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DETECTING MULTIPLE STRUCTURAL BREAKS IN THE
MEAN OF A TIME SERIES

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The Application of Regression Trees to Detecting Multiple Structural Breaks in the Mean of a Time Series

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A non parametric approach is proposed for dating structural breaks whose number and dates of occurrence are *a priori* unknown. In particular, the case of level shifts is considered. For the purpose of locating the breakdates the method exploits, in the framework of least square regression trees, the contiguity property introduced by Fisher for grouping a single real variable. The proposed approach is applied to study the changes in growth rates in Campito Mountain bristlecone pines, a standard example of a long memory time series.

Key Words: Fisher's algorithm; Least square regression trees; CUSUM; Bai and Perron procedure.

1 INTRODUCTION

The detection of structural breaks is an important problem in time series analysis that has attracted the attention of both statisticians and econometricians for more than forty years (for a review see Hansen 2001). In this

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paper we focus on the problem of detecting multiple breaks occurring at unknown dates. To this aim, recently, Bai and Perron (1998, 2003) have proposed an estimation procedure that makes use of a dynamic programming approach that can be traced back to Fisher's method of exact optimization (Fisher, 1958) for grouping a single real variable into mutually exclusive and exhaustive subsets having maximum homogeneity, i.e. minimizing the within-group sum of squares.

Regression trees (RT) are a non-parametric method for fitting piece-wise constant functions to a data set. The boundaries between the constant functions are steps. These are often regarded as a drawback or weakness if the variable being modeled is continuous. However, change point analysis is undertaken when it is believed that the underlying data generating process is discontinuous, or at least changes rapidly between two distinct states, at the change or break point. In this context it seems natural to interpret the discontinuities in the RT as the break points in the process. There is a substantial body of literature dealing with change point detection and location in time series. However, apart from the papers of Cooper (1998) and Cappelli, Penny, Rea and Reale (2007) the application of RT to this problem appears to be an overlooked tool.

In this paper we describe the use of RT in the location of structural breaks in time series to define contiguous partitions in section (2). Section

(3) presents the results of a simulation study on using RT to find structural breaks in the mean. Section (4) presents an illustrative application to the changes in the tree-ring index of Campito Mountain, California. Brief concluding remarks follow in Section (5).

2 REGRESSION TREES

Hyafil and Rivest (1976) showed that the problem of obtaining an optimal binary decision tree is NP-complete. Thus, with the exception of small data sets, it is computationally impractical to search for an optimal tree. Despite their sub-optimal results, RT have found wide application owing to their computational efficiency. This allows them to handle large data sets with relative ease. Probably the best known regression tree methodology is the Classification and Regression Tree (CART) of Breiman, Friedman, Olshen and Stone (1984) to which the reader is referred for a detailed description of RT.

The recently proposed procedure of Bai and Perron (1998, 2003) (BP) is based on Fisher's (1958) method of exact optimization. While the BP produces an optimal partition of a time series, it is a computationally expensive procedure.

A number of financial and geophysical time series such as stock market

volatilities, tree-ring indices, mud-varve sequences, and ice core data are very long. Sometimes the geophysical series exceed 10,000 data points with annual resolution. The long compute times and large memory requirements of the BP makes its use impractical on these types of series. We show that RT provide a practical alternative for locating structural breaks in the mean of long time series data. In the context of RT time assumes the role of the predictor variable when, in fact, it is merely a counter. A common source of poor predictive performance in RT is that the distribution of the response variable is not orthogonal or parallel to the predictor variables (see Fig 8.12 of Hastie, Tibshirani and Friedman 2001, for an example). This problem does not arise in univariate time series. This gives us reason to suspect they will perform well in the location of structural breaks.

There are several questions to be addressed in applying RT to time series.

These are:-

1. As RT fit piece wise constant functions to data do RT discover or impose breaks on time series?
2. What is the effect of serial correlations on RT performance in detecting structural breaks?
3. Given that observations in time series are, in general, non-interchangeable can cross-validation be used in tree selection?

Consider the time series model:

$$y_t = \mu_g + \epsilon_t, \quad g = 1, \dots, G, \quad t = T_{g-1} + 1, \dots, T_g, \quad (1)$$

where G is the number of regimes (and $G - 1$ the number of breakdates), y_t is the observed response variable and ϵ_t is the error term at time t (we adopt the common convention that $T_0 = 0$ and $T_G = T$ where T is the series length). This is a pure structural breaks model because all the model coefficients are subject to change and it has been employed by Bai and Perron (2003) to detect abrupt structural changes in the mean occurring at unknown dates. The problem is to estimate the set of breakdates $(T_1, \dots, T_g, \dots, T_{G-1})$ that define a partition of the series

$$P(G) = \{(1, \dots, T_1), \dots, (T_{g-1} + 1, \dots, T_g), \dots, (T_{G-1} + 1, \dots, T)\},$$

into homogeneous intervals such that $\mu_g \neq \mu_{g+1}$. The BP estimation method is based on the least squares principle: for each G -partition, the corresponding least square estimates of the μ_g 's are obtained by minimizing the within-group sum of squares

$$WSS_{y|P(G)} = \sum_{g=1}^G \sum_{t=T_{g-1}+1}^{T_g} (y_t - \mu_g)^2. \quad (2)$$

The estimated breakdates $(\hat{T}_1, \dots, \hat{T}_g, \dots, \hat{T}_{G-1})$ are associated with the partition $P^*(G)$ such that $P^*(G) = \arg \min_{P(G)} WSS_{y|P(G)}$. In this approach, the breakdate estimators are global minimizers since the procedure considers

all possible partitions by using the dynamic programming approach proposed by Fisher's (1958) to find the least squares partition of T *contiguous* objects into G groups. His efficient algorithm exploits the additivity of the sum of squares criterion using a dynamic programming approach (Bellmann and Dreyfus 1962) that when applied to ordered data points finds the global minimum. Despite the computational saving, the method cannot deal with high values of T and G and the same remark holds for the BP's procedure, even with today's computing power.

In the case of time series data Hartigan (1975) provides an excellent justification in favor of the (faster) binary division algorithm: suppose that the observed time series consists of G segments within each of which the values are constant, i.e. model (1) becomes a piecewise constant model with $\epsilon_t = 0$. The series can be partitioned into G segments where for each segment the within-group sum of squares is zero. This partitioning can be identified by a sequential splitting algorithm such as the one in RT.

The binary splitting used by RT does not necessarily provide the optimal partition, however if the correct number of partitions is identified, because the observations are ordered by time, misplacements can occur only on the boundaries. As discussed in Hansen (2001), although structural breaks are treated as immediate, it is more reasonable to think that they take a period of time to become effective, thus misplacements on the boundaries are not a

concern.

Despite the potential suboptimal solutions, RT has the advantage over global search algorithms as used in BP method in being computationally fast. The global search algorithm requires $O(n^2)$ steps, whereas RT, at any tree node requires $O(n(h))$ steps to identify the best split, where $n(h)$ is the number of values in node h .

Another distinction between RT and the global search algorithm is in the selection of the final partition and the consequent set of break dates. Indeed, partitioning methods such as Fisher's (and BP's) have the drawback of producing a single partition for a prespecified value of G and, in general, it is advisable to produce and compare more partitions by varying G . In the case of RT this is not a concern because the method produces a hierarchical tree structure associated with the breaks. The selection of the final set of breakdates can be handled within the framework of tree methods by pruning. Pruning is the process of retrospectively discarding branches whose contribution to the reduction of the error is negligible (for details see Breiman *et al.* 1984, Chap. 3). In this way a nested sequence of partitions and candidate breakdates is created. In order to select the optimal sequence corresponding to the actual number of break dates and distinct subperiods present in the data, cross validation (CV), the sequential testing procedure of BP and model selection criteria can be employed (for details on the use of model

selection criteria in regression trees see Su, Wang and Fan 2003). Moreover, the inspection of the tree structure allows an insight into the partitioning process. Breakdates can be ordered based on their position in the tree and the reduction of the error function achieved. For this reason manual pruning based on subjective choices of the analyst can be preferred to an automatic procedure, see Zhang and Singer (1998, Chap. 4).

Finally, note that if estimation is not the sole concern and one wants to test for structural breaks or model the observations in the segments, it can be appropriate to consider restrictions on the possible values of the breakpoints as suggested by BP. Indeed, extra conditions on the reduction in deviance and/or on the length of the subperiods are easily handled within the tree growing recursive partitioning approach of RT.

3 SIMULATION EXPERIMENTS

In this section we present the results of simulation experiments using RT to detect structural breaks. For comparison purposes we used the estimation procedure proposed by BP based on the Fisher's method of exact optimization.

For the RT method we used tree growing and pruning procedures as implemented in `tree` as a contributed package in the R software. For the

BP method we used the contributed package `strucchange` (Zeileis, Leisch, Hornik and Kleiber 2002) in R.

3.1 UNCORRELATED SERIES WITH A SINGLE BREAK

A set of simulations were run with series of uncorrelated observations drawn from standard Normal, geometric and gamma distributed populations with a single break point at the midpoint of the series giving two equal lengthed regimes. In most simulations there were 16 regime sizes, 5^2 to 20^2 observations in length. Thus in the graphs of the results the axis labeled Regime Number is non-linear in scale. The break sizes ranged from 0.05 to 2 standard deviations in steps of 0.05 standard deviations. 1,000 replications of each combination of regime length and break size were run. For the comparable results from the BP the break sizes ranged from 0.1 to 2 standard deviations in steps of 0.1 standard deviations. The longest series were composed of 256 data points per regime (regime 16). 100 replications were run of each parameter combination. The results for the standard normal simulations are presented here, the remainder are available on request from the authors.

Figure (1) presents the results for the RT and BP for a single break at the mid point of the series. Note that direction of the regime number axis is

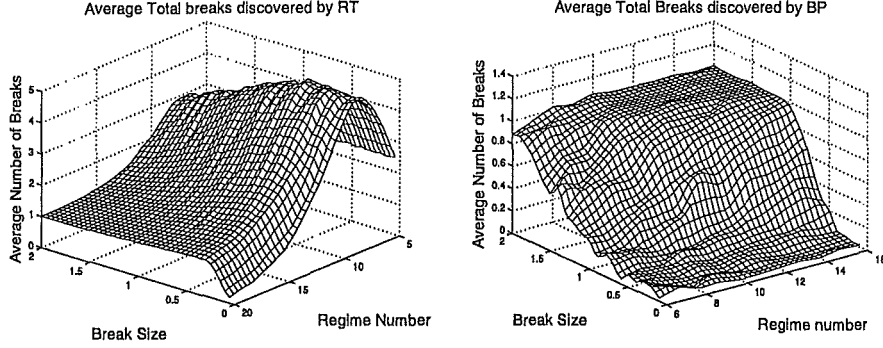


Figure 1: *Left panel: Average total number of breaks found by RT. Right panel: Average total number of breaks found by BP. Simulated series of uncorrelated observations with Gaussian noise and a single break. Pruning based on cost complexity using deviance. Break size is measured in terms of standard deviations. Regime number refers to the length of the series, regime 5 is length 5^2 and the series is 2×5^2 long, regime 20 is length 20^2 and the series is 2×20^2 long.*

reversed in the BP results compared to the RT results. When the series are short the RT is very prone to over-fitting but that this tendency gradually disappears by a series length of approximately 700 data points (regime 18 or 19). RT had a tendency to over-fit for smaller breaks and for shorter regimes. BP tended to under-fit for smaller breaks and for shorter regimes.

We tested the RT ability to find the location of the break when the break was not at the mid-point but was within the first half of the series. We

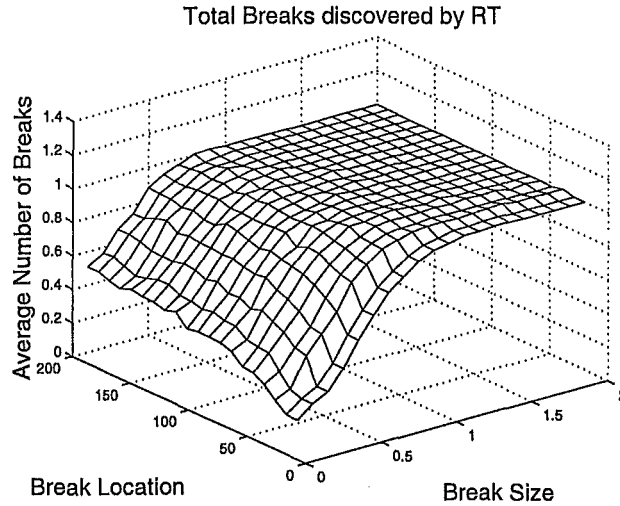


Figure 2: *Average total number of breaks found by RT. Simulated series of 400 uncorrelated observations, Gaussian noise and a single break at different locations. Pruning with Bayesian Information Criterion.*

examined series with 100, 400, and 1600 observations. The BP was not run for comparison. We present the results for the 400 data point series, the remainder are available on request from the authors.

The results are presented in Figure (2). The dominant factor in locating a break is its size rather than its location. Unsurprisingly, it was more difficult for the RT to locate the break when it is close to the end of the series.

3.2 UNCORRELATED SERIES WITH MULTIPLE BREAKS

To investigate the performance of RT for series with multiple breaks we simulated series with 4 breaks:

$$y_t = \mu_{r_i} + \epsilon_t \quad (3)$$

where

μ_{r_i} = the mean of regime r_i ; $i = 1, \dots, 5$

ϵ_t = noise terms drawn from an $N(0,1)$, gamma, or geometric distribution.

In all simulations $\mu_{r_i} = 0$ for $i = 1, 3, 5$ and $\mu_{r_4} = -\mu_{r_2}$. The value of μ_{r_2} started at 2 standard deviations and was decremented to 0.05 in steps of 0.05.

When the BP was used to detect breaks in the series, because the amount of computation required, the value of μ_{r_2} was sometimes decremented to 0.1 in steps of 0.1.

The resultant series were square waves with an amplitude of break size with Gaussian (or other) noise of constant variance imposed on them. We present the results for the Gaussian noise series. The remainder are available on request from the authors.

We also examined three tree-pruning methods. The deviance-based cost complexity, the default method in the R package, the Bayesian Information Criterion (BIC) (Schwarz 1978) and cross-validation. Of the range of infor-

mation criteria available we selected the BIC on the basis of Su *et al.* (2004) and because it is more robust to non-Gaussian error structures than the AIC (Akaike 1970).

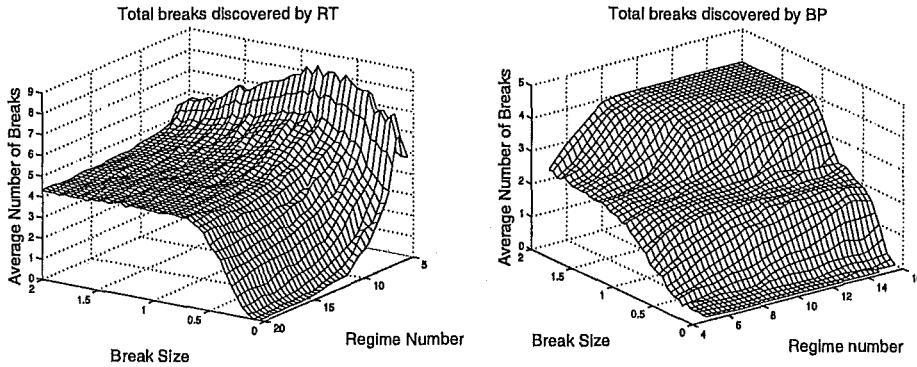


Figure 3: *Left panel: Total number of breaks found by RT in the noisy square wave simulations. Deviance based cost-complexity pruning. Right panel: Total number of breaks found by BP. The series have four breaks and Gaussian noise.*

It is well-known that tree-based procedures over-fit small data sets (Cooper 1998). This can be seen in the left panel of Figure (3). However, as the series lengthens the problem of overfitting reduces and is not evident by regime 15 (length of about 1000 data points). This is where the compute times of the BP begin to become excessive. The BP method underfit for small breaks particularly for for short series.

The problem of overfitting in the RT method can be reduced by a more

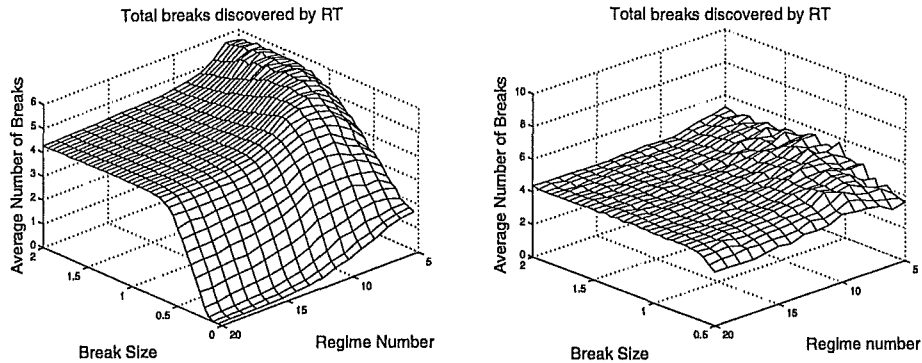


Figure 4: *The average total number of breaks found by RT when using BIC pruning (left panel) and leave-one-out cross-validation (right panel). The series have four breaks and Gaussian noise.*

aggressive pruning criteria than the usual R-default cost-complexity pruning introduced by Breiman *et al.* (1984). The results of BIC and leave-one-out cross-validation are presented in the left and right panels of Figure (4) respectively.

In time series data observations are usually not interchangeable. Thus the common 10-fold cross-validation cannot be used. The alternative we considered was leave-one-out cross-validation. This minimizes the disturbance to any correlation structure in the data but it is much more computationally expensive than the BIC, requiring N trees to be constructed where N is the number of data points. The leave-one-out cross-validation pruning reduced model overfit considerably. For routine tree selection we recommend the BIC

for its robustness to non-Normality, but note that for series with more than approximately 600 data points the BIC becomes indistinguishable from the default cost-complexity pruning.

3.3 SERIES WITH CORRELATED DATA

To investigate the ability of RT and BP to detect structural breaks in correlated data we analyzed series with AR(1), AR(2), AR(5), and MA(1) correlations. The results for the AR(1) series are presented here and the remainder are available on request from the authors.

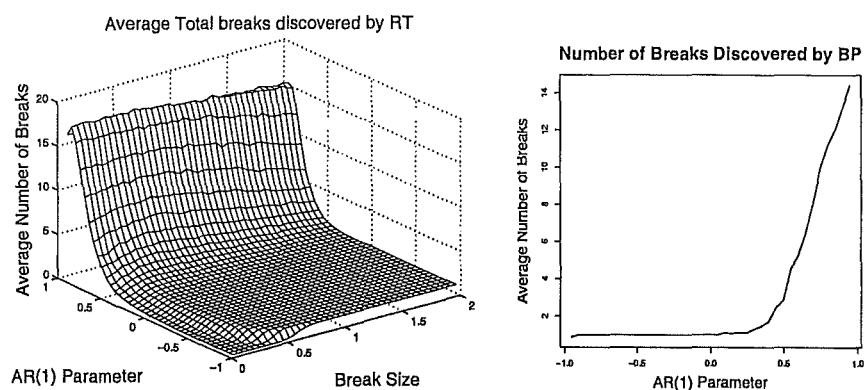


Figure 5: *Left panel: Average total number of candidate breaks found by RTs in series with AR(1) correlations and a single break. Default pruning. Right panel: Comparable results from the BP for a break size of two.*

The left panel of Figure (5) show the results for the RT. The right panel the results of the BP for a break size of two standard deviations. All series

were 1024 observations long. It should be noted that the series standard deviation changes with the magnitude of the AR parameter. The break size was measured in terms of the input noise series.

Both RT and the BP are robust to negative values of the AR parameter and to small positive values (less than 0.25). However, neither break detection method is robust to larger positive values, each finds increasing numbers of spurious breaks as the AR parameter approaches unity.

We found that the RT had similar robustness to MA correlations except that they induced far fewer spurious breaks when the MA parameter was higher than 0.25. In the worst case an average of less than one spurious break per series was reported.

3.4 SERIES WITH HETEROSCEDASTICITY

We examined RT robustness to heteroscedasticity by simulating series with a break at the mid-point and different standard deviations in the two halves. The first half always had a standard deviation of one. The break size is stated in standard deviations of the first half. The second half had a standard deviation ranging from one to 2.95. We examined two lengths of series, 800 and 1800 data points. We did not run BP for comparison due to the excessive computational times it would require.

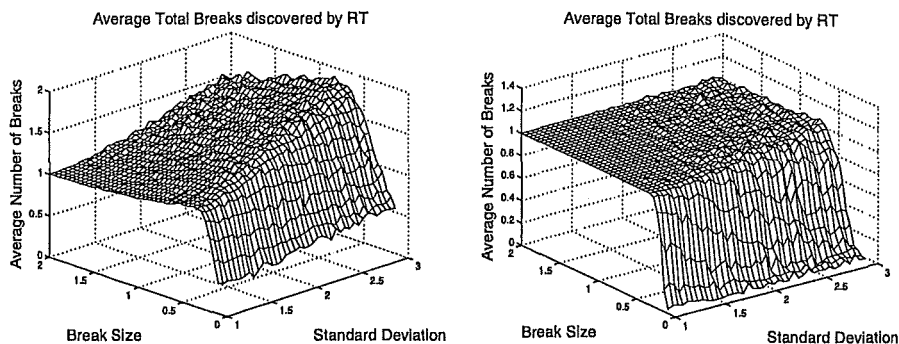


Figure 6: *Average total number of breaks found by RT in series with heteroscedasticity. Left panel - 800 data point series. Right panel - 1800 data point series. Deviance-based cost-complexity pruning. The series had one true break.*

The results are presented in Figure (6). RT is more robust to heteroscedasticity in the longer series than in the shorter series. This is consistent with the other observations presented in this paper that the problem of over-fitting declines with increasing series length.

4 APPLICATION: CAMPITO MOUNTAIN TREE RING INDEX

To illustrate the RT method in comparison to the BP method we applied both methods to the Campito Mountain bristlecone data. The dataset is available from the website <http://www.sci.usq.edu.au/staff/dunn/Datasets>

`/Books/HipelMcLeod/lamarche/campito.1` and is also available in the library of the contributed package `tseries` in R. These data are regarded as a standard example of a long memory process (see Doukhan, Oppenheim and Taqqu 2003). Klemes (1974) argued that the appearance of long memory in geophysical time series was often a statistical artefact caused by a non-stationary mean. Despite Klemes' arguments and numerical experiments he could not demonstrate the correctness of his proposal from data analysis. The physical cause or causes of long memory is still an open question. We examine the Campito data with RT and BP to see if a non-stationary mean can be detected.

We ran the BP on the Campito data twice. The first time with the minimum segment size set to 0.05 times the length of the series or approximately 270 data points. The second time with the minimum segment size set to 0.01 or approximately 54 data points. The second run took almost 243 hours of CPU time on a SunBlade 1000 with 750Mhz UltraSPARC-III processor and 2Gb of memory. By comparison the RT took 0.16 seconds. This supports our contention that regression trees are a practical, if sub-optimal, tool for analyzing long time series.

Figure (7) presents the Campito data with the break points marked for both the regression tree and the BP with the minimum segment size set to 0.05. The RT reported 12 break points, while the BP reported 13. Some of

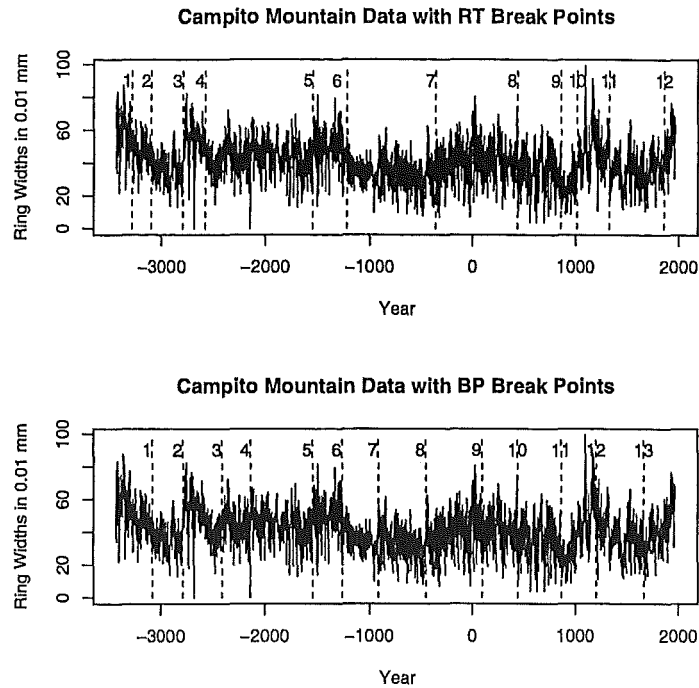


Figure 7: *Regression (RT) and Bai and Perron (BP) break points marked on the Campito data.*

the break points are essentially identical. With a minimum segment size of 0.01 the BP reported 40 break points. We regard 40 breaks to be an excessive number and do not report them here. They are available on request from the authors. We attribute the difference in the reported numbers of break points between the RT and the BP to be due to the penalty terms applied in pruning the tree. The pruning phase of tree selection favors small (parsimonious)

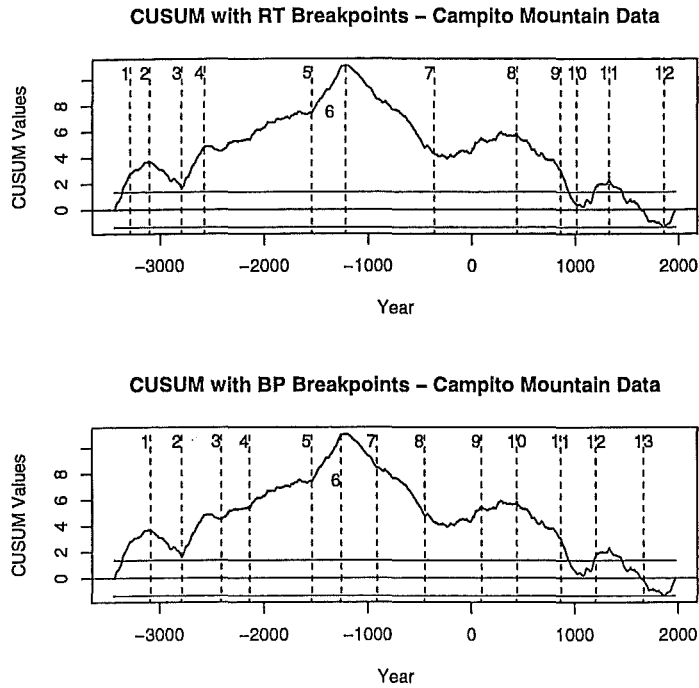


Figure 8: *Regression tree (RT) and Bai and Perron (BP) break points marked on CUSUM graphs for the Campito data.*

trees. It is not clear how to select the optimal number of breaks in the BP.

Figure (8) presents both the tree and BP break points on a cumulative summation (CUSUM) (Brown, Durbin and Evans 1974) graph. The CUSUM graph is interpreted subjectively by examining the slope of the plot with regard to the five percent significance lines plotted parallel to the horizontal axis. When the slope of the CUSUM line is positive the tree is experiencing

an above average growth rate. Conversely, when the slope is negative the tree is experiencing below average growth. In these plots there are several places where the RT breaks appear more physically reasonable than the BP breaks.

The first is the regime between breaks three and four in RT, which most closely relate to breaks two and three in the BP. The RT places the break at the end of a period of above average growth while BP includes a short period of below average growth. Similarly RT breaks nine through 12 separate out the above and below average growth periods. In the corresponding period BP breaks 11, 12, and 13 generate regimes which mix above and below average growth rates.

On the other hand, for break point six, the RT places this at the highest point on the CUSUM plot whereas the BP displaces this slightly to the left. Examining the break points in Figure (7) the BP break point appears the more reasonable.

5 CONCLUDING REMARKS

We have proposed a new application of RT by using them as a data driven nonparametric procedure for detecting multiple structural breaks in the mean occurring at unknown dates. The simulations have provided us with answers

to our three original questions. They are,

1. RT do impose spurious breaks when the series is short but this tendency disappears as the series becomes longer. This was seen in both single and multiple break simulations.
2. RT are robust to negative serial correlation and a small amount of positive correlation, but in this regard they are no worse than the BP.
3. Leave-one-out cross-validation can be used for tree selection but is computationally expensive.

The main advantages of the proposed approach are:

1. *simplicity* - it can be easily implemented or run with packages containing routines to grow and prune least squares regression trees;
2. *feasibility* - it can be used to find the least squares partition of an ordered sequence and is particularly suited to long series which are currently not practical to analyse with the BP;
3. *visualization* - it results in a nested hierarchy represented by a tree diagram that displays the whole partitioning process and allows the scientist to interact with the tree and make use of *a priori* knowledge.

Although RT do not necessarily find the global minimum, their results are comparable to those obtainable by applying the established BP when the

series are long. In the example data set the breakdates for both the RT and BP coincide at a number of points. Also, in our example data set some of the RT break points are more physically reasonable than the optimal break points of the BP.

The application to the Campito Mountain data shows that Klemes' contention that the Hurst effect is caused by a non-stationary mean is supported by both the RT and BP. However, the RT is computationally orders of magnitude faster than the BP for this series.

As with any statistical test or modelling procedure, regression trees must be used with care and discernment. For an experienced time series analyst who must deal with long series, RT provide a complementary procedure to the BP when detecting and locating structural breaks in the mean.

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