), is a linear function of the normal compressive stress, \( f_n \), defined by Eq. 4.5. It was mentioned previously that this expression represents adequately the shear strength of the mortar joints. Therefore, Eq. 4.5 cannot be considered as a general expression, being valid only for low to medium values of \( f_n \), where a failure by debonding of the mortar-brick interfaces is expected. It is very important to indicate the limit value of \( f_n \) up to which Eq. 4.5 is valid. Typically, this limit ranges from 2 to 4.5 MPa [H2, S1]. Beyond this limit, different mechanisms of failure control the strength of the masonry panels and Eq. 4.5 will overestimate the real strength.

### 4.2.3 Mann and Müller’s Theory

#### 4.2.3.1 General Assumptions

Mann and Müller [M2] developed a failure theory which explains the behaviour of unreinforced masonry subjected to shear and compressive stresses based on equilibrium considerations. They considered a cut-out specimen (see Fig. 4.8) and assumed that:

- The stress acting on the direction parallel to the bed joint, \( f_p \), is small enough to be neglected.
- The shear stress \( \tau \) and the axial compressive stress \( f_n \) are uniform in the masonry panel. Therefore, they represent an average values of the stresses.
- No shear stresses can be transferred by the head joints. This hypothesis may be considered as valid because head joints are often improperly filled with mortar and it is assumed that no perpendicular stresses act on them (\( f_p = 0 \)). Furthermore, the shrinkage of the mortar, especially under poor curing conditions, may affect the bond strength between mortar and brick. Other researchers [W1] have pointed out the same conclusion. This assumption represents the most unfavourable situation.

Shear stresses in the bed joints produce a torque in each individual brick which must be equilibrated by a vertical couple (see Fig. 4.8 (b)). This couple modifies the vertical stress distribution and it is assumed that one brick half is subjected to a stress \( f_{n1} \) and the other half to a smaller stress \( f_{n2} \). Thus:

\[
\begin{align*}
  f_{n1} &= f_n + \frac{2b}{d} \tau \\
  f_{n2} &= f_n - \frac{2b}{d} \tau
\end{align*}
\]  

(4.8)
where \( b \) and \( d \) are the height and the length of the brick, respectively. It is worth noting that, according to the convention used in this section, a negative value of \( f_{a1} \) or \( f_{a2} \) represents a tensile stress. The combined state of stress \((\tau, f_{n1}, f_{n2})\) produces failure in different ways, depending on the relative values of the axial and shear stresses. Three distinct cases are considered in the next sections.

**Figure 4.8.** Stress state in the masonry panel and stress distribution in the brick, according to Mann and Müller's assumptions.

### 4.2.3.2 Shear-Friction Failure

The shear-friction failure in the joint occurs for low levels of axial stress and Eq. 4.5 can be applied for the evaluation of the strength. In this case, the cracks usually begin in those parts of the mortar joints where the axial stress is smaller. Introducing Eq. 4.8 into Eq. 4.5 (with \( f_n = f_{a} \)) the following expression is found to represent the failure condition:

\[
\tau_m = \tau_o^* + \mu^* f_n
\]  

(4.9)

in which \( \tau_o^* \) and \( \mu^* \) are the reduced cohesion and the coefficient of friction, respectively:

\[
\tau_o^* = \frac{\tau_o}{1 + \mu - \frac{2b}{d}}
\]  

(4.10)

\[
\mu^* = \frac{\mu}{1 + \mu - \frac{2b}{d}}
\]
It must be mentioned that Eq. 4.9 represents a lower limit to the real shear strength of masonry because it was derived on the basis that the head joints cannot transfer any shear force.

This theory can accurately explain the observed development of cracks in stepped form, Fig. 4.2 (a), because the brick halves with smallest stresses, \( f_n \), are positioned diagonally opposite one another, as Fig. 4.8 (b) indicates.

### 4.2.3.3 Diagonal Tension Failure

Cracking of the bricks can occur as a result of the combined effect of compressive and shear stresses. It was assumed that the failure occur when the principal tensile stress, \( f_t \), in the brick is equal to the tensile strength \( f_n \). Mann and Müller [M2] reported that "precise testing of an individual brick" indicated that the maximum shear stress, \( \tau_{\text{max}} \) was 2.3 \( \tau \). Considering that the centre of the brick is subjected to a normal compressive stress \( f_n \) (the increment of the normal stresses shall be zero at this point) and a shear stress 2.3 \( \tau \):

\[
f_2 = f_t^b = \frac{f_n}{2} - \sqrt{\left(\frac{f_n}{2}\right)^2 + (2.3 \ \tau)^2}
\]

From the previous equation, it is found that the shear strength is:

\[
\tau_m = \frac{f_t^b}{2.3} \sqrt{1 + \frac{f_n}{f_t^b}}
\]

Eq. 4.12 indicates that the shear strength augments for increasing values of the normal stress, \( f_n \). In this case, the cracks crossing the bricks are approximately perpendicular to the direction of the principal tensile stress, as shown in Fig. 4.2 (b).

### 4.2.3.4 Compressive Failure

This type of failure occurs for very high values of the normal stress, when the greater normal stress, \( f_n \), exceeds the compressive strength of the masonry. From Eq. 4.8 and assuming \( f_n = f_n^t \):

\[
\tau_m = (f_n^t - f_n) \frac{d}{2 b}
\]

### 4.2.3.5 Failure Envelope Curve

Eqs. 4.9, 4.12 and 4.13 represent three different criteria for each type of failure, depending on the value of the normal compressive stress \( f_n \). The failure envelope curve is given for the lowest values, as Fig. 4.9 indicates. A similar shape of the failure envelope was obtained by Lafuente and Genatos [L13], who used finite element analysis to investigate the effect of vertical loads in masonry panels. It is observed in Fig. 4.9 that the Mohr-Coulomb criterion (Eq. 4.5) overpredicts the shear strength of the masonry wall. The differences are very large in the range of medium to high normal stresses.
Figure 4.9. Envelope curve for Mann and Müller's failure theory compared with Mohr-Coulomb criterion.

In spite of the generality of this theory, the expressions presented here were developed for stretcher bond (bricks laid with their long side along the face of the wall) and they are only strictly valid for this type of bond. However, this failure theory can be also applied for walls with cross bond (bricks laid with their long side at right angles to the face of the wall) without significant error [M2].

Dialer [D25] generalized Mann and Müller's theory by considering the effect of the axial stress $f_p$ and different shear strength parameters (bond shear strength and coefficient of friction) in the directions perpendicular and parallel to the mortar joints. This modification leads to a more general failure theory, but the complexity of the formulation increases significantly. Dialer also conducted an experimental investigation and verified the rotation of the bricks resulting from the uneven distribution of vertical stresses (see Fig. 4.8(b)).

4.2.4 Modification of Mann and Müller's Theory

4.2.4.1 General Formulation

The validity of the Mann and Müller's theory depends mainly on the assumed distribution of the normal stresses. Eq. 4.8 was formulated on the basis that the vertical couple is formed by uniform stresses acting on each brick half. It seems that this consideration is rather improbable. Such stress distribution could only occur if the mortar behaves plastically. For this reason, a general distribution of the vertical stresses induced by the couple, $\Delta f_n$, is assumed here (see Fig. 4.10 (a)). Therefore, the shear strength corresponding to the shear-friction failure can be calculated from Eq. 4.5, considering half of the brick where the normal stress is reduced ($f_n$ and $\Delta f_n$ have contrary directions):
\[ \tau_m = \frac{2}{d} \int_0^{d/2} \left[ \tau_o + \mu \left( f_n - \Delta f_n \right) \right] dL \quad (4.14) \]

in which \( dL \) is an increment of length. Assuming that \( \tau_o \) and \( \mu \) are constant (which is valid in the range of low to medium normal stresses):

\[ \tau_m = \tau_o + \mu \left( f_n - \frac{2}{d} \int_0^{d/2} \Delta f_n \, dL \right) \quad (4.15) \]

The second term within the parenthesis in Eq. 4.15 represents an average value of the normal stresses \( \Delta f_n \) acting on each half brick, \( \Delta f_{n,av} \). To maintain the form of the equations presented in the original theory (see Eq. 4.8), \( \Delta f_{n,av} \) is expressed as a function of the nominal shear stress, \( \tau \):

\[ \Delta f_{n,av} = \frac{2}{d} \int_0^{d/2} \Delta f_n \, dL = C_n \frac{b}{d} \tau \quad (4.16) \]

where \( C_n \) is a coefficient representing the variation of the normal stresses \( \Delta f_n \) (in Mann and Müller's theory, \( C_n = 2.0 \)). Therefore, Eq. 4.10 becomes:

\[ \tau_o^* = \frac{\tau_o}{1 + \mu C_n \frac{b}{d}} \quad (4.17) \]

\[ \mu^* = \frac{\mu}{1 + \mu C_n \frac{b}{d}} \]

In a similar way, Eq. 4.13 can be modified to consider the general distribution of \( \Delta f_n \) introducing the average value of these stresses. Therefore, the shear strength in the case of compressive failure is:

\[ \tau_m = (f_m' - f_n) \frac{d}{C_n b} \quad (4.18) \]

Mann and Müller [M2] considered that the maximum shear stress in the brick, \( \tau_{\text{max}} \), is 2.3 times greater than the nominal shear stress, \( \tau \). This assumption is also revised and a coefficient \( C_s \) is introduced to represent, in a general way, the ratio of the maximum to nominal shear stress:

\[ C_s = \frac{\tau_{\text{max}}}{\tau} \quad (4.19) \]
Introducing this modification to calculate the shear strength in the case of diagonal tension failure, Eq. 4.12 transforms to:

\[
\tau_m = \frac{f_{ib}}{C_s} \sqrt{1 + \frac{f_a}{f_{ib}}}
\]

(4.20)

Figure 4.10. General and linear normal stress distributions acting on a brick.

### 4.2.4.2 Evaluation of the Coefficients \(C_a\) and \(C_s\)

The modification of the Mann and Müller's theory presented in the previous section, and expressed in Eqs. 4.17, 4.18 and 4.20, requires the evaluation of the coefficients \(C_a\) and \(C_s\), which take into account the variation of the normal stresses and the effect of the maximum shear stress, respectively. In order to evaluate these coefficients two different approaches were followed. Firstly, \(C_a\) was estimated assuming a linear variation of the normal stresses (see Fig. 4.10 (b)) and \(C_s\) was derived on the basis that the increase of the shear stress is inversely proportional to the reduction of the effective area across vertical section of the panel. Secondly, a parametric study, based on finite element models, was conducted to investigate the stress distribution in masonry panels subjected to pure shear. Both approaches are described in detail in Appendix 3.

The results presented in Appendix 3 indicate that \(C_a = 1.5\) and \(C_s = 2.0\) can be adopted for the practical application of Eqs. 4.17 and 4.20. The proposed values are smaller than those considered in Mann and Müller's theory, resulting in an increase of the shear strength. The shear strength associated with a compressive failure can be evaluated with Eq. 4.18 using a value of the coefficient \(C_a\) between 1.8 and 2.0, which agrees well with the expression originally developed by Mann and Müller [M2] (Eq. 4.13).
It is worth noting that a compressive failure is prone to occur in those cases where the normal stress is significantly higher than the shear stress \( f_n > 8 \tau \), see section 4.1.2.4). This situation, therefore, has practical importance only for masonry structures primarily subjected to compression.

### 4.2.4.3 Total Participation of the Head Joints

The shear theory developed by Mann and Müller [M2] and the modification proposed in this study represent a lower limit to the real strength of masonry panels because the contribution of the head joints is not considered. Even though it is generally accepted that the properties of the head joints are poorer than those of the head joints, a partial transfer of shear stresses may occur through the head joints, especially in those cases where the compressive stress \( f_n \) is not neglected. It is interesting to investigate the contrary situation, in which a total participation of head joints is assumed. In this case, the head joints can resist shear stresses like a homogeneous material and the variation of the normal stress, \( \Delta f_n \), is zero. Therefore, the following modifications must be considered for each type of failure:

- **Shear-friction failure**: Eq. 4.9 becomes equal to Eq. 4.5, thus \( \tau_n^* = \tau_n \) and \( \mu^* = \mu \) (which implies that \( C_s = 0.0 \)).
- **Diagonal tension failure**: Eq. 4.20 is valid considering \( C_s = 1.5 \).
- **Compressive failure**: there is no increment of the normal stress, therefore, the failure occurs when \( f_n = f_n^* \). This condition indicates that the failure is independent of the values of the shear stress, which seems to be unrealistic.

### 4.2.4.4 Influence of the Axial Stress \( f_p \)

It has been assumed in the previous sections that the axial stress \( f_p \), in the direction parallel to the bed joints, is zero. This situation may be rather unrealistic in some cases where the stress state in the masonry panel is more complex. For this reason, the effect of the compressive axial stress \( f_p \) on the shear strength of the panel is discussed here for each mode of failure:

- **Shear-friction failure**: it may be assumed that the stress \( f_p \) has no significant effect for this type of failure. This observation agrees with the failure surface proposed by Ganz and Thürlimann [G9] (see section 4.4.3.3).
- **Diagonal tension failure**: the axial stress \( f_p \) modifies the stress state in the brick. In this case, the principal tensile stress is equal to:

\[
f_2 = -f_{tb}^* = \frac{f_n + f_p}{2} + \sqrt{\left(\frac{f_n + f_p}{2}\right)^2 + (C_s \tau)^2}
\]  \hspace{1cm} (4.21)

From Eq. 4.21, it is found that the shear strength of the masonry panels is:

\[
\tau_m = \frac{f_{tb}^*}{C_s} \sqrt{\frac{1}{1 + \frac{f_n + f_p}{f_{tb}^*} + \frac{f_n f_p}{f_{tb}^*}}} \hspace{1cm} (4.22)
\]
It is worth noting that, according to the convention adopted in section 4.2, the compressive stresses \( f_n \) and \( f_p \) are considered in absolute values. Thus, tensile stresses \( f_p \) must be introduced as a negative value. The comparison of the Eqs. 4.20 and 4.22 indicates that the consideration of the compressive normal stress \( f_p \) increases the strength of the masonry panels.

**Compressive failure:** according to experimental results (see section 4.4.3.3), the compressive strength of masonry loaded in the direction perpendicular to the bed joints is modified by the presence of the axial stress \( f_p \) (biaxial stress state). The strength of masonry increases when \( f_p \) is compressive (for low values of \( f_p/f_m^* \)), whereas tensile stresses produce the contrary effect. Therefore, Eq. 4.18 is valid, assuming a realistic value for the compressive strength, \( f_m^* \), obtained from the biaxial failure envelope.

It must be also considered that the axial stress \( f_p \) may affect the participation of the head joints in resisting shear stresses. When the head joints are compressed, the conditions in these joints improve. Shear stresses can be resisted by friction, even if the bond strength has been previously broken. Contrarily, the participation of head joints can be completely neglected in the case that \( f_p \) induces tension. This effect is taken into account in the proposed model by changing the values of the coefficients \( C_n \) and \( C_s \). Based on the numerical results reported in Appendix 3, the values of these coefficients should be increased from 1.5 to 1.8 and from 2.0 to 2.4, for \( C_n \) and \( C_s \) respectively, when \( f_p \) is tensile (no participation of the head joints is expected). In the case that the head joints are compressed, the values of \( C_n \) and \( C_s \) should be adopted between 0.0 to 1.5 and 1.5 to 2.0, respectively. The considerations presented in this section are valid for low to medium values of the axial stress \( f_p \).

### 4.2.4.5 Comments and Discussion of Results

The reduction of the parameters \( \tau_o \) and \( \mu \), expressed by Eqs. 4.10 and 4.17, reflects the influence of the variation of the normal stresses in some zones of the mortar joints. There is experimental evidence that the shear parameters evaluated from couplet and triplet tests are consistently higher than those obtain from wall tests [R1]. This fact was also observed by Wan and Yi [W1] who proposed an empirical expression to reduce the bond strength, \( \tau_o \), to half of the value calculated from direct shear tests. These researchers explained the decrease of the real bond strength in masonry panels considering that the head joints cannot transfer the stresses \( f_p \) which originates additional shear stresses in the bed joints. Eq. 4.17, considering \( C_n = 1.5 \), indicates that the shear strength parameters of a masonry panel are 65% to 79% of the values measured in direct shear tests, for the usual cases in which \( 0.25 < b/d < 0.5 \) and \( \mu = 0.7 \).

According to Mann and Müller's theory [M2] (see Eq. 4.10) the reduction of the shear strength parameters ranges from 58% to 74%.

Tensile stresses may occur in the mortar joint (\( \Delta f_n > f_n \)), when the masonry panel is subjected to very low normal stresses \( f_n \). Therefore, the failure takes place where either the tensile bond strength between mortar and brick or the tensile strength of the mortar are exceeded. The crack pattern observed in this case is similar to that described in section 4.1.2.2 for stepped debonding of the mortar joints, although Eq. 4.9 is not valid.
When diagonal tension failure controls the behaviour, the proposed modification (Eq. 4.20 with $C_n = 2.0$), leads to shear strengths 15% higher than the original theory (Eq. 4.12). In this case, the inclination of the cracks, $\varphi$ (measured from the direction of the normal stress $f_n$) can be evaluated according to the elasticity theory:

$$\tan 2\varphi = \frac{2 \tau}{f_n - f_p}$$  \hspace{1cm} (4.23)

Eq. 4.23 indicates that the inclination of the cracks decreases (becomes closer to the vertical direction) when the normal stress $f_n$ increases.

Fig. 4.11 illustrates the envelope curve obtained according to the proposed modification (Eqs. 4.17, 4.18 and 4.20, $C_n = 2.0$ and $C_s = 1.5$) in comparison with that assuming total participation of the head joints (see section 4.2.4.3). Common values of the strength parameters are considered in this example, as shown in Fig. 4.11. It is observed that the shear strength calculated on the basis of total participation of the head joints (solid line) is consistently higher. The shear strength corresponding to the shear-friction and compressive failures reduces for increasing values of the ratio b/d. The ratio b/d does not affect the strength for the diagonal tension failure (see Eq. 4.20).

![Figure 4.11](image.png)

**Figure 4.11.** Comparison of envelope curves according to the modification of Mann and Müller's theory and assuming total participation of the head joints.

Mann and Müller [M2] carried out a comparison between experimental data obtained in Darmstadt, Stuttgart and Edinburgh using different materials and theoretical values obtained from their failure theory. This comparison showed a good agreement, however, the analytical values were consistently smaller than the experimental data. These differences are reduced when the modification proposed in section 4.2.4 is taken into account.
Other failure theories have been proposed by different researchers. Ghazali and Riddington [G1] conducted an experimental programme testing brickwork triplets under different stress distributions (this was achieved by introducing bending into the samples). They proposed an analytical model only for low normal stresses and suggested that the mode of failure changes from joint slip to mortar tensile failure as precompression increases. Nevertheless, there is not enough experimental evidence of this type of failure and the diagonal tension failure of the brick is generally accepted.

Page [P3] carried out a parametric study to investigate the influence of masonry strength properties on the behaviour of masonry walls subjected to shear forces. He considered a non-linear finite element model and assumed a combined criterion to represent two different modes of failure. The general failure criterion used in this investigation (see section 4.4.3.3) was obtained from a large number of biaxial tests on solid clay brick panels. Using the finite element models, Page reported failure envelopes similar in shape to those obtained from Mann and Müller's theory, and showed that the shear strength decreased at high normal stress levels.

Wan and Yi [W1] proposed a failure criterion for the diagonal tension failure similar to that expressed in Eq. 4.12, however the factor 2.3 was not considered by these authors. Consequently, the values resulting from this criterion are markedly higher than those obtained from Eq. 4.12.

4.2.5 Masonry Subjected to Shear and Tensile Stresses

Mohr-Coulomb's theory, expressed by Eqs. 4.5 and 4.9, is not applicable for masonry subjected to shear and tensile stresses. Even though this case has little practical importance, some considerations are mentioned here to complete a general failure criterion. Essawy and Drysdale [E1] and Page [P7] have proposed the following expression to represent the shear strength for masonry subjected to tensile normal stresses:

\[
\tau_m = \tau_n \left(1 - \frac{f_n}{f_{to}} \right) \tag{4.24}
\]

where \( f_{to} \) is the pure tensile bond strength between bricks and mortar (see section 4.3) and \( f_n \) is the absolute value of the tensile normal stress. This equation is also plotted in Fig. 4.9, where it can be observed that there is a discontinuity in the failure envelope at \( f_n = 0 \). In order to avoid this problem, the reduced bond strength \( \tau_n^* \) should be considered in Eq. 4.24. Rahman and Anand [R7] considered that \( f_n^* = \tau_m \) in the lack of experimental information, and adopted a similar failure criterion to that represented by Eq. 4.24.

4.3 MASONRY SUBJECTED TO TENSION

4.3.1 Introduction

The resistance of masonry to tensile stresses is an important aspect of the behaviour when flexural effects are significant, for example, in masonry infills subjected to out-of-plane loading. The tensile strength of masonry is primarily controlled by the bond strength developed at the mortar-brick interfaces. Flexural tests conducted by Decanini and Ochat [D14] showed that the strength of the masonry panels was not affected by the compressive strength of the mortar or of the masonry units, whereas the water absorption
of the masonry units had a strong influence. The nature of tensile bond is similar to the shear bond (see section 4.1.4.3), however, the values of tensile bond strength, $f_{to}^*$, are usually smaller than those of the shear bond strength, $\tau_o$. Sinha [S11] investigated the relationship between both parameters and proposed the following empirical expression:

$$\tau_o = 0.80 \ f_{to}^{0.56} \quad \text{(MPa)}$$  \hspace{1cm} (4.25)

valid for $f_{to}^* \leq 0.6 \text{ MPa}$. He mentioned that Eq. 4.25 presented good agreement with experimental results and with other expressions suggested by various researchers. Sinha also found that the tension and shear bond strength were affected in a similar way by distinct parameters. Page [P6] reported that the ratio of the shear to tensile bond strength was 2.3 for solid bricks with 1:1:6 mortar (proportion of cement:lime:sand, by volume).

4.3.2 Modes of Failure and Strength in Direct Tension

Different types of failure can occur according to the direction of the tensile load and the relative magnitude of the bond resistance and the tensile strength of the brick [S9]. Figs. 4.12 (a) and (b) show two typical crack patterns for masonry subjected to tensile stresses parallel to the bed joint. In the first case, cracks occur through brick in alternate courses and the strength is controlled by the tensile strength of the masonry units. In the second case, the cracks do not affect the bricks and only occur along the mortar joints, being the shear bond strength and the overlap length, $L_\text{o}$, the determinant factors. Schubert [S9] observed these types of failure in tensile tests and proposed an analytical method to determine the tensile strength of the masonry in both cases. Drysdale and Hamid [D6] also developed expressions to calculate the tensile strength parallel to the bed joints considering two possible modes of failure.

![Figure 4.12. Modes of failure of masonry subjected to direct tension.](image-url)
The mode of failure, for tensile loads acting perpendicularly to the bed joints, usually occurs by debonding of the mortar-brick interfaces (see Fig. 4.12 (c)). However, the tension failure of the bricks could also occur, as illustrated in Fig. 4.12 (d).

4.3.3 Testing Procedures
Different test techniques have been used to determine the tensile strength of the masonry. The test of masonry panels under direct tension presents some difficulties, for this reason this parameter is often investigated using flexural or splitting tests.

The modulus of rupture, defined in the flexural test, has great practical importance in relation to the resistance of masonry walls subjected to lateral loads. Hendry [H2] summarized experimental results from several researchers and different specimens used in flexural strength tests. These tests reveal that the flexural strength is different for bending in a plane perpendicular to the bed joints than for bending in a plane parallel to this direction, being several times greater in the latter case.

It seems that the first splitting tests were conducted by Johnson and Thompson in the 1960s, using circular disks of masonry. Posteriorly, several researchers have used hexagonal-shaped [B5, D6] or octagonal-shaped [A6] specimens to simplify the procedure. It is worth noting that the splitting test was initially developed to evaluate the tensile strength of concrete cylinders. However, the conditions in masonry specimens are different due to the heterogeneous characteristics of this material and this test provides only an average value of the resistance. The tensile strength obtained from the splitting test, \( f'_t \), is evaluated by the following expression, based on linear elastic theory:

\[
 f'_t = \frac{2}{\pi} \frac{P}{A_t} \quad (4.26)
\]

where \( P \) is the failure load and \( A_t \) is the area of the splitting section.

Test results [A6, D6] indicated that the tensile strength, when the splitting plane is perpendicular to the bed joints, is about twice greater than the strength parallel to this direction. In the former case the cracks cross the masonry units (see Fig. 4.12 (a)), whereas in the latter case the failure occurs by debonding of the mortar-brick interface (see Fig. 4.12 (c)).

4.4 MASONRY SUBJECTED TO BIAXIAL STRESSES
4.4.1 Introduction
The consideration of masonry under a biaxial stress state leads to a more general approach to the study of strength of masonry structures. Most of the masonry walls subjected to in-plane loads, e.g. shear walls, infill walls, etc., are biaxially stressed. Based on this idea, many attempts have been made in the last 20 years, to obtain a general failure criterion. These criteria have been based on experimental results [D4, H6, P5, P6, S33], on combinations of simple failure theories [A16, G9], on finite element modelling [P7], on isotropic material failure theories [Y1], on composite material failure theories [H5, P8, W1], on macroscopic physical interpretations [E1], and on the theory of mixtures [M22, M23, M24, M25].
The finite element method for masonry structures has been often used with different levels of refinement. Simple models consider a homogeneous material law, accounting for the effects of mortar joints in an average sense, whereas in the most refined approaches both the masonry units and the mortar joints are represented separately with continuum elements. Interface elements are also used to consider the potential separation between both materials (see section 6.3.1). Lotfi and Shing [L12] gave a good revision of the different methodologies used for unreinforced masonry. The discussion of these models and their constitutive relations is not considered here, however, some general criteria of failure are presented to explain the overall response of masonry structures.

The experimental investigation of masonry subjected to biaxial stresses requires the use of a more complex equipment. In order to provide a uniform, well defined state of stress, a biaxial test rig should be employed. This test technique has been used by several researchers [D24, D25].

4.4.2 Modes of Failure
Some modes of failure of masonry subjected to a general biaxial stress state have been presented in previous sections as particular cases, such as uniaxial compressive failure (section 3.1), uniaxial tension failure (section 4.3) and different shear failures (section 4.1.2).

The mode of failure in the tension-compression range is similar to that of masonry in direct tension, where the cracks are perpendicular to the direction of the tension stress. However, the strength is reduced in this case due to the action of the compressive stresses which contribute to the formation of the cracks.

In the case of biaxial compression, the failure occurred by splitting in a plane parallel to the free surface of the panel. The presence of the second principal stress prevents the formation of cracks in a plane normal to the panel, as usually occurs under uniaxial compression. Page [P12] reported that the failure occurs suddenly in a brittle manner, regardless of the bed joint direction. The biaxial compressive strength is obviously greater than the uniaxial strength. Experimental results show that the beneficial effect of the biaxial stress state is more significant in increasing the uniaxial compressive strength in the direction parallel to the bed joints. The biaxial compressive strength for the case \( f_1 = f_2 \) is about 30% higher than the uniaxial masonry strength \( f_m \) perpendicular to the bed joints.

The behaviour of masonry subjected to biaxial tension is not clearly understood due to the lack of experimental information. Numerical simulations conducted by Page [P7] indicated that the failure occurred either suddenly along the plane of a bed joint or progressively through a number of bed and head joints in a stepped pattern. Gazzola et al. [G2] proposed a failure criterion for masonry in a tensile biaxial stress state, in which two different cases are considered, namely, stepped failure along the mortar joints and cracking of the masonry units. This criterion was used to evaluate the strength of masonry walls subjected to out-of-plane bending.

4.4.3 Failure Envelope
4.4.3.1 General
Masonry is an anisotropic, two phase material, which exhibits different directional properties due to the influence of the mortar joints, being this effect more significant in terms of strength. Therefore, it is not
possible to define the in-plane failure of this material considering only the principal stresses at any point and a third variable must be introduced. The different failure criteria have been usually expressed either in terms of the principal stresses, $f_1$ and $f_2$, and their orientation referred to the bed joints $\theta$, or in terms of a stress state related to the mortar joints considering the normal and parallel stresses to the bed joints, $f_n$ and $f_p$, and the shear stress, $\tau$.

It is assumed in the discussion of the failure envelopes for masonry that the tensile and compressive stresses are positive and negative, respectively, which is customarily accepted in the literature. Note that this convention is different from that adopted in section 4.2 for shear failure theories.

4.4.3.2 Representation in the Space $f_1$, $f_2$, $\theta$

The representation in terms of principal stress is useful to present experimental results and has been used for numerous researchers [D3, D4, H3, P5, P6, P7, P12, R2, S33, Y1]. The test technique usually employed to evaluate the failure envelope is similar to that shown in Fig. 4.5 (a), although in this case the load is applied in two perpendicular directions, which requires specialised equipment. Square panels are constructed with varying bed joint angles by cutting individual bricks to the shape required to obtain plane edges. Then the panel is loaded with hydraulic actuators in orthogonal directions. The loads are often applied with steel brush platens to minimise the effect of platen restraint and to ensure more uniform distribution of the stresses in the panel.

One of the first attempts to represent the biaxial strength of masonry was developed by Samarasinghe and Hendry [S33], who tested 1/6 reduced scale panels (150 mm x 150 mm and 230 mm x 240 mm) in the compression-tension range considering five different values of the angle $\theta$. They proposed a failure surface derived from a family of hyperbolic curves, which were obtained from the experimental curves. The curve for $\theta = 90^\circ$ is concave, showing a similar shape to the brick failure envelope proposed by Khoo and Hendry [K12], Eq. 2.3. This fact suggests that the testing device would have affected the results (see section 2.3.5).

Page and co-workers conducted a large experimental program to investigate the general behaviour of masonry under biaxial stresses. They tested numerous specimens in the compression-compression and compression-tension range [D4, P6, P12]. In the tension-tension range, only numerical simulations were considered using finite element analysis [P7], due to the sophisticated instruments required to obtain experimental data in this range. The tension-compression region is more important from the practical point of view, particularly for the analysis of infilled frames. Experimental results in this region are presented in Fig. 4.13. It can be observed that the biaxial strength is significantly reduced as the angle $\theta$ decreases, with a minimum at about $\theta = 22.5^\circ$; then the strength increases slightly. This effect has been also observed by Samarasinghe (as reported in reference [H2]), Wan and Yi [W1], and Hamid and Drysdale [H3]. Contrarily, Hegemier et al. [H6] reported that "experimental data reveals a weak dependence on strength on the angle $\theta$" based on tests of concrete masonry panels (ungroouted and grouted). In the author's opinion, there is analytical and experimental evidence that the strength of the masonry is significantly affected by the angle $\theta$. 
The validity of the results presented by Page et al. could be discussed due to the method used to build the masonry panels. According to the research reports [P6, P12], the panels were constructed horizontally on a rigid form and the mortar was poured into the joints and thoroughly compacted. This method was applied with the aim of minimizing the effects of workmanship. However, the characteristics of the mortar joints in the testing panels of this research programme could have been rather different from those in actual masonry walls. Usually, the head joints are not properly filled and compacted, and their capacity to transfer shear forces is smaller than that of the bed joints. It is expected, consequently, that the reduction of the strength with the angle $\theta$ will be greater than that presented in Fig. 4.13. Previous biaxial tests conducted by Page [P5] showed that the uniaxial compressive strength for $\theta = 22.5^\circ$ was about one tenth of the strength for $\theta = 90^\circ$. It must be also noted that the failure envelope in the plane $\theta = 90^\circ$ is rather different from the usual failure envelope in the tension-compression region. In this case, the strength of masonry increased for small values of the compressive stress $f_1$. According to Fig. 4.13 this effect occurred for $\theta > 67.5^\circ$.

The variation of the compressive strength for different loading angles, $\theta$, was investigated by Gallegos [A15, G8], who tested octagonal specimens built with sand-lime and clay bricks. In this investigation, however, only three points of the failure envelope were considered and the envelope could be partially plotted ($\theta$ equal to $0^\circ$, $45^\circ$ and $90^\circ$, and $f_2$ equal to zero). Fig. 4.14 shows the test results for two different types of mortar. It is observed that the compressive strength for $\theta = 90^\circ$ is the same for both types of
mortar, which can be explained considering that the splitting failure that occurred in this case was controlled by the tensile strength of the brick. The shape of the failure envelope shows partial agreement with the results illustrated in Fig. 4.13 in the plane $f_z = 0$.

![Graph](image)

**Figure 4.14.** Experimental results of masonry strength at different loading angles, according to Gallegos [G8].

Bernardini et al. [B5] proposed an anisotropic failure criterion for hollow ceramic masonry. The model is simple enough for numerical computations, but suitable to account for anisotropic effects due to the presence of mortar joints and perforations in the bricks. This criterion was expressed in terms of strength envelopes in the $f_1 - f_2$ plane considering different values of the angle $\theta$.

### 4.4.3.3 Representation in the Space $f_n$, $f_p$, $\tau$

Several of the modes of failure observed in masonry involve some form of joint failure and most of the inelastic deformations develop in the mortar joints. Therefore, it is convenient in some cases to express the failure criteria in terms of stresses related to these jointing planes. This type of representation is generally better suited for modelling masonry using the finite element method. The experimental results, given in terms of the principal stress system $(f_1, f_2, \theta)$, may be transformed to this system using the following relationships [D4]:

\[
\begin{align*}
    f_n & = f_1 \sin^2 \theta + f_2 \cos^2 \theta \\
    f_p & = f_1 \cos^2 \theta + f_2 \sin^2 \theta \\
    \tau & = -(f_1 - f_2) \sin \theta \cos \theta
\end{align*}
\]  

(4.27)  
(4.28)  
(4.29)

where $f_1$ and $f_2$ are positive for tension, and $\theta$ is the angle measured from the stress $f_1$ to the bed joint direction.
Based on large amount of experimental data obtained from biaxial tests, Dhanasekar et al. [D4, D5, D10] derived a general criterion in which the failure surface is represented by three intersecting elliptic cones. The perspective view of this surface is shown in Fig. 4.15.

![Failure surface diagram](image)

**Figure 4.15.** Failure surface in terms of $f_n$, $f_p$, and $\tau$, according to Dhanasekar et al. [D4].

The comparison between the experimental values and the idealized surface is better represented by plotting the strength contours in the $f_n - f_p$ plane. As Fig. 4.16 indicates, there is a good agreement between the series of biaxial tests and the proposed failure surface, especially in the region of practical interest (close to the plane $f_p = 0$). It is worth noting that the contour for $\tau = 0$ presents a similar shape to the contour found by Kupfer et al. [K2] for unconfined concrete. However, there are some differences in this case due to the nonisotropic characteristics of the masonry. The shear contours are not symmetric with respect to the plane $f_n = f_p$ because the uniaxial compressive strengths are different. Furthermore, these contours depend on the values of the shear stress acting along the bed joints (or on the inclination of the mortar joint defined by the angle $\theta$). All tests were carried out on half-scale brick panels using 1:1:6 mortar (proportion of cement:lime:sand, by volume). The mean compressive strengths for bricks and mortar were 15.4 and 5.1 MPa, respectively.

Fig. 4.17 presents the projection of the failure surface on the $f_n - \tau$ plane for different values of the stress parallel to the bed joint, $f_p$. It is observed that the curve for $f_p = 0$ presents a similar shape to the envelope resulting from Mann and Müller's theory (see Fig. 4.9). When masonry is subjected to biaxial compression ($f_n < 0$, $f_p < 0$), the strength increases considerably.
The general equations for each of the three cones are defined by six constants. Therefore, it is necessary to determine a total number of eighteen constants. This procedure is rather complicated for practical purposes, and a simplified method was developed by the Dhanasekar et al. to approximate the failure surface. The simplified failure surface, indicated in Fig. 4.18, can be determined using results from six different tests. The uniaxial compressive strengths in both directions define the points A and B. It is
conservatively assumed that the compressive stress in both directions cannot exceed the value of the maximum compressive strength (segment AE), then the segments AG and GH are drawn parallel to the axes $f_p$ and $f_n$, respectively, being AE equal to AG. Points D and C represent the uniaxial tensile strength. Point J and K are defined on the basis that the segments JJ and IK must be parallel to the coordinate axes and the segment JK parallel to DC. The polygon AGHKIJ represents the contour $\tau = 0$. In order to complete the failure surface two biaxial tests with $\theta = 45^\circ$ are required; points E and F represent tension-compression ($f_1 = -f_2 - f_n = f_p = 0$, $\tau = f_1$) and compression-compression ($f_1 = 4 f_2 - f_n = f_p = 0.625 f_1$, $\tau = 0.375 f_1$) stress state, respectively. Dhanasekar et al. [D3] also proposed a complete description of the biaxial stress-strain relationships for brick masonry, considering anisotropic behaviour.

**Figure 4.18.** Simplified formulation of the failure surface proposed by Dhanasekar et al. [D4].

The criterion developed by Dhanasekar et al. [D4] is essentially phenomenological and it is not directly based on physical considerations. Essawy and Drysdale [E1], on the other hand, proposed a criterion considering only equilibrium and compatibility conditions. A set of sub-criteria was used to account for different modes of failure. However, this criterion is not completely general because it does not consider the case of compressive stresses $f_n$ and $f_p$ with medium to high values.

Ganz and Thürlimann [G9] developed a failure criterion based on five equations to account for different types of failure. These equations depend on four parameters, namely, masonry compressive strength perpendicular and parallel to the bed joints, bond shear strength and angle of friction of the bed joints. This criterion was proposed for the design of masonry walls and is extremely conservative. For example, it is assumed that the shear strength is null, independently of the value of the compressive stress $f_n$, when
f_p = 0. The shape of the failure envelope, however, agrees conceptually with the Mann and Müller's theory [M2] and the modification proposed in section 4.2.4.

Hamid and Drysdale [H5] developed a failure criterion for ungrouted and grouted concrete block masonry. They considered that the failure theories for isotropic materials are not applicable for masonry and derived a different theory based on the mechanics of the composite materials. This theory accounts for three possible modes of failure, namely, shear failure along the bed joints, shear failure along the head joints and tension failure of the blocks. The latter failure was represented considering Hoffman's criterion for brittle anisotropic materials. The resultant equations are very complex, especially for the case of tension failure. In this case the final expression depends on nine constants, which are material parameters determined from strength tests (for the case of in-plane loading the number of constants reduces to six). Even though this criterion considers the anisotropic and composite nature of the masonry and accounts for different modes of failure, the comparison with experimental results shows a poor agreement.

Andeaus [A16] combined the failure criteria proposed by Mohr-Coulomb, Saint Venant and Navier to predict the failure of masonry according to three different modes, namely, splitting of the mortar joints, cracking of the bricks and spalling in the middle plane. The use of this failure theory requires the experimental evaluation of numerous parameters, such as uniaxial compressive and tensile strength, coefficient of friction, shear bond strength, elastic modulus and Poisson's ratio in the direction perpendicular and parallel to the bed joints.

In 1992, Pietruszczak and Niu [P8] presented an alternative approach which consider the masonry as a composite medium intercepted by sets of head and bed joints. The head joints are treated as aligned, uniformly dispersed weak inclusions, whereas the bed joints represent continuous planes of weakness. The numerical results obtained from this theory follow the qualitative trends indicated by experimental data. However, these authors have not provided any quantitative comparison to assure the validity of their criterion.

4.5 CONCLUSIONS
• The behaviour of masonry in shear, or in a general sense under biaxial stress state, is very complex due to the distinct mechanical properties of the constitutive materials. As a result, the common failure theories for homogeneous materials are not valid for masonry.

• Different types of failure can occur depending on the properties of the materials and on the stress state. Three basic modes of failure can be distinguished, namely, shear failure along mortar joints, diagonal tension failure and compressive failure. In other cases, the final failure is a combination of these basic modes.

• Numerous testing procedures are used to evaluate the shear strength of masonry. This situation leads to experimental results which usually are not comparable due to different stress distributions and boundary conditions. Consequently, it is necessary to define standard procedures, which represent adequately the actual situation of masonry walls.
The shear strength is usually measured from direct shear tests, using small specimens formed by a few masonry units. Even though this is a simple and inexpensive procedure, it is not representative of the real strength of masonry walls. The results from these tests represent just the behaviour of the mortar joints under direct shear.

The testing method in which the masonry panel is loaded in compression for different inclination of the bed joints seems to be more realistic method to investigate the strength of masonry in infilled frames.

The Mohr-Coulomb criterion, widely used for the design of masonry structures, overestimates the shear strength of masonry panels. The differences can be very large in the range of medium to high normal compressive stresses.

The shear failure theory proposed by Mann and Müller is a rational approach based on equilibrium conditions, which allows the representation of the observed modes of failure and to evaluate the shear strength of masonry walls. The material parameters required to determine the enveloping curve can be easily evaluated from laboratory tests.

Mann and Müller's theory was modified in this study to take into account a more realistic distribution of the normal and shear stresses acting on the brick. In developing this modification, simplicity has been prioritized instead of generality in order to apply posteriorly this theory to the evaluation of the strength of masonry panels in infilled frames. Numerical results obtained from finite element analysis agree reasonably well with the hypothesis assumed in this modification.
5. BEHAVIOUR OF INFILLED FRAMES

5.1 INTRODUCTION
This chapter describes the observed behaviour of infilled frame structures subjected to lateral forces, based on experimental results obtained from previous research. An extensive revision of the literature is conducted, under the conviction that a clear understanding of the structural behaviour is required in order to develop rational theoretical models and adequate design procedures.

Emphasis is given to reinforced concrete frames with masonry infills, although some test results related to infilled steel frames are also included, especially when the frame material does not significantly affect the topic under consideration.

The terminology related to infilled frames is discussed in section 1.3.

5.2 INFILLED FRAMES AS STRUCTURAL SYSTEM
It has been generally recognized that infilled frame structures exhibit poor seismic performance, since numerous buildings have failed in past earthquakes. However, experimental observations, analytical studies and the performance of infilled frames in real earthquakes also indicate that masonry infills may produce some beneficial effects on the response of the building. These contradictory conclusions indicate that masonry infilled frames exhibit a poor or good performance depending on how the masonry is used in the earthquake-resistant structure [T4].

Unreinforced masonry walls present a completely brittle behaviour with low resistance to seismic action. Failure occurs due to flexural or shear cracks at the base of the wall. Thus, the combination with the frame improves the behaviour of the unreinforced wall and the strength of the resultant structure is usually greater than the sum of the two components separately [M5, S21]. Due to the composite action developed between panel and frame, the ductility of the infilled frame is larger than that of the unreinforced masonry wall structure. Furthermore, the in-plane bracing action of the masonry panel increases the stiffness, which reduces the lateral deformation when compared with that of the bare frame. The dynamic behaviour may also improve because the system is able to dissipate energy through friction and slip at the structural interfaces. Nonlinear dynamic analyses conducted by Fardis et al. [F7] indicated that, in the absence of irregularities, the effect of the masonry infills is generally beneficial for the structure.

The main disadvantage of infilled frames is the degradation of stiffness, strength and energy dissipation capacity observed under cyclic loading. This generalized degradation of the most important parameters of the system is due to the progressive damage of the panel as well as the frame, and the deterioration of
the interface conditions. Furthermore, only low to medium displacement ductilities can be achieved due to the fragile behaviour of the masonry panel. In spite of these disadvantages, the overall response of infilled frames is definitively better than that of unreinforced masonry walls [M13].

Masonry materials present an ample dispersion in their characteristic values and the compressive and shear strengths of the wall are markedly affected by the workmanship. The conditions in the construction site are different from those at the laboratory, therefore, a strict quality control during the construction is required to assure the minimum strength requirements. Furthermore, the strength of the masonry panels is also reduced by openings and perforations for service ducts. All these facts contributed to consider the infill panels as a non-structural elements.

Two different philosophies have been used for seismic design of the infilled frames. The first philosophy is to isolate structurally the masonry panel from the frame, considering that its effect can be neglected. The masonry infill is constructed as a secondary element separated from the frame, with a gap big enough to allow the free deformation of the frame during an earthquake. The second philosophy assumes that the infill panel is in contact with the frame and considers all the effects that this interaction produces [P1, Z1]. It seems that the latter alternative offers more conceptual and practical advantages. The effective isolation of the panel is difficult to achieve in practice. It is also necessary to provide support against out-of-plane forces, and thermal and sound insulation. Furthermore, considering the basic concepts for seismic design "avoid unnecessary masses" and "if a mass is necessary, uses it structurally to resist seismic effects", attempts should be made to consider these infill panels as structural elements [B6].

Unfavourable consequences may occur when the influence of infill panel is neglected. Masonry panels are very stiff, even if the thickness is small, and they can alter drastically the expected response of the structure. These possible consequences are [B10, M16, P1]:

- Modification of the global response of the structure due to the decrement of the natural period. Thus, the seismic forces will be modified.
- Unexpected failure of reinforced concrete members, as a result of the increase of shear forces produced by partial infilled of the frame (for example, "short column").
- Alteration of the torsional response of the building when the masonry panels are asymmetrical distributed in the building.
- Formation of soft-storey sway mechanism due to non-uniform distribution of the masonry walls along the height of the frame.

Seismic behaviour of infilled frames has been unsatisfactory during several earthquakes [K9, P1] and this poor behaviour may be attributed to one or more of the problems mentioned above. However, these are not intrinsic problems of infilled frames and can be avoided with a careful design and distribution of the masonry panels. Therefore, it is very important to emphasize that masonry panels cannot be considered as non-structural partitions. The presence of masonry panels significantly changes the structural characteristics of the building and creates new potential failure mechanisms. Thus, their influence should be always considered in the evaluation of the structural stiffness, the torsional response and the possible
formation of local failure mechanisms. However, the contributions of each panel to the resistance of the entire structure should be carefully checked, considering all the factors which can affect the strength.

The adequate comprehension of the behaviour of infilled frames is a vital step either in designing seismic resistant structures or in repairing and retrofitting existent buildings. Infilled frames are largely used for low and medium-height buildings all over the world in regions of high seismicity, especially in developing countries where the labour costs are not very high or where masonry structures are used for traditional reasons. Thus, adequate knowledge of the seismic response is very important to reduce the loss of life and property damage, associated with the failure of infilled frames.

Infilled frames are composite structures and their behaviour is complex and markedly nonlinear. Such a behaviour is a consequence of the brittle behaviour of the masonry panel, the ductile nonlinear characteristics of the frame, the different deformational properties and strengths of both components, and the variable conditions at the panel-frame interface. The presence of the infill panel constrains the deformation of the frame, then the stiffness and the strength of the combination are usually greater than the sum of its components taken individually [K9].

The nonlinear finite element program ABAQUS [A12] was used in this work to model infilled frames. Numerous results and observations obtained from these analyses are included in this chapter. The objective is to complement the experimental information available about the behaviour of infilled frames. The characteristics of the finite element models used in the numerical simulations are presented in section 8.4.

Different types of force can act over the infilled frames when they form part of a building, as indicated in Fig. 5.1. Gravity loads produce compressive stresses in the vertical direction, whereas lateral forces (induced by wind or earthquakes) induce shear and flexural effects. These actions are usually grouped in out-of-plane and in-plane forces.

![Figure 5.1. Actions in the infilled frame.](image)
In this study, the effect of out-of-plane forces will not be analysed, although its consideration is very important during the design of infilled frames to ensure the stability of the whole structure and to avoid losing of the panel from the surrounding frame. The interaction between in-plane and out-of-plane response of infilled frame structures is very complex and the numerical modelling of this behaviour requires the use of sophisticated constitutive relations and elements [B10]. Vertical loads rarely produce the failure of the infilled frame because the masonry sections are usually big and this material is able to resist compressive stresses. Therefore, special attention is dedicated to the study of infilled frames under lateral in-plane forces induced by earthquakes.

5.3 EXPERIMENTAL RESEARCH

5.3.1 General Considerations

Infilled frame structures were, and still are, a common form of construction. However, their behaviour is one of the least understood and investigated among all the major construction systems. For a long time, these structures were built with a general lack of conclusive research and design information. The rational study of infilled frames started in the middle years of this century. One of the first experimental works was conducted by Benjamin and Williams [B9] who tested steel and reinforced concrete frames infilled with brick masonry panels, and compared the response of these structures with that of the unframed walls. Whitney et al. [W5] also investigated the strength of the infilled frames related to the design of blast-resistant structures. In the UK, the experimental work conducted by Thomas (as reported by Liao [L11]) showed the stiffening effect of the brick infills, and it was the beginning of a useful investigation continued by Wood [W4], Holmes [H7, H9], Stafford Smith [S15, S16, S19, S22] and Mallick and Severn [M6, M10]. They tested numerous reduced scale models using steel frames infilled with concrete. The behaviour of reinforced concrete frames with masonry panels was also study in other countries. It is worth mentioning the work conducted by Polyakov [P9] in USSR, Sachanski [S28] in Bulgaria, and Esteva and Meli [E3, M13] in Mexico. Most of this research was oriented to develop practical methods for estimating the stiffness and strength of infilled frames. Many empirical and semi-empirical expressions were proposed by different authors which generated polemic discussions among the researchers [S20]. Both, during and after the 70s, numerous researchers from different countries continued the investigation of infilled frames.

Different testing arrangements have been used in the experimental investigation of framed masonry and infilled frames. Initially, square or rectangular panels were tested under diagonal forces [H7, H9, P9, S15, S16, S20, W4], as indicated in Fig. 5.2 (a). This simple method is not very representative of the real behaviour of infilled frames under lateral forces. The boundary conditions are different and the flexural effect, which induces axial forces in the columns of the frame, is not considered. However, it was a first approximation and some interesting conclusion could be obtained from these tests.

A more realistic model was used by Stafford Smith [S16] and Mallick and Severn [M10]. In this arrangement, shown in Fig. 5.2 (b) and (d), the central line can be considered as a fixed boundary. Thus, these models are able to represent the behaviour of cantilever walls of one or two storeys under a shear force V. Finally, the cantilever test is the most common arrangement and represents the best solution (see
Fig. 5.2 (c)). In this case, the boundary conditions are more realistic and the effect of gravity loads can be also included.

![Testing arrangements used in the experimental investigation of infilled frames.](image)

**Figure 5.2.** Testing arrangements used in the experimental investigation of infilled frames.

### 5.3.2 Experimental Simulation of the Seismic Response of Infilled Frame Structures

The simulation of seismic forces in laboratory specimens is usually achieved by three different techniques: quasistatic, dynamic or pseudodynamic tests (a description of these techniques can be found in reference [C12]). The first technique is adequate in those tests aimed at improving the understanding of the structural behaviour and developing mathematical models, whereas the second technique is often conceived for verifying the response implying higher complexity and cost of the experiment. The third technique represents an alternative solution, which combines characteristics of the other two. The advantages and disadvantages of each testing technique should be carefully analysed when designing a test programme or interpreting experimental results.

The most important problem related to the pseudodynamic and dynamic tests is the selection of one or more adequate ground motions to be used in the experiments. The selected accelerograms should consider the seismicity and the local soil conditions of the region. However, it is clear that no single accelerogram can represent a general class of expected ground motions [C12]. Another alternative is the use of artificially generated accelerograms, whose response spectrum is similar to that specified by seismic codes for design. Calvi and Kingsley [C12] pointed out that smooth design spectra define a curve of equal probability of exceedance rather than representing a single real event. In order to approximate
this type of response spectra, a large, unrealistic number of high frequency components must be considered in the generation of the artificial ground motion.

In quasistatic or pseudodynamic tests, the lateral forces are applied to the specimen with actuators, as a concentrated force, either at one or both ends of the top beam. However, in real structures subjected to seismic motions, part of the inertia forces induced in the building may be transferred to the infilled frame along the beams connected with the floor slabs. In the case of multistorey infilled frames, the shear forces of the upper storeys reach the foundation following a truss mechanism. Therefore, the beams are subjected to large tensile forces, independent of the direction of the lateral forces. This situation is rarely considered in laboratory tests, as discussed in section 9.5.2. Quasistatic tests of multistorey structures present the problem that the distribution of the lateral forces is adopted a priori, usually based on code specification or numerical analyses, and the effect of the higher modes of vibration cannot be considered.

Paulson and Abrams [P21] compared the response of two identical reinforced masonry structures subjected to dynamic and quasistatic tests. The first specimen was subjected to a simulated ground motion on a shaking table, whereas the second specimen was quasistatically tested by imposing the displacement history measured in the first test. The comparison of the results indicates that the rate of strain clearly affects the crack propagation in the masonry. In the quasistatic test, it was common to observe that cracks grew over the course of several minutes. This slow dispersion of the cracks could not have occurred in a dynamic test. The stiffness and strength of the specimens were also affected, being both parameters usually smaller in the response measured during the quasistatic test. These researchers concluded that quasistatic tests conducted in the laboratory would represent a more demanding condition than an actual seismic motion. Even though these results were obtained from reinforced masonry specimens, they could be also valid for infilled frame structures, in which the masonry panels strongly affect the response of the system.

5.4 BEHAVIOUR OF SINGLE INFILLED FRAMES UNDER MONOTONIC LATERAL LOADING

5.4.1 Introduction
Before analysing in detail the behaviour of infilled frame structures, it must be noted that the number of influential parameters is rather large. Numerous combinations can be considered by changing the materials of the masonry panel and the frame, the constructive techniques used to build the structure and the interface conditions. Despite the numerous experimental results now available, it is very rare to find in the literature comparable results from two or more distinct sources to draw definitive conclusions about the behaviour of infilled frames and the role of the different parameters. Furthermore, the cost of the laboratory tests limits the repetition of identical specimens to avoid the scatter of data. Reduced scale specimens have been used for investigation of infilled frames. However, the results obtained from these tests should be considered carefully because masonry materials are very sensitive to scale effects.

The general behaviour of masonry infilled frames subjected to lateral forces is presented in the following sections grouped in two cases. First, the structural response under monotonic loading is discussed for integral and non-integral infilled frames. Particular characteristics of multistorey and multibay structures
are also included. Secondly, the behaviour of infilled frames subjected to cyclic and dynamic lateral forces is considered.

5.4.2 Response of Framed Masonry and Integral Infilled Frames

The behaviour of infilled frames in the initial stage is almost elastic and largely controlled by the characteristic of the masonry panel. The structure behaves as a monolithic element due to the bond strength developed along the structural interfaces [E3, M13]. In this stage, the columns act as tension or compression boundary members and the infill panel acts as a connecting shear element. It may be approximately considered that the system is similar to a cantilever wall. Finite element analysis indicates that stress concentration occurs at all the four corners, whereas an approximately pure shear stress state develops at the central region of the panel. As an example of this effect, Fig. 5.3 illustrates the distribution of the principal stresses in the masonry panel, obtained from a typical infilled frame subjected to lateral forces. The length of the vectors is proportional to the magnitude of the stress, whereas the direction of the arrows indicates whether the stress is tension or compression. In this model, the foundation beam is assumed to be fixed and no vertical loads are considered in the analysis.

![Figure 5.3. Stress distribution in a masonry infill before separation occurs.](image)

As the lateral force increases, some parts of the panel-frame interface crack due to the incompatible displacement resulting from different deformational characteristics. Thus, the masonry panel separates from the surrounding frame, except at the diagonally opposite compression corners, as indicated in Fig. 5.4. The force level at which separation occurs mainly depends on the conditions of the panel-frame interfaces. Leuchars and Scrivener [L5, L14] indicated that this force can be up to 50% of the ultimate force when the frame is built with reinforced concrete. Higher forces could be expected in framed masonry structures, where better bond conditions develop at the interfaces. The boundary separation does not significantly affect the resistance of the structure and only decreases its stiffness [K9]. There is not enough information to predict when the separation will occur and in numerous cases the researchers did not report the values to define this point. Esteva [E3] tested clay brick and concrete block masonry framed with reinforced concrete members and found that the separation occurred for a storey drift ranging between 0.01% and 0.3%.
After the separation occurs, the stresses at the tensile corners are relieved while those near the compressive corners are significantly increased, as observed in Fig. 5.5. The masonry panel is mainly subjected to compressive stresses, $f_1$, along the loaded diagonal. In this stage, the principal stress perpendicular to the compressed diagonal, $f_2$, is compressive at the loaded corners and tensile at the centre of the panel. Therefore, the loaded corners are subjected to biaxial compression, whereas the centre of the panel is under a tension-compression stress state. Finite element analyses conducted by the author indicated that the principal stress $f_1$ is relatively low. Results obtained from different models, in which the dimensions and the mechanical properties were changed, showed that the ratio of the principal stresses, $f_1/f_2$, varied from 2 to 4, in the loaded corners, and from -7 to -10, in the central zone of the panel.

The masonry panel reacts against each side of the frame over a reduced length extending from the loaded corners. As a result, the composite structure behaves approximately as a braced frame. This behaviour suggested to early researchers that the effect of the panel could be approximately represented by a compressive diagonal strut, as shown in Fig. 5.6. When the direction of the lateral force is reversed, the compressive strut will develop along the direction of the other diagonal. This idea was firstly considered.
by Polyakov in 1958 (as reported by Mallick and Severn [M10]) and later took up by Holmes [H7] who proposed that the diagonal strut should have a width equal to one third of the diagonal length of the panel and the same thickness. Stafford Smith [S15, S16] and many other researchers improved this method posteriorly.

![Diagram](image)

**Figure 5.6.** Equivalent truss mechanisms for infilled frames.

Along the contact length, shear and compressive stresses are generated. Polyakov assumed a parabolic distribution of the normal stresses, whereas Stafford Smith considered a linear variation (as reported by Leuchars [L14]). Fig. 5.7 shows the nonlinear normal stress distribution reported by Leuchars [L14], which can vary depending on the force level. The shear stress due to friction between the masonry panel and the frame is proportional to the normal stress, therefore it may be considered that both stress distributions are similar. The length of the interface subjected to compressive normal stresses depends on the relative stiffness of the masonry panel and the frame. In the elastic stage, the normal stresses are highly concentrated at the loaded corners. As the lateral force increases, the normal stresses redistribute, therefore the line of action of the resultant of the interface normal stresses shifts away from the compressive corners. This causes a significant increase in the bending moments acting on the frame [K4].

![Diagram](image)

**Figure 5.7.** Normal and shear stresses acting on a loaded corner.
Due to the composite interaction, explained with the truss mechanism, one of the columns is subjected to tensile axial efforts. The axial force in the beams depends on the loading system used for the application of the lateral force, as discussed in section 9.5.2. Shear forces and bending moments also appear in the members of the frame. The bending moments in the frame members are drastically reduced in comparison with those corresponding to the bare frame for the same load level \([S19]\) because the lateral force is predominately transferred to the foundation by the truss mechanism.

Fig. 5.8 (a) presents the typical bending moment, shear and axial forces diagrams for the members of the frame after separation occurs. In this case, the lateral force was applied along the top beam and no vertical load was considered. The efforts in the foundation beam are not included in this figure. The shape of these diagrams agrees very well with those measured by Dhanasekar et al. \([D5]\) from tests on infilled frames. In the example illustrated in Fig. 5.8 (a), the maximum bending moment (at the bottom of the right column) was about six times smaller than the moment which would occur in the bare frame. It is important to mention that the bending moment and shear forces acting on the frame cannot be properly predicted with the equivalent strut model (see section 8.2.2).

(a) Bending moment, shear and axial force diagrams for a typical infilled frame.

(b) Bending moment diagrams for flexible and rigid frame

Figure 5.8. Typical bending moment, shear and axial forces diagrams obtained from an infilled frame, after separation occurs.
The bending moments, and consequently the shear force, are affected by the relative stiffness between the frame and the masonry panels. Fig. 5.8 (b) shows the bending moment diagrams obtained from two different situations, in which the stiffness of the frame was reduced and increased, respectively. It is observed that the maximum bending moments in the columns move from the zone near to the loaded corner (flexible frames) to the opposite end of the column (rigid frames). This fact is very important to analyse the potential location of plastic hinges in the frame. The maximum bending moments of the flexible frame were much smaller than those of the rigid frame. Boundary conditions and characteristics of the foundation beam also affect the bending moment distribution. In the examples presented here, it was assumed that the beam foundation was fixed. The relative stiffness between the frame and the masonry panels, evaluated according to the parameter $\lambda_h$ (see section 5.8.2), was $\lambda_h = 4.7$ in the case presented in Fig. 5.8 (a). In the examples shown in Fig. 5.8 (b) this parameter was 17.4 and 3.2 for flexible and rigid frame, respectively.

As the lateral force increases, further separation of the masonry panel and frame occurs, with contact finally restricted to regions adjacent to loaded corners. Cracking of the masonry panel is observed following different patterns, which causes a significant decrease in the stiffness until the maximum shear resistance is attained. The effect of the masonry panels, even if they are cracked, can be represented by diagonal struts [G8]. Dawe and Hatzinikolas [D17] observed that the formation of major cracks in the masonry panel may produce a sawtooth effect in the force-displacement relationship. In this stage, the masonry panel exhibits damage and plastic hinges usually develop in the frame due to the increase of the member bending moments. The final behaviour is mainly controlled by the frame, which restrains the cracked infill panel.

The characteristics of the falling branch of the force-displacement relationship, which occurs after the lateral resistance has been achieved, depend on the mode of failure of the infilled frame. Experimental results indicate that the falling branch is very smooth in those cases where the masonry panel is weak when compared with the surrounding frame, and a ductile mechanism develops in the frame [B10]. The falling branch is steeper either when the frame is filled with a strong panel [S26], which leads to a premature failure of the frame members, or when the masonry panel degrades very rapidly. Therefore, when the infilled frame is adequately designed, the structural system is able to sustain low to medium plastic deformations with limited strength decay. Different types of failure that are usually observed in infilled frames are described in section 5.7.

In conclusion, four different stages can be distinguished. During the initial stage the structure behaves as a monolithic cantilever wall until separation occurs. Then the behaviour is characterized by the composite interaction between the panel and the frame, although the materials remain mainly uncracked. The induced state of stress into the panel produces different cracking patterns, with significant damage until the maximum lateral resistance is achieved. Finally, the lateral strength decreases and the response is mainly controlled by the frame. Fig. 5.9, line A, shows the typical force-displacement relationship for an integral infilled frame under monotonic loading. It is observed that the response can be approximately represented by a trilinear curve [D13, Z4].
Figure 5.9. Force-displacement relationship for integral and non-integral infilled frames tested by Liauw and Kwan [L1].

Klingner and Bertero [K9] reported that walls bounded with very weak frames were seen to fail suddenly at very low lateral forces by shear and tension across their base, in a mode of failure similar to that observed for unframed masonry walls [S29]. This brittle behaviour is different from that described here, and should be completely avoided with an adequate design to obtain a more ductile response of the structure.

It must be noted that framed masonry and integral infilled frames with shear connectors are not completely equivalent. In the former case, the interface conditions depend on the bond strength and friction between both surfaces. When separation occurs, the bond strength is broken and only the friction effect remains in the zones where panel and frame are in contact. In infilled frames with shear connectors, the mechanism of interaction is different and shear forces can be transferred even if there is a gap between the panel and the frame. However, this mechanism induces high concentration of stresses, which produce local damage in the zones of the panel in contact with the connectors. Then, the effectiveness of the system to transfer shear forces is significantly reduced under increasing lateral forces, especially for cyclic or dynamic loading.

5.4.3 Response of Non-Integral Infilled Frames
The behaviour of non-integral infilled frames is, in a general sense, similar to that of the integral infilled frames presented in the previous section. However, some differences can be pointed out, especially at the initial stage. Tests conducted by different researchers indicated that the initial stiffness was unpredictable and erratic [S22], and the separation occurred almost immediately after the specimens were loaded [L11] or for very low values of the storey drift (0.003% to 0.007% according to Polyakov's results [B10]). It was observed in some tests [D20, L1] that the stiffness increased after the force was applied, as shown in Fig. 5.9, line B. Such a behaviour was explained considering the initial lack of fit due to
unintentional gaps between the frame and the masonry panel. This effect was not observed by Mallick and Severn [M10] who tested similar structures, but using an expansible material for the infill panel, to avoid the formation of unintentional gaps.

After separation occurs, the infill panel shows a tendency to rotate inside the frame and the length of contact between panel and frame is generally smaller than that observed in framed masonry or integral infilled frames. The conditions in the interface between the panel and the top beam are expected to be more unfavourable than in the other interfaces. When no special constructive techniques are applied (for example, filling the interface with no-shrink mortar) the strength of the interface is reduced. Eventually, a small unintentional gap may form in this interface. This circumstance is discussed in section 5.8.8.

Cracking of the panel usually occurs along the diagonal with a narrow spread and concentrated in a few major cracks. The formation of the crack is accompanied by an audible sound and a sudden drop in the force. Beyond this point, loading resumes with no significant decrease in the stiffness until the maximum force is achieved [D7]. After the peak force, the structure continues to sustain loading, however, with significant strength softening. The system can achieve an ultimate displacement many times greater than the displacement at the maximum force level.

The typical response of an infilled frame tested by Dawe and Seah [D7] is illustrated in Fig. 5.10. It is clearly observed the initial uncracked stage (line O-A), the posterior behaviour with a slight decrease in the stiffness until reaches the peak force (line A'-B) and the final stage which leads to the collapse of the structural system (line B-C). The force reduction (line A-A') observed in the experimental response is consequence of the relocation of the panel after the first crack occurs; this effect is more significant in non-integral infilled frames, where there is no perfect fit between the frame and the panel. The behaviour of the non-integral infilled frames until failure (line O-A-A'-B) can be approximately assumed as linear [K9].

![Graph](image)

**Figure 5.10.** Force-displacement relationship for infilled and bare frames, according to tests conducted by Dawe and Seah [D7].
The experimental force-displacement relationship of the bare frame is also included in Fig. 5.10 to compare the behaviour of both systems. A similar response was obtained by Leuchars and Scrivener [L5, L14] from two reinforced concrete frames tested with and without masonry infill. Fig. 5.9, line B, presents also the experimental response obtained by Liauw and Kwan [L1] from a non-integral infilled frame. In this case, after the peak force was reached, the load capacity of the model dropped rapidly, however, when the deflection increased the panel gained firm contact with the columns enabling the structure to regain part of the strength.

5.4.4 Behaviour of Infilled Frames with Horizontal Shear Cracking

In the previous sections, the general behaviour of infilled frames was discussed independently of the different modes of failure. However, a rather different behaviour is observed when the failure of the masonry panel occurs due to horizontal shear sliding (this type of failure is explained in detail in section 5.7.2.2). Fig. 5.11 illustrates the deformed shape of the structure after the formation of the horizontal crack, located approximately at the middle height of the panel, due to the breakdown of the bond strength of the bed joints. This figure was obtained using a finite element model, in which the horizontal crack was simulated with interface elements (see section 8.4).

![Deformed shape of an infilled frame with horizontal shear cracking.](image)

Analytical results indicate that the interaction between the surrounding frame and the infill panel is more complex. The masonry panel, divided into two parts for the horizontal crack, is also in contact with columns at the span length. Consequently, the mechanism for transferring the lateral forces is different. The flow of the principal stresses (see Fig. 5.12) shows that the horizontal force is not only transferred by compressive stresses along the panel diagonal, but also by additional compressive fields, which develop between the loaded corners and the opposite columns. Consequently, the behaviour of the infilled frame can be approximately represented by a multi-braced frame, as suggested in Fig. 5.13. However, the truss mechanism is not completely clear in this case, due to the friction forces that develop along the horizontal crack.
Figure 5.12. Principal stresses in a masonry panel after the formation of a horizontal shear crack.

Figure 5.13. Approximate mechanism for describing the response of the infilled frame subsequent to the horizontal cracking.

The actions induced in the surrounding frame are modified after the formation of the horizontal crack, as the comparison of Figs. 5.8 (a) and 5.14 indicates. The variation of the shear and axial forces of the columns at middle height reflect the frame-panel interaction that occurs in those zones. The variation of the shear forces in the columns is much more significant than the variation of the axial forces, which indicates that the resultant force introduced at the span length of the columns is practically horizontal. The flexural demand in the columns at middle height is significantly increased and plastic hinges can develop in these regions (see the bending moment diagram plotted in Fig. 5.14).

5.4.5 Discussion on Other Experimental Investigations
It is worth noting that the differences in loading methods and boundary conditions used in some studies, led to conclusions about the behaviour of infilled frames and which require a careful interpretation. Smolira [S21] used qualitative models constructed with very flexible frames and bricks free of mortar or any other adhesive. Based on these tests, he explained the behaviour of infilled frames at high lateral
forces assuming that a double arching effect occurred in the panel (see Fig. 5.15 (a)). The testing models were able to sustain the lateral force even when the bricks at the centre of the panel were removed. Similar results were reported by Mainstone [M5] and Stafford Smith et al. [S20], who tested diagonal loaded panels and proposed that the effect of the infill panel at high lateral forces could be simulated by replacing the wall by a linkage of two pinned diagonals, as indicated in Fig. 5.15 (b).

![Diagram of forces](image)

**Figure 5.14.** Bending moment, shear and axial forces diagrams for the surrounding frame after the formation of a horizontal shear crack.

![Diagram of arching effect](image)

**Figure 5.15.** Double arching effect and proposed equivalent model.

Mander et al. [M12, M20] conducted experimental work testing infilled steel frames with semi-rigid beam-columns connections. They used the arrangement illustrated in Fig. 5.16 to represent the behaviour of multistorey infilled frames. From the observed behaviour, these researchers concluded that the corner-to-corner diagonal strut was not able to sustain high forces and then a secondary strut mechanism
developed into the panel. This proposed mechanism was similar to that previously suggested by Mainstone [M5].

![Diagram of testing device](image)

**Figure 5.16.** Testing device used by Mander et al. [M12, M20].

In the author's opinion, the double arching effect as a general representation of the behaviour of infilled frames should be carefully considered. This mechanism has been observed in tests where one or more of the following situations occurred: the frame or the beams were very flexible, the testing arrangement was not realistic or the surrounding beams were unrestrained. In real buildings, the deformations of the beams are usually restrained by the foundation or by other masonry panels in multistorey structures.

The testing program carried out by Fiorato et al. [F4] has been considered by numerous researchers as a reference work. This study was one of the first attempts to understand the behaviour of infilled reinforced concrete frames and the influence of different structural parameters. However, it is worth noting the following limitations:

- the masonry walls were cast with the frame laying in a horizontal position. The bricks were located into the frame, and then the mortar was poured over the panel and vibrated into the joints. Obviously, the conditions of the mortar joints obtained according to this method are rather different from those in real masonry walls (see section 4.4.3.2).
- the columns of the surrounding frame had no transverse reinforcement. Therefore, shear behaviour controlled the response of the reinforced concrete frame. This reinforcement detail is not representative of common infilled frame structures.
- several specimens were tested with vertical loads acting on the columns. These loads were applied by 6 mm diameter prestressing strand inserted through the columns. This situation modified the stiffness and the strength of the columns and could have affected the response of the structure.
Consequently, the results obtained from these tests are discussed in this work only in a general sense, especially when there is insufficient experimental information about certain topics. It is believed that no definitive conclusions can be drawn from these investigations.

5.5 BEHAVIOUR OF LARGE INFILLED FRAMES

5.5.1 Behaviour of Multistorey Structures

Most of the experimental research related to infilled frames has been carried out considering only a single panel, even though the structural configuration of real buildings is usually rather different. Laboratory tests of large structures are more expensive and require complex testing facilities. However, there is some experimental and analytical information available on multibay and multistorey structures which allows the evaluation of the behaviour in these cases.

The first tests on two-storey structures were conducted by Stafford Smith [S14, S15, S22] and Holmes [H9]. They tested steel frames with concrete or masonry infills, using the test device illustrated in Fig. 5.2 (d). It was observed in those tests that cracking occurred in all the panels, but no comments about the final mode of failure were given in the reports. The main objective was only to evaluate the stiffness and the strength of the infilled frames. Both authors concluded that the methods proposed to analyse one-storey infilled frame are valid for multistorey structures.

Tests conducted by Fiorato et al. [F4] with five-storey reinforced concrete frames with masonry infill indicated that flexural cracking occurred initially at the base of the structure, similarly to the cracking of slender cantilever beams. The strains measured in the column reinforcement showed that there was no significant gradient across the columns, therefore, the independent bending of the columns was considerably reduced. In the final stage, the braced system formed in a similar way to that observed in the one-storey specimens, however the final mechanism of failure was slightly different due to the influence of the higher axial forces in the columns. Damage in the panel and in the columns concentrated in the first storey, even though some cracks occurred in other parts of the structure. This conclusion was also reported by Klingner and Bertero [K5, K9] based on tests of three-storey infilled frames.

Liauw and Kwan [K4, L1, L4, L8, L9, L15] conducted a large amount of experimental and analytical work related to multistorey infilled frames; most of the tested specimens were steel frames with concrete panels. In some cases (two-storey frames), the cracks ran from the windward top corner to the leeward bottom corner, crossing the internal beam. This can be explained because the stiffness of the frame members was very small when compared with that of the concrete panel. Thus, the interior beams were not rigid enough to allow the independent behaviour of each panel. Furthermore, in a material like concrete, which can be considered as homogeneous, the shear cracks occur approximately at 45°, and the inclination of the diagonal of each panel was about 26° (h/L = 0.5). In spite of these facts, the authors reported that the final mode of failure of one-storey and multistorey structures was similar. Other experimental results related to this type of structure can be found elsewhere [D20, G3, G4, M27, S17, S18, S19].

Nonlinear analysis conducted by May and Naji [M4] and by the author indicate that, in a general sense, the behaviour of multistorey infilled frames is similar to that of one-storey structures. Results obtained
From the finite element analysis of a two-storey infilled frame conducted by the author are illustrated in Figs. 5.17 and 5.18. The former figure shows the deformed shape of the structure subsequent to the separation of the panel-frame interfaces and the latter figure represents the principal stresses developing into the masonry panel.

**Figure 5.17.** Deformed shape corresponding to a two-storey infilled frame.

**Figure 5.18.** Principal stresses obtained from finite element analysis of a two-storey infilled frame.
The mechanism for transferring the lateral forces is similar to that for one-storey frames (see Fig. 5.19). The beams are subjected to tensile axial forces approximately equal to the total shear force applied in the upper storeys. This fact is very important for reinforced concrete frames, where the beams should be designed to resist that tensile force. It must be noted also, that the ratio of the overturning moment to the shear force at the base of the multistorey structures increases considerably when compared with that of one storey structure. Thus, the axial efforts induced in the columns are higher, which affect the behaviour of the structure and can modify the mode of failure. These axial forces decrease or increase the stiffness and the flexural strength of the tension and compression columns, respectively. Furthermore, large vertical deformation may occur whether the longitudinal reinforcement of the windward columns yields in tension.

![Figure 5.19. Equivalent truss mechanism for multistorey frames.](image)

For a multistorey structure, the ratio of the total overturning moment, $M_o$, to the base shear force, $V$, may be expressed as:

$$
\frac{M_o}{V} = \frac{\sum_{i=1}^{n} F_i h_i}{\sum_{i=1}^{n} F_i} \quad (5.1)
$$

where $n$ is the number of storeys, $F_i$ the lateral force acting at level $i$, and $h_i$ the $i$-storey height. The axial force, $P$, induced in the tension column at the lowest storey can be approximately calculated by dividing the total overturning moment by the separation between the columns $L$. According to the braced frame model, this assumption is acceptable for the tension column, however, the axial force in the compression column is smaller (this difference reduces when the number of storeys increases). Therefore, the ratio $P/V$ can be estimated as:
The ratio $P/V$ is plotted in Fig. 5.20 for a linear and a uniform distribution of the lateral forces, assuming that the storey height is constant. It is observed that the axial force in the columns increases considerably with the number of storeys, and its value depends also on the dimensions of the frame, $h$ and $L$.

$$\frac{P}{V} = \frac{M_o}{V L}$$  \hspace{1cm} (5.2)

**Figure 5.20.** Variation of the ratio $P/V$ for multistorey infilled frames.

Damage of the masonry panel and the surrounding frame concentrates in the lowest storey, where the maximum efforts induced by lateral forces develop. Therefore, most of the inelastic behaviour occurs in this part of the structure and the formation of a soft-storey mechanism can be expected. A high displacement ductility capacity in the lowest storey is required to achieve a reasonably displacement ductility factor for the whole structure [P1].

Based on these results, it is concluded that behaviour of multistorey infilled frames can be explained according to the mechanisms observed for one-storey infilled frames. The damage concentrates mainly at the first storey, where the columns and masonry panel are the most affected elements. However, more research is necessary to investigate the effect of concentration of damage in the lowest storey and the danger of soft-storey mechanisms.

### 5.5.2 Behaviour of Multibay Structures

The behaviour of multibay infilled frames is more complex due to the presence of interior columns which influence the response of the system. In infilled frames, the columns tend to distort as in a bare frame, however, the deformation is restrained by the masonry panels. After separation occurs, the external columns are in contact with the panel along short lengths near to the loaded corners, but internal columns are restrained at both ends.
There are few reports in the literature about the behaviour of multibay infilled frames. Fiorato et al. [F4] tested a three-bay two-storey structure and reported that the behaviour was approximately similar to that observed for one-bay specimens. Shear cracking of the masonry walls initiated in the windward panel of the specimens, and then the cracks formed in the middle panel and, finally, in the leeward panel. This fact was explained considering that the leeward panel was slightly compressed (based on measurements of vertical deformations), hence, the shear strength was increased. After cracking, the behaviour of multibay structures was more complex due to the presence of the interior columns, which inhibited the displacements of the cracked walls. The observed response indicated that, in the stage prior to failure, the structure behaved as a braced system.

The experimental results, complemented with finite element analyses, conducted by Liauw, Kwan and Lo [K4, L3] showed that the panel and interface distribution in multibay infilled frames are very similar to the respective stress distribution in single bay infilled frames. They concluded that the simplified mechanism proposed for single-bay infilled frames is also valid for multibay structures.

In order to obtain more information about the behaviour of multibay infilled frames, nonlinear finite element analyses were performed by the author. Figs. 5.21 and 5.22 illustrate the deformed shape of the model and the stress state in the panel, respectively. The mechanical and geometrical properties considered in this example were similar to those used in the examples presented in sections 5.4.2 and 5.5.1 for single and multistorey infilled frames, respectively. It is observed that the truss mechanism develops clearly and the effect of each panel may be represented by a diagonal strut. The central column of the frame is more restrained because it is in contact with the masonry panels at both ends. This fact leads to a more severe effort regimen in the central column, as Fig. 5.23 indicates.

![Figure 5.21. Deformed shape obtained from a two-bay infilled frame.](image)

5.6 BEHAVIOUR OF INFILLED FRAMES UNDER CYCLIC AND DYNAMIC LOADING

5.6.1 General

The behaviour of infilled frames subjected to alternating lateral forces has been investigated by numerous researchers, mainly using quasistatic cyclic tests. Pseudo-dynamic [C14, N9, S17] and dynamic tests [C4, D9, D20, L15, M6, M15, M17, M20, M27, M30, S18] have been also performed in different countries. In a general sense, the conclusions presented in the previous sections are also valid in this case. However,
there are some particular aspects related to the response of infilled frames under cyclic actions, which are discussed here.

![Stress regimen in the masonry panels of a two-bay structure.](image)

**Figure 5.22.** Stress regimen in the masonry panels of a two-bay structure.

 ![Bending moment diagram for a two-bay infilled frame after separation.](image)

**Figure 5.23.** Bending moment diagram for a two-bay infilled frame after separation.

### 5.6.2 Characteristics of the Hysteretic Response

Infilled frames are shear sensitive systems, therefore, the hysteresis loops present a pronounced pinching effect. The differences in the hysteretic behaviour between the bare frame and the infilled frame are obvious. The hysteresis loops of the bare frame are fatter, due to the inelastic behaviour of flexural plastic hinges. Fig. 5.24 shows two typical force-displacement relationships, according to the results obtained by Kato et al. [K3]. Both specimens had the same characteristics, except for the amount of longitudinal reinforcement of the columns. Specimen A had 6-D13 (reinforcement ratio $\rho_s = 3.8\%$) whereas specimen C had only 4-D8 ($\rho_s = 0.1\%$). The envelope of the hysteresis loops obtained from a cyclic test is very similar to the force-displacement relationship measured under monotonic loading. The hysteresis loops show a significant pinching effect and strength degradation. Furthermore, the tangent stiffness is very low in the initial branch of each half-cyclic, corresponding to the closing of the cracks due to the force applied in the opposite direction. When the lateral force increases, the tangent stiffness also increases. Fig. 5.25 illustrates the cyclic response obtained by Lechars and Scrivener [L5, L14] from a reinforced concrete infilled frame in which horizontal shear sliding occurred. Despite the variation
in the type of failure and the resistance mechanism, the global response of the structural was similar to that usually observed when diagonal cracking of the panel occurs.

![Diagram](image_url)

**Figure 5.24.** Typical force-storey drift curve under alternating forces, according to the results obtained by Kato et al. [K3].

![Diagram](image_url)

**Figure 5.25.** Cyclic response corresponding to an infilled frame in which horizontal shear sliding occurred (according to Leuchars and Scrivener [L5, L14]).

Table 5.1 presents the statistical evaluation of numerous experimental results obtained from infilled reinforced concrete frames, which was reported in reference [B10]. The variables considered in this study are: (i) the ratio of the shear resistance of the infilled frame to the shear resistance of the bare frame, \( \phi \),
(ii) the ratio of the residual resistance of the infilled frame (defined at a storey drift of 2-3%) to the residual resistance of the bare frame, $\phi_{\text{res}}$, (iii) the storey drift corresponding to the maximum force, $\delta_u$, and (iv) the displacement ductility ratio defined at force level, $V$, equal to 0.85% of the maximum force, $V_{\text{res}}$. It is observed that the shear resistance of the infilled frame may be several times greater than that of the bare frame in those cases where no significant damage of the reinforced concrete of the frame occurred. The storey drift corresponding to the maximum lateral resistance of the infilled frame, $\delta_u$, is considerably smaller than in the case of a bare frame. However, the surrounding frame markedly influences the storey drift at failure, which is several times higher than the maximum storey drift of unreinforced masonry walls (about 0.1%). In this investigation, the ductility factor $\mu_\Delta$ was calculated as the ratio between the lateral displacements corresponding to $V = 0.85 V_{\text{res}}$ on the falling branch and on the ascending branch of the envelope curve (see Fig. 5.26). The average values of the ductility are quite satisfactory, taking into account that infilled frames are usually considered as "brittle" structures. Higher values can be achieved if the structure is adequately designed [G5]. The experimental results considered in the study are very scattered, as the high value of the coefficient of variation for $\delta_u$ indicates. For the other parameters presented in Table 5.1, the coefficient of variation is acceptable (about 10% of the main value).

**Table 5.1. Statistical evaluation of test results for infilled frames [B10].**

<table>
<thead>
<tr>
<th>Type of Infilled Frame</th>
<th>$\phi$</th>
<th>$\phi_{\text{res}}$</th>
<th>$\delta_u$ (%)</th>
<th>$\mu_\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MV</td>
<td>CV</td>
<td>MV</td>
<td>CV</td>
</tr>
<tr>
<td>No local damage in frame</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integral I.F.</td>
<td>3.34</td>
<td>0.43</td>
<td>3.92</td>
<td>0.21</td>
</tr>
<tr>
<td>Non-integral I.F.</td>
<td>3.80</td>
<td>0.40</td>
<td>2.01</td>
<td>0.15</td>
</tr>
<tr>
<td>Local damage in frame</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integral I.F.</td>
<td>1.44</td>
<td>0.10</td>
<td>1.44</td>
<td>0.10</td>
</tr>
<tr>
<td>Non-integral I.F.</td>
<td>1.79</td>
<td>0.26</td>
<td>1.49</td>
<td>0.05</td>
</tr>
</tbody>
</table>

- $\phi$: ratio of the shear resistance of the infilled frame to the shear resistance of the bare frame.
- $\phi_{\text{res}}$: ratio of the residual resistance of the infilled frame, at $\delta = 2-3\%$, to the residual resistance of the bare frame.
- $\delta_u$: storey drift corresponding to the maximum force, $V_{\text{res}}$.
- $\mu_\Delta$: displacement ductility ratio defined at $V = 0.85 V_{\text{res}}$.
- MV: mean value.
- CV: coefficient of variation.

5.6.3 **Stiffness Degradation**

As a consequence of the damage in the infilled frame and the deterioration of the panel-frame interfaces, the stiffness significantly decreases when the lateral displacement increases. In order to evaluate this effect, the secant stiffness is usually calculated from peak to peak, as shown in Fig. 5.27, and compared with the elastic stiffness. In other cases, the secant stiffness is defined considering only half of the cycle [Y2]. If the cyclic response is symmetric with respect to the origin, both definitions are identical.
Fig. 5.26. Definition of the displacement ductility ratio $\mu_\Delta$.

Fig. 5.27. Definition of the secant stiffness, $K_s$, from peak to peak.

Fig. 5.28 shows the variation of the secant stiffness as a function of the storey drift, $\delta$. The values plotted in this figure were obtained from the experimental data reported by Sánchez et al. [S35], Valiásis and Styliánidis [V2] and Žarnič and Tomažević [Z2], who tested masonry walls surrounded by reinforced concrete frames. The secant stiffness was calculated according to the definition illustrated in Fig. 5.27, and normalized with a reference stiffness $K_{s0}$ defined at a $\delta = 0.05\%$ (which approximately corresponds to the separation of the infill panel from the surrounding frame). Even though the materials of the masonry panel, dimensions and amount of reinforcement were different, the stiffness degradation measured in these tests was very similar. The results plotted in Fig. 5.28 also agree with experimental values reported by other researchers [G4, H1, Y2]. It is worth noting that the initially high stiffness of the infilled frames decreases rapidly. For example, the secant stiffness for a storey drift, $\delta = 1\%$ is about ten times smaller than the stiffness when separation occurs.
5.6.4 Strength Degradation

Alternating forces affect the capacity of the infilled frame to sustain the lateral resistance at large deformations. Even though the strength softening is a common characteristic of infilled frames, this effect is more pronounced when the structures are subjected to cyclic loading, because the unreinforced masonry panel degrades more rapidly. Bertero and Brokken [B6] found that the lateral strength under cyclic loading was smaller than that obtained under monotonically increasing forces. The storey drift at which strength decay commences depends on the characteristics of the infilled frame and, consequently, cannot be indicated in a general sense. Tests conducted by Meli [M13] indicated that masonry panels constructed with hollow masonry units showed local crushing and spalling of the shell of the masonry units. This produced a gradually increasing deterioration, and the lateral resistance was greatly affected.

Valiasis and Stylianidis [S25, V2] conducted cyclic tests in which the loading program included full cycles of gradually increasing displacements; two reversals were applied for each displacement level. They observed that the loss of strength in infilled frames during the second cycle was about 16% and 12%, in relation to the strength of the first cycle. These values correspond to specimens tested without axial load and with axial load applied in the columns, respectively. The lateral force-displacement curve for one of the specimens tested in this investigation is illustrated in Fig. 5.29. Similar cyclic tests were carried out by Žarnič and Tomaževič [Z2], however, in this case four reversals were applied for each displacement level. Results indicated that the average losses of strength for the second, third and fourth cycle (in relation to the first cycle) were 18%, 25% and 28%, respectively. Calvi et al. (as reported in [B10]) concluded that the strength degradation approximately equal to 20% occurs during the second loading cycle. This value seems to be independent of the amplitude of the deformation imposed to the structure. However, higher strength degradation has been measured (about 36% to 60%), when a premature shear failure of the reinforced concrete members occurred.
Despite the numerous cyclic tests carried out by different researchers, there is no general conclusion on how to estimate the strength degradation of infilled frames. Klingner and Bertero [K5, K6, K9] proposed a decaying exponential curve which reflects the desire characteristics of decreasing strength with increasing deformation. However, the proposed curve is function of an empirical parameter, selected on the basis of experience. More research related to this aspect of the behaviour is necessary to predict adequately the strength envelope in the falling branch of the force-displacement relationship.

5.6.5 Energy Dissipation Capacity

When a structural system deforms, the work done in this process is stored as strain energy. Part of this energy is released in the unloading process, whereas the remaining energy is dissipated through different mechanisms. In infilled frames, energy dissipation can arise from the following causes [M15, M17]:

- material damping,
- cracking and crushing of the panel,
- friction at the panel-frame interface and cracks,
- impact resulting from rocking of the infill panel inside the frame,
- hysteretic work of the frame members.

It is worth noting that the friction mechanism is very active at low distortions, tending to disappear due to the rapid degradation of the panel material [V2].

Different methods have been used to evaluate the energy dissipation capacity based on results obtained from cyclic tests. Mallick and Severn [M6] calculated the energy dissipated in each cycle, represented by the area surrounded by the hysteresis loops, and plotted this value against the maximum lateral displacement achieved in the cycle. The cumulative energy dissipated by the infilled frame during the
entire cyclic test has been also used as an indicator of the structural behaviour [V2, G4]. Values of this parameter reported by Valasis and Stylianidis [V2] indicate that the cumulative energy dissipation capacity of the filled frames tested in this study was about 1.4 to 2 times greater than that of the bare frame. These methods, however, cannot follow the changes of the energy dissipation capacity considering the imposed displacements. For this reason, Valasis and Stylianidis [V2] proposed a different parameter. They calculated the energy dissipated per cycle, $E_D$, divided by the corresponding total displacement of the cycle, $2 \Delta_m$. Fig. 5.30 shows the results obtained from an infilled frame and the corresponding bare frame. For both specimens, two curves are plotted to distinguish the response of the first and second cycles applied for each displacement level. It may be observed that the energy per unit of total displacement dissipated by the bare frame increased along the entire test. However, this parameter decreases in the range of large displacements for the case of the infilled frame, as a consequence of the degradation of the masonry panel. Despite this fact, the energy dissipation capacity of the infilled frame system is considerably higher than that of the corresponding bare frame. The energy dissipation capacity, in both frame and infilled frame, decrease significantly from the first cycle to the second cycles.

![Figure 5.30. Energy dissipated per cycle, $E_D$, according to the tests conducted by Valasis and Stylianidis [V2].](image)

Great emphasis has been given to the energy dissipation capacity as a structural parameter to evaluate the cyclic response, although the measurement of energy capacity is only useful from a qualitative viewpoint. Other parameters, such as displacement at maximum force, strength degradation and ductility, are equally important to judge the behaviour of the structure and its capacity to sustain alternating lateral forces.

5.6.6 Dynamic Properties of Infilled Frames
The presence of the masonry panels significantly affects the dynamic response of the infilled frames, because the lateral stiffness, the mass and the damping ratio are changed. The increase of the lateral stiffness is much more important than the increase of the mass. Consequently, the fundamental period of infilled frame buildings is smaller than that of the corresponding frame building. This observation agrees with the results obtained by Brokken and Bertero [B6, B8], who concluded that the effect of the added mass due to infill panels on the period is very small. It is worth noting, however, that the lateral
stiffness of the system deteriorates rapidly. Thus, the period of the structure increases, compared with the initial period, even at service load levels.

The variation of the period due to the presence of the masonry infills may change the inertial forces induced in the structure as a consequence of the seismic actions. Therefore, the influence of masonry panels should be always considered when designing or analysing infilled frame buildings because this situation can lead to more unfavourable conditions. Paulay and Priestley [P1] recommended that the natural period should be calculated considering the structural stiffness after separation occurs. The mathematical models used to represent the infilled frames are discussed in Chapters 6 and 8.

The damping ratio, $\xi$, (defined as the ratio of the damping to the critical damping value) has been measured from dynamic tests, using some of the existing methods to evaluate this parameter [C7]. Mallick and Severn [M6] investigated the behaviour of steel frames filled with concrete panels. Results of forced vibration tests showed that the damping ratio varied from 1.0% to 2.3%. It was not reported, however, if these values correspond to the initial stage or to nonlinear behaviour. Carydis et al. [C4] tested steel infilled frames and reported a damping ratio of about 4.2% (the specimens were excited with an acceleration of 0.05 g of steadily increasing frequency). Brokken and Bertero [B6, B8] reported that values as high as $\xi = 12\%$ has been measured in the range of large deformations.

The damping ratio can be evaluated based on energy considerations. For linear elastic systems with one dynamic degree of freedom, the absorbed strain energy, $E_A$, in half cycle is:

$$E_A = \frac{K \Delta^2}{2}$$  \hspace{1cm} (5.3)

where $K$ is the stiffness of the structural system. Assuming linear viscous damping, the relationship between the damping force and the displacement are elliptic curves, whose area represents the dissipated energy, $E_D$, also equal to the variation of the kinetic energy between the beginning and the end of the cycle. Based on these hypotheses the following expression can be derived to calculate the damping ratio [C7]:

$$\xi = \frac{E_D}{4 \pi E_A}$$  \hspace{1cm} (5.4)

In those cases where the damping is of a nonlinear viscous form, the damping force-displacement relationships are not elliptical. Nevertheless, $E_D$ can be obtained as the area of the hysteresis loop, whereas $E_A$ is calculated from Eq. 5.3. Therefore, it is possible to evaluate an equivalent viscous damping ratio, which when used in the linear viscous form will dissipate the same amount of energy as in the nonlinear case [C7].

The concept of equivalent viscous damping has been generalized for nonlinear systems [B10, L1, M4, Y2]. In this case, $\xi$ represents not only the effect of damping but also the energy dissipated due to plastic work. It could be considered as the viscous damping factor of an equivalent linear elastic structure. For
this reason, $\xi$ is also an indirect measurement of the dissipated energy. Fig. 5.31 indicates the significance of $E_A$ and $E_D$ in this case. Values of the equivalent damping ratio reported by Yamin and García [Y2] varied from 3% to 16%, for different specimens. Even though there was a wide scatter of results, it is possible to observe a reduction of this parameter for increasing values of the lateral displacement, which indicates a diminution of the energy dissipation capacity. On the contrary, the results obtained by Liauw and Kwan [L1] indicated that the equivalent damping ratio significantly increased when the lateral displacement was increased. Measured values ranged from 2.3% to 12.8% of the critical damping.

![Figure 5.31. Evaluation of the absorbed and dissipated energy.](image)

(a) Absorbed strain energy in a linear equivalent system  
(b) Dissipated energy

5.7 MODES OF FAILURE OF INFILLED FRAMES

5.7.1 Introduction

The type of failure that will occur in an infilled frame is normally difficult to predict, depending on several factors, such as the relative stiffness of the frame and the infill panel, the strength of their components and the dimensions of the structure. The collapse of the system usually involves one or more simple types of failure, which occurs in the masonry infill, as well as in the frame. The description of these modes of failure is the main objective of this section.

The different mechanisms of failure affecting the components of the infilled frames are referred, in a general sense, as modes of failure. In some cases, however, the local failure of one component do not represent the failure of the whole system and should be regarded only as a serviceability limit state. The figures presented in the following sections illustrate separately these mechanisms for sake of clarity. The final failure of the infilled frame usually results from a combination of them.

5.7.2 Failure of the Masonry Panel

5.7.2.1 General

The failure of the masonry panel can develop by debonding of the mortar joints, cracking or crushing of the masonry units or a combination of these. The occurrence of the different types of failure depends on the material properties and the stress state induced in the panel. Fig. 5.32 summarizes the different modes
of failure which may occur in masonry infills, according to a detailed review of the experimental results available in the literature.

![Diagram showing modes of failure in masonry infills]

Figure 5.32. Modes of failure observed in masonry infills.

### 5.7.2.2 Shear Cracking

Cracking in the masonry panel due to shear stresses is the most common type of failure observed in experimental work and also in infilled frame buildings affected by earthquakes. This type of failure is mainly controlled by the shear strength of the mortar joints (bond strength and coefficient of friction), the tensile strength of the masonry units and the relative values of the shear and normal stress. Depending on these parameters, the combination of shear stresses with vertical axial stresses can produce either cracks crossing the masonry units or debonding along the mortar joints (also termed as shear friction failure).

In infilled frames subjected to lateral forces, the stress state can be approximately represented by a compressive stress field along the diagonal (see Fig. 5.5). The ratio of the normal stress, \( f_n \), to the shear stress, \( \tau \), can be estimated using Eqs. 4.27 and 4.29 and assuming that, in the central zone of the panel, the effect of the principal tensile stress can be neglected in comparison with the principal compressive stress. Thus, it is found that:

\[
\frac{f_n}{\tau} = \tan \theta = \frac{h_m}{L_m}
\]  

(5.5)

where \( \theta \) is the angle between the diagonal of the panel and the horizontal axis, \( L_m \) is the length of the masonry panel and \( h_m \) is the height. Since the stress \( f_n \) is normally compressive in masonry panels of
infilled frames, its absolute value is considered in Eq. 5.5 and in the discussion presented here. It is worth noting that the same notation, $\theta$, is expressly used in Chapter 4 to indicate the angle between the principal compressive stress and the direction of the bed joints and here to represent the inclination of the diagonal of the panel respect to the horizontal axis (also the bed joint direction). Eq. 5.5 does not consider the effect of gravity loads acting on the masonry panel, which is normally unimportant.

The ratio of the normal stress to shear stress has a significant influence on the failure of the masonry panel, as discussed in section 4.1.2. Eq. 5.5 indicates that the ratio $f_r/f$ increases in direct proportion to the aspect ratio $h/L_w$. For typical infilled frames the aspect ratio, and consequently the ratio $f_r/f$, can vary from 0.5 to 1.25. These values were obtained assuming a panel height of 2.5 m and a panel length between 2.0 and 5.0 m, which are the common limit values recommended by seismic codes [C9, C10, N7]. Smaller values of the aspect ratio can be found in older existing buildings.

Alcocer and Klingner [A15] pointed out that failure has been customarily considered as the force which produces the first diagonal crack, rather than the maximum force. Experimental data indicates that the lateral resistance can increase after cracking. The ratio of the maximum force, $V_{u}$, to the cracking force, $V_c$, obtained from tests with framed masonry and infilled reinforced concrete frames is summarized in Table 5.2. These tests cover different constructive and testing techniques, masonry materials, reinforcement ratios and dimensions. Therefore, shear cracking does not necessarily represent a failure condition because the cracked panel is restrained by the surrounding frame. The formation of diagonal cracks is regarded only as a serviceability limit state. For hollow masonry, however, cracking of the masonry units can produce the failure of the structure.

### Table 5.2. Ultimate to cracking force ratio obtained from different tests.

<table>
<thead>
<tr>
<th>Reference</th>
<th>No. of specimens</th>
<th>Type of loading</th>
<th>$V_{u}/V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Govindan et al. [G4]</td>
<td>1</td>
<td>cyclic</td>
<td>2.09</td>
</tr>
<tr>
<td>Klingner and Bertero [K9]</td>
<td>2</td>
<td>cyclic</td>
<td>1.27 to 1.95</td>
</tr>
<tr>
<td>Lechairs and Scrivener [L5]</td>
<td>1</td>
<td>cyclic</td>
<td>1.26</td>
</tr>
<tr>
<td>Liauw and Kwan [L15]</td>
<td>1</td>
<td>dynamic</td>
<td>1.62</td>
</tr>
<tr>
<td>Meli [M13]</td>
<td>10</td>
<td>cyclic</td>
<td>1.0 to 1.69</td>
</tr>
<tr>
<td>San Bartolomé [S7]</td>
<td>4</td>
<td>monotonic</td>
<td>1.09 to 1.54</td>
</tr>
<tr>
<td>Sánchez et al. [S35]</td>
<td>3</td>
<td>cyclic</td>
<td>1.0 to 1.41</td>
</tr>
<tr>
<td>Sánchez et al. [S36]</td>
<td>3</td>
<td>cyclic</td>
<td>1.0 to 1.68</td>
</tr>
<tr>
<td>Yamin and García [Y2]</td>
<td>8</td>
<td>cyclic</td>
<td>1.01 to 1.29</td>
</tr>
</tbody>
</table>
Three different types of shear cracking observed in masonry panels are discussed in the following sections.

(i) Stepped Cracking Along the Mortar Joints
When the mortar joints are weak in comparison with the masonry units or when the shear stress predominates over the normal stress (low to medium aspect ratios), cracking usually occurs by debonding along the mortar joints (details regarding this type of cracking were given in section 4.1.2). Fig. 5.33 illustrates the case in which one or two large cracks run along the diagonal with stepped pattern. This mode of cracking has been widely observed in laboratory tests, as well as in infilled frame buildings subjected to earthquakes, and it can be regarded as the most common type of failure.

![Figure 5.33. Shear cracking along the mortar joints, stepped cracks.](image)

(ii) Horizontal Sliding Along the Mortar Joints
It has been also observed a different mechanism, in which the panel fails by shear sliding due to the formation of a horizontal crack, as shown in Fig. 5.34. This type of failure was reported by Fiorato et al. [F4], Leuchars [L14] and Brokken and Bertero [B6, B8] in tests of infilled reinforced concrete frames. Tests results indicate that the major crack usually starts a few courses below of the upper loaded corner and continues diagonally downwards to approximately the centre of the panel. Then, the cracks propagates horizontally. When the direction of the force is reversed, the horizontal crack increases its length, crossing the panel.

A factor which could contribute to the formation of horizontal cracks is the relative dimensions of the masonry units and the infill panel. The diagonal stepped crack tends to run from one loaded corner to another, because this is the zone where the higher shear stresses develops. The horizontal projection of the length of the potential stepped crack, \( L_{\text{proj}} \), depends on the dimensions of the brick and it can be approximately evaluated using the following equation (see Fig. 5.35):
\[ L_{sc} = \frac{d}{2} \frac{h_m}{b} \]

(5.6)

where \(d\) and \(b\) are the length and height of the masonry unit, respectively. When the length of the panel, \(L_m\), is larger than \(L_{sc}\), the stepped crack cannot completely develop. Hence, a horizontal crack may also form. From Eq. 5.6, the condition for the formation of the horizontal crack \((L_{sc} < L_m)\) can be expressed as:

\[ \frac{h_m}{L_m} < \frac{2b}{d} \]

(5.7)

Figure 5.34. Failure due to horizontal sliding.

Figure 5.35. Horizontal and stepped cracks in a masonry panel.
Table 5.3 presents the geometric parameters required for the evaluation of Eq. 5.7, for those tests in which this type of failure occurred. It is observed that Eq. 5.7 matches the results in all cases.

Table 5.3. Comparison of data for tests where horizontal cracks along the mortar joints occurred.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$b_y/L_{m}$</th>
<th>$2b/d$</th>
<th>Type of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brokken-Bertero [B6] Specimens 1, 4, 7, 8</td>
<td>0.38</td>
<td>1.00*</td>
<td>Horizontal sliding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50*</td>
<td></td>
</tr>
<tr>
<td>Fiorato et al. [F4] Most of the specimens</td>
<td>0.50</td>
<td>0.55</td>
<td>Horizontal sliding</td>
</tr>
<tr>
<td>Klingner-Bertero [K9] Test # 3</td>
<td>0.38</td>
<td>0.50</td>
<td>Horizontal sliding combined with stepped cracks</td>
</tr>
<tr>
<td>Leuchars [L14] Unit 2</td>
<td>0.65</td>
<td>0.67</td>
<td>Horizontal sliding</td>
</tr>
</tbody>
</table>

* Information not available, value estimated from photographs.

The formation of an unintentional gap between the masonry panel and the floor beam could be also an important factor in the occurrence of horizontal sliding. This aspect is further discussed in section 5.8.8.

The constructive techniques commonly employed to build masonry structures lead to distinct conditions between the horizontal and vertical mortar joints, the latter usually being the weakest. However, the use of different techniques to construct laboratory specimens may improve the conditions of the head joints. It seems that this factor can also contribute to the formation of horizontal cracks, instead of stepped cracks.

Experimental results indicate that panels built with hollow masonry units can be prone to fail due to horizontal sliding cracks. The use of masonry units with ungrouted vertical cells leads to a shear strength of the bed mortar joints smaller than that of the head joints, contrary to the usual situation. Therefore, the bed joints represent a plane of weakness in the masonry panel and cracks tend to occur along them. Test conducted by Castilla and Pose [C8] on framed masonry specimens built with hollow concrete blocks showed this problem. Similar results were observed by Mehrabi et al. [M32]. The ratio of the net area to the gross cross-sectional area of the masonry units was 0.63 and 0.52, respectively. In the tests reported by Mehrabi et al. [M32], the horizontal cracks were distributed over a large zone of the panel, instead of one large mid height crack.

This type of failure was also observed by Gergely et al. [G6] when testing a reduced scale specimen under cyclic loading. The specimen was a one-storey two-bay semi-rigidly connected steel frame infilled with unreinforced masonry panels. In this case, the formation of the horizontal crack could be due to unintentional gaps between the frame and the masonry panel (see section 5.8.8).
Shear sliding along horizontal joints also occurred during the Dinar, Turkey earthquake of October 1995 in a four-story infilled frame [T5]. Unfortunately, there is no information available about the masonry properties needed in the evaluation of Eq. 5.7. The damaged masonry panels had an aspect ratio, $h_m/L_m$, equal to 0.52.

The review of experimental data suggests that horizontal sliding has mainly occurred in non-integral infilled frames with low to medium aspect ratios ($h_m/L_m < 2/3$), which also agrees with the condition expressed in Eq. 5.7. Other factors, such as unintentional gaps, improved conditions of the head mortar joints or the use of ungrouted hollow masonry units, may contribute to the formation of horizontal cracks.

When shear cracking occurs along the mortar joints, following either a stepped or a horizontal pattern, the strength of the panel is controlled by a shear friction mechanism. It is worth noting that this mechanism degrades rapidly, affecting the response of the structure under cyclic lateral forces [P11].

(iii) Cracking Due to Diagonal Tension

The stress state induced in the masonry by the lateral forces (see Fig. 5.36), can produce diagonal cracks which occur because the stress state exceeds the tensile strength of the masonry unit. These cracks start in the central zone of the panel, where the tensile principal stresses are higher (see Fig. 5.5), and then propagate towards the corners, running with an inclination approximately equal to the angle $\theta$.

![Figure 5.36. Biaxial compression-tension stress state in the masonry panel.](image)

This type of cracking usually occurs when the mortar joints are strong in comparison with the masonry units or when the normal stress predominates over the shear stress (medium to high aspect ratios). The distribution of the diagonal cracks depends on the characteristics of the masonry wall and the panel-frame interface. When the panel is horizontally reinforced or when the conditions of the panel-frame interfaces are improved, as occur in framed masonry, the cracks are usually small and distributed in a wide zone along the diagonal (see Fig. 5.37 (a)). In other cases, the damage concentrates in one or two large cracks, as shown in Fig. 5.37 (b). However, this is not a general conclusion and it is difficult to predict the cracking pattern for a particular case.

Priestley and Calvi [P17] pointed out that the occurrence of diagonal tension cracking should be regarded as the failure of the structure, even if it does not produce the collapse, because of the possibility of an out-
of-plane expulsion of the panel material when diagonal cracks develop along both diagonals. Experimental results and observations of the damage produced by earthquakes indicated that hollow masonry units collapse quickly after cracking occurs. However, when solid masonry units are used to build the panel, it appears that diagonal tension cracking represents just a serviceability limit.

![Diagram](image)

(a) Wide spread of the diagonal cracks  
(b) Major diagonal crack

**Figure 5.37.** Cracking induced by diagonal tension.

### 5.7.2.3 Compressive Failure

Failure of the masonry due to compression has been observed following two mechanisms, resulting of the different stress states which develop in the infill panel at the loaded corners and along the diagonal.

**(i) Crushing of the Loaded Corners**

The first mechanism of compressive failure can occur in the regions close to the loaded corners, where a biaxial compression-compression stress state develops due to the lateral loading (see Fig. 5.5). The biaxial stress regimen improves the strength of the masonry, however, the values of the stress are higher in these zones. Fig. 5.38 illustrates this case.

![Diagram](image)

**Figure 5.38.** Failure due to crushing of the masonry in the corners of the panels.
This type of failure was observed in the earliest investigations [L1, M5, S15, S19], which were conducted using steel frames infilled with concrete. Most of the tested specimens had a very flexible frame, which produce the reduction of the contact length and the increase of the compressive stresses near the corners. Dawe et al. [D9] also observed this type of failure in steel frames infilled with masonry panels. However, the characteristics of the surrounding frame were rather unusual, because they used a frame made from 50 mm wide flat-bar stock (the thickness was 6 and 12 mm) to enclose a 3.0 x 3.0 m panel. Stafford Smith and Carter [S19] pointed out that, because of the weakness of the shear and diagonal tension modes of failure, crushing of the corners would be unlikely to occur in masonry panels. This mode of failure has been rarely observed in masonry infilled reinforced concrete frames, except in some specimens tested by Brokken and Bertero [B8].

**(ii) Compressive Failure of the Diagonal Strut**

This mechanism is associated with diagonal cracking (see Fig. 5.37). After the cracks occur, the tensile stresses along the diagonal are relieved and the masonry between the cracks behaves like small prisms axially loaded. As a result of the increase of the lateral displacement, the separation of the cracks grows leading to a failure of the panel by instability of the cracked masonry. Therefore, this type of failure occurs as a consequence of diagonal tension cracking, as indicated in Fig. 5.32 with a vertical arrow. In infill panels built with hollow masonry units, a sudden (occasionally "explosive") compressive failure is prone to occur after cracking because this type of masonry units is normally made of brittle, high-strength materials to compensate the large area of voids.

It is very important to note that the compressive strength of the diagonal strut is not the standard compressive strength of masonry $f'$, because the axial load is not perpendicular to the bed joints.

**5.7.2.4 Flexural Cracking**

In those cases where flexure effects are predominating (such as multistorey infilled frames) and the columns of the frame are very weak, flexural cracks can open in the tensile side of the panel due to the low tensile strength of the masonry [H8, L14]. Fig. 5.39 illustrates this type of failure, which has been rarely observed because separation at the panel-frame interface usually occurs first and the overturning moment is then mainly resisted by the truss mechanism.

![Figure 5.39. Flexural cracking of masonry panel.](image)
5.7.3 Failure of the Reinforced Concrete Boundary Frame

5.7.3.1 General

This section focuses on the failure of the surrounding reinforced concrete frame. Different failure mechanisms can develop, depending on the characteristics of the frame members and on the effects resulting from the panel-frame interaction. Damage in the frame members usually occurs from flexural plastic hinges, shear failure, yielding under axial forces, compression failure or a combination of these. Fig. 5.40 summarizes the modes of failure observed in previous experimental work.

![Diagram of failure modes](image)

**Figure 5.40.** Modes of failure observed in reinforced concrete boundary frames.

5.7.3.2 Flexural Collapse Mechanism

The typical collapse mechanism is illustrated in Fig. 5.41. Flexural plastic hinges usually develop at the ends of both columns, where the maximum bending moments occur. Before the failure of the masonry panel, the mechanism illustrated in Fig. 5.41 is not really a "collapse mechanism", because the system behaves as a pinned braced frame. Considering the different structural characteristics of both components, frame and masonry panel, the collapse mechanism usually develops after the panel has failed. The formation of plastic hinges in the beams is rarely observed, even in multistorey infilled frames.
Figure 5.41. Flexural collapse mechanism with plastic hinges at member ends.

When sliding shear occurs in the masonry panel (see Fig. 5.34), a different flexural collapse mechanism may form as a result of the modification of the bending moment diagram (see Fig. 5.14). Plastic hinges also develop in both columns, one hinge at the end of the member and another one approximately at middle height, as shown in Fig. 5.42. Fiorato et al. [F4] termed this mechanism as "knee braced frame".

Figure 5.42. Flexural collapse mechanism with plastic hinges at span length.

In order to achieve a reasonably ductile response of the structure, the plastic hinge regions of the frame members need to be provided with adequate rotational capacity. For multistorey infilled frames, the ductility demand is higher, since the plastic displacement occurs mainly in the first storey. Paulay and Priestley [P1] showed that the displacement ductility demand in the first storey increases in direct proportion to the number of storeys. Therefore, large plastic deformations can be expected in the flexural hinges. It should be noted that the shear forces induced in the columns after the formation of the flexural mechanism increases when the distance between the plastic hinges decreases. Therefore, higher shear forces are resisted in the case illustrated in Fig. 5.42, in which the plastic hinges form at middle height. The inelastic deformation demand in the plastic hinges is also greater.
5.7.3.3  Failure Due to Axial Loads

Axial forces are induced in the frame members as a result of the truss mechanism. Gravity loads also produce axial compressive forces in the columns, however, this effect is usually less important. Compressive failure of the columns is very rare, probably due to the high compressive strength of the reinforced concrete sections when compared with the tensile strength. However, buckling of the longitudinal reinforcement could occur under severe cyclic loading resulting in a compressive failure. Tensile axial forces cause the cracking of the reinforced concrete members and, when the lateral forces increase, yielding of the members subjected to tension can occur. Two modes of failure due to tensile axial load are discussed below:

(i) Tension Failure with Yielding of the Longitudinal Reinforcement

Yielding of the longitudinal reinforcement of the column may occur in infilled frames with high aspect ratio, such as multistorey structures. Horizontal cracks form along the tension column due to tensile strains (see Fig. 5.43). As the lateral displacement increases, plastic deformations can develop in the longitudinal reinforcement leading to a large elongation of the columns. The masonry panel tends to rotate within the frame and a wide crack usually appears at the base. Since this type of failure occurs when the flexural effects are dominant, it has been also called flexural failure [S18].

![Figure 5.43. Tension failure of the column.](image)

The axial forces in the columns can be greater than those acting on the floor beams. Static analysis using the equivalent truss model indicates that the column axial load increases with the number of storeys and can be several times larger than the base shear resisted by the structure (see section 5.5.1). The axial force of the beam, however, is equal to the shear force resisted by the storeys located above this beam. The axial force in the columns changes depending on the direction of the lateral forces, whereas the beams are always subjected to tension.

The yielding of the longitudinal reinforcement, which produces large elongations of the columns, has unfavourable consequences for the infilled frame. As a result of the distinct properties of the masonry panel and frame, the panel cannot match such large deformations, resulting in loss of fit between both components. When the force is reversed, the elongated column requires significantly compression to achieve its initial length. A similar problem occurs in the floor beams, which are elongated by tensile
forces resulting from the braced frame mechanism. Under these circumstances, the interaction between panel and frame may significantly degrade or completely disappear.

(ii) Bar Anchorage Failure
This type of failure is due to the slip of the longitudinal reinforcement of the tension column, as shown in Fig. 5.44. Bar anchorage failure can be avoided by providing the adequate development length for the longitudinal bars. Even in those cases where this type of failure does not occur, the vertical movement resulting from the slip causes a rotation of the structure. This effect can be approximately considered as a rigid body rotation about the compression column [F4]. Under the action of cyclic loading, shear sliding at the base of the columns can control the response of the infilled frame.

![Figure 5.44. Bar anchorage failure.](image)

5.7.3.4 Shear Failure of the Columns
The columns can fail due to the shear forces resulting from the composite interaction with the infill panel. The maximum shear forces occur along the contact length, near the loaded corners (see Fig. 5.8). In this case, one or more large diagonal cracks form crossing the column section, as illustrated in Fig. 5.45.

The shear strength of the column is mainly controlled by the amount of transverse reinforcement and the shear strength of the concrete. The axial loads also affect the shear strength of the columns. Compressive forces increase the shear resistance, hence, the tension column is weaker than the compression column to resist shear forces. However, the shear force is usually larger at the bottom of the compression column.

Sliding shear failure of the columns was observed during the experimental programme conducted by the author. In this case, a horizontal shear crack formed in the column close to the beam face, starting in the region in contact with the masonry panel. The crack propagated and increased its width as the test progressed. In the final stage, the crack almost completely crossed the column section and propagated vertically towards the external face of the beam-column joint. A detailed description of this type of failure is given in section 10.2.1.
5.7.3.5 Beam-Column Joint Failure

High normal and tangential stresses develop along the contact lengths in the zones near to the loaded corners, resulting in large shear forces and bending moments in the loaded corners (see Figs. 5.7 and 5.8). The stress state induced in these beam-column joints may cause the formation of wide diagonal cracks running across the joint from the interior to the exterior corner. Fig. 5.46 illustrates this situation, showing the crack pattern reported by Leuchars [L14]. Diagonal cracks in joints only occur in one direction. When the lateral force is reversed, the effects significantly decrease in these joints and the diagonal force acts on the opposite joint.
Minor attention has been given to this mode of failure, even though it has been observed in different investigations [K9, L14]. The failure of the beam-column joint causes unfavourable effects in the behaviour of infilled frames, because the lateral forces cannot be transferred from the floor beam to the columns and the masonry panel. Furthermore, the formation of diagonal cracks causes the opening of the joint. Therefore, the contact length at the loaded corners and the width of the equivalent strut decrease, resulting in an increase of the stresses in the masonry panel.

5.8 FACTORS AFFECTING THE BEHAVIOUR OF INFILLED FRAMES

5.8.1 Introduction

This section attempts to illustrate the influence of several parameters which can affect the behaviour of infilled frames. This is a difficult task, however, due to the number of geometric and mechanical parameters involved in the problem, the different details and constructive techniques used for infilled frames, and the difficulty to isolate the effect of each parameter. For these reasons, the comparison of test results should be conducted with caution in order to obtain valid conclusions.

The most important parameters which may affect the structural response of infilled frames are:

- **Masonry units**: mechanical properties, dimensions, type of brick (hollow or solid), characteristics of the brick surface.
- **Mortar joints**: mechanical properties, lime content, joint thickness, presence of horizontal reinforcement.
- **Frame**: mechanical properties of the concrete and reinforcing steel, dimensions of the members, amount of longitudinal and transverse reinforcement.
- **General**: vertical loads, number of storeys, number of bays, presence of openings, conditions of the panel-frame interface, constructive techniques.

The influence of these parameters is discussed in the following sections, based on experimental results available in the literature.

5.8.2 Mechanical Properties and Dimensions of the Infilled Frame

Since the earliest experimental work, researchers tried to relate the mechanical and geometrical properties of infilled frames with some structural parameters, such as the stiffness or the strength. Stafford Smith [S15, S19] proposed a dimensionless parameter, \( \lambda_h \), which expresses the relative stiffness of the infill panel to the frame:

\[
\lambda_h = h \sqrt{\frac{E_m t \sin \theta}{4 E_c I_c h_m}}
\]

(5.8)

where \( t \) and \( h_m \) are the thickness and the height of the masonry panel (see fig. 5.35), respectively, \( \theta \) is the inclination of the diagonal of the panel (see Fig. 5.36), \( E_m \) and \( E_c \) are the modulus of elasticity of the masonry and the concrete, respectively, and \( I_c \) is the moment of inertia of the columns. The parameter \( \lambda_h \) can be also calculated considering the length of the masonry panel instead of the height. However, it has been found that the parameter given by Eq. 5.8 is more suitable [M5]. Eq. 5.8 may be simplified assuming that \( h = h_m \) and \( \sin 2 \theta = 1 \) (\( \theta \) varies usually from 25° to 50°). Thus [R3]:
\[
\lambda_h = \sqrt[4]{\frac{E_m \ t \ h_m^3}{4 \ E_c \ I_c}}
\]  
(5.9)

The typical values for \( \lambda_h \), according to Eq. 5.9, range from 3 to 10. The smaller values indicate that the frame is much stiffer than the infill panel. The parameter \( \lambda_h \) was initially proposed for steel frames, in which \( I_c \) can be uniquely defined. For reinforced concrete frames, however, the value of \( I_c \) significantly decreases after cracking develops in the columns.

Numerous empirical equations have been proposed to evaluate the stiffness, the lateral strength or the contact length between the frame and the infill panel as a function of the parameter \( \lambda_h \) [K7, L9, M5, S14, S19, S23]. Some of these expressions are discussed in section 6.2.1.3. It has been observed that for low values of \( \lambda_h \) (the frame is stiffer than the infill panel), the lateral strength of the infilled frame increases, as does the contact length between the panel and the frame. The latter observation implies that the width of the equivalent diagonal strut also increases.

Bazán and Meli [B7] proposed a dimensionless parameter to evaluate the relative stiffness of the reinforced concrete frame to the panel using the following expression:

\[
\beta = \frac{E_c \ A_c}{G_m \ A_m}
\]  
(5.10)

in which \( A_c \) is the gross area of the column and \( A_m = (L_m \ t) \) is the area of the masonry panel in the horizontal plane. They correlated the numerical results obtained from finite element analyses with the parameter \( \beta \). It was observed that the width of the equivalent diagonal strut and the ratio of the secant stiffness at failure to the initial stiffness increased for increasing values of \( \beta \). The values of \( \beta \) considered in this study varied from 1 to 12, although practical values range from 1 to 5. The smaller values indicate that the masonry panel is stiffer than the frame, contrarily to values of the parameter \( \lambda_h \).

The parameters \( \lambda_h \) and \( \beta \) are adequate for the evaluation of the elastic behaviour, because they express the relative stiffness between frame and panel. However, it seems more appropriate to define a different parameter, associated with strength properties, in order to evaluate the resistance of the structure and its mode of failure. For these reasons, Liauw and Kwan [L4, L6, L7] introduced a relative strength parameter, \( m \):

\[
m = \sqrt[4]{\frac{4 \ M_u}{f_m \ t \ h^2}}
\]  
(5.11)

where \( M_u \) is the ultimate moment of the weakest member of the frame. This parameter was used to calculate the lateral resistance of steel infilled frames for different types of failure. The results indicated
that crushing of the loaded corners occurs only for low values of the parameter m (weak frame). Otherwise, the failure occurs due to compression of the diagonal strut.

The aspect ratio, \( h_m/L_m \), is also an important factor which could influence the mode of failure of the masonry infill. As mentioned in section 5.7.2.2, the ratio of the normal stress to shear stress in the masonry panel depends on the aspect ratio. Polyakov [P9] reported experimental results in which the cracks crossed the bricks for values of \( h_m/L_m > 1.0 \), whereas the cracks formed along the bed joints for \( h_m/L_m < 2/3 \).

5.8.3 Strength of the Masonry Infill

The compressive and shear strength of masonry panels depend on the properties of their constitutive materials, mortar and brick, as discussed in Chapters 3 and 4. The hydration conditions at the mortar-brick interface, the characteristics of the brick surface and the compressive strength of the mortar are the most important factors to be controlled in order to obtain adequate values of shear bond strength. The quality of the masonry units, especially the tensile strength, and the compressive strength of the mortar control the compressive strength of masonry structures. Therefore, a better quality of the constitutive materials leads to a stronger masonry, but not necessarily to increase the lateral strength of the infilled frame. This fact is explained considering that when the masonry panel becomes excessively strong when compared with the surrounding frame, premature failure of the frame members may occur and the strength of the system can be significantly reduced [B10]. Furthermore, the variation of the masonry strength associated with certain types of failure will only affect the response of the infilled frame whether this type of failure is involved in the final mode of failure.

Tests conducted by Dawe and Hatzinikolas [D17] showed that the use of poor quality mortar considerably reduces the force at initial cracking and the lateral strength of the infilled steel frames. Additionally, the crack pattern was very extensive. Liauw [L11] reported that the use of high bond strength mortar produced much higher strength and stiffness as a result of no separation. Contrarily, test results obtained by Stylianidis (as reported in reference [B10]) were not affected by the variation of the mortar strength. The analysis of these tests indicates that the cracks in the reinforced concrete columns appeared before than in the masonry infill, and the formation of the plastic hinges preceded the failure of the infill panel, even in the case of mortars with very low compressive strength. Consequently, the variation of the mortar strength did not affect the response of the specimens in this case.

Dawe and Seah [D7] correlated the compressive masonry strength with the cracking and ultimate force of infilled steel frames, considering groups of specimens with similar characteristics. The results show that there is a random scatter of the data within each group, indicating no correlation between these variables.

The type of masonry is also an important factor. Panels built with solid masonry units usually show a higher compressive strength when compared with that of panels of hollow masonry units. Furthermore, the panels exhibit a better behaviour when subjected to alternating forces [B10, M13]. For this reason, several codes [C9, N7] increases the seismic lateral forces for the design of infilled frame buildings when hollow masonry units are used.
5.8.4 Characteristics of the Reinforced Concrete Frame

The concrete area of the members and the amount of longitudinal and transverse reinforcement are the most important parameters which can affect the response of infilled frames. The influence of these parameters has been experimentally studied by different researchers.

In their investigations of the behaviour of brick panels surrounded by reinforced concrete frames, Benjamin and Williams [B9] observed that the variation in the concrete and steel area did not influence the stiffness in the uncracked range. However, the strength of the composite structure depended on the resistance of the frame (particularly the columns) to bending moment, axial force and shear.

The effect of the relative beam-to-column stiffness was investigated by Parducci and Mezzi [P13], who tested infilled frames with two different top beams (800 x 100 mm and 230 x 400 mm) and similar columns. The experimental results indicated no influence of this parameter on the lateral strength of the structure.

The amount of reinforcement in the frame can affect the capacity of the system and it is an important factor influencing the response of the infilled frame, depending on the relative strength of the frame and the masonry panel. When columns are lightly reinforced, the tension column can yield before the strength of the masonry panel is reached. In this case, the increase of the amount of longitudinal steel enhance the resistance of the infilled frame. Further increases in the longitudinal reinforcement, however, do not affect the strength of the structure, because the failure is mainly controlled by the strength of the masonry panel.

Experimental results reported by Fiorato et al. [F4], based on tests of five-storey infilled frames, indicate that the strength increased about 100% when the reinforcement ratio \( \rho_t \) (area of the total longitudinal reinforcement, \( A_{lt} \), divided by the gross area of the section, \( A_g \)) was increased from 0.011 to 0.022. However, the strength improved slightly for the case of \( \rho_t = 0.034 \). Meli [M29] reported that the amount of longitudinal reinforcement at the columns had only a limited effect on increasing the ultimate capacity of framed masonry structures, but it significantly increased the ductility that could be attained. Similar results were obtained by Kato et al. [K3] and Valiassis and Stylianidis [V2] testing several infilled frames with different amount of longitudinal reinforced in the columns.

Tests conducted by Valiassis and Stylianidis [V2] showed that the effect of the longitudinal reinforcement of the columns also depends on the level of axial load acting on these members. The improvement of the lateral strength due to the increase of the amount of longitudinal steel (\( \rho_t = 0.01 \) and \( 0.018 \)) was more important in those specimens tested without axial compressive load. This fact can be explained considering that the application of low axial loads (0.14 of the ultimate compressive capacity of the columns, in these tests) improved the strength of the tension column. Therefore, the effect of the variation of the longitudinal reinforcement was less significant.

The amount of transverse reinforcement of the columns can modify the response of infilled frames in their ultimate stage. After separation at the panel-frame interface occurs, large shear forces are resisted by the
columns near the loaded corners, as indicated in Figs. 5.7 and 5.8 (a). It is necessary to provide sufficient transverse reinforcement, especially in the columns, to sustain large deformations without a brittle shear failure. Experimental results obtained by Kato et al. [K3], using infilled frames with different spacings (50 and 220 mm) of the stirrups of the columns, showed no significant influence of this parameter on the strength of the structure. However, the cyclic behaviour of the specimens was considerably improved in the case of small spacing of the stirrups. Alcocer and Klingner [A15] and Flores and Alcocer [F8] concluded that the amount of longitudinal and transverse reinforcement in the surrounding frame does not affect the pre-cracking behaviour. When large lateral displacements are imposed to the structure, the reinforcement of the columns governs the deformation and the energy dissipation capacity, the residual strength and the strength degradation.

5.8.5 Horizontal Reinforcement within the Masonry Panel

The presence of horizontal reinforcement located within the bed joints of the masonry panel normally improves the response of infilled frames. Only a few experimental works have been conducted to investigate the influence of the horizontal reinforcement, and contradictory conclusions have been reported in some cases. In order to compare experimental results, the ratio of the horizontal reinforcement, the characteristics of the steel (yield strength and type of bar) and the anchorage of the reinforcing bars should be considered.

It is clear from test results [A8, S36, Z1, Z3] that the horizontal reinforcement does not affect the initial stiffness or the force at which shear cracking of the masonry panel occurs. This fact was explained by Moghaddam and Dowling [M18] considering that the measured values of the vertical slip in the cracked panel were significantly greater than the horizontal slip. Therefore, the horizontal reinforcement is ineffective in preventing the shear cracking. Measurements of strains at the horizontal reinforcement [Z3] showed that the steel bars yield in tension at the ultimate stage, which indicated that the reinforcement effectively contribute to resist the lateral force.

Sánchez et al. [S36] tested three framed masonry specimens built with solid clay bricks. Two specimens were reinforced with a ladder-shaped prefabricated reinforcement (made of 3.4 mm diameter high strength cold-drawn wires, $f_y = 490$ MPa) and with high strength deformed bars (two 4 mm diameter bars every three courses, $f_y = 580$ MPa). The ratio of horizontal reinforcement in the masonry panel was 0.1% in both cases. The deformed bars were anchored around the longitudinal bars of the columns with 180° hooks. The use of deformed bars as horizontal reinforcement increased the strength of the system about 70% when compared with the strength of the unreinforced specimen. In contrast, prefabricated cold-drawn wire reinforcement did not significantly improve the strength of the structure, due to the premature failure of the wires at the weld points. Aguilar et al. [A20] continued this research program and tested framed masonry walls with horizontal reinforcement ratios of 0%, 0.07% and 0.19%. The presence of horizontal bars increased the strength of the structure by about 35% when compared with that of the unreinforced panel. The influence was more significant on the ductility, which increased significantly with increasing values of the reinforcement ratio. They also reported that the anchorage of the horizontal bars by 90° hooks inside the columns had a satisfactory behaviour. This detail is easier to construct than the anchorages with 180° hooks.
A similar investigation was conducted by Žarnič and Tomaževič [Z1, Z3], who tested infilled frames in 1:2 reduced scale. Two specimens had horizontal reinforcement formed by two 6 mm diameter bars every three courses (reinforcement ratio equal to 0.22%). In one specimen, the horizontal bars were anchored in the reinforced concrete columns with 180° hooks, whereas in the other specimen the bars were not connected to the frame. In the latter case, the specimen showed no significant difference when compared with the unreinforced specimen. However, when the horizontal reinforcement of the infill panel was anchored in the frame, the behaviour of the structure improved and the strength increased by about 18%. The increase in the strength was smaller than that reported by other researchers [A8, A20, S36], probably due to the partial slipping of the horizontal reinforcement, which indicates that the horizontal bars were not effectively anchored into the columns.

Alvarez [A19] compared experimental results obtained from different research programmes, mainly in Mexico, conducted to evaluate the effect of the horizontal reinforcement placed in the masonry. Despite the large dispersion of the experimental data, he proposed an empirical equation in which the ultimate shear strength of the masonry wall increases in direct proportion to horizontal reinforcement ratio and the yield strength of the steel. The reinforcement ratio of the specimens considered in this comparative study ranged from 0% to 0.16%.

The cyclic behaviour of the infilled frames is significantly improved by horizontal reinforcement if the steel bars are properly anchored in the surrounding frame. In this case, the diagonal cracks are well distributed in the panel and the width of these cracks is smaller when compared with the case of unreinforced masonry panels [A8, S36]. The horizontal reinforcement prevents the premature disintegration of the infill panel, which increases the lateral deformation capacity and the ductility of the system [A15, B10, G5]. San Bartolomé et al. [S18] recommended that a minimum horizontal reinforcement ratio of 0.1% should be placed in the masonry panel to control the size of the diagonal cracks and to retard masonry deterioration under cyclic loading. Furthermore, the horizontal reinforcement enhances the strength of the masonry panel for resisting out-of-plane actions.

It must be mentioned that the presence of horizontal reinforcement anchored in the surrounding frame also modified the conditions of the panel-frame interface, producing a similar effect to that of the shear connectors (see section 5.8.7.2).

The behaviour of infilled frames with both vertical and horizontal reinforcement within the masonry panels has been studied by several researchers [B8, K9, L5, L14, M29]. Experimental results indicate that the strength and ductility of the structure is markedly improved. It can be considered that this system behaves in an intermediate stage between usual infilled frames and reinforced concrete walls. However, this type of constructive technique is not a common practice.

5.8.6 Gravity Loads
5.8.6.1 Transfer of the Gravity Loads
Gravity loads acting on the floor beams are compositely transmitted towards the foundations through the infill panels and the columns of the frame. The amount of vertical load transmitted by the panel depends on the relative stiffness of both components, the dimensions of frame and the conditions at the panel-
beam interfaces. In non-integral infilled frames, the presence of unintentional gaps between the floor beam and the masonry panel implies that the gravity loads are primarily transferred through the columns.

Several researchers [L10, R4, R10, S24, W3] have conducted experimental and analytical studies to evaluate the behaviour of vertically loaded masonry walls supported by simple or continuous beams. The vertical load is transferred to the beam near the supports, where concentration of compressive stress occurs. The beam and the masonry panel separate, or tend to separate, in the central region. This composite action is explained considering that the system behaves in a way similar to a tied arch. The wall arches across the span and the beam serves as a tie to prevent the arch from spreading [D8, S24]. However, this mechanism could not be totally valid for infilled frames because the panel is restrained by the frame and the gravity load is also transferred through the panel-column interfaces.

The investigation of the behaviour of infilled frames subjected to gravity loads is outside the scope of this work, although some basic characteristics are presented here. A two-storey infilled frame was analysed using a finite element model, in which vertical uniformed distributed loads were applied on the beams. Fig. 5.47 illustrates the deformed shape obtained from the analysis. It is observed that the different deformation fields induced in the frame and the masonry panels produce a partial separation between both components of the structure. This effect depends on the magnitude of the vertical load and the characteristics of the panel-frame interfaces. The separation is more significant in the upper storey, where the rotational flexibility of the beam-column joint is larger. Fig. 5.48 shows the principal stresses in the masonry panels of the same model. The flow of the principal stresses indicates that the tied arch mechanism does not develop completely in this case.

Figure 5.47. Deformed shape of a two-storey infilled frame subjected to gravity loads.
The mechanism of transfer of the gravity load is schematically represented in Fig. 5.49 (a). Part of the load acting on the panels is transferred to the columns through the interfaces along their length. The remaining part is resisted by the bottom beam. The axial forces in the columns increase from one storey to another and also increase from top to bottom of each column, as shown in Fig. 5.49 (b). The precise evaluation of the stress state induced by gravity loads requires the use of refined tools, such as finite element computer programs. However, for infilled frames subjected to seismic actions, the effect of gravity load is usually secondary. Therefore, simplified methods [W3] can be applied to estimate the effect produced on the infilled frame.

5.8.6.2 Effect on the Behaviour of Infilled Frames
The presence of gravity loads modified the stress state of the surrounding frame, especially in the columns, and in the masonry panels. The effect of this parameter in the different components of the structural systems is discussed in the following paragraphs:

- **Tension column:** the gravity loads reduce, or even eliminate, the tension force induced by the lateral forces. Therefore, the flexural and shear strength of the member increase.
- **Compression column:** the gravity loads increase the axial compressive force in the column. Thus, the flexural strength can either increase or decrease, depending on the level of the axial load. The increase of the compressive axial load enhances the shear strength of the column.
• **Beam**: gravity loads acting on the beam produce bending moments and shear forces. However, these efforts are usually small when compared with those produced by lateral forces.

• **Masonry panel**: the increase of axial stresses normal to the bed joints usually leads to a more favourable condition in the masonry panel, within the practical range of stresses induced by gravity loads (see section 4.1). This fact explains the experimental observation [F4] that the cracking force of the infilled frame increases when gravity loads are applied.

These observations are valid assuming that the lateral forces are the dominant loading condition. In other cases, the effect of gravity load may control the behaviour of the structure.

![Diagram](image)

**Figure 5.49.** Transfer of gravity loads in infilled frames and axial forces in the columns.

The influence of the gravity loads has been experimentally studied by several researchers [F4, M13, M29, S7, S37, V2]. The gravity loads considered in these tests were either uniform distributed loads acting on the beams or concentrated forces applied directly to the columns. The former case is more realistic for one-storey structures, where the gravity load is partly transmitted through the panel, whereas the latter case is more appropriate for multi-storey infilled frames, because most of the gravity load acting on the lower storey is resisted by the columns (see Fig. 5.49).
Stafford Smith [S37] investigated the behaviour of steel frames infilled with concrete panels subjected to gravity and lateral loads. He observed that the lateral strength of the structure increases when gravity load was applied, for gravity loads less than approximately one-half of the vertical compressive strength. The mode of failure was similar to that observed in laterally loaded infilled frames. For higher load values, the lateral strength decreases because the response is controlled by the compressive strength of the panel. This observation conceptually agrees with the behaviour of masonry panels subjected to normal and shear stresses (see section 4.1.2). According to the results reported by Stafford Smith [S37], and also by Meli [M29] for framed masonry, it can be considered that the lateral capacity increases almost linearly with the vertical load in the range of low to moderate gravity loads.

Valiasis and Stylianidis [V2] carried out a test program with reinforced concrete frames infilled with unreinforced masonry panels. The specimens were constructed in 1/3 reduced scale. The level of axial load applied to the columns was set to either 0 or 80 kN (which represents 14% of the compressive strength of the column). The specimens tested with axial load showed variable improvement of the lateral strength, with increases ranging between 26% and 91%. The more significant increase occurred for the infilled frame with a greater aspect ratio, \( h_m/L_m \).

The influence of gravity loads in the ductility of the infilled frame is not clear. Fiorato et al. [F4] reported that the ductility decreased due to the presence of gravity loads, whereas the tests conducted by Valiasis and Stylianidis [V2] showed significant increase of the ductility. The level of the axial load in the columns was similar in both tests, however, the system used by Fiorato et al. [F4] to apply the gravity load was not very suitable (see section 5.4.5). According to experimental results obtained by Meli [M13], the presence of gravity load enhanced the behaviour of framed masonry subjected to alternating lateral loads. When a low level of precompression (0.3 to 0.5 MPa) was applied to the specimens, the deterioration decreased for all the cases considered.

Paulay and Priestley [P1] pointed out that the beneficial effect of gravity loads can be neglected in the evaluation of the shear strength of the masonry panel. This assumption is adequate for non-integral infilled frames, in which it is not clear the amount of gravity load transferred through the roof beam-panel interface. The effect of gravity loads on the flexural capacity of the columns should be always taken into account.

Gravity loads acting eccentrically to the plane of the infilled frame induced out-of-plane bending moments in the structure. This effect can significantly reduce the strength of the system, especially for unreinforced masonry.

5.8.7 Conditions of the Panel-Frame Interface

5.8.7.1 General

The characteristics of the panel-frame interfaces depend on the properties of the materials and the constructive techniques used to build the infilled frame. When the masonry infill is built after the frame, unintentional gaps can form due to shrinkage of the infill material or to defects in workmanship. Therefore the masonry panel is not effective until these gaps are closed, producing the initial slackness observed in Fig 5.9, line B. After the frame has gained firm contact with the panel, there is no appreciable
difference [L9]. The effect of shrinkage can be minimized using adequate curing techniques [S21] or non-shrink mortar. Liauw and Kwan [L9] studied the effect of the friction developed at the panel-frame interfaces using nonlinear finite element models. The coefficient of friction was increased from 0 to 0.8, resulting in higher values of both the stiffness and strength, with an increase of the lateral resistance up to 20%. They also showed that the effect of friction is more important in the nonlinear range and suggested that the interface friction is very useful in dissipating energy. King and Pandey [K8] carried out a similar study and found that changes in the coefficient of friction had no significant influence on the behaviour.

The conditions at the panel-frame interface can be modified using shear connectors or toothed connections. The characteristics of these types of interface system are discussed in the following sections.

5.8.7.2 Shear Connectors

The effect of shear connectors between the frame and the masonry panel has been studied for several researchers and there is a general agreement about their beneficial influence in improving the structural behaviour. A comprehensible study of the effects of the shear connectors should include a quantitative evaluation of the connector properties relative to the infilled frame characteristics. Obviously, the behaviour of the system could be different depending on the type of shear connector and its stiffness and strength. According to the author's knowledge, this quantification has not been conducted yet. Despite this problem, some experimental observations are presented below.

The presence of connectors restrains the tendency of the infill panel to rotate inside the frame, increases the length of contact [M6] and causes a more spread distribution of the cracks. The cracking force usually is smaller [D17], and the strength of the structure generally increases [L8]. Experimental results indicate that, apart from the differences in the crack pattern and in the strength of the models, the interaction mechanisms and the modes of failure of integral and non-integral infilled frames are similar in most of the cases [K4]. However, some important differences have been observed. For example, Liauw and Lee [L8] reported that specimens with shear connectors showed no separation along the panel-frame interfaces.

Mallick and Garg [M11] noted that infilled steel frames with shear connectors were stiffer and stronger, and that the presence of connectors reduced the effect of lack of fit in reducing the initial stiffness. Furthermore, they observed that the specimens with shear connectors failed due to the crushing of one of the loaded corners after diagonal cracking occurred. The ultimate lateral force was usually higher than that for infilled frames without shear connectors. In this work, the spacing of the connectors did not appreciably affect the behaviour of infilled frames. However, this would not be regarded as a general conclusion because it was based on two tests only (with connectors spaced at 50 and 100 mm).

Liauw and Kwan [L1] conducted an interesting series of tests to compare the static and cyclic behaviour of infilled steel frames with and without shear connectors. Three different connectors were tested: U hooks with a separation of 30 and 50 mm, and J hooks with a separation of 50 mm. They reported that the specimens with shear connectors maintained the initial uncracked stage up to a higher force and more
abundant cracks were observed. As the corners of the panel crushed, the normal pressure at the interfaces evened out and propagated away from the corners. At the same time the shear strength of the interface was mobilized and the connectors yielded.

Cyclic tests indicated that infilled frames with shear connectors maintained a gradual and fairly slow degradation rate and presented much higher energy dissipation capacity due to the improved characteristics of the interface. The equivalent hysteretic damping ratio measured from cyclic test for infilled frames with shear connectors was initially smaller than that obtained from the same model but without connectors, in agreement with the results reported by Mallick and Severn [M6]. However when the deflection amplitude was increased the equivalent hysteretic damping ratio also increased and the models with connectors showed a higher ratio at large amplitudes. Measured values ranged from 2.5% to 12.8% of the critical damping depending on the type of structure, deflection amplitude and the cycle considered.

It must be pointed out that the presence of shear connectors could improve the stability of the infill panel inside the frame when the structure is subjected to out-of-plane actions. Even though there is insufficient experimental information about this topic, conceptual considerations indicate that it is difficult for the masonry panel to be shifted out of the frame when both are linked by connectors.

5.8.7.3 Toothed Connections
In framed masonry structures, the shear strength of the panel-frame interface can be improved using toothed connections [S7]. The panel is built first with toothed edges and then the surrounding frame is cast directly against the toothed edges (see Fig. 5.50). Therefore, the transfer of shear forces through the interface depends on a shear key mechanism rather than on the bond and the friction strength. It is worth noting that the effectiveness of this connection may be affected due to honey comb formations, which can reduce the lateral strength of the structures by up to 50% [S7]. Consequently, the size of the concrete aggregate should be adequately selected and vibration techniques should be used to avoid this problem.

![Figure 5.50. Toothed connection between the masonry panel and the reinforced concrete frame.](image-url)
Toothed connections are rarely used to improve the conditions at the panel-frame interface. There is clear evidence from experimental results and from observed behaviour during earthquakes, that good integration between the masonry panel and the surrounding frame can be achieved using framed masonry without toothed connections.

5.8.8 Unintentional Gaps in Non-integral Infilled Frames
Unintentional gaps between the masonry panel and the surrounding frame can develop in non-integral infilled frames due to shrinkage of the mortar and constructive problems. The presence of vertical gaps between the columns and the masonry panel provides slackness and reduced stiffness in the initial stage (see section 5.4.3). When the gaps are small, the effect on the response is not very important because these gaps close rapidly when the lateral force is applied and the diagonal strut mechanism develops. In other cases, however, the effect of the vertical gaps can significantly modify the response of the structure. For example, Fig. 5.51 illustrates the cyclic response of a two-bay infilled steel frame tested by Gergely et al. [G6], in which the shapes of the hysteresis loops indicates that unintentional gaps formed between the panel and the frame.

The effect of a horizontal gap between the panel and the beam can also be important because it modifies the transfer mechanism of the lateral force. When there is no contact between the beam and the masonry panel, the lateral force is mainly transferred by normal stresses developing in the contact zone between the columns and the panel. As a result, the stress state in the panel changes in comparison to that of the integral infilled frames (see Fig. 5.7), especially in the loaded upper corner zone.

![Graph](image)

**Figure 5.51.** Cyclic response of a two-bay infilled steel frame tested by Gergely et al. [G6].

Dawe et al. [D7, P10] tested a steel infilled frame, in which a 20 mm gap between the panel and the roof beam was provided. The results indicated that the cracking and ultimate force was reduced about 50% when compared with similar specimens without the gap. The failure of the masonry panel was mostly
Due to horizontal shear cracks. Even though in these tests the gap was intentionally provided, the results suggested that unintentional gaps may produce a shear sliding failure. A similar mode of failure was observed in one of the specimens tested by Gergely et al. [G6], in which the measured response illustrated in Fig. 5.51 clearly indicates that an intentional gap formed between the masonry panel and the surrounding frame.

Since there is insufficient experimental data related with this aspect, numerical simulations were conducted using the finite element method. The model considered in this study was similar to that used in the examples presented in the section 5.4.2, but a small gap was introduced in the interface between the panel and the roof beam. Several models were analysed, in which the width of the gap was changed from 0.01 to 1.0 mm. The numerical results indicate that for very small gaps the behaviour of the infilled frame is only slightly affected in the initial stage. When the lateral force increases, the gap closes in the zone near the corner and the interfaces are able to transfer normal and shear stresses. For wider gaps, the lateral force is primarily transferred to the masonry panel through the vertical interfaces. Consequently, the diagonal strut mechanism develops partially and the stress distribution in the panel changes. These results also showed that, in the zone close to the upper loaded corner, the principal compressive stress becomes almost horizontal. This fact explains the experimental observation that the presence of the horizontal crack may lead to a horizontal shear sliding failure in the masonry panel. Furthermore, the shear force in the tension column of the frame can significantly increase, because the transfer of the lateral force to the panel occurs primarily through the columns. It was not possible to obtain a general conclusion regarding the width of the gap which significantly affect the response. Numerical results indicated that this width depends on numerous parameters, such as dimensions of the masonry panel, mechanical properties of the materials, panel-frame interface conditions, etc. Furthermore, it was observed that the finite element analysis was very sensitive to the width of the gap considered in the analysis.

5.8.9 Presence of Openings
Openings usually appear in infilled frame buildings to provide adequate space for windows and doors. The effect of openings in the structural response is difficult to evaluate. Even though the experimental results available in the literature are contradictory in some cases, there is evidence that the dimensions of the opening and the position into the panel are the most important factors.

The influence of openings on the lateral force response of reinforced concrete frames with masonry infill panel was studied analytically by Durrani and Luo [D18]. Using finite element analysis, they considered different sizes of concentric openings and observed that the transfer of shear is possible with a diagonal strut mechanism for relative small openings. However, for large openings, the strut mechanism cannot develop.

Mallick and Garg [M11] tested infilled frames with square openings in different positions (the dimension of the openings was equal to one quarter of the length of the panel). They concluded that stiffness and especially the lateral strength reduce when the opening is located along the loaded diagonal. The most unfavourable situation occurs when the opening is provided in one of the loaded corners, being the lateral strength in this case about half of that of a solid panel. Similar results were obtained by Dawe et al. [D7,
D11, D17] and Mosalam et al [M36] from steel infilled frames with openings located in different positions. Both stiffness and strength were significant reduced, especially when the opening was located close to the tension column. They also observed that the cracking pattern in the masonry panel was drastically changed.

The analysis of infilled frames with opening is complex. The use of a simplified method is usually not possible and more sophisticated tools, such as computer programs based on the finite element method, are required. A detail investigation of the effect of the openings is outside the scope of this thesis. Additional information can be found in references [L8, M7, P11, T2].

5.8.10 Three-Dimensional Configuration of the Building
Most of the research related to infilled frames has been conducted on two-dimensional structures. Even though the conclusions obtained from these studies are useful, particular aspects associated with the three-dimensional behaviour should be also investigated. Unfortunately, there is almost no experimental information about three-dimensional behaviour of infilled frame buildings, except for a few cases in which full-scale specimens were tested under lateral loading [A9, N12]. However, some conclusions can be drawn from analytical studies and conceptual considerations.

Several researchers [B10, M16, P1] have pointed out the effects of the masonry infills, which can drastically modify the response of the structure due to changes in the stiffness and strength of the structural planes. Irregular distributions of the masonry panels, either in height or in plant, can produce unfavourable effects and change the expected mechanism of failure (see section 5.2).

The interaction mechanism between the masonry panel and the surrounding frame described in previous sections is also applicable for three-dimensional structures, however, the situation is more complex in this case. Under the action of skew seismic attacks, bi-axial bending moments can develop in the columns of the infilled frame accompanied with important modifications in the axial forces. These effects should be considered with caution.

The floor slab is an important factor in the spatial behaviour, influencing the torsional response of the building and the distribution of seismic forces among the structural planes. It is customarily assumed in the analysis of common buildings that the floor slabs behave as infinitely rigid diaphragms. However, the large stiffness of infilled frames can make it necessary to assume more realistic models, considering deformable floors.

5.9 CONCLUSIONS

Infilled frames are composite structures which exhibit a complex behaviour. The masonry panel provides stiffness to reduce the deformation demand, whereas the surrounding frame confers ductility. Due to the interaction between panel and frame, the strength of the resultant structure is usually greater than the sum of the two components separately. The influence of the masonry infill panel need to be considered when evaluating the structural response, otherwise unfavourable consequences can occur.
CHAPTER 5

The structural response of infilled frames subjected to lateral forces are markedly nonlinear. Four different stages may be usually distinguished. Initially, the structure behaves like a cantilever beam until separation occurs between the frame and the masonry panel. Then, a braced frame mechanism develops. The increase of lateral forces produces cracking of the masonry panel following different patterns. The failure of infilled frames occurs due to a combination of simple mechanisms of failure that form in both the masonry panel and the surrounding frame. Finally, the strength and the stiffness significantly degrade and the structure collapse.

The behaviour of multi-storey and multibay infilled frames is approximately similar to that observed for a single storey infilled frame. In multi-storey structures, the inelastic deformation concentrates mainly in the first storey. More experimental research is necessary to investigate the effect of concentration of damage and soft-storey mechanisms.

The envelope of the hysteresis loops obtained from cyclic tests is similar to the lateral force-displacement relationship for infilled frames subjected to monotonic loading. Usually, the hysteresis loops exhibit pinching and strength degradation.

Infilled frames exhibit a large energy dissipation capacity due to material damping, inelastic behaviour of the masonry panel and the surrounding frame, and friction at the panel-frame interfaces.

When infilled frames are adequately designed and constructed, brittle modes of failure can be avoided. In this case, it is possible to obtain a reasonably ductile response, even under alternating forces.

Infilled frames are redundant structures, therefore, the final mode of failure is complex and usually results as a combination of different types of failure that occur in both the masonry panel and the surrounding frame. Shear cracking, either along the mortar joints or through the brick, is the most common type of failure in masonry panels. However, compressive failure or flexural cracking may also occur. Failure of the reinforced concrete frame can be produced by axial or shear forces acting on the columns or due to a flexural collapse mechanism.

A large number of parameters affect the behaviour of infilled frames. For this reason, it is very difficult to compare experimental information obtained from different researchers and to draw qualitative conclusions. Experimental results indicate that the most important parameters affecting the response of infilled frames are the relative stiffness of the panel to the frame, the strength of the masonry infill panel and the flexural, shear and axial capacity of the surrounding frame.
It seems that an adequate response, especially for structures subjected to cyclic forces, is obtained from framed masonry built with solid bricks. The use of horizontal reinforcement in the panel also contributes to improve the behaviour.
6. REVIEW OF PROCEDURES FOR ANALYSIS OF INFILLED FRAMES

6.1 INTRODUCTION

The aim of this chapter is to review the approaches used for the analysis of infilled frame structures. The different techniques proposed in the literature for idealizing this structural type can be divided into two groups, namely, local or micro-models and simplified or macro-models. The first group involves the models in which the structure is divided into numerous elements to take account of the local effects in detail, whereas the second group includes simplified models based on a physical understanding of the behaviour of the infill panel. In the latter case, a few elements are used to represent the effect of the masonry infill as a whole. Both types of model will be discussed in the following sections.

It is evident from experimental observations that these structures exhibit a highly nonlinear inelastic behaviour, as discussed in Chapter 5. The most important factors contributing to the nonlinear behaviour of infilled frames arise from material nonlinearity. These factors can be summarized as follows:

- **Infill Panel:** cracking and crushing of the masonry, stiffness and strength degradation.
- **Surrounding Frame:** cracking of the concrete, yielding of the reinforcing bars, local bond slip.
- **Panel-Frame Interfaces:** degradation of the bond-friction mechanism, variation of the contact length.

Geometric nonlinear effects can also occur in infilled frames, especially when the structure is able to resist large horizontal displacements. However, these effects do not present any particularity and can be considered in the analysis using the same methodologies applied to reinforced concrete or steel structures. The nonlinear effects mentioned above introduce analytical complexities which required sophisticated computational techniques to be properly considered in the modelling. Furthermore, the material properties are difficult to define accurately, especially for masonry. These facts complicate the analysis of infilled frames and represent one of the principal reasons to explain why infill panels has been considered as "non-structural elements", despite the strong influence on the global response.

It is worth noting that infilled frame structures cannot be modelled as elasto-plastic systems due to the stiffness and strength degradation occurring under cyclic loading. More realistic models should be used to obtain valid results, especially in the dynamic analysis of short period structures, such as infilled frames, where the energy dissipation capacity and shape of the hysteresis loops may have strong influence in the response.
6.2 MACRO-MODELS FOR INFILLED FRAMES

6.2.1 Diagonal Strut Model

6.2.1.1 General Description

Polyakov (as reported by Klinger and Bertero [K9] and Mallick and Severn [M10]) conducted one of the first analytical studies based on elastic theory. From his study, complemented with tests on masonry walls diagonally loaded in compression, he suggested that the effect of the masonry panels in infilled frames subjected to lateral loads could be equivalent to a diagonal strut (see Fig. 6.1). Later, Holmes [H7] took up this idea and proposed that the equivalent diagonal strut should have a width equal to one third of the length of the panel. Stafford Smith [S16, S17] refined the approach and started a series of tests to investigate more precisely the width of the equivalent strut. This task was continued by many other researchers. Nowadays, the diagonal strut model is widely accepted as a simple and rational way to describe the influence of the masonry panels on the infilled frame.

![Figure 6.1. Diagonal strut model for infilled frames.](image)

When the structure is subjected to cyclic or dynamic loading, the use of only one diagonal strut resisting compressive and tensile forces cannot describe properly the internal forces induced in the members of the frame. In this case, at least two struts following the diagonal directions of the panel must be considered to represent approximately the effect of the masonry infill. It is usually assumed that the diagonal struts are active when compressive forces develop in them. However, compression only elements are not available in common elastic computer programs. In this case, Flanagan et al. [F3] recommend the use of tension-compression truss members with half of the equivalent strut area in each diagonal direction. The use of this simplified model results in significant changes in the internal forces in the surrounding frame, especially the axial forces in the columns (tensile forces decrease, whereas compressive forces increase).

The assumption of a compression only strut is acceptable on the basis that the bond strength at the panel-frame interfaces and the tensile strength of the masonry are very low. Tensile forces, therefore, can be transferred through the interfaces only for small levels of seismic excitation. This consideration may not be valid when either shear connectors are used at the interfaces or the masonry panel is reinforced with
horizontal or vertical bars. Refined models, however, can consider the tensile behaviour, which usually does not affect significantly the results.

### 6.2.1.2 Modification of the Diagonal Strut Model

The single diagonal strut model illustrated in Fig. 6.1 is simple and capable of representing the influence of the masonry panel in a global sense. This model, however, cannot describe the local effects resulting from the interaction between the infill panel and the surrounding frame. As a result, the bending moments and shear forces in the frame members are not realistic and the location of potential plastic hinges cannot be adequately predicted. For these reasons, the single diagonal strut model has been modified by different researchers, as illustrated in Fig. 6.2. For simplicity, the struts acting just on one direction have been indicated in this figure.

![Figure 6.2](image)

**Figure 6.2.** Modification of the diagonal strut model and multiple struts models.

Žarnić and Tomaževič [Z1, Z2, Z5] proposed the model illustrated in Fig. 6.2 (a) based on their experimental results. In these tests, the damage in the upper zone of the masonry panel occurred off the diagonal, probably due to perturbation introduced by the devices used to apply the lateral and vertical loads in the corners of the frame. Consequently, in the proposed model the upper end of the diagonal strut is not connected to the beam-column joint. This model could be applied in those cases where a shear failure develops at the top of the columns, although it does not represent the mechanism usually observed in laboratory tests.

Figs. 6.2 (b), (c) and (d) show multiple struts models proposed by Schmidt (as reported by König [K7]), Chrysostomou [C11] and Syrmakesis and Vratsanou [S39], respectively. The main advantage of these models, in spite of the increase of complexity, is the ability to represent the actions in the frame more accurately.
San Bartolomé [S7] proposed a nine-strut model for the analysis of framed masonry structures. A more complex model was developed by Thiruvengadam [T2] for the dynamic analysis of infilled frames. The model consists of a moment resisting frame with a number of pin joined diagonals and vertical struts uniformly distributed in the panel. These diagonals represent the shear and axial stiffness of the masonry infill. In order to take into account the partial separation at the panel-frame interfaces, the contact length is calculated and those ineffective struts are removed. In a similar way, the effect of openings can be considered by removing the struts crossing the opening area. Due to the complexity and refinement involved in this multiple strut model, it may be considered as an intermediate approach between the micro-models and macro-models.

The strut models presented above are not capable of describing the response of the infilled frame system when horizontal shear sliding occurs in the masonry panel (see sections 5.4.4 and 5.7.2.2). For this case, Fiorato et al [F4] proposed a "knee braced frame" to represent the behaviour, and Leuchars and Scrivener [L5] suggested the model illustrated in Fig. 6.3. The double strut can depict the large bending moments and shear forces induced in the central zone of the columns. Furthermore, it is possible to consider the friction mechanism developing along the cracks, which mainly controls the strength of the system. According to the author's knowledge, this model was just a suggestion, which was never implemented to verify its accuracy.

![Figure 6.3. Model suggested by Leuchars and Scrivener [L5] for describing the response of the infilled frame subsequent to horizontal shear sliding.](image)

Andreas et al. [A14] generalized the idea of the diagonal strut and assumed that masonry can be represented using a truss-like system, in order to generate a sort of finite element mesh formed by "cells" (see Fig. 6.4). Each of these cells represent a four node element, whose mechanical behaviour is defined by two truss members located along the diagonal directions of the element. This approach can be considered as a micro-model, due to the refinement involved in the representation of the structure. However, it is included here because the formulation of the model was based on the diagonal strut concept. D'Asdia et al. [D22] applied this approach to model infilled frame structures.
6.2.1.3 Properties of the Diagonal Strut

The use of the equivalent strut model is attractive from the practical point of view. Consequently, much experimental research has been directed to define the relationships between the characteristics of the infilled frame system and this simplified model. The properties required for defining the strut model depend on the type of analysis (linear elastic or nonlinear) and the type of loading (monotonic, cyclic or dynamic). For linear elastic analysis only the area and length of the strut, and the modulus of elasticity are needed to calculate the elastic stiffness. When nonlinear behaviour of the material is considered, the complete axial force-displacement relationship is required. Even more complex is the problem for cyclic or dynamic loading, because the hysteretic behaviour of the material must be established. In this section, only the evaluation of the elastic stiffness is discussed, whereas the hysteretic models are presented in the next section.

It is usually assumed that the ends of the diagonal members coincide with the intersection of the centre lines of the beams and columns of the surrounding frame. This implies that the diagonal length in the model is longer than the diagonal length of the masonry panels. The difference, however, is not significant in most of the cases. The thickness, $t$, and the elastic modulus, $E_m$, of the strut are equal to those of the masonry infill. The value of $E_m$ adopted in the analysis obviously depends on the stress level expected in the panels, since the behaviour of masonry is nonlinear, as indicated in section 3.4. There is some disagreement concerning to the width of the equivalent strut, $w$ (see Fig. 6.5), to be considered in the analysis. Numerous expressions have been suggested in the literature to calculate the width $w$ as a function of one or more properties of the infilled frame system. A complete review of these expressions is outside the scope of this work, although several of them are discussed here.

Two approaches have been used to calculate the equivalent width of the equivalent strut. The first approach is based on measurements from tests of infilled frame structures, whereas in the second procedure analytical results (for example, from finite element analysis) are utilized. Unfortunately, the conditions and methodologies considered in the evaluation of $w$ were not clearly reported in several investigations. This information is indicated in the following review, in those cases in which is available.
Figure 6.5. Effective width of the diagonal strut.

The first approximation to calculate the width of the equivalent strut was proposed by Holmes [H7], assuming that:

$$w = \frac{d_m}{3}$$  \hspace{1cm} (6.1)

where $d_m$ is the diagonal length of the masonry panel. Eq. 6.1 was proposed as the first approach to evaluate the equivalent width in the lack of experimental data.

Stafford Smith [S15, S16, S19, S22] was one of the most active developers of the equivalent strut model and conducted a large series of tests using infilled steel frames. Based on these results and considering also analytical data, he proposed different charts to calculate the equivalent width, $w$. In the first investigations [S16], it was found that the ratio $w/d$ varied from 0.10 to 0.25 depending on the relative value of the length and height of the panel. Later, Stafford Smith [S19, S22] included additional experimental information which was summarized in charts considering the parameter $\lambda_h$ (defined in Eq. 5.8) and the ratio $h_m/L_m$. Inspection of these charts shows that the ratio $w/d_m$ decreases for increasing values of either $\lambda_h$ or $h_m/L_m$.

Paulay and Priestley [P1] pointed out that a high value of $w$ will result in a stiffer structure, and therefore potentially higher seismic response. They suggested a conservative value useful for design proposal, given by:

$$w = 0.25 \ d_m$$  \hspace{1cm} (6.2)

This equation is recommended for a lateral force level of 50% of the ultimate capacity.
Mainstone [M5] conducted a series of tests using masonry panels surrounded by steel frames. Most of the tests were conducted on small scale specimens (h = 406 mm) diagonally loaded in compression. The following expression was obtained from these tests:

\[ w = 0.16 \lambda_h^{-0.3} d_m \]  

(6.3)

where \( \lambda_h \) is a dimensionless parameter defined in Eq. 5.8, which takes account of the relative stiffness of the masonry panel to the frame. Mainstone [M5] proposed also two other expressions for the evaluation of \( w \) in order to calculate the first-crack and the ultimate strength of the panel.

Liauw and Kwan [L9] analysed previous experimental data obtained from steel frames infilled with mortar and found that the test results could be approximated by the following expression:

\[ w = \frac{0.95 h_m \cos \theta}{\sqrt{\lambda_h}} \]  

(6.4)

The ratio \( h_m/L_m \) considered in this investigation varied from 1.0 to 1.5.

Fig. 6.6 illustrates the variation of the ratio \( w/d_m \) according to the previous expressions. In order to include in this figure the expression proposed by Liauw and Kwan [L9], Eq. 6.4, it is assumed that \( \theta \) is equal to 25° and 50°, representing the limit values for practical situations. Eqs 6.1 and 6.2 are independent of the parameter \( \lambda_h \) and they represent just an approximation useful for simplified analysis. Eqs. 6.3 and 6.4 indicate that the ratio \( w/d_m \) decreases when the parameter \( \lambda_h \) increases, because the stiffness of the masonry panel is large, when compared with the stiffness of the frame, and the contact length is smaller.

![Figure 6.6. Variation of the ratio w/d_m for infilled frames as a function of the parameter \( \lambda_h \).](image_url)
Based on results obtained from framed masonry tested under lateral forces, Decanini and Fantin [D13] proposed two sets of equations considering different states of the masonry infill:

Uncracked panel:

\[
\begin{align*}
  w &= \left( \frac{0.748}{\lambda_h} + 0.085 \right) d_m & \text{if } \lambda_h \leq 7.85 \\
  w &= \left( \frac{0.393}{\lambda_h} + 0.130 \right) d_m & \text{if } \lambda_h > 7.85
\end{align*}
\]  

(6.5)

Cracked panel:

\[
\begin{align*}
  w &= \left( \frac{0.707}{\lambda_h} + 0.010 \right) d_m & \text{if } \lambda_h \leq 7.85 \\
  w &= \left( \frac{0.470}{\lambda_h} + 0.040 \right) d_m & \text{if } \lambda_h > 7.85
\end{align*}
\]

(6.6)

Decanini and Fantin [D13] indicated that the modulus \( E_m \) to be used in the calculation of the parameter \( \lambda_h \) (see Eq. 5.8) is the modulus corresponding to the considered state (uncracked or cracked masonry). These equations are plotted in Fig. 6.7 as a function of the parameter \( \lambda_h \).

![Graph](image-url)

**Figure. 6.7.** Ratio \( w/d_m \) for framed masonry structures according to Decanini and Fantin [D13].

The principal advantage of the approach proposed by Decanini and Fantin [D13] is the distinction between the uncracked and cracked stages. This fact has been rarely considered by other researchers. Fig. 6.8 illustrates the ratio of the equivalent width in the cracked stage to that corresponding to the uncracked panel, according to Eqs. 6.5 and 6.6. It can be observed that \( w \) reduces significantly after
cracking to a value ranging from 50% to 80% of the initial width. The higher reductions occur for large values of the parameter $\lambda_m$ because the influence of the infill panel in the response of the system is greater in these cases.

![Graph](image)

**Figure 6.8.** Reduction of the equivalent width due to cracking of the masonry panel.

Bazán and Meli (as reported in reference [S7]), proposed also an empirical expression to calculate the equivalent width $w$ for framed masonry:

$$w = (0.35 + 0.22 \beta) \ h$$  \hspace{1cm} (6.7)

where $\beta$ is a dimensionless parameter defined in Eq. 5.10. This expression is valid when $0.9 \leq \beta \leq 11.0$ and $0.75 \leq h/L \leq 2.50$. Fig. 6.9 illustrates the ratio $w/d_m$ according to Eq. 6.7. It is difficult to compare the results obtained from Eq. 6.7 with previous expressions because they are related to two different parameters. Despite this fact, it is observed that Eq. 6.7 leads to higher ratios $w/d_m$ than Eqs. 6.5 and 6.6 in the case of stiffer masonry panels ($\lambda_m$ and $\beta$, in the range of 7 to 10 and 1 to 3, respectively).

![Graph](image)

**Figure 6.9.** Ratio $w/d_m$ for framed masonry structures according to Bazán and Meli [S7].
It is also important to note that the equivalent width for framed masonry is usually higher than that for non-integral infilled frames, according to the empirical equations presented above. This conclusion is not surprising since framed masonry exhibits better conditions, bond strength and friction, at the panel-frame interfaces.

The simplified expression proposed by Paulay and Priestley [P1], Eq. 6.2, can be considered as an upper limit for the ratio \( w/d_a \). The expressions recommended for framed masonry (Eqs. 6.5, 6.6 and 6.7) lead to higher results only when limit conditions are considered.

Stafford Smith [S15, S19] pointed out that the length of contact, \( z \), between the frame and the panel (see Fig. 6.5) can be used as a reference parameter to evaluate either the stiffness or the strength of the infilled frame. They found that the contact length is governed by the relative stiffness parameter, \( \lambda_h \), and proposed that \( z \) can be approximated by the following expression:

\[
z = \frac{\pi}{2 \lambda_h} h
\]

(6.8)

It is worth noting that Eq. 6.8 was developed from tests conducted on small specimens diagonally loaded in compression. The frames were built with mild steel flat bars of different sizes and the panels were made of mortar. The panel dimension were 150 x 150 x 19 mm. In the author's opinion, the validity of Eq. 6.8 for infilled frame structures should be verified considering more realistic experimental data.

For the model illustrated in Fig. 6.2 (a), Žarnič [Z5] proposed an analytical procedure to calculate the area of the strut. It was assumed that the axial stiffness of the brace is equal to the stiffness of the triangular part of the masonry wall (considering shear and flexural deformations). This triangular part forms in the wall after cracking of the masonry. Therefore, it is possible to obtain the area of the strut as a function of the geometric and mechanical properties of the masonry infill. The equation proposed by Žarnič [Z5], however, did not consider that both stiffnesses are related to different displacements (axial displacement of the strut and horizontal displacement at the top of the triangular part of the panel). As a result, one of the stiffness should be transformed as a function of the inclination of the strut.

Chrysostomou [C11] used a different approach to calculate the stiffness of the strut elements of his model, represented in Fig. 6.2 (c). The compressive force resisted by the masonry panel and their stiffness were calculated as a function of the storey drift, using a modification of the expression proposed by Sorourshian et al. [S29] for masonry shear walls. In order to define the properties of the three struts the following approach was implemented. The behaviour of the central strut was represented by expressions similar to those corresponding to the entire masonry panel. However, it was assumed that the central part of the infill panel deteriorates faster than the other parts. The properties of the off-diagonal struts were evaluated by considering that the forces and stiffnesses of the three struts should be equal to the force and stiffness of the entire masonry wall. The principle of virtual work was used to derive these expressions, assuming one particular displacement field. Chrysostomou's procedure to evaluate the properties of the off-diagonal struts implies that plastic hinges form only at the end of the columns or beams and that the
internal work produced in these plastic hinges is negligible. The influence of these hypotheses should be checked to verify the validity of the model.

6.2.1.4 Hysteretic Behaviour of the Diagonal Struts

In order to conduct nonlinear cyclic or dynamic analysis, the force-displacement relationships corresponding to the equivalent strut must be adequately defined. The representation of the hysteretic behaviour increases not only the complexity of the analysis but also the uncertainties of the problem, since there is not enough information about the cyclic response of masonry subjected to compressive and shear loading.

Klingner and Bertero [K9] developed three different hysteretic models to represent the diagonal strut, each of them involved a slight increase in the complexity. Fig. 6.10 illustrates the characteristics of the third model, in which the envelope was represented by a linear elastic ascending branch followed by an exponential descending curve. Unloading was assumed to be linear with stiffness equal to the initial stiffness, whereas the effect of stiffness degradation was considered for reloading. Even though the model assumed that tensile forces could be resisted by the strut, Klingner and Bertero considered no tensile forces in the numerical calculations conducted in their investigation. The comparison of the analytical results with experimental data showed poor agreement, although this model was the first approach to include the nonlinear response of infilled frames and represented the basis for further developments. The strength envelope proposed by Klingner and Bertero has been also used for nonlinear static analysis in order to represent the effect of strength degradation [C2].

![Figure 6.10. Hysteretic behaviour of the strut model proposed by Klingner and Bertero [K9].](image)

The hysteretic model proposed by Doudoumis and Mitsopoulos [D21] is shown in Fig. 6.11. This model was developed for non-integral infilled frames, in which a gap normally forms between the masonry panel and the surrounding frame. The envelope curve considered the effect of strength degradation. The hysteresis cycles were described in a very simplistic way assuming that reloading occurs following the elastic branch.
Figure 6.11. Hysteretic model developed by Doudoumis and Mitsopoulou [D21] for non-integral infilled frames.

Fig. 6.12 shows the force-displacement relationship adopted by Andreaus et al. [A14] for representing the mechanical behaviour of the diagonal struts. This model assumes that strength degradation starts immediately after the strength of the strut has been reached. Reloading occurs when the axial deformation is equal to the plastic deformation of the previous loop.

Figure 6.12. Force-displacement relationship assumed by Andreaus et al. [A14].

The comparison of the three hysteretic models illustrated in Figs. 6.10, 6.11 and 6.12 shows that the envelope curves are similar. The effect of strength degradation appears to be significant and has been considered in all the cases. The representation of the hysteresis loops, however, exhibits important differences. Doudoumis and Mitsopoulou [D21] (Fig. 6.11) assumed that reloading occurs following the initial loading branch. This assumption leads to fat hysteresis loops with high energy dissipation capacity. On the contrary, Andreaus et al. [A14] (Fig. 6.12) considered that unloading and subsequent reloading
follow the same line, which reduces considerably the area of the loops. The model proposed by Klingner and Bertero [K9] (Fig. 6.10) represents and intermediate situation, which includes also the effect of stiffness degradation.

A different approach was proposed by Soroushian et al. [S29] for masonry walls, which was later modified by Chrysostomou [C11] for representing the behaviour of masonry infills. The hysteretic response is modelled by combining two equations. The first equation (a logarithm exponential function) defines the strength envelope, whereas the second equation (a quartic polynomial function) represents the hysteretic loops, as shown in Fig. 6.13. These expressions, which describe the mechanical behaviour of the infill, were used to derive the force-displacement relationships for the central and off-diagonal struts of the model proposed by Chrysostomou [C11] (see Fig. 6.2 (c)).

![Diagram](image)

**Figure 6.13.** Hysteretic model proposed by Soroushian et al. [S29] for masonry shear walls and adopted by Chrysostomou [C11] for the diagonal strut.

Reinhorn et al. [R12] developed a hysteretic model which combines two mathematical functions to provide a smooth force-displacement relationship. Strength degradation, stiffness decay and pinching of the hysteresis loops can be considered by selecting the proper values of the nine parameters included in the model. Some of these parameters are empirical, whereas the others depend on energy considerations. The implementation of this model is not straightforward and the solution requires the numerical integration of a differential equation.

### 6.2.2 Storey Mechanism Model

The storey mechanism model is a simplified approach developed to investigate the global response of infilled frame structures. According to this approach, the response of a complete storey, or even the entire structure, is represented using a nonlinear relationship between the lateral force and the storey drift (or lateral displacement). The model does not consider any distinction between the frame and the masonry panel.
Moroni et al. [M33] used the storey mechanism model to conduct nonlinear static analysis with the objective of comparing the displacement capacity of infilled frames with that required by major earthquakes. Flores and Alcocer [F8] proposed a hysteric rule, which was calibrated from experimental results obtained from infilled frames with and without horizontal reinforcement into the masonry panel. The model was used for nonlinear dynamic analyses aimed at investigating the influence of several parameters in the response of infilled frames. Panagiotakos and Fardis [P22] proposed a shear force-storey drift relationship which has a multilinear envelope to represent the most important characteristics of the response (cracking, ultimate strength, post-ultimate falling branch and residual strength). The hysteretic response is controlled by three empirical parameters which define the unloading and reloading branches. These parameters were calibrated to model the pinching effect observed during tests of infilled frames. The general characteristics of the proposed hysteretic model is illustrated in Fig. 6.14.

![Shear force-storey drift hysteretic model proposed by Panagiotakos and Fardis (P22) for infilled frame structures.](image)

**Figure 6.14.** Shear force-storey drift hysteretic model proposed by Panagiotakos and Fardis [P22] for infilled frame structures.

### 6.2.3 Other Macro-Models

Despite the advantages of the diagonal strut model, other approaches have been used to analyse infilled frame structures. Most of these models were developed to evaluate the stiffness of the structure assuming elastic behaviour, consequently, their applicability is very limited.

The initial uncracked behaviour of infilled frames, providing that there is no gap between the frame and the masonry panel, can be evaluated by assuming that the total system behaves as a single monolithic member [F4, L5]. Therefore, the structure can be analysed using standard elastic theory considering the contribution of the flexural and shear deformations of the system to evaluate the horizontal displacements. The validity of this model, usually called "the beam analogy", depends on the bond strength developed at the panel-frame interface. Leuchars and Scrivener [L5] reported that the initial uncracked mode can resist forces up to 50% of the ultimate force. Using the same concept of the beam analogy,
Thiruvenkadam [T2] applied a shear-flexure cantilever model to evaluate the natural periods of infilled frames.

Smolira [S21] developed an approximate method for analysing infilled frame structures, assuming that the materials obey the Hooke's law. The indeterminate variables were the bending moments at the ends of the columns, the diagonal force in the masonry infill and the horizontal displacement. These values were evaluated using a set of equations obtained from conditions of compatibility of deformations and equilibrium of forces.

The equivalent frame method was proposed by Liauw and Lee [L8] for infilled frames with openings and shear connectors at the panel-frame interfaces. They assumed that an analogous model can be set up by representing the structure with an equivalent frame. Using the ratio of the elastic modulus of the two materials (masonry and concrete), the actual members are transformed into equivalent sections of infill material. The dimensions of the equivalent frame are obtained from the centroidal axes of the actual infilled frames. The validity of the method depends on the capacity of the shear connectors to sustain the composite action without allowing the separation of the infill.

An approximate substructuring technique, called constraint approach, was developed by Axley and Bertero [A17] to investigate the influence of masonry infills in reinforced concrete frames. Three steps were considered in the formulation of the constraint approach. Firstly, the system is modelled separately, using finite elements for the frame and the panel, and the stiffness matrixes are formed. Then, the stiffness of the masonry infill is reduced to the boundary degrees of freedom by applying static condensation. In the final step, a new transformation is conducted to obtain the stiffness matrix of the panel related to the degrees of freedom of the four corners, which are also the degrees of freedom of the frame. For the transformation to be possible, a constraint relation between the displacements at the boundaries of the panel and those of the corners must be adopted. Obviously, the accuracy of the approach depends on the constraint relation assumed in the formulation. Axley and Bertero [A17] considered two sets of constraints: conforming constraints (which assure the deformation of the infill will be contained within that of the frame) and nonconforming constraints (based on experimental observations). The method was applied to conduct linear elastic analyses and was implemented as a four node element in a computer program for structural analysis. Even though this technique was formulated on the basis of the finite element method, it can be considered as a macro-model from a practical point of view.

Pires et al. [P18] idealised the infilled frames using a parallel association between frame and the masonry infills (represented by a shear cantilever beam), as shown in Fig. 6.15. Both parts of the model were connected with rigid links. Nonlinear behaviour was considered for the reinforced concrete frame and masonry infill. This model, however, does not consider that separation between the frame and the panel occurs when the lateral load increases. It is believed that this factor may completely distort the behaviour of the model when compared with the real structure. Furthermore, the complexity of the model increases significantly for large infilled frames.
Figure 6.15. Structural idealization of the infilled frames proposed by Pires et al. [P18].

Valiassis et al. [V3] developed a phenomenological model in which the relationship between the average shear stress in the panel and the angular deformation was defined. The envelope was represented by two linear ascending branches and one exponential descending branch. The hysteresis rules incorporated the effects of stiffness degradation and pinching using linear branches to approximate the unloading-reloading curves. This model was based on experimental data obtained from bare and infilled frames tested under alternate cycles of gradually increasing lateral displacements. It was assumed that the contribution of the infill panel is equal to the difference of the response of the infilled and bare frame for each level of lateral displacement. Valiassis et al. [V3] pointed out that this model includes the influence of the interaction between the masonry panel and the frame and is strongly dependent on the geometric and mechanical characteristics of both the surrounding frame and the infill panel. Posteriorly, Michailidis et al [M31] implemented this model into a general program for the analysis of plane structures. The entire masonry panel was represented by a four-node isoparametric element with two degrees of freedom at each node.

6.3 MICRO-MODELS FOR INFILLED FRAMES

6.3.1 Finite Element Models

6.3.1.1 Introduction

The finite element method has been extensively used for modelling infilled frame structures, since Mallick and Severn [M10] applied this approach in 1967. Due to the composite characteristics of infilled frames, different elements are required in the model: beam or continuum elements for the surrounding frame, continuum elements for the masonry panel and interface elements for representing the interaction between the frame and the panel. Finite element models exhibit obvious advantages for describing the behaviour of infilled frames and the local effects related to cracking, crushing and contact interaction. This implies a greater computational effort and more time in preparing the input data and in analysing the results. It is worth noting the importance of defining the constitutive relationships of the elements. For the model to be realistic, the nonlinear phenomena which occur in the masonry infill and in the panel-frame interfaces must be adequately considered. Otherwise, the validity of the results is jeopardized, despite the great computational effort involved in the analysis.
Even though three-dimensional continuum elements are available for the analysis, it is commonly considered that the use of two-dimensional continuum elements leads to acceptable results. In this case, a state of plane stress is a reasonable assumption for most of the cases of in-plane loading [A18].

The modelling of the frame and the masonry panel with finite elements has been amply investigated. Only a brief description of these models is presented here. More attention is given to the modelling of the panel-frame interfaces, which represents a particular characteristic of infilled frame structures.

6.3.1.2 Modelling of the Masonry Panel
The analytical model used for the masonry panel should reflect the nonlinear nature of this material and the influence of the mortar joints. Different approaches have been implemented for representing the masonry panel, which are primarily based on the modelling techniques developed for concrete and rock mechanics [S32]. Nevertheless, the behaviour of masonry is more complex due to the planes of weakness introduced by the mortar joints. These approaches can be grouped according to the level of refinement involved in the model [L12, P19].

The first approach is the least refined, in which the masonry is represented as a homogeneous material. Consequently, the effect of the mortar joint is considered in an average sense. This approach is suitable for modelling large masonry structures, where a detail stress analysis is not required. The material model should represent the mechanical behaviour of masonry by adequately defining the stress-strain relationship and the failure criterion. Several failure criteria specifically developed for masonry structures are described in section 4.4. Other criteria have also been used, for example, the Von Mises criterion with tension cut-off [P19] or the Drucker-Prager criterion [M14].

In the second approach, masonry is represented as a two-phase material. Both masonry units and mortar joints are modelled with continuum elements [A18, L13]. The model usually requires a large number of elements and the mechanical behaviour of masonry units and mortar is separately defined. Interface elements should be used to represent the mortar-brick interfaces, where debonding, slip or separation can occur. The model is capable of capturing the different modes of failure, if it is adequately implemented and calibrated. Analyses with such level of refinement require a great computational effort. Consequently, this approach is mainly applied to small structures, usually as a research tool.

The third approach used for modelling masonry panels represents an intermediate situation between the two previous approaches. In this case, masonry units are represented with continuum elements, while the mortar joints are modelled with interface elements [L12, M19, P20]. The interface elements not only represent the behaviour of the mortar-brick interfaces, but they also take into account the elastic and plastic deformations occurring in the mortar. In the initial implementation of this approach, conducted by Page [P20], the masonry units were assumed to behave elastically. Later developments of the methodology allow the consideration of a more realistic behaviour for the masonry units, including cracking.

Cracking is an important feature that should be also considered in the analysis, independent of the approach used in the discretization of the panel (masonry as homogeneous material or as two-phase
The smeared crack model is commonly implemented for considering the effect of cracking [K7, M14, M19]. This model does not track each individual crack. Instead, the overall cracking within an area is simulated by changing the stress and the material stiffness associated with the integration points. Schnobrich [S44] pointed out that there are some doubts about the independence of the solution relative to the grid size used in the analysis (mesh sensitivity). Furthermore, the use of low order finite elements (for example, constant strain triangles) may confuse the cracking situation, due to the inadequate characteristics of these elements to respond to steep stress gradients. It is worth noting that the smeared crack model is a valid tool only for those structures where multiple cracks occur and the response is not sensitive to the precise geometry of cracking. This model should not be used in problems where a few isolated cracks control the behaviour. Furthermore, Shing et al. [S32] pointed out that the smeared crack approach alone is not able to capture the brittle shear failure of masonry panels and to account for the influence of the mortar joints.

### 6.3.1.3 Modelling of the Surrounding Frame

The analytical representation of the frame can be done either with beam elements [D12, K8, M6, M9, M10, M11] or with a more refined discretization using continuum elements (two or three-dimensional elements) [K7, M14, M19, R3]. The use of these different representations implies increasing levels of complexity in the analysis, resulting in a better accuracy when the model is properly implemented.

Beam elements are line elements, whose stiffness is associated with the deformation of the beam axis. These deformations are the curvature change and the axial deformation (torsion is also considered in three-dimensional elements). The main advantage of the beam elements is that they are geometrically simple and have a few degrees of freedom. The effect of the steel bars, in reinforced concrete members, is implicitly considered in the definition of the flexural and axial relationships assumed in the analysis. When nonlinear analysis is conducted, the effect of slip of the reinforcing bars can be also taken into consideration using rotational springs located at the ends of the member [F2].

The use of continuum elements for modelling the frame allows a better description of its behaviour, although many more elements are needed in the discretization. Reinforced concrete members require additional elements to represent the effect of the reinforcing bars. This can be done by using a smeared overlay or discrete bar elements, assuming a hypothesis for the strain compatibility between the steel and the concrete [K7, M14, M19].

### 6.3.1.4 Modelling of the Interfaces

The structural interfaces between the surrounding frame and the infill panel have been represented in the analytical models by using tie-link or interface elements. The function of these elements is to represent the interaction between deformable structures, along surfaces, where separation and sliding may arise. They allow for geometric discontinuity to occur in the structure. The adequate description of the contact effects developing at the panel-frame interfaces is very important to obtain a realistic response of the model.

The first attempt to take into account the behaviour of the interfaces was developed by Mallick and Severn [M10]. They implemented an iterative scheme using a finite element model, in which additional
contact forces were introduced in those zones where the panel-frame interfaces were closed. Several researchers [D12, F6, M9] used tie-link elements to connect the boundary nodes of the panel with the surrounding frame. These elements enable two adjacent nodes to be held together or released according to specified conditions. Each node of the element has two translational degrees of freedom. The element is able to transfer compressive and bond forces, but incapable of resisting tensile forces. Large values of the normal and tangential stiffnesses are adopted when the link is active. Conversely, the link is released by setting these values to zero. Fig. 6.16 illustrates schematically the characteristics of the tie-link model.

![Figure 6.16. Tie-link element used to represent the behaviour of panel-frame interfaces.](image)

A more accurate description of the interaction between the panel and the frame can be achieved by using interface elements [A12, K7, K8, M14]. These elements were introduced by Ngo and Scordelis in the area of concrete mechanics and by Goodman et al. in the area of rock mechanics (as reported by Lofti and Shing [L12]). Each element requires at least four nodes (with translational degrees of freedom in the directions normal and tangential to the interface) to represent two adjacent surfaces, as schematically illustrated in Fig. 6.17 (a). The degrees of freedom are related to the normal and shear stresses developed between the surfaces, $f_n$ and $\tau$, respectively. The traction transmitted between the surfaces and the relative displacement can be represented using different constitutive relationships. However, the friction theory proposed by Coulomb is usually implemented.

King and Pandey [K8] developed a modified interface element in which one of the surfaces presents two perpendicular, rigid links to represent the depth of the frame member (see Fig. 6.17 (b)). The nodes related to the rigid links have also a rotational degree of freedom. This modification is useful when the surrounding frame is modelled with beam elements. A similar approach was implemented by Liaum and Kwan [L2].

Mosalam et al. [M14] pointed out that interface elements may be sensitive to the mesh implemented in the analysis. Consequently, the characteristics of the finite element mesh must be carefully selected. A preliminary study with different mesh configurations is recommended.
The constitutive model implemented in the interface element must assure the impenetrability condition, when the surfaces are in contact. The normal stress is usually defined with a linear elastic model. However, the finite value of the normal stiffness violates the impenetrability requirement. By taking a large value of the normal stiffness (in relation to other stiffnesses in the model), this violation is not significant [M14].

The friction theory proposed by Coulomb in 1781 is commonly accepted to represent the behaviour of the interfaces. This theory asserts that "relative sliding between two bodies in contact along plane surfaces will occur when the net shear force parallel to the plane reaches a critical value proportional to the net normal force pressing the bodies together. The constant of proportionality is called the coefficient of friction" [O2].

Oden and Pires [O2] pointed out that the Coulomb's theory is capable of describing only friction effects between rigid bodies. They also considered that is very important to represent adequately the non-local character of the mechanism by which normal stresses are distributed along the surfaces. The stresses are transmitted over junctions formed by asperities on the contact surface. Small tangential displacements occur due to the elastic and inelastic deformation of these junctions. They indicated that, from the mathematical point of view, Coulomb's theory introduced problems in the formulation of the model. Oden and Pires [O2] formulated a non-local friction law, which proposed that "impending motion at a point of a contact between two deformable bodies will occur when the shear stress at that point reaches a value proportional to a weighted measure of the normal stresses in a neighbourhood of the point". An additional parameter was introduced to consider a small, but non-zero, elastic tangential displacement for shear stresses below the sliding limit. Further discussion about the implementation of the non-local theory is outside the scope of this work.
It is worth noting that Coulomb's theory does not consider any shear bond between the surfaces in the shear friction mechanism. This effect is not significant in the case of steel infilled frames. However, the effect of the shear bond strength could be important for mortar-concrete or brick-concrete interfaces, especially in framed masonry. Despite this problem, Coulomb's theory has been widely used in the implementation of interface models.

According to Coulomb's theory, the adequate modelling of the behaviour of structural interfaces requires the consideration of three different stages:

- **Firm contact and no slip**: when the surfaces are in contact, a compressive normal stress, \( f_n \), develops at the interface. It is considered that slip does not occur if the shear stresses, \( \tau \), satisfies:

\[
|\tau| < \mu f_n
\]  

(6.9)

where \( \mu \) is the coefficient of friction of the interface and \( f_n \) is positive when the surfaces are compressed. The condition of no slip is usually approximated by elastic behaviour and the friction theory is implemented with the stiffness method. Consequently, some relative motion (an "elastic slip") can occur between both surfaces. Permitting a relative motion when the surfaces are stuck makes convergence of the solution more rapid, at the expense of solution accuracy. It must be pointed out that no shear deformation should occur when the surfaces are in contact and Eq. 6.9 holds, since the interface has no thickness. This fact indicates that, in a strict sense, the shear stiffness of the interface should be infinite.

Different approaches have been implemented to define the shear stiffness of the interface. The non-local friction models assume that impeding motion at a point of contact between deformable bodies will occur when the shear stress at a point reaches a value proportional to a weighted measure of the normal stresses in a neighbourhood of the point [O2]. In the model implemented in the program ABAQUS [A12], the shear stiffness is chosen in order that the elastic slip (when \( \tau = \mu f_n \)) is limited to an allowable value. This value is selected as a small fraction (for example, 0.005) of the length of the interface element. The shear stiffness will change during the analysis because it depends on the normal stress \( f_n \). It can be shown that this implementation represents a non-local friction model.

King and Pandey [K8] suggested the use of experimental data to define the shear stress of the interface. They reported values of the shear stiffness, which were obtained from tests of square specimens sliding on steel or concrete surfaces. However, it is not clear what method was used for the evaluation of the shear stiffness. Consequently, the validity of these results cannot be discussed.

Other formulations can be implemented, instead of the stiffness method, to assure that zero relative slip will occur at this stage. For example, it is possible, using by a Lagrange
multiplier method, to impose constraints or to prescribe relationships between degrees of freedom in the mathematical formulation of the model [A12].

- **Firm contact with slip**: the surfaces remain in contact, although slip occurs because the shear stress is equal to the friction strength of the interface:

\[
|\tau| = \mu f_n
\]  
(6.10)

It is usually assumed that the shear stress remains constant (for a constant level of normal stress) as the surfaces slip. However, there is experimental evidence that the coefficient of friction for mortar-brick interfaces decreases after the strength has been achieved (see section 4.1.4.5). Similar behaviour could be exhibited by other materials as concrete or steel. The decrease of the shear stress after slip occurs could be more significant for those interfaces which exhibit shear bond strength.

- **No contact**: in this stage the surfaces are separated. Neither normal nor shear stresses develop at the interface. It is usually assumed that separation occurs when tensile stresses develop in the normal direction [M9, M10]. Other models [A12] consider that tensile stresses can be resisted without separation up to some allowable level. In this way, it is possible to represent the effect of tensile bond strength between both surfaces.

The models described in this section indicate that adequate analytical tools have been developed for representing the behaviour of the interfaces. Unfortunately, this effort has not been accompanied by a similar improvement in the knowledge of the mechanical properties of panel-frame interfaces. In the author's knowledge, there is no data on the tensile and shear bond strength of the interfaces. Only one report has been found with experimental values of the coefficient of friction. These values, obtained by King and Pandey [K8], are presented in Table 6.1. It is believed that some of these results should be revised. For example, the coefficient of friction for concrete on concrete is significantly smaller than that for brick on concrete, which seems to be unrealistic.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Coefficient of friction, ( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick on steel</td>
<td>0.50</td>
</tr>
<tr>
<td>Mortar on steel</td>
<td>0.44</td>
</tr>
<tr>
<td>Concrete on steel</td>
<td>0.41</td>
</tr>
<tr>
<td>Brick on concrete</td>
<td>0.62</td>
</tr>
<tr>
<td>Mortar on Concrete</td>
<td>0.42</td>
</tr>
<tr>
<td>Concrete on concrete</td>
<td>0.44</td>
</tr>
</tbody>
</table>
The modelling of panel-frame interfaces with shear connectors is more complex. It has been suggested [D12, M9] that the tie-link elements can be used to simulate the effect of shear connectors or steel reinforcement connecting the masonry panel and the frame.

6.3.2 Other Micro-Models
The early analytical investigations related to the behaviour of infilled frames were based on the application of the theory of elasticity. For example, Stafford Smith [S16] used a finite difference approximation to solve the stress function for the masonry panel. Polyakov [P9] found the stress distribution inside the panel by using variational methods. With the improvement of computer capabilities and the development of the finite element method these types of model have been discarded.

Other researchers [D16, M8, S43] have applied the distinct element method, originally developed for fractured rocks, to the analyses of masonry walls. This method allows the study of jointed media subjected to static or dynamic loads. The media is simulated as an assemblage of discrete blocks, which interact through edge contacts. The mathematical model is established by considering two set of equations. The first group of equations is formed by constitutive relationships between the force in the blocks and the relative displacement. The second group represents the kinematic equations which define the motion of the blocks [S43]. This numerical method is capable of following the complete process of fracture. A further discussion about this analytical procedure is outside the scope of this work.

6.4 Evaluation of the Strength of Infilled Frames
6.4.1 General
This section describes the simplified methods and equations specifically proposed for the design and assessment of infilled frame structures. Early attempts to evaluate the strength were made by Stafford Smith [S15, S19, S22] and Mainstone [M5], who proposed empirical expressions derived from tests on infilled steel frames diagonally loaded. These expressions allow the calculation of the diagonal forces which produce cracking or crushing of the masonry panel as a function of the parameter $\lambda_s$ (see Eq. 5.8) and the dimensions of the panel.

Plastic theory has been used for several researchers [K4, L4, L7, M7, W2] to evaluate the shear strength of infilled frame structures, especially when steel frames are considered. The main problem of this procedure arises from the fact that masonry does not behave, even approximately, as a plastic material. Plastic theory can be an useful tool when the failure is due to a flexural mechanism in the surrounding frame with crushing at the corners of the panel. However, other types of failure are difficult to represent.

Decanini and Fantin [D13, D15] proposed a series of empirical equations to estimate the capacity of the equivalent compressive strut for four different modes of failure, namely, diagonal tension, sliding shear, crushing of the corners and diagonal compression. The general expression to calculate the compressive strength of the equivalent strut, $R_s$, is given by:

$$R_s = f_m \times w$$  \hspace{1cm} (6.11)
where \( f'_m \) is a fictitious compressive strength for each mode of failure, depending on the parameter \( \lambda_n \) (see Eq. 5.8) and on the mechanical properties of the masonry.

Several expressions proposed for the design and assessment of infilled frames are discussed in detail in the following sections. Emphasis is given to equations derived from rational grounds. It is worth noting that empirical procedures are valid only for materials and conditions similar to those considered when they were obtained. In this review, the original notation has been modified in order to obtain a consistent set of expressions which can be easily compared.

### 6.4.2 Sliding Shear Failure of the Masonry Panel

Stafford Smith and Riddington [S23] investigated the stress state in infilled frames and concluded that the sliding shear failure initiates at the centre of the panel when the shear strength of the mortar joints is exceeded. Introducing in Eq. 4.5 the values of the shear and normal stresses obtained from stress analyses [R3], they found that the shear force to initiate sliding shear, \( V_s \), is:

\[
V_s = \frac{\tau_o A_m}{1.43 - \mu \left( 0.8 \frac{h_m}{L_m} - 0.2 \right)} \tag{6.12}
\]

where \( A_m \) is the area of the masonry panel in the horizontal plane. Priestley [P23] proposed a similar equation to fit the results obtained by Stafford Smith and Carter [S19], which agrees very well with Eq. 6.12 considering a coefficient of friction \( \mu = 0.3 \).

A more rational approach was developed by Paulay and Priestley [P1], who calculated \( V_s \) based on Eq. 4.5, and considering that the clamping force acting on the potential sliding surface is provided by the vertical component of the diagonal compressive force \( R \) (see Fig. 6.5). Therefore:

\[
V_s = \tau_o A_m + \mu R \sin \theta \tag{6.13}
\]

Substituting \( R = (V_s / \cos \theta) \) and \( \cos \theta = (L_m/d_m) \) in the previous equation:

\[
V_s = \frac{\tau_o A_m}{1 - \mu \frac{h_m}{L_m}} \tag{6.14}
\]

Fig. 6.18 illustrates the variation the shear force \( V_s \) as a function of the aspect ratio \( h_m/L_m \), according to Eqs. 6.12 and 6.14. The values of the shear force are expressed in relative terms, \( V_s / (\tau_o A_m) \), which represents the ratio of the average shear strength in the panel to the bond shear strength, \( \tau_o \). It is observed that \( V_s \) increases with the aspect ratio as a result of the clamping action over the bed joints. Eq. 6.14 leads to significantly higher results because in the derivation of this equation it was assumed that a uniform distribution of the shear stress occurs along the transverse section of the panel, whereas the maximum value at the centre of the masonry panel, equal to \( 1.43 V_s / A_m \), was considered in Eq. 6.12. Paulay and
Priestley [P1] recommended a value of $\mu = 0.3$ for design purposes, although experimental results indicate that the coefficient of friction usually varies from 0.7 to 0.85 (see section 4.1.4.5). When these values are considered, the shear strength $V_s$ obtained from Eq. 6.14 is extremely large. Despite the rational grounds involved in Eq. 6.14, it seems that this expression can result in an overestimation of the shear force $V_s$, especially when medium to large values of the coefficient of friction are considered. Therefore, the validity of Eq. 6.14 should be revised.

![Graph showing the variation of shear force $V_s$ with $h_m / L_m$](image)

**Figure 6.18.** Variation of the shear force $V_s$ according to Eqs. 6.12 and 6.14.

The effect of the gravity loads acting on the masonry panels has not been considered in Eqs. 6.12 and 6.14. In framed masonry, this beneficial effect could be taken into account when evaluating the shear strength of the mortar joints. However, there is no clear criterion to estimate the amount of gravity load transferred through the masonry panel (see section 5.8.6.1).

Priestley [P23] pointed out that the lateral strength of the infilled frame, $V_u$, results by the combined strength of the masonry panel and the shear resisted by the columns:

$$V_u = \frac{2(M_{uc} + M_{ut})}{h_e} + V_s$$  \hspace{1cm} (6.15)

where $M_{uc}$ and $M_{ut}$ are the flexural strength of the compression and tension columns, respectively, and $h_e$ is the effective height between plastic hinges. The effect of axial forces resulting from overturning moments and gravity loads should be considered in the evaluation of the flexural strength. When a horizontal sliding shear failure is expected, plastic hinges may form approximately at the centre of the columns (see Figs. 5.34 and 5.42). In this case, $h_e$ can be significantly smaller than the column height, increasing the shear force $V_u$. For multibay infilled frames, the shear capacity of all the columns and panels should be considered in Eq. 6.15.
The shear friction mechanism in the masonry panel degrades rapidly under cyclic loading. For this reason, Priestley [P23] indicated that the shear resistance of the infilled frame should be conservatively calculated taking into account only the contribution of the surrounding frame. This condition represents a lower limit of the strength of the structures, likely to apply only after severe cycling at large displacements.

6.4.3 Diagonal Tension Failure of the Masonry Panel

Stafford Smith and Riddington [S23] calculated the shear force, $V_t$, at which a diagonal crack occurs considering an approximate value of the principal tensile stress at the centre of the masonry panel obtained from finite element analyses [R3]. Thus, they found that:

$$V_t = 1.72 \ A_m \ f_{tm}$$  \hspace{1cm} (6.16)

where $f_{tm}$ is a permissible tensile stress in the masonry panel, whose value was recommended to be 0.07 MPa.

Priestley and Calvi [P17] derived an expression to evaluate $V_t$ using the relationship for tensile stress in a disk diagonally loaded:

$$V_t = \frac{\pi}{2} \ d_m \ t \ f_{tm} \ \cos \theta = \frac{\pi}{2} \ A_m \ f_{tm}$$  \hspace{1cm} (6.17)

where $f_{tm}$ is the "direct tensile strength of the panel". It is important to mention that Eq. 6.17 is based on a relationship derived from elastic stress analyses considering a homogeneous isotropic material. The validity of this relationship for an anisotropic material like masonry is not assured. Furthermore, it is not clear how to evaluate the "direct tensile strength of the panel". Direct tensile tests are difficult to perform and the failure can occur by debonding of the mortar joints, as Fig. 4.12 indicates, depending on the direction of the tensile force. It seems more appropriate to evaluate $f_{tm}$ from splitting tests of small masonry panels, in which the angle between the applied force and the bed joint direction is similar to the angle $\theta$ in the infilled frame.

A similar criterion was used by Sancinejad and Hobbs [S38], who adapted the elastic analysis of a cube under diagonal load, described by Chen [C15], and proposed:

$$V_t = 2 \ \sqrt{2} \ h_m \ t \ f_{tm} \ \cos^2 \theta = \sqrt{2} \ A_m \ f_{tm} \ \sin 2\theta$$  \hspace{1cm} (6.18)

where $f_{tm}$ is the "effective direct tensile strength of the infill". The same problems discussed for Eq. 6.17 also apply in this case. The difference between Eqs. 6.17 and 6.18 mainly arises from the fact that, according to elastic and limit analyses [C15], the splitting force in a cylinder is 1.34 times greater than that in a diagonally loaded cube. Fig. 6.19 compares numerical results obtained from Eqs. 6.16, 6.17 and 6.18, expressed as the ratio of the average shear stress in the panel to the tensile strength $f_{tm}$. Eqs. 6.16 and 6.17 are independent of the aspect ratio of the panel, $h_m/L_m$ (or the angle $\theta$). The aspect ratio has a direct influence on the ratio of the normal to shear stress, $f_d/t$, as Eq. 5.5 indicates, and could also affect...
the shear resistance under diagonal tension failure. The shear force \( V_e \) evaluated according to Eq. 6.18 exhibits a maximum value for \( h_m/L_m = 1 \) (or \( \theta = 45^\circ \)) and decreases for other values. This variation seems to be contrary to the results obtained from Mann and Müller's theory and the proposed failure theory presented in section 4.2.4 for the evaluation of the shear strength of masonry panels.

![Comparison of the cracking shear force, \( V_e \), according to Eqs. 6.16, 6.17 and 6.18.](image)

**Figure 6.19.** Comparison of the cracking shear force, \( V_e \), according to Eqs. 6.16, 6.17 and 6.18.

### 6.4.4 Compressive Failure of the Masonry Panel

Early research conducted by Stafford Smith [S15, S22] on steel frames infilled with concrete panels showed that the diagonal force to produce the compressive failure of the panel, \( R_e \), can be estimated as a function of the contact length \( z \) (see Eq. 6.8), the dimensions of the panel and the compressive strength \( f_m' \).

\[
R_e = z \times f_m' \sec \theta 
\]  

(6.19)

This equation was originally developed for square infilled frames, assuming that the loaded corners of the infill panel are in a plastic state, at a uniform stress \( f_m' \) and a linear distribution of the contact forces in the panel-frame interfaces. Later, it was modified to consider rectangular infilled frames [S22]. The shear force \( V_e \) at which crushing occurs can be calculated from Eq. 6.19, considering that \( V_e = R_e \cos \theta \). Therefore:

\[
V_e = z \times f_m' 
\]  

(6.20)

which agrees with the experimental observation that \( V_e \) was independent of the aspect ratio of the concrete infill. The validity of this observation for masonry infilled frames has not been proved.
Stafford Smith and Carter [S19] pointed out that this type of failure is unlikely to occur in masonry panels due to the weakness of other modes of failure, such as sliding shear or diagonal tension failure. In order to generalize the proposed design procedure, they suggested that $R_e$ can be estimated from Eq. 6.20 assuming that the compressive strength of the masonry panel is equal to the compressive strength of the mortar. Later, Stafford Smith and Riddington [S23] recommended the use of the empirical expression proposed by Mainstone [M5] to calculate the shear force at which compressive failure occurs.

Trigo (as reported by Lecharns and Scrivener [L5]) modified Eq. 6.20 to consider a parabolic load distribution along the contact length between the columns and the panel, and proposed that:

$$V_e = \frac{2}{3} z f_m'$$  (6.21)

Lecharns and Scrivener [L5] reported a good agreement between the experimental results and Eq. 6.21. However, the actual contact length measured during the test was used in the evaluation of the strength. The use of Eq. 6.8 for the calculation of the contact length $z$, as suggested by Priestley and Calvi [P17, P23], would have resulted in an overestimation of the compressive strength of the panel.

In the author's opinion, Eqs. 6.19 and 6.21 do not reflect the physical phenomenon involved in the compressive failure of the masonry panel. It is worth noting that the compressive strength $f_m'$ obtained from the standard test, does not consider the effect of the angle $\theta$ between the diagonal force and the bed joints. There is clear evidence (see section 4.4.3) that the strength of the masonry panel decreases when the compressive load is not perpendicular to the bed joint direction. A rational expression, consequently, should consider this effect. Furthermore, since the equivalent strut mechanism is able to represent the interaction between the surrounding frame and the infill, the strength of the diagonal strut should be associated with the width of the equivalent strut, $w$, instead of the contact length, $z$.

### 6.4.5 Tensile Failure

In multistorey infilled frames with high aspect ratios, a flexural failure can occur producing yielding of the longitudinal reinforcement of the columns. In this case, the strength of the structure can be estimated using the equivalent truss model. It is worth noting that this mechanism requires that the masonry panel be able to resist the diagonal compressive force. Depending on the amount of longitudinal reinforcement of the columns, large compressive forces can develop in the masonry panel, producing the failure of the panel before that yielding in the columns occurs.

### 6.4.6 Sliding Shear Failure of the Columns

A sliding shear failure can occur in the columns, especially if they are subjected to tensile axial forces. In this case, the lateral resistance of the infilled frame is controlled by the shear strength of the columns, which can be evaluated according to the mechanisms of shear transfer by aggregate interlock and dowel action [P2]. San Bartolomé [S7] recommended that, in order to avoid this type of failure, the nominal shear stress in the columns should not exceed $0.2 f'_c$ and that additional longitudinal reinforcement should be placed in the columns to provide a clamping action.
6.4.7 General Design Expressions

Seismic codes procedures and proposed design methods [C9, F8, N7, S7] usually estimate the ultimate lateral strength, $V_u$, of framed masonry structures according to simple general equations, which are not intended to describe any particular mode of failure. These equations usually have a term related to the shear strength and another term associated with the applied vertical load, being conceptually similar to Eq. 4.5. For example, the following expressions have been adopted by San Bartolome [S7] and the seismic code of Mendoza, Argentina [C9], respectively:

$$V_u = (0.5 \psi \tau' + 0.23 f_n) A_m$$  \hspace{1cm} (6.22)

$$V_u = \tau' A_m + 0.3 P$$  \hspace{1cm} (6.23)

where $\psi$ is a reduction factor function of the aspect ratio of the panel, $\tau'$ is the design shear strength, which depends on the masonry materials, and $f_n$ is the axial stress induced by the vertical load, $P$, acting on the masonry panel. The second term in these expressions represents the effect of the vertical load, which increases the shear resistance of the structure due to the friction mechanism. This beneficial effect is only valid if shear cracking along the mortar joints is the controlling type of failure. In other cases, such as diagonal tension or compressive failure, the presence of the vertical load can result in a decrease of the lateral resistance of the infilled frame.

It must be noted that the strength reduction factor $\psi$, proposed by different researchers and design codes [A19, B13, N7, S7, T7], decreases when the aspect ratio $h_m/L_m$ increases. This factor is intended to consider the "flexural effects" on the masonry panel, which reduces the shear strength. This reduction, however, contradicts the variation of the shear strength obtained from Eqs. 6.12 and 6.14 (see Fig. 6.18). Furthermore, Eq. 5.1 indicates that the increase of the aspect ratio leads to higher values of the ratio $f_n/\tau'$, improving the shear strength of the panel. In the author's opinion, the strength reduction factor represents a misconception of the problem. The analysis of the experimental results used to evaluate the strength reduction factor [A19, B13, T7] indicates that the shear strength increases in the range of aspect ratios varying from 0.5 to 1.0, as Eqs. 6.12 and 6.14 predict. For high aspect ratios, between 2.0 and 2.5, a different type of failure occurred, which cannot be adequately modelled with expressions developed for sliding shear failure, such as Eqs. 6.22 and 6.23.

6.5 CONCLUSIONS

- Different analytical models have been used to describe the behaviour of infilled frames. These models can be divided into two groups: micro-models and macro-models. The finite element formulation and the equivalent truss mechanism are the typical examples of the first and second group, respectively.

- Macro-models exhibit obvious advantages in terms of computational simplicity and efficiency. Their formulation is based on a physically reasonable representation of the infilled frame.
The single strut model is a simple representation, but it is not able to describe the local effects occurring in the surrounding frame. The use of multi-strut models can overcome this problem without a significant increase in the complexity of the analysis.

Micro-models can simulate the structural behaviour with great detail, providing that adequate constitutive models are used. However, they are computational intensive and difficult to apply in the analysis of large structures.

The analysis of infilled frames with finite element models requires the use of at least three types of element to represent the surrounding frame, the masonry panels and the panel-frame interfaces. In a more refined analysis, the masonry panels can be considered as a two-phase material, in which the masonry units and the mortar joints are modelled separately.

There is not enough experimental information on the mechanical properties of panel-frame interfaces. More research is required to evaluate these properties.

The evaluation of the shear strength of infilled frames is usually conducted with several expressions resulting from empirical and rational considerations. In some cases, however, the physical phenomenon is not clearly represented in the design equations. Intensive research should be conducted to obtain a more consistent and rational procedure.
7. PROPOSED MODEL FOR THE CYCLIC BEHAVIOUR OF MASONRY

7.1 INTRODUCTION

In this chapter, hysteretic stress-strain relationships for masonry subjected to axial and shear stresses are proposed. These relationships are intended to define the cyclic response of the structural elements which form the macro-model proposed in Chapter 8 for representing the effect of the masonry panels in infilled frames.

The cyclic axial behaviour of masonry has received little investigation, resulting in a lack of information for the development of an adequate hysteretic model, particularly for the proper representation of the response of small cycle hysteresis. Laboratory tests usually follow a simple stress-strain path, in which the specimen is completely unloaded from the virgin curve and then reloaded. This condition is useful for the analysis and the comparison of experimental results, but it is very different from the stress-strain paths imposed to the structure by seismic actions. For these reasons, the information required for the calibration of the model can be completed considering that masonry behaves similarly to concrete, from a qualitative point of view, since both are fragile materials.

The shear response of masonry is mainly controlled by the behaviour of the mortar joints and can be adequately represented with a simple model. This aspect is discussed in section 7.3.

It is assumed, for the development of the constitutive equations of the proposed model, that compressive strains and stresses are negative.

7.2 ANALYTICAL MODEL FOR CYCLIC AXIAL BEHAVIOUR OF MASONRY

7.2.1 General

A new model is proposed here, aimed at representing the hysteretic behaviour of masonry subjected to axial loading. However, the model is general enough to model other fragile materials, such as concrete. The general characteristics of the proposed model are illustrated in Fig. 7.1. The cyclic compressive behaviour of masonry is represented by several hysteresis rules to consider different behaviours for loading, unloading or re-loading. The relationship between stress and strain, at a given state, depends on the actual strain and some parameters related to the previous stress-strain history. These different rules are discussed in detail in the next sections. For simplicity, the compressive behaviour is discussed in sections 7.2.2, 7.2.3 and 7.2.4 without considering the tensile behaviour and the local contact effects of the cracked material. These topics are investigated in sections 7.2.5 and 7.2.6.
7.2.2 Rule 1: Envelope Curve in Compression

The envelope curve can be defined as the limiting curve within which all the strain-stress curves lie regardless of the load pattern [K11]. It is assumed that the envelope curve is independent of the loading history and coincides approximately with the stress-strain curve obtained under monotonic loading [K11, N2]. Combescure et al. [C13] proposed that the compressive strength of masonry, \( f_m' \), should be reduced as a result of the cyclic loading. However, there is not sufficient experimental information, in the author's knowledge, to quantify this reduction.

The strain-stress curves proposed in the technical literature for unreinforced masonry (see section 3.3) are just valid up to the maximum stress \( f_m' \). Furthermore, these equations (Eqs. 3.1, 3.2 and 3.3) do not consider the particular characteristics of masonry behaviour, such as the influence of mortar strength in the shape of the curves. For this reason, it is assumed here that the expression proposed by Sargin et al. [S31], originally for concrete, can approximately represent the envelope curve for masonry. Hence, the relationship between the masonry axial strain, \( \varepsilon_m \), and the compressive stress, \( f_m' \), is given by:

\[
f_m = f_m' \frac{\frac{\varepsilon_m}{\varepsilon_m'}}{1 + (A_1 - 2) \frac{\varepsilon_m}{\varepsilon_m'} + A_2 \left( \frac{\varepsilon_m}{\varepsilon_m'} \right)^2} \tag{7.1a}
\]

where \( f_m' \) is the maximum stress and \( \varepsilon_m \) the corresponding strain. The constants \( A_1 \) and \( A_2 \) are function of the mechanical properties of the material, as explained below. It is considered in the model that the descending branch of the strength envelop can be alternatively represented by a parabolic curve given by:
\[ f_m = f'_m \left[ 1 - \left( \frac{\varepsilon - \varepsilon'_m}{\varepsilon_u - \varepsilon'_m} \right)^2 \right] \]  

(7.1b)

The consideration of Eq. 7.1b is intended to allow a better control of the descending branch of the envelope when the cyclic model is applied to the representation of masonry struts (see sections 8.2 and 8.3). In the computational implementation of the cyclic model both options are considered, being the user able to decide the more convenient option for each particular case. Fig. 7.2 illustrates the shape of the strength envelope according to both alternatives. The tangent modulus, \( E_t \), is obtained by differentiation of equations 7.1a and 7.1b, respectively, with respect to the strain \( \varepsilon_m \):

\[ E_t = \frac{f'_m}{\varepsilon'_m} \left( A_1 + 2 \left( A_2 - 1 \right) \frac{\varepsilon_m}{\varepsilon'_m} + (2 - A_1 - 2 A_2) \left( \frac{\varepsilon_m}{\varepsilon'_m} \right)^2 \right) \left[ 1 + (A_1 - 2) \frac{\varepsilon_m}{\varepsilon'_m} + A_2 \left( \frac{\varepsilon_m}{\varepsilon'_m} \right)^2 \right] \]  

(7.2a)

\[ E_t = -2 f'_m \frac{\varepsilon - \varepsilon'_m}{\left( \varepsilon_u - \varepsilon'_m \right)^2} \]  

(7.2b)

The coefficient \( A_1 \) was calculated by Sargin et al. [S31] assuming that the modulus \( E_t \) (Eq. 7.2a) is equal to the initial modulus, \( E_{mo} \), at \( \varepsilon_m = 0 \), thus:

\[ A_1 = \frac{E_{mo} \varepsilon'_m}{f'_m} \]  

(7.3)

The coefficient \( A_2 \), which controls the falling branch of the curve, was originally adopted as an empirical function of the confined strength of concrete in order to fit experimental results. In this thesis, however, the coefficient \( A_2 \) is calculated by imposing the condition that \( f_m = 0 \) at \( \varepsilon_m = \varepsilon_u \), where \( \varepsilon_u \) is the ultimate strain at zero stress level, therefore:

\[ A_2 = 1 - A_1 \frac{\varepsilon'_m}{\varepsilon_u} \]  

(7.4)

The consideration of the falling branch of the stress-strain curve could be important when modelling infilled frames. Failure of the masonry panel usually occurs at a small lateral displacement, before the frame reaches its strength, and the composite system is able to resist increasing lateral loads. Furthermore, the beneficial effect of the frame which restrains the cracked panel could lead to a smoother decrease of the resistance of the panel than the sudden decrease of strength observed in masonry prisms.
7.2.3 Unloading and Reloading

7.2.3.1 Proposed Equation

Unloading and reloading from the envelope curve is a complex phenomenon which is very difficult to model accurately. Different analytical models have been proposed for masonry and concrete assuming that the unloading-reloading cycles are represented by either a piecewise linear curve [Y3], a combination of linear and nonlinear curves [D23, M28, O1] or empirical equations [N8, S40]. Naraine and Sinha [N8] and Subramaniam and Sinha [S40] proposed a family of curves to represent the unloading-reloading cycles, based on tests of clay masonry prisms. These expressions (second order polynomials) were obtained using the least-square method to fit the points observed in the tests. Different equations were given depending on the strain level and the type of load (normal or parallel to the bed joints). Even though the researchers reported a very good agreement with their experimental results, the generality of the method should be verified due to some problems discussed below.

Based on the experimental response of masonry and concrete specimens subjected to cyclic loading, it seems that a good analytical model can be developed using a general curve which pass through two predefined points, 1 and 2, where the slope of the curve, $E_1$ and $E_2$, is known (see Fig. 7.3). Hence, the following nonlinear continuous expression is proposed to represent the unloading-reloading curves:

$$f_m = f_1 + (f_2 - f_1) \frac{B_1 \chi + \chi^2}{1 + B_2 \chi + B_3 \chi^2}$$  \hspace{1cm} (7.5)

where $(\varepsilon_1, f_1)$ and $(\varepsilon_2, f_2)$ are the coordinates of the points 1 and 2, respectively, and

$$\chi = \frac{\varepsilon_m - \varepsilon_1}{\varepsilon_2 - \varepsilon_1}$$  \hspace{1cm} (7.6)
The normalized strain $\chi$ varies from 0 to 1. The tangent modulus, $E_t$, is the derivative of the stress-strain function with respect to the strain $\varepsilon_m$. Hence, from Eq. 7.5:

$$E_t = E_s \frac{(B_1 + 2 \chi) (1 + B_2 \chi + B_3 \chi^2) - (B_1 \chi + \chi^2) (B_2 + 2 B_3 \chi)}{(1 + B_2 \chi + B_3 \chi^2)^2}$$  \hspace{1cm} (7.7)

where

$$E_s = \frac{f_2 - f_1}{\varepsilon_2 - \varepsilon_1}$$  \hspace{1cm} (7.8)

is the secant modulus defined between points 1 and 2.

![Diagram of stress-strain curve](image)

**Figure 7.3.** Proposed curve to represent the unloading and reloading curves.

The unloading-reloading curve and its derivative must satisfy four limiting conditions (see Fig. 7.3):

<table>
<thead>
<tr>
<th>Condition</th>
<th>Limiting Condition</th>
<th>Tangent Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>at $\varepsilon_m = \varepsilon_1 (\chi = 0)$</td>
<td>$f_m = f_1$ and $E_t = E_1$</td>
<td></td>
</tr>
<tr>
<td>at $\varepsilon_m = \varepsilon_2 (\chi = 1)$</td>
<td>$f_m = f_2$ and $E_t = E_2$</td>
<td></td>
</tr>
</tbody>
</table>

The analysis of Eq. 7.5 indicates that the first condition always holds. The remaining three conditions allow the calculation of the values of the coefficients $B_1$, $B_2$ and $B_3$. Therefore, it is found:

$$B_1 = \frac{E_1}{E_s}$$

$$B_2 = B_1 - B_3$$

$$B_3 = 2 - \frac{E_2}{E_s} (1 + B_1)$$  \hspace{1cm} (7.9)
The unloading-reloading process can be adequately described by Eq. 7.5. The principal advantage of this equation is that the slope of the curve can be imposed at both ends. Previous analytical models [D23, M28] have used an equation similar to that proposed by Popovics [P16], which requires a horizontal slope at one of the extremes of the curve.

7.2.3.2 Rule 2: Unloading from the Envelope Curve

The unloading curve starts from the envelope curve \((e_{un}, f_{un})\) and finishes with a residual or plastic deformation, \(e_{pl}\), when the compressive stress is reduced to zero (see Fig. 7.4). Experimental results indicated that these curves exhibit a simple curvature and their shapes depend on the level of strain at which unloading occurs [N8]. In the proposed model, Eq. 7.5 is used, considering that point 1 \((e_1, f_1)\) coincides with the unloading point and point 2 \((e_2, f_2)\) with the plastic point. The shape of the curve may be controlled by changing the initial and final modulus.

![Stress-strain curves for the unloading branch.](image)

**Figure 7.4.** Stress-strain curves for the unloading branch.

The plastic strain \(e_{pl}\) seems to be the most important parameter in determining the unloading curve. However, it is difficult to predict this value when unloading starts. Subramaniam and Sinha [S40] proposed the following expressions valid for loading perpendicular and parallel to the bed joints, respectively:

\[
\frac{e_{un}}{e_m} = 0.184 + 3 \frac{e_{pl}}{e_m} - 2 \left( \frac{e_{pl}}{e_m} \right)^2
\]

(7.10)
\[
\frac{\varepsilon_{un}}{\varepsilon^*_m} = 0.25 + 2.5 \left( \frac{\varepsilon_{pl}}{\varepsilon^*_m} \right) - 1.1 \left( \frac{\varepsilon_{pl}}{\varepsilon^*_m} \right)^2
\]

(7.11)

These empirical equations cannot be considered as general and their validity for different cases should be verified. Furthermore, the method proposed by Subramaniam and Sinha [S40] was developed to calculate \(\varepsilon_{un}\) for a given value of \(\varepsilon_{pl}\). However, the nonlinear analysis requires an inverse procedure, in which \(\varepsilon_{pl}\) must be evaluated as a function of \(\varepsilon_{un}\). The inspection of Eqs. 7.10 and 7.11 shows that there is no real solution when \(\varepsilon_{un}/\varepsilon^*_m\) is greater than 0.552 and 0.750, respectively.

Karsan and Jirsa [K11] investigated the behaviour of concrete under compressive loadings. Based on test results obtained from short concrete column (the dimensions at the critical section were 125 x 75 mm), the following expression was proposed to calculate the plastic strain:

\[
\frac{\varepsilon_{pl}}{\varepsilon^*_m} = 0.13 \frac{\varepsilon_{un}}{\varepsilon^*_m} + 0.145 \left( \frac{\varepsilon_{un}}{\varepsilon^*_m} \right)^2
\]

(7.12)

A more general approach was proposed by Mander et al. [M28] for unconfined and confined concrete. They assumed that the plastic strain \(\varepsilon_{pl}\) lies on the unloading secant slope define between the unloading point \((\varepsilon_{un}, f_{un})\) and an auxiliary point A, as shown in Fig. 7.4. The stress corresponding to point A is \(f_a = E_{un} \varepsilon_a\), whereas the strain \(\varepsilon_a\) is given by:

\[
\varepsilon_a = a \sqrt{\varepsilon_{un} \varepsilon^*_m}
\]

(7.13)

where a is an empirical factor defined by the greatest value of:

\[
a = \frac{1}{1 + \frac{\varepsilon_{un}}{\varepsilon^*_m}} \quad \text{or} \quad \frac{\varepsilon_{un}}{\varepsilon^*_m}
\]

(7.14)

(7.15)

The inspection of Eqs. 7.14 and 7.15 indicates that both are equal when \(\varepsilon_{un}/\varepsilon^*_m = 2.87\). Values obtained from Eq. 7.14 are valid in the range \(\varepsilon_{un}/\varepsilon^*_m < 2.87\) and from Eq. 7.15 in other cases. The plastic strain, therefore, is given by (as corrected by Dodd [D23]):

\[
\varepsilon_{pl} = \varepsilon_{un} - \frac{(\varepsilon_{un} - \varepsilon_a) f_{un}}{f_{un} - E_{mo} \varepsilon_a}
\]

(7.16)

Note that \(\varepsilon_a\) is a tensile strain and, consequently with the convention adopted here, is positive. The remaining strains and stresses correspond to compressive values.
Yankelevsky and Reinhardt [Y3] developed a model for unconfined concrete, in which the unloading curve is represented by a piecwise linear approximation. The method to calculate the plastic strain \( \varepsilon_{pl} \) is similar to that used by Mander et al. [M28]. It was assumed that the auxiliary point A (see Fig. 7.4) is independent of the unloading strain \( \varepsilon_{un} \), being its coordinates \( \varepsilon = \left| f_{u} \right| / E_{mo} \) and \( f_{u} = \left| f_{u} \right| \). Therefore, the plastic strain is given by:

\[
\varepsilon_{pl} = \varepsilon_{un} - \frac{\left| f_{u} \right|}{E_{mo}} \frac{f_{u}}{\varepsilon_{un} - \left| f_{u} \right|}
\]  

(7.17)

The main disadvantage of the previous methods is that they cannot be easily modified to fit different or new experimental data. For this reason, a modified approach is proposed here, in which it is assumed that the stress corresponding to point A (see Fig. 7.4) is equal to:

\[
f_{u} = \beta_{s} \left| f_{u} \right|
\]  

(7.18)

where \( \beta_{s} \) is an empirical factor which allows the calculation of the plastic strain in a more general manner. Based on this modification, Eq. 7.17 becomes:

\[
\varepsilon_{pl} = \varepsilon_{un} - \frac{\beta_{s} \left| f_{u} \right|}{E_{mo}} \frac{f_{u}}{\varepsilon_{un} - \beta_{s} \left| f_{u} \right|}
\]  

(7.19)

The different analytical methods proposed to calculate the plastic strain \( \varepsilon_{pl} \) are compared in Fig. 7.5 (a). According to Eqs. 7.16, 7.17 and 7.19, the evaluation of \( \varepsilon_{pl} \) depends on the coordinates of the envelope curve where unloading occurs (\( \varepsilon_{un}, f_{u} \)). Consequently, it is assumed in this comparison that the stress-strain relationship of the envelope curve is represented by Eq. 7.1a. The results are plotted for the case that \( A_{1} = 2.5 \) and \( A_{2} = -0.25 \) (see Eqs. 7.3 and 7.4). Four different values of \( \beta_{s} \) are considered for plotting Eq. 7.19. Note that the case \( \beta_{s} = 1.0 \) corresponds to the method developed by Yankelevsky and Reinhardt [Y3] (Eq. 7.17). The proposed expression, Eq. 7.19, can approximate very well the values obtained from Mander’s method [M28] and hopefully future tests results. In order to complete the comparison illustrated in this figure, the results obtained by Otter and Naaman [O1] are also included. These researchers proposed an exponential expression to calculate the plastic strain \( \varepsilon_{pl} \).

It can be observed in Fig. 7.5 (a) a significant scatter of results between the different analytical methods, especially when unloading begins at medium to large strains (\( \varepsilon_{un} / \varepsilon_{un}^{*} > 1 \)). This fact justifies the use of an expression, such as Eq. 7.19, which can be adjusted to represent different materials. The results obtained from Eqs. 7.16, 7.17 and 7.19 are affected by the value assumed for the ultimate strain, \( \varepsilon_{un} \) and for the initial modulus of elasticity, \( E_{mo} \), which controls the location of point A (see Fig. 7.4). Fig. 7.5 (b) illustrates the comparison between experimental results from one masonry prism tested by Naraine and Sinha [N2] (these results are later discussed in section 7.2.8) and two concrete specimens tested by the author (see section 7.2.4.2). The experimental values obtained from the masonry prism are included in
Fig. 7.5 (b) just to indicate the shape of the curve. In this case, experimental and analytical results cannot be strictly compared because the ultimate strain $\varepsilon_u$ is unknown (the tests were conducted up to a strain slightly greater than $\varepsilon_u$). For plotting Fig. 7.5 (a) and (b), it is assumed $\varepsilon_p/\varepsilon_u = 2$. The test results plotted in Fig. 7.5 (b) suggest that the plastic deformation for masonry would be greater than that for concrete. This fact seems to be logical considering that more damage, at the same strain level, is normally expected in masonry. However, more experimental data is required to verify the validity of the analytical method and to calibrate adequately the proposed expression for masonry.

![Graph showing comparison between experimental and analytical results](image)

**Figure 7.5.** Comparison between experimental data and analytical procedures proposed for the evaluation of plastic strain $\varepsilon_p$ as a function of the unloading strain $\varepsilon_u$.

Some experimental results show that, immediately after unloading starts, the compressive strain increases, resulting in a negative slope of the unloading curve. It is believed that this phenomenon is due to creep and relaxation of the specimen and the loading system while preparations are made to reverse the direction of the loading. This effect is not expected to occur under seismic loading and, consequently,
is not considered in the proposed model. The tangent modulus corresponding to the beginning of the unloading curve, $E_{un}$, is given by:

$$E_{un} = \gamma_{un} E_{mo}$$  \hspace{1cm} (7.20)

where $\gamma_{un}$ is an empirical constant. Values of $\gamma_{un}$ between 1.5 and 2.5 appear to be realistic.

The tangent modulus corresponding to the plastic strain of the unloading curve, $E_{pl,u}$, is given as a function of the initial modulus $E_{mo}$. It is also considered that the modulus $E_p$ decreases with increasing values of the unloading strain $\varepsilon_{un}$, as clearly indicates experimental data (see Figs. 7.12 and 7.13).

$$E_{pl,u} = \frac{\gamma_{pl,u} E_{mo}}{\left(1 + \frac{\varepsilon_{un}}{\varepsilon_m}\right)}$$  \hspace{1cm} (7.21)

in which $\gamma_{pl,u}$ and $\varepsilon_1$ are empirical constants. The exponent $\varepsilon_1$ controls the influence of $\varepsilon_{un}$ in the degradation of the stiffness. If $\varepsilon_1 = 0$ this influence is null, whereas for increasing values of $\varepsilon_1$, the effect of the unloading strain $\varepsilon_{un}$ in reducing the modulus $E_{pl,u}$ is more significant. Values of $\varepsilon_1$ between 1.5 and 2.0 fit adequately the available experimental data. The constant $\gamma_{pl,u}$ can vary between 0 and 1.

### 7.2.3.3 Rule 3: No Stress

The third hysteresis rule considers the cases in which no stress develops in the masonry. This situation can occur under tensile strains ($\varepsilon_m \geq 0$), when the ultimate compressive strain is exceeded ($\varepsilon_m \leq \varepsilon_a$) or after the tensile strength has been reached and $\varepsilon_m \geq \varepsilon_p$ (this aspect is explained in section 7.2.5).

### 7.2.3.4 Rule 4 and 5: Reloading after Complete Unloading

Reloading in compression starts when the compressive strain $\varepsilon_m$ reaches the plastic strain $\varepsilon_p$. Then, the compressive stress increases, following a path different to that corresponding to unloading, towards the envelope curve. The shape of the reloading curve is complex, showing a double curvature with mild concavity in the low stress region and a sharp reversal in curvature near the envelope [O1].

The coordinates of the point where the reloading curve contacts the envelope curve are $\varepsilon_m = \varepsilon_a$ and $f_m = f_a$ (see Fig. 7.6). It is assumed in the model that the reloading curve is represented by the combination of two curves defined by Eq. 7.5. The first curve (rule 4) goes from the point where reloading begins ($\varepsilon_p, 0$) to an intermediate point ($\varepsilon_{cb}, f_{cb}$). Then, a second curve (rule 5) is used until the strength envelope is reached, as shown in Fig. 7.6. The resultant curve and its derivative are continuous. In this way, the changes of curvature observed in tests of masonry and concrete specimens can be adequately represented.

### Rule 4: This part of the reloading curve is defined according to Eq. 7.5, in which points 1 and 2 correspond to the change point ($\varepsilon_{cb}, f_{cb}$) and plastic point ($\varepsilon_p, 0$), respectively. The former point is located on the line defined by the points ($\varepsilon_{mo}, f_{mo})$ and ($\varepsilon_p, 0$), as illustrated in Fig. 7.7, at a stress level, $f_{cb}$, smaller than $f_{re}$:

$$f_{cb} = \beta_{cb} f_{re}$$  \hspace{1cm} (7.22)
Figure 7.6. Reloading curve and associated parameters.

Figure 7.7. Definition of the change point for the reloading curve.
The evaluation of the reloading strain, $\varepsilon_{re}$, and stress, $f_{re}$, is explained below. The factor $\beta_{ch}$ varies from 0.5 to 0.9 to obtain a reloading curve similar to that observed in tests. The strain corresponding to the point of change, $\varepsilon_{ch}$, is:

$$
\varepsilon_{ch} = \varepsilon_b + \frac{f_{ch}}{E_{ch}} \quad (7.23)
$$

where

$$
\varepsilon_b = \varepsilon_{pl} + \alpha_{ch} \left( \varepsilon_{un} - \frac{f_{un}}{E_{un}} - \varepsilon_{pl} \right) \quad (7.24)
$$

$$
E_{ch} = \frac{f_{un}}{\varepsilon_{un} - \varepsilon_b} \quad (7.25)
$$

and $\alpha_{ch}$ is a factor which controls the fatness of the hysteresis loops. For practical applications, the limits of this parameter are fixed between 0.1 and 0.7, for fat and thin loops, respectively.

The modulus $E_{ch}$ (Eq. 7.25) is used as final modulus for rule 4 and initial modulus for rule 5, assuring the continuity of the resultant curve. When reloading begins, the modulus corresponding to the plastic point is (see Fig. 7.6):

$$
E_{pl,r} = \gamma_{pl,r} E_{pl,u} \leq E_{un} \quad (7.26)
$$

where the factor $\gamma_{pl,r}$ is greater than 1.0. Values ranging from 1.1 to 1.5 can be used to fit experimental data.

**Rule 5:** For this hysteresis rule, Eq. 7.5 applies considering point 1 equal to the change point and point 2 equal to the reloading point on the envelope curve (see Fig. 7.6). The prediction of the strain $\varepsilon_{re}$ represents a difficult task. Mander et al. [M28] used a linear reloading curve combined with a parabolic transition. The strain $\varepsilon_{re}$ was computed using an empirical expression depending on the points where unloading and reloading occur and on the ratio of the confined to unconfined concrete strength. In the method proposed by Yankelevsky and Reinhardt [Y3], the reloading strain $\varepsilon_{re}$ is calculated as the intersection of a predefined line with the envelope curve. Consequently, a nonlinear equation must be solved to compute $\varepsilon_{re}$ when reloading starts. This method exhibits some problems in the evaluation of $\varepsilon_{re}$ when unloading occurs at low or medium strain levels. Otter and Naaman [O1] suggested a linear expression to approximate the actual behaviour observed in the reloading process. These researchers used also a linear expression to calculate $\varepsilon_{re}$ depending on the reloading strain and the strain at peak stress. Otter and Naaman [O1] pointed out that this expression holds well at large values of the unloading strain, although some modification may be required when unloading occurs at small strains.
The method proposed in this study assumes that the reloading strain, \( \varepsilon_{re} \), is proportional to the difference of the unloading strain, \( \varepsilon_{un} \), and the strain where reloading begins, in this case \( \varepsilon_{pl} \) (for small cycle hysteresis the expression is different, see section 7.2.4.5):

\[
\varepsilon_{re} = \varepsilon_{un} + \alpha_{re} (\varepsilon_{un} - \varepsilon_{pl})
\]
(7.27)

where \( \alpha_{re} \) is an empirical factor greater than zero, whose value usually ranges from 0.1 to 0.8. It is found that the method proposed by Mander et al [M28] (as corrected by Dodd [D23]) is equivalent to Eq. 7.27 with \( \alpha_{re} = 0.26 \) (assuming that there is no increase of the masonry strength due to confinement). According to the experimental data reported in Table 7.1, the values of \( \alpha_{re} \) range from 0.28 to 0.75. Nevertheless, for medium to large strain this factor is almost constant with an average value of 0.33. The stress \( f_{re} \) corresponding to the strain \( \varepsilon_{re} \) can be found from Eq. 7.1.

It is also necessary to define the tangent modulus at both ends of the curve. The initial modulus \( E_1 \) is equal to \( E_{ch} \) (see Eq. 7.25), whereas the final modulus \( E_2 \) depends on the unloading strain:

\[
E_2 = \begin{cases} 
\varepsilon_{un} > \varepsilon_{m} & E_2 = E_{re} \\
0.5 \ E_1 & E_2 = 0.5 \ E_1 
\end{cases}
\]
(7.28)

being \( E_{re} \) the tangent modulus of the envelope curve corresponding to the reloading point (see Fig. 7.6). Eq. 7.28 intends, as far as possible, to reduce sudden changes of stiffness.

It must be pointed out that the proposed reloading curve is more complex than other procedures previously developed [D23, K11, M28, S40, X1, Y3]. This increment in the complexity of the algorithm was necessary in order to gain generality. Thus, the reloading curves can be adjusted to represent hysteresis loops with different characteristics.

7.2.4 Small Cycle Hysteresis

7.2.4.1 General

In previous sections, constitutive relations have been proposed for cyclic behaviour with loops that start from and return to the envelope curve with only one reversal after complete unloading. Reversals may occur, however, at any place in the loading history. Therefore, for the sake of completeness, the proposed model also includes a description for these so called inner loops (small cycle hysteresis).

The effect of inner loops has been scarcely considered in the development of analytical methods. One of the first attempts to take into account this effect was due to Otter and Naaman [O1]. They assumed that the unloading strain, \( \varepsilon_{un} \), increases each time that reloading begins, which indicates that the unloading strain is considered as a damage index. In this model, the plastic strain also increases as a result of the small cycle hysteresis (the plastic strain is directly related to the unloading strain, see section 7.2.3.2), which is contrary to experimental results reported in section 7.2.4.2 and references [K11, N2]. The complex behaviour observed during laboratory tests with small cycle hysteresis can only be represented analytically by keeping a record of the previous loading history. It seems that only one parameter, as
assumed by Otter and Naaman [O1] is not enough to describe the observed response. For this reason, in the proposed model the number of cycles is also considered.

Due to the lack of data about the behaviour of inner loops, some tests were conducted using standard concrete cylinders. The main objective of these experiments is to collect qualitative information for the implementation of the model. Despite the fact that the analytical model is developed for modelling masonry panels, plain concrete shows a similar behaviour to masonry, since both are fragile materials with very low tensile strength.

7.2.4.2 Cyclic Tests

Five standard concrete cylinders were tested under cyclic compressive loading in an Avery Universal Testing Machine at the University of Canterbury. A loading frame formed by four steel columns was used to control the response when stress softening occurred. During the tests, the compressive force applied to the specimen was measured with a cell load and the corresponding deformation with a LVDT. The gauge length of 100 mm was set by two steel rings. An analog to digital data acquisition board was connected to a personal computer to convert the signals and to save the data for posterior analysis.

The specimens were subjected to different combinations of complete loops and inner loops. The loading paths were developed with the aim of investigating the response of the material when subjected to inner loops and the effect of these loops in the global response. Based on the results obtained, it can be concluded that:

- The plastic deformation $\varepsilon_p$ is not affected by the occurrence of inner loops.
- Successive inner loops increase the reloading strain $\varepsilon_{re}$.
- The inner loops remain inside of the cycle defined for the complete unloading and reloading curves.
- The unloading curves show no inflection point, whereas the reloading curves can exhibit a change in the direction of its concavity, depending on the point where the loading curve started.

Results obtained from these tests are presented in section 7.2.8.

7.2.4.3 Unloading from Rule 4 or 5

When unloading occurs from rule 5 ($\varepsilon_s < \varepsilon_m < \varepsilon_{cu}$), Eq. 7.5 is used with the same parameters defined in section 7.2.3.2 (unloading from the envelope curve). However, for unloading from rule 4 ($\varepsilon_{ch} < \varepsilon_m < \varepsilon_{pu}$) it is considered that the initial modulus is equal to:

$$E_1 = 2 \; E_t \leq E_{un} \tag{7.29}$$

where $E_t$ is the tangent modulus corresponding to the point where unloading begins and $E_{un}$ was defined in Eq. 7.20. This case is represented in Fig. 7.8 by curves C.
7.2.4.4  Reloading from Rule 2

This case occurs when $\varepsilon_m < \varepsilon_{ph}$ and two different situations are considered depending on the location of the change point and the point where strain reversal occurs and reloading begins (the coordinates of this point are $\varepsilon_m = \varepsilon_{m,i}$ and $f_m = f_{m,i}$, see Fig. 7.9):

Case A: if $\varepsilon_{m,i} > 0.9 \varepsilon_{ch}$ and $f_{m,i} > 0.9 f_{ch}$, then reloading follows rule 4 up to the change point and then goes to the reloading point using rule 5. The coordinates corresponding to point 1 (see Eq. 7.5) are the same as those indicated in section 7.2.3.4. Point 2, in this case, is the point where reloading begins, being the tangent modulus $E_2$ the smaller value between:

$$E_2 = 1.2 \ E_1 \quad \text{or} \quad E_2 = 0.9 \ E_s$$  \hspace{1cm} (7.30)

where $E_1$ is defined at the point where reloading begins and $E_s$ is the secant modulus between points 1 and 2 (see Eq. 7.8). This case is illustrated in Fig. 7.8 by the reloading lines A.

Case B: if $\varepsilon_{m,i} < 0.9 \varepsilon_{ch}$ and $f_{m,i} < 0.9 f_{ch}$, then reloading follows rule 5 directly (see curves B in Fig. 7.8), from the point where the reversal occurs (point 1, in Eq. 7.5) up to the reloading point (point 2). The initial modulus is adopted as the greatest value of:

$$E_1 = 2.0 \ E_1 \leq E_{un} \quad \text{or} \quad E_1 = 1.2 \ E_s \leq E_{un}$$ \hspace{1cm} (7.31)

where $E_s$ is defined in Eq. 7.8.
7.2.4.5 Accumulative Damage Due to Small Cycle Hysteresis

Inner cycles produce accumulative damage, which results in an increase of the reloading strain $\varepsilon_{re}$ and a slight variation of the shape of the reloading curves. This effect is clearly observed in the experimental results reported by Naraine and Sinha [N2] and Karsan and Jirsa [K11]. The consideration of this effect in the analytical model is cumbersome, especially due to the lack of complete experimental information. The following procedure is implemented in the proposed model:

Step 1: when unloading occurs from the envelope curve the reloading strain is initialized as $\varepsilon_{re} = \varepsilon_{un}$.

Step 2: when unloading occurs either from the envelope curve or from an inner loop at a strain level greater than the previous unloading strain ($\varepsilon_{un,i} < \varepsilon_{un}$), the counter of cycles, $n_{cy}$, is set to zero. In the latter case, the unloading strain is also updated, thus, $\varepsilon_{un} = \varepsilon_{un,i}$ (see Fig. 7.9).

Step 3: each time that a strain reversal occurs and reloading starts, the counter of cycles is increased, i.e. $n_{cy} = n_{cy} + 1$.

Step 4: the increment of the reloading strain, $\Delta\varepsilon_{re}$, corresponding to the cycle $n_{cy}$ is evaluated according to the following expression:

$$\Delta\varepsilon_{re} = \frac{\alpha_{re}}{\varepsilon_z} (\varepsilon_{un,i} - \varepsilon_{re,i})$$

(7.32)

where $\varepsilon_{un,i}$ and $\varepsilon_{re,i}$ are the unloading and reloading strains for the inner loop, respectively, $\varepsilon_z$ is an empirical exponent greater than zero and $\alpha_{re}$ is a factor used in Eq. 7.27. The stress corresponding to the reloading point, $f_{re}$, is computed from Eq. 7.1 with $\varepsilon_m = \varepsilon_{re}$.

Step 5: the strain corresponding to the change point, $\varepsilon_{ch}$ is calculated from Eq. 7.23, whereas the stress $f_{ch}$ is given by the following expression:

$$f_{ch} = \frac{\beta_{ch} f_{re}}{n_{cy}^{0.4}}$$

(7.33)

According to Eq. 7.33, the stress $f_{ch}$ decreases as the number of inner cycles increases. However, this reduction is limited by the condition:

$$f_{ch} \leq 0.5 f_{re}$$

(7.34)

The procedure proposed here is able to represent the effects of strain accumulation and cyclic degradation observed in the laboratory tests. A comparison with experimental results is presented in section 7.2.8.

7.2.5 Tensile Behaviour of Masonry - Rule 6

The behaviour of masonry in tension has been scarcely investigated. Even though there is some experimental information related to the tensile behaviour under monotonic loading (see section 4.3), it
has not been possible to find, in the literature, results obtained from cyclic tests. For this reason, the model proposed here is mainly based on experimental data corresponding to plain concrete. It is assumed that, when no previous compression has taken place, the strain-stress relationship in tension is linearly elastic until the tensile strength, $f'_t$, is reached. At that point, a brittle failure occurs and the material cannot resist any tensile stress. The elastic modulus in tension is assumed to be equal to the initial modulus in compression, $E_{mo}$ (see Fig. 7.10, line O-A). The effect of tension softening is not considered in the model. Tension softening is a local material softening observed during strain controlled experiments (this effect should not be confused with tension stiffening, which is a global property of reinforced concrete, even though it manifests as a softening problem) [H13, S44].

![Graph](image)

**Figure 7.9.** Limit strains associated with small cycle hysteresis.

The effect of pre-loading in compression on the tension behaviour of concrete was investigated by Moria and Kaku (as reported by Mander et al. [M28]). These researchers found that previous compressive strain histories produce degradation of the tensile strength and the elastic modulus. The tensile strength is assumed to be zero when the plastic strain, $\varepsilon_{pl}$, exceeds the magnitude of the strain at the compressive strain, $\varepsilon'_m$. In the case that $\varepsilon'_m < \varepsilon_{pl} < 0$, this effect can be evaluated according to the following expressions:

$$f'_t = f'_t \left(1 - \frac{\varepsilon_{pl}}{\varepsilon'_m}\right) \tag{7.35}$$

$$E'_t = E_{mo} \left(1 - \frac{\varepsilon_{pl}}{\varepsilon'_m}\right) \tag{7.36}$$

where $f'_t$ is the reduced tensile strength, $E'_t$ is the elastic modulus and $\varepsilon_{pl}$ is the largest plastic strain developed in previous compressive cycles. The strain-stress relationship in tension is assumed to follow a linear relationship:
Eq. 7.37 is also valid if compression has not previously taken place by considering that $\varepsilon_{pl} = 0$.

Fig. 7.10 shows an example of the tensile behaviour following the proposed model. Initially, a compressive cycle is applied (curve O-B-C-D), which affects the subsequent tensile response. As the magnitude of the strain decreases, the tensile loading occurs following Eq. 7.37 (line D-E). When the strain is reversed before reach the reduced tensile strength, $f_t$ (point E), linear unloading occurs following the same line. Subsequently, a second compressive cycle is applied (curve D-F-G-H). Tensile loading occurs again, however, the tensile strength and the elastic modulus are smaller in this case because the plastic strain has increased. The tensile stress augments until the strength $f_t$ is reached and the tensile failure occurs (line H-I). After that, no tensile stress can be resisted in further cycles.

![Diagram of tensile behaviour](image)

**Figure 7.10.** Model assumed for the tensile behaviour of masonry.

### 7.2.6 Local Contact Effects of Cracked Material on the Hysteretic Response

It is usually assumed that the cracked masonry (or concrete) cannot carry any compressive stress until the cracks are completely closed. However, there is experimental evidence [B11, S42, X1] which indicates that the compressive stress starts to increase when the strain is reversed, following a soft response. Bolong et al. [B11] observed during the testing of reinforced concrete members that the contact effects are more important as the width of the cracks become larger. This behaviour is explained considering that small particles flake off during cracking and remain in the cracks. Furthermore, misalignment of the crack surfaces causes a progressive contact and the compression is transferred across the cracks gradually. The effect of crack closing with development of compressive stresses has been previously considered for the analytical modelling of concrete response [B11, F5, X1].
Based on the previous considerations, it is assumed that reloading starts at a given strain $\varepsilon_m$ if the following condition is satisfied (see Fig. 7.11):

$$\varepsilon_m \leq \varepsilon_{pl} \quad \text{or} \quad \varepsilon_m \leq \varepsilon_{cl}$$  \hspace{1cm} (7.38)

where $\varepsilon_{cl}$ is the limit strain at which it is assumed that the cracks are partially closed and compressive stresses can be resisted. The value of $\varepsilon_{cl}$ is given as input data of the model. A large negative value (for example $\varepsilon_{cl} < \varepsilon_{c}$) implies that contact effects in the cracks are not considered because the first condition of Eq. 7.38 always controls the reloading process.

The reloading curve is defined using Rule 4, as indicated in section 7.2.3.4. However, the elastic modulus $E_2$ is defined in a different way. When reloading starts at a strain $\varepsilon_m > \varepsilon_{pl}$ (which implies that $\varepsilon_{cl} > \varepsilon_{pl}$), the elastic modulus at the beginning of the curve is:

$$E_2 = E_{cl} \quad \text{if} \quad 0 < \varepsilon_m \leq \varepsilon_{cl}$$

$$E_2 = E_{pl} + \left(E_{plr} - E_{cl}\right) \frac{\varepsilon_m}{\varepsilon_{pl}} \quad \text{if} \quad \varepsilon_{pl} < \varepsilon_m \leq 0$$  \hspace{1cm} (7.39)

where $E_{plr}$ is the reloading modulus defined by Eq. 7.26 and $E_{cl}$ is equal to 0.15 $E_{plr}$. The inspection of Eq. 7.39 indicates that $E_2$ can vary from $E_{cl}$ to $E_{plr}$. The tangent modulus corresponding to point 2 is limited by the conditions $E_2 \leq 0.2 E_1$ and $E_2 \leq 0.9 E_4$ in order to obtain a smooth transition curve.

Fig. 7.11 illustrates an example of the cyclic response, in which contact effects are considered. After the first cycle, reloading starts when the strain is equal to $\varepsilon_{pl}$. The response is soft at the beginning and becomes stiffer until the normal reloading curve is reached. The second reloading process occurs immediately after the strain is reversed because the strain is smaller than $\varepsilon_{cl}$ (Eq. 7.38). The consideration of the contact effects produces wider hysteresis loops and a gradual increase of the compressive stress in the reloading process. Another example is shown in Fig. 7.12, in which successive cycles with increasing strain levels are imposed.

![Figure 7.11](image-url)

**Figure 7.11.** Local contact effects for the cracked masonry according to the proposed model.
Figure 7.12. Example of cyclic response with local contact effects for the cracked masonry.

A special situation occurs when the strain is reversed after the tensile failure has occurred without previous compression. In this case, it is assumed that compressive stress can be resisted by the cracked material when Eq. 7.38 is satisfied. The strain-stress relationship is defined by Rule 4, as mentioned in the previous paragraphs. Nevertheless, some differences are considered in this particular case. The transition curve goes to a point located on the compressive envelope curve (point 1), where the strain is arbitrarily defined as $\varepsilon_t = -f_t/E_{mo}$. The tangent moduli at the ends of the transition curve are adopted as follows: $E_1$ is equal to the tangent modulus of the envelope curve (therefore, the continuity of the slope is assured) and $E_2 = 0.01 E_{mo}$.

7.2.7 Summary of the Hysteresis Rules

The envelope curve in compression, Rule 1, is given by Eq. 7.1, whereas the behaviour under tensile stresses is represented by Eq. 7.37. Five properties of masonry are needed to define the envelope curve: the compressive and tensile strengths, $f'_c$ and $f'_t$, the strain at maximum stress, $\varepsilon_{m}'$, the ultimate strain, $\varepsilon_{u}$, and the elastic modulus, $E_{mo}$. Contact effects in the cracked material can be considered by setting an appropriate value for the variable $\varepsilon_{cl}$.

The unloading and reloading curves are represented in the proposed model according to a general expression (Eq. 7.5). This curve is defined in each case by giving the coordinates and the tangent modulus at both ends of the curve. Table 7.1 summarizes the parameters used for the unloading and reloading curves. Nine empirical constants are used in the model to calculate different parameters associated with the cyclic behaviour. In this way, it is possible to adjust the models to fit a wide range of experimental results, even for different materials. Table 7.2 presents suggested and limit values for these constants.
Table 7.1. Parameters for unloading or reloading using Eq. 7.5.

<table>
<thead>
<tr>
<th>Case</th>
<th>$e_1$</th>
<th>$f_1$</th>
<th>$E_1$</th>
<th>$e_2$</th>
<th>$f_2$</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 2 (from 1)</td>
<td>$e_{sa}$</td>
<td>$f_{sa}$</td>
<td>$E_{sa}$</td>
<td>$e_{pl}$</td>
<td>0</td>
<td>$E_{pl,s}$</td>
</tr>
<tr>
<td>Rule 2 (from 4)</td>
<td>$e_{sa}$</td>
<td>$f_{sa}$</td>
<td>$2 E_s \leq E_{sa}$</td>
<td>$e_{pl}$</td>
<td>0</td>
<td>$E_{pl,s}$</td>
</tr>
<tr>
<td>Rule 2 (from 5)</td>
<td>$e_{sa}$</td>
<td>$f_{sa}$</td>
<td>$E_{sa}$</td>
<td>$e_{pl}$</td>
<td>0</td>
<td>$E_{pl,s}$</td>
</tr>
<tr>
<td>Rule 4 (from 2)</td>
<td>$e_{ch}$</td>
<td>$f_{ch}$</td>
<td>$E_{ch}$</td>
<td>$e_{pl}$</td>
<td>0</td>
<td>$1.2 E_s \leq E_{ch}$, or $0.9 E_s \leq E_{sa}$</td>
</tr>
<tr>
<td>Rule 4 (from 3)*</td>
<td>$e_{ch}$</td>
<td>$f_{ch}$</td>
<td>$E_{ch}$</td>
<td>$e_{pl}$</td>
<td>0</td>
<td>$E_{pl,c}$</td>
</tr>
<tr>
<td>Rule 5 (from 2)</td>
<td>$e_{re,i}$</td>
<td>$f_{re,i}$</td>
<td>$2 E_s \leq E_{sa}$, or $1.2 E_s \leq E_{su}$</td>
<td>$e_{re}$</td>
<td>$f_{re}$</td>
<td>$E_{re}$ or $0.5 E_s$</td>
</tr>
<tr>
<td>Rule 5 (from 4)</td>
<td>$e_{ch}$</td>
<td>$f_{ch}$</td>
<td>$E_{ch}$</td>
<td>$e_{re}$</td>
<td>$f_{re}$</td>
<td>$E_{re}$ or $0.5 E_s$</td>
</tr>
</tbody>
</table>

* The parameters corresponding to point 1 and 2 for Rule 4 (from 3) may be different when local effects due to cracked material are considered (see section 7.2.6).

Table 7.2. Suggested and limit values for the empirical constants used in the model.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Suggested values</th>
<th>Limit values</th>
<th>Associated with equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ch}$</td>
<td>0.3 - 0.6</td>
<td>0.1 - 0.7</td>
<td>7.24</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>0.2 - 0.4</td>
<td>&gt; 0</td>
<td>7.27 - 7.32</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>1.5 - 2.0</td>
<td>&gt; 0</td>
<td>7.18 - 7.19</td>
</tr>
<tr>
<td>$\beta_{ch}$</td>
<td>0.6 - 0.7</td>
<td>0.5 - 0.9</td>
<td>7.22 - 7.33</td>
</tr>
<tr>
<td>$\gamma_{un}$</td>
<td>1.5 - 2.5</td>
<td>$\geq$ 1</td>
<td>7.2</td>
</tr>
<tr>
<td>$\gamma_{pl}$</td>
<td>0.5 - 0.7</td>
<td>0 - 1</td>
<td>7.21</td>
</tr>
<tr>
<td>$\gamma_{pl,c}$</td>
<td>1.1 - 1.5</td>
<td>$\geq$ 1.0</td>
<td>7.26</td>
</tr>
<tr>
<td>$e_1$</td>
<td>1.5 - 2.0</td>
<td>$\geq$ 0</td>
<td>7.21</td>
</tr>
<tr>
<td>$e_2$</td>
<td>1.0 - 1.5</td>
<td>$\geq$ 0</td>
<td>7.32</td>
</tr>
</tbody>
</table>

7.2.8 Comparison with Experimental Results

Fig. 7.13 shows the comparison between experimental results obtained by Naraine and Sinha [N2] for clay masonry and analytical data obtained from the proposed model. It is observed that a good agreement exists between both curves. However, some differences appear due to errors in the prediction of the plastic strain and the envelope curve (in the range of medium strains). Table 7.3 presents the numerical results obtained from these tests. In this comparison, the following values are assigned to the empirical constants of the analytical model:

\[
\begin{align*}
\alpha_{ch} &= 0.60 \\
\alpha_s &= 0.35 \\
\beta_s &= 2.00 \\
\beta_{ch} &= 0.60 \\
\gamma_{un} &= 2.00 \\
\gamma_{pl,c} &= 1.20 \\
e_1 &= 2.00 \\
e_2 &= 1.00
\end{align*}
\]
Table 7.3. Experimental values obtained from a cyclic test conducted by Naraine and Sinha [N2].

<table>
<thead>
<tr>
<th>Cycle</th>
<th>ε&lt;sub&gt;um&lt;/sub&gt;</th>
<th>f&lt;sub&gt;um&lt;/sub&gt; (MPa)</th>
<th>ε&lt;sub&gt;pl&lt;/sub&gt;</th>
<th>ε&lt;sub&gt;pl/ε&lt;sub&gt;um&lt;/sub&gt;&lt;/sub&gt;</th>
<th>ε&lt;sub&gt;re&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60</td>
<td>1.57</td>
<td>0.07</td>
<td>0.12</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>2.24</td>
<td>0.14</td>
<td>0.14</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>2.79</td>
<td>0.25</td>
<td>0.17</td>
<td>1.85</td>
</tr>
<tr>
<td>4</td>
<td>1.85</td>
<td>3.15</td>
<td>0.39</td>
<td>0.21</td>
<td>2.50</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
<td>3.96</td>
<td>0.61</td>
<td>0.24</td>
<td>3.30</td>
</tr>
<tr>
<td>6</td>
<td>3.30</td>
<td>4.31</td>
<td>0.86</td>
<td>0.26</td>
<td>4.00</td>
</tr>
<tr>
<td>7</td>
<td>4.00</td>
<td>4.76</td>
<td>1.07</td>
<td>0.27</td>
<td>4.80</td>
</tr>
<tr>
<td>8</td>
<td>4.80</td>
<td>5.03</td>
<td>1.50</td>
<td>0.31</td>
<td>5.75</td>
</tr>
<tr>
<td>9</td>
<td>5.75</td>
<td>5.17</td>
<td>2.20</td>
<td>0.38</td>
<td>6.95</td>
</tr>
<tr>
<td>10</td>
<td>6.95</td>
<td>5.27</td>
<td>2.95</td>
<td>0.42</td>
<td>8.20</td>
</tr>
<tr>
<td>11</td>
<td>8.20</td>
<td>5.06</td>
<td>4.50</td>
<td>0.55</td>
<td>---</td>
</tr>
</tbody>
</table>

Figure 7.13. Comparison between analytical and experimental results obtained by Naraine and Sinha [N2].
One of the stress-strain curves measured by the author from a concrete cylinder and the analytical results obtained from the proposed model are plotted in Fig. 7.14. Again a very good agreement is observed, which indicates that the analytical model can be adapted to fit different cases, for example distinct types of masonry. The values assigned to the empirical factors are:

\[
\begin{align*}
\alpha_{ch} &= 0.35 & \beta_a &= 2.00 & \gamma_{un} &= 1.70 & \gamma_{pl} &= 1.10 & e_2 &= 1.00 \\
\alpha_{me} &= 0.20 & \beta_{ch} &= 0.60 & \gamma_{pl \alpha} &= 1.00 & e_1 &= 1.50
\end{align*}
\]

It is worth noting that many of the values used in this case (concrete specimens, Fig. 7.14) and in the previous comparison (masonry specimens, Fig. 7.13) are different, which reflects the distinct characteristics of the cyclic response of both materials. According to the results considered here, masonry exhibits hysteretic loops with larger area and more significant strength degradation. However, both concrete and masonry present similar behaviour, from the qualitative viewpoint, and their behaviour can be represented with the same analytical model.

![Graph showing experimental and analytical results for a concrete specimen.](image)

**Figure 7.14.** Analytical and experimental results corresponding to a concrete specimen.

The proposed model was used to simulate the cyclic response obtained by Karsan and Jirsa [K11] for concrete specimens, which were subjected to different load histories. Unfortunately, only a qualitative comparison was conducted because the complete information regarding these tests could not be found. Two particular cases are considered here. In the first case, the load cycles varied between maximum and minimum stress levels, which remained constant during the test. Fig. 7.15 illustrates the analytical response obtained from the proposed model. In the second case, the specimen was subjected to complete unloading in each cycle and then reloaded until the curve of the previous cycle was reached. This load history, as shown in Fig. 7.16, produced a degradation of the cyclic response. After the specimen was cycled several times the response stabilized. The shape of the hysteresis loops, the effect of strain
accumulation and the degradation of the response observed experimentally are well modelled by the proposed algorithm. It is worth noting that none of the previous models developed for concrete or masonry [D23, K11, M28, S40, X1, Y3] has the capability of simulating these tests.

Figure 7.15. Analytical response for cycles between constant maximum and minimum stresses.

Figure 7.16. Analytical response for cycles inside the initial loop.
The results plotted in Figs. 7.15 and 7.16 were obtained using the following values for the nine parameters required by the analytical model:

\[
\begin{align*}
\alpha_n &= 0.25 & \beta_n &= 2.00 & \gamma_\text{un} &= 1.70 & \gamma_\text{pl,r} &= 1.20 & e_2 &= 1.50 \\
\alpha_r &= 0.40 & \beta_{\text{ch}} &= 0.60 & \gamma_{\text{pl,s}} &= 0.70 & e_1 &= 2.00
\end{align*}
\]

The comparison with the experimental data indicates that the proposed model represents adequately the cyclic compressive behaviour of masonry. However, it must be mentioned that the good agreement observed in the previous comparisons was reached after several trials in which the experimental response was known. The most important features of the proposed model are the ability to keep a record of the loading history and the use of empirical constants to fit experimental results obtained from different fragile materials. The latter fact represents an important advantage when compared with other analytical models, which were calibrated considering only one set of experimental results. It is also important to note that all the unloading and reloading curves are represented with the same expression, despite the complexity of the cyclic loading response. A further advantage of the proposed algorithm is that no iterations are required and all the calculations are straightforward.

7.3 ANALYTICAL MODEL FOR CYCLIC BEHAVIOUR OF MASONRY IN SHEAR

7.3.1 General

The model developed here is intended for representing the shear behaviour when bond failure occurs along the mortar joints, or in other words, for those cases in which shear is dominant.

The cyclic response of masonry in shear is simple in comparison with the behaviour under compression. In the proposed model, the shear behaviour is represented by two hysteresis rules (see Fig. 7.17), which are explained in the following sections.

7.3.2 Elastic Response - Rule 1

It is assumed that shear behaviour of mortar joints is linear elastic while the shear strength is not reached. Therefore, the relationship between the shear stress, \( \tau \), and the shear deformation, \( \gamma \), is given by:

\[
\tau = G_m \gamma
\]  

(7.40)

where \( G_m \) is the shear modulus. Unloading and reloading in the elastic range also follows Eq. 7.40.

The shear strength is evaluated following a bond-friction mechanism, in which the bond strength, \( \tau_0 \), is combined with a frictional component depending on the coefficient of friction, \( \mu \), and the compressive stress, \( f_n \), in the direction perpendicular to the mortar joints (see section 4.1.4), thus:

\[
\begin{align*}
\tau_m &= \tau_0 + \mu |f_n| \leq \tau_\text{max} & \text{if } f_n < 0 \\
\tau_m &= \tau_0 & \text{if } f_n \geq 0
\end{align*}
\]  

(7.41)
The stress $\tau_{\text{max}}$ represents an upper limit for the shear strength because there is experimental and analytical evidence indicating that Eq. 7.41 is not valid for medium to high values of the compressive stresses $f_{\text{n}}$. It is worth noting that the values of the shear parameters $\tau_0$ and $\mu$ should be adopted to reflect the real strength of the masonry panel. Values obtained from direct shear tests can overestimate the shear strength, as mentioned in section 4.2.3.5.

![Diagram of shear stress and strain](image)

**Figure 7.17.** Analytical response for cyclic shear response of mortar joints.

### 7.3.3 Sliding - Rule 2

When the shear strength is reached, the bond between mortar and brick is broken and cracks form in the affected region. In this stage, part of the masonry panel slides, with respect to the other part, and only the frictional mechanism remains. Consequently, the shear stress is given by:

$$
\tau = \mu_{\text{r}} |f_{\text{n}}| \leq \tau_{\text{max}} \quad \text{if } f_{\text{n}} < 0
$$

$$
\tau = 0 \quad \text{if } f_{\text{n}} \geq 0
$$

(7.42)

where $\mu_{\text{r}}$ is the residual coefficient of friction (see section 4.1.4.5).

In this analytical model, shear strains are considered as the deformational variable. Nevertheless, the use of strains is not strictly correct because the total deformation is built up not only by elastic strains but also by relative displacement along the cracks. This problem can be solved using a formulation based on a force-displacement relationship. Thus, the shear strains are transformed to horizontal displacements which can be directly added to the relative horizontal displacement between the cracked surfaces.
7.3.4 Unloading and Reloading after the Bond Failure - Rule 2

It is assumed in the model that unloading and reloading after the bond failure follow a linear relationship. Therefore, this process can be represented by Rule 1, using Eq. 7.40. The reloading line increases the magnitude of the shear stress until the shear strength (given by Eq. 7.42) is reached and sliding starts again. This situation is illustrated in Fig. 7.17.

7.3.5 Effect of the Normal Stress $f_n$

According to the bond-friction mechanism adopted for the representation of the behaviour of masonry in shear, the normal stress $f_n$ controls the shear stress level which can be resisted by the mortar joints. As Eqs. 7.41 and 7.42 indicate, the shear stress increases in direct proportion to the compressive stress $f_n$. This aspect was not considered when plotting Fig. 7.17, in which it was assumed that $f_n$ remains constant in order to describe the basic behaviour of the model. In masonry panels subjected to cyclic or dynamic loading, the normal stress $f_n$ usually changes as the panel deforms in shear. As a result, the shape of the hysteresis loops can be significantly different from that illustrated in Fig. 7.17.

7.4 CONCLUSIONS

- A new analytical model is proposed to represent the hysteretic axial behaviour of masonry. The envelope in compression is defined with the Sargin's equation [S31], whereas the behaviour in tension is assumed to be linearly elastic. The unloading-reloading curves are represented with a general expression which pass through two predefined points where the slope of the curve is also imposed. Contact effects in the cracked material may be considered. The comparison between different experimental results and the analytical response obtained from the proposed model shows very good agreement.

- The cyclic axial behaviour of masonry is very complex and there is insufficient experimental information related to this aspect. Consequently, experimental data obtained from concrete specimens is also used for the calibration of the model. Tests results indicate that fragile materials, such as concrete and masonry, exhibit a similar behaviour.

- The proposed algorithm can be easily adjusted to describe the behaviour of different fragile materials. Furthermore, the model is able to keep information about the previous loading history, which is required to represent properly the dynamic response under earthquake induced actions.

- Concrete specimens were tested under cyclic compression to obtain experimental information about the characteristics of inner loops. This information was used for the calibration of the analytical model.

- The cyclic behaviour of masonry subjected to shear is mainly controlled by the behaviour of the mortar joints. This behaviour can be represented according to a bond-friction model, which account for the debonding of the mortar joints and the variation of the shear strength depending on the axial stress level.
8. PROPOSED MODELS FOR THE ANALYSIS OF INFILLED FRAMES

8.1 INTRODUCTION
Different types of models have been used for the analysis of infilled frame structures. As discussed in Chapter 6, these models present advantages and disadvantages and the selection of the more adequate option depends on the characteristics of each case. According to the objectives of this study, it is believed that a simple, but physically reasonable model constitutes the best alternative. In developing an analytical model for infilled frames, the principal premises are the rational consideration of the particular characteristics of masonry and the adequate representation of the hysteretic response. It is also desirable that the model be able to represent the different modes of failure observed for infilled frames. Taking into account these premises, two analytical models with different degree of refinement are presented in this chapter. First, a simple procedure is proposed on the basis of the equivalent truss mechanism, which is particularly useful for design purposes. This model, however, can be also applied for the analysis of infilled frames when the objective focuses in the global response of the structure. Second, a more refined macroscopic model is developed, considering a multi-strut formulation, in order to achieve a better representation of the effect of the masonry panel on the surrounding frame.

This chapter also describes the general characteristics of the finite element models implemented with the general computer program ABAQUS [A12]. These models are used in several parts of this thesis in order to obtain additional qualitative information regarding some particular aspects of the behaviour of infilled frames.

8.2 SIMPLE PROCEDURE FOR THE DESIGN AND ANALYSIS OF INFILLED FRAMES

8.2.1 General
Simplified procedures used for the evaluation of the strength of the masonry panel in infilled frames were discussed in section 6.4. In some of these procedures, however, the physical phenomenon is not clearly represented in the equations, resulting in large discrepancies when compared with experimental results (as shown in section 10.2.6). A new approach is proposed in this section based on the equivalent truss mechanism and on a rational consideration of the strength of the masonry panel. This is intended to provide a general framework in which the different modes of failure are considered in a similar way.

8.2.2 Preliminary Study
A preliminary study was conducted to investigate the limitations of the single strut model, which is the simplest rational representation used for the analysis of infilled frames. Furthermore, the influence of
different multi-strut models on the structural response of the infilled frame was studied, with particular interest in the stiffness of the structure and in the actions induced in the surrounding frame. Numerical results obtained from three strut models were compared with those corresponding to an equivalent finite element model using the program ABAQUS [A12] (the general characteristics of this model are described in section 8.4). Fig. 8.1 illustrates the strut models, which are referred as Model A, B and C, respectively. The total area of the equivalent masonry struts, $A_{ms}$, was the same in all the cases. It was assumed in Model C that the sectional area of the central strut was the double of that corresponding to the off-diagonal struts. Several series of models were analysed considering a 2.5 m high masonry panel with a length of 3.6 or 5.0 m, and an elastic modulus for the masonry of 2500 or 10000 MPa. The dimensions of the frame members were 200 x 200 mm and the elastic modulus of concrete was 25000 MPa.

![Diagram of strut models](image)

**Figure 8.1.** Different strut models considered in the preliminary study.

According to the objectives of the study, the analyses were conducted under static lateral loading assuming linear elastic behaviour, except for the finite element model in which nonlinear effects were considered to represent the separation of the panel-frame interfaces. Results are presented in the following paragraphs in qualitative terms.

The stiffness of the infilled frame was similar in all the cases considered, with smaller values for model B and C. It must be noted that for the multi-strut models, especially Model C, the stiffness may significantly change depending on the distance $h_z$ (see Fig. 8.1). This distance was evaluated as a fraction of the contact length, $z$, defined by Eq. 6.8. When $h_z$ increases, the stiffness of the infilled frame reduces, being chiefly controlled by the mechanical properties of the columns.
Fig. 8.2 compares the bending moment diagrams obtained from one typical example according to the different models used in this study. Model A underestimates the bending moment because the lateral forces are primarily resisted by a truss mechanism. On the other hand, Model B leads to much larger values than those corresponding to the finite element model. A better approximation is obtained from Model C, although some differences arise at the ends of both columns. Similar conclusions can be drawn regarding the shear forces. The maximum axial forces in the frame members are approximately equal in all the models, even though the variation of the axial forces along the columns shows some discrepancy at the top end of the tension column and at the bottom end of the compression column.

![Figure 8.2. Comparison of the bending moments diagrams corresponding to different strut models.](image)

It can be concluded that the single strut model, despite its simplicity, gives an adequate estimation of the stiffness of the infilled frame and the axial forces induced in the frame members by lateral forces. However, a more refined model, Model C, is required in order to obtain realistic values of the bending moments and shear forces in the frame. The results obtained here indicate that the single strut model represents an adequate tool when the analysis is focussed on the overall response of the structure.

8.2.3 Formulation of the Analytical Model

8.2.3.1 General

The equivalent strut model allows the calculation of the stiffness of the infilled frame and the actions in the different components of the structure when subjected to lateral forces. The main problem, however, for the assessment of the strength of the infilled frame is the evaluation of the resistance of the masonry panel. Following the criterion introduced by Decanini and Fantin [D13, D15] (see section 6.4.1), it is assumed that the compressive strength of the diagonal strut can be estimated as:

\[ R_c = f'_{m0} A_{ms} \]  \hspace{1cm} (8.1)

where \( A_{ms} \) is the area of the equivalent strut, \( A_{ms} = w t \), and \( f'_{m0} \) is the strength of masonry when diagonally loaded at an inclination \( \theta \). It is to be noted that only one expression, Eq. 8.1, is used to estimate the
strength of the masonry panel independently of the mode of failure. This aspect is taken into account in the evaluation of \( f_{\text{inf}} \).

In section 4.2.4, a failure theory was proposed to represent the shear strength of masonry under the assumption that failure can occur according to three different mechanisms, namely, shear-friction, diagonal tension and compressive failure. This theory was developed considering the shear and normal stresses in the bed joint, \( \tau \) and \( f_n \), and assuming that the axial stress parallel to the bed joint can be neglected. Based on equilibrium considerations, three equations were obtained to evaluate the shear strength as a function of the normal stress and the mechanical properties of the masonry. It is possible to transform these equations to express the failure criterion in terms of the principal stresses, \( f_1 \) and \( f_2 \), and their orientation with respect to the bed joints, \( \theta \) (see section 4.4.3.3). In order to obtain this transformation, the following equations can be used according to the notation presented in Fig. 8.3:

\[
\begin{align*}
  f_n &= f_1 \sin^2 \theta \\
  \tau &= f_1 \sin \theta \cos \theta
\end{align*}
\]  

Figure 8.3. Stress state considered to evaluate the strength of masonry.

It is worth noting that the compressive stress \( f_1 \) is assumed to be positive in Eqs. 8.2 and 8.3 and in the subsequent discussion. This is because in this particular case \( f_1 \) is always compressive and, therefore, simplicity is put ahead of generality. The previous equations are derived under the assumption that the principal stress \( f_1 \) developed in the masonry panel is not significant. According to numerical results obtained by the author (see section 5.4.2), the compressive stress \( f_1 \) is about 7 to 10 times greater than the tensile stress \( f_2 \) at the centre of the panel. Fig. 8.4 illustrates the variation of the principal stresses along the diagonal of the masonry panel obtained from a typical example. A similar conclusion can be drawn from results reported by Riddington and Stafford Smith [R3]. They found, based on finite element analysis of infilled frames, that the stress \( f_2 \) at the central part of the panel is approximately equal to 0.58 of the average shear stress, which can be expressed in terms of the strut force \( R \) as follows:
\[ f_2 = 0.58 \frac{R}{d_m t} \]  \hspace{1cm} (8.4)

where \( d_m \) and \( t \) are the diagonal length and the thickness of the masonry panel, respectively. The comparison of Eq. 8.4 with the principal compressive stress given by:

\[ f_1 = \frac{R}{w t} \]  \hspace{1cm} (8.5)

shows that the stress \( f_1 \) is about 6.7 to 11.5 times greater than the stress \( f_2 \), for values of the equivalent width of strut, \( w \), equal to 0.25 and 0.15 of \( d_m \), respectively. Therefore, the assumption of \( f_2 = 0 \) seems to be justified from the engineering point of view.

**Figure 8.4.** Typical variation of the principal stresses along the diagonal of the masonry panel of an infilled frame subjected to lateral forces.

The evaluation of the diagonal strength \( f'_{mb} \) is discussed in the following sections taking into account the different types of failure observed in masonry panels subjected to shear and axial stresses (see section 4.1.2). According to experimental results [M2], the compressive failure described in section 4.1.2.4 usually occurs when the normal stress \( f_n \) is significantly higher than the shear stress \( \tau \), particularly when \( f_n > 8 \tau \). It can be shown, using Eqs. 8.2 and 8.3, that the previous condition is reached when \( \tan \theta > 8 \), or \( \theta > 82.9^\circ \), which indicates that masonry panels of infilled frames are not prone to fail under this type of failure, except when crushing of the corners occurs (see section 8.3.3).

### 8.2.3.2 Shear-Friction Failure

When a shear-friction failure occurs, the shear strength can be estimated according to Eq. 4.9. Introducing Eqs. 8.2 and 8.3 into Eq. 4.9, and considering \( f_i = f'_{mb} \), it is found that:
\[ f'_{m\theta} = \frac{\tau^*_{\theta}}{\sin \theta (\cos \theta - \mu^* \sin \theta)} \]  

(8.6)

where \( \tau^* \) and \( \mu^* \) are the reduced shear strength parameters obtained from Eq. 4.17.

### 8.2.3.3 Diagonal Tension Failure

The shear strength for this case is given by Eq. 4.20, which is a function of the square root of the normal stress. In order to facilitate the derivation of a simple expression, Eq. 4.20 is approximated using a linear function:

\[ \tau_m = \frac{f'_{tb}}{C_s} \left( 1 + 0.27 \frac{f_n}{f'_{tb}} \right) \]  

(8.7)

Fig. 8.5 shows the comparison between Eqs. 4.20 and 8.7, indicating that the latter expression gives a good approximation for a large range of ratios \( f_n/f'_{tb} \). The differences between both expressions are less significant for medium values of \( f_n/f'_{tb} \), which is the case where this type of failure is likely to occur. Therefore, following a similar procedure to that used in the previous section, Eq. 8.7 is transformed as follows:

\[ f'_{m\theta} = \frac{f'_{tb}}{\sin \theta (C_s \cos \theta - 0.27 \sin \theta)} \]  

(8.8)

![Figure 8.5. Comparison of Eq. 4.20 and approximated values obtained from Eq. 8.7.](image-url)
8.2.3.4 Failure Envelope

The compressive strength of the strut, \( f_{\text{mb}}^* \), depends on the mechanical properties of masonry, on the angle \( \theta \) and on the type of failure expected in the infill panel. According to Eqs. 8.6 and 8.8, it is possible to obtain a strength envelope as a function of the inclination \( \theta \). Fig. 8.6 illustrates this envelope for a typical example in which sliding shear controls the resistance of the masonry panel. In this case, the strength \( f_{\text{mb}}^* \) does not significantly change in the range \( 30^\circ < \theta < 50^\circ \). This fact can be explained considering that, as the angle \( \theta \) increases, both the shear and the axial stresses, \( \tau \) and \( f_n \), also increase. As a result, the shear strength, which directly depends on \( f_n \) (see Eq. 4.9) remains approximately constant.

![Graph showing the strength envelope as a function of angle \( \theta \).](image)

**Figure 8.6.** Typical strength envelope as a function of the angle \( \theta \).

The strength envelope allows the prediction of the type of failure and the evaluation of the diagonal force required to produce it (using Eq. 8.1). Theoretical values obtained from this approach are compared with experimental results corresponding to two units tested by the author in sections 10.2.6 and 10.3.6.

It is worth noting that Eqs. 8.6 and 8.8 exhibit a singularity when the angle \( \theta \) matches the following conditions: \( \tan \theta = 1/\mu^* \) and \( \tan \theta = C_s/0.27 \), respectively. The latter condition occurs at \( \theta = 82.3^\circ \) (considering \( C_s = 2.0 \), see section 4.2.4.2), whereas the former condition depends on the values of the reduced coefficient of friction, \( \mu^* \). For usual cases, this parameter varies between 0.3 and 0.6, which leads to a singularity in Eq. 8.8 when \( \theta \) lies between 73.3\(^\circ\) and 59.0\(^\circ\), respectively. Therefore, this problem is not important for the evaluation of the strength of masonry panels in infilled frames, with typical values of \( \theta \) ranging from 25\(^\circ\) to 50\(^\circ\).

8.2.4 Hysteretic Behaviour

An adequate consideration of the hysteretic behaviour of masonry is required in order to conduct nonlinear dynamic analyses of infilled frames subjected to earthquake ground motions. In the proposed model, the response of the axial springs is represented according to the hysteretic stress-strain relationship.
presented in section 7.2. The axial force, \( R \), and the axial displacement, \( \Delta_R \), in the masonry strut are related to the stress, \( f_m \), and the strain, \( \varepsilon_m \), according to the following expressions:

\[
R = f_m A_{ms} \\
\Delta_R = \varepsilon_m d_m
\]  
(8.9)  
(8.10)

The stiffness of the axial spring, \( K_a \), depends on the area and length of the strut and on the tangent modulus of the masonry, \( E_m \), thus:

\[
K_a = \frac{E_m A_{ms}}{d_m}
\]  
(8.11)

According to the stress-strain relationship considered in the model, the tangent modulus, and consequently the stiffness \( K_a \), are negative in the descending branch of the curve. In practical applications however, it is assumed that the axial stiffness of the strut is zero in order to avoid numerical problems associated with negative values of the diagonal terms of the global stiffness matrix.

The area of the equivalent strut, \( A_{ms} \), can decrease as the lateral displacement, and consequently the axial displacement, increase as a result of the reduction of the contact length between the panel and the frame, and due to the cracking of the masonry infill. In order to consider this fact, it is assumed in the proposed model that the area of the equivalent varies as a function of the axial displacement, according to the following relationships (the notation is self-explanatory in Fig. 8.7):

\[
A_{ms} = A_{ms1} \quad \text{if } \Delta_R \geq \Delta_{R1}
\]

\[
A_{ms} = A_{ms1} - (A_{ms1} - A_{ms2}) \left( \frac{\Delta_{R1} - \Delta_R}{\Delta_{R1} - \Delta_{R2}} \right) \quad \text{if } \Delta_{R1} < \Delta_R > \Delta_{R2}
\]  
(8.12)

\[
A_{ms} = A_{ms2} \quad \text{if } \Delta_R \leq \Delta_{R1}
\]

![Figure 8.7. Variation of the area of the masonry strut as a function of the axial displacement.](image)
The variation of the strut area $A_{mn}$ is introduced in the model to gain generality, even though there is insufficient information to estimate the practical values for the parameters required to apply Eq. 8.12. According to experimental results reported by Decanini and Fantin [D13] (see section 6.2.1.3) the equivalent width of the strut decreases by about 20% to 50% due to cracking of the masonry panel. However, these values were derived under the assumption that the modulus $E$ remains constant, whereas the proposed model considers a variable modulus, which decreases as the axial compressive strain increases. The main advantage of Eq. 8.12 is that it allows the user to control the variation of the stiffness and the axial strength of the masonry strut.

8.2.5 Computational Implementation and Numerical Examples

The proposed analytical model was implemented using the computer program RUAUMOKO [C16]. A subroutine was incorporated to this program in order to represent the hysteretic behaviour of masonry in compression, according to the model described in section 8.2.4 (the source code of the subroutine is presented in Appendix 4). The masonry struts, one for each diagonal of the panel, are represented with spring elements, whereas the members of the frame are modelled with beam-column elements available in the element library of the program.

Several geometrical and mechanical parameters are required to define the behaviour of the masonry struts. A list of the variables needed as input data is presented below, with recommendations for the selection of their values (the meaning of the variables associated with the cyclic axial behaviour of masonry is given in section 7.2):

- **Compressive strength**, $f_{mn}':$ this is the main parameter to control the resistance of the strut. It must be noted that $f_{mn}'$ does not represent the standard compressive stress of masonry. The value of $f_{mn}'$ can be adopted from Eqs. 8.6 or 8.8, depending on the mode of failure expected in the masonry panel.

- **Tensile strength**, $f_{t}:$ this variable represents the tensile strength of the masonry or the bond strength of the panel-frame interfaces, whichever is smaller. The consideration of the $f_{t}$ has been introduced in the model in order to gain generality. However, results obtained from different examples indicates that the tensile strength, which is usually much smaller than the compressive strength, has no significant influence on the overall response. Therefore, $f_{t}$ can be assumed to be zero in the lack of more detail information.

- **Strain at maximum stress**, $e_{um}:$ this parameter usually varies between -0.002 and -0.005 and its main effect on the overall response of the infilled frame is the modification of the secant stiffness of the ascending branch of the stress-strain curve.

- **Ultimate strain**, $e_{u}:$ this parameter can be used to control the descending branch of the stress-strain relationship. When a large value is adopted, for example $e_{u} = 20 e_{m}'$, a smooth decrease of the compressive stress is obtained. It has been found that the descending branch is more adequately described using a parabolic curve instead of Sargin's equation (see Fig. 7.2).

- **Closing strain**, $e_{c}:$ defines the limit strain at which the cracks partially closed and compressive stresses can be resisted. Values of $e_{c}$ ranging between 0 and 0.003 lead to
results which agree adequately with experimental data. If a large negative value is adopted, for example $\varepsilon_a = -\varepsilon_a$, this effect is not considered in the analysis.

- **Elastic modulus, $E_{ma}$**: this parameter represents the initial slope of the strain-stress curve (see Fig. 7.2) and its value can exhibit a large variation. Various expressions have been proposed for the evaluation of the elastic modulus of masonry, as discussed in section 3.4. It is worth noting, however, that these expressions usually define the secant modulus at a stress level between 1/3 to 2/3 of the maximum compressive stress. Therefore, the use of these values can underestimate the initial stiffness of the infilled frame. In order to obtain an adequate ascending branch of the strength envelope it is assumed that $E_{ma} \geq 2 f'_{cd}/\varepsilon'_{ma}$.

- **Unloading stiffness factor, $\gamma_{un}$**: this parameter controls the slope of the unloading branch, with usual values ranging from 1.5 to 2.5. It is assumed that $\gamma_{un} \geq 1.0$.

- **Reloading strain factor, $\alpha_r$**: defines the point where the reloading curves reach the strength envelope. The calibration of the hysteretic model for the axial behaviour of masonry showed that good results are obtained using values of $\alpha_r$ between 0.2 and 0.4 (see sections 7.2.7 and 7.2.8). However, higher values are required to model adequately the cyclic response of the infilled frames, because other sources of nonlinear behaviour (such as sliding shear) need to be indirectly considered in the response of the masonry struts. For this reason, a value of $\alpha_r$ equal to 1.5 was adopted in the nonlinear analyses conducted by the author.

- **Area of the strut**: four parameters are required to represent the variation of the area of the masonry strut, namely, initial and final area, $A_{m1}$ and $A_{m2}$, and the axial displacements at which the area changes, $\Delta_{R1}$ and $\Delta_{R2}$ (see Fig. 8.7). In a simplified model, it can be assumed $A_{m1} = A_{m2}$, considering a low value of the strut area to avoid an excessive increase of the axial strength. In a more refined analysis, a higher value of the initial area can be adopted, whereas the final area is reduced about 10 to 30%. The displacement $\Delta_{R1}$ and $\Delta_{R2}$ can be estimated as $\varepsilon'_a d_m/10$ and $\varepsilon'_a/2 d_m$ (where $d_m$ is the length of the masonry strut), respectively, at least until more precise information becomes available. Several empirical expressions, which were described in section 6.2.1.3, have been proposed for the evaluation of the equivalent width of the masonry strut, whose value normally ranges from 0.1 to 0.25 of the diagonal length of the infill panel.

The calibration of the model was conducted in two steps. First, several examples were analysed to evaluate the general response, to observe the shape of the hysteresis loops and to investigate the effect of the different parameters associated with the masonry struts. Fig. 8.8 illustrates the results obtained from dynamic analyses in which an infilled frame was subjected to a sinusoidal ground acceleration and to the ground motion recorded during El Centro 1940 earthquake. These curves were obtained by combining the shear forces resisted by the columns and the struts. The axial force-displacement relationships for one strut of the model are shown in Fig. 8.9 considering the two loading cases indicated in Fig. 8.8. In the second step, a comparison was made between analytical results and the experimental response recorded during a test conducted by the author (see Chapters 9 and 10). This comparison is aimed at demonstrating that the proposed model is general enough to consider different situations. The results are illustrated in Fig. 8.10, where it is observed that the model can properly describe the more important features of the experimental response, such as stiffness and strength degradation, pinching of the hysteresis loops and accumulative damage due to repetition of cycles.
Figure 8.8. Dynamic response of an infilled frame subjected to (a) sinusoidal ground acceleration and (b) El Centro 1940 N-S earthquake.

The characteristics of the model used in the previous comparison were adopted taking into account the material properties measured in the test programme. The strength of the frame members were evaluated using sectional analysis and their cyclic flexural response was modelled according to the Modified Takeda Degrading Stiffness rule [C16]. Rigid ends were provided to the frame members to represent the effect of the tapered beam-columns joints used in the test unit. The response was primarily controlled by the masonry struts and, therefore, the model was not very sensitive to changes in the representation of the frame. The parameters related to the axial cyclic behaviour of the masonry struts were:

\[ f'_{um} = 1.2 \text{ MPa} \quad f'_i = 0.1 \text{ MPa} \quad \varepsilon'_a = -0.002 \quad \varepsilon_v = -0.02 \]
\[ \varepsilon_0 = 0.002 \quad E_{mo} = 1.2 \text{ GPa} \quad \gamma_{sm} = 2.0 \quad \alpha_e = 1.5 \]
\[ A_{mol} = 0.101 \text{ m}^2 \quad A_{mol} = 0.072 \text{ m}^2 \quad \Delta_{R1} = 0.2 \text{ mm} \quad \Delta_{R2} = 2.0 \text{ mm} \]
The initial and final areas, $A_{\text{init}}$ and $A_{\text{exit}}$, were evaluated assuming an equivalent width of the strut equal to 0.35 and 0.25 of the diagonal length of the panel, respectively. The infilled frame model was subjected to a displacement history similar to that considered in the test using pushover analysis option of the program RUAUMOKO.

![Axial displacement, $\Delta R$ (mm)](image)

(a) Sinusoidal ground acceleration

![Axial displacement, $\Delta R$ (mm)](image)

(b) El Centro 1940 earthquake

**Figure 8.9.** Typical axial force-displacement relationships for the masonry struts.

The comparison presented in Fig. 8.10 indicated that a good agreement can be obtained between the experimental data and the analytical results. This was achieved after several adjustments in which different parameters were successively modified to fit the experimental results. However, it is important to note the limitations of the analytical formulation. Most of the pinching of the hysteresis loops observed in the model results from the nonlinear behaviour of the masonry struts, whereas in the test unit it was due to sliding shear of the masonry panel and the columns of the surrounding frame. The representation of this phenomenon with a macro-model is very complex and requires the use of a refined formulation, which is not usually available in most of the existing computer programs. Further research is required to incorporate shear failure mechanisms of reinforced concrete members in the analytical modelling, considering their influence in the structural response can be very important.

**8.3 PROPOSED MACRO-MODEL FOR REFINED ANALYSIS OF INFILLED FRAMES**

**8.3.1 Introduction**

The single-strut model discussed in the previous section is capable of describing the overall response of infilled frames. However, a refined model is required to consider more accurately the interaction between the masonry panel and the surrounding frame, and to represent a particular type of failure. Even though the development of a general model is desirable, it seems very difficult to achieve this objective with a macro-model when a detail representation of the structure is required. As a result, several models are discussed in the following sections, intended to represent different types of failure of the masonry panel. The use of these models implies a preliminary study in order to investigate the mode of failure expected in each particular case.
8.3.2 Diagonal Tension Failure
When a diagonal tension failure is expected in the panel, the triple-strut model (Model C) described in Fig. 8.1 can be implemented. The modelling of each masonry panel and the surrounding frame requires the use of 6 struts elements and 9 beam or beam-columns elements, which obviously increases the complexity of the analysis. The strength of the masonry struts can be evaluated according to Eq. 8.8.

8.3.3 Crushing of the Corners
The failure of the masonry panels due to crushing of the masonry at the corners is uncommon for infilled reinforced concrete frames, although it has been observed in infilled steel frames (see section 5.7.2.3). This situation can be approximately represented with the macro-model illustrated in Fig. 8.11. The central strut is divided into two parts with different area, in order to consider approximately the increase of axial stresses occurring in the corners of the panels (see Fig. 8.4). It is worth noting that this idea is presented here in a general sense. The practical application of the model requires further research to investigate the convenient values of the area and length of each part of the central strut.

8.3.4 Shear Failure
8.3.4.1 General Considerations
The proper representation of the shear failure of the masonry panel by stepped debonding of the mortar joint is a complex problem because the shear strength varies during the analysis depending on the compressive stress level induced in the panel. This is the most common type of failure of the masonry panel according to the experimental data discussed in Chapter 5 and to the theoretical results obtained from the comparison of Eqs. 8.6 and 8.8. For this reason, a new multi-spring model is proposed in this section, which is intended to represent specifically the shear failure of the masonry panel. The model...
accounts separately for the compressive and shear behaviour of masonry using a double truss mechanism and a shear spring in each direction, as illustrated in Fig. 8.12 (for simplicity, only the struts and the shear spring active in one direction are represented in this figure). It is assumed that both struts are parallel and separated by a vertical distance equal to \( h_x \). Values of \( h_x \) between \( z/3 \) and \( z/2 \) seem to lead to adequate results, being \( z \) the contact length between the panel and the frame (see Eq. 6.8).

**Figure 8.11.** Multi-strut model proposed to consider the increase of the compressive stress in the corners of the panel.

![Multi-strut model](image1)

**Figure 8.12.** Multi-strut model proposed for the analysis of infilled frame when a shear failure is expected in the panel.

According to this model, the response in the initial stage is primarily controlled by the shear spring and the shear forces and bending moments in the frame are similar to those obtained from the triple-strut model (Model C, Fig. 8.1). After the shear strength is reached and sliding shear starts, the mechanism changes resulting in a significant increase of the actions induced in the frame.

The properties of the shear spring are evaluated on the basis of a simplified approach, which tries to reflect the actual behaviour of the masonry panel. The hysteretic response is modelled following an elasto-plastic rule with a variable shear strength. The stiffness of the shear spring, \( K_s \), can be calculated as a fraction \( \gamma \), of the total stiffness of the masonry strut:
\[
K_s = \gamma_s \frac{A_m}{d_m} \frac{E_m}{d_m} \cos^2 \theta
\]  

(8.13)

where the term \( \cos^2 \theta \) (being \( \theta \) the inclination of the diagonal of the infill panel) is introduced to express the stiffness in the horizontal direction [L16]. The factor \( \gamma_s \) defines the proportion of the total stiffness of the panel assigned to the shear spring. The comparison of theoretical results and data recorded in the experimental programme shows that a value of \( \gamma_s \) ranging from 0.5 to 0.75 can be used in the lack of further information.

It is worth noting that the procedure considered for the evaluation of the stiffness of the shear spring does not reflect the actual shear behaviour of the masonry panel, but represents a practical approach which leads to adequate values of the lateral stiffness of the infilled frame. In this way, it is possible to use the existing empirical expressions proposed for the calculation of the equivalent width of the masonry strut. Additional experimental and analytical research is required in order to develop a more rational procedure which be capable of taking into account the shear response of the infill panel not only in terms of strength but also in terms of stiffness.

The shear strength of the spring is controlled by the shear-friction mechanism described in section 7.3. For the application of the model, however, the strength has to be expressed in terms of forces instead of stresses. The evaluation of the shear strength, \( V_s \), is conducted considering two different stages:

- **Elastic response (before the bond-shear strength is reached)**

\[
V_s = \frac{1}{\alpha_s} \left( \tau_o L_m t + \mu \left| \sum R_i \right| \right) \quad \text{if} \quad \sum R_i < 0
\]

\[
V_s = \frac{\tau_o}{\alpha_s} L_m t \quad \text{if} \quad \sum R_i \geq 0
\]

(8.14)

- **Sliding (after the bond-shear strength is reached)**

\[
V_s = \mu \left| \sum R_i \right| \quad \text{if} \quad \sum R_i < 0
\]

\[
V_s = 0 \quad \text{if} \quad \sum R_i \geq 0
\]

(8.15)

where \( \sum R_i \) is the sum of the axial forces acting on the four masonry struts. The coefficient \( \alpha_s \) is introduced in Eq. 8.14 to take into account the nonuniform distribution of the shear stresses along the horizontal section of the panels. This coefficient is defined as the ratio of the maximum shear stress, \( \tau_{max} \), to the average shear stress, \( \tau_{av} \):

\[
\alpha_s = \frac{\tau_{max}}{\tau_{av}} = \frac{L_m}{V} \frac{t}{\tau_{max}}
\]

(8.16)
Fig. 8.13 illustrates the shear stress contours in a masonry panel subjected to lateral forces and shows the variation of the shear stress along a horizontal plane located at the centre of the panel. It can be observed that the maximum stress is markedly higher than the average shear stress, with typical values ranging from 1.40 to 1.65. These results agree very well with those reported by Riddington and Stafford Smith [R3], who proposed that \( \tau_{\text{max}} \) can be estimated as 1.43 \( \tau_{\text{m}} \) for design purposes (see section 6.4.2).

In order to avoid a large shear strength due to high axial forces in the struts, the values of \( V_s \) obtained from Eqs. 8.14 and 8.15 are limited by the following condition:

\[
V_s \leq \frac{\tau_{\text{max}}'}{\alpha_s} L_m t
\]

(8.17)

where \( \tau_{\text{max}}' \) is the maximum permissible shear stress in the masonry panel. The value \( \tau_{\text{max}}' \) can be adopted from shear failure envelope obtained from the modification of the Mann and Müller's theory [M2] (see section 4.2.4), as indicated in Fig. 8.14.

![Figure 8.13. Variation of the shear stress along a horizontal plane at the centre of the panel.](image)

![Figure 8.14. Approximate definition of the maximum permissible shear stress, \( \tau_{\text{max}}' \).](image)
The response of the masonry struts, aimed at representing the behaviour of masonry in compression, is governed by the analytical model described in section 8.2.4. The areas of both struts are assumed to be equal, with a value assigned in order that the combination of the shear spring and the two masonry struts results in a total stiffness approximately equal to the stiffness of the single strut model. Therefore, the area of each masonry strut is equal to \((1 - \gamma_s) A_m/2\), being the axial stiffness given by:

\[
K_a = \frac{(1 - \gamma_s) A_m E_t}{2 d_m}
\]  \hspace{1cm} (8.18)

The model described in this section can be also applied for the particular case in which the failure is due to sliding shear with the formation of an approximately horizontal crack (see section 5.7.2.2). The general characteristics of the model, as illustrated in Fig. 8.15, are similar to those described in the previous paragraphs, but it is considered that \(h_r = h_w/2\), since the horizontal crack usually occurs at the centre of the masonry panel.

![Diagram](image)

**Figure 8.15.** Proposed model for the particular case of horizontal sliding shear.

The values of the factors \(\gamma_s\) and \(\alpha_s\) recommended in this section were obtained from several finite element analyses and represent only a first approximation adopted in the lack of more information. Additional experimental and theoretical research is required to calibrate adequately these parameters.

### 8.3.4.2 Formulation of the Proposed Macro-Model

The practical implementation of the proposed model requires the use of several spring elements and beam or beam-column elements to represent the masonry panel and the surrounding frame. In order to simplify the application of the proposed model, from the user point of view, a 4-node panel element has been formulated and implemented in the structural program RUAUMOKO [C16] (this new element is currently under testing). In this way, the user only needs to define the characteristics of the masonry infill as a whole element, whereas the program evaluates internally the properties of the struts and the shear spring. Fig. 8.16 illustrates the main characteristics of the proposed panel element.
Three different sets of nodes are considered for the development of the panel element, namely, external nodes, internal nodes and dummy nodes. The external nodes are those connected to the remaining structure, whereas the internal nodes are defined by a horizontal and a vertical offset, $x_{oi}$ and $y_{oi}$ respectively, measured from the external node i. This is intended to represent the reduction of the dimensions of the panel due to the depth of the frame members. Three degrees of freedom, the translations $u$ and $v$ and the rotation $\phi$, are considered in each of the external and internal nodes. Four dummy nodes, with 2 translational degrees of freedom per node, are required to define one end of the strut members, which is not connected to the corners of the panel.

The formulation of the stiffness matrix and the nodal forces of the panel element is conducted considering the equilibrium and compatibility equations between the forces and displacement of the different coordinate systems. These relationships, derived from the principle of Virtual Displacement, are presented in a general way in Fig. 8.17. They indicate that if a transformation matrix $[Q]$ relates the
displacements \{u\}_A and \{u\}_B expressed in two different systems of rectangular coordinates, the transpose of this matrix, \([Q]^T\), also transforms the nodal forces \{F\}_B to \{F\}_A. Applying virtual work considerations, it can be shown that the stiffness matrix in the coordinates system "B" is equal to the double product of the matrix \([Q]\) applied to the stiffness matrix in the coordinates system "A". A more detail description and the derivation of these relationships can be found elsewhere [L16]. These relationships are successively applied to transform the structural parameters of each strut to the global system of coordinates, associated with the external nodes of the panel element.

![Diagram](image)

**Figure 8.17.** Relationships between displacement, forces and stiffness expressed in different coordinates systems.

The first transformation required in the analysis of the panel element relates the axial displacement of the strut \(\Delta R\) (local system) to the horizontal, \(u\), and vertical, \(v\), displacements at the ends \(j\) and \(k\) of the strut (global system), according to the following relationship:

\[
\Delta R = [Q]_{GL} \begin{bmatrix} u_j \\ v_j \\ u_k \\ v_k \end{bmatrix} = \begin{bmatrix} -\cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}
\]

where \(\theta_i\) is the inclination of the strut referred to the global system of coordinates (see Fig. 8.16). The transformation matrix \([Q]_{GL}\) is used to obtain the nodal forces and the stiffness matrix related to each node of the strut, according to the relationships given in Fig. 8.17. Each strut has one end connected to a dummy node, and consequently, the structural parameters related to this node need to be transformed to the adjacent internal nodes \(m\) and \(n\) (see Fig. 8.18):
\[
\begin{bmatrix}
\{u\} \\
\{v\}
\end{bmatrix}_D = [Q]_{ID} \begin{bmatrix}
\{u_m\} \\
\{v_m\} \\
\{\theta_m\} \\
\{u_n\} \\
\{v_n\} \\
\{\theta_n\}
\end{bmatrix}
\]

(8.20)

where the sub-indexes D and I refer to dummy nodes and internal nodes, respectively. The matrix \([Q]_{ID}\) can be evaluated using interpolation functions which relate the displacements \(u\) and \(v\) corresponding to a point located at a distance \(s\) from the node \(m\), with displacements, \(u, v,\) and \(\phi\), of the adjacent internal nodes, \(m\) and \(n\). The displacement perpendicular to the edge and the rotation are interpolated with cubic hermitian polynomials, whereas the displacement in the direction of the edge is interpolated with a linear function. In the proposed model, the matrix \([Q]_{ID}\) is calculated using a subroutine of the library of the computer program RUAUMOKO [C16] according to the following expressions:

\[
\begin{align*}
Q(1,1)_{ID} &= Q_1 \cos^2 \theta_E + Q_3 \sin^2 \theta_E \\
Q(1,2)_{ID} &= Q(2,1)_{ID} = (Q_1 - Q_3) \cos \theta_E \sin \theta_E \\
Q(1,3)_{ID} &= Q_2 \cos \theta_E \\
Q(1,4)_{ID} &= Q_3 \cos^2 \theta_E + Q_6 \sin^2 \theta_E \\
Q(1,5)_{ID} &= Q(2,4)_{ID} = (Q_3 - Q_6) \cos \theta_E \sin \theta_E \\
Q(1,6)_{ID} &= Q_4 \cos \theta_E \\
Q(2,2)_{ID} &= Q_1 \sin^2 \theta_E + Q_5 \cos^2 \theta_E \\
Q(2,3)_{ID} &= Q_2 \sin \theta_E \\
Q(2,5)_{ID} &= Q_3 \sin^2 \theta_E + Q_6 \cos^2 \theta_E \\
Q(2,6)_{ID} &= Q_4 \sin \theta_E
\end{align*}
\]

(8.21)

where \(\theta_E\) is the inclination of the edge (see Fig. 8.18),

\[
\begin{align*}
Q_1 &= 1 - 3 \chi^2 + 2 \chi^3 \\
Q_2 &= (\chi - 2 \chi^2 + \chi^3) L_E \\
Q_3 &= 3 \chi^2 - 2 \chi^3 \\
Q_4 &= (-\chi^2 + \chi^3) L_E \\
Q_5 &= 1 - \chi \\
Q_6 &= \chi
\end{align*}
\]

(8.22)

\(L_E\) is the length of the edge and \(\chi = s/L_E\) (\(s\) is the distance between the dummy node and the internal node, as indicated in Fig. 8.18). The final transformation relates the displacements of the internal nodes to the displacements of the external nodes:
\[
\begin{pmatrix}
u \\ v \\ \psi \\
\end{pmatrix}_I = \begin{pmatrix} \mathbf{Q}_{EI} \end{pmatrix} \begin{pmatrix}
u \\ v \\ \psi \\
\end{pmatrix}_E
\]

(8.23)

where the sub-indexes I and E indicate internal and external nodes, respectively, and the matrix \([\mathbf{Q}_{EI}]\) is a function of the horizontal and vertical offsets, \(x_{oi}\) and \(y_{oi}\) corresponding to the node i:

\[
\begin{pmatrix} \mathbf{Q}_{EI} \end{pmatrix} = \begin{pmatrix}
1 & 0 & -y_{oi} \\
0 & 1 & x_{oi} \\
0 & 0 & 1 \\
\end{pmatrix}
\]

(8.24)

![Diagram of panel element](image)

**Figure 8.18.** Detail of one edge of the panel element.

This procedure of successive transformations should be applied for the four struts which form the panel element in order to assemble the stiffness matrix (12 x 12 terms) and the vector of nodal forces (12 terms).

The consideration of the shear behaviour of the panel element is simpler, since only one spring element is used. This spring is connected to two diagonally opposite internal nodes depending on the direction of the shear force (see Fig. 8.16 (b)). It is worth noting that the implementation of the proposed model in a panel element allows the calculation of the axial forces in the struts to be used for evaluating the strength of the shear spring. This inter-relationship between different members is not possible to be considered in most of the existing programs for structural analysis. The stiffness and shear force are evaluated according to Eqs. 8.13, 8.14 and 8.15. These parameters are associated with the horizontal displacement \(u\), and consequently the only transformation required in this case is that which relates the internal nodes to the external nodes (see Eqs. 8.23 and 8.24).

It is worth noting that a limitation of the proposed panel element is that the potential plastic hinges which could develop along the length of the columns cannot be considered in the model, even though the nodal forces (moment and shear forces) take into account the effect due to the eccentricity of the struts. When the formation of plastic hinges at span length is important, the proposed model needs to be implemented.
as described in Figs. 8.12 or 8.15, which requires the use of 4 strut elements, 2 shear spring elements and 6 beam or beam-columns elements to represent each masonry panel and the surrounding frame.

The input data required for the application of the panel element includes the parameters described in section 8.2.5 for defining the cyclic axial behaviour of masonry and the properties of the equivalent strut. In addition, the following variables need to be defined in relation to the shear behaviour of the masonry panel:

- **Vertical separation between struts, \( h_z \):** values of \( h_z \) between \( z/3 \) and \( z/2 \) seems to lead to adequate results, being \( z \) the contact length between the panel and the frame (see Eq. 6.8).
- **Horizontal and vertical offset, \( x_{al} \text{ and } y_{al} \):** These parameters define the horizontal and vertical distance, respectively, measured from the external nodes to the internal nodes. This is intended to represent the reduction of the dimensions of the panel due to the depth of the frame members.
- **Bond shear strength, \( \tau_o \) and coefficient of friction, \( \mu \):** these parameters are usually obtained from direct shear tests or following design specifications. It is recommended, however, the use of the reduced values resulting from the modification of the Mann and Müller's theory [M2] proposed in section 4.2.4 (see Eq. 4.17). This theory takes into account the complex stress state in the panel due to the composite nature of masonry.
- **Maximum shear stress, \( \tau_{\max} \):** this is the maximum shear stress permissible in the masonry panel, whose value can be selected from the shear failure envelope resulting from the modification of the Mann and Müller's theory, as indicated in Fig. 8.14.
- **Shear stress factor, \( \alpha_s \):** this parameter, defined by Eq. 8.16, normally varies from 1.40 to 1.65.
- **Shear stiffness factor, \( \gamma_s \):** which represents the fraction of the total stiffness assigned to the shear spring, ranging from 0.5 to 0.75.
- **Thickness of the panel, \( t \).**

### 8.3.4.3 Numerical Examples

In order to verify the principal characteristics of the model proposed for the analysis of infilled frames when a shear failure is expected, a nonlinear static analysis was conducted using the multi-strut model illustrated in Fig. 8.12. This analysis was aimed at representing the experimental response measured during the test of Unit 2 (see Chapter 9 and section 10.3). The lateral and vertical forces were applied in successive increments, in which the mechanical properties of the structure were updated according to the strains and displacements induced in the different components of the model in the previous step. The separation of the struts was adopted as \( h_z = 0.23 \text{ m}, \) which is equivalent to \( z/2 \). The properties of the model associated with the axial cyclic behaviour of masonry were the same used in the comparison presented in section 8.2.5, whereas the parameters required to define the behaviour of the shear spring were:

\[
\begin{align*}
\tau_o &= 0.31 \text{ MPa} \\
\mu &= 0.51 \\
\gamma_s &= 2/3 \\
\tau_{\max} &= 1 \text{ MPa}
\end{align*}
\]
Fig. 8.19 compares experimental and analytical results in the range of small displacement in order to observe clearly the response in the initial stage. The force level at which shear cracking occurs according to the analytical procedure, 59.8 kN, agrees very well with the measured value equal to 65.0 kN (see section 10.3.6). The overall theoretical response is compared in Fig. 8.20 with the strength envelope measured during the test, indicating that the proposed model can estimate the lateral resistance of the infilled frame and the strength degradation observed for large displacements. In addition, the failure mechanism is properly represented, being the analytical model capable of describing the shear cracking of the masonry panel and the yielding of the tension column of the frame.

![Graph showing shear cracking and lateral displacement](image)

**Figure 8.20.** Comparison between the experimental strength envelope and the analytical results.

![Graph showing yielding and lateral displacement](image)

**Figure 8.19.** Comparison of experimental and analytical results in the range of small displacements.
8.4 MODELLING OF INFILLED FRAMES WITH THE PROGRAM ABAQUS

In this study, the information related to the behaviour of infilled frames was complemented with some numerical results obtained from the general computer program ABAQUS [A12], based on a finite element formulation. The same computer program was used to investigate the behaviour of masonry subjected to compression and shear (see Chapter 3 and 4, respectively). The main characteristics of these models are described below:

- **Reinforced Concrete Frame**: the frame was modelled with two-node linear beam elements, which are based on Timoshenko beam theory. Each member of the frame was subdivided into a number of elements according to the characteristic of the mesh adopted for the discretization of the panel. The elements coincided with the geometrical axis of the frame members. Therefore, there was an error in the formulation of the model because the depth of the reinforced concrete members was not considered. Since the aim of these analyses was to investigate conceptually the behaviour of infilled frames, it is believed that this error is not significant. In some examples, the behaviour of the frame members was assumed to be elastic, considering the mechanical properties on the basis of cracked sections. However, nonlinear behaviour was considered in other examples. A trilinear bending moment-curvature relationship was adopted to consider the flexural behaviour of the members. The axial behaviour was represented with a linear elastic relationship in compression and elasto-plastic relationship in tension.

- **Masonry Panel**: the panel was modelled with plane stress, four node, bilinear elements. The material was represented with the model provided by ABAQUS [A12] for plain concrete. The uniaxial compressive behaviour was defined with a polygonal curve to approach the curve given by Sargin's equation (see Eq. 7.1 (a)). Cracking is the most dominant aspect of the behaviour of this analytical model. It is considered that cracking occurs when the stresses reach a failure surface, taken as a simple Coulomb line, written in terms of the first and second stress invariants. Once cracking develops, a yield surface is introduced to model the opening cracks. The model uses the smeared approach to simulate the effect of cracking. In order to avoid mesh sensitivity and convergence problems, stress softening was modelled with a nonlinear curve from the point of maximum tensile stress to zero, at a strain five times greater than the strain at the maximum tensile stress. The response of the material in compression was modelled using elasto-plastic theory. Associated flow and isotropic hardening were used in the model.

- **Panel-Frame Interfaces**: interface elements were used all around the masonry panels. A general formulation is provided in ABAQUS [A12] to analyse contact problems and different options can be selected by the user. It was assumed a classical Coulomb friction model, with a limit on the shear stress. The purpose of this limit is to prevent the shear stress from being excessive when the pressure in the interface is very high. The Lagrange multiplier formulation for sticking friction was selected in the tangential direction of the surface, instead of the stiffness method. Consequently, there is no slip between surfaces any time the surfaces are stuck. The model assumes that the surfaces are not stuck when the
axial tensile stress is greater than the bond strength. In the analyses conducted in this work, it was considered a coefficient of friction 0.45 and the tensile bond strength was 0.10 MPa.

The boundary conditions for the model were set assuming that the base is completely fixed. Horizontal forces were applied to the structure uniformly distributed along the top beam. The static nonlinear analysis was conducted using the modified Riks method (also called Constant-Arc-Length method) [C6, R8, R9], which obtains equilibrium solutions by controlling the path length along the load-displacement curve within each increment. This method provides solutions even in the cases of complex, unstable response. It was originally developed to analyse structures exhibiting a decreasing post-limit characteristic (buckling and snapping problems). In the analysis of the infilled frames, the modified Riks method was used to overcome numerical problems occurring during the execution of the ABAQUS program.

The author's experience using finite element models for infilled frames indicates that many uncertainties and problems still remain, despite the enormous advance achieved in analytical modelling in recent years. The proper representation of the cracking of the panel is difficult and the response is very sensitive to the finite element mesh and the tension stiffening model assumed for the panel material. Shear cracking along mortar joints cannot be considered using a unique material for masonry. Therefore, bricks and joints should be modelled separately, which leads to large models with thousands of degrees of freedom. The response of interface elements is rather unstable and very susceptible to the values adopted for the parameters of the model. Numerical problems may occur when interface elements are joined in different directions. Due to these problems, a real prediction of the response of infilled frames seems to be difficult to obtain. After the test results are available, the finite element model can be successively modified until "a very good agreement" between the experimental and analytical data is achieved. Finite element models, however, represent a proper tool for qualitatively research and good results can be obtained when the model is adequately calibrated with experimental data.

8.5 CONCLUSIONS

• A comparative study was conducted to evaluate the limitations of the single strut model. The results of this study indicate that the single strut model, despite its simplicity, gives an adequate estimation of the stiffness of the infilled frame and the axial forces induced in the frame members by lateral forces, and represents an adequate procedure for simplified analyses focussed on the overall response of the structure. However, a more refined model is required in order to obtain realistic values of the bending moments and shear forces in the frame.

• A simple approach is proposed for the design and analysis of infilled frames based on the truss mechanism. The strength of the masonry infill is evaluated according to a transformation of a failure theory for masonry which considers the mechanical properties of the masonry and the inclination of the diagonal, \( \theta \). This theory represents the most common modes of failure of the panel, namely, shear-friction or diagonal tension failure. In order to conduct nonlinear dynamic analysis based on the proposed model, a new
hysteresis rule aimed at representing the cyclic behaviour of the masonry struts was implemented in the computer program RUAUMOKO.

- Since debonding of the mortar joints is the most common type of failure observed in the masonry panel, a multi-spring model is developed to represent specifically this situation. The model accounts separately for the compressive and shear behaviour of masonry using a double truss mechanism and a shear spring in each direction. This concept is implemented in a 4-node panel element which is being incorporated in the computer program RUAUMOKO.

- Recommendations are also given for the analysis of infilled frames when a failure due to crushing of the corners is expected in the panel. Further research is required in order to implement this model.

- Reinforced concrete columns of infilled frames are prone to exhibit a sliding shear failure. The representation of this phenomenon is difficult to incorporate in the analytical modelling because the shear strength depends on different resistance mechanisms and it can be affected by the axial force acting on the frame member. Since sliding shear can significantly affect the response of infilled frames, additional investigation needs to be conducted.

- The examples and comparisons conducted in this chapter show that the experimental cyclic response of infilled frames can be properly described with theoretical results obtained from the proposed models. This, however, requires a fine calibration of the analytical model which is usually achieved after several adjustments of the model, indicating that a detail prediction of the experimental cyclic response is still a difficult task.
9. TEST PROGRAMME ON FRAMED MASONRY STRUCTURES

9.1 INTRODUCTION
Infilled frame structures, of which framed masonry is a particular case, have been experimentally and analytically investigated in the last forty years, although a number of uncertainties still persist. The literature review presented in previous chapters indicates that the behaviour of this type of structure is not completely understood, that there are no simple and reliable models for the analysis and that earthquake-resistant design procedures should be improved. Therefore, more research is required to design safe buildings in those seismic regions where infilled frames are still used as a structural system and to assess and retrofit existing structures.

It was pointed out in section 5.8 that a great number of mechanical and geometric parameters can affect the structural response of infilled frames. This fact makes it difficult to obtain general conclusions and to compare experimental results. A careful planning of the research programme is required to select the characteristics of the test units and the test setup according to the main objectives of the study.

9.2 OBJECTIVES OF THE TEST PROGRAMME
The development of the test programme was conditioned by limitations in time and funding. For these reasons, only two framed masonry structures were tested under simulated seismic loading. The reduced number of test units obviously narrowed the scope of the programme and limited the parameters to be investigated. Despite this problem, it is believed that useful conclusions can be obtained from the experimental results.

The principal objectives of this test programme can be summarized as follow:
• To observe the performance of framed masonry structures and to improve the understanding of the structural behaviour.
• To use a realistic loading system for the application of the lateral forces intended to simulate seismic actions.
• To investigate the behaviour of new reinforcement details proposed to improve the seismic response.
• To obtain experimental data to calibrate and to validate analytical models.
9.3 TEST UNITS

9.3.1 Design Considerations

The research programme involved the testing of two single-bay single-storey framed masonry structures. The units, constructed to a reduced scale of 3/4, represented the lower part of a two-storey structure. The dimensions of the masonry panel and reinforced concrete frame were selected according to the common practice followed in regions where framed masonry is used as a structural system.

According to the objectives described above, the criteria followed in the design of the Unit 1 were:

- cracking of the masonry panel should occur before large plastic deformations develop in the frame,
- the columns should yield in tension as a result of the axial forces induced by the equivalent truss mechanism,
- the tensile strength of the top beam should be greater than the applied horizontal force to avoid yielding of the longitudinal reinforcement,
- the separation of the transverse reinforcement at the ends of the columns, which represent potential plastic hinge regions, was reduced to increase the shear strength, to provide confinement to the concrete and to avoid premature buckling of the longitudinal reinforcement,
- no special considerations were taken to design the beam-column joints of Unit 1.

Unit 2 was designed according to a new criterion proposed here for infilled frames, in which ductile behaviour is achieved by yielding of the longitudinal reinforcement of the columns. Yielding is limited to occur only at the base of the columns, avoiding large elongations of these members. Furthermore, tapered beam-columns joints with diagonal reinforcement are used to reduce the opening of the joints and increase the width of the compressive strut. This design approach is described in detail in section 11.4.

9.3.2 Description of the Test Units

9.3.2.1 Test Unit 1

Fig. 9.1 illustrates the reinforcing details and dimensions of Unit 1. The reinforcement of the columns and the beam was similar and consisted of 4 - 10 mm diameter longitudinal bars (reinforcement ratio equal to 1.4% and 1.1%, respectively) with 6 mm diameter stirrups. The separation of the stirrups was reduced at the end of the frame members, where plastic hinges could develop. The dimensions of the concrete sections were 150 x 150 mm and 150 x 200 mm for the columns and the beam, respectively. The masonry panel was 2.00 m height and 2.52 m long, which represents a ratio $h_m/L_m$ equal to 0.80. The panel was formed by 23 courses of solid concrete bricks with 10 ½ bricks per course.

9.3.2.2 Test Unit 2

The overall dimensions of both test units were identical and similar materials were employed in the construction. However, different reinforcing details were used in Unit 2, as a result of the criterion considered in the design. A 10 mm diameter bar was added to both lateral faces of each column, as Fig. 9.2 illustrates. These bars were not anchored to the base of the unit to assure that the axial and flexural strengths of the bottom end of the columns were significantly weaker. Therefore, plastic deformations due to tensile axial loads or bending moments should develop mainly in these regions. The top beam was also reinforced with two additional 10 mm bars to reduce the elongation produced by the axial force.
Figure 9.1. Reinforcing details and dimensions of Unit 1
Tapered beam-columns joints, with an inclination selected to be perpendicular to the diagonal of the masonry panel, were used for Unit 2. A diagonal 10 mm diameter stirrup was placed in each joint with three perpendicular U-shaped 6 mm bars tied to the longitudinal reinforcement of the frame, as Figs. 9.2 and 9.3 illustrate. The tapered joints improve the transfer of the lateral force from the frame to the masonry panel. In this way, the width of the equivalent compressive strut can be increased resulting in a reduction of the stresses in the panel. Additionally, the shear force at the top of the column decreases, delaying or avoiding a shear failure in this region.

![Image of reinforcement detail](image)

**Figure 9.3.** Reinforcement detail of the tapered beam-column joints used in Unit 2.

### 9.3.3 Construction of the Test Units

The construction of the test units started with the preparation of the reinforced concrete beams used as a base for the frame and the masonry panel. Two undamaged beams, previously constructed for another research project, were used in these tests. In order to anchor the longitudinal reinforcement of the columns, four 50 mm diameter 350 mm length holes were drilled close to each end of the bases. Additionally, three more holes were drilled in one of the bases to perform pull-out test for the evaluation of the anchor capacity. These holes were filled with cement grout and 10 mm diameter bars were placed into the holes leaving a length of 300 mm protruding from the base. After four days, pull out tests were conducted using a hydraulic actuator. It was observed in these tests that the reinforcing bars were able to develop the full tensile strength without an anchorage failure. The protruding bars were cut and the top surface of the beam was mechanically roughened with a pneumatic scrambler.

The masonry walls were built before casting the frame, as usual in framed masonry structures. A professional bricklayer constructed the walls on the top of the reinforced concrete bases following common
techniques used in New Zealand for masonry structures. Solid concrete bricks were layed with mortar. The thickness of the mortar joints was 10 mm. In order to fit the length of the masonry wall, some of the bricks were cut by half with a circular saw. During the construction of the masonry walls, small gaps were observed in some head joints due to the difficulty of filling completely the space between bricks. This observation confirms the assumption presented in sections 4.2.3 and 4.2.4 about the poor quality of the head joints in comparison with the bed joints. These gaps could not be detected after the bricklayer finished the exposed surface of the mortar joints with a steel rod.

The reinforcing cages were constructed in the laboratory using stirrups and ties which were bent and cut by a commercial firm. The reinforcing bars selected for instrumentation were prepared before the cages were made. Steel stubs for supporting the clip gauges were welded and the strain gauges were attached to the bars. The top end of the longitudinal bars of the columns was threaded and designed to protrude 75 mm from the concrete. When the cages were finished, the column reinforcement was placed in the units by inserting the longitudinal bars in holes drilled in the base. The holes were previously cleaned with air, wetted and filled with cement grout. The columns cages were kept in the vertical position by steel braces fixed to the floor of the laboratory. Finally, the ends of the beam cage were inserted in the column reinforcement and between the columns and the stirrups at the top of the columns. For Unit 2, the diagonal reinforcement in the beam columns joints (see Fig. 9.2) was positioned and fixed to the longitudinal reinforcement of the frame with tying wire.

The formwork for columns and beams was constructed with 19 mm plywood sheets and timber battens to stiffen the mould. All the edges of the mould were sealed with parcel tape and silicone RTV to avoid leaking of the water from the fresh concrete. The internal faces of the mould were coated with mould realisers. Before pouring the concrete 30 mm diameter cylinders of polystyrene were placed around the tack welded stubs to provide a void in the concrete which allows the free movement of the stub. Furthermore, eight threaded 16 mm diameter steel rods were placed passing through the beam. These rods were used to bolt the steel plates needed to apply the horizontal forces.

The entire reinforced concrete frame for both units was cast in one operation, using fresh concrete provided by a local company. When the concrete arrived at the laboratory, the slump was checked and superplasticising admixture based on melamine formaldehyde was added to the mix to produce a highly flowable concrete. Initially, the concrete was fed to the lower parts of the columns using small openings left in one face of the mould. When this part was filled with concrete, the openings were closed and the placing continued from the top of the mould. The concrete was placed with a hopper and was mechanically vibrated. The units were cured with damp hessian fabrics and covered with plastic sheets to avoid excessive evaporation and retain the moisture. The moulds were released after seven days and the units were painted with a water based white paint to ease the observation of cracks. Finally, the steel plates used for the application of the vertical and horizontal forces were bolted to the unit.

The masonry walls and reinforcing cages of both units were constructed simultaneously. However, Unit 2 was cast after testing the first unit.

Fig. 9.4 shows different stages during the construction of the tests units.
9.4 MATERIAL PROPERTIES

9.4.1 Masonry Units

Solid concrete bricks, with dimensions 230 x 90 x 75 mm, were used for the construction of the masonry panels. The compressive strength of the bricks was evaluated following different standards and testing techniques in order to investigate the effect of capping and the aspect ratio of the specimen. A series of 10
bricks were tested with either a thin coat of gypsum, a piece of 12 mm softboard or a 4 mm sheet of cork between the specimen and the testing machine platens. Furthermore, the bricks were placed in different positions which lead to aspect ratios equal to 0.83, 3.07 and 5.00. Based on these results, it was concluded that the compressive strength of the bricks was 26.2 MPa (details of this study are presented in section 2.3.3). The measured value of the elastic modulus was 12980 MPa, defined as the secant modulus at 1/3 of the compressive strength. It is worth noting that this value is equal to 495 $f_{cm}$, which is much smaller than usual values for the elastic modulus of concrete masonry units (see section 2.3.6).

Splitting tests were conducted using two 6 mm diameter steel rods to apply the compressive load (see section 2.3.4). The tensile strength, obtained as the average value from five tests, was 2.8 MPa.

9.4.2 Mortar
The mortar used in the construction of the masonry panels was prepared by mixing one part of cement, two parts of bricklayer lime mortar and two parts of sand. The mechanical properties of the mortar were evaluated from compressive tests conducted on 50 mm diameter by 100 mm high cylinders cast during the construction of the masonry panels. The cylinders were cured under the same conditions that the panels. The compressive strength obtained from the test of five specimens was 8.0 MPa. The compressive strength was rather low when compared with usual values ranging from 5 to 20 MPa (see section 2.4.1). The elastic modulus of the mortar was 8540 MPa. This value was defined as the secant modulus at 1/3 of the compressive strength.

9.4.3 Masonry
Compressive and shear tests were conducted on small masonry specimens in order to evaluate the mechanical properties. The specimens were constructed by the same bricklayer and with the same materials used for Units 1 and 2. After construction, they were placed close to the units to assure similar curing conditions.

Five masonry prisms, formed by three bricks laid in stack bond, were used to measure the compressive strength. These prisms were tested in an Avery universal testing machine using two pieces of softboard between the prism and the loading plates. The compressive strength, obtained as average value of five tests and affected by the correction factor presented in Table 3.2 for $h/p = 2.72$, was 19.3 MPa. The elastic modulus, in the direction perpendicular to the mortar joints was 11550 MPa, defined as the secant modulus at 1/3 of the compressive strength. The analytical value obtained from Eq. 3.4 is 12340 MPa, which agrees well with the measured value.

Shear strength parameters were assessed by testing masonry triplets, similar to that illustrated in Fig. 4.3 (c), with three different levels of axial load. The shear force was applied with an Avery universal testing machine, whereas the axial load (perpendicular to the mortar joints) was applied with a hydraulic actuator mounted on a special steel rig, as Fig. 9.5 illustrates. A load cell was used to measure the axial load, which was keep constant during the test. The supports on which the masonry triplets were placed during the test were designed to allow the lateral movement of the bricks and to assure that the resultant of the reactive forces passed through the plane of the mortar joints (in order to obtain a pure shear state in the joints). Fig. 9.6 shows experimental results of the shear strength, $\tau_{m}$ as a function of the normal stress, $f_{n}$, induced by the
axial load. Based on these results and using linear regression analysis, it was found that the initial bond strength was $\tau_0 = 0.41$ MPa and the coefficient of friction was $\mu = 0.70$.

The strength envelope of the masonry was evaluated according to the procedure described in section 8.2.3 and considering the measured mechanical properties of the composite material. Fig 9.7 illustrates the results of the compressive strength of the panel, $f_n^{\text{th}}$, as a function of the angle $\theta$. These results indicated that for Units 1 and 2, in which $\theta = 38.4^\circ$, a sliding shear failure is expected at a stress level $f_n^{\text{th}} = 1.05$ MPa. The reduced shear parameters of the masonry considered in this evaluation were $\tau_0^* = 0.30$ MPa and $\mu^* = 0.52$ (see section 4.2.4).

![Diagram](image)

**Figure 9.5.** View of the test set-up for the evaluation of the shear strength.

![Graph](image)

**Figure 9.6.** Experimental results of shear strength for different levels of axial load.
Figure 9.7. Failure envelope of masonry obtained from the measured mechanical properties.

9.4.4 Concrete
The concrete was provided by a commercial ready-mix supplier with a specified compressive strength of 20 and 30 MPa at 28 days for Units 1 and 2, respectively. A higher strength was required for Unit 2 because the test started 17 days after casting the reinforced concrete frame. The maximum aggregate size was 13 mm because the congestion of the reinforcing cages and the reduce scale factor of the test units. Concrete samples were taken during the construction and 100 mm diameter by 200 mm high cylinders were cast over a vibrating table. The cylinders were cured in a fog room at 20°C and 100% relative humidity and left to dry for three hours before testing. Table 9.1 presents the mechanical properties of the concrete obtained from compressive and splitting tests conducted in a 2500 kN Avery universal testing machine.

<table>
<thead>
<tr>
<th>Description</th>
<th>$f'_c$ at 28 days (MPa)</th>
<th>$f'_c$ at test (MPa)</th>
<th>$f'_t$ at test (MPa)</th>
<th>$E_c$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>14.6</td>
<td>22.5</td>
<td>2.4</td>
<td>22.1</td>
</tr>
<tr>
<td>Unit 2</td>
<td>---</td>
<td>31.2</td>
<td>2.9</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Table 9.1. Mechanical properties of the concrete.

$f'_c$: compressive strength
$E_c$: Elastic modulus

9.4.5 Reinforcing Steel
Fig. 9.8 illustrates the strain-stress relationship measured for samples taken from 6 mm and 16 mm diameter bars used in the construction of the surrounding frame. The mechanical properties of the reinforcing steel are given in Table 9.2. The tests were conducted in a 100 kN Avery universal testing machine at a quasistatic rate. The strains were measured with a clip gauge attached to the reinforcing bars. An XY plotter, properly calibrated before the test, was used to record the tensile stress-strain curves.

Additional tests were conducted to verify the weldability of the longitudinal reinforcement considering that steel stubs were welded to support the clip gauges. Two steel samples were tested in tension with a
transverse stub welded to the reinforcing bars. It was observed in both cases that the weld did not affect the results in comparison to those obtained from samples without welding.

![Graph showing stress-strain relationship](image)

**Figure 9.8.** Strain-stress curves for reinforcing steel.

<table>
<thead>
<tr>
<th>Description</th>
<th>$f_y$ (MPa)</th>
<th>$\varepsilon_y$</th>
<th>$E_Y$ (GPa)</th>
<th>$\varepsilon_{sh}$</th>
<th>$f_u$ (MPa)</th>
<th>$\varepsilon_{mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 mm diameter plain bars (transverse reinforcement)</td>
<td>353</td>
<td>0.0017</td>
<td>207.6</td>
<td>0.031</td>
<td>466</td>
<td>0.151</td>
</tr>
<tr>
<td>10 mm diameter deformed bars (longitudinal reinforcement)</td>
<td>323</td>
<td>0.0016</td>
<td>201.9</td>
<td>0.026</td>
<td>441</td>
<td>0.253</td>
</tr>
</tbody>
</table>

$f_y$: yield strength  
$\varepsilon_y$: yield strain  
$E_Y$: elastic modulus  
$\varepsilon_{sh}$: strain at the beginning of strain hardening  
$f_u$: ultimate (maximum) strength  
$\varepsilon_{mu}$: strain at maximum load

### 9.5 TEST ARRANGEMENT

#### 9.5.1 General Description

The loading frame consisted of two steel columns connected with a steel beam at the top. Two diagonal braces were used to stiffen the columns at the level where the lateral forces were applied. The base of the columns and braces were bolted to the floor of the laboratory. The test units were anchored to the floor, using four RHS steel beams and eight 38 mm diameter steel rods, which fixed the ends of the reinforced concrete base. In order to avoid the horizontal movement, the base was compressed between the steel columns with two hydraulic actuators loaded with a constant force of 120 kN. A RHS steel beam was located at each side of the unit to restrain it against out-of-plane displacements. The restraint was provided by four nylon shims glued to the internal faces of the steel beams, which were in contact with the top of the columns. Small steel plates were glued on the columns at the contact points. In this way, the units were able to move under the action of the lateral forces without significant friction at the contact points. The general details of the loading frame are illustrated in Figs. 9.9 and 9.10.
Figure 9.9. Loading frame used in the tests.

Figure 9.10. View of the test set-up for Unit 1.
The lateral forces were applied using two hydraulic actuators connected to the units at 2.50 m from the testing floor. Four threaded steel bars protruding from the concrete were located close to the each end of the top beam and steel plates were bolted to them. From these plates, 24 mm diameter high strength steel rods were connected to the hydraulic actuators to apply the lateral forces. The ends of these rods were bolted with special nuts and washers, which had spherical surfaces, to allow small rotations of the rods due to potential vertical displacements of the reinforced concrete beam.

During the test, the hydraulic actuators were used alternatively depending on the direction of the applied force or displacement. This procedure was adopted to represent more adequately the conditions in real multistorey framed masonry structures, in which the beams of the frame are subjected to tensile forces independent of the direction of the lateral force. In order to simulate the gravity loads and overturning moment corresponding to a two-storey structure, vertical forces were also applied at the top of each column. The vertical hydraulic actuators were automatically controlled by servo-valves, which were operated by a computer controlled system. Fig. 9.11 illustrates schematically the test setup and the electric and hydraulic connections of the system. The computer was permanently monitoring the applied horizontal force and adjusting the values of the vertical forces, according to the following expression:

\[ P = -20\text{kN} \pm 0.52\text{ V} \] (9.1)

where the first term represents a constant compressive load and V indicates the applied lateral force. In this way, it was assured that the ratio of overturning moment to shear force remained constant during the test.

The additional overturning moment applied by the vertical hydraulic actuators was calculated to represent a two-storey structure in which the total weight of the building was concentrated at the floor levels in equal proportions. It was assumed that the equivalent lateral forces induced by seismic action followed a linear variation with height, as Fig. 9.12 shows. The total overturning moment, measured at the base of the infilled frame, was equivalent to that produced by the same lateral force applied at 3.48 m. The constant compressive loads of 20 kN (see Eq. 9.1) applied with the vertical hydraulic actuators were adopted to represent the gravity loads of a typical building with masonry panels.

### 9.5.2 Discussion on the Loading System

A chief objective of this experimental programme was to use a realistic procedure for the application of the lateral forces. It is useful, therefore, to compare the real path of the seismic forces and the loading system used in the laboratory. Inertial forces resulting from seismic actions are distributed between the resisting elements depending on the structural configuration of the building. In the lower storey, a large part of the shear force is transferred from the upper storeys through a truss mechanism (see section 5.5.1), whereas the remaining part is transmitted to the beams from the floor slabs as Fig. 9.12 (b) illustrates. Hence, the equivalent seismic force should be applied at a point close to the end of the beam. The exact location of the resultant, which can vary during an earthquake, depends on the structural configuration and on the dynamic properties.

The review of previous quasistatic experiments indicated that two loading systems are commonly used to apply the lateral forces. In the most common system, two hydraulic actuators alternately push the unit at the
external faces of the beam-column joints. The second system employs only one hydraulic actuator capable of pulling and pushing the unit to apply cyclic loading. This actuator is connected either to the external face of the beam column joint or to a rigid steel beam bolted along the top of the unit. In this research programme, the lateral forces were applied by pulling the top beam at 0.50 m from the end of the beam, with the aim of representing the resultant shear force transmitted by the floor slab and from the upper storey.

In the lack of experimental evidence related to this topic, finite element analyses were conducted to investigate the effect of the different loading systems on the structural response. This study is only aimed at obtaining qualitative results about the stress state and efforts in the infilled frame. The finite element model was similar to that considered in section 5.4.2. Fig. 9.13 illustrates the bending moment diagrams in the surrounding frames obtained from two different loading conditions, namely, (i) when the force is applied by pushing the face of the beam-column joint, and (ii) by pulling the top beam close to one of its ends. These results are given at the same force level. It can be observed in Fig. 9.13 that the magnitude of the maximum bending moments are similar, however, their location and direction markedly change. The distribution of the shear forces exhibit some variation, although the maximum values are comparable and occur at the ends of the members. The axial forces in the columns of the frame are similar in both cases. In the beam, however, the difference is significant. Large tensile axial forces developed when the force is applied by pushing the infilled frames, whereas reduced compressive forces occur in the other case.

![Schematic representation of the test setup.](image-url)
Figure 9.12. Horizontal and vertical forces applied to the test units.

Figure 9.13. Bending moment diagrams for infilled frames subjected to pushing and pulling lateral forces.

It is worth noting that monotonically increasing forces were considered in these examples. When the structure is subjected to cyclic loading, the elongation of the beam, which can be significant even if longitudinal reinforcement does not yield, remains constant or increases because the beam is subjected to tension axial forces independent of the loading direction. The lengthening of the beam would be
conceptually similar to the effect of unintentional gaps in the panel-column interfaces, producing pinching of the hysteresis loops.

The model considered in the previous analyses is not adequate to investigate the local effects in the beam-column joints, since two-nodes beam elements were used to model the frame. For this reason, a more refined model was developed in which each member of the frame was represented with mesh of 5 x 12 four-node plane stress elements. The numerical results indicate that, when the lateral force is applied by pulling the beam, the core of the beam-column joint is subjected to a stress state similar to pure shear. Contrarily, when the force is applied by pushing the face of the joint, the stress state in the core is predominantly compressive, with tensile stresses developing only in the region close to the internal corner of the joint. In the later case, the behaviour of the joint can be significantly improved.

The stress state in the masonry panel was, in a general sense, similar for both loading cases. Some differences occurred, however, in the upper corner in contact with the frame, where most of the shear is transferred. When the lateral force is applied by pulling the beam, the direction of the principal compressive stresses in the region adjacent to the beam is closer to the horizontal than in the other case.

The considerations discussed above indicate that special precautions should be taken in the application of the lateral forces. Furthermore, the test results should be interpreted taking into account these considerations. The loading system used in this research programme does not reproduce exactly the conditions in real structures. However, it seems to be more realistic than the other procedures commonly used in laboratory tests.

9.6 INSTRUMENTATION

9.6.1 Measurement of Loads
The magnitude of the vertical and horizontal forces was measured using four load cells placed between the hydraulic actuators and the test unit. These load cells consisted of high strength steel hollow cylinders with two independent full bridges (Poisson) circuits. One of the circuits was connected to the data logger unit and the other to the valve controller system. The load cells were calibrated in compression before the test in an Avery universal testing machine.

9.6.2 Measurements of Displacements and Deformations
Displacements and deformations of the test units were measured with Sakae linear potentiometers of 10 kΩ of resistance. The lay out of these devices is shown in Fig. 9.14.

The horizontal displacement of the centre of the top beam was registered with one linear potentiometer, LP1, mounted on steel angles fixed to a concrete wall of the laboratory. During the initial part of the test, a linear potentiometer with a travel length of 15 mm was used. Later, this linear potentiometer was changed to increase the travel length to 100 mm with the aim of ensuring an adequate resolution relative to the magnitude of the displacements being measured.

Considering that the lateral displacement is a very important parameter of the test, another linear potentiometer with a travel length of 100 mm, LP2, was used to verify the results. LP2 was mounted on a
steel column placed outside the testing floor slab. A Kevlar string connected the unit and the linear potentiometer. The wire was kept in tension by a weight attached to the linear potentiometer. Since the lateral displacement registered by the LP2 was referred to one edge of the frame, the displacement measured for equal force levels with contrary directions would be different due to the elongation of the beam. For this reason, a linear potentiometer with 30 mm travel length, LP3, was used to measure the elongation of the beam subjected to tensile axial forces. This potentiometer was mounted on an aluminium bracket connected to both ends of the beam.

![Diagram of linear potentiometers](image)

**Figure 9.14.** Lay out of the linear potentiometers used in Units 1 and 2.

The measurement of the beam elongation, obtained from LP3, could have been affected by temperature variations changing the length of the 2.6 m long aluminium rod. Consequently, an electronic thermometer was attached to the aluminium rod. Readings from this linear potentiometer were automatically corrected by the software used to read and store the data.

The diagonal deformation of the masonry panel was registered by measuring the change of length of two Kevlar wires with a length of 3.2 m. These wires run along the diagonals of the panel, having one end fixed to the wall and the other connected to linear potentiometers with a 50 mm travel (named LP4 and LP5 in Fig. 9.14). The linear potentiometer and a small aluminium wheel were mounted on a steel plate glued to a corner of the masonry panel. The Kevlar wires were kept in tension by a weight attached to the linear potentiometer. These linear potentiometers were placed on the back of the test unit to avoid interference with other instruments.

In order to measure the relative movement of the frame members at the beam-column joints, two linear potentiometers with 30 mm travel, LP6 and LP7, were mounted on aluminium brackets, whose ends were bolted to steel rods welded to the longitudinal reinforcement of the frame members. These measurements
were taken because the aperture of the beam-column joints of Unit 1 can modify the interaction between the frame and the masonry panel. In Unit 2, tapered joints were used and the linear potentiometers LP6 and LP7 were attached in a similar position in order to compare the results in both cases.

Two linear potentiometers, LP8 and LP9, with 30 mm travel were used to monitor relative vertical displacement between the base and the testing floor due to deformations of the anchor system. A similar linear potentiometer, LP10, was located on the floor of the laboratory, in contact with the reinforced concrete base, in order to measure any significant movement produced by horizontal sliding of the base.

An inclinometer was attached to a steel channel fixed to the base to measure the rotation of the reinforced concrete base due to deformations of the testing floor and the anchorage system. This instrument was developed at the University of Canterbury and consisted of a 10 N capacity load cell able to measure forces in its transverse direction. The load cell was mounted in vertical position on a triangular steel rig fixed to the base. A 20 N weight was attached to the bottom of the load cell. If the base rotated, the transverse component of the weight changed in direct proportion to the angle of rotation. The resolution of the inclinometer was 0.01° per data logger unit.

9.6.3 Measurements of Local and Average Reinforcement Strains
Local strains in the longitudinal and transverse reinforcement were measured with 120Ω 5 mm electrical foil strain gauges type FLA-5-11 manufactured by Tokyo Sokki Kenkyujo Co, with a nominal gauge factor of 2.14. Details of the positions of the strain gauges for Unit 1 are given in Fig. 9.15, where only half of the strain gauges are shown due to symmetry in the arrangement. In Unit 2 the positions of the strain gauges were similar.

![Figure 9.15. Position of the strain gauges and clip gauges in Unit 1.](image-url)
The surface of the reinforcing bars was properly prepared in those parts where the strain gauges were to be attached. The deformation of the bar was removed with a file, avoiding an excessive reduction of the bar section. Then the surface was abraded by cross hatching at 45°, using emery tape grade 60 and 200, and was cleaned with Methyl-ethyl-ketone to remove dust and grease. The strain gauges were glued to the bars with Loctite 401 cyanoacrilate adhesive, which was applied in a thin layer on the prepared surface. Three layers of waterproofing cement Hamite No.A-862-B were coated over the strain gauges at intervals of at least 12 hours. Finally, the strain gauges were covered with a vinyl mastic tape to provide mechanical protection and additional insulation. The wires from the strain gauges were tied to the bars with waxed cotton thread. In this region the wires were protected with a PVC flexible sleeve. Before the test, all the strain gauges were checked for continuity and resistance to earth. Only one strain gauge attached to the transverse reinforcement of Unit 2 was rejected.

Average strains in the longitudinal reinforcement were also measured with 36 clip gauges. Fig. 9.16 illustrates the characteristics of the clip gauges and the way in which they were attached to the reinforcing bars. The clip gauges were placed at the ends of the frame members, which represented potential plastic hinge regions (see Fig. 9.15). The location of the clip gauges attached to Unit 2 was similar to that of Unit 1, although the diagonal deformation of the beam-column joints was not measured in Unit 2. The average strain was calculated as the displacement measured with the clip gauges divided by the gauge length, equal to 150 mm.

Figure 9.16. Clip gauges used to measured average strains
The strains measured in the longitudinal reinforcement were used to calculate the curvature, $\varphi$, and axial strain, $\varepsilon$, of the frame members, according to the following expressions:

$$\varphi = \frac{\varepsilon_x - \varepsilon'_x}{d_s} \tag{9.2}$$

$$\varepsilon = \frac{\varepsilon_x + \varepsilon'_x}{2} \tag{9.3}$$

where $\varepsilon_x$ and $\varepsilon'_x$ are the strains in the longitudinal bars measured in the same section and $d_s$ is the distance between the bars.

Four clip gauges were used to measure the diagonal deformation of the beam-column joints in Unit 1. These readings enable the calculation of the average shear distortion. It can be demonstrated that the shear strain, $\gamma$, in the joints is given by (see Fig. 9.17):

$$\gamma = \frac{\Delta_d - \Delta'_d}{2 L_d} \left( \tan\theta + \frac{1}{\tan\theta} \right) \tag{9.4}$$

where $L_d$ is the original length of the diagonals, $\Delta_d$ and $\Delta'_d$ are the changes in length of the diagonals and $\theta$ is the angle of the diagonals to the horizontal. All the clip gauges were calibrated before the test with a metric calibrator and then placed in position in the test units.

![Figure 9.17. Shear distortion in beam-column joints.](image-url)
9.6.4 Data Acquisition System
Data measured from load cells, linear potentiometers, clip gauges and strain gauges was read and stored by computer, using a data logger unit with analogue-to-digital converter cards. The characteristic voltage of the data logger units was 20 V / 4096 units. An uninterruptable power supply was used to provide regulated, transient free AC power to the data acquisition system. A tailored computer software package was used to read in real time forces, displacements and strains, allowing the plot of the results during the test. An algorithm, based on statistical analysis, was included in the computer program to avoid problems related to electrical noise affecting the readings. Table 9.3 summarises the main characteristics of the setup used for connecting the transducers to the data logger unit.

Table 9.3 Setup for transducers in the data logger unit.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Bridge configuration</th>
<th>Input voltage</th>
<th>Gain</th>
<th>Nominal resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical load cells</td>
<td>Full</td>
<td>7.5</td>
<td>1000</td>
<td>16 dlu/kN</td>
</tr>
<tr>
<td>Horizontal load cells</td>
<td>Full</td>
<td>7.5</td>
<td>2</td>
<td>12 dlu/kN</td>
</tr>
<tr>
<td>Linear potentiometers</td>
<td>Half</td>
<td>10.0</td>
<td>2/10</td>
<td>(*)</td>
</tr>
<tr>
<td>Strain gauges</td>
<td>Quarter</td>
<td>2.5</td>
<td>200</td>
<td>0.1 dlu/με</td>
</tr>
<tr>
<td>Clip gauges</td>
<td>Full</td>
<td>4.0</td>
<td>1000</td>
<td>200 dlu/mm</td>
</tr>
<tr>
<td>Thermometer</td>
<td>Full</td>
<td>10.0</td>
<td>2</td>
<td>8 dlu/°C</td>
</tr>
<tr>
<td>Inclinometer</td>
<td>Full</td>
<td>10.0</td>
<td>2</td>
<td>100 dlu/degree</td>
</tr>
</tbody>
</table>

(*) Resolution depends on travel length. dlu: data logger unit.

9.7 TEST SEQUENCE
The test units were subjected to reversed quasistatic cyclic loading. This testing technique, extensively used to investigate the seismic response of reinforced concrete structures, does not attempt to simulate the displacement sequence induced by a specific earthquake record. Quasistatic actions allow the precise observation of the progression of damage and the degradation of strength and stiffness, which is the main objective when the tests are intended to improve the understanding of the structural behaviour [C12] (see section 5.3.2).

The initial cycles applied to the units were conducted to evaluate the initial stiffness. In Unit 1, one cycle was applied to a force level equal to 20 kN, whereas in Unit 2, three cycles were imposed to 20, 40 and 60 kN. It should be noted that framed masonry structures exhibit an early nonlinear behaviour due to separation of the frame-panel interfaces and cracking of the masonry. Furthermore, this type of structure are usually very stiff, having a small initial yield displacement, and any small change in the definition of this point may result in large variation of the ductility factor. As a result, the definition of displacement ductility factors is rather arbitrary in this case. In the inelastic range, the test was displacement controlled and several cycles with increasing storey drift, δ, were applied. For each displacement level, three consecutive cycles were repeated to capture adequately the effect of strength and stiffness degradation up to a storey drift of 0.5% and 0.75%, for Unit 1 and 2 respectively. The test was finished after applying 34 and 39 cycles up to a maximum storey drift of 1.5% and
2.0%, for Units 1 and 2, respectively. Fig. 9.18 illustrates the test sequence of cyclic displacements applied to both units.

9.8 LIMITATIONS OF THE TESTS

Special care was taken to design the units and the test setup in order to simulate accurately the conditions in real structures. This proposal, however, could not be completely fulfilled due to some practical limitations, as described below.

Figure 9.18. Test sequence of cyclic displacement applied to the Units 1 and 2.
The effect of a floor slab in the prototype structure was not considered in the test units, resulting in reduced flexural and axial strengths of the beam when compared with those of the prototype. Furthermore, the presence of the floor slab and additional reinforcement could decrease the elongation of the beam due to tensile axial forces.

Even though the units were loaded to simulate the actions in a two-storey structure, the upper masonry panel was not constructed. When the structure is subjected to lateral forces, the panel usually separates from the surrounding frame, except at diagonally opposite compression corners (see section 5.4). In those corners, the masonry panel restricts the deformation of the beam-column joint and this effect was not represented in the test.

The additional overturning moment applied with the vertical actuators was calculated considering a linear variation of the equivalent seismic forces (see Fig. 9.12). It was also assumed that this relationship remains constant. Obviously, when the structure is subjected to real seismic actions, the shear forces induced at each level depend on the dynamic properties of the building. In order to overcome this problem, pseudodynamic or shaking table tests should be conducted.

Hydraulic actuators were used to apply the gravity load at the top of the columns (see section 9.5.1). Part of these loads should have been applied as a distributed load in the top beam to represent the load transferred through a floor slab. This fact can modify the stress state in the masonry panel and axial forces in the columns. According to the transfer mechanism explained in section 5.8.6.1, the vertical axial stresses in the masonry panel of the units would be slightly smaller than those in a real structure and the compressive axial forces in the columns would be higher. The former consideration leads to a reduction of the shear strength of the masonry, whereas the latter produces an increase in the shear resistance of the columns. The conditions modelled in the tests represent a framed masonry structure connected to a floor slab with the main reinforcement parallel to the structure, as can occur when precast slabs are used. In this case, gravity loads are transferred directly to the columns.

The limitations of the loading system used for the application of the lateral forces was discussed in section 9.5.2.

The mechanical properties of the concrete used in Units 1 and 2 were different, as Table 9.1 indicates. This could invalidate a proper comparison of results obtained from both units. However, the variation of the concrete strength, within a normal range, cannot significantly affect the response of infilled frame structures. It is believed, therefore, that the different behaviour observed in the tests was mainly due to the effect of the tapered joints and the modifications in the longitudinal reinforcement.

9.9 CONCLUSIONS

- This chapter describes the construction and testing procedure of two framed masonry structures. The test units, constructed to a reduced scale of 3/4, were designed to represent a typical structure from a two-storey building.
The main objectives of the test programme were to investigate the behaviour of framed masonry structures using a realistic procedure to apply the lateral forces, to compare the behaviour of new reinforcement details for framed masonry to that of customarily designed structures and to obtain experimental data for the validation of analytical models.

The influence of different loading system was discussed on the basis of analytical results and conceptual considerations. It is concluded that the loading system can significantly affect the test results and, consequently, a realistic procedure should be adopted for laboratory tests.
10. TEST RESULTS

10.1 INTRODUCTION
This chapter contains the experimental results obtained from two framed masonry structures tested under quasistatic cyclic forces. The first unit was designed and detailed according to common practice for this type of structure. Tapered beam-column joints and special reinforcement details of the columns were used in the second unit to improve the structural response. The lateral forces, intended to simulate seismic actions, were applied to the units by pulling the top beam. Vertical loads were also applied at the top of the columns to represent gravity loads and to introduce additional overturning moment to the units. Complete details of the test units, material properties, test setup and instrumentation are described in Chapter 9.

The test sequence, except for the first cycles, was displacement controlled. Due to the characteristic response of framed masonry, the storey drift was selected as the controlling parameter instead of the displacement ductility factor. Test results reported in this chapter are usually referred to by the load run number. The correlation between the storey drift and the load run number is given in Fig. 9.18. The positive direction for the lateral displacement is defined from South to North (see Fig. 9.9).

Test results obtained from Unit 1 and 2 are presented separately in sections 10.2 and 10.3, whereas the comparison of the response of both units is given in section 10.4.

10.2 UNIT 1
10.2.1 General Behaviour
The test of Unit 1 was performed over a period of 3 days, in which 34 cycles were applied with a maximum storey drift $\delta = 1.5\%$. In this stage, the test was finished due to the excessive damage observed in the masonry panel. Linear potentiometers and clip gauges were removed from the unit and a final cycle with very large displacements was applied to intensify the damage in the unit and to emphasize the failure mechanism. The unit was able to resist the applied history of cyclic displacements without significant strength decay in the positive direction. However, the residual strength in the negative direction was 79% of the lateral capacity attained in load run 58. The hysteresis loop showed severe pinching only in the final stage of the test. Unit 1 was designed with the aim of obtaining a reasonably ductile response, involving tensile yielding of the longitudinal reinforcement of the tension column. However, this objective was not achieved. Severe cracking of the upper part of the masonry panel occurred and large elongations developed in the frame members, which led to a sliding shear failure at the top of the columns. The lateral resistance measured during the test was only 50% of that predicted assuming the hypothesis of tensile failure.

Cracking commenced during load run 3. A long crack started in the south beam-column joint and continued horizontally through the mortar joint closest to the top beam. Below the steel plate used for the application
of the lateral force, the crack changed to a stepped crack, running alternatively through head and bed joints. Another crack formed in the beam, between the steel plates connected to one of the hydraulic actuators. A similar crack pattern in the masonry panel was observed in two units tested by Fiorato et al [F4], in which two lateral forces were applied simultaneously at the thirds of the beam. When the lateral displacement was reversed, the horizontal crack crossed the entire section of the panel and a similar stepped crack developed in the opposite corner. During load runs 9 and 10 another stepped crack formed in each corner of the panel and several vertical cracks developed in the central region of the beam. Cracks also formed at the column and beam faces of the joints, leaving the core uncracked. Fig. 10.1 (a) illustrates the cracking of Unit 1 during load run 10. It is worth noting that the behaviour of the unit was rather unusual (see section 5.4.2) because significant cracking of the masonry panel occurred without visible separation at the panel-frame interfaces. This is probably due to the poor properties of the mortar joints when compared with the bond strength of the interfaces. Furthermore, the difference in the thicknesses of the masonry panel, 90 mm, and the width of the frame, 150 mm, made the observation of cracks at the interfaces difficult.

As the test progressed, cracking in the upper part of the masonry panel continued and propagated through the interfaces between the columns and the panel. In load runs 15 and 16 tensile cracks developed along the columns and yielding commenced at the ends of the bottom reinforcement of the beam. This was a local effect and yielding did not propagate to other regions of the beam. Flexural cracks formed at the top region of the columns during load runs 21 and 22, and yielding started in the external longitudinal bars during load runs 39 and 40, indicating that a plastic hinge was developing. In this stage, the uneven opening of the cracks in the central part of the beam indicated that this member was bent upward, as a result of an inclined compressive field developing in the masonry panel. Fig. 10.1 (b) shows a photograph of Unit 1 during the application of load run 52. It can be observed that the cores of the beam-column joints were almost undamaged and most of the cracks in the masonry panel formed along the mortar joints.

In the final cycles, the beam, one layer of bricks and the corner of the panel slid over the rest of the panel, forming large gaps between the bricks. Sliding also occurred along the shear crack formed at the top of the column. This crack suggest that dowel action in the longitudinal reinforcement of the column became the main mechanism for transferring the shear force. A crack occurred in load run 63 at the bottom of the masonry panel, running initially as a stepped crack and then propagating horizontally. Fig 10.1 (c) shows Unit 1 at the end of the test. It can be observed that the south column separated from the panel and is bent in double curvature, whereas the north column remained in contact with the panel and most of the flexural deformation is concentrated at the top, where a plastic hinge developed as a result of the bracing effect induced by the masonry panel. The central region of the beam was bent upwards and the horizontal crack in the mortar joint was slightly opened.

Fig. 10.2 shows details of both beam-column joints after the test was finished and large displacements were imposed to emphasize the failure mechanism. The photograph presented in Fig. 10.2 (b) was taken after removing the concrete to expose the reinforcement. The cracks formed in these regions and the local deformation of the reinforcing bars indicate that the force applied to the beam was mainly transferred to the column through the bottom longitudinal reinforcement of the beam. From the column, the shear force was transferred progressively to the masonry panel.
(a) At $\delta = -0.075\%$, load run 10.

(b) At $\delta = -0.3\%$, load run 52.

(c) At $\delta = -1.5\%$, load run 68.

Figure 10.1. Cracking of Unit 1 in different stages.
Figure 10.2. Detail of the beam-column joints of Unit 1 after the test.

It is worth noting that the type of failure of Unit 1 has been rarely observed in laboratory tests. This fact could be explained considering that the most common loading system employed in previous research consists of two hydraulic actuators which are alternately used to push the unit. As discussed in section 9.5.2, this loading system can markedly modify the stress state in the beam-column joints. Fig. 10.3 illustrates schematically the transfer mechanism of the lateral force corresponding to two testing procedures. When the force is applied by pushing the test unit, the joint is mainly subjected to a diagonal compressive field, which can be adequately resisted by the concrete. However, a different mechanism develops when the force is applied by pulling the top beam. In this case, the force is primarily transferred to the joint by the longitudinal reinforcement because the beam is already cracked. As a result, large shear stresses are induced at the bottom of the joint in order to transfer the force to the column and then to the masonry panel.

Readings from the linear potentiometer LP10 indicate that no significant horizontal movement of the base occurred during the test. The vertical uplift of the base due to elongation of the steel rods of the anchor system never exceeded 0.06 mm. In the first cycle, the component of the lateral displacement due to rigid rotation of the unit was 8% of the total displacement. During subsequent cycles, this deformation had no practical influence. The inclinometer mounted on the base of the unit did not register any rotation during the test.
Figure 10.3. Schematic representation of the transfer mechanism of the lateral force for different loading systems.

10.2.2 Lateral Force-Displacement Response
The lateral force-displacement response of Unit 1 is shown in Fig. 10.4. Only the initial 20 cycles, up to a storey drift of $\delta = 0.2\%$, are included in this figure, focussing in the range of low to medium lateral displacements. Fig. 10.5 presents the overall response. The first cycle exhibited nonlinear behaviour, even though no cracks were observed. During the second cycle, load runs 3 and 4, the stiffness significantly changed at a force level of 30 kN due to cracking of the masonry panel. The lateral force resisted by the unit slightly increased in subsequent cycles up to 43.0 and 42.9 kN, respectively, in the direction of the positive and negative displacements. These values were achieved in load runs 15 and 58. Even though the lateral capacity was very similar in both directions, the envelope of the hysteresis loop shows different behaviours. In the direction of the positive displacements, the envelope exhibits an almost constant strength from load run 15 to 67, with a small decrease in load run 65. In the negative direction, the strength increased up to load run 58 and then decays continuously.

It is to be noted that the lateral resistance of the bare frame, evaluated according to a push over analysis, is approximately 13.5 kN. This fact demonstrates the significant influence of the masonry panel in increasing the lateral resistance of the infilled frame, even though the panel was severely cracked.

The elastic stiffness of Unit 1 was evaluated according to two simplified models, namely, the beam analogy and the truss model. In the first case, it was assumed that the frame and the panel can be represented as a monolithic member. The stiffness, therefore, was calculated according to standard elastic theory considering the contribution of the flexural and shear deformations (see section 6.2.3). In the second model, the panel was represented with an equivalent compressive strut (see section 6.2.1), in which the width of the strut was adopted as 0.25 of the diagonal length of the panel. The use of Eqs. 6.3, 6.4 and 6.5 (for $\lambda_k = 7.2$) leads to values of w/d equal to 0.09, 0.17 and 0.19, respectively. The reinforced concrete frame was assumed to remain uncracked. Fig. 10.6 compares the experimental response during the initial stage with the analytical results. It is observed that the beam analogy agrees very well with the experimental results up to a force level of 10 kN, which represents 23% of the maximum lateral force. The sudden change in the force-displacement
relationship indicates that partial separation between the panel and the frame occurred in this stage, even though cracks were not observed at the interfaces. The second model gave an adequate value of the secant stiffness at a force level equal to 80% of the lateral capacity of the unit.

Figure 10.4. Lateral force-displacement response of Unit 1 up to a storey drift of $\delta = 0.2\%$.

Figure 10.5. Overall response of Unit 1.
Figure 10.6. Comparison of the initial response of Unit 1 with analytical results.

Strength and stiffness degradation occurred when the cycles were repeated to the same displacement level. These effects were more significant between the first and second cycle of the series, with a typical strength reduction ranging from 16% to 5% in the initial and final cycles, respectively.

In order to compare the shape of the hysteresis loops, Fig. 10.7 illustrates four loops in different stages of the test, all of them scaled to have the same maximum displacements and forces. It can be clearly seen in this figure the progressive degradation of the hysteresis loops resulting from the cyclic displacement history applied to the unit. The shape of the loops was satisfactory up to storey drifts of 0.3%, even though some pinching occurred. In the final cycles, however, the response was typical of sliding systems (see Fig. 10.7 (d)). In this case, reloading started with a very low slope which suddenly increased when the dowel action mechanism is mobilized. The slope of the unloading branches were almost vertical and large deformation remained even when the lateral force was removed.

The lateral displacement, measured at the centre of the beam, is mainly due to the elongation of the beam, flexural deformation in the columns and sliding shear in the columns and masonry panel. Other sources of displacement, such as shear deformations in the columns and rigid body movements, had no practical importance in this case. Fig. 10.8 illustrates the variation of the lateral displacement and the beam elongation during the test. These results are plotted using two scales, which allow a clearer observation of the response in the range of small displacements. The first part includes load runs 1 to 50 (up to storey drift of $\delta = 0.25\%$), whereas the second part shows the results up to the end of the test. In runs 1 to 4 the lateral displacement was principally due to the elongation of the beam. Then, cracking occurred in the beam and the total beam elongation was about twice the lateral displacement. From load run 33 the beam elongation slightly decreased and the lateral displacement was larger because the flexural deformations in the columns and sliding shear in the masonry panel became more important. In subsequent cycles, the maximum value of the beam elongation remained almost constant, although in the last cycles increased up to 22 mm.
Readings of the displacement along the diagonals of the masonry panel, registered with linear potentiometers LP4 and LP5, indicated that the contribution of the panel deformation to the total lateral displacement was significant, especially in the final stage of the test when large gaps formed due to sliding shear. The shape of the hysteresis loops, obtained by plotting the lateral forces against the horizontal component of the diagonal displacement, was very similar to that of the response presented in Figs. 10.4 and 10.5, and showed the same degradation pattern through the test (see Fig. 10.7).
10.2.3 Beam-Column Joint Deformation

Fig. 10.9 shows the variation of the shear distortion in the south beam-column joint obtained from readings of the clip gauges located along the diagonals of the joint (see Eq. 9.4). The peaks of the shear distortion in each cycle remained almost constant up to load run 48, increasing up to 0.0093 at the end of the test. In relative terms to the storey drift of the unit, the shear distortion was more significant in the initial cycles. Each joint of the frame was mainly stressed in one direction of loading. For this reason, the joint distorted only in one direction. Similar values, with contrary sign, were obtained from the north beam-column joint.

![Graph showing shear distortion](image)

**Figure 10.9.** Variation of the shear distortion in the south beam-column joint.

As a result of the interaction between the frame and the masonry panel, the beam-column joints opened, producing an unfavourable effect on the structural behaviour due to the reduction of the restraint provided by the frame to the cracked masonry panel and to the modification of the transfer of the shear forces in the loaded corner. The variation of the relative opening of the south joint, measured with the linear potentiometer LP6, is shown in Fig. 10.10 up to load run 63 (in the next cycle the travel length of the linear potentiometer was exceeded). It can be observed that the opening started in the early cycles and the joint closed when the forces were applied in the reverse direction. As the test progressed, the opening increased, showing permanent deformations when the forces were reversed.

![Graph showing joint opening](image)

**Figure 10.10.** Opening of the north beam-column joint registered during the test.
10.2.4 Curvature and Axial Strain in the Frame

Data obtained from clip gauges and strain gauges was used to evaluate the strain state in the longitudinal reinforcement of the frame. In each instrumented section, the strain in both layers of reinforcement was measured, which allowed the calculation of the curvature and axial strain of the frame members according to Eqs. 9.2 and 9.3. Figs. 10.11 and 10.12 show the curvatures and axial strains along the frame members evaluated in two different stages of the test for positive displacements. The curvature diagram is plotted in the side of the frame members where tensile strains develop. The small circles indicate the sections where the curvatures were calculated. Fig. 10.11 (b), corresponding to a stage of the test in which the resistance mechanism was evidently developed, indicates that the maximum flexural deformations occurred at the top of the tension column and at both ends of the compression column.

10.2.5 Stresses in the Transverse Reinforcement

Strain gauges were attached only to the transverse reinforcement located at the bottom of the columns. Recorded strains indicate that the stress level in these stirrups remained very low. The maximum stresses, registered in the last cycle, were 70 and 132 MPa in the south and north column, respectively. The stress was higher in the north column, probably due to the formation of an inclined crack crossing the transverse reinforcement (see Fig. 10.1(c)), which did not occur in the other column.

![Curvature diagram](image)

**Figure 10.11.** Curvature of the reinforced concrete frame members.
10.2.6 Evaluation of the Cracking Force and the Lateral Strength

The cracking force at which sliding shear starts can be evaluated according to the equations described in section 6.4.2 and considering the material properties given in section 9.4. The expressions proposed by Stafford Smith and Riddington [S23] (Eq. 6.12) and Paulay and Priestley [P1] (Eq. 6.14) lead to a cracking force of 82.6 and 209.2 kN, respectively, which are several times higher than the measured value of 31.6 kN. According to the procedure proposed in section 8.2.3 and considering $f_{cd} = 1.05$ MPa, the cracking force is 35.8 or 47.7 kN; for values of the equivalent strut width of 0.15 or 0.20 of the diagonal length, respectively. These values show a better agreement with the measured cracking force.

The lateral strength of Unit 1 can be estimated according to the mechanism illustrated in Fig. 10.13, considering the imposition of a positive displacement. In the ultimate stage, the top beam and the upper part of the masonry panel moved sideways. The north column separated from the lower part of the masonry panel and bent in double curvature. The south column remained in contact with the masonry panel and large curvatures developed at the top, due to the bracing effect induced by the panel. The lateral force applied to the beam, $V$, is mainly transmitted through the columns, since a large horizontal gap developed at the top of the masonry panel and continued open during the last cycles of the test. Therefore, no significant force could be transferred from the beam directly to the masonry panel.
In the north column, the shear force $V_c$ can be calculated under the assumption that the flexural strength is developed at both ends of the member:

$$V_c = \frac{2 \, M_{col}}{h_c}$$  \hspace{1cm} (10.1)

where $M_{col}$ is the flexural strength of the compression column, equal to 7.8 kNm, and $h_c$ is the effective height between plastic hinges, which can be taken as 1.83 m. This value was obtained considering the usual expressions for the evaluation of the plastic hinge length [P1]. According to Eq. 10.1, it is found that $V_c = 8$ kN.

The shear force resisted by the cracked section of the south column results from a combined mechanism due to friction between the crack surfaces and dowel action in the reinforcing bars crossing the crack. The participation of each mechanism in the total strength depends on the applied displacement and on the previous loading history. Initially, friction controls the response and gradually decreases as the roughness of the crack degrades owing to the cyclic loading. In contrast, the development of dowel action requires large relative displacements at the horizontal joint. Different mechanisms, namely, flexure, shear or kinking of the longitudinal reinforcement [P2], can occur depending on the magnitude of the relative sliding and the elongation of the reinforcing bars. Therefore, the shear in the cracked section is not equal to the sum of the maximum shear forces developed by friction and dowel action because they occur at different displacement levels. In the ultimate stage, it can be considered that the dowel action is mainly due to a kinking mechanism in the reinforcement. Therefore, the shear force $V_d$ can be calculated as [P2]:

$$V_d = A_{\text{str}} \, f_y \, \cos \alpha$$  \hspace{1cm} (10.2)

where $A_{\text{str}}$ is the total area of longitudinal reinforcement and $\alpha$ is the angle of kinking (measured between the direction of the shear force and the longitudinal bar). It is to be noted that the angle $\alpha$ cannot be measured
accurately in the test unit. The equilibrium of horizontal forces requires that the force \( V_d \) be equal to 31 kN, which implies a kinking angle of 72° according to Eq. 10.2. Assuming that kinking occurs over a length equivalent to 5 times the diameter of the bar, in this case 50 mm, the horizontal relative displacement between the beam and the column should have been 16 mm to obtain a kinking angle of 72°. This value agrees with the relative displacement measured in the last cycle of approximately 14 mm (see Fig. 10.2 (a)). Therefore, it is believed that the failure mechanism represented in Fig. 10.13 gives an adequate explanation of the response of Unit 1.

10.3 UNIT 2

10.3.1 General Behaviour

The test of Unit 2 took 4 days to complete, after the application of 39 cycles with a maximum storey drift of 2.0%. The test was finished in this stage because the travel length of the main linear potentiometers was close to their limits. The overall response of Unit 2 was excellent when compared with that of Unit 1. The lateral resistance was 89.7 and -91.5 kN in each direction, representing an increase of about 110% in relation to Unit 1. Even though large lateral displacement were imposed, strength degradation was only observed in the final cycles. The maximum storey drift imposed to the unit was markedly higher than typical values measured in similar tests [A9] or recommended for design purposes [V1], which normally range from 0.4% to 0.6%. The crack pattern in the masonry panel was also improved, with a wide spread of thinner cracks. No significant damage occurred in the tapered beam-column joints.

The first cracks, induced by tensile axial forces, formed at the lower region of the columns and at the top beam during load run 7 and 8. Furthermore, a horizontal crack developed in the mortar joint closest to the top beam and then propagates, as a stepped crack, parallel to the tapered joints. When the storey drift was increased to 0.08%, additional cracks formed at the frame members and in the masonry panel, running diagonally with an X-shaped pattern, as Fig. 10.14 (a) illustrates. As the test progressed, cracking of the masonry panel continued, especially in its central region, clearly showing the development of the equivalent truss mechanism. During load runs 25 and 26, small vertical cracks were observed at the bottom of the columns as a result of the splitting stresses induced by the reinforcing bars. In this stage, cracks formed at the ends of the tapered joints, being perpendicular to the diagonal reinforcement. Yielding of the longitudinal reinforcement started at the bottom of the columns during load runs 43 and 44, in which a storey drift of 0.2% was applied to the unit. In subsequent cycles, yielding also occurred at the top of the columns, produced by the combined effect of bending moment and tensile axial force. Fig. 10.14 (b) shows a photograph of Unit 2 when load run 56 was applied.

In the final cycles of the test, additional cracks formed in both the frame and the panel. The crack located at the base of the columns considerably grew due to the large plastic deformations induced in the longitudinal reinforcement. These deformations concentrated mainly at the base of the column as a result of the special reinforcement detail used in the construction of Unit 2 (see section 9.3.2.2). During load runs 61 and 62, yielding developed at the ends of the bottom reinforcement of the top beam. It was clearly observed that the tension column bent in double curvature. Independent of the direction of the applied displacement, the top beam remained bent upwards and a 1-2 mm gap formed between the two layers of bricks located at the top of the masonry panel. Fig. 10.14 (c) shows the cracking of Unit 2 during the application of load run 76, corresponding to a storey drift of 1.5%. It is worth comparing this figure with Fig. 10.1 (c) in order to observe the different cracking pattern in both units.
(a) At $\delta = -0.08\%$, load run 14.

(b) At $\delta = -0.3\%$, load run 56.

(c) At $\delta = -1.5\%$, load run 76.

Figure 10.14. Cracking of Unit 2 in different stages.
A detail of a tapered beam-column joint is shown in Fig. 10.15, where it can be observed that no significant damage occurred, except for a few cracks in the direction perpendicular to the diagonal reinforcement. The comparison of Figs. 10.2 and 10.15 clearly indicates that the objective of restraining the opening of the joints was achieved with the proposed detail.

![Image](a) ![Image](b)

**Figure 10.15.** Detail of the tapered beam-column joints of Unit 2 after the test.

Fig. 10.16 presents a photograph of the base of the north columns after the test was finished and the cracked concrete removed to expose the reinforcement. The shear transferred to the columns, directly through the top beam and through the masonry panel by the compressive strut mechanism, was resisted across the cracked interfaces by a combination of friction and dowel action. The deformation of the reinforcing bars at the base of the column suggested that kinking of the longitudinal reinforcement was probably the main source of the dowel strength in the final stage of the test.

Data recorded from linear potentiometer LP10 indicates that the unit slipped 0.08 mm in the direction of the negative displacements during the application of load run 4 and remained in this position afterwards. This rigid body movement introduced an error in the application of the displacement controlled cycles because the actual storey drift imposed to the unit was slightly larger in the positive direction than in the negative direction. As the lateral displacement increased, this error had no significant effect. The vertical uplift of the base due to elongation of the steel rods of the anchor system was similar to that observed during the test of Unit 1 (see section 10.2.1). The inclinometer mounted on the base of the unit did not measure any rotation during the test.
10.3.2 Lateral Force-Displacement Response

Fig. 10.17 shows the lateral force-lateral displacement response of Unit 2 measured during cycles 1 to 24, with a maximum storey drift of 0.2%, whereas the overall response is presented in Fig. 10.18. Cracking of the masonry panel and the reinforced concrete frame commenced at a force level of about 65 kN, producing a significant decrease of the stiffness. As the imposed displacement increased, the lateral strength of the unit augmented up to a maximum value of 89.7 and -91.6 kN in load runs 67 and 74, respectively. Afterwards, the resistance continuously decayed to 70.3 and -75.6 in the final cycle. The lateral resistance of the bare frame, obtained from a push over analysis, is approximately 17 kN. The large difference between this value and the measured strength of Unit 2 is due to the interaction between the panel and the frame.

The comparison of the experimental response measured during load runs 1 to 5 with analytical values of the lateral stiffness are given in Fig. 10.19. The initial stiffness of Unit 2 agrees well with that obtained from the cantilever wall model (see section 6.2.3) up to a force level of 12 kN. Afterwards, the stiffness decreased markedly as a result of partial separation between the panel and the frame. In the load range from 40 to 70 kN, the secant stiffness is well represented by the equivalent truss model taking a width of the compressive strut of between 0.25 and 0.35 of the diagonal length of the panel, \( d_m \). In this model, rigid end blocks were provided to represent the stiff region at the tapered joints where the beam and column intersect. The length of this rigid region was set to be half of the dimension of the joint. The effect of rigid end blocks was not significant due to the large axial stiffness of the strut, which controls the stiffness of the model.
Figure 10.17. Lateral force-displacement response of Unit 2 up to a storey drift of $\delta = 0.2\%$.

Figure 10.18. Overall response of Unit 2.
The hysteresis loops measured during the test showed strength and stiffness degradation. These phenomena were observed when the cycles were repeated to the same displacement level as well as when the lateral displacement was increased. The degradation of the hysteresis loops observed in Unit 1 (see Fig. 10.7) also occurred in this case. The repetition of the cycles to equal lateral displacements was accompanied by a decrease in the lateral strength of the unit and stiffness degradation in the reloading branches. These effects were more important between the first and second cycle at each displacement level.

Fig. 10.20 shows the variation of the lateral displacement and the beam elongation during the test. These results are plotted using two different scales to illustrate more clearly the response in the range of small displacements. In the initial cycles, the beam elongation represented about 50% of the lateral displacement. As the test progressed, the beam elongation became less significant. The comparison of Figs. 10.8 and 10.20 indicates that the additional beam reinforcement placed in Unit 2 was very effective in reducing the axial deformation of the beam.

### 10.3.3 Beam-Column Joint Deformation

The deformation of the beam-columns joints of Unit 2 was efficiently controlled by providing tapered joints with adequate reinforcement (see section 9.3.2.2). Readings from linear potentiometers LP2 and LP3, located in similar position in both units in order to compare results, indicated that the opening of the joint was less than 0.1 mm up to load run 60 ($\delta = 0.3\%$). Afterwards, the opening progressively increased up to a maximum value of 1.2 mm in the final cycle. These values are markedly smaller than those measured in Unit 1 (see Fig. 10.10).

### 10.3.4 Curvature and Axial Strain in the Frame

Strains measured in the longitudinal reinforcement were used to evaluate curvatures and axial strains in the frame members, according to the procedure discussed in section 10.2.4. These results corresponding to load run 62, are illustrated in Figs. 10.21 and 10.22. The maximum curvatures developed at both ends of the
tension column whereas no significant curvatures occurred in the compression column. The top beam bent upwards as a result of the rotation induced in the joint in which the lateral force was applied. The variation of the axial strains illustrated in Fig. 10.22 conceptually agrees with axial forces predicted according to the equivalent strut mechanism. Tensile strains concentrate at the base of the south column due to the cut off of the longitudinal reinforcement used in Unit 2.

The stress-strain response in the longitudinal bars of the columns were evaluated following the analytical model proposed by Dodd and Restrepo [D27]. As an example of these calculations, Fig. 10.23 shows the stress-strain response corresponding to the external bar of the south column, based on the strain history registered by a strain gauge located at the base of the column. In the initial cycles the stresses remained in the elastic range and in load run 55 yielding occurred. In subsequent cycles, the tensile strains increased and the bar yielded alternatively in tension and compression. The maximum tensile strain developed during the application of load run 61.

![Graph](image)

**Figure 10.20.** Variation of the lateral displacement and the beam elongation during the test.

![Graph](image)

**Figure 10.21.** Curvature of the reinforced concrete frame members.
10.3.5 Stresses in the Transverse Reinforcement

Strains recorded in the stirrups closest to the base of each column indicate that the stress did not exceed 80 MPa during the application of load runs 1 to 60, with maximum values arising when the column was subjected to tension. In subsequent cycles, the stress increased and the maximum values occurred in both loading directions. During the two final cycles of the test, the yield strain was slightly exceeded. In the other instrumented stirrups, the stresses were smaller than 20 MPa.

10.3.6 Evaluation of the Cracking Force and Lateral Strength

Different equations were used to predict the force level that initiates sliding shear in the masonry panel by debonding of the mortar joints and these results were presented in section 10.2.6. The cracking force, evaluated according to the procedure proposed in section 8.2.3, is 59.6 or 83.5 kN for values of the equivalent strut width of 0.25 or 0.35 of the diagonal length, respectively. These results agree well with the
measured force equal to 65.0 kN. The comparison between the cracking forces corresponding to Units 1 and 2 (31.6 and 65.0 kN, respectively) clearly indicates that the tapered joints contributes to increase the width of the equivalent compressive strut, reducing the stresses in the masonry panel.

The lateral resistance of Unit 2 was primarily controlled by the tensile axial strength of the columns. This is one of the chief objectives considered in the design, which allows the use of simple considerations for the evaluation of the lateral strength. The lateral force applied to the unit induced in one of the columns a tensile axial force which can be estimated using the equivalent truss mechanism. In addition, the axial force applied by the vertical actuators should be also considered (see section 9.5, Eq. 9.1). Therefore, the tensile axial force in the column, $T_o$, is given by:

$$T_o = V \frac{h}{L} + 0.52 V - 20 \text{ kN}$$

(10.3)

where $V$ is the applied lateral force and $L$ and $h$ are the length and height of the frame, respectively. The lateral strength, $V'$, will develop when the longitudinal reinforcement of the column (with area $A_r$) yields. Thus:

$$V' = \frac{A_r f_y + 20 \text{ kN}}{h/L + 0.52}$$

(10.4)

From Eq. 10.4, considering that $A_r = 316 \text{ mm}^2$ and $f_y = 323 \text{ MPa}$, it is found that the shear strength of the unit is $V' = 93.4 \text{ kN}$. The comparison of this value with the measured response of Unit 2 indicates a good agreement, although the predicted strength is slightly higher (see Fig. 10.18). This fact could be explained considering that the bending moment acting on the base of the column slightly decreases the tensile axial strength of the member.

10.4 COMPARISON OF THE BEHAVIOUR OF UNITS 1 AND 2

In the previous sections, results obtained from the test units have been presented separately. Since Unit 2 was designed with the aim of improving the structural behaviour under lateral loading, it is useful to discuss some aspects of the response of both units.

In the initial stage, the lateral stiffness of Unit 2 was higher than that corresponding to Unit 1 at the same force level (see Figs. 10.6 and 10.19). This increase can be explained by the use of the tapered beam-columns joint augmenting the width of the equivalent compressive strut. Furthermore, the deformable length of the frame members was smaller. Fig. 10.24 compares the envelope of the cyclic lateral force-lateral displacement relationship. The maximum lateral force resisted by Units 1 and 2 was 43.0 and 91.6 kN, respectively, which represents an increment of 113%. This increment was mainly achieved by limiting the deformation of the surrounding frame, whereas the mechanical properties of the masonry panel and the tensile axial capacity of the columns were the same. Even though both units exhibited some strength degradation, Unit 2 was able to sustain a storey drift of 2%, which is a high value for framed masonry structures.
The comparison of the cyclic response of both units indicates that the shape of the hysteresis loops was similar, changing from approximately linear loops at the beginning of the test to highly pinched loops at the end of the test. Fig. 10.25 shows hysteresis loops of Units 1 and 2, scaled to have the same maximum displacements and forces, corresponding to two stages of the test. The main difference is observed in the initial cycles, in which the area of the hysteresis loops of Unit 1 was larger due to more severe cracking and nonlinear behaviour.

The energy dissipation capacity (see section 5.6.5) is an important parameter which can affect the dynamic response, especially for structures with short natural period of vibration. In order to quantify this parameter, the energy absorbed and dissipated in each cycle, $E_A$ and $E_D$, respectively, was calculated by numerical integration, according to the criteria illustrated in Figs. 10.26 and 5.31 (b). Even though different procedures have been used for the evaluation of the absorbed energy $E_A$, the definition represented in Fig. 10.26 is not
adopted on the basis of a mathematical demonstration, but on that of a physical consideration. In laboratory tests, work is applied to the unit by pumping the hydraulic actuators during the loading process (from A to B and from C to D in Fig. 10.26). This criterion has been also used by Tomažević and Lutman [T6]. The ratio $E_p/E_A$ is plotted in Fig. 10.27(a) for both units. It can be observed that the ratio $E_p/E_A$ remained approximately constant or even increased in the final stage of the cycle. The repetition of cycles to equal lateral displacement produced a decrease in the energy dissipation capacity. Fig. 10.27 (b) illustrates the variation of the equivalent damping, evaluated from Eq. 5.4, which also gives an indication of the capacity of the structure to dissipate energy, as discussed in section 5.6.6.

![Figure 10.26. Definition of the absorbed energy $E_A$.](image)

![Figure 10.27. Variation of the ratio $E_p/E_A$ during the test of Units 1 and 2.](image)
The capacity to dissipate energy decreases, in relative terms, as the test progressed due to the reduction of the area of the hysteresis loops produced by pinching. However, the absorbed energy also decreases as a result of stiffness degradation. This observation raises some questions about the effect of pinching on the dynamic response of the structure, suggesting that its influence perhaps is not as detrimental as is usually considered. The answer to this issue should be given on the basis of experimental and analytical results considering dynamic loading. In Chapter 12, results from a parametric study are presented with the objective of clarifying this problem.

10.5 CONCLUSIONS

- The test of Unit 1 showed severe cracking of the masonry panel, which occurred mainly in the upper part by debonding of the mortar joints, and significant elongation of the frame members. The failure mechanism involved sliding shear at a column top and along the horizontal mortar joints. The lateral resistance was primarily determined by dowel action of the longitudinal reinforcement.

- Unit 1 was designed to obtain a mode of failure involving tensile yielding of the longitudinal reinforcement of the columns. This objective was not achieved. The lateral resistance measured during the test was about 50% of that predicted under the hypothesis of a tensile failure of the columns.

- The mode of failure of Unit 1 was different from that normally observed in similar tests, which suggests that the loading system used for the application of the lateral force could markedly influence the results.

- Significant opening of the joints, produced by the relative rotation of the beam with respect to the column, was observed in Unit 1. This has an adverse influence on the structural behaviour due to the reduction of the restraint provided by the frame to the cracked masonry panel and to the modification of the transfer of the shear forces in the loaded corner of the panel.

- Unit 2 showed a very good response, and the lateral resistance was about 110% higher than that of Unit 1, even though the tensile and shear strengths of the columns and the characteristics of the masonry panel were the same. This improvement of the response was achieved by a rational design, in which ductile behaviour was assured by yielding of the longitudinal reinforcement of the columns, and the deformation of the surrounding frame was limited in order to preserve the geometry of the masonry panel. The cracks in the masonry panel of Unit 2 mostly formed along the mortar joints and were spread widely over the panel, with a smaller width than those observed in Unit 1.

- The use of tapered beam-column joints with diagonal reinforcement was found to be an efficient method of restraining the opening of the joints. Furthermore, this type of joint improves the transfer of the lateral forces from the beam to the masonry panel and increases the width of the equivalent compressive strut.
According to comparisons between measured and analytical values of the stiffness and cracking force, the width of masonry strut to be considered in the equivalent truss mechanism in Unit 1 can be taken between 0.15 and 0.25 of the diagonal length. In Unit 2, with tapered joints, the width of the strut varies between 0.25 and 0.35 of the diagonal length. In both cases, smaller width values were required for an adequate evaluation of the cracking force than for the stiffness. More experimental research is needed to investigate this aspect in detail and consider the influence of different parameters, such as dimensions, mechanical properties and relative stiffness of the masonry infill and the frame.

The lateral resistance of Unit 2, which was designed according to a new criterion, can be adequately predicted on the basis of a simple procedure based on an equivalent truss mechanism and the tensile strength of the columns.

Both units exhibited significant pinching of their force-displacement hysteresis diagrams and moderate strength degradation in the final stages of the test. The maximum storey drifts imposed during the test were 1.5% and 2.0% for Unit 1 and 2, respectively.
11. DESIGN CONSIDERATIONS FOR INFILLED FRAMES

11.1 INTRODUCTION
Infilled frames are still used as structural systems for commercial and residential buildings, especially in seismic regions where masonry is a convenient material due to economical and traditional reasons. Even though infilled frames can exhibit an adequate response, when properly designed, severe damage and loss of life have occurred in past earthquakes. The usual problems associated with the damage or collapse of infilled frame buildings are irregular distribution of the masonry infills, inappropriate detailing of the reinforced concrete frame, partial infill, and deficiencies in materials and workmanship. However, inadequate design criteria and the lack of comprehension of the structural behaviour are also critical issues. It is very important, therefore, to develop simple and rational design procedures in order to obtain a safe and economical solution.

In this chapter, some aspects related to the current design of infilled frames are discussed and the basis of a new design criterion is presented. Unfortunately, some of the recommendations given here could not be experimentally verified due to limitations in funds and time. However, it is expected that these ideas will motivate other researchers to continue the work.

11.2 DESIGN CONSIDERATIONS
11.2.1 General
The design of infilled frames is often conducted by controlling the average shear stress in the masonry panel according to very simple procedures (see section 6.4.7). The surrounding reinforced concrete frame is usually designed according to the common practice for this type of structure, without taking into account the particular aspects resulting from the interaction between the panel and the frame. These aspects, however, should be carefully considered when evaluating the member actions and the appropriate ductile mechanism.

A detailed discussion of the design specifications for infilled frames according to a particular seismic code is outside the scope of this work. However, two important issues related to the customary design are examined in the following sections, before introducing a different approach for the ductile design of infilled frames.

11.2.2 Tensile Failure of the Reinforced Concrete Frame
It has been suggested [P1, S18] that infilled frames can be designed to fail in a "flexural mode" by yielding of the tension column in order to obtain a reasonably ductile behaviour, avoiding other brittle types of failure. However, yielding of the tension column could produce lack of restraint and stability problems, as discussed below.
When the columns of the infilled frame are subjected to tensile forces, numerous near-horizontal cracks cross the width of the column. Increasing forces will produce yielding of the longitudinal reinforcement, if the masonry panel is able to resist the diagonal compressive force resulting from the equivalent truss mechanism. Since the tensile force, and consequently the strain, is approximately constant along the column, yielding of the reinforcement results in a significant elongation of the member. For example, if yielding occurs and a tensile strain of 0.005 develops in the longitudinal reinforcement of a 3.0 m column, the total elongation of the member will be about 15 mm, which is usually incompatible with the brittle characteristics of the masonry panel. The elongation of the columns reduces or eliminates the beneficial effect of the frame in restraining the masonry panel and jeopardizes the stability of the panel against out-of-plane actions. There is some experimental evidence to sustain this observation. Liao and Kwan [L15] tested a four-storey reinforced concrete frame infilled with brick masonry. The model consisted of two parallel structures connected by a reinforced concrete slab at each floor level. Dynamic tests were carried out using the El Centro, 1940 record on a shaking table, with increasing values of the peak acceleration. Seismic excitations were applied only in the plane of the structure. According to characteristics of the structure, large tensile forces could have developed in the columns of the infilled frame. In the final stage of the test, the masonry panel of the lowest storey was shifted out of the plane of the frame and finally fell, resulting in the total collapse of the structure.

It is also to be noted that, when the direction of the lateral displacement is reversed, the tensile stress in these bars reduces to zero, while the cracks are still open due to the residual strains in the steel bars. An increase in the lateral displacement will induce a compressive axial force in the column and, thus, will cause the cracks to close uniformly. However, small dislocated particles of concrete and out-of-plane displacements may produce the uneven closure of cracks. In this case, transverse curvature develops in the columns and buckling can occur. Paulay and Priestley [P1, P14] pointed out that instability due to out-of-plane buckling may occur in reinforced concrete or masonry structural walls, when part of the sections are subjected to alternate cycles of tensile and compressive strains. The danger of instability also arises for the columns of infilled frames, in case that large cyclic axial forces occur in these members.

In multistorry structures, it is very important to estimate adequately the distribution in height of the equivalent static lateral forces used in the design. The variation of this distribution affects the ratio of the overturning moment to the shear force and can modify the failure mode. San Bartolomé et al. [S18] designed a three-storey infilled frame, assuming a triangular distribution of the lateral forces, to obtain a ductile failure by yielding of the longitudinal reinforcement of the columns. However, the shaking table test of the specimen resulted in a shear type failure. The most important reason for this behaviour was that at the time of the shear failure, the lateral force distribution changed from triangular to approximately uniform with height. This variation obviously decreased the overturning moment and also the ratio of the axial load to the shear force.

11.2.3 Sliding Shear Failure of the Columns

Sliding shear failure of the columns is likely to occur in infilled frames as a result of the unfavourable combination of shear with tensile axial force. It is believed that this type of failure has been seldom observed in laboratory tests due to the use of unrealistic procedures for the application of the lateral forces (see sections 9.5.2). It should be also noted that the reinforcing detail commonly used for the beam-column joints in framed structures, does not represent an adequate solution for infilled frames. Fig. 11.1 (a) illustrates a
customary detail and shows the axial forces resulting from the equivalent truss mechanism (bending moments and shear forces in the frame members have been omitted for sake of simplicity). It is clearly observed that the top of the tension column is prone to develop a sliding shear crack, as occurred in one of the tests conducted by the author (see section 10.2). Fig. 11.1 (b) shows an alternative design, intended to improve the anchorage of the longitudinal reinforcement of the beam and to avoid the sliding shear failure in the columns. However, the beam-column joint is weakened because the strut and truss mechanism of shear transfer cannot properly develop in the joint. Since shear is normally not significant in beam-columns joints of infilled frames, this detail could be used when assuring that the joint remains uncracked. Tests to confirm this behaviour are required before recommending the use of this detail in infilled frames. Tapered beam-columns joints with diagonal reinforcement represent a better solution, as discussed in section 11.4.1.

![Potential shear crack](image)

**Figure 11.1.** Reinforcing details of the beam-column joints.

The shear strength of a cracked section results from a combined mechanism of friction between the concrete surfaces and dowel action in the reinforcing bars crossing the crack (see section 10.2.6). Since the development of dowel action requires large relative displacements, it is usually assumed in design that a conservative estimation of the shear capacity of the cracked section is given by the shear force resisted only by the friction mechanism, \( V_f \), which can be calculated according to the following expression:

\[
V_f = \mu |C_c| = \mu (\Sigma A_{si} f_{si} - P)
\]  

(11.1)

where \( \mu \) is the coefficient of friction of concrete (normally equal to 1.4 for concrete cast monolithically [P2]), \( C_c \) is the compressive force carried by the concrete, \( P \) is the gravity load acting on the section and \( A_{si} f_{si} \) represents the axial force in the longitudinal bars which provide a clamping action. The forces considered in Eq. 11.1 are positive when tensile. If the cracked section is predominantly subjected to axial forces, it
can be assumed for design that the total of the longitudinal reinforcement, $A_{sr}$, at yield contributes to the clamping action:

$$V_f = \mu (A_{sr} f_y - P)$$  \hspace{1cm} (11.2)

Eq. 11.2 indicates that tensile axial loads reduce the shear strength of the section and additional reinforcement should be provided. Shear friction tests reported by Paulay and Priestley [P1] show that, independently of the amount of reinforcement or external compression, there is an upper limit for the shear force to be transferred for a cracked section. This limit, which considers the detrimental effect of cyclic loading, can be expressed as:

$$V_f = 0.25 f'_c A_f$$  \hspace{1cm} (11.3)

where $f'_c$ is the compressive strength of the concrete and $A_f$ is the effective area of the shear friction.

11.3  CLASSIFICATION OF INFILLED FRAMES

11.3.1 General

Infilled frames can be divided into three categories based on their expected response, following a similar criterion to that proposed by Paulay and Priestley [P1] for reinforced concrete walls. This classification, schematically represented in Fig. 11.2, is described in the following sections.

![Classification of infilled frames](image)

**Figure 11.2.** Classification of infilled frames according to their response.

11.3.2 Elastic Infilled Frames

The presence of masonry panels confers to the infilled frame a rather large lateral strength and, consequently, the structure may remain in the elastic range when subjected to severe earthquakes. This situation can be found in low-rise buildings, especially those with a high ratio of masonry walls to floor area. No special requirements for ductile behaviour need to be provided in this case. However, it should be assured that masonry panels remain uncracked and the reinforced concrete frame is able to resist the actions resulting from the panel-frame interaction without significant damage. Furthermore, the foundation system should
be adequately designed to withstand the base shear and overturning moment transmitted by the superstructure.

11.3.3 Rocking Infilled Frames
An alternative design approach is to allow the rocking of the entire infilled frame over the foundations. The main advantage of permitting the structure to rock relies on the efficiency of the rocking phenomena as an energy dissipating mechanism [E2], being conceptually similar to the use of mechanical devices for base isolation. In this way, it is possible to limit the actions on the infilled frame, avoiding undesirable modes of failure which can lead to a brittle behaviour.

Rocking seems to be an adequate solution for infilled frames subjected to high seismic forces and low gravity loads. Unfortunately, this interesting approach has not received enough attention and there is not much experimental information available. Consequently, Paulay and Priestley [P1] recommended a careful consideration of the different aspects that may affect the response of the structure. A special study, including realistic dynamic analyses, should be conducted and the foundation system should be properly designed to assure that the supporting soil is able to resist the large forces that may result from the rocking structure without significant plastic deformations. The design should be conducted to assure that the rocking parts of the superstructure and their foundations remain elastic. A further discussion of rocking infilled frames is outside the scope of this study.

11.3.4 Ductile Infilled Frames
In some cases, it is not practically possible to design the infilled frames to respond elastically and, therefore, a reasonably ductile response should be assured. Following principles of capacity design, undesirable modes of failure in the surrounding frame or in the masonry panels can be avoided, while plastic deformations are deliberately induced in special parts of the structure, which are adequately detailed to this purpose. In this case, it has to be assured that the foundation system is able to resist elastically the actions transmitted by the superstructure.

Infilled frames normally exhibit limited ductility due to the brittle characteristics of the masonry panel. In the inelastic range, the surrounding frame is able to resist large deformations whereas the masonry panel cracks and fails under relatively low distortions, which may result in an unexpected failure of the frame. Furthermore, the lateral strength and the ductility capacity of the infilled frame are very difficult to predict due to uncertainties related to the behaviour of the masonry panel. In the last years, significant efforts have been made to improve the response of infilled frames by providing horizontal reinforcement in the mortar joints of the panel. This reinforcement is properly anchored in the surrounding columns, and design recommendation based on this criterion has been proposed [A19, A20, S36, Z1, Z3]. Other researchers believe that a reliable computational model, which would allow a precise prediction of the response, is the best approach to obtain an adequate design. In the author's opinion, however, the solution for a proper design is achieved in a different way using capacity design. Consequently, a new approach, based on the principles of capacity design, is proposed in the following sections for cantilever and squat infilled frames.

The term "cantilever" is used here to refer to infilled frames in which flexural effects are significant and yielding of the longitudinal reinforcement of the columns can occur. It is difficult, however, to indicate
quantitatively the separation between cantilever and squat infilled frames, which depends not only on the dimensions of the structure but also on the total shear force and gravity load acting on the infilled frame. According to the design criteria proposed in this study, it is assumed that squat infilled frames are those in which yielding of the longitudinal reinforcement of the tension columns cannot occur under the action of the largest expected earthquake in the locality.

11.4 PROPOSED DESIGN OF CANTILEVER INFILLED FRAMES WITH FLEXURAL YIELDING

11.4.1 General Description of the Procedure

The seismic response of cantilever infilled frames can be improved by a rational design method, in which ductile behaviour is achieved by controlled yielding of the columns subjected to tensile axial forces, and the masonry panels are prevented from suffering severe damage. In order to preserve the geometry of the masonry panel, the elongation of the columns is controlled with additional longitudinal reinforcement, as Fig. 11.3 illustrates. These additional bars, which are not anchored to the foundation, assure the formation of a weak region at the base of the columns, where most of the plastic deformations will develop. According to this criterion, the capacity of the column to resist tensile axial forces is determined by the amount of anchored reinforcement. Therefore, the lateral strength of the infilled frame can be easily evaluated, considering a simple flexural mechanism and avoiding some of the uncertainties related to the panel-frame interaction. Based on this mechanism, the amount of longitudinal reinforcement of the columns anchored in the foundations, $A_{in}$, can be determined to resist the earthquake design forces. This criterion was followed for the design of Unit 2 and good results were obtained when the unit was tested under cyclic lateral forces (see section 10.3), confirming the applicability of the procedure.

The use of tapered beam-columns joints with diagonal reinforcement contributes to a reduction of the distortion of the masonry panel by limiting the opening of the joints, improving the transfer of the lateral forces from the frame to the panel, and increasing the width of the compressive strut. Furthermore, the formation of a sliding shear crack at the top of the column (see Fig. 11.1 (a)) is improbable this case, because most of the lateral force is transmitted to the panel by the diagonal surface of the joint. Fig. 11.4 shows the proposed reinforcing detail for the tapered beam-column joints.

According to the principles of capacity design, it is required to estimate the maximum shear force that could be sustained by the infilled frame, in order to assure that the selected plastic mechanism is achieved and brittle modes of failure are avoided. The design shear force, $V_{ud}$, can be determined from the shear demand specified by the code, $V$, considering the flexural overstrength of the infilled frame and the influence of higher modes of response [P1]:

$$V_{ud} = \omega \phi_v V$$

(11.4)

where $\phi_v$ is the flexural overstrength factor defined as the ratio of the flexural overstrength to the overturning moment resulting from code forces, and $\omega$ is the dynamic shear magnification factor, defined as a function of the number of storeys of the building, $n$ [P1]:

$$\omega_v = 0.9 + \frac{n}{10} \quad \text{if } n \leq 6$$

$$\omega_v = 1.3 + \frac{n}{30} \quad \text{if } n > 6$$

(11.5)
It is to be noted that this equation was originally derived for reinforced concrete walls. Since no information is available for infilled frame buildings, Eq. 11.5 is a reasonable approximation to evaluate the effect of higher modes until further research be conducted.

Figure 11.3. Reinforcing details for infilled frames designed according to the proposed approach.

Figure 11.4. Detail of the tapered beam-column joints.
Sliding shear can occur at the bottom of the compression columns, due to the main crack previously formed when the column was subjected to tension. The designer should be aware of this problem and the cracked section should be verified according to the procedure described in section 11.2.3 (Eqs. 11.1, 11.2 and 11.3). However, the amount of shear force applied to the column is not clear (see Fig. 11.5). Most of the lateral forces are transferred through the masonry panel by a diagonal compressive field, and only part of this force is taken by the column. Additional shear is induced as a result of the flexural deformation of the column, although it is normally insignificant due to the large difference between the shear stiffness of the column and the axial stiffness of the truss mechanism. Therefore, it can be conservatively assumed that, in usual cases, the shear force resisted by the columns is about 70% of the design force $V_u$.

![Figure 11.5. Forces acting at the bottom of the compression column.](image)

Fig. 11.6 illustrates a further development in the design of infilled frames, in which a reinforced concrete beam is constructed between the masonry panel and the foundation. In this way, the frame completely encloses the lower masonry panel, thus preserving the geometry of the panel. When large displacements are imposed on the structure, yielding of the longitudinal reinforcement concentrates at the base of the column and the bottom beam separates from the foundation, allowing the masonry panel to accompany the deformation of the frame with a markedly reduced distortion. In order to achieve this behaviour, it must be assured that a crack forms along the beam-foundation interface. A pre-cracked connection can be obtained by applying an adequate sealant over the foundation to break the adhesion between hardened and fresh concrete, avoiding the development of tensile stresses at the interface. The top surface of the foundation should be properly roughened to allow the transfer of shear stresses by friction.

Plain round dowels can be embedded into the foundation and into the bottom beam to control sliding shear of the infilled frame, in those cases where the frictional strength of the pre-cracked connection is exceeded. The plain bars can resist part of the total shear force by a combination of shear and flexure mechanism, whereas they do not contribute to the flexural strength of the infilled frame. For design purposes, it can be assumed that the shear forces resisted by the plain round dowels is:
where $A_{sd}$ is the total area of the plain round dowels and $f_{yd}$ is the yield strength. This equation was proposed by Restrepo et al. [R16] under the assumption that the shear force in the dowel is transferred by a flexure mechanism [P2], with a distance between plastic hinges equal to the diameter of the dowel. It is to be noted that Eq. 11.6 implies that all the dowels are equally effective to transfer the shear force. However, the dowels close to the compression side are able to resist a higher shear force than those located in the opposite side, because the opening of the flexural cracks increases the length between plastic hinges in the dowels. This fact is compensated because strain hardening of the steel is not considered.

\[
V_d = 0.42 \, A_{sd} \, f_{yd} \quad (11.6)
\]

**Figure 11.6.** Recommended design for infilled frames with a pre-cracked connection at the base.

The total area of the dowels, $A_{sd}$, can be obtained from Eq. 11.6 to resist a shear force $V_d$, equal to the difference between the design shear force $V_u$ and the frictional strength of the connection,

\[
V_d = V_u - 0.75 \, V_f \quad (11.7)
\]
where the factor 0.75 is introduced to consider the detrimental effect of cyclic loading in the frictional strength. It is to be noted that Eq 11.7 conservatively neglects any contribution of the longitudinal reinforcement of the columns. In order to calculate \( V_t \) (see Eq. 11.1) the coefficient of friction can be adopted as \( \mu = 1.0 \), considering that the pre-cracked connection is not monolithic concrete [P2]. Since \( V_a \) is derived from flexural overstress development of the pre-cracked connection, it is not required to reduce the ideal strength of the dowels by the use of a strength reduction factor [P1].

The plain round dowels should be located in a symmetrical arrangement, preferably along the center line of the bottom beam, with a separation larger than 15 times the diameter of the dowel. This value is conservatively assumed, in the lack of experimental information, in order to avoid local cracking of the surrounding concrete due to the splitting effect induced by the dowels. The use of several dowels of small diameter could be preferred. The bottom beam has to transfer part of the shear force from the joints of the frame to the dowels. In some cases, depending on the configuration of the infilled frame and on the location of the dowels, tensile axial forces may develop in the bottom beam. This situation should be also considered in the design.

It is worth noting that, based on Eq. 11.6, plain round dowels are provided to assure an adequate strength for the pre-cracked connection. However, dowel action represents a very flexible mechanism and significant pinching of the hysteresis loops can be expected under cyclic loading. When the lateral forces reverse after significant plastic deformations have developed in the longitudinal reinforcement, the shear has to be entirely transferred by dowel action until contact between the surfaces of the pre-cracked connection is restored. In order to reduce this effect, additional dowels can be placed in the connection to increase the stiffness.

The use of multiple shear key connections could be thought as an adequate solution to control sliding shear. It must be noted, however, that the predominant action in the connection is the overturning moment, originating a significant rotation around the compression edge. As a result, the shear forces are primarily transferred at the corner of the keys, which can produce the crushing of the concrete in this part of the connection. Since most of the experiments related to shear key connections have been conducted under predominantly shear, this detail is not recommended until realistic test results be available.

Even though the design approach proposed in Fig. 11.6 has not yet been tested, it is believed that it represents an adequate solution to limit the deformation of the lower masonry panels and to control sliding shear with additional plain round dowels. Therefore, the use of this design approach is recommended and further experimental research is encouraged.

11.4.2 Ductility Requirements
According to the design procedure outlined in the previous section, most of the plastic deformations are concentrated in a small region of the structure, where large strains can be induced in the longitudinal reinforcement of the columns. In order to assure that fracture of the steel does not occur, the deformation capacity of those bars should be compared with the demand imposed by severe seismic actions. It is useful, therefore, to find a relationship between the maximum strain in the reinforcing bars and the global ductility of the structure. Due to the problems discussed in section 9.7, ductility is not the most appropriate damage
index to be associated with infilled frames. However, this parameter is normally adopted in seismic codes as a measure of the degree of plastic deformation permissible in the structure.

Fig. 11.7 (a) shows the deformed shape of a typical infilled frame designed according to the proposed procedure. The lateral displacement can be evaluated taking into account the contribution of two components, one due to shear and flexural deformation of the infilled frame, $\Delta_s$, and the other due to deformation occurring at the base of the columns, $\Delta_c$. The latter component depends on the elongation of the bars, $\Delta_\nu$, which originates a rigid body rotation equal to $\Delta_c/L$. The elongation $\Delta_\nu$ can be approximately evaluated assuming an equivalent uniform distribution of the strains, $\varepsilon_\nu$, along a length equal to 12 times the bar diameter, $d_\nu$, as indicated in Fig. 11.8 (a). This simplified criterion is adopted to represent the effect of tensile strain penetration into the foundation and into the columns. The length of the equivalent strain block is adopted, in the lack of more precise experimental data, considering typical values of strain penetration in flexural plastic hinges [P1] and results reported by Restrepo et al. [R14]. By integrating the strains, it is found that:

$$\Delta_s = 12 d_\nu \varepsilon_s$$  \hspace{1cm} (11.8)

It is worth noting that the strain penetration can be significantly reduced if reinforcing steel with no strain hardening is used. In this case, the equivalent length considered in the deduction of Eq. 11.8 should be shorter, resulting in a marked reduction of the deformation capacity of the bar.

![Figure 11.7. Deformed shape and displacement profiles.](image-url)
Figure 11.8. Variation of the strain in the reinforcing bars of the columns.

The displacement ductility ratio, $\mu_\Delta$, related to the lateral displacement at a height, $h^*$, of the resultant of the seismic lateral forces, $V$ (see Fig. 11.7 (b)), can be expressed as:

$$\mu_\Delta = 1 + \frac{\Delta_p}{\Delta_y} \quad (11.9)$$

where $\Delta_y$ is the lateral displacement at first yield and $\Delta_p$ is the plastic lateral displacement. In order to calculate $\Delta_y$ both components of the lateral displacement should be defined at first yield. In this stage, it can be assumed that the lateral displacement is mainly due to the deformation of the infilled frame, which can be evaluated using the equivalent truss mechanism or by any other suitable method. Thus:

$$\Delta_y = \Delta_{fy} \quad (11.10)$$

After yielding starts, the component $\Delta_{fy}$ remains constant because the lateral forces do not increase, whereas the component associated with $\Delta_p$ increases. Therefore, the plastic displacement of the infilled frame originates primarily in the plastic elongation of the longitudinal reinforcement at the base of the columns. When the steel strain reaches the maximum value in the ultimate stage ($\varepsilon_s = \varepsilon_{\text{max}}$), the plastic displacement is:

$$\Delta_p = 12 \, d_b \left( \varepsilon_{\text{max}} - \varepsilon_y \right) \frac{h^*}{L} \quad (11.11)$$
Introducing Eqs. 11.10 and 11.11 into Eq. 11.9 and rearranging, it is found that the ratio of the maximum strain to the yield strain of the steel, \( \frac{\varepsilon_{\text{max}}}{\varepsilon_y} \), is equal to:

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_y} = \frac{(\mu_\Delta - 1) \Delta_{\text{fy}} L}{12 d_b \varepsilon_y h^*} + 1
\]  

(11.12)

The deformation capacity can be improved, without increasing the maximum strain in the steel, by partially unbonding the reinforcing bars along a distance \( L_{\text{ub}} \), as Fig. 11.8 (b) illustrates. This can be achieved, for example, using a plastic tube to cover the bar, avoiding the development of bond stresses between the reinforcement and the surrounding concrete. Following a similar procedure to that used in deriving Eq. 11.12, it is found that for partially unbonded bars:

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_y} = \frac{(\mu_\Delta - 1) \Delta_{\text{fy}} L}{(12 d_b + L_{\text{ub}}) \varepsilon_y h^*} + 1
\]  

(11.13)

Eqs. 11.12 and 11.13 allow the verification of the maximum strain expected in the longitudinal reinforcement for a given value of the displacement ductility ratio, taking into account the particular characteristics of each case. It is interesting, however, to obtain some conclusions based on a general evaluation of these equations. Fig. 11.9 presents results of the ratio \( \frac{\varepsilon_{\text{max}}}{\varepsilon_y} \) as a function of \( L/d_b \) for different values of the global ductility. In order to calculate the yield displacement \( \Delta_{\text{fy}} \), it was assumed that the storey drift, \( \delta_s \), is uniform along the height of the infilled frame. Under this hypothesis, \( \Delta_{\text{fy}} = \delta_s h^* \), where \( \delta_s \) is the storey drift at first yield. The values of \( \delta_s \) were estimated from experimental results (see sections 10.2.2. and 10.3.2). Fig. 11.9 indicates that large strains can be induced in the longitudinal reinforcement when medium to large values of the displacement ductility \( \mu_\Delta \) are required, or when the infilled frame is more flexible (higher values of \( \delta_s \)). Similar effects result from increasing the ratio \( L/d_b \). It is also observed that the use of partially unbonded bars leads to a significant reduction in the maximum strain.

![Graphs](image)

**Figure 11.9.** Strain demand in the reinforcing bars for different levels of ductility \( \mu_\Delta \) and ratios \( L/d_b \).
The maximum strain permissible in the reinforcing steel when subjected to tension depends on the following parameters:

- **Characteristics of the steel**: the ultimate (uniform) strain, $\varepsilon_{\text{tu}}$, corresponding to the maximum stress is the main parameter.
- **Strain history**: the number of cycles and the maximum compressive strain experimented during the loading history can affect the fracture strain.
- **Temperature**: at very low temperature, the reinforcing steel may exhibit a reduction in the deformation capacity. However, this effect is not significant in most of the regions where infilled frames are used.

Dodd and Restrepo [D27] concluded that, for small number of cycles such as those expected to occur in a reinforcing bar of a reinforced concrete member during an earthquake, the fracture strain is equal to the ultimate strain, $\varepsilon_{\text{tu}}$, less the maximum plastic strain previously developed in compression. At the base of the columns, sliding shear can bend the longitudinal reinforcement in double curvature (as occurred in Unit 2, see section 10.3.1), inducing compressive strains in the concave side of the bar which can significantly reduce the fracture strain. Consequently, it is recommended that the maximum strain in the longitudinal reinforcement should not exceed $0.5 \varepsilon_{\text{tu}}$.

### 11.4.3 Multibay Infilled Frames

The design approach proposed in the previous section is also applicable for multibay infilled frames, provided that tensile yielding of the longitudinal reinforcement of the columns occurs. Two particular aspects require further discussion.

The evaluation of the actions in the frame members and in the masonry panels can be conducted using a simple model, such as the equivalent truss mechanism. It is important, however, to consider a realistic distribution of the seismic forces along the beams. An inadequate representation, such as assuming that the total lateral force corresponding to a certain floor level is applied at one end of the beam, can lead to significant errors in the axial forces of the structural members.

In those cases where design criteria presented in Fig. 11.6 is used and a pre-cracked connection is induced at the base, the infilled frame behaves similarly to a monolithic member with a flexural crack at the base. Consequently, the flexural strength of the connection can be calculated applying a similar procedure to that considered for reinforced concrete columns with multiple reinforcing steel layers. Equilibrium and compatibility equations allow the calculation of the stresses and strains of the longitudinal reinforcement crossing the connection.

### 11.4.4 Summary of the Design Procedure

Based on the considerations discussed in the previous sections, a general framework for the design of cantilever infilled frames with a pre-cracked connection at the base is proposed in order to achieve a reasonably ductile behaviour. The proposed procedure can be summarized as follows:
Evaluate the actions in the infilled frame based on a simple model, such as the equivalent truss mechanism, assuming an adequate distribution of the lateral forces along the beams.

Calculate the longitudinal reinforcement of the columns of the lower storey to provide adequate flexural strength to the pre-cracked connection, taking into account the gravity loads and the strength reduction factor prescribed by the design code. This reinforcement, with total area $A_{as}$ should be properly anchored in the foundations.

Provide additional longitudinal reinforcement in the lower columns to induce a "weak region" at the base (see Fig. 11.6). The area of the additional reinforcement should be at least equal to $0.5 A_{as}$.

Evaluate the design shear force $V_{as}$, taking into account the flexural overstrength of the pre-cracked connection and the influence of the higher modes of response, according to Eq. 11.4.

Verify that the masonry panel is able to resist the diagonal compressive forces induced by the design shear $V_s$ without failure. When solid masonry units are used, cracking of the panel is regarded as a serviceability limit state.

Design the floor beams and the columns of the upper storeys to resist the axial tensile forces (considering flexural overstrength) without excessive elongation. Yielding of the longitudinal reinforcement should be avoided.

Provide tapered beam-columns joints with diagonal reinforcement, assuring that the inclined surface of the joint is perpendicular to the diagonal of the masonry panel. The vertical and horizontal dimensions of the tapered joint, $d_v$ and $d_h$, should be greater than 1.5 times the depth of the beam and the column, respectively. The total area of the diagonal reinforcement can be taken as $0.5 A_{as}$.

Design the transverse reinforcement of the frame members to resist the shear forces, to provide confinement to the concrete and to avoid premature buckling of the longitudinal reinforcement.

Produce a pre-cracked connection between the infilled frame and the foundation, assuring that adhesion is broken and that the top surface of the foundation is properly roughened. The connection should be able to transfer the total shear force by a friction mechanism. Otherwise, plain round dowels should be placed to connect the foundation and the infilled frame to control sliding shear. The total area of the plain round dowels should be assigned to resist the shear force $V_d = V_s - 0.75 V_r$. The frictional strength of the connection can be evaluated according to Eq. 11.1, with a coefficient of friction $\mu = 1.0$. The dowels should be located in a symmetrical arrangement, preferably in the centre line of the bottom beam, with a separation larger than 15 times the diameter of the dowel. The bottom beam should be designed to transfer the shear forces from the infilled frame to the dowels.
Check the maximum tensile deformation expected in the longitudinal reinforcement of the columns, according to Eq. 11.12. This value should not exceed 0.5 ε_{sec}. When required, the deformation capacity of the structure can be improved by partially unbonding the longitudinal reinforcement (see Eq. 11.13).

Design the foundations of the infilled frame to resist elastically the gravity loads and the actions resulting from the plastic mechanism of the superstructure.

11.5 PROPOSED DESIGN OF SQUAT INFILLED FRAMES WITH SHEAR YIELDING

11.5.1 General Description of the Procedure

According to the philosophy of capacity design, the seismic forces in the structure can be limited by selecting an adequate plastic mechanism, in which ductile flexural failures occur in carefully detailed regions, whereas shear strength is supplied in excess to prevent a brittle behaviour. This approach, however, is difficult to apply to squat infilled frames. The flexural failure, involving yielding of the longitudinal reinforcement, requires very large lateral forces which will lead to a different, and usually undesired, mode of failure of the infilled frame before the flexural mechanism develops. It is important, therefore, to discuss design recommendations for this case, since squat infilled frames are widely used as structural systems for low-rise buildings.

Under certain conditions of geometry and loading, squat infilled frames can be designed to produce a "shear fuse" at the base, using a pre-cracked connection between the foundation and the infilled frames (see Fig. 11.10) where sliding shear is deliberately provoked. Sliding shear is normally regarded as an inadequate mechanism, which should be avoided. For squat infilled frames, however, it represents an alternative solution for obtaining a reasonably ductile behaviour and to prevent the masonry panels from suffering severe damage. As mentioned in section 11.4.1, the pre-cracked connection at the bottom of the infilled frame can be provided by breaking the adhesion between the concrete surfaces. In this case, the top surface of the foundation beam should be smoothed to facilitate the sliding of the infilled frame. The use of tapered beam-columns joints with diagonal reinforcement is also recommended in this case to reduce the opening of the joints and to improve the transfer of the lateral forces from the frame to the masonry panel (see section 11.4.1).

This design criterion can be applied only if the total base shear, V, applied to the structure can overcome the frictional resistance of the pre-cracked connection, V_f. Therefore:

$$ V > V_f $$  \hspace{1cm} (11.14)

It is to be noted that this equation neglects the contribution of the reinforcing steel by dowel action, because this mechanism is mobilized after the imposition of significant displacements. Eq. 11.14 limits the applicability of the proposed design procedure to those cases in which the shear force is equal to or greater than the gravity loads acting on the infilled frame. This situation is rarely found in practice, except for buildings with structural systems formed by a combination of flexible frames and infilled frames, the latter being the main source of earthquake resistance. Since the connection is not subjected to significant overturning moment, the frictional strength can be evaluated according to:

$$ V_f = \mu P $$  \hspace{1cm} (11.15)
Figure 11.10. Longitudinal reinforcement at the base of squat infilled frames.

In some particular cases, tensile strains can develop in the connection when the maximum strain due to the overturning moment exceeds the uniform compressive strain induced by the gravity loads. The condition for development of tensile strains can be derived using customary expressions of strength of materials and neglecting the contribution of the reinforcing steel in the pre-cracked connection. Therefore, the shear force required to produce this situation is given by the following condition:

\[ V > \frac{P L_{if}}{6 h^*} \]  

(11.16)

where \( L_{if} \) is the total length of the infilled frame (see Fig. 11.10). In this case, part of the longitudinal reinforcement of the columns will be subjected to tension and the frictional strength of the connection may be increased. Consequently, Eq. 11.1 should be used to calculate \( V_f \), in those cases in which tensile strains can develop in the connection. Either Eq. 11.1 or Eq. 11.12 apply, the coefficient of friction \( \mu \) can be taken as 0.7 for smooth surfaces [R16] and 1.0 in other situations [P2].

In order to control sliding shear, the longitudinal reinforcement of the column crossing the connection should be able to resist the force \( V_o \), equal to the difference between the total shear force and the frictional strength of the connection (without consideration of the frictional overstrength), thus:
\[ V_d = \frac{V - 0.75 \, V_f}{\psi} \]  

(11.17)

where the coefficient of 0.75, affecting the frictional strength, is introduced to considering a probable degradation of the strength due to cyclic loading, and \( \psi \) is the strength reduction factor usually prescribed in design codes. The shear strength due to dowel action in the longitudinal reinforcement can be evaluated according to (see Eq. 11.6):

\[ V_d = 0.42 \, A_{\text{ac}} \, f_y \]  

(11.18)

where \( A_{\text{ac}} \) is the total area of the reinforcement crossing the connection and \( f_y \) is its yield strength. If tensile strains are expected to develop in the vertical reinforcement, Eq. 11.18 does not lead to conservative results because the dowel strength of the longitudinal reinforcement cannot be fully utilized when the same bars also provide a clamping force. Following the criterion suggested by Paulay and Priestley [P1], the dowel shear resistance can be estimated as:

\[ V_d = 0.25 \, A_{\text{ac}} \, f_y \]  

(11.19)

When required, additional plain round dowels can be placed in the connection, as described in section 11.4.1. Diagonal reinforcement, similar to that used for the seismic design of coupling beams and squat walls [P1], could represent an alternative solution to avoid pinching of the response due to sliding shear. However, further research is required to verify the applicability of this detail.

The infilled frame should be designed to avoid severe damage of the masonry panels and yielding of the frame members, considering that the code lateral forces can be magnified due to the shear overstrength of the pre-cracked connection. When evaluating this condition, it is necessary to assess the maximum feasible overstrength due to probable enhancement of the frictional properties and of the steel strength. Consequently, the design shear force \( V_u \) (conceptually similar to that derived in section 11.4.1 to consider flexural overstrength) is calculated according to:

\[ V_u = \lambda_{\text{of}} \, V_f + \lambda_{\text{os}} \, V_d \]  

(11.20)

where \( \lambda_{\text{of}} \) and \( \lambda_{\text{os}} \) are the frictional and steel overstrength factors, respectively. A tentative value of \( \lambda_{\text{of}} = 1.5 \) is recommended in the lack of more information. The steel overstrength factor depends on the variability of the local supply and on the yield strength and the steel composition [P1]. Typical values of \( \lambda_{\text{os}} \) ranging from 1.25 to 1.40 are used in New Zealand, although they should be locally verified. The dowel strength \( V_d \) in Eq. 11.20 needs to be evaluated considering all the longitudinal reinforcement of the columns and the plain round dowels, if any, crossing the connection.

11.5.2 Ductility Requirements

Squat infilled frames with shear yielding can be designed to concentrate the plastic deformations at the base of the structure, where sliding shear occurs and longitudinal reinforcement crossing the connection controls
the response by dowel action. In order to verify the lateral displacement, $\Delta_d$, required in the pre-cracked connection to achieve a specified displacement ductility ratio, $\mu_d$, a procedure conceptually similar to that presented in section 11.4.2 can be followed. In this case, the displacement corresponding to "first yield" is defined just before sliding starts in the connection. The lateral displacement in this stage is entirely due to the flexural and shear deformation of the infilled frame, thus $\Delta_y = \Delta_{sy}$. In the inelastic range, the deformation of the infilled frame itself does not increase because the lateral forces remain constant, and the plastic displacement is equal to the sliding shear displacement at the base, $\Delta_s = \Delta_{sb}$ (see Fig. 11.11). Based on these considerations, it is found from Eq. 11.9 that:

$$\Delta_d = (\mu_d - 1) \Delta_{sy}$$

(11.21)

There is not enough experimental information to evaluate the displacement capacity of the connection when sliding shear occurs. However, tests results reported by Park and Paulay [P2] for construction joints and by Restrepo et al. [R16] for tilt-up walls with a pre-cracked connection at the base indicate that the shear transfer by dowel action is a mechanism capable of sustaining large deformations. According to the latter results, displacements up to 5 mm were imposed to 8 mm diameter bars resisting shear by dowel action in the connection without strength degradation. Similar results were reported by Soudki et al. [S38], who conducted shear tests on horizontal connection for precast reinforced concrete panels and observed that 25 mm diameter bars subjected to dowel action were able to resist cyclic lateral displacements up to 10 mm. Even though simple equations have been proposed to evaluate the shear strength due to dowel action [P2], more research is required to understand the deformation mechanism and to develop adequate expressions to predict the deformational capacity of the reinforcing bars.

It is also to be noted that the displacement $\Delta_{sy}$ is very small in usual cases, particularly for squat infilled frames, and that infilled frames are designed as structures of limited ductility. Consequently, it seems that the ductility capacity of the connection is normally adequate to sustain the demand imposed in the infilled frame by severe earthquakes.

![Figure 11.11. Displacement profiles for squat infilled frames.](image-url)
11.5.3 Summary of the Design Procedure
The design approach proposed for squat infilled frames is intended for those cases in which the flexural strength of the structure largely exceeds its shear resistance. This approach is summarized in the following steps:

- Verify that sliding shear can develop at the base of the infilled frame using Eq. 11.14.

- Evaluate the actions in the infilled frame based on a simple model, such as the equivalent truss mechanism, assuming an adequate distribution of the lateral forces along the beams.

- Produce a pre-cracked connection between the infilled frame and the foundation, assuring that adhesion is broken and the top surface of the foundation is adequately smoothened to reduce the friction between the surfaces. The longitudinal reinforcement crossing the connection should be able to resist by dowel action the shear force given by Eq. 11.17. The shear strength of the reinforcement can be verified using Eq. 11.18 or Eq. 11.19, whereas the frictional strength of the connection can be evaluated according to Eq. 11.1 or Eq. 11.2.

- If the previous condition is not satisfied, provide additional reinforcement in the connection, in the form of plain round dowels. These dowels should be located in a symmetrical arrangement, preferably in the centre line of the bottom beam and with a separation larger than 15 times the diameter of the dowel. The bottom beam should be designed to transfer the shear forces from the infilled frame to the dowels.

- Evaluate the design shear force $V_u$ taking into account the shear overstrength of the pre-cracked connection, according to Eq. 11.20.

- Verify that the masonry panel is able to resist the diagonal compressive forces induced by the design shear $V_u$ without failure. When solid masonry units are used, cracking of the panel is regarded as a serviceability limit state.

- Design the floor beams and the columns to resist the axial tensile forces (considering shear overstrength) without excessive elongation. Yielding of the longitudinal reinforcement should be avoided.

- Provide tapered beam-columns joints with diagonal reinforcement, assuring that the inclined surface of the joint is perpendicular to the diagonal of the masonry panel. The vertical and horizontal dimensions of the tapered joints, $d_v$ and $d_h$, should be greater than 1.5 times the depth of the beam and the column, respectively. The total area of the diagonal reinforcement can be taken as half of the longitudinal reinforcement of the columns.

- Design the transverse reinforcement of the frame members to resist the shear forces, to provide confinement to the concrete and to avoid premature buckling of the longitudinal reinforcement.
• Check the maximum displacement expected in the connection when the displacement ductility \( \mu_A \) is imposed, according to Eq. 11.21.

• Design the foundations of the infilled frame to resist elastically the gravity loads and the actions resulting from the plastic mechanism of the superstructure.

11.6 RECOMMENDATIONS FOR THE CONSTRUCTION OF INFILLED FRAMES

Based on the observations regarding the behaviour of infilled frames presented in this research project, a brief summary of recommendations for the construction of infilled frames is indicated here.

Framed masonry seems to be the best construction technique to integrate effectively the reinforced concrete frame and the masonry infill. In this way, it is possible to achieve adequate interface conditions without the use of complicated procedures or special devices. When the application of this method is not viable, the surrounding frame should be cautiously filled to avoid the formation of unintentional gaps between the infill and the frame. It is also important to provide a minimum amount of horizontal reinforcement adequately anchored in the surrounding frame, which improves the response under alternating lateral forces and out-of-plane actions. The horizontal reinforcement should be continuous, without lapped splices, because bond forces transmitted to the surrounding mortar cannot be resisted through the small thickness of the bed joints (usually ranging from 8 to 12 mm). Furthermore, the bars should be properly covered to avoid corrosion.

The use of solid masonry units is recommended to prevent a brittle crushing failure and to delay the panel degradation. The dimensions of the panel should be adopted avoiding aspect ratios, \( h_m/L_m \), smaller than 1/2.

Special attention should be given to the quality control of both materials and workmanship. Masonry units with uniform characteristics should be used. The mix proportions for the mortar should be adopted according to the properties of the materials available in each locality. Adequate hydration conditions should be assured, both when building the masonry panel and during the curing process, to obtain higher bond strength in the mortar joints.

11.7 CONCLUSIONS

• It has been pointed out that reasonably ductile behaviour can be achieved when the longitudinal reinforcement of the columns yield in tension and the design of infilled frames to obtain this type of failure has been proposed. There is evidence, however, that loss of fit between the frame and the panel and instability problems may degrade the response or produce the failure due to excessive elongation of the columns.

• Shear and axial forces acting on the columns of the reinforced concrete frame can cause a sliding shear failure. This possibility should be carefully considered when designing the infilled frame. Additional longitudinal reinforcement represents an adequate solution to control sliding shear, especially when tensile axial forces are expected.

• Infilled frames can be categorized in three groups depending on their response: ductile, elastic or rocking infilled frames. A reasonably ductile response can be obtained from cantilever or
squat infilled frames, when certain geometric and loading conditions are met, by producing a pre-cracked connection at the base, where plastic deformations are concentrated to preclude the infilled frame from suffering severe damage. In other cases, the infilled frame should be designed to remain in the elastic range or to rock on specifically designed foundations.

A new design approach for cantilever infilled frames is proposed, in which ductile behaviour is achieved by yielding of the longitudinal reinforcement of the columns. Yielding is limited to the base of the columns, avoiding large elongations of these members. A pre-cracked connection is induced between the infilled frame and the foundation, where plain round dowels can be placed to control shear sliding. The use of tapered beam-columns joints with diagonal reinforcement is recommended to reduce the opening of the joints and to improve the transfer of the lateral forces from the frame to the masonry panel.

A similar design criterion is proposed for squat infilled frames, in which ductility is conferred to the structure by allowing controlled sliding of the infilled frame over the foundation. The applicability of this approach is limited to those cases where the total shear force exceeds the frictional strength of the pre-cracked connection.

Infilled frames, with both high and low aspect ratios, can be designed to obtain a reasonable ductile response. With this aim, a design framework has been proposed in this chapter, based on rational considerations and the experimental results obtained from Unit 2. It must be recognized, however, that more research is needed to clarify some aspects of the design, such as convenient dimensions of the tapered joints, area of the additional longitudinal reinforcement and response of the pre-cracked connection, and to verify the behaviour of multibay and multistorey structures.

If the design procedures proposed in this study are corroborated by subsequent investigation, efforts should be made to incorporate these recommendations in seismic codes.
12. EFFECT OF HYSTERETIC PINCHING ON THE SEISMIC RESPONSE

12.1 INTRODUCTION

12.1.1 Objectives

Infilled frames subjected to cyclic loading normally exhibit severe pinching of the hysteretic response. It seems convenient, therefore, to investigate the influence of this phenomenon in the seismic response and its implications in the design, considering that seismic codes usually specify the level of seismic actions based on results obtained from dominantly flexural nonlinear systems. A parametric study, based on nonlinear dynamic analyses, was conducted with the following objectives:

- Verify the validity of the concept of equal displacement for structural systems with significant pinching, as occurs in infilled frames.
- Study the influence of the shape of the hysteresis loops by comparing the response of ideal elasto-plastic models, which are usually considered for design purposes, and pinched models.

12.1.2 Background

The concept of equal displacement indicates that an elastic and inelastic system with the same initial stiffness have approximately the same maximum displacement when the system has a period greater than a given value (usually called "characteristic period of the ground motion"). For example, the New Zealand's Code of Practice for General Structural Design and Design Loadings for Buildings [N11] assumes that the concept of equal displacement is applicable for structures with a natural period of free vibration greater than 0.7 s. As a result, the yield displacement of the nonlinear system is equal to the elastic displacement affected by the reduction factor $1/\mu_d$, where $\mu_d$ is the displacement ductility of the system. The fact that nonlinear and elastic response are similar was introduced by Veletsos and Newmark [V4] on the basis of nonlinear dynamic analyses. Later, several researchers conducted more complete studies taking into account different hysteretic models and earthquake records [C3, Q1, R15, S46]. Since the concept of equal displacement has implications on the seismic design, it is useful to discuss in some detail the results reported in previous research.

The analyses conducted by Veletsos and Newmark [V4] considered an elastic and elasto-plastic one-degree-of-freedom oscillator with 0% and 10% critical damping subjected to two different ground motions. In the 10% damping case, it was observed that the ratio of the maximum inelastic to elastic displacement "generally lie above 1.0, and is substantially above 1.5 over a portion of periods considered, running to values as high as 1.8 for the Vernon earthquake". The maximum values did not necessarily occur in the range of short periods. Similar observations can be made by analysing the results reported by Qi and Moehle [Q1], who
conducted a more complete study considering also stiffness degrading systems and nine different ground motions. In this case, some results corresponding to systems with periods smaller than 2.5 s shows that the concept of equal displacement underestimates the nonlinear displacement.

The inspection of previous results reveal a "general tendency" to follow the concept of equal displacement. However, there are several exceptions in which the nonlinear displacements are markedly higher than those corresponding to elastic systems. These exceptions are hidden when the results are analysed by means of statistical analysis, in which conclusions are formulated on the basis of average values. It is believed that design considerations for structures subjected to seismic actions should provide a conservative upper limit. Furthermore, efforts should be made to clearly defined the period range in which concept of equal displacement is valid. This aspect is the significant importance for infilled frame structures, which usually exhibit short to medium natural periods of free vibration.

Several researchers have investigated the effect of different types of resistance functions on the inelastic behaviour, usually considering a simple oscillator with one degree of freedom. One of the first studies related to this aspect was carried out by Clough [C3] to determine the influence of stiffness degradation of the cyclic response, similar to that observed in tests of reinforced concrete structures. It was concluded that, for long period structures (greater than 0.6 s), the ductility requirement for stiffness degrading systems was slightly higher than that for non-degrading systems. This study was based on five earthquakes records registered in California, United States. Otani [O3] conducted a parametric study using dominantly flexural hysteretic models and also observed that the maximum displacements and the response waveform did not change appreciably, except in the short period range. Similar conclusions were obtained by Moss et al. [M35], even though a pinched hysteretic model was included in the study to represent nailed timber joints.

Based on the analysis of a large number of cases, in which a degrading stiffness model was included, Riddell and Newmark [R15] concluded that "At the low frequency end of the spectrum, say below 0.05 or 0.1 Hertz depending on the ground motion record, the responses are practically independent of the force deformation law" and "For intermediate frequencies (from 0.05 or 0.1 to 10.0 Hertz) the responses of stiffness degrading systems are generally between about 0.5 to 1.5 times the response of the associated elasto-plastic systems". It is worth noting that these conclusions were obtained from the comparison of average spectra, corresponding to 10 different earthquakes, which indicate that larger differences could occur for some particular cases. Furthermore, the range of "intermediate frequencies" (from 0.05 or 0.1 to 10.0 Hertz, or from 0.1 to 10 or 20 s) includes most of the typical buildings, suggesting that the classification considered in this study was not very realistic from the practical point of view.

Stewart [S45] investigated the influence of a pinched hysteresis loop for plywood sheathed walls and found no important effect of pinching in the response to earthquake motions. The conclusions of this study cannot be generalized to infilled frames due to the limitations in the damping model and the particular characteristics of the strength envelope assumed by Stewart to represent the response of plywood sheathed walls.

In most of the studies revised above, viscous damping model proportional to the initial stiffness was adopted.
12.2 Parametric Study of Inelastic Systems Subjected to Earthquake Motions

12.2.1 Earthquake Accelerograms

The selection of the earthquakes accelerograms to be used in time-history analyses remains a difficult task. Due to the complex interrelation between the dynamic characteristics of the seismic ground motions and the energy dissipation properties of the structure, one single parameter is insufficient to characterize the damage potential of different earthquakes. Consequently, the selection of the accelerograms should be based on the consideration of several parameters, such as peak ground acceleration and velocity, duration of the strong motion, frequency content, spectral response, etc, and taking into account the characteristics of the structures to be analysed.

Peak ground acceleration, $A$, has been customarily taken as a measure of the damage potential of strong motion. The reasons for that are probably the association between acceleration and inertial forces, which appeals to structural engineers, and the simplicity to obtain this parameter directly from the record. However, peak ground velocity, $V$, seems to be a more reliable parameter, especially for engineered structures with periods of 0.5 s and longer [H14, N10]. In this range of periods, velocity is the parameter which controls the response (region of conservation of energy).

The examination of the response spectra, expressed in terms of acceleration, velocity and displacement, also contributes to an adequate selection of the earthquakes records. The shape of the spectral curves, and not only the maximum values, indicates the range of periods more significantly affected by the earthquake.

The varying frequency content of the ground motion is recognized as an adequate parameter to take into account for the classification of different earthquakes. Heidebrecht [H14] pointed out that the ratio $A/V$ is a simple but good measurement of the frequency content of the earthquake. When $A$ is expressed as decimal percentage of the gravity and $V$ in m/s, the ratio $A/V$ for typical strong motion records varies from 0.7 to 1.3. Values as low as 0.3 can be obtained from records with significant content of low frequencies, typically recorded at the surface of soft soils. Intraplate earthquakes, which are rich in high frequencies, normally exhibit a ratio $A/V$ between 2.0 and 3.0. Zhu et al. [Z6] classified a large number of earthquake records in three groups according to the ratio $A/V$, and found that the ductility demands of one-degree-of-freedom systems were similar for each group.

The characteristic period of the ground motion, $T_g$, is also an important parameter to be considered. According to the theory equal displacement, when the natural period of a structure is longer than $T_g$ the maximum inelastic displacement is approximately equal to the elastic response of a system with the same initial stiffness. However, there is no general agreement in the procedure to calculate $T_g$. Newmark and Hall (as reported by Miranda and Bertero [M34]) proposed the following expression to calculate the characteristic period:

$$ T_g = 2 \pi \frac{\phi_s \ V}{\phi_s \ A} \quad (12.1) $$
where \( V \) and \( A \) are the peak ground velocity and acceleration, respectively, and \( \phi_v \) and \( \phi_a \) are amplification factors, which applied to \( V \) and \( A \) give the ordinates of the elastic design spectrum in the velocity and acceleration spectral regions, respectively. Values of the amplification factors calculated for different researchers [M34, N10, R15] indicate that the ratio \( \phi_v/\phi_a \) varies between 0.67 and 0.80. Therefore, an average value of 0.75 is assumed here to calculate the characteristic period \( T_g \) based on Eq. 12.1, which leads to:

\[
T_g = 4.71 \frac{V}{A}
\]  

(12.2)

Miranda and Bertero [M34] proposed that the characteristic period \( T_g \) can be defined as the period at which the velocity spectrum of a 5\% damped linear oscillator is maximum. This definition leads to values of \( T_g \) equal to or higher than those obtained from Eq. 12.1 and it is adopted in this study as a reference parameter to normalize the results presented in section 12.3. Other criteria, which lead to similar results, has been proposed on the basis of energy consideration [Q1, S46]. The values of \( T_g \) obtained from the different criteria described above are usually significantly higher than the period corresponding to the peak of the elastic acceleration spectrum.

Finally, it is worth noting that rupture directivity effects have a strong influence on the ground motion. This effect, which is manifested by large long-period pulses of motion, occurs when the rupture front propagates towards the site, and the direction of the slip of the fault is aligned with the site, resulting by the coincidence of the maximum in the radiation pattern of the tangential motion [S47]. These conditions are frequently met in strike-slip faulting. Soil conditions and topographic amplification also influence the ground motion and should be considered in the selection of the earthquakes.

Five earthquake accelerograms recorded in different parts of the world were used in this study (see Fig. 12.1). Tables 12.1 and 12.2 describe the general characteristics of the earthquakes records, whereas Fig. 12.2 shows the elastic response spectra, in terms of displacement, velocity and acceleration, for a damping ratio of 5\%. These earthquakes records were selected based on the following reasons:

- **Llolleo, 1985 (Chile):** the main features of this accelerogram, recorded near the epicentre of the earthquake, are long duration of the strong motion and high peak ground velocity.

- **San Juan, 1977 (Argentina):** this is also a record with long duration of the strong motion. Even thought the peak ground acceleration and velocity are not very high, the spectral velocity remains approximately constant for periods between 0.4 and 3.0 s, which indicates a damage potential in a wide range of periods.

- **Bucharest, 1977 (Romania):** this record was selected due to its large acceleration pulse and the low A/V ratio. Furthermore, the maximum spectral response occurs in the range of intermediate natural periods of vibration.
Pacoima Dam, 1971 (United States): this accelerogram exhibits large peak ground parameters, with high frequency content. There is evidence that rupture directivity, as well as topographic amplification, give long-period pulses of motion [S47]. The record is regarded now as a strong motion representative of severe near-field earthquakes [J1].

El Centro, 1940 (United States): this record represented for many years the largest registered ground acceleration and velocity, and it has been extensively used in dynamic studies and in the development of seismic codes. However, it is considered now as being not very typical of an earthquake excitation.

Figure 12.1. Earthquake records considered as input ground motion in the parametric study.
Figure 12.2. Elastic response spectra for the earthquake records (damping ratio is 5%).
### Table 12.1. General description of the earthquake records

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Site</th>
<th>Date</th>
<th>Component</th>
<th>Focal depth (km)</th>
<th>Epicentral distance (km)</th>
<th>Duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valparaiso, Chile</td>
<td>Lloloe</td>
<td>3/3/85</td>
<td>N10E</td>
<td>33</td>
<td>--</td>
<td>83.2</td>
</tr>
<tr>
<td>Caucete, Argentina</td>
<td>San Juan</td>
<td>23/11/77</td>
<td>E-W</td>
<td>13</td>
<td>80</td>
<td>69.7</td>
</tr>
<tr>
<td>Romania</td>
<td>Bucharest</td>
<td>4/3/77</td>
<td>N-S</td>
<td>110</td>
<td>165</td>
<td>16.2</td>
</tr>
<tr>
<td>San Fernando, California</td>
<td>Pacoima Dam</td>
<td>9/2/71</td>
<td></td>
<td>254</td>
<td>13</td>
<td>41.8</td>
</tr>
<tr>
<td>Imperial Valley, California</td>
<td>El Centro</td>
<td>18/5/40</td>
<td>N-S</td>
<td>16</td>
<td>9</td>
<td>20.1</td>
</tr>
</tbody>
</table>

### Table 12.2. Earthquake parameters

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Magnitude</th>
<th>Peak motion</th>
<th>Ratio</th>
<th>Period</th>
<th>$T_s$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acceler. (g)</td>
<td>Velocity (m/s)</td>
<td>Displac. (m)</td>
<td>A/V</td>
<td>[a]</td>
</tr>
<tr>
<td>Valparaiso, Chile</td>
<td>$M_s = 7.8$</td>
<td>0.66</td>
<td>0.90</td>
<td>----</td>
<td>0.75</td>
</tr>
<tr>
<td>Caucete, Argentina</td>
<td>$M_s = 7.4$</td>
<td>0.19</td>
<td>0.20</td>
<td>----</td>
<td>0.95</td>
</tr>
<tr>
<td>Romania</td>
<td>$M_s = 7.2$</td>
<td>0.21</td>
<td>0.73</td>
<td>0.20</td>
<td>0.29</td>
</tr>
<tr>
<td>San Fernando, California</td>
<td>$M_s = 6.4$</td>
<td>1.25</td>
<td>0.53</td>
<td>0.38</td>
<td>2.35</td>
</tr>
<tr>
<td></td>
<td>$M_s = 6.6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imperial Valley, California</td>
<td>$M_s = 6.4$</td>
<td>0.35</td>
<td>0.38</td>
<td>0.11</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>$M_s = 7.1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[a]: defined according to Eq. 12.2.  
[b]: period at which the velocity spectrum of a 5% damped linear system is maximum [M34].

It is worth noting that procedures normally used to correct the earthquake records may lead to initial values for the ground motion, because some part of the motion is lost since a certain input level is required to trigger the recording device. Consequently, these initial conditions should be considered in the nonlinear dynamic analyses, otherwise distortions of the response can occur in the high period range [R15]. Since the initial conditions of the records employed in this study were unknown, it was assumed zero initial velocity and displacement to perform the analyses. This simplification seems to be valid because the study is focussed on structures in the short to medium period range.
12.2.2 Structural Model

Nonlinear dynamic analyses of one-degree-of-freedom systems (see Fig. 12.3) were conducted with the computer program RUAUMOKO [C16]. The mass of the system (corresponding to a weight $W = 5000$ kN) was constant in all the analyses and the initial natural period of vibration, $T_o$, was changed by considering different values of the initial stiffness, $K_o$. The periods $T_o$ considered in this parametric study were 0.1 to 1.0 (with intervals of 0.1), 1.2, 1.5 and 2.0 s. For the case of earthquake records with large characteristic periods $T_p$ such as Liolio, 1985, Bucharest, 1977 or El Centro, 1940, values of 2.5 and 3.0 s were also included to obtain a similar range of ratios $T_p/T_o$ in all the cases.

![Figure 12.3. One-degree-of-freedom system subjected to ground motions.](image)

Rayleigh damping, in which the damping results from a combination of the mass and stiffness, was adopted in the mathematical model. It was assumed in the application of this model that the damping was proportional to the tangent stiffness. Otani [O3] pointed out that, as the tangent stiffness degrades with damage, the damping associated with the mass leads to an apparent increase of the damping ratio, whereas the damping related to the tangent stiffness decreases. As a result, the damping ratio in the structure tends to be constant when both damping mechanism are combined, as indicated by Carr [C16]. In the parametric study, a damping ratio of 5% of the critical damping was adopted, considering that this is the normal value specified by seismic codes.

The hysteretic behaviour of the lateral force-lateral displacement relationship was represented according to two different models, namely, elasto-plastic and Stewart's model. In the first case, the model is defined by the initial stiffness, $K_o$, and the yield strength, $V_y$. The hysteretic model proposed by Stewart [S45], which was originally developed for plywood sheathed walls, represents adequately the pinching effect in the cyclic response observed in infilled frames. The resistance envelope is represented by a trilinear curve, which is used in this case to define the cracking and yielding point. Pinching is mainly controlled by the force $V_i$ at which the reloading curves intercept the axis of zero displacement. Fig. 12.4 shows the general characteristics of this hysteretic model and indicates the values of the various parameters required in the analyses. The post-yielding stiffness was assumed to be $0.001K_o$ in both models, considering that strength hardening is not normally observed in infilled frames.
In order to compensate the difference of the five earthquake records used in the parametric study, the yield strength of the system, $V_y$, was defined as:

$$V_y = \frac{V_e}{R}$$  \hspace{1cm} (12.3)

where $V_e$ is the maximum base shear developed in an elastic oscillator having the same initial natural period $T_o$ and subjected to the same earthquake record as the nonlinear oscillator, and $R$ is a strength reduction factor. The analyses were conducted for values of $R$ equal to 2 and 4, which were adopted considering that infilled frames are usually designed as structures with limited ductility.

The numerical integration of the dynamic equation of equilibrium was conducted using the Newmark Constant Average Acceleration method. The time step used in the integration process was 0.005 s and the total duration of the analysis was equal to the duration of the earthquake plus 5.0 s.

Results obtained from dynamic analyses using different earthquakes cannot be compared on an absolute basis because the ground motions differ from each other. For this reason, it is customary to scale the earthquake records to some predetermined parameter, such as peak ground acceleration or velocity [R15], or to fit a particular design spectrum. A preliminary study conducted as part of this research project showed that the ratio of the maximum displacements obtained from different hysteretic models changed when the accelerograms were scaled. As a result of this problem and considering the objectives of the parametric study, the dynamic analyses were carried out using the original ground motion records, without applying any normalization procedure. In order to make meaningful comparisons, results are presented in the next sections in the form of displacement ratios.
12.3 RESULTS
12.3.1 Verification of the Concept of Equal Displacement

In order to verify the concept of equal displacement, the maximum displacements in the inelastic system, $\Delta_{ep}$ and $\Delta_p$, for the elasto-plastic and pinched model, respectively, are compared with the maximum elastic displacement, $\Delta_e$. Results obtained from the parametric study are presented in Fig. 12.5 as a function of the initial period $T_o$, normalized using the characteristic period $T_e$ of each earthquake record (see Table 12.2). According to the concept of equal displacement, the ratios $\Delta_{ep}/\Delta_e$ and $\Delta_p/\Delta_e$ should be equal to 1.0 in the range of periods $T_o/T_e > 1.0$. The maximum values corresponding to periods of 0.1 and 0.2 s, as high as 100, are not included in Figs. 12.5 and 12.6 in order to emphasize the range of periods in which this theory is valid. The use of logarithmic scales has been deliberately avoided, considering that this type of scale may confuse the interpretation of the results because significant changes appear to be just small differences.

Results obtained for a strength reduction factors $R = 2$ (Figs. 12.5 (a) and (b)) indicate that the concept of equal displacement is satisfied in a general sense, independent of the nonlinear model considered in the analyses. Only a few cases exhibited displacement ratios $\Delta_{ep}/\Delta_e$ and $\Delta_p/\Delta_e$ greater than 1.0 in the range $T_o/T_e > 1.0$, with values up to 1.8, for some of the earthquakes considered in the study. For systems with smaller yield strength, $R = 4$ (Figs. 12.5 (c) and (d)), the differences between the maximum nonlinear and elastic displacements are significant for the Pacoima Dam, 1977 and San Juan, 1977 excitations. From the design point of view, this observation indicates that the ductility demand implied in seismic codes based on the concept of equal displacement can be exceeded greatly if certain type of earthquakes occur.

The large ratios $\Delta_{ep}/\Delta_e$ and $\Delta_p/\Delta_e$ obtained from the San Juan, 1977 record could be explained considering the particular characteristics of the velocity spectra (see Fig. 12.2), which exhibit a maximum value at 0.5 s and another peak very similar to the maximum at 2.05 s. Furthermore, a significant increase in the displacement spectral response occurs between 1.5 and 2.0 s.

Fig. 12.6 presents the same results illustrated in Fig. 12.5, although the natural period of vibration, $T_o$, is the variable plotted along the horizontal axis. In this way, it is possible to draw some conclusions independent of the characteristic period $T_e$, considering that this information is normally not available for the designers when applying code procedures. The results presented here show that, for the earthquakes considered in this study, the concept of equal displacement holds for periods greater than 2.0 s. Consequently, the theory is not valid over the range of periods that the design codes usually assume it to be. A similar conclusion was also reported by Moss et al. [M35].

12.3.2 Effect of the Shape of the Hysteresis Loops

Fig. 12.7 shows the results obtained from the parametric study considering the ratio of the maximum displacements corresponding to the pinched and elasto-plastic model, $\Delta_p/\Delta_{ep}$, with the objective of investigating the effect of the shape of the hysteresis loops on the seismic response. The variation of this displacement ratio, as a function of the period $T_o$, shows a similar tendency to that observed in Fig. 12.6. The influence of hysteretic pinching is significant for structures with natural periods smaller than 1.0 s. This conclusion seems to contradict previous results obtained by Stewart [S45], who reported that the response of the elasto-plastic system, expressed in terms of ductility demand, was very similar to that of the pinched system.
Figure 12.5. Variation of the ratios $\frac{\Delta_{EP}}{\Delta_{E}}$ and $\frac{\Delta_{P}}{\Delta_{E}}$ as a function of the normalized period, for systems with 5% critical damping.
Figure 12.6. Comparison of the maximum displacements for nonlinear and elastic systems with 5% critical damping.
Figure 12.7. Comparison of the maximum displacements for different nonlinear models with 5% damping.

The different behaviour observed in this study can be explained on the ground of the damping model, considering that Stewart assumed damping proportional to the initial stiffness (this aspect is discussed in more detail in section 12.3.4). Furthermore, the minimum period considered in Stewart's study was 0.3 s. The comparison of results corresponding to different strength reduction factors indicates that, in the range of short periods, the differences are larger for system with $R = 2$, whereas for medium periods this situation occurs for systems with $R = 4$.

The inspection of the hysteresis curves, of which Fig. 12.8 shows some examples, indicates that the pinched model is usually able to recover in the opposite direction after the imposition of a large pulse of ground motion, with a good balance between positive and negative deformations. On the contrary, the elasto-plastic model commonly leads to a one-sided behaviour, with large permanent deformations after the end of the earthquake. This situation is clearly seen in Fig. 12.8 (a), where the final displacements are 0.5 and 88.0 mm for the pinched and the elasto-plastic system, respectively.
The response of a particular system to earthquake excitations depends on a complex interaction between the hysteretic behaviour, the characteristics of the ground motion and the damping model. It is observed in some cases that a slight change in one of the variables may result in an entirely different response. For this reason, it is difficult to obtain general conclusions which allow a precise interpretation of the results. As part of the process intended to a conceptual understanding of the problem, the variation of the energy components, namely, damping, kinetic and strain energy, was analysed for several cases. Four examples of these analyses are illustrated in Fig. 12.9, corresponding to different oscillators subjected to the same earthquake record. These graphs represent the total applied work and the energy dissipated by damping and hysteretic behaviour. The difference between the lines representing the applied work and the damping energy is due to kinematic and elastic strain energy.
In a general sense, the hysteretic behaviour is the most efficient mechanism to dissipate energy, even for pinched systems. The effect of damping is less important for short periods, because large ductilities develop in this case and most of the energy is dissipated through hysteretic behaviour. In those cases where the maximum nonlinear displacements are similar (see Fig. 12.9 (b)), the total applied work and the hysteretic energy in the elasto-plastic system are usually higher than those in the pinched system, in order to accommodate the same displacement with different loop areas. In the short period range, the dynamic behaviour is more erratic, indicating that the system is more sensitive to the effect of the different parameters.

12.3.3 Ductility Demand
The displacement ductility ratio, $\mu$, defined as the ratio of the maximum to the yield displacement, is evaluated for all the cases considered in the parametric study. For systems represented with the pinched model, an equivalent yield displacement, $\Delta^*_{\nu}$, has to be adopted because this model exhibits a trilinear envelope. Following the criterion usually accepted in experimental research, the yield displacement $\Delta^*_{\nu}$, is defined at the intersection point between the yield strength level, $V_{\nu}$, and the secant stiffness associated with
a force of 0.75 $V_y$ (see Fig. 12.10). The displacement $\Delta_{75}$, corresponding to the force level of 0.75 $V_y$, is given by:

$$
\Delta_{75} = \frac{V_c}{K_o} + \frac{0.05 V_y}{0.2 K_o} = 0.95 \frac{V_y}{K_o} \quad (12.4)
$$

The equivalent yield displacement, therefore, can be obtained by linear interpolation as:

$$
\Delta^*_y = \frac{4}{3} \Delta_{75} = 1.27 \frac{V_y}{K_o} \quad (12.5)
$$

The displacement ductility factors, normalized using the strength reduction factor $R$, are plotted in Fig. 12.11. The use of this ratio allows a more concise presentation of the results and also a better interpretation according to the objectives of the study, considering that the ratio $\mu/R$ should be equal to 1.0 in the range of periods where the concept of equal displacement is valid. Excepting the Bucharest, 1977 record, the variation of the $\mu/R$ curves is rather divergent, with numerous cases in which the values are significantly larger than 1.0. The maximum value usually occurs for elasto-plastic oscillators and for the systems with strength reduction factors $R = 4$. As discussed in section 12.3.1, these results indicate that the ductility demand expected to develop under certain earthquakes can significantly exceed the values implied in seismic codes based on the concept of equal displacement.

![Diagram](image-url)

**Figure 12.10.** Definition of the equivalent yield displacement $\Delta^*_y$ for the pinched model.
Figure 12.11. Ratio $\mu_\Delta / R$ for oscillators subjected to different earthquakes.
Figure 12.11. (Cont.) Ratio $\mu_A / R$ for oscillators subjected to different earthquakes.

It can be argued that the procedure applied in this study to assign the yield strength of the oscillator did not follow the requirements of any seismic code. The application of a particular design spectrum would have led to different yield strengths. In this case, however, each earthquake record should have been scaled to fit the design spectrum, which could modify the ductility demands. The verification of the requirements of a particular seismic code is outside the scope of this work, even though it is strongly recommended. Another limitation of this study is that the strength reduction factors were assumed to be constant, which is approximately the case of the New Zealand's Code of Practice for General Structural Design and Design Loadings for Buildings [N11]. Other seismic codes usually consider that the strength reduction factor is a function not only of the ductility but also of the period of the structure, leading to smaller factors in the range of short periods where the concept of equal displacement does not hold.
12.3.4 Influence of the Damping Model

It was assumed in the previous nonlinear analyses that the damping is proportional to the tangent stiffness (see section 12.2.2). In order to investigate the effect of the damping model on the response, a number of cases were analysed with a different model in which the initial stiffness is considered in the damping model. The single oscillator described in section 12.2.2, with natural periods between 0.1 and 2.0 s, was subjected to three different strong motions, San Juan, 1977, El Centro, 1940 and Bucharest, 1977.

Results are presented in Fig. 12.12 by comparing the maximum nonlinear displacements obtained from the damping model proportional to the tangent stiffness, $\Delta_{\text{tan}}$, and to the initial stiffness, $\Delta_{\text{in}}$. It can be seen that the effect of damping model on the inelastic response becomes very important in the range of short periods, being $\Delta_{\text{tan}}$ significantly higher than $\Delta_{\text{in}}$. The reason for this consistent difference is that the damping model proportional to the tangent stiffness reduces the energy dissipated by damping, leading to an increase of the energy dissipated by plastic work and, thus, of the maximum displacement. It is observed in Fig. 12.12 that the ratio $\Delta_{\text{tan}}/\Delta_{\text{in}}$ markedly decreases as the period $T_o$ augments and then increases slightly. The larger differences between the maximum response for both damping models occur for the highest strength reduction factor ($R=4$, black circles and triangles) and for the pinched model (dashed lines).

Since the damping model can markedly affect the response, it is instructive to compare again the maximum displacements $\Delta_p$ and $\Delta_{\text{EP}}$ corresponding to systems with damping proportional to the initial stiffness. This comparison is presented in Fig. 12.13, where it can be observed that the values of the ratio $\Delta_p/\Delta_{\text{EP}}$ are significantly smaller than those plotted in Fig. 12.7 for systems with tangent stiffness damping. Only for very short periods both damping models lead to high $\Delta_p/\Delta_{\text{EP}}$ ratios. Therefore, the different conclusions related to the influence of pinching obtained from previous research work (see section 12.3.2) can be chiefly attributed to the effect of the damping model.

![Figure 12.12](image)

**Figure 12.12.** Comparison of the maximum nonlinear displacements for different damping models.
The use of tangent stiffness damping seems to be more appropriate because it considers the variations of the stiffness with the cyclic loading. However, it must be recognized that any type of viscous damping model is traditionally adopted on the grounds of mathematical convenience rather than physical considerations. Numerous uncertainties still remain and more research, especially that based on experimental work, is required to clarify this aspect of the analysis. The results presented here draw attention about the significant differences in the response that the damping model may produce, particularly for infilled frames which are structures with short to medium periods of vibration.

12.4 CONCLUSIONS

- A parametric study was conducted to investigate the validity of the concept of equal displacement and the effect of pinching in the structural response. Five strong motions, recorded in different earthquakes, were used in the dynamic analyses of simple oscillators, having one degree of freedom. The force-displacement relationship was represented with elasto-plastic and pinched hysteretic models and the damping coefficient was 5%.

- Results obtained from dynamic analyses confirm that the concept of equal displacement generally holds for structures with initial periods greater than the characteristic period of the earthquake $T_x$. However, some exceptions to this theory are observed for certain types of earthquakes, especially when the strength reduction factor is increased. Additional research is needed to identify the characteristics of the ground motion that lead to this situation.

- For infilled frames, which normally exhibit short initial periods, say below 1.5 s, the concept of equal displacement appears to be not applicable.
The effect of pinched hysteresis loops has a strong influence on the response of short period structures when tangent stiffness damping is used. This effect is less significant for systems with initial stiffness damping.

The ductility demands imposed by certain ground motions can be significantly larger than those expected according to seismic codes based on the concept of equal displacement, suggesting that the evaluation of the strength reduction factors for ductile response should be revised.

Special caution is required for the nonlinear dynamic analysis of infilled frames, considering that this type of structures normally has a short to medium initial period of vibration. The effect of pinching and the damping model can significantly affect the response to ground motions.
13. CONCLUSIONS AND RECOMMENDATIONS

13.1 GENERAL
A study of the seismic behaviour of reinforced concrete frames infilled with masonry panels has been carried out, which considered primarily the behaviour, analysis and design of this type of structure. Acknowledging that the masonry panels strongly affect the response of the infilled frames, the basic properties of masonry and its constitutive materials were also investigated, and reliable models for the assessment of the strength were developed.

The conclusions drawn as a result of the theoretical and experimental work carried out in this thesis have been given at the end of each chapter. A summary of these conclusions and recommendations for further research are presented in the following sections. Since an extensive revision of previous research related to the behaviour of masonry and infilled frames was conducted, the conclusions obtained from the literature review are presented separately.

13.2 CONCLUSIONS BASED ON THE LITERATURE REVIEW
13.2.1 Behaviour of Masonry and Constitutive Materials
The behaviour of masonry and the constitutive materials were reviewed, giving special emphasis to those aspects which contribute to a better understanding of the strength mechanism. Due to the wide range of mortars and masonry units commonly used, masonry exhibits a large variation in its mechanical properties and behaviour, which makes it difficult to obtain general conclusions and to calibrate analytical models.

As a result of extensive research conducted in the last decades, numerous empirical expressions have been proposed for the evaluation of different mechanical properties of masonry and its constitutive materials. The comparison of these expressions shows a wide scatter of the results. Consequently, they should be applied carefully taking into account the materials and conditions used when these equations were derived.

Various testing procedures are employed for the investigation of the shear strength of masonry. This situation leads to experimental results which usually are not comparable due to different stress distributions and boundary conditions. It is required, therefore, that standard procedures which represent adequately the conditions of masonry walls be defined.

Failure criteria for masonry units subjected to biaxial tension-compression and for confined mortar were discussed, considering that these criteria are required for the development of theoretical procedures for the evaluation of the compressive strength of masonry. Some of the failure criteria proposed for masonry units under a tension-compression stress state should be revised using more realistic testing techniques because they are not compatible with the general biaxial envelope observed in fragile materials.
Various theories have been developed with the objective of determining the compressive and shear strengths of masonry panels, based on the properties of the mortar and the masonry units. These theories assume different hypotheses and criteria, and occasionally lead to completely different results.

13.2.2 Behaviour of Infilled Frames
A large amount of experimental work was reviewed and it was noted that results obtained by distinct researchers may lead to rather different conclusions. Numerous parameters can affect the structural response and usually more than one variable is changed from one test to the other. Furthermore, different dimensions, construction techniques, infill materials and interface conditions were used to build the specimens. For these reasons, it is difficult to evaluate the role of each separate parameter and to understand its influence in the structural response.

The presence of the masonry panels strongly affects the structural response of the infilled frames and unfavourable consequences may occur when the influence of these panels is neglected. These possible consequences are: (i) the modification of the global response due to the decrease of the natural period of the vibration, (ii) the unexpected modes of failure of the surrounding frame, (iii) the alteration of the torsional response of the building, and (iv) the formation of a soft-storey sway mechanism caused by a non-uniform distribution of the masonry panels. Therefore, it is very important to recognize that infilled frames are a different type of structural system, which require specific procedures for design and analysis.

The structural response of infilled frames subjected to lateral forces are markedly nonlinear. Four different stages may be usually distinguished. Initially, the structure behaves like a cantilever beam until separation between the frame and the panel occurs. In this stage, the structural behaviour can be explained by a braced frame mechanism. A further increase of the lateral forces produces cracking of the masonry panel following different patterns. Finally, the strength and the stiffness significantly degrade and the structure collapses. When the infilled frames are subjected to cyclic or dynamic loading, the hysteresis loops normally exhibit severe pinching.

The failure of infilled frames results from a combination of simple failure mechanisms that may occur in both the masonry panel and the surrounding frame. A comprehensive classification of these modes of failure was presented in order to facilitate a clear understanding of the behaviour of infilled frames. Failure of the reinforced concrete boundary frame can be produced by axial or shear forces acting on the columns or due to a flexural collapse mechanism. In the masonry panel, shear cracking, either along the mortar joints or through the masonry units, is the most common type of failure. However, compressive failure or flexural cracking may also occur.

13.2.3 Analysis of Infilled Frames
Various procedures used for the analysis of infilled frames were reviewed and their advantages and disadvantages were assessed. These procedures can be categorized in two main groups, namely, macro-models and micro-models. The diagonal strut model and the finite element formulation are typical examples of the first and second group, respectively. Macro-models are based on a physical representation of the observed behaviour of infilled frames, having significant advantages in terms of simplicity and computational efficiency. On the other hand, micro-models are able to represent the structural behaviour in great detail,
but with a notable increase in the complexity of the analysis. The response is very sensitive to the different structural parameters involved in the problem, especially in nonlinear analyses when the effect of cracking needs to be considered.

Several expressions obtained from empirical and rational considerations have been proposed for the evaluation of the strength of infilled frames, taking into account different modes of failure. These simple expressions are useful for design purposes, although they may lead to significantly different results in some cases.

13.3 CONCLUSIONS DRAWN FROM THIS PROJECT

13.3.1 Masonry Materials

Current standard procedures for the evaluation of the compressive strength of masonry units are not able to predict the real uniaxial strength. A new testing technique is proposed using a stack of three masonry units with cork sheets in between. In this way, it is possible to reduce significantly the confinement effect produced by friction between the loading platens and the specimen without inducing additional lateral stresses in the masonry units. The proposed technique was experimentally verified with a series of compressive tests of solid concrete bricks.

Based on experimental results obtained from previous research, a mathematical model was developed to represent the mechanical properties of mortar, namely, the tangent modulus and Poisson's ratio, when it is subjected to biaxial compression.

13.3.2 Evaluation of the Masonry Strength

Hilsdorf's theory [H10] provides an adequate explanation for the understanding of the behaviour of masonry in compression. This theory, however, suffers from some conceptual mistakes and normally underestimates the compressive strength. For these reasons, a modified procedure was developed in this thesis to consider a rational distribution of the lateral stresses and a more realistic representation of the constitutive materials. Furthermore, a nonlinear procedure was implemented to predict not only the compressive strength, but also the complete stress-strain path up to failure. In this procedure, the compressive stress is applied in successive increments, taking into account, at each step, the variation of the mechanical properties of the mortar and masonry units. Theoretical results obtained from both procedures agree well with the experimental data measured during three different series of compressive tests.

The shear failure theory proposed by Mann and Müller [M2] is a rational approach based on equilibrium conditions, which allows the representation of the observed modes of failure, namely, shear-friction, diagonal tension and compressive failure. The failure envelope gives an adequate estimation of the shear strength of masonry panels subjected to a combined state of shear and compressive stresses. This theory was modified in this thesis to consider a more realistic distribution of the normal and shear stresses acting on the masonry unit, based on a parametric study conducted with a finite element analysis.

13.3.3 Analysis of Infilled Frames

A preliminary study was conducted to investigate the limitations of the single strut model and to compare the response with more refined procedures, such as multi-struts models or finite element analysis. It is
concluded that the single strut model, despite its simplicity, gives an adequate estimation of the stiffness of the infilled frame and the axial forces induced in the frame members by lateral forces, and represents an adequate procedure for simplified analyses focussed on the overall response of the structure. However, a more refined model is required in order to obtain realistic values of the bending moments and shear forces in the surrounding frame member.

Two theoretical procedures, with different degree of refinement, are proposed in this study for the analysis of infilled frames. The first procedure is a simple approach, based on the equivalent truss mechanism, which allows the evaluation of the lateral resistance of the infilled frames, considering two different types of failure in the masonry panel, namely, shear-friction and diagonal tension failure. The compressive strength of the diagonal strut is assessed by transforming the shear failure envelope obtained from the modification of the Mann and Müller's theory [M2], which was developed earlier in this thesis. This transformation takes into account the inclination of the diagonal strut and neglects the effect of the tensile principal stresses acting on the masonry panel. The proposed procedure is suitable for the design of infilled frames or for the assessment of existing structures. However, it can be also applied for the dynamic nonlinear analysis of infilled frames when the objective is to investigate the global response of the structure. The comparison of theoretical results and experimental data obtained from the test programme showed a good agreement.

The second procedure is a refined macroscopic model based on a multi-strut formulation, which is intended to represent more accurately the effect of the masonry panel on the surrounding frame. Since debonding of the mortar joints is the most common type of failure observed in the masonry panel, the formulation of the procedure is specifically developed to represent this situation. The model accounts separately for the compressive and shear behaviour of masonry using a double truss mechanism and a shear spring in each direction. This concept is implemented in a 4-node panel element which is being incorporated in the computer program RUAUMOKO. Recommendations are also given for the analysis of infilled frames when a failure due to diagonal tension or crushing of the corners is expected in the panel.

In order to conduct nonlinear dynamic analysis based on the proposed model, a new hysteresis rule aimed at representing the cyclic behaviour of the masonry struts was implemented in the computer program RUAUMOKO. In this hysteretic model, the strength envelope in compression is defined according to Sargin's equation [S31], whereas the behaviour in tension is assumed to be linearly elastic. The unloading-reloading curves are represented with a general expression which pass through two predefined points where the slope of the curve is also imposed. Contact effects in the cracked material are included in the model considering that the compressive stresses can be transferred as the cracks gradually close. The proposed algorithm is general enough to represent the behaviour of different fragile materials and can be simply modified to fit new experimental results. Special emphasis is given to consider properly the effect of small cycle hysteresis in order to represent the response under earthquake induced actions. The model was calibrated using experimental results obtained from masonry and concrete specimens tested by the author and other researchers.

13.3.4 Test Programme
Two framed masonry units, with a single infill panel, were constructed to a reduced scale of 3/4 full size using solid concrete bricks. The dimensions of the masonry panel and the reinforced concrete frame were
selected according to the customary practice for this type of structure. The main criteria followed for the design were that the reinforced concrete columns should yield in tension, as a result of the axial forces induced by the equivalent truss mechanism, and that the tensile strength of the top beam should be greater than the applied lateral force to avoid yielding of the longitudinal reinforcement. New reinforcing details were provided in Unit 2, aimed at enhancing the structural response. These details consisted in the use of tapered beam-column joints with diagonal reinforcement, and additional longitudinal reinforcement in the frame members. The additional bars placed in the columns were not anchored to the foundation in order to produce a weak region at the base of the columns, where most of the plastic deformations were expected to occur.

The units were tested under quasistatic cyclic lateral forces, following a loading history in which the imposed lateral displacement was gradually increased up to a storey drift of 1.5% and 2.0%, for Units 1 and 2, respectively. In this experimental programme, the lateral forces were applied by pulling the top beam at 0.5 m from the ends, with the aim of representing the resultant of the shear force transmitted by the floor slab and from an upper storey. Vertical forces were also applied in the columns to represent the effect of gravity loads and to introduce additional overturning moment. In this way, the forces acting on the units simulated those corresponding to a two-storey structure. Special emphasis was given to use a realistic procedure for the application of the lateral forces, under the conviction that customary experimental loading systems do not represent properly the conditions in actual structures. In most of the previous quasistatic tests, two hydraulic actuators have been used to push alternatively the external face of the beam-columns joints. Theoretical results obtained from finite element analysis showed that, when this loading system is used, the axial forces in the top beam and the variation of the bending moments and shear forces in the columns are affected markedly. In addition, the stress state in the beam-column joints is predominantly compressive, resulting in an enhancement of the joint strength.

The test of Unit 1 showed an unexpected mode of failure. Cracking along the mortar joint occurred mainly in the upper region of the masonry panel, and a large shear crack formed at the top of each column. In the final stages of the test, the lateral resistance of the unit was mainly controlled by dowel action in the longitudinal reinforcement crossing this crack. The lateral strength of Unit 1 was half of that predicted in the design, and yielding of the longitudinal reinforcement of the columns did not occur, except in a reduced region close to the ends of the member. Measurements taken during the test indicate that the top beam was subjected to tensile axial forces independently of the direction of the imposed displacement, which resulted in increasing elongations of this member. In addition, it was observed that the beam-column joints opened significantly as a result of the cracks formed at the faces of the joint core and slip of the longitudinal reinforcement of the beam.

The type of failure of Unit 1 has been observed rarely in previous tests. The main reason for this difference seems to be due to the use of unrealistic loading systems. Consequently, the experimental programmes related to infilled frames should be carefully planned, otherwise important aspects of the behaviour can be modified or altered by the devices used for the application of the lateral forces.

The overall response of Unit 2 was very satisfactory, showing widely distributed cracking of the masonry panel which occurred mainly along the mortar joints with a stepped pattern. Yielding of the longitudinal reinforcement concentrated at the base of the columns, according to a typical "flexural mechanism". Unit
2 exhibited a lateral resistance about 110% higher than that measured for Unit 1, even though both units were built with similar materials and had the same theoretical strength. The equivalent truss mechanism was used to predict the lateral strength, showing a good agreement with the measured value.

Both units exhibited a hysteretic response affected by severe pinching of the loops for storey drifts larger than 0.5%. This was due to the shear cracking of the masonry panels and to sliding shear occurring at the cracked columns. The lateral strength was adequately sustained up to storey drifts of about 0.75% and 1.5%, for Units 1 and 2, respectively. Beyond these levels, a strength degradation of about 20% was observed.

The comparison between measured and analytical values of the stiffness and cracking force, revealed that the width of the masonry strut to be considered in the equivalent truss mechanism for Unit 1 can be taken between 0.15 and 0.25 of the diagonal length. The presence of tapered joints produced an increase of the strut width in Unit 2, with values ranging from 0.25 to 0.35 of the diagonal length. In both cases, smaller width values were required for an adequate evaluation of the cracking force than for the stiffness.

The most important conclusion of the experimental programme is that the response of reinforced concrete frames with masonry infills can be significantly improved by a rational design aimed at reducing the distortion of the masonry panels while plastic deformations are concentrated in selected regions of the structure. The use of tapered beam-column joints with diagonal reinforcement was shown to be very effective in reducing the opening of the joints and enhancing the transfer of the lateral forces from the top beam to the masonry panel.

13.3.5 Design of Infilled Frames
One of the chief objectives of this research project is to propose design recommendations for infilled frames, since these structures are still used in various regions with high seismic activity. Severe damage and loss of life have occurred in infilled frame buildings during past earthquakes, although a reasonably ductile response can be obtained when the structure is properly designed. It is believed that a clear understanding of the structural behaviour and a rational design procedure are the critical issues to be developed in order to obtain a safe and economical solution.

Taking into account the expected response, infilled frames can be categorized in three groups, namely, ductile, elastic and rocking infilled frames. According to the results obtained from the experimental programme and using the principles of capacity design, an adequate behaviour can be achieved by inducing the plastic deformation at the base of the infilled frame, while the remaining part of the structure is prevented from suffering severe damage. This procedure also assures that the lateral resistance can be evaluated according to a simple mechanism, avoiding the uncertainties associated with the complexity of the panel-frame interaction. In other cases, the infilled frame should be designed to remain elastic or to rock on specifically designed foundations.

A new design approach is proposed for infilled frames, in which two cases are considered: cantilever and squat infilled frames. In the first case, the ductile behaviour is achieved by yielding of the longitudinal reinforcement, which is limited to occur only at the base of the columns, and by avoiding large elongations of the remaining parts of the surrounding frame. A pre-cracked connection is induced between the infilled
frame and the foundation, where plain round dowels can be placed to control shear sliding. The use of tapered beam-column joints with diagonal reinforcement is recommended in order to reduce the opening of the joints and to improve the transfer of the lateral forces from the frame to the masonry panel. In the second case, ductility is conferred to the structure by allowing controlled sliding of the infilled frame over the foundation. The applicability of this approach is limited to those cases where the total shear force exceeds the frictional strength of the pre-cracked connection.

The effect of pinching of the hysteresis loops in the response of infilled frames subjected to earthquakes was investigated, since this phenomenon has been observed in most of the laboratory tests reported in the literature. A parametric study was conducted using a one-degree-of-freedom oscillator subjected to ground accelerations recorded in five different earthquakes. The force-displacement relationship was represented with the elasto-plastic and pinched models. Rayleigh damping was considered in the analysis, assuming two different models in which damping was proportional to either the tangent or the initial stiffness. Results obtained from the dynamic nonlinear analyses indicated that the effect of pinching and the damping model used can significantly influence the response of infilled frames, which normally exhibit a short to medium initial period of free vibration. Therefore, the displacement demand imposed by the earthquake can be larger than that assumed by the seismic codes if they are based on the theory of equal displacement. Some unsafe exceptions to the theory of equal displacement were observed, even for structures with initial periods of vibration greater than the characteristic period of the earthquake. This fact could be explained considering that conclusions in previous studies have usually been drawn considering average results from different earthquakes, even though large standard deviations were obtained.

13.4 SUGGESTIONS FOR FUTURE RESEARCH
13.4.1 Masonry
A more complete investigation is necessary to verify the testing technique proposed in this study for the experimental evaluation of the compressive strength of masonry units. Such investigation should consider different types of masonry units in order to probe the generality of the procedure.

The complete behaviour of mortar under biaxial compression, especially the variation of its properties at high stress levels, is not yet clearly understood. Therefore, additional information could be obtained from triaxial compressive tests to improve the theoretical procedures used in the evaluation of the masonry strength. It is also very important to investigate the behaviour of the mortar in the bed joints, considering that the water absorption of the units, water retentivity of the mortar, moisture content and curing conditions may strongly affect the mortar strength in situ.

A systematic investigation of the different procedures used for testing masonry specimens under shear is also recommended. Such investigation should be aimed at proposing a standard testing technique able to represent realistically the conditions in masonry panels.

The modification of Mann and Müller's theory [M2] proposed in this thesis represents a rational procedure which allows the evaluation of the strength of masonry panels subjected to a combined state of shear and compression. Even though this theory was originally verified using test results obtained from different researchers, a more complete comparison is required considering a variety of masonry units and types of
mortar. Emphasis should be given to tests considering low to medium normal stresses, which is the usual situation in masonry panels of infilled frames.

13.4.2 Analysis of Infilled Frames
The use of a macro-model for the analysis of infilled frames represents a simple and adequate solution. This type of model usually expresses in a rational way the complex interaction between the masonry infill and the surrounding frame. However, the practical application of the analytical procedures requires the evaluation of several parameters, such as the width of the equivalent strut, the variation of this width resulting from cracking, the contact length between the frame and the panel, etc. The values of these parameters usually exhibit a large variation and they can be affected by different circumstances which are not completely understood in some cases. More research is required to validate some of the existing empirical expressions and to develop general equations which reflect the physical considerations involved in the problem.

Experimental and analytical results indicate that infilled frames are prone to fail due to sliding shear occurring at the end of the columns. This type of failure is difficult to be modelled using most of the existing analysis procedures. Consequently, reliable models need to be developed for assessing the shear strength of the frame members and for considering the effect of this type of failure in the overall response of the structure.

Results obtained from nonlinear dynamic analyses showed that the damping model has a marked influence on the response of infilled frames subjected to ground motions. Even though extensive theoretical research has been conducted, numerous uncertainties still remain which usually lead researchers and designers to assume "5% of critical damping" without realising about the importance of this aspect of the analysis. Despite the significant advance in analytical procedures and computational algorithms, it is believed that a systematic investigation, based on simple dynamic tests, should be conducted to calibrate the mathematical models.

13.4.3 Design of Infilled Frames
Infilled frames, with both high and low aspect ratios, can be designed in order to obtain a reasonable ductile response. With this aim, a design framework has been proposed in this thesis, based on rational considerations and on the experimental results obtained from Unit 2. It must be recognized, however, that more research is needed to clarify some aspects of the design related to the response of the pre-cracked connections and the amount of additional longitudinal reinforcement to be provided to the frame members. In addition, the verification of the behaviour of multibay and multistorey structures is also required. If the advantages of the proposed design procedure are experimentally corroborated, efforts should be made to incorporate these criteria in seismic codes.

The use of tapered beam-columns joints is proposed in this study to enhance the response of infilled frames. However, only one unit was tested with this detail due to the limitations in both cost and time of the experimental programme. It could be valuable, therefore, to conduct additional research aimed at investigating varied dimensions of the tapered joints and the amount of diagonal reinforcement. Another important issue which requires more investigation is the effect of the tapered joints in increasing the width of the equivalent masonry strut associated with the truss mechanism. Since this mechanism is the basis of
a reliable procedure widely used for the design and the analysis of infilled frames, a parametric study could be conducted to determine the influence of different structural parameters on the equivalent width and to propose expressions for its evaluation.

The concept of equal displacement is a key aspect in the seismic design of structures according to current design standards. Results obtained from dynamic analyses showed that some unconservative exceptions to this theory occur for certain types of earthquakes. As a result, more research is required to identify the characteristics of the ground motions which lead to this situation and to modify the concept of equal displacement, if required, to fit the results. The damping models and the strength reduction factor seem to be important parameters whose effect could be investigated more deeply. When results are interpreted on the basis of statistical analyses, the conclusion should be drawn considering a conservative upper limit, instead of average values.
REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[R5] Richart, F. E. and Brandtaeg, A., A Study of the Failure of Concrete under Combined Compressive Stresses, University of Illinois, Engineering Experiment Station, Bulletin No. 185, November, 1928.


REFERENCES


REFERENCES


APPENDIX 1: EVALUATION OF THE STRESS DISTRIBUTION FACTOR $C_d$

A1.1 FINITE ELEMENT ANALYSES OF MASONRY PRISMS

A1.1.1 Introduction
The stress distribution factors, $C_{ab}$ and $C_{ij}$, introduced in Eqs. 3.30 and 3.32 for the evaluation of the compressive strength $f'_c$, were derived from a parametric study based on finite element models of masonry prisms and individual bricks and mortar joints. The general purpose finite element program ABAQUS [A12] was used. It should be pointed out that the main interest of these analyses was to investigate the stress distribution in the masonry prism, and not the precise values of the stress nor the ultimate load.

A1.1.2 Finite Element Model
The characteristics of the model used in the parametric study were adopted after a series of analyses in which different alternatives were considered:

- **Number of bricks and boundary conditions:** stack-bonded prisms of masonry units formed by 3, 5 and 7 bricks were analysed in order to investigate whether the application of the compressive load affects the stress distribution. It was observed that perturbation of the stress distribution occurred in the zone near to the loaded face, but this disturbing effect disappeared quickly. The stress distribution observed in the central brick and in the neighbouring mortar joint were very similar in all cases. Consequently, a model formed by 3 bricks was adopted. Based on symmetry considerations only one quarter of the stack-bonded prism was analysed. The boundary conditions assumed that there was no translational motion perpendicular to a plane of geometric symmetry (see Fig. A1.1). Consequently, the model is free to expand laterally at the load faces.

- **Type of element:** before selecting a plane element for both materials, linear three-dimensional analyses were conducted using a 8-node brick element. The model was formed by $15 \times 8 \times 5$ elements in order to represent $1/8$ of a rectangular masonry prism. It was observed from the analyses that the lateral stresses were almost constant along the perpendicular direction. In other words, according to the notation of Fig. 3.3, $f_x$ was practically constant along the $z$ direction, and $f_z$ along the $x$ direction. The shape of the stress distribution was similar for both lateral stresses. The observed stress state may not be strictly considered as either plane stress or plane strain state. Even though the stress $f_y$ was not equal to zero, a plane stress, four node, bilinear element was adopted to model the masonry prism in the parametric study, based on the fact that $f_y$ was almost constant along the thickness of the prism.
**Finite element mesh**: two main factors influenced the selection of the nodal mesh. First, the gradient of the lateral stress was high in some parts of the model, particularly at the mortar-brick interface and near the vertical edge of the prism. Second, the algorithm employed to represent the material model of bricks required a refined mesh to avoid numerical and convergence problems after cracking. Preliminary analyses were carried out with $9 \times 6$, $15 \times 8$, $24 \times 10$, $27 \times 10$ and $46 \times 20$ meshes of elements. The finest mesh was adopted, as Fig. A1.1 indicates. The mortar joint was modelled with four horizontal layers of elements to represent adequately the parabolic displacement shape observed in the free edge. This model was named Model A.

**Mortar-brick interface**: the brick-mortar interface was modelled with 4-node linear interface elements, whose mechanical behaviour is described in section 6.3.1.4. The bond and friction strength are expected to be sufficiently large to avoid relative displacements between both materials, when the masonry prism is subjected to compression. The results obtained from this study indicate that sliding did not occur because the frictional strength ($\mu f_s$) at the interface was much higher than the shear stress. A different model was also implemented to study the effect of relative displacement between the surfaces before the strength of the interface is broken. Even though there is no experimental evidence that such a behaviour occurs, it would be very difficult to observe or measure this effect, if it occurred. In this model, interface elements were combined with nonlinear springs linking the horizontal displacement of the interface nodes; the shear strength of the mortar-brick interface was assigned to the springs. The results indicate that relative lateral displacements occur at the interface before its strength is exceeded. The relative
displacements begin at the free edges of the prism and propagate towards the centre. The mortar is "extruded" between the bricks and cracking never occurs. Consequently, this model was not adopted for the parametric study.

A1.1.3 Material Properties

It is difficult to represent accurately the general behaviour of the constitutive materials of masonry. The models used in this study were chosen with the aim of considering the most important aspects of the real behaviour when the bricks are subjected to biaxial tension-compression and the mortar joints to biaxial compression-compression. Despite these simplifications, it is believed that the model is sufficiently accurate to represent the stress distribution in a masonry prism. The material properties were modelled as follows:

- **Mortar:** this material was modelled using the incremental plasticity model available in ABAQUS program [A12]. The model assumes a standard von Mises yield surface with associated plastic flow. The program provides different possibilities to consider work hardening, and in this study, isotropic hardening was adopted (the yield surface changes size uniformly in all directions as plastic straining occurs). Due to the lack of analytical models specifically developed for mortars, the stress-strain relationship for mortar in compression was represented by an expression proposed by Sargin et al. [S31] for concrete (see Appendix 2, Eq. A2.5). This curve was approximated using five linear segments, in order to represent adequately the change of the tangent modulus. Poisson's ratio was assumed constant throughout the load increments, even though experimental results indicate that the Poisson's ratio increases when mortar is compressed (see section 2.4.3). Therefore, the values of the compressive load at failure could be overestimated. The model, however, can represent adequately the distribution of the lateral stresses induced as a consequence of the more deformable behaviour of the mortar.

- **Brick:** the model provided in ABAQUS [A12] for plain concrete was adopted for modelling the brick behaviour. Based on the considerations presented in section 2.3.6, the stress-strain relationship in uniaxial compression was assumed as linearly elastic until the compressive strength is reached. The effect of cracking was represented by using a smeared approach (see section 8.4).

A1.1.4 Procedure and Results

The masonry prisms considered in this study were subjected to compressive loads applied at the top and bottom faces. The static nonlinear analysis was conducted using the modified Riks method [C6, R8, R9] (see section 8.4).

In order to calculate the stress distribution factor \( C_d \) (see Eqs. 3.29 and 3.32), the distribution of the lateral stresses \( f_x \) along a vertical section needs to be evaluated. It is expected that the dimensions of brick and joint, \( b, d, j \), the mechanical properties of the brick, \( f_{ub}, E_b, \nu_b \) and the mechanical properties of the mortar, \( f'_p, E_p, \nu_j \) could affect the response of the model. On the ground of the great number of variables involved in the problem, it would have been very difficult to consider all of them in a parametric study based on nonlinear finite element analyses. Hence, the study was divided into two parts. In the first part, several nonlinear analyses were conducted to understand conceptually the influence of the different variables using the finite
element model described previously, Model A. Results obtained from Model A indicated that the distribution of lateral stresses at the ultimate compressive load was not significantly affected when the mechanical properties of both materials were changed, even though the absolute values of the stresses were strongly influenced. This observation was valid for all the cases in which the mortar was more deformable than the brick. For this reason, in the second part of the study only five cases were analysed considering different geometric ratios, \( b/d \) and \( d/j \). The mechanical properties of both materials were the same in all cases. It is worth noting that the aim of the finite element analyses was to evaluate the variation of the lateral stresses and not to calculate the compressive strength of the masonry. This fact also justifies the assumption that the Poisson's ratio of the mortar remains constant in the nonlinear analysis.

Fig. A1.2. illustrates the contours of the lateral, \( f_{xx} \), and vertical, \( f_{yy} \) stresses, for the case with dimensions \( b = 60 \, \text{mm}, \ d = 120 \, \text{mm} \) and \( j = 10 \, \text{mm} \). These graphs correspond at two different stages of the loading process: (a) and (b) before cracking; (c) and (d) at failure. Only one quarter of the prism is shown due to vertical and horizontal symmetry (see Fig. A1.1). It can be observed in Fig. A1.2 that the vertical stresses are almost constant, although some stress concentration occurs at the brick-mortar interface near the vertical edge of the prism. In this area, large lateral displacements develop in both materials and the mortar is subjected to tension at the edge. It is also evident in Fig. A1.2 that the distribution of the lateral stresses cannot be considered to be uniform. The maximum tensile stress, where the first crack will develop, occurs at the mortar-brick interface near the vertical edge of the prism. Then, the values of the tensile stress decrease rapidly towards the centre of the brick. After the first crack developed, the tension stresses in the cracked zone are released and the stress distribution changes progressively to a more uniform state.

The lateral stress distributions depicted in Fig. A1.2 suggest that splitting of the brick should occur near the vertical edges, where the maximum tensile stresses are induced. Test results of masonry prisms conducted by the author (see Fig. 3.5) and by Noland et al. [N4] clearly confirm this assumption.

Fig. A1.3 shows the lateral stress distribution obtained from a particular case of Model A (the same case depicted in Fig. A1.2), before the first crack occurred and at failure stage. These stress distributions correspond to a vertical section placed at distance 0.46 \( d \) (before first crack) and 0.30 \( d \) (failure stage) from the vertical axis of symmetry (\( d \) is the length of the brick). It is clearly observed that the stress distribution tends to be more uniform at failure, which means that the factors \( C_{b} \) and \( C_{q} \) increase and approach a value of 1.0.

The stress distributions obtained from this study agree well with the results reported by Rots [R11], who analysed a masonry prism subjected to compressive loading. The comparison is valid only in the uncracked stage, because Rots considered an elastic finite element model in his investigation. Despite this fact, Rots’ results confirm the conclusion that the stress distribution in both, bricks and mortar joints, cannot be considered as uniform.
Figure A1.2. Stresses contours obtained from finite element analysis, Model A.

Figure A1.3. Distribution of lateral stresses obtained from finite element analyses.
Ganesan and Ramamurthy [G7] conducted three-dimensional linear finite element analyses of concrete hollow-block masonry prisms. They observed that the maximum lateral tensile stress occurred at the mortar-block interface. In the case of stack-bonded prisms, the distribution of the lateral stresses at the central web was very similar to that presented in Fig. A1.3. However, both studies are not strictly comparable because the presence of the hollows in the masonry units affect the behaviour of the prism, especially for face-shell bedded or running bond prisms.

A1.2  **FINITE ELEMENT ANALYSES OF BRICKS AND MORTAR JOINTS**

Results obtained from nonlinear analyses with Model A indicate that, independent of the properties assumed for the model, the distribution of the shear stress at the mortar-brick interface presented some particular characteristics. Before the first crack occurred, the shear stresses at the interface had an approximately triangular distribution with the maximum at the vertical edge. At failure, the shear stresses were almost constant along the interface mortar-brick interface. Consequently, it was assumed that these limit situations could represent approximately the conditions in the initial stage and at failure. A series of simplified linear elastic models was used to represent individual bricks and mortar joints, in which shear forces were applied at the horizontal faces to simulate the effect of the composite interaction between bricks and joints in the masonry prism.

Fig. A1.4 illustrates the model for individual bricks, called Model B, subjected alternatively to linear (LD) and uniform (UD) shear force distribution. Due to double symmetry conditions, only one quarter of the brick was modelled using a mesh of 20 x 20 plane stress elements; six different geometric ratios b/d were considered. A similar model was formulated for the mortar joints, called Model C, with a mesh of 5 x 20 elements. These models were also subjected to linear and uniform shear force distributions, but with opposite direction to those considered for model B.

The lateral stress contours for four cases obtained from elastic analyses of the Model B-UD are plotted in Fig. A1.5. The stress levels are not shown in this figure because the objective is only to understand the variation of the lateral stresses. It is clearly observed the strong influence of the ratio b/d on the lateral stress distribution in the brick. The distribution of stresses tends to be more uniform for smaller values of the ratio b/d. This result is expected considering that the lateral stress in the brick is induced as a consequence of the interaction in the mortar-brick interface. Therefore, when the height of the brick is reduced, the effect of this perturbation affects a bigger part of the brick and the coefficient $C_{lb}$ tends to 1.0.

A1.3  **EVALUATION OF THE STRESS DISTRIBUTION FACTORS**

From the results of the three series of finite element analyses, the stress distribution factors were calculated according to Eq. 3.29. The factor $C_{lb}$ was evaluated at the vertical section of the brick where the maximum lateral stress occurs. Values obtained from Model B-LD and Model A (before the first crack) were very similar in all cases, with an approximate value of about 0.10. However, this value has no practical interest because $C_{lb}$ must be defined at failure according to the hypothesis made in the derivation of Eq. 3.30. For Model B-UD, the maximum lateral stress always occurred at the centre of the brick and the stress distribution factor $C_{lb}$ decreased as a function of the ratio b/d.
Model B

Case (a): UD
Uniform shear force distribution

Mesh: 20 x 20 elements

Case (b): LD
Linear shear force distribution

Mesh: 20 x 20 elements

Figure A.1.4. Finite element model for individual bricks.

Figure A.1.5. Lateral stress contours for Model B-UD considering different ratios b/d.
Factor $C_{eb}$ for the mortar joint was evaluated at the same vertical section in which the maximum lateral stress $f_{eb}$ occurred in the brick, in order to be coherent with the equilibrium condition (see Eq. 3.28 and Fig. 3.13). In some cases the maximum compressive stress in the mortar joint, $f_{eb}$, did not coincide with the vertical section at which the maximum stress $f_{eb}$ occurred. The results from Model C-UD indicated that the stress distribution tends to be constant when the mortar joint thickness decreases.

Fig. A1.6 presents the results of $C_{eb}$ and $C_{ej}$ obtained from models A, B and C. Values of the stress distribution factors plotted in Fig. A1.6 for Model A correspond to the distribution of stresses at the ultimate compressive load. It may be observed that the results from the simplified models, Model B-UD and Model C-UD, agree reasonably well with those obtained from a more sophisticated nonlinear analysis, Model A. Based on this conclusion, the following expressions are proposed to calculate stress distribution factors:

\[
C_{eb} = \frac{0.2}{b} \quad (A1.1)
\]

\[
C_{ej} = 1 - \frac{1.1}{d} \quad (A1.2)
\]

Both equations were obtained from linear regression analysis using the numerical results. These two equations are also plotted in Fig. A1.6 by dotted lines.

The results from Model A (see Fig. A1.6) indicate that the interaction of the different variables is more complex than it was assumed in the simplified models. Factor $C_{eb}$ varies mainly as a function of the ratio $b/d$. However, the thickness of the mortar joint also affects the results. In this case, it seems that the Model A represents an upper limit of the real values and leads to conservative values of the compressive strength (see the comparison with experimental results presented in section 3.5.6). Factor $C_{ej}$ depends principally on the $d/j$ ratio, although the height of the brick, $b$, also influences the stress distribution. Consequently, Eqs. A1.1 and A1.2 represent just an approximation. Nevertheless, it is believed that these expressions constitute a first approach in considering the real distribution of lateral stresses, which are far from a uniform state as was initially assumed in Hilsdorf's work.

A1.4 SUMMARY OF THE MECHANICAL AND GEOMETRICAL PROPERTIES
The values of the mechanical and geometrical properties used in the finite element analyses conducted in this parametric study were:

**Masonry Units:**
- $f'_{eb} = 15.0^\circ$, 20.0 and 25.0$^\circ$, MPa, $f'_{ib} = 1.5^\circ$, 2.0 and 2.5$^\circ$ MPa, $E_b = 10^\circ$ 15 GPa, $v_b = 0.15$

**Mortar joints:**
- $f'_{j} = 10.0^\circ$, 12.0 and 15.0$^\circ$ MPa, $\varepsilon'_j = 0.003$ $E_s = 5^\circ$ and 10 GPa, $v_j = 0.20$. 
Geometrical Properties:
Model A: $b/d = 1/3, 1/2$ and 1; $d/j = 9, 12$ and 24.
Model B: $b/d = 1/3, 2/3, 1, 4/3, 5/3$ and 2.
Model C: $d/j = 5, 10, 20, 30$ and 40.
* These values were only used in a preliminary study conducted with Model A.

Figure A1.6. Stress distribution factors for brick, $C_{db}$, and mortar, $C_{dj}$. 
APPENDIX 2: NONLINEAR ANALYSIS OF MASONRY PRISMS IN COMPRESSION

A2.1 BASIS OF THE PROCEDURE
A2.1.1 General Formulation
The procedure implemented for the nonlinear analysis of masonry in compression is similar to that proposed by Atkinson et al. [A3] (section 3.5.3.4), but some modifications are introduced to consider a nonuniform distribution of the lateral stresses. Furthermore, analytical expressions are proposed to consider the variation of the mechanical properties of mortar as a function of the strain state.

The increment of the lateral stress in the brick, $\Delta f_{sb}$, and mortar, $\Delta f_{sj}$, induced by an increment of the vertical compressive stress, $\Delta f_y$, can be calculated from the compatibility condition in plane stress:

$$\frac{1}{E_b} \begin{bmatrix} \Delta f_{sb} - u_b \\ \Delta f_y \end{bmatrix} = \frac{1}{E_j} \begin{bmatrix} \Delta f_{sj} - u_j \\ \Delta f_y \end{bmatrix}$$  \hspace{1cm} (A2.1)

and the equilibrium equation considering nonuniform distribution of the lateral stresses (see Fig. 3.13):

$$C_{db} \Delta f_{sb} + C_{dj} \Delta f_{sj} \quad j = 0$$  \hspace{1cm} (A2.2)

Rearranging Eq. A2.1 and substituting in Eq. A2.2, it is found that:

$$\Delta f_{sb} = \frac{u_b - \beta \frac{u_j}{1 + C_d \beta r}}{1 + C_d \beta r} \Delta f_y$$  \hspace{1cm} (A2.3)

$$\Delta f_{sj} = -\frac{b}{j} \frac{C_d \Delta f_{sb}}{C_{dj}}$$  \hspace{1cm} (A2.4)

where $\beta = E_y/E_j$ and $r = b/j$. Eqs. A2.3 and A2.4 allow the implementation of a nonlinear procedure, in which the compressive stress is applied in successive increments taking into account the variation of the mechanical properties of the constitutive materials.

The analysis of Eq. A2.3 shows that $f_{sb}$ increases when the lateral deformability of the mortar increases or when the $b/j$ ratio decreases. The stress distribution factor $C_d$ has also a significant influence on the value of the lateral stress. For usual cases, $f_{sb}$ is less than 10% of the applied vertical stress $f_y$. Eq. A2.4 indicates that the magnitude of $f_{sj}$ can be several times greater than $f_{sb}$ for normal cases in which $C_d > 0.3$ and $b/j > 5$. These results agree with those obtained by Shrive and Jessop (as reported in reference [S14]), who conducted finite element analyses using three-dimensional anisotropic models. They found that the lateral tensile stress
is about 0.02 of the magnitude of the applied compressive stress. Other researchers have reported values ranging from 0.025 to 0.033 for the same parameter [S14].

Based on the observed behaviour of the constitutive materials, the mechanical properties were assumed as follows:

### A2.1.2 Analytical Model for Mortar

The stress-strain relationship for confined mortar was represented with the expression proposed by Sargin et al. [S31] for concrete (following the notation used in this thesis):

\[
f_j = f'_{ij} \frac{A_1 \frac{\varepsilon_j}{\varepsilon'_{ij}} + (A_2 - 1) \left( \frac{\varepsilon_j}{\varepsilon'_{ij}} \right)^2}{1 + (A_1 - 2) \frac{\varepsilon_j}{\varepsilon'_{ij}} + A_2 \left( \frac{\varepsilon_j}{\varepsilon'_{ij}} \right)^2}\tag{A2.5}
\]

where \( \varepsilon_j \) is the mortar strain, \( \varepsilon'_{ij} \) is the strain corresponding to the maximum stress \( f'_{ij} \), the coefficients \( A_1 \) and \( A_2 \) are defined as:

\[
A_1 = \frac{E_p \varepsilon'_{ij}}{f'_{ij}}
\]

\[
A_2 = 0.65 - 7.25 \times 10^{-3} f'_{ij} \text{ (MPa)}
\]

and \( E_p \) is the initial modulus of elasticity of the mortar.

The influence of the confinement pressure in the mortar strength is represented by Eq. 2.10 (\( \sigma_i = f_i \)), whereas for the strain \( \varepsilon'_{ij} \) it is assumed, according to Atkinson's results [A3]:

\[
\varepsilon'_{ij} = \varepsilon'_{ij,0} \left( 1 + 6 \frac{f_{ij}}{f'_{ij}} \right)\tag{A2.6}
\]

In the above equation, \( \varepsilon'_{ij,0} \) is the strain at maximum stress for unconfined mortar. A similar expression has been used for confined concrete by different researchers: Balmer, Mander et al. and Saatcioglu and Razvi (as reported in reference [S34]). For confined concrete, however, the factor affecting the ratio of the lateral confinement stress, \( f_{ij} \), to the unconfined compressive strength, \( f'_{ij} \) is 5.0 instead of 6.0. This modification was introduced for confined mortar based on the experimental results reported by Atkinson et al. [A3].

The tangent modulus of the mortar is calculated from Eq. A2.5:

\[
E_j = \frac{\partial f_j}{\partial \varepsilon_j} (\varepsilon_j, f_{ij})
\]
\[ E_j = \frac{f'_{ij}}{e_j'} \frac{A_1 + 2 (A_2 - 1) \frac{e_j}{e_j'} + (2 - A_1 - 2 A_2) \left( \frac{e_j}{e_j'} \right)^2}{\left[ 1 + (A_1 - 2) \frac{e_j}{e_j'} + A_2 \left( \frac{e_j}{e_j'} \right)^2 \right]^2} \]  

(A2.7)

Numerical results obtained from this expression agree very well with the variation of the tangent modulus measured in laboratory tests [A3, M26].

The variation of Poisson's ratio for confined mortar (see section 2.4.3) is represented by the following expression, derived to fit the experimental results reported by Atkinson et al. [A3] and McNary and Abrams [M26],

\[ u_j = \frac{u_{jo}}{1 - \left( 1 - \frac{1}{5 - 6 \frac{f'_{ij}}{f''_{ij}}} \right) \left( \frac{f_y}{f''_{ij}} \right)^{10}} \]  

(A2.8)

where \( u_{jo} \) is the initial Poisson's ratio for unconfined mortar. This expression considers the increase of the Poisson's ratio at medium to high stress levels, and the beneficial effect of the confinement pressure which restrains the dilatation of the mortar. The Poisson's ratio at failure \( (f'_y = f''_{ij}) \) is assumed to vary from 5 \( u_{jo} \) for unconfined mortar, to 2 \( u_{jo} \) for confined mortar with \( f'_{ij}/f'_y = 0.5 \). For greater values of the confinement pressure there is no enough experimental data, therefore the validity of the proposed equation is limited to this range. In this way, it is possible to determine the properties of the mortar for a given stress state, \( f_y, f'_y \). Fig. A2.1 (a), (b) and (c) plot the variation of the properties of the mortar according to the model mentioned here, for the particular case in which \( f'_y = 8 \) MPa, \( u_{jo} = 0.0035 \), \( E_{jo} = 8000 \) MPa and \( u_{jo} = 0.20 \). Three different values of the relative confinement pressure \( f'_{ij}/f'_y \) are considered in this example.

A2.1.3 Analytical Model for Masonry Units

The bricks were considered as a brittle material with linear behaviour under tensile and compressive stresses, as shown in Fig. A2.1 (d). This model is acceptable for clay bricks, but it could not be very realistic for concrete units. The failure criterion for combined stress states was modelled according to Eq. 2.4, assuming that the coefficient \( n \) is variable. In this way, different situations can be represented.

Using these hypotheses, a nonlinear incremental procedure was implemented to solve Eq. A2.3. The vertical stress \( f_y \) was increased from zero until failure occurred, according to the failure criteria (Eqs. 2.4 and 2.10). The bisection method was used to solve the problem for each increment of vertical stress \( \Delta f_y \). This procedure was adopted on the ground that it assures convergence when the initial interval contains the solution. The Newton-Raphson method is widely used for its faster convergence, but in some particular conditions it does not converge. Furthermore, the first derivative of the nonlinear equation, which appears in the Newton-Raphson algorithm, is rather difficult to calculate in this case. The problem analysed contains only one nonlinear equation to solve. Consequently, the computer time was not an important factor and a very small
increment was selected in order to avoid the effect of accumulative errors. It was assumed that the stress distribution factor $C_d$ remained constant through the analysis. This is not strictly valid because numerical results indicate that $C_d$ increases from a small value before cracking up to 0.2-0.7 at failure.

![Graphs showing material properties](image)

**Fig. A2.1.** Material properties considered in the nonlinear analysis.

According to this model, the masonry prism failure will be often due to the failure of the brick. When the compressive stress in the mortar increases, its deformational properties, $E_j$ and $\nu_j$, are changed and the lateral stresses significantly increases. Therefore, cracking in the brick usually develops before the mortar strength is reached. However, if the mortar strength is very low, it is also possible that the mortar reaches its maximum stress first. In this stage, $f_{sj} = f'_{sj}$ and $E_j = 0$. From Eq. A2.3 the lateral stress in the brick is:

$$
\Delta f_{xb} = \lim_{E_j \to 0} \Delta f_y = -\Delta f_y \frac{\nu_j}{C_d} \frac{b}{j}
$$

(A2.9)
This expression indicates that for large values of \( b/j \) (for example, \( b/j = 20 \)) the mortar could reach the maximum stress in compression without a cracking failure in the brick, because \( f_{ob} \) is small enough. However, the mortar is able to sustain large plastic deformation which invariably leads to a splitting failure of the brick.

### A2.2 NUMERICAL EXAMPLES

The complete results obtained from an application of this procedure are illustrated in Figs. A2.1 and A2.2. The properties of the material are indicated in the former figure. The dimension of the bricks and mortar joints were: \( b = 60 \) mm, \( d = 120 \) mm and \( j = 10 \) mm. Fig. A2.2 shows the stress path described by the lateral stresses when the vertical stress increases up to failure.

![Failure envelope and stress paths obtained from nonlinear analysis.](image)

Fig. A2.2. Failure envelope and stress paths obtained from nonlinear analysis.

Two different values of the stress distribution factor were considered, \( C_d = 0.1 \) for uncracked bricks and \( C_d = 0.44 \) (from Eqs. 3.33 and 3.34) for the failure state. In this case, the mortar strength \( f_y \) was much smaller than the brick strength \( f_{ob} \). Thus, both materials reach the failure envelope almost simultaneously. Fig. A2.1 also plots the trajectory described by different variables associated with the mortar (these trajectories are indicated by dotted lines with small circles). When the vertical stress \( f_y \) increases, the mortar becomes more deformable: the tangent modulus falls and the Poisson's ratio increases at medium to high stress levels. However, the lateral stress \( f_{sj} \) also increases and tends to produce a contrary effect, which smooths the variation of these parameters. The final values at failure, \( E_j = 441 \) MPa, \( \nu_j = 0.38 \), are far different from those at the initial unloaded stage.
A2.3 COMPUTER PROGRAM

The computer program, based on the formulation described above, was written using Microsoft Quick BASIC.

For sake of simplicity, the subroutines related to input/output of data are not included here.

**SUB Compression (CF1, CF2, CB1, CB2, XSer, YSer)**

This subroutine calculates the compressive strength of masonry,

according to the nonlinear procedure proposed in this thesis.

Written by: Francisco J. Crisafulli - Programmed in Microsoft Quick BASIC


The stress-strain relationship of the mortar is represented with the

Sargin's equation, in which the empirical coefficient D is calculated

as a function of the compressive strength in MPa.

The units used in the program are: MPa and mm.

The stresses fcb, fj and fy are considered in absolute value,

independent if they are compressive or tensile stresses.

**LIST OF VARIABLES:**

A, C, B : Constants of a 2nd order equation

ASteel, DSteel : Constants of Sargin's equation

Bb : Brick height

Beta : Ratio Eb/Ej

BetaO : Ratio Eb/Ejo

Cib, Cdj : Stress distribution factor for brick and mortar

Cd : Total stress distribution factor

Defxb, Defxj : Lateral deformation of brick and mortar

Db : Length of the brick

DePa : Variable associated with deformational parameters

Dfj1,2,3 : Variables used in the bisection method

Dfy1,3 : Variables used in the bisection method

Dfj1,3 : Increment of the vertical stress

Djco : Strain at maximum stress for unconfined mortar

Djcc : Strain at maximum stress for confined mortar

Dy : Vertical strain in the mortar

Eb : Modulus of elasticity of the brick

Ejo, Ej : Initial and tangent modulus of elasticity of the mortar

Err1 : Admissible error for interaction of Dfj

Err2 : Admissible error for the failure

ErrDfj : Error in the interaction of Dfj

Fail : Error in reaching the failure envelope

Failb, Failj : Error in reaching the brick and mortar failure envelope

fcb, fcbj : Compressive and tensile strength of brick

fj, fjcc : Unconfined and confined strength of mortar

fxb, fjx : Lateral stress in brick and mortar

fy : Vertical stress

Hilsb : Masonry strength according to Hilsdorf's theory

HilsMo : Masonry strength according to the modified theory

j : Joint thickness

MM : Confinement factor for mortar

NN : Exponent for the failure envelope of the brick

Ninc : Number of increments

NItMa : Maximum number of iterations

NItter : Number of iterations

Nub : Poisson's ratio of the brick

Nujo, Nuj : Initial and tangent Poisson's ratio of the mortar

Rbj : Ratio bj

Rfj : Ratio of the lateral stress fjb to the strength fj

Rfy : Ratio of the vertical stress fy to the strength fjcc

S1, S2 : Constants to define the failure envelope of the brick (proposed equation)
' X1, X2 : Solutions of the 2nd order equation
' XX : Selected solution (between X1 and X2)

'----- Call subroutines to input data
CALL SUBROUTINE FOR INPUT OF REQUIRED DATA:
  Mortar: j, f, Djco, Ejo, Nujo, MM
  Brick: Db, Bb, fcb, fth, Eb, Nub, NN, S1, S2
  Errors and increment: Err1, Err2, Dfy, NlMa

'----- Calculate some variables
BetaO = Eb / Ejo
Rbj = Bb / j
IF Djco = 0 OR Djco < f / Ejo THEN Djco = 2 * f / Ejo

'----- Calculate stress distribution factors
Cdb = .2 * Db / Bb
Cdj = 1 - 1.1 * j / Db
C = Cdb / Cdj

'----- Compressive strength according to Hilsdorf's theory and its modification
Hils = fcb * (fth + f / (4.1 + Rbj)) / (fcb + f / (4.1 + Rbj)) / 1.5
HilsMo = fcb * (S2 * fth + S1 * f / (MM * Rbj * Cd))
HilsMo = HilsMo / (fth + S1 * fcb / (MM * Rbj * Cd))

'----- Deformational parameter = (μb - β μj) / (1 + β Cd r)
DePa = -(Nu - BetaO * Nujo) / (1 + Cd * Rbj * BetaO)
IF DePa <= 0 THEN
  PRINT * ERROR: Elastic Parameter <= 0
END
END IF

'----- Assign initial values
fy = 0: fxb = 0: fxj = 0: Dfj3 = 0
Defxb = 0: Defxj = 0
Nlnc = 0

'----- Main loop for increments of vertical stress, fy, up to failure
DO
  fy = fy + Dfy
  Nlnc = Nlnc + 1
  NIter = 0

'----- Initial values of the increment of lateral stress fxj
Dfj1 = 0
Dfj2 = .5

'----- Loop to calculate nonlinear properties of mortar using the bisection method
DO
  NIter = NIter + 1

'----- Calculate mortar properties and increment of vertical stress at two points:
CALL Mortar(fy, f, f + Dfj1, fjo, f, MM, Ejo, Ej, Nujo, Eb, Nub, Beta, Cd, DePa,
Dy, Djco, Rbj)
Dfy1 = Dfy - Dfj1 / (DePa * Cd * Rbj)
Dfj3 = (Dfj1 + Dfj2) / 2
CALL Mortar(fy, f, f + Dfj3, fjo, f, MM, Ejo, Ej, Nujo, Eb, Nub, Beta, Cd, DePa,
Dy, Djco, Rbj)
Dfy3 = Dfy - Dfj3 / (DePa * Cd * Rbj)

'----- Select the points of the interval to assure that the solution is between them.
IF SGN(Dfy3) ∝ SGN(Dfy1) THEN
  Dfj2 = Dfj3
ELSE
    Dfxj1 = Dfxj3
END IF

'----- Error of the iteration and print results
ErrDfxj = ABS(2 * (Dfxj1 - Dfxj2) / (Dfxj1 + Dfxj2))

LOOP WHILE ErrDfxj > Err1 AND NitIer < NitMa

'----- Lateral stresses in mortar and brick
fxj = fxj + Dfxj3
fxb = fxb + Dfy * DePa

'----- Lateral strains in mortar and brick
Defxj = Defxj + (-Dfxj3 + Nuj * Dfy) / Ej
Defxb = Defxb + (Dfxj3 / (Rbj * Cd) + Nub * Dfy) / Eb

'----- Evaluation of failure of mortar or brick.
' If ABS(Fail) < Err2 the incremental process ends
FailI = fy / fjec - 1
FailB = ((fxb / ftb) * NN + fy / fcb - 1)
IF FailB > FailI THEN
    Fail = FailB
ELSE
    Fail = FailI
END IF

IF Fail > 0 THEN
    fxj = fxj - Dfxj3
    fxb = fxb - Dfy * DePa
    Defxj = Defxj - (-Dfxj3 + Nuj * Dfy) / Ej
    Defxb = Defxb - (Dfxj3 / (Rbj * Cd) + Nub * Dfy) / Eb
    fy = fy - Dfy; Dfy = Dfy / 2
END IF

LOOP WHILE ABS(Fail) > Err2

'----- Print numerical results and plot
CALL SUBROUTINE FOR PRINTING RESULTS
END SUB

SUB Mortar (fy, fxj, fjec, fj, MM, Ejo, Ej, Nuj, Nujo, Eb, Nub, Beta, Cd, DePa, Dfy, Djec, Rbj)

******************************************************************************
' This subroutine calculates mechanical properties of the mortar
******************************************************************************

fjec = fj + MM * fxj
Rfy = fy / fjec
IF Rfy > 1 THEN Rfy = 1
Djec = Djco * (1 + 6 * fxj / fj)
ASar = Ejo * Djec / fjec
DSar = .65 . .00725 * fj

'----- Evaluation of Ej using a second order equation. For a given
' value of the vertical stress fy, the deformation XX=eyj/edic
' is calculated by solving a second order equation (Eq. A2.5)
A = Rfy * DSar - (DSar - 1)
B = Rfy * (ASar - 2) - ASar
C = Rfy
CALL Eq2ndOrder(A, B, C, X1, X2)
XX = X1: IF X1 > X2 THEN XX = X2
Dy = XX * Djec

'----- Young’s modulus for confined mortar (Eq. A2.7)
Ej = fjcc / Djcc * (ASar + 2 * (DSar - 1) * XX + (2 - ASar - 2 * DSar) * XX ^ 2)
Ej = Ej / (1 + (ASar - 2) * XX + DSar * XX ^ 2) ^ 2
IF Ej = 0 THEN
  Ej = 0.001
  Beta = 1E+29
ELSE
  Beta = Eb / Ej
END IF

----- Poisson's ratio (Eq. A2.8) and
DEformational Parameter = (μb - β μj) / (1 + β Cd r)
RFxj = ffxj / fj
IF RFxj > .5 THEN RFxj = .5
Nuj = Nujo / (1 - (1 - 1 / (5 - 6 * RFxj)) * RFy ^ 10)
DeFx = -(Nub - Beta * Nuj) / (1 + Cd * Beta * RbF)
END SUB

SUB Eq2ndOrder (A, B, C, X1, X2)
**************************************************************************
' Solve a second order equation A x^2 + B x + C = 0
**************************************************************************
R = B ^ 2 - 4 * A * C
IF R < 0 THEN
  X1 = 0; X2 = 0
ELSE
  R = SQR(R)
  X1 = (-B + R) / (2 * A)
  X2 = (-B - R) / (2 * A)
END IF
END SUB
APPENDIX 3: EVALUATION OF THE COEFFICIENTS $C_n$ AND $C_s$

A3.1 INTRODUCTION
The failure theory developed by Mann and Müller [M2] was modified in section 4.2.4 by introducing two coefficients, $C_n$ and $C_s$, which take into account the variation of the normal stresses and the effect of the maximum shear stress, respectively. The evaluation of these coefficients is described in this Appendix.

The procedure to evaluate the coefficients $C_n$ and $C_s$ was divided into two parts. First, a simplified approach, based on conceptual considerations, was used. In the second part, finite element models were developed to investigate the stress state in masonry panels subjected to pure shear.

A3.2 SIMPLIFIED EVALUATION OF THE COEFFICIENTS $C_n$ AND $C_s$
Mann and Müller [M2] considered a uniform distribution of the normal stresses $\Delta f_n$ (see Fig. 4.8). This hypothesis would be valid only if the mortar behaves like a fully plastic material. It seems more realistic to assume, as a first approximation, that the normal stresses $\Delta f_n$ follows a linear variation, as shown in Fig. 4.10 (b). In this case, the average value of the stress block acting on each half of the brick can be estimated as:

$$\Delta f_{n,av} = 1.5 \frac{b}{d} \tau$$  \hspace{1cm} (A3.1)

The comparison of Eqs. 4.16 and A3.1 indicates that $C_n$ is equal to 1.5 when a linear variation of the normal stresses is assumed.

The maximum shear stress in the masonry unit, and consequently the factor $C_s$ (see Eq. 4.19), may be approximately calculated considering that the increase of the shear stress is inversely proportional to the reduction of the effective shear area across vertical sections:

$$C_s = \frac{A_v}{A_{v,eff}}$$  \hspace{1cm} (A3.2)

Since it is assumed that the head joints are able to transfer no shear stresses, in vertical sections such as A-A or B-B with total area $A_v$ (see Fig. A3.1), the effective shear area, $A_{v,eff}$, decreases considerably.
Figure A3.1. Effective shear area across vertical sections.

The reduction of the vertical shear area depends on the number of brick layers, \( n_b \), the height of the brick, \( b \), and the thickness of the mortar joint, \( j \). The area of the vertical section, \( A_v \), can be assumed as:

\[
A_v = n_b \ (b + j) \ t
\]  
(A3.3)

where \( t \) is the thickness of the masonry wall. Three different equations are developed to calculate \( C_s \), considering that the number of brick layers is either even or odd. In the former case, the number of head joints crossing the vertical sections is equal to \( n_b/2 \), thus:

\[
C_s = \frac{b}{j} + \frac{1}{2j} + 1
\]  
(A3.4)

whereas in the latter case is \((n_b+1)/2\) or \((n_b-1)/2\):

\[
C_s = \frac{b}{(n_b+1)\ j} + 1, \ \text{or} \ \ C_s = \frac{b}{(n_b-1)\ j} + 1
\]  
(A3.5)

Assuming that \( n_b > 10 \), the coefficient \( C_s \) ranges from 1.7 to 2.0, for the common dimensions used in masonry construction (10 ≤ \( b/j \) ≤ 20). When the number of brick layers, \( n_b \), is significantly greater than 1.0, Eq. A3.5 approaches Eq. A3.4 and the coefficient \( C_s \) varies from 1.83 to 1.91, for \( b/j \) equal to 10.0 and 20.0,
respectively. However, it is expected that the factor \( C_s \) could be slightly greater than these values, because the brick is usually stiffer than the mortar, and hence, the stress distribution is not uniform.

The results presented in this section suggest that the hypothesis of uniform distribution of the vertical stresses and the values of maximum shear stress adopted by Mann and Müller [M2] are rather conservative.

**A3.3 FINITE ELEMENT ANALYSIS OF MASONRY PANELS SUBJECTED TO PURE SHEAR**

The validity of Mann and Müller's theory and the proposed modification depend greatly on the hypotheses assumed to develop it, especially the stress state considered in the panel and in each individual brick. To check these hypotheses, finite element analyses of masonry panels subjected to pure shear were conducted, using the program ABAQUS [A12]. The masonry panel used in the analytical model had nine courses with five bricks in each course. Only one quarter of the panel was modelled due to the symmetric properties of the structure and antisymmetric load distribution, as indicated in Fig. A3.2 (a). Linear elastic behavior of the constitutive materials and continuous mortar-brick interfaces were assumed. The hypothesis of linear elastic behavior is acceptable because the maximum normal stress induced by the pure shear state is only about one to three times greater than the nominal shear stress, \( \tau \), whereas the compressive strength of the constitutive materials is many times greater.

The bed joints were modelled with three horizontal layers of elements, using the same type of elements adopted for the bricks. Two distinct situations were considered to model the head joints: in some cases they were completely removed and in others were modelled with a different material than that used for the bed joints. In the latter case, the modulus of elasticity of the head joints was changed to modify their stiffness, and consequently the amount of shear stresses transferred in these joints.

In a preliminary study, each individual brick was modelled using three finite element meshes, namely, 4 x 9, 8 x 18 and 16 x 36 four-node elements. In order to check the convergence of the models, the resultant shear force (obtained from the integral of the shear stresses) in the vertical edge of the brick was considered as reference parameter. This parameter should be zero in the model without head joints. The results indicate that the solution converges slowly and the error was not close to zero even for the more refined mesh. This fact was due to the stress concentration that occurred at the corners of the brick, although was a local effect which vanished rapidly. In order to avoid the above mentioned problem, a more realistic model to represent the mortar-brick interface should be adopted. Since the objective of these finite element analyses is only to investigate the stress distribution within the panel, it is believed that the model is accurate enough. The stress states obtained from the second and third meshes were very similar, consequently the 8 x 18 mesh was adopted for further analyses. The final models were formed by 2192 and 2352 elements for analyses with or without head joints, respectively. The geometric and mechanical properties of the model were changed to analyse different cases. In this parametric study, the following variations were considered:

- dimension of the masonry units: \( b/d = 0.25 \) and 0.50,
- thickness of the bed joints: \( b/j = 8, 16 \) and 24,
- brick and mortar modulus of elasticity: \( E_b/E_j = 4, 8, 16 \) and 20,
- modulus of elasticity of the head joints equal to zero (no head joints) or equal to that of the bed joints.
The analysis of the results obtained from different cases confirms the validity of the mechanism proposed by Mann and Müller [M2] for the representation of the behaviour of masonry subjected to shear. The different properties of mortar and brick, and the poorer conditions of the head joints lead to a complex stress state in each individual masonry unit. The typical deformed shape of the model is shown in Fig. A3.2 (b), where the elements representing the mortar joints are removed to simplify the plot. Typical results of the contours of normal and shear stress are plotted in Fig. A3.3, corresponding to the models without and with head joints. In the latter case, the properties of the bed and head joints were identical. In both models it was considered that $b/d = 0.5$, $b/j = 8$ and $E_v/E_s = 4$.

(a) Masonry panel subjected to pure shear

(b) Deformed shape of the model

**Figure A3.2.** Characteristics of the masonry panel and deformed shape of the model.
Figure A3.3. Contours of normal and shear stresses.

It is worth pointing out the following observations regarding the stress state into the panel:

- The distribution of the normal stress, $\Delta f_n$, on the top and bottom faces of the brick was not linear, with the maximum value at the corners and zero at the centre. Fig. A3.4 shows the distribution of $\Delta f_n$ (normalized to the nominal shear stress, $\tau$) for three different cases. In this figure, the stresses acting on one half of the brick are plotted. The increase of the stresses observed near the central point is produced by the influence of the corner of the neighbouring brick.

- The average normal stress varied from 1.55 to 1.80 times $\tau$ b/d, for the cases where no head joints were considered. Smaller values, between 0.85 to 1.65 times $\tau$ b/d, were obtained considering head joints with a stiffness 2 to 10 times smaller than that of the bed joints. It was observed that the stress distribution was closer to the uniform distribution when the mortar was much more deformable than the brick (see curve B, $E_b/E_j = 16$, in Fig. A3.4).

- The value of the normal stresses decreased when the modulus of elasticity of the head joints was increased. However, these stresses were not zero for the limit case in which both vertical and bed joint had the same properties (see curve C in Fig. A3.4). This fact indicates that the normal stresses at the bed joint are induced not only due to the poor characteristics of the head joints, but also because the mortar joints are more deformable than the bricks.
The maximum shear stress occurred at the centre of the bricks. The values of the coefficient $C_\tau$, calculated from numerical results ranged from 2.05 to 2.50, for the model without head joints. However, the coefficient $C_\tau$ varied from 1.5 to 2.1 when head joints were considered with reduced stiffness. The smaller values correspond to the cases with either large joint thickness (small ratio $b/j$) or small $d/b$ ratio.

The axial stress $f_a$ (in the direction parallel to the bed joints) was not zero in the zone close to the corners of the brick. These stresses were smaller than the stresses $\Delta f_a$ and presented the same sign. Consequently, near the corners there is a biaxial (compression-compression or tension-tension) stress state. The effect of this on the failure mechanism of the brick depends on the final stress state, which result from the superposition with the normal stress $f_n$ (the finite element analyses considered only the effect of the shear stress $\tau$).

The results obtained from finite element analyses agree reasonably well with the hypothesis assumed in the modification of Mann and Müller's theory [M2]. However, more research, especially experimental work, is needed to consider the real stress state in the panel and the factors that affect it. Based on the numerical results presented in this appendix, it is suggested that $C_n = 1.5$ and $C_\tau = 2.0$ be adopted for the practical applications of Eqs. 4.17 and 4.20, respectively.

It is worth noting that the superposition of results obtained from finite element analyses for a pure shear state with the compressive stress $f_n$ is usually valid considering that the bricks behave approximately in the linear elastic range before fracture occurs in a brittle mode (see section 2.3) and the mortar is not compressed at high stress levels. The latter condition, however, is not valid in the case of compressive failure and the coefficient $C_n$ could be greater than the value obtained from the finite element analyses. Therefore, Eq. 4.18 should be applied considering a value of the coefficient $C_n$ between 1.8 and 2.0, which agrees well with the equation originally developed by Mann and Müller [M2].

![Figure A3.4. Distribution of the stress $\Delta f_n$ on the face of the brick.](image-url)
APPENDIX 4: COMPUTER PROGRAM FOR THE HYSTERETIC RESPONSE OF MASONRY STRUTS

A4.1 GENERAL
A subroutine, written in FORTRAN, was incorporated in the computer program RUAUMOKO [C16] in order to represent the hysteretic response of masonry struts, which is required for the modelling of infilled frames using the equivalent truss model. The formulation of this hysteretic model is given in sections 6.2 and 8.2.

A4.2 COMPUTER PROGRAM

SUBROUTINE AXXS21(S,R,DR,K0,F,SR,FC,FT,UC,UUL,UCL,EMO,GUN,ARE,AREA1,AREA2,
* R1,R2,SLEN,RBP,RBN,ECH,ECL,EPLU,EPLR,ERE,ET,E1,E2,FTR,PST,PU,STCH,STRE,STUN,
* ST1,ST2,UCH,UPI,URE,UUNI,UUN,U1,U2,NCYIN,KK,NCYC,IPLOT)

C
C **********************************************************************
C Diagonal Strut Model for Masonry
C
C DATA VARIABLES (for stress-strain relationship) HYST
C FC = Compressive strength 1
C FT = Tensile strength 2
C UC = Strain at FC 3
C UUL = Ultimate strain 4
C UCL = Closing strain 5
C EMO = Initial modulus of elasticity 6
C GUN = Factor to calculate EUN 7
C ARE = Strain reloading factor 8
C
C DATA VARIABLES (for strut model)
C AREA1 = Initial Area of the strut 9
C AREA2 = Final Area of the strut 10
C R1 = Displacement at which AREA1 changes 11
C R2 = Displacement at which AREA2 changes 12
C SLEN = Length of the strut 13
C
C HISTORY VARIABLES
C RBP,RBN = Positive and Negative displacement envelopes 5-6
C ECH = Modulus at the change point 7
C ECL = Modulus at closing point 8
C EPLU = Modulus at plastic strain for unloading 9
C EPLR = Modulus at plastic strain for reloading 10
C ERE = Modulus at the reloading point on the envelope 11
C ET = Tangent module 12
C E1 = Modulus at point 1 13
C E2 = Modulus at point 2 14
C FTR = Reduce tensile strength 15
C PST = Previous stress 16
C PU = Previous strain 17
STCH = Stress at change point 18
STRE = Reloading stress on the envelope 19
STUN = Unloading stress on the envelope 20
ST1 = Stress at point 1 21
ST2 = Stress at point 2 22
UCH = Strain at change point 23
UPL = Plastic strain 24
URE = Reloading strain on the envelope 25
UUNI = Unloading strain for internal loops 26
UUN = Unloading strain on the envelope 27
U1 = Strain at point 1 28
U2 = Strain at point 2 29
NCYIN = Internal cycle number 30
IVIR = Flag indicate the 1st time that Sargin's Eq. is used 31
LL = Rule number 31

OTHER EXTERNAL VARIABLES
S = Current force
R = Current displacement
DR = Increment of displacement
F = Stiffness factor
K0 = Initial stiffness
SR = Overshoot force
NCYC = Number of Inelastic Cycles*2
IPLT = Plot flag

DATA VARIABLES INTERNALLY DEFINED
ACH = Factor to calculate UCH
BA = Factor to calculate UPL
BCH = Factor to calculate FCH
GPLU = Factor to calculate EPLUI and EPLU
GPLR = Factor to calculate EPLR
EX1 = Exponent for EPLU
EX2 = Exponent for FCH

INTERNAL VARIABLES
AREA = Area of the strut
RNEW = New displacement
SNEW = New force
ST = Actual stress
U = Actual strain

SUBROUTINES:
AXSS22: Calculate the stress for a given strain.
AXSS23: Sargin's equation for the strength envelope.
AXSS24: Proposed equation for unloading and reloading curves.
AXSS25: Calculate plastic strain and reduce tensile strength.
AXSS26: Calculate reloading point in the skeleton curve and change point in the reloading curve.
AXSS27: Calculate parameters for reloading from rule 3.
AXSS28: Calculate parameters for unloading/reloading rules 2,4,5.

Programmed by: Francisco J. Crisafulli
Data/Version: 05-June-1997/1.0
06-July-1997/2.0

******************************************************************************************

INTEGER IPLT,NCYC,NCYIN,KK,IVIR,LL
REAL FC,FT,UC,UUL,UCL,EMO,GUN,AREA1,AREA2,R1,R2,SLEN
REAL ACH,ARE,BA,BCH,GPLU,GPLR,EX1,EX2,AREA,ST,U
REAL RBP,RBN,ECH,ECL,EPLU,EPLR,ERE,ET,E1,E2,PTR,PST,PU
REAL STCH,STRE,STUN,ST1,ST2,UPL,URE,UUNI,UUN,U1,U2
REAL DR,F,K0,R,S,SR,RNEW,SNEW
C *Hysteresis control variables which can be modified to adjust the shape of the loops. See Table 7.2 for limit values
DATA ACH, BA, BCH /0.4, 2.0, 0.6/
DATA GPLU, GPLR, EX1, EX2 /0.6, 1.2, 1.75, 1.25/
C
C *Initial values
LL = KK/10
IVIR = KK-LL*10
IF(KK.EQ.0) THEN
  IVIR = 1
  LL = 3
  FTR = FT
  NCYIN = 1
  K0 = AREA1*EMO/SLEN
ENDIF
C *Calculate new values of displacement and force
RNEW = R+DR
SNEW = S+K0*F*DR
C *Calculate the strain, stress and tangent modulus
U = RNEW/SLEN
CALL AXSS22(LL, FC, FT, UC, UUL, UCL, EMO, ARE, ACH, BA, BCH, GUN, GPLU, GPLR, EX1, EX2, ECH,
  ECL, EPLU, EPLR, ERE, ET, E1, E2, FTR, PST, PU, ST, STCH, STRE, STUN, ST1, ST2, U, UCH, UP,
  L, URE, UUNI, UUN, U1, U2, IVIR, NCYIN, NCYC)
C *Update strain and stress values
PST = ST
PU = U
C *Calculate force and stiffness
IF(AREA1.EQ.AREA2) THEN
  AREA = AREA1
ELSE
  IF(RNEW.GT.R1) THEN
    AREA = AREA1
  ELSEIF(RNEW.LT.R2) THEN
    AREA = AREA2
  ELSE
    AREA = AREA1-(AREA1-AREA2)*(R1-RNEW)/(R1-R2)
  ENDIF
ENDIF
ENDIF
S = ST*AREA
F = ET*AREA/EMO/AREA1
C *Update variables
R = RNEW
RBP = MAX(R,RBP)
RBN = MIN(R,RBN)
SR = SNEW-S
KK = LL*10+IVIR
IPLOT = 1
IF(LL.EQ.1.AND.U.LT.0.75*UC) IPLOT = 3
IF(LL.EQ.3.AND.FTR.EQ.0.0) IPLOT = 4
C RETURN
END
C SUBROUTINE AXSS22(LL, FC, FT, UC, UUL, UCL, EMO, ARE, ACH, BA, BCH, GUN, GPLU, GPLR, EX1, EX2,
  ECH, ECL, EPLU, EPLR, ERE, ET, E1, E2, FTR, PST, PU, ST, STCH, STRE, STUN, ST1, ST2, U,
  UCH, UPL, URE, UUNI, UUN, U1, U2, IVIR, NCYIN, NCYC)
C
C ********************************************
C Axial cyclic strain-stress relationship for fragile materials.
C This subroutine calculate the stress, ST, for a given value
of the strain, U. Compressive stresses and strains are (-),
The relative strain, XX, is always (+).

INTERNAL VARIABLES
A1, A2  =  Coefficients for Sargin's Eq.
EUN    =  Unloading modulus
DELTAU =  U - UP = Strain increment

Programmed by:  Francisco J. Crisafulli
Data/Version :  01-Sep-1995/1.0
               05-Jun-1997/2.0 (Considering tensile strength)

*********************************************************************

INTEGER      IVIR, LL, NCYC, NCYIN
REAL          EMO, FC, FT, UC, UUL, GUN, UCL, ACH, ARE, BA, BCH, GPLU, GPLR
REAL          ECH, ECL, EPLU, EPLR, ERE, ET, E1, E2, FTR, PST, PU, ST, STCH
REAL          STRE, STUN, ST1, ST2, U, UCH, UPL, URE, UUNI, UUN, U1, U2
REAL          EX1, EX2, A1, A2, EUN, DELTAU

A1 = EMO*UC/FC
A2 = 1-A1*UC/UUL
DELTAU = U-PU
EUN = GUN*EMO

*RULE 1: Skeleton curve, compression (Sargin)
IF(LL.EQ.1) THEN
  IF(DELTAU.LT.0.0) THEN
    CALL AXSS23(U, ST, UC, FC, A1, A2, UUL, ET)
    IF(U.LE.UUL) THEN
      LL = 3
      UPL = 1000*UC
      ENDIF
  ELSE
    LL = 2
    NCYC = NCYC+2
    NCYIN = 1
    UUN = PU
    STUN = PST
    URE = UUN
    CALL AXSS25(UPL, FC, UC, FT, FTR, EMO, UUN, STUN, BA)
    E1 = EUN
    CALL AXSS28(PU, PST, UC, U1, ST1, E1, U2, ST2, E2, EMO, UPL, UUN, UUNI, EPLU, GPLU, EX1)
    CLEAR AXSS24(U, ST, U1, ST1, E1, U2, ST2, E2, ET)
  ENDIF
ELSEIF(LL.EQ.2) THEN
  IF(U.GE.U2) THEN
    IF(FTR.GT.0.0) THEN
      LL = 6
      U1 = UPL
      ET = EMO*FTR/FT
      ST = ET*(U-U1)
    ELSE
      LL = 3
      ST = 0.0
      ET = 0.0
    ENDIF
  ELSEIF(DELTAU.GT.0.0) THEN
    CALL AXSS24(U, ST, U1, ST1, E1, U2, ST2, E2, ET)
  ELSE
    CLEAR AXSS24(U, ST, U1, ST1, E1, U2, ST2, E2, ET)
  ENDIF
ENDIF
CALL AXSS26( PU, UC, FC, UPL, UUL, URE, STRE, ERE, UCH, STCH, ECH, UUN, STUN, EUN, UUNI, NCYIN, A1, A2, ARE, ACH, BCH, EX2)

* IF(STRE.EQ.0.0) THEN
  LL = 3
  UPL = 1000*UC
ELSE
  IF(PST.GE.0.9*STCH.AND.U.GE.0.9*UCH) THEN
    LL = 4
    U1 = UCH
    ST1 = STCH
    E1 = ECH
    ST2 = PST
    U2 = PU
    E2 = MIN(1.2*ET,0.9*(ST2-ST1)/(U2-U1))
  ELSE
    LL = 5
    U1 = PU
    ST1 = PST
    U2 = URE
    ST2 = STRE
    E1 = MIN(MAX(2.0*ET,1.2*(ST2-ST1)/(U2-U1)),EUN)
    E2 = ERE
    IF(E2.EQ.0.0) E2 = 0.5*(ST2-ST1)/(U2-U1)
  ENDIF
ENDIF
CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
ENDIF
ENDIF

* RULE 3: No stress
ELSEIF(LL.EQ.3) THEN
  IF(DELTAU.GT.0.0) THEN
    IF(FTR.GT.0.0) THEN
      LL = 6
      U1 = 0.0
      ET = EMO
      ET = EMO+FTR/FT
      ST = ET*(U-U1)
    ELSE
      ST = 0.0
      ET = 0.0
    ENDIF
  ELSE
    IF(U.LE.UPL.OR.U.LE.UCL) THEN
      IF(IVIR.EQ.1) THEN
        IF(U.GT.0.0) THEN
          LL = 4
          U1 = -FT/EMO
          IF(FT.EQ.0.0) U1 = FC/10.0/EMO
          CALL AXSS23(U1,ST1,UC,FC,A1,A2,UUL,E1)
          U2 = U
          ST2=0.0
          E2=0.01*EMO
          CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
        ELSE
          LL = 0
          IVIR = 0
          CALL AXSS23(U,ST,UC,FC,A1,A2,UUL,ET)
        ENDIF
      ELSE
        CALL AXSS26( PU, UC, FC, UPL, UUL, URE, STRE, ERE, UCH, BCH, EX2)
      ENDIF
    ENDIF
  ENDIF
ENDIF

ELSE
* STCH,ECH,UUN,STUN,EUN,UUNI,NCYIN,A1,A2,
* ARE,ACH,BCH,EX2)
IF(STRE.LT.0.0) THEN
  LL = 4
  CALL AXSS27(U,EMO,UPL,UCL,ECL,PU,PST,U1,ST1,E1,U2,ST2,E2,UUN,ECH,STCH,ECH,
               EPLU,GPLU,EPRL,GPLR,EX1)
  CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
ELSE
  LL = 3
  UPL = 1000*UC
  ST = 0.0
  ET = 0.0
ENDIF
ENDIF
ELSE
  ST = 0.0
  ET = 0.0
ENDIF
ENDIF
C *RULE 4: First part of reloading curve
ELSEIF(LL.EQ.4) THEN
  IF(DELTAU.LT.0.0) THEN
    IF(U.LE.U1) THEN
      IF(IVIR.EQ.0) THEN
        LL = 5
        U2 = URE
        ST2 = STRE
        E2 = ERE
        IF(E2.EQ.0.0) THEN
          E2 = 0.5*(ST2-ST1)/(U2-U1)
        ENDIF
        CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
      ELSE
        LL = 1
        IVIR = 0
        CALL AXSS23(U,ST,UC,FC,A1,A2,UUL,ET)
      ENDIF
    ELSE
      CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
    ENDIF
  ELSE
    LL = 2
    E1 = MIN(2*ET,EUN)
    CALL AXSS28(PU,PST,UC,U1,ST1,E1,U2,ST2,E2,EMO,UPL,UUN,UUNI,EPLU,GPLU,EX1)
    CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
  ENDIF
ENDIF
C *RULE 5: Second part of reloading curve
ELSEIF(LL.EQ.5) THEN
  IF(U.LE.URE) THEN
    LL = 1
    CALL AXSS23(U,ST,UC,FC,A1,A2,UUL,ET)
  ELSE
    IF(DELTAU.GT.0.0) THEN
      LL = 2
      IF(UUN.GT.PU) THEN
        UUN = PU
        URE = PU
        STUN = PST
        NCYIN = 1
        CALL AXSS25(UPL,FC,UC,FT,FTR,EMO,UUN,STUN,BA)
      ENDIF
      E1 = EUN
      CALL AXSS28(PU,PST,UC,U1,ST1,E1,U2,ST2,E2,EMO,UPL,UUN,UUNI,EPLU,GPLU,EX1)
C *RULE 6: Tensile Behaviour (elastic)
ELSE
ET = EMO*FTR/FT
ST = ET*(U-U1)
IF(ST.GE.FTR) THEN
   LL = 3
   ST = FTR
   FTR = 0.0
ELSEIF(ST.LT.0.0.AND.IVIR.EQ.1.AND.U.LE.0.0) THEN
   LL = 1
   IVIR = 0
   CALL AXSS23(U,ST,UC,FC,A1,A2,UUL,ET)
ELSEIF(ST.LT.0.0) THEN
   LL = 4
   CALL AXSS26(PU,UC,FC,UPL,UUL,URE,STRE,ERE,UCH,STCH,ECH,UUN,STUN,EUN,UUNI, * NCRY,1,A1,2,ARE,ACH,BCH,EX2)
   IF(STRE.LT.0.0) THEN
      CALL AXSS27(UC,EMO,UPL,UCL,ECL,PU,PST,ST1,ST1,E1,U2,ST2,E2,UC,STCH,ECH,EPLU, * GPLU,EPLR,GPLR,EX1)
      CALL AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)
   ELSE
      LL = 3
      UPL = 1000*UC
      ST = 0.0
      ET = 0.0
   ENDIF
ENDIF
ENDIF

C RETURN
END

C SUBROUTINE AXSS23(U,ST,UC,FC,A1,A2,UUL,ET)
C
C ******************************************************
C Axial cyclic stress-strain model.
C Calculate stress according to Modified Sargin's equation
C
C INTERNAL VARIABLES
C IENV = Flag to select descending branch of the envelope curve
C XX = Relative strain
C
C Programmed by: Francisco J. Crisafulli
C Data/Version : 01-Sep-1995/1.0
C ******************************************************

C INTEGER  IENV
REAL      U,ST,UC,FC,A1,A2,UUL,ET,XX,Z0

C DATA Z0,IENV /0.0, 1/
IF(U.GT.UC.AND.IENV.EQ.1.OR.U.GT.UUL.AND.IENV.EQ.2) THEN
   *Sargin's Equation
   XX = U/UC
   ST = FC*(A1**XX+(A2-1.0)**XX**2.0)/(1.0+(A1-2.0)*XX+A2**XX**2.0)
   ET = FC/UC*(A1+2.0*(A2-1.0)**XX+(2.0-A1-2.0)*A2)**XX**2.0
   ET = MAX(ET/((1.0+(A1-2.0)**XX+A2**XX**2.0)**2),Z0)
ELSEIF(IENV.EQ.1) THEN
*Parabola - Descending branch if IENV=1
ST = MIN (FC*(1-((U-UC)/(UUL-UC))**2),Z0)
ET = 0.0
ELSE
ST = 0.0
ET = 0.0
ENDIF

RETURN
END

SUBROUTINE AXSS24(U,ST,U1,ST1,E1,U2,ST2,E2,ET)

* Axial cyclic stress-strain model.
* Proposed equation for unloading and reloading curves

INTERNAL VARIABLES
A1,A2 = Auxiliary variables
B1,B2,B3 = Coefficients for proposed Eq.
ESEC = Secant modulus between points 1 and 2
XX = Relative strain

Programmed by: Francisco J. Crisafulli
Data/Version : 01-Sep-1995/1.0

REAL U,ST,U1,ST1,E1,U2,ST2,E2,ET
REAL A1,A2,B1,B2,B3,ESEC,XX

ESEC = (ST2-ST1)/(U2-U1)
B1 = E1/ESEC
B3 = 2.0-E2/ESEC*(1+B1)
B2 = B1-B3
XX = (U-U1)/(U2-U1)
A1 = (B1*XX+XX**2)
A2 = (1.0+B2*XX+B3*XX**2.0)
ST = ST1+(ST2-ST1)*A1/A2
ET = ESEC*(B1+2.0*XX)*A2-A1*(B2+2.0*B3*XX)/A2**2.0

RETURN
END

SUBROUTINE AXSS25(UPL,FC,UC,FT,FTR,EMO,UUN,STUN,BA)

* Axial cyclic stress-strain model.
* Calculate plastic strain and reduce tensile strength

Programmed by: Francisco J. Crisafulli
Data/Version : 01-Sep-1995/1.0
05-Jun-1997/2.0 (Considering tensile strength)

REAL UPL,FC,UC,FT,FTR,EMO,UUN,STUN,BA,ZERO
DATA ZERO /0.0/

UPL = MIN(UUN-STUN*(UUN-BA*ABS(FC)/EMO)/STUN-BA*ABS(FC))/ZERO)
IF(UPL.GT.UC.AND.FTR.GT.0.0) FTR = MAX(FTR*(1.0-UPL/UC),ZERO)

RETURN
SUBROUTINE AXSS26(PU,UC,FC, UPL, UUL, URE, STRE, ERE, UCH, STCH, ECH, 
    * 
    UUN, STUN, EUN, UUNI, NCYIN, A1, A2, ARE, ACH, BCH, EX2)

INTERNAL VARIABLES
UA = Strain at point A, used to calculate the UPL
XX = Relative strain

Programmed by: Francisco J. Crisafulli
Data/Version: 01-Sep-1995/1.0

INTEGER NCYIN
REAL PU, UC, FC, UPL, UUL, URE, STRE, ERE, UCH, STCH, ECH
REAL UUN, STUN, EUN, UUNI, A1, A2, ARE, ACH, BCH, EX2, UA, XX

IF(UUNI.LT.UPL) THEN
    XX = MIN(UPL, PU)
    URE = URE+ARE*(UUNI-XX)/NCYIN**2
    NCYIN = NCYIN+1
ENDIF
CALL AXSS23(URE, STRE, UC, FC, A1, A2, UUL, ERE)

UA = UPL+ACH*(UUN-STUN/EUN-UPL)
ECH = STUN/(UUN-UA)
STCH = BCH*MAX(STUN, STRE)/NCYIN**2
STCH = MAX(STRE, MIN(STCH, 0.5*STRE))
UCH = UA+STCH/ECH

RETURN
END

SUBROUTINE AXSS27(UC, EMO, UPL, UCL, ECL, PU, PST, U1, ST1, E1, U2, ST2, E2, 
    * 
    UUN, UCH, STCH, ECH, EPLU, GPLU, EPLR, GPLR, EX1)

INTERNAL VARIABLES
ESEC = Secant modulus between points 1 and 2

Programmed by: Francisco J. Crisafulli
Data/Version: 01-Sep-1995/1.0

REAL UC, EMO, UPL, UCL, ECL, PU, PST, U1, ST1, E1, U2, ST2, E2
REAL UUN, UCH, STCH, ECH, EPLU, GPLU, EPLR, GPLR, EX1, ESEC, Z0

DATA Z0 /0.0/
U1 = UCH
ST1 = STCH
E1 = ECH
ST2 = MIN(PST, Z0)
EPLU = GPLU*EMO/(1.0+UUN/UC)**EX1
U2 = PU
EPLR = GPLR*EPLU
ESEC = (ST2-ST1)/(U2-U1)
IF(UPL.GT.UCL) THEN
  E2 = EPLR
ELSE
  ECL = 0.15*EPLR
  IF(UCL.GT.0.0.AND.U2.GT.0.0) THEN
    E2 = ECL
  ELSE
    E2 = ECL+ABS(U2/UPL)*(EPLR-ECL)
  ENDIF
ENDIF
E2 = MIN(E2,0.2*E1,0.9*ESEC)
C
RETURN
END
C
SUBROUTINE AXSS28(PU,PST,UC,U1,ST1,E1,U2,ST2,E2,EMO,UPL,UUN,UNI,EPLU,GPLU,EX1)
C
******************************************************************************
C Axial cyclic stress-strain model.
C Calculate parameters for unloading and reloading rules 2,4,5.
C E1 is defined in different ways for each case
C
C INTERNAL VARIABLES
C ESEC = Secant modulus between points 1 and 2
C
C Programmed by: Francisco J. Crisafulli
C Date/Version : 01-Sep-1995/1.0
C******************************************************************************
C
REAL  PU,PST,UC,U1,ST1,E1,U2,ST2,E2,EMO,UPL,UUN,UNI
REAL  EPLU,GPLU,EX1,ESEC

UUNI = PU
U1  = PU
ST1 = PST
ST2 = 0.0
U2  = MAX(U1-1.5*ST1/E1,UPL)
ESEC = (ST2-ST1)/(U2-U1)
EPLU = GPLU*EMO/(1.0+UUN/UC)**EX1
E2  = MIN(EPLU,0.5*ESEC)
C
RETURN
END