Harmonic State Estimation
and
Transient State Estimation

By

Kent K.C. Yu  B.E (Hons)

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List of symbols and abbreviations

\[ A \]  \quad \text{coefficient matrix}

\( A_{al} \)  \quad \text{auxiliary alpha node-inductance matrix}

\( \text{col}[A] \)  \quad \text{column space of matrix } A

\( C_{aa} \)  \quad \text{auxiliary capacitance matrix of alpha node}

\( C_{ba} \)  \quad \text{branch-node incidence matrix}

\( C_{cc} \)  \quad \text{branch capacitance matrix}

\( C_{lb} \)  \quad \text{line-branch incidence matrix}

\( C_{nl} \)  \quad \text{node-line incidence matrix}

\( \frac{dV_c}{dt} \)  \quad \text{rate of change of capacitor voltage}

\( \frac{dI_l}{dt} \)  \quad \text{rate of change of inductor current}

\( f \)  \quad \text{harmonic frequency}

\( E_i \)  \quad \text{emf source}

\( G_{ar} \)  \quad \text{auxiliary alpha node-resistance matrix}

\( h \)  \quad \text{harmonic order}

\( h \)  \quad \text{measurement matrix}

\( H_0 \)  \quad \text{null hypothesis}

\( H_A \)  \quad \text{alternative hypothesis}

\( H1 \)  \quad \text{phase A of high voltage side}

\( H2 \)  \quad \text{phase B of high voltage side}

\( H3 \)  \quad \text{phase C of high voltage side}

\( [h_{\text{reduced}}] \)  \quad \text{reduced measurement matrix}

\( [H] \)  \quad \text{measurement matrix}

\( [H_{\text{diag}}] \)  \quad \text{measurement matrix formed using diakoptical formulation}
$[H_{op}]$ measurement matrix formed using systematic formulation

$I$ identity matrix

$I_b$ branch current vector

$I_c$ current vector of capacitive branch

$I_{li}, I_{lj}$ line current vector

$I_n$ vector of node injection current

$I_r$ current vector of resistive branch

$J$ indication index

$K_{lb}$ branch-node incidence matrix for current

$K_{vb}$ branch-node incidence matrix for voltage

$K_{bn}$ branch-node incidence matrix

$K_{cn}^T$ capacitive branch-node incidence matrix

$K_{cc}^T$ capacitive branch- alpha node incidence matrix

$K_{lu}^T$ inductive branch- alpha node incidence matrix

$K_{ib}^T$ inductive branch- beta node incidence matrix

$K_{ln}^T$ inductive branch-node incidence matrix

$K_{iy}^T$ inductive branch- gamma node incidence matrix

$K_{rm}^T$ resistive branch-node incidence matrix

$K_{rb}^T$ resistive branch- beta node incidence matrix

$K_{ra}^T$ resistive branch- alpha node incidence matrix

$K_k$ Kalman filter gain

$k_i$ coefficient of nullspace vector

$L_{ii}$ self inductance

$L_{ij}, L_{ji}$ mutual inductance
\( L_{ii} \) inductance matrix

\( L_{yy} \) auxiliary inductance matrix of gamma node

\([L,]\) inductance matrix

L1 phase A of low voltage side

L2 phase B of low voltage side

L3 phase C of low voltage side

\( M_g \) auxiliary inductance matrix

\( N \) dimension of subspace

\( \text{null}[A] \) nullspace of matrix A

\( P \) real power

\( P^{' k} \) \textit{a priori} estimate error covariance at time \( k \)

\( P^{' k} \) \textit{a posteriori} estimate error covariance at time \( k \)

\( P^{' k+1} \) \textit{a priori} estimate error covariance at time \( k+1 \)

\( P^{' k+1} \) \textit{a posteriori} estimate error covariance at time \( k+1 \)

\( P_{\Delta x_k} \) probability of \( \Delta x_k \)

\( Q \) reactive power

\( Q_k \) process noise covariance matrix

\( R_k \) measurement noise covariance matrix

\( r \) residual vector

\( r'' \) residual test

\([R,]\) resistance matrix

\( R_{\beta} \) auxiliary resistance matrix of beta nodes

\( R_i \) core losses

\( R_{\beta}, R_{\alpha} \) auxiliary inductive branch resistance matrix

\( R_{rr} \) resistance matrix

\( R_{\ldots} \) residual covariance matrix

\( R_{\xi} \) state estimate covariance matrix

\( R_{\ldots} \) measurement covariance matrix

\( s \) sample std. deviation

\( S \) sensitivity matrix
\( t \)  
\( UR_{sh} \)  
\( UR_{shr} \)  
\( UC_{sh} \)  
\( UC_{shr} \)  
\( U_n \)  
\( U_n^r \)  
\([U]\)  
\( u \)  
\([V]^T\)  
\( V \)  
\( V_\alpha \)  
\( V_b \)  
\( V_\beta \)  
\( V_c \)  
\( V_l \)  
\( V_n \)  
\( v_k \)  
\([W]\)  
\( w_k \)  
\( x \)  
\( \hat{x} \)  
\( \hat{x}^- \)  
\( x_{r-p} \)  
\( x_{m} \)  
\( \dot{x}_{t+h} \)  
\( x_{t+h} \)  
\( x_k \)  
\( x_{k+1} \)
\[ \dot{x} \] rate of change of state vector

\[ y \] dependent variable vector

\[ Y_{pp} \] primitive branch admittance matrix

\[ Y_{ln} \] line-node admittance matrix

\[ Y_{nn} \] nodal admittance matrix

\[ z_k \] measurement vector at time-step k

\[ \bar{z} \] measurement vector

\[ \Delta z_{k,k-1} \] mean measurement difference between time k and k-1

\[ \Delta t \] change in time

\[ \Delta x \] change in state vector

\[ \alpha \] alpha node

\[ \beta \] beta node

\[ e \] error vector

\[ \phi_k \] state transition matrix

\[ \phi_k^T \] transpose of the state transition matrix

\[ \mu \] population mean

\[ \theta^2 \] variance

\[ \gamma \] gamma node

**Abbreviations**

ATP Alternative Transient Program
dim dimension
GPS Global Positioning System
EMTP ElectroMagnetic Transient Program
EMTDC ElectroMagnetic Transient for DC
HSE Harmonic State Estimation
HV High Voltage
Inv Invercargill
inf infinity
kV kiloVolt
LV Low Voltage
Man Manapouri
MW MegaWatt
MVar MegaVolt-ampere reactive
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<tr>
<td>pdf's</td>
<td>probability distribution functions</td>
</tr>
<tr>
<td>PI</td>
<td>Pie</td>
</tr>
<tr>
<td>Rox</td>
<td>Roxburgh</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
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<tr>
<td>TCS</td>
<td>Transient converter Simulation</td>
</tr>
<tr>
<td>Tiw</td>
<td>Tiwai</td>
</tr>
<tr>
<td>TSE</td>
<td>Transient State Estimation</td>
</tr>
<tr>
<td>UMIST</td>
<td>University of Manchester, Institute of Science and Technology</td>
</tr>
<tr>
<td>$\mu s$</td>
<td>micro second</td>
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List of Publications

Publications associated with this research work (Appendix V):


K. K.C. Yu and N. R. Watson, “An approximate method for Transient State Estimation,” *accepted by IEEE*

K. K.C. Yu and N. R. Watson, “Transient state estimation via a diakoptical formulation methodology,” *under review by IEEE*
Abstract

This thesis describes the algorithms and techniques developed for harmonic state estimation and transient state estimation, which can be used to identify the location of disturbance sources in an electrical power system.

The previous harmonic state estimation algorithm is extended to include the estimation of time-varying harmonics using an adaptive Kalman filter. The proposed method utilises two covariance noise models to overcome the divergence problem in traditional Kalman filters. Moreover, it does not require an optimal covariance noise matrix of the Kalman filter to be used. The common problems faced in harmonic state estimation applications due to the influence of measurement bad data associated with measurements and the lack of measurement points, hence the system being partially observable, are investigated with reference to the Lower South Island of the New Zealand system.

The state estimation technique is also extended to transient state estimation. Two formulation methods are outlined and the development of the proposed methodology is presented. Fault scenarios with reference to the Lower South Island of the New Zealand system are simulated to demonstrate the ability of transient state estimation in estimating the voltages and currents of the unmeasured locations, and applying the estimated results to search for the fault location. The estimation results are compared with PSCAD/EMTDC simulations to justify their accuracy.
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Chapter 1  Introduction

1.1 Motivation of Research

The operation of an electrical power system is constantly at risk due to frequent occurrence of electromagnetic disturbances such as system faults, load/converter switching, lightning strikes and other intentional or unintentional events. Their undesirable effects propagate throughout the network and degrade the quality of service, even at points remote to the source of the disturbance. Concerns over such power quality problems have been increasing since they affect the normal operation of equipment that is connected to the network. Poor power quality also causes malfunction of protective devices and metering equipment, interference with communication circuits and control devices and overheating of components. Thus, maintaining adequate power quality has become an important task for electric utility companies.

In order to minimize the disruption caused by these electromagnetic transients, the disturbance source must be quickly identified and rapid remedial action taken. It is important that efficient tools and analysis methods are available to meet this requirement. Full simulation techniques such as HARMAC (for harmonics) and EMTP/EMTDC/ATP (for transients) have been widely used to study the behaviour of ac/dc systems subject to various conditions. By simulation, a system can be checked thoroughly to ensure the satisfactory operation and optimization of component parameters and controller settings. Switching operations, transients and other disturbance events can be simulated to ensure that the system is capable to withstand the resulting over-voltages and that protective devices and control schemes limit their effect. Despite its advantages, digital simulation requires complete knowledge of the system and event prior to the simulation and is therefore inadequate.

As an alternative to digital simulation, global assessment can be performed by full monitoring of the system states. Meters are installed at each location to record the busbar voltages and branch currents. With the use of Global Positioning System (GPS) technology, the measurement samples taken at the individual locations can be synchronized to the GPS signals within an accuracy of 1 µs. Complete monitoring, however has practical limitations, owing to the restricted number of monitoring points which is impractical for large systems. A better solution would be to combine partial measurements at optimal locations with state estimation technique to derive complete knowledge of the system states.
In the context of power quality, the state estimation concept has only been discussed with reference to harmonic distortion. Previously, a static Harmonic Static Estimation (HSE) algorithm has been developed by Zhen-Ping Du at the University of Canterbury, New Zealand. His work involved the development of practical models and algorithms for static HSE, which uses partial measurements and knowledge of the power system to derive harmonic flows at locations that are not measured directly. The algorithm was also extended to partially observable systems. However, his work assumed no time variation of harmonics, which is often not the case. Moreover, the models and algorithms are based on a noise-free environment. In practice, measurements are different from their true value, they may be affected by noise associated with transducers, transmissions links, etc. Some of the measurements might even contain bad data. It is therefore important to:

1. Investigate the effect of measurement noise/bad data on the performance of HSE;
2. To further extend the existing algorithm to allow estimation of time-varying harmonics.

Similarly, the state estimation technique can be extended to Transient State Estimation (TSE) to estimate the transient voltages and currents at the unmeasured locations. It would provide an effective method to assess the system transient response and identify the possible location of the disturbance/fault sources.

1.2 Aim of present work

With current computer technology and reliable mathematical models and numerical techniques, the two popular methods used for transient simulation of electric power systems are Electromagnetic Transient type Programs and State Variable Analysis. A major disadvantage of the computer simulation methods is that the complete system topology must be known for accurate assessment. In a practical situation, an operating problem is usually encountered first and the ideal solution is to identify the source location of the problem as quickly as possible. Given that global assessment is impractical in a real-size system due to cost considerations, an optimal location of measurement transducer placement, combined with power system computer modelling can be used to derive acceptable estimation of the voltages and currents at unmeasured locations. This forms the basis of harmonic state estimation and transient state estimation. This concept has only been discussed recently, and further work is still needed to achieve general acceptance by the industry.

The purpose of the research is to further develop the existing HSE algorithm to
estimate dynamic harmonic injection and investigate the general HSE performance under the influence of bad data associated with bad data and the solvability of partially observable system. The harmonic injection in a power system is variable by nature mainly because power electronic devices operate at variable power level continuously. To be capable of tracking harmonic content with time, the use of Kalman Filter is investigated. Although Kalman Filter has become a popular choice in many tracking applications, it inherently suffers from divergence problem caused by the “dropping off” effect of the filter. As the filter resides in the steady-state condition, it becomes less sensitive to parameter variations and begins to lose its ability in tracking dynamic properties. To overcome this problem, a carefully selected process noise covariance matrix (Q) of the filter is needed. The design of the Q matrix in a dynamic system is subject to many variables such as operating schedule and the behaviour of the system and is therefore difficult to obtain. The aim of the present work is to develop a dynamic HSE method that overcomes the aforementioned problems.

Another aspect of the research is the development of practical models and algorithms for transient state estimation. At present, when a disturbance event is recorded, there is a lack of efficient fault identification procedure. Traditional identification process is performed using exhaustive search method, where the possible disturbance events are simulated and then comparing them with the actual transient response recorded. A good indication of the disturbance sources is usually achieved but it is time consuming to simulate all possible disturbance scenarios. The aim of TSE is to provide an alternative method to obtain a full system transient response from partial measurements and thereby providing an effective fault identification method.

1.3 Thesis Outlines

The thesis is divided into two main parts: Harmonic State Estimation and Transient State Estimation. The thesis presents the development of the existing algorithm to include estimation of variable harmonic injection and electromagnetic transients.

Chapter 2, a brief review of the previous work on HSE is given. This chapter also looks at the performance of HSE under the influence of bad measurement data. A method based on Monte Carlo simulation is used to assess the impact of bad data measurements. The influence of different transformer configurations on the observability and solvability of HSE, as well as solving partially observable systems using the Singular Valued Decomposition (SVD) technique are discussed. Three
separate simulations are performed to justify the findings.

A new HSE implementation that incorporates the ability to track varying harmonic injection using an adaptive Kalman filter is also presented. The dynamic HSE algorithm is applied to the test system taken from the Lower South Island of New Zealand and the results are compared to those obtained from PSCAD/EMTDC [1].

Chapter 3 extends the state estimation technique to transient state estimation. The proposed TSE algorithm models the power system as an equivalent lumped circuit and two different formulation methods are introduced to form the system equations. The performance of TSE based on the two formulation methods is investigated and their development and methodology are discussed. At the end of the chapter, a test system taken from the Lower South Island of the New Zealand is used to validate the proposed TSE algorithms.

Finally, Chapter 4 draws the concluding remarks and offers suggestions for future work.
Chapter 2  Harmonic State Estimation

2.1  Introduction of Harmonic State Estimation

State estimation has traditionally been used for fundamental frequency estimation taking information from the numerous real and reactive power meters used for revenue. Due to the large number of real and reactive power measurement points available, over-determined state estimation has been used to compute the probable values of the state variables by filtering out noise and bad data among the measurements. Only recently, the technique has been extended to harmonic state estimation to identify harmonic sources. With the increased use of power electronic devices, harmonic distortion has become a growing concern to electric power utilities and operating problems have been encountered as a result. Their ability to maintain the quality of service is greatly affected if the location of the harmonic sources is unknown. Traditionally, the location of the harmonic injection has been determined by installing harmonic meters at suspicious nodes and the harmonic content derived by inspecting the harmonic frequency spectra of the voltage and current waveforms. However, such an approach is impractical for large power systems due to the large number of harmonic meters required.

An alternative method for locating harmonic sources is through harmonic state estimation technique, which is a reverse process of harmonic penetration simulation. Harmonic state estimation has the ability to determine the harmonic flows in parts of the network where they are not directly metered, using only partial measurements. It provides an efficient approach in identifying harmonic injection within a power system. The framework of the harmonic state estimation described in Fig. 2.1 is based on a multi-frequency system model. It is formulated from the system admittance matrix and network topology at harmonic frequencies using synchronised measurement of harmonic voltages, harmonic branch currents and harmonic current injection at selected locations. Based on the system nodal admittance matrix and the placement of measurement points, the harmonic state estimator describing the measurements as a function to the state variables can be formed as:

\[ z = h(x) + \xi \quad (2.1) \]

where \( z \) and \( x \) are the measurements and state variables vectors respectively, \( \xi \) is the measurement error vector and \( h \) is the measurement matrix.

In a three-phase HSE model, asymmetrical conditions such as circuit mutual coupling,
system unbalance can also be considered.

![Diagram showing inputs and outputs of a Harmonic State Estimator](image)

**Fig. 2.1. Process of Harmonic state estimation**

HSE problems can be categorized as over-determined, completely-determined or under-determined depending on whether the number of independent measurement equations is greater, equal or less than the number of state variables. For an over-determined system, the system is said to be fully observable since there exists a unique solution for the state variables. An over-determined system contains redundant measurements that can minimise the effect of any bad data and overcome the problem when one or more measurement information are temporary lost. For an under-determined system, the system is partially observable and only the state variables corresponding to the observable islands can be uniquely determined.

All state estimation techniques rely on accurate measurement data and network models to produce consistent estimation results. The measurements require adequate synchronisation; this can be achieved using Global Positioning System (GPS) time stamping, where the snapshot scans of the measurement points are synchronised with the GPS signals. Due to the limited number of harmonic meters used for revenue, HSE is usually performed in under-determined condition. Therefore, the quality and placement of measurements as well as the accuracy of modelling network topology can greatly affect the performance of HSE. Metering errors resulting from communication failure or large noise variance may produce erroneous HSE results and cause false alarm actions. This chapter looks at the influences of such errors on the performance of HSE. A new dynamic HSE implementation is also presented to identify time varying harmonic injection sources.

### 2.2 Previous work in Harmonic State Estimation

State estimation technique was first applied to electric power systems by Schewpe and Wilde [2]. The structure of the state estimation was based on single-phase, single frequency model and non-synchronised P, Q, V measurement set. This restricts the system to have pure sinusoidal current and voltage waveforms with constant
frequency and magnitude. Moreover, the system must be symmetric and operate under balanced three-phase conditions. Due to these restrictions, the applicability of state estimation techniques in asymmetrical power systems has been very limited. In the last decade, the use of the technique has been extended for harmonic state estimation. The main focuses have been multi-phase model, measurement and solution optimisation, observability and performance analysis and implementation of the technique. Dynamic harmonic state estimation has also become very popular as a result of the increased use of non-linear loads which draw time varying harmonic currents.

In the past, harmonic state estimation has been shown to be an efficient tool in analyzing the propagation of harmonic signals throughout the network. It has the ability to system-wide estimate busbar harmonic voltages and branch harmonic currents of the network. Harmonic current injections can be subsequently determined to reveal the location of harmonic sources. Heydt [3] and Najjar & Heydt [4] described a reverse power flow procedure to identify the sources of harmonic signals in electric power systems. The method estimates the harmonic spectrum of injection currents using real and reactive power which require no local phasor reference if the line measurements are used. However, the use of reactive power is always questionable as it can be misleading due to its lack of a generally accepted definition in the presence of waveform distortion.

Meliopoulos, Zhang and Zelingher [5] proposed a power system state estimation algorithm based on multi-phase and multi-frequency model to account for waveform distortion and harmonics. The method utilises three-phase voltage and current measurements only. The estimation is divided into real and imaginary parts where the Lagrangian function is used to optimize the solution and is solved via Cholesky factorization with a forward and back substitution. Sensitivity analysis is also presented to show the impact of asymmetry and imbalance on the state estimation performance.

Matair *et.al* [6] suggested a method for remote harmonic assessment using harmonic state estimation in a deregulated network. The measurement model developed is based on measurements of nodal current injections, nodal voltages and branch currents. In addition, zero nodal currents at non harmonic source nodes are used as virtual measurement to increase the observability of the estimator. The paper showed that location of measurements can be changed from HV level to MV level without compromising the performance of HSE.
Du, Arrillaga, Watson and Chen [7] proposed a method of harmonic source identification via harmonic state estimation. The same harmonic estimator described in [6] is used for the estimation. The estimated harmonic voltages and current injection are used to determine whether the suspicious sources are harmonic injectors or harmonic absorbers with respect to the overall impact to the network. The suspicious harmonic sources are interpreted as Norton equivalent circuit at harmonic frequencies in which unambiguous identification of the type of source, e.g. a passive load, can be achieved by inspecting the solution of their corresponding Norton impedance at several frequencies.

Farach, Grady and Arapostathis [8] included an optimal procedure for optimal sensors placement in HSE. The estimation model is basically formed by postulating a linear relationship between the Fourier transforms $V(\omega)$ and $I(\omega)$ of busbar voltages and busbar injections respectively through the use of bus-admittance and bus-impedance matrices of the network. Two approaches (Sensitivity analysis and minimum variance) were used in solving the estimation. The procedure for an optimal sensor placement presented used a sequential technique. It was shown from simulation of all combinations of one, two and three measurements that the optimal sensor location problem is sequential. This approach significantly reduces the number of possible combination as compared to a complete enumeration.

In Hartana and Richards [9], a simple HSE method is used. Current measurements at any harmonic order are expressed as a linear combination of busbar injected harmonic currents and measurement noise. Neural networks are applied to make initial estimates of harmonic sources behaviour in a power system. The initial estimates are then used as pseudo-measurements for the HSE. This approach, although reduces the number harmonic measuring instruments needed, requires neural networks to learn to associate the available power network data pattern using patterns of harmonic source behaviour.

Du, Arrillaga, Watson and Chen [10] introduced a fundamental methodology in the implementation of their HSE in [7]. The implementation included a software development environment, object-oriented model, graphical user interface and a database and algorithm.

Several papers have extended the HSE technique to monitor dynamic harmonic injections. Beides and Heydt [11] presented a Kalman filter methodology to obtain
optimal estimates of the power system harmonic content. The method estimates the power system bus voltage magnitudes and phase angles at different harmonic levels using real and reactive power measurements resulting in a Jacobian matrix as the measurement matrix.

Ma and Girgis [12] suggested the use of Kalman filter in HSE for identification and tracking of harmonic sources. The state variable used is the injection current. The model included system admittance matrix information that allows harmonic state estimation across the power network. Their proposed technique is frequency dependent and requires the fundamental frequency to be fixed and known. This is rarely the case in practice as the fundamental frequency is time varying in dynamic systems. Similarly, Girgis, Chang and Makram [13] also proposed a fundamental frequency dependent state model using a Kalman recursive measurement scheme for harmonic tracking.

The use of Kalman filter in dynamic harmonic state estimation has gained acceptance in recent years, and has been applied in a wide variety of applications. However it has become apparent that the recursive Kalman algorithm is strongly dependent on the a priori information of the process and measurement noise, both of which are usually not exactly determined. In addition, Kalman filters often suffer from a divergence problem after an extended period of filter operation. This phenomenon is due to the calculated covariance matrix becoming unrealistically small, so that undue confidence is placed in the estimates and subsequent measurements effectively have very small weight.

Fitzgerald [14] showed that under certain conditions, the mean-square errors may become unbounded with time and cause divergence. It was concluded that there was increased likelihood of divergence when the gain matrix Gs had a zero eigenvalue; a situation which is dependent on the selection of the process noise covariance matrix Q. The simulation showed that manipulation of the matrix Q can partially compensate or prevent the deterioration of the filter performance.

In the analysis of the manipulation of process noise covariance matrix Q, Saab [15] and Sangsuk-iam & Bullock [16] investigated the behaviour of the Kalman filter when incorrect noise covariances are used. Saab showed by simulation that the Kalman gains can be insensitive to the scaling of covariance matrices under incorrect noise covariances for a special case. Sangsuk-iam and Bullock also showed that the
Kalman filter is asymptotically optimal, even though the noise covariance used in the design is incorrect, but may cause the filter to diverge if the state matrix has an unreachable mode outside or on the unit circle.

Liu [17] considered the divergence problem in Kalman filter and introduced an adaptive Kalman filter based on correlation analysis to overcome the problem. The method utilizes a transient indication function to detect system transient. Once a transient has been detected, the error covariance is reset to increase the sensitivity of the Kalman filter, so that the new parameter variations can be matched again.

2.3 Harmonic Measurement Model

In the task of establishing the harmonic measurement model, harmonic voltage phasors at the busbars are chosen as the state variables. When these state variables are known, the system is completely specified as all the branch currents, shunt currents and current injection at each node can be determined. The harmonic measurement model becomes linear if busbar harmonic voltages, branch harmonic currents and harmonic injection currents are used as measurement data and hence Eqn.2.1 can be linearised as:

\[ \mathbf{z} = [H]\mathbf{x} + \mathbf{e} \quad (2.2) \]

where \([H]\) is the measurement matrix.

Each measurement taken adds a row to the measurement equation. The formulation of each measurement equation is dependent on the type of measurement. Consider a three-phase power system modelled as an oriented graph, in which let \( n \) to be a set of all nodes excluding the reference node, \( B \) to be a set of all branches and \( L \) to be a set of all lines connected to the nodes. Then the node-line incidence matrix \( C_{nl} \), line-branch incidence matrix \( C_{lb} \) and branch-node incidence matrix \( C_{bn} \) can be described as follows:

\[
C_{nl}(i,j) = \begin{cases} 
+1, & \text{if line } j \text{ is from node } i \text{ with injection arrow} \\
-1, & \text{if line } j \text{ is to node } i \text{ with injection arrow} \\
0, & \text{otherwise}
\end{cases}
\]
\[ C_{bi}(i, j) = \begin{cases} +1, & \text{if branch } j \text{ is from line } i \\ -1, & \text{if branch } j \text{ is to line } i \\ 0, & \text{otherwise} \end{cases} \]

\[ C_{bn}(i, j) = \begin{cases} +1, & \text{if branch } j \text{ is from node } i \text{ with injection arrow} \\ -1, & \text{if branch } j \text{ is to node } i \text{ with injection arrow} \\ 0, & \text{otherwise} \end{cases} \]

For each harmonic frequency, let \( V_n \), \( I_n \) be phasor vectors of node voltages and injection currents, \( I_l \) be phasor vector of line currents, \( V_b \), \( I_b \) be phasor vectors of branch voltages and currents, and \( Y_{pp} \) be primitive branch admittance matrix. Based on Kirchhoff’s voltage and current laws, and Ohm’s law,

\[ V_b = C_{bn} V_n \quad (2.3) \]

\[ I_n = C_{nl} I_l \quad (2.4) \]

\[ I_l = C_{lb} I_b \quad (2.5) \]

\[ I_b = Y_{pp} V_b \quad (2.6) \]

Given that

\[ C_{bn} = (C_{cn} C_{lb})^T \quad (2.7) \]

then for harmonic injection measurements, the measurement matrix \( [H] \) is the nodal admittance matrix \([Y_{nn}]\).

\[ I_n = [C_{bn}^T Y_{pp} C_{bn}] V_n = [Y_{nn}] V_n \quad (2.8) \]

Similarly for branch current measurements, described in Appendix I, the measurement matrix is

\[ I_l = [C_{lb}^T Y_{pp} C_{bn}] V_n = [Y_{ln}] V_n \quad (2.9) \]

and for nodal voltage measurements

\[ V_n = [I] V_n \quad (2.10) \]

In an under-determined case where the solution is not solved uniquely, additional measurement information such as virtual measurements and pseudo measurements can be added to the estimator. But, due to the lack of harmonic data, pseudo measurements are not normally viable for HSE. Instead, virtual measurements that provide additional information without metering are used, e.g. zero injection at
busbars without any load directly connected to it. Then (2.8) becomes

\[ 0 = [C_{ln}^T Y_{pp} C_{ln}] V_n = [Y_{nn}] V_n \]  \hspace{1cm} (2.11)

Given the nature of the interconnection between the branches and lines to the nodes, the resulting measurement matrix is mostly sparse. By using a sparsity technique, storing and computing only the non-zero elements, computation speed and memory requirement can be significantly reduced.

To solve the estimator, out of all the different optimization criteria such as maximum likelihood, conditional expected value, minimum variance etc, Weighted Least Squares (WLS) is commonly used in practical HSE problems. The method minimises the weighted sum of the squares of the residuals between the estimates and the actual measurements. This can minimise the effect of bad measurement data such as measurement noise and gross errors. The WLS solves the estimate in the sense as given by Eqn. 2.12 and is the only optimization criteria considered in the scope of this study.

\[ \text{Min } J(x) = \frac{1}{2} [z - h(x)]^T R^{-1} [z - h(x)] \]  \hspace{1cm} (2.12)

where \( E(ee^T) = R \) is the covariance matrix

The solution is then obtained by solving the equation

\[ x = [H^T R^{-1} H]^{-1} [H^T] z \]  \hspace{1cm} (2.13)

where \([R]\) is diagonal and contains the covariance of the measurements. It is possible to apply higher weighting to measurements that are known to be more accurate. The solution is uniquely defined if the measurement system is over-determined or completely-determined i.e. if the rank of \([H]\) is equal to the dimension of the state variable vector \(x\).

## 2.4 Influence of Measurement Placement

The design of a measurement system for HSE is a very complex problem due to the size of the power system and the conflicting requirements of estimator accuracy, reliability in the presence of transducer and data communication noise and failures,
solvability when changes occur in the network topology and cost minimisation. In fundamental frequency state estimation, power is the measured quantity due to the abundance of revenue meters. However, no pre-existing metering is available for HSE, thus harmonic voltage and harmonic current measurements are used, as this is what power quality instrumentation measures. Moreover, there is no generally accepted definition of harmonic power. The number of harmonic instruments available is always limited due to cost, and the quality of the estimate is a function of the number and location of the measurement points. In general, an increase in the number of measurement points would lead to an increase in the observability and solvability of the estimator. In some cases, measurements that are redundant do not provide additional information but incur unnecessary measurement cost. Therefore, it is important that the measurement placement is optimised. The design of optimal measurement placement is a compromise between the accuracy, the cost and the system observability. The design criteria can be any one of the following:

1. Minimise measurement placement for network observability.
2. Minimise the sum of the state estimate variances.
3. Maximise measurement system reliability.
4. Minimise the condition number of the gain matrix.
5. Minimise or limit measurement system cost.

Traditionally, measurements have been limited to symmetrical placement only. This restriction reduces the different combinations of measurement placements which may result in redundant measurements. It would be more appropriate if asymmetrical placements are used in the search of an optimal measurement placement. In the past, optimal measurement placement algorithms were achieved through heuristic solution, brute force searches or non-rigorous formulations such as constrained non-linear optimisation [18], Monte Carlo simulation and perturbation [19], sequential elimination [20] and sequential addition [21].

An alternative method to search for an optimal measurement placement is by locating the independent measurement points for each type of a.c system component. As a first step, the independence of the measurement equations that would be incorporated in the measurement matrix [H], from current measurements made on a transformer is investigated. This development can be used to search for the best transformer measurement placement and therefore limit the possible combinations when searching for the optimum measurement placement in HSE. Three transformer configurations investigated are star-g/star-g, star-g/star and star-g/delta.
2.4.1 Measurement Placement on Transformers
An equivalent π model is used for the transformer in the HSE algorithm, with the shunt components representing the off-nominal turns ratio and the series impedance is scaled for skin effect. The coupling between the different phases is dependent on the transformer winding configuration. For each measurement set the measurement equations that would be incorporated in the measurement matrix [H] used to produce \[h_{reduced}\] are given in Appendix II.

To test the independence of the measurement equations the rank of \[h_{reduced}\] is checked. Three transformer winding configurations are studied to show the effect of the winding connection on measurement placements. A three-phase two-winding transformer has six possible current measurement sites, measurement L1, L2, L3 and H1, H2, H3 are phase A, B and C measurements on the LV and HV sides respectively.

2.4.1.1 Star-g/Star-g Configuration
Testing the rank of the transformer measurement matrix \[h_{reduced}\] shows that there exist only three independent measurements for a star-g/star-g connected transformer out of six possible measurements (3 HV and 3 LV). Table 2.1 shows equation independence and observability for various combinations of measurement points. The first Yes or No in a cell shows whether the equations are independent, and the second indicates whether all the transformer currents are observable. For example although the two measurement points L1, L2 are independent (row 1 column 1 of Table 2.1), the transformer circuits are not observable due to being rank deficient. A rank of three will allow the transformer circuits to be observable. Measurement points L1, L2 and L3 are independent and the transformer is observable under these measurements (thus the Y,Y entry).

Symmetric line current measurements on either the LV side (L1, L2, L3) or HV side (H1, H2, H3) are adequate, since the zero sequence current flowing on either side will be evident from symmetric measurements made on either the LV or HV side. However, asymmetric line measurements e.g. L1, L2, H2, are inadequate since the phase C current and zero sequence are unknown. On the other hand, any combination of phase A, B and C measurements on LV and HV side are sufficient to give a fully observable transformer circuit.
Table 2.1 Measurement properties for star-g/star-g transformer

<table>
<thead>
<tr>
<th>Third measurement point</th>
<th>First two measurement points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1,L2</td>
</tr>
<tr>
<td>L1</td>
<td>*</td>
</tr>
<tr>
<td>L2</td>
<td>*</td>
</tr>
<tr>
<td>L3</td>
<td>Y,Y</td>
</tr>
<tr>
<td>H1</td>
<td>N,N</td>
</tr>
<tr>
<td>H2</td>
<td>N,N</td>
</tr>
<tr>
<td>H3</td>
<td>Y,Y</td>
</tr>
</tbody>
</table>

2.4.1.2 Star-g/star Configuration

Using the same approach as in Section 2.4.1.1, the results for star-g/star connected transformers are shown in Table 2.2. The transformer currents are observable with only two appropriate current measurements either, on the LV or HV side, due to the absence of zero sequence current flow in the transformer.

Table 2.2 Measurement properties for star-g/star transformer

<table>
<thead>
<tr>
<th>Third measurement point</th>
<th>First two measurement points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1,L2</td>
</tr>
<tr>
<td>L1</td>
<td>*</td>
</tr>
<tr>
<td>L2</td>
<td>*</td>
</tr>
<tr>
<td>L3</td>
<td>N,Y</td>
</tr>
<tr>
<td>H1</td>
<td>N,Y</td>
</tr>
<tr>
<td>H2</td>
<td>N,Y</td>
</tr>
<tr>
<td>H3</td>
<td>N,Y</td>
</tr>
</tbody>
</table>

2.4.1.3 Star-g/Delta Configuration

The observability of circuits with a star-g/delta transformer requires special consideration. Line current measurements taken only on the delta side are inadequate since the zero sequence current can flow into the star-g winding and can flow as a circulating current in the delta that is never measured. At least one measurement on the star-g side is required for the system to be observable, but if sufficient measurements are made on the star-g side, which is normally the LV side on the load transformers, then delta side measurements can be eliminated completely. Table 2.3 shows the measurement properties for a star-g/delta connected transformer.
Table 2.3 Measurement properties for star-g/delta transformer

<table>
<thead>
<tr>
<th>Third measurement point</th>
<th>First two measurement points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1, L2</td>
</tr>
<tr>
<td>L2</td>
<td>*</td>
</tr>
<tr>
<td>L3</td>
<td>Y, Y</td>
</tr>
<tr>
<td>H1</td>
<td>Y, N</td>
</tr>
<tr>
<td>H2</td>
<td>N, N</td>
</tr>
<tr>
<td>H3</td>
<td>Y, Y</td>
</tr>
</tbody>
</table>

2.4.2 Case Study: The influence of transformer configuration on measurement placement

This simulation focuses on the effect of transformer winding configuration on measurement placement and its influence towards the overall system observability. So far no rules have been given as to whether measurements are preferred on HV side, LV side, based on winding configuration, or asymmetrical placements. With reference to Tables 2.1-2.3, the best transformer measurement placement for the three common transformer configurations is suggested. This information reduces the number of possible placement arrangements needing investigation if a search technique is used.

The New Zealand Lower South Island test system

The test system used in this chapter is taken from the actual 220kV grid below Roxburgh in the South Island of New Zealand. The system consists of nine buses, eight transmission lines, three generators and five transformers as shown in Fig 2.2.

The nine buses are Roxburgh 220kV, Manapouri 220kV, Invercargill 220kV, Tiwai 220kV, Invercargill 33kV, Manapouri-1 14kV, Manapouri-2 14kV, Roxburgh-1 11kV and Roxburgh-2 11kV.

The eight three-phase transmission lines which are modelled with mutual coupling are:

One double circuit between Manapouri 220kV and Tiwai 220kV.
One double circuit between Manapouri 220kV and Invercargill 220kV.
One double circuit between Invercargill 220kV and Tiwai 220kV.
Two separate single circuits between Roxburgh 220kV and Invercargill 220kV.
Three generators of 11kV, 14kV and 14kV are connected to Roxburgh-1 11kV, Manapouri-1 14kV and Manapouri-2 14kV respectively.

The configurations of the five transformers are:
Delta (LV)/grounded star (HV) is connected between Manapouri-1 14kV and Manapouri 220kV.
Delta (LV)/grounded star (HV) is connected between Manapouri-2 14kV and Manapouri 220kV.
Delta (LV)/grounded star (HV) is connected between Roxburgh-1 11kV and Invercargill 220kV.
Delta (HV)/grounded star (LV) is connected between Roxburgh 220kV and Roxburgh-2 11kV.
Delta (HV)/grounded star (LV) is connected between Invercargill 220kV and Invercargill 33kV.

The harmonic sources used in the test system are 6 and 12 pulse rectifiers located at Invercargill 33kV and Tiwai 220kV respectively. A linear load is also placed at Roxburgh-2 11kV.

For the purpose of demonstrating the influence of transformer configuration on measurement placement, the transformer at Roxburgh 220kV/11kV is simulated as a test transformer with three different transformer winding configurations. Various combinations of measurement placement were looked at to investigate their effect on system observability.

The system contains 27 state variables taken as the harmonic barbar voltages. 18 virtual measurements are used, 6 measurements are taken as shown in Fig. 2.3 and 3 measurements are taken at the test transformer. For comparison, the “true” values are determined by performing a detailed three phase harmonic penetration study of the system. Results from the simulation at the measurement locations are supplied to the HSE algorithm as measurement data. The algorithm solves over-determined and under-determined systems using Gaussian elimination and Singular Value Decomposition (SVD). Three transformer winding configurations (star-g/star-g, star-g/star, and star-g/delta) are simulated.
Fig. 2.2 Three-phase diagram of the test system
**Simulation 1: Star-g/Star-g**

In the first simulation the test transformer is modelled as a star-g/star-g winding configuration. The measurements taken at the test transformer are shown in Fig. 2.4. From section 2.4.1, it was found that any measurement combination of phases A, B and C taken on LV or HV is sufficient. This condition is only satisfied with the measurement arrangement shown on the left hand side of Fig. 2.4. One possible way to identify unobservable islands within the system is to inspect the null space vector of the measurement matrix. Fig. 2.5 shows the entries of the nullspace vector (27x1) of the measurement matrix [\text{h}_{\text{reduced}}] for the two different sets of measurement placement in Fig. 2.4. For the asymmetrical placement, the non-zero entries in the
nullspace vector suggest that there are infinite number of possible solutions for the corresponding state variables (busbar voltages in this case) and hence the system is fully unobservable. Fig 2.6 shows the HSE results obtained using SVD.

![Diagram of two different sets of measurement locations for star-g/star-g transformer](image)

Fig. 2.4 Two different sets of measurement locations for star-g/star-g transformer

The results show the difference between the actual and the estimated values for all nodes, but these are small in magnitude due to the powerful solvability of SVD, a subject discussed in section 2.6. On the other hand, symmetric measurements taken at branches 16, 17, 18 allow the system to be fully observable and the estimation results show no difference between the actual and estimates as evident from Fig. 2.7. This demonstrates that the observability is highly influenced by the measurement placement on transformers even though only one measurement position has been changed.
Fig. 2.5. Nullspace vector of $[h_{\text{reduced}}]$, (star-g/star-g)

Fig. 2.6. Harmonic voltage magnitude for current measurements at branches 16,17,20 (star-g/star-g)
Fig. 2.7. Harmonic voltage magnitude for current measurements at branches 16,17,18 (star-g/star-g)

Simulation 2: Star-g/Star

In the second simulation, measurement placement on a star-g/star transformer winding configuration is shown in Fig. 2.8. Fig. 2.9 shows the nullspace vectors for both configurations. The system network is now partially observable for both cases with an unobservable island at busbar 4,5,6 (Roxburgh 11kV). This is indicated by the non-zero entries in the nullspace vector. With the chosen measurement placement, the transformer circuits are observable when the transformer is in isolation (refer to Table 2.2), however, the measurements are dependent resulting in rank deficiency (a rank of 24) in the system measurement matrix [H]. The system is therefore under-determined. The algorithm utilizing SVD is able to produce a least square solution that matches with the actual values even though the system is under-determined. The HSE results are identical for both cases (Fig. 2.10).
Fig. 2.8 Two different sets of measurement locations for star-g/star transformer

Fig. 2.9 Nullspace vector of $[h_{\text{reduced}}]$, (star-g/star)

**Simulation 3: Star-g/Delta**

The test transformer is star-g/delta connected, which is the normal configuration of the Roxburgh 220kV/11kV transformer. The measurement points taken in this case are symmetric, being either on the HV side or the LV side as shown in Fig. 2.11. The null-space vector in Fig. 2.12 shows that current measurements taken on the LV side (star-g) allow the test system to be fully observable, but only partially observable when measurements are taken at the HV side (delta). This is because the zero sequence current is not known when measurement are taken on the delta side of the star-g/delta transformer.
Fig. 2.10 Harmonic voltage magnitude for the 2 sets of current measurement (star-g/delta, Fig. 2.8)

The unobservable area is located on the LV side of the test transformer, Roxburgh 11kV, busbar 4,5,6. The HSE results for both cases are shown in Fig. 2.13. The results are identical due to the powerful solvability of SVD. Even though the system observability is different in both cases, SVD is able to produce a solution that matches with the actual values in this case. From these observations, the preferred measurement placement for star-g/delta transformers is found to be on the star-g side since it gives a better observability for the same number of measurement points.

Fig. 2.11 Two different sets of measurement locations for star-g/delta transformer
Fig. 2.12 Nullspace vector of $[h_{\text{reduced}}]$

Fig. 2.13 Harmonic voltage magnitude for the 2 sets of current measurement (star-g/delta, Fig. 2.11)
The harmonic state estimation algorithm using SVD has the benefit of obtaining a particular solution even for an under-determined system. Unobservable islands can be easily identified by looking at the position of the non-zero entries in the nullspace vector of the measurement matrix. It is important to know that the influence of transformer configuration on measurement placement needs to be considered when taking transformer measurements. It can reduce the possible combinations when searching for the optimum measurement placement in harmonic state estimation. The simulation results showed that the choice of measurement locations on transformers can highly degrade the overall system observability, and hence the solvability of the estimation. Use of independent measurement locations is beneficial, especially in the case when the numbers of measurements are limited. Unnecessary measurements can also be avoided without affecting the system's observability by ensuring that the best measurement locations around transformers are used.

2.5 Bad Data Processing

An issue that needed addressing is the sensitivity of state estimation to bad data, incorrect system parameters or incorrect network topology. Bad data can be divided into two groups: 1) Gross errors, which are usually caused by equipment failure or data communication problems such as loss of communication, are often large in magnitude. 2) Noise errors, which emanate from transducers, transmission lines, etc. occur in all measurements but are relatively small in magnitude. Generally, simple preliminary checks on incoming measurements may eliminate excessively large data errors, however, these checks may not be totally reliable and some bad data will remain undetected in the pre-filtering stage. The main concern is the extent to which they affect the estimation accuracy and what can be done to reduce the estimation errors.

Extensive research has been performed in the past in the detection, identification and removal of bad data [22]–[25] to improve the accuracy of the fundamental frequency state estimation. This usually requires the system to be over-determined and is therefore of limited applicability to harmonic state estimation. Without redundant measurements, the estimation is very subject to the presence of bad data. The basic techniques normally used for detection and identification of bad data include:

1. $J(x)$-test

$$J(x) = (z - Hx)^T R^{-1}(z - Hx)$$  \hspace{1cm} (2.14)
2. $r^n$-test (Weighted)

$$r(x) = \sqrt{R^{-1}} (z - Hx)$$

(2.15)

3. $r^n$-test (Normalized)

$$r(x) = \sqrt{\sum_r} (z - Hx)$$

(2.16)

where $\sum_r = \text{diag}(R_r)$

$R_r$ is the residual covariance matrix

$R_r = SR_s$

$S$ is the sensitivity matrix

$S = I - HR_sH^TR^{-1}$

$R_t$ is the state estimate covariance matrix

$R_t = (H^TR^{-1}H)^{-1}$

$R_z$ is the measurement covariance matrix

$R_z = HR_zH^T$

For fast and accurate detection of bad data, the $J(x)$ and $r^n$ test pair are normally used. Identification simply involves removing suspect measurements and then re-estimating. Using the ordered residual search, suspect measurements are arranged in order of their residuals. The measurement with the largest residual is removed first and then re-estimated. The removal and re-estimation procedure is repeated until all the bad data is eliminated. Although bad data such as gross errors can be filtered using the aforementioned techniques, measurement noise which is incorporated in all measurements is not easily filtered. Therefore, measurements containing noise are usually the input to the state estimation. It is important to understand the influence of measurement noise on the accuracy of HSE.

A statistical simulation method - Monte Carlo Simulation can be used to inspect the effect of this type of bad data on the accuracy of HSE from a statistical approach. Generally, the primary components of a Monte Carlo simulation include the following:
- Probability distribution functions (pdf's) - the physical (or mathematical) system must be described by a set of probability density functions.
- Random number generator - a source of random numbers uniformly distributed on the unit interval must be available.
- Sampling rule - a prescription for sampling from the specified pdf's, assuming the availability of random numbers on the unit interval, must be given.
- Scoring (or tallying) - the outcomes must be accumulated into overall tallies or scores for the quantities of interest.
- Error estimation - an estimate of the statistical error (variance) as a function of the number of trials and other quantities must be determined.
- Variance reduction techniques - methods for reducing the variance in the estimated solution to reduce the computational time for Monte Carlo simulation
- Parallelization and vectorization - algorithms to allow Monte Carlo methods to be implemented efficiently on advanced computer architectures.

Monte Carlo methods randomly select values to create scenarios of a problem. These values are taken from within a fixed range and selected to fit a probability distribution. In the simulation, the random selection process is repeated many times to create multiple scenarios. For each run, the outcome forms one possible scenario or solution to the problem. Together, these scenarios give a range of possible solutions.

The exact noise level is difficult to measure and is usually determined from experience and historical data in practice. Typically, measurement noise is assumed to be uncorrelated, unbiased and normally distributed with a variance of $\theta^2$. Prior to each Monte Carlo run, it is assumed that a new set of measurement $\tilde{z}$ with random noise is available and the true values are available for comparison. The sequence steps for the proposed Monte Carlo simulation are:

1. Use a random number generator to select noise to be added to the actual data at the measurement points to simulate measurement noise. The random number generator can be weighted to give whatever distribution is most appropriate (e.g. noise that is uniformly or normally distributed).
2. Compute $\hat{x}$ using SVD and pseudo-inverse

$$z_{true} + \epsilon = [H]\hat{x} \; ; \; [H] = [U][W][V]^T$$

$$\hat{x} = [V][W]^{-1}[U](z_{true} + \epsilon)$$
3. Compute the estimation error $\Delta x(n)$, and tally chart $\Delta x(n)$ into levels of estimation error such as: estimation error 0.1-0.2%, 0.3-0.4%, etc.

$$\Delta x(n) = x(n) - \hat{x}(n)$$

4. Repeat steps 1-3 for enough Monte Carlo runs. For each run, incorporate random measurement noise into the measurements.

5. Calculate probability $P_{\Delta x}$ for each level of estimation error caused by measurement noise.

$$P_{\Delta x}(n) = \frac{\# \text{ of occurrence } \Delta x(n)}{\# \text{ of simulation runs}}$$

A flow chart of the Monte Carlo simulation is illustrated in Fig. 2.14.
The result is a cumulative probability density curve for each level of estimation error. They are used to show the probability of an estimated error being below a certain level. This information is useful in determining the range of state estimation error under the influence of measurement noise and hence the confidence in the estimate.

2.5.1 Case Study: HSE with Bad Data Measurements

The influence of bad data such as measurement noise and gross errors is expected to have a significant effect on the performance of HSE. These outlying data can have a disproportionate effect on the minimum least squares fit. Although bad data detection and filtering is used to minimise its effect, it is a difficult task to completely remove all the bad data particularly for measurement noise. The Monte Carlo technique described in Section 2.5 is used to examine the effects of bad data on HSE accuracy. The test system is taken from the Lower South Island of the N.Z system described in Section 2.4.2. 6 pulse and 12 pulse rectifiers are placed at Invercargill 33kV and Tiwai.
220kV respectively. A linear load is also placed at Roxburgh 11kV. By taking symmetric branch current measurements at Manapouri 220kV, Roxburgh-2 11kV and Invercargill 33kV, the system is fully observable. A single line diagram of the measurement locations is shown in Fig 2.15.

![Diagram showing measurement placement](image)

**Fig. 2.15 Measurement placement of the test system**

The algorithm solves the solution (busbar voltages) in the weighted least squares sense, where the residuals between the actual harmonic measurements and estimated values are minimized; any measurement noise and gross error can affect the solution directly. The first part of the simulation looks at the effect of measurement noise. Random noise is added to the measurement set. The measurement noise is assumed to
be uncorrelated, unbiased and normally distributed with a 5% standard deviation. The level of noise is usually unknown in practice, but it can be determined from experience and historical data. The results from the Monte Carlo computation are shown in Figs. 2.16-2.20. The cumulative probability density curves depict the characteristics of estimation errors for the harmonic orders $h = 6n\pm1$. This provides information on the susceptibility of an estimate to noise and hence the confidence that can be placed on the estimate. The $y$-axis gives the probability of the error in the harmonic estimate is below the corresponding level (as shown on the $x$-axis). For example, with reference to Fig. 2.16, 80% of the time (probability of 0.8) the estimation error for the $5^{th}$ harmonic voltage is less than 7.5%. Conversely, there is a probability of 0.2 that the estimation error for the $5^{th}$ harmonic voltage will exceed 7.5%.

Fig. 2.16 Harmonic voltage estimation error at Roxburgh 11kV phase A
Fig. 2.17 Harmonic voltage estimation error at Roxburgh 220kV phase A

Fig. 2.18 Harmonic voltage estimation error at Tiwai 220kV phase B
Fig. 2.19 Harmonic voltage estimation error at Invercargill 33kV phase A

Fig. 2.20 Harmonic voltage estimation error at Manapouri 220kV phase A
A summary of the expected mean harmonic voltage and harmonic current injection error is shown in Tables 2.4 and 2.5. This helps to identify areas that are sensitive to measurement noise. The inf (infinite) entry in Table 2.5 indicates that there is no harmonic injection at this point in the system but is estimated with a non-zero injection. Note that

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Table 2.5 Mean harmonic current injection error

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Despite the measurement noise, the estimator achieves a good estimate of the system. The voltage and current injection magnitudes shown in Fig. 2.21 and 2.22 demonstrate that the effect of measurement noise in HSE is minimal. The differences between the actual and estimated values are small and the harmonic sources can still be correctly identified by their significant characteristic harmonics i.e. $h = 6n \pm 1$ for
6-pulse and 12n±1 for 12-pulse rectifier. The results showed harmonic injections of the order 6n±1 and 12n±1 are located at busbars 1-3 (Tiwai 220kV) and busbars 22-24 (Invercargill 33kV) respectively. This suggests that the presence of two harmonic injectors consisting of a 6-pulse and 12-pulse rectifier.

The influence of measurement noise on the state estimates is system-wide. Since the HSE model used is linear, the estimation error caused by the measurement noise is also linear, i.e. for a 10% measurement error, the estimation results will also have a 10% error. Nevertheless, with Gaussian noise in each of the measurement, it is difficult to predict the error in the state estimates and hence an analysis based on Monte Carlo simulation is the most suitable to assess the HSE performance under the influence of measurement noise.

![Graph showing harmonic busbar voltages](image)

Fig. 2.21 Harmonic busbar voltages
Another type of bad data, gross error is also simulated. The impact of one or more gross errors in HSE is investigated. The harmonic current measurement taken at branch 77 (phase B) and 34 (phase A) are replaced with an artificially introduced gross error at a level of 3 times its original value. The HSE results are shown in Figs. 2.23-2.26. Tables 2.6 and 2.7 show the mean harmonic estimation error in the presence of two gross errors in the measurement set. The figures show that gross error causes estimation discrepancy on busbar voltages, branch currents and injection currents that are significant and will give false identification of harmonic sources. Additional points of harmonic injection at busbars 1-3 were erroneously identified. Significant levels of 5th, 7th, 11th, 13th, 23rd and 25th harmonic are estimated at Tiwai 220kV (busbars 1-3). With the presence of 5th and 7th harmonic, the characteristic harmonics of a 12-pulse rectifier are no longer apparent.
Fig. 2.23 Harmonic busbar voltages with one gross error at branch 77

Fig. 2.24 Harmonic current injections with one gross error at branch 77
Fig. 2.25 Harmonic busbar voltages with two gross errors at branch 77 and 34

Fig. 2.26 Harmonic current injections with two gross errors at branch 77 and 34
Table 2.6 Mean harmonic voltage estimation error

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These effects caused by bad data can be minimized if redundant measurements are used. The trade off is that an additional base unit may be required to monitor extra measurements, which would dramatically increase the instrumentation cost. Figs. 2.27-2.30 show the improvement in state estimation accuracy using redundant measurements. The figures display the estimation error for harmonic orders $h=6n\pm1$.
up to the 25th harmonic. A significant improvement on the HSE accuracy is achieved through the first nine redundant measurements (grey bar) and a diminished improvement with additional nine redundant measurements (white bar), i.e. a total of eighteen redundant measurements.

<table>
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<th>Bus No.</th>
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Fig. 2.27 Mean harmonic voltage estimation error in Tiwai 220kV

Fig. 2.28 Mean harmonic voltage estimation error in Roxburgh 11kV
Fig. 2.29 Mean harmonic voltage estimation error in Invercargill 33kV

Fig. 2.30 Mean harmonic voltage estimation error in Manapouri 220kV

The effectiveness of HSE is largely dependent on the quality of the measurements. The effect of random measurement noise has shown to be less significant in
comparison to gross errors. The latter caused system-wide discrepancies on the state estimates that are significant enough to mis-identify the harmonic sources. However, the simulation results show that the injection locations at Tiwai 220kV and Invercargill 33kV remain correctly identified. Significant improvement using redundant measurements requires the system to be over-determined. When the system is under determined or completely-determined, gross errors have a large impact on the estimates which cannot be easily mitigated. Hence, a judicious trade off between accuracy and measurement costs has to be made.

2.6 Under-determined estimation problem

State estimation problems can be classified as over-determined, completely-determined or under-determined, depending on whether the number of independent measurement equations is greater, equal or less than the number of state-variables respectively. The standard techniques used in solving over-determined systems are LU decomposition and back substitution, Cholesky decomposition and Gauss-Jordan elimination. Over-determined systems are desirable as they allow for bad data filtering, however, obtaining sufficient harmonic measurements in a practical power system is often impractical. This leads to HSE problem being solved for the under-determined condition. In such a case, the system will have infinite number of solutions due to the ill-conditioning of the gain matrix $H^TR^{-1}H$. The infinite solution is in the form of

$$
x = x_p + \sum_{i=1}^{n-rank(A)} k_i x_{n_i} \tag{2.17}
$$

where $[x_p]$ is the particular solution, $k_i$ is a constant and $[x_{n_i}]$ is a null space vector.

In this circumstance, standard techniques used in over-determined system are insufficient. To overcome this, Singular Value Decomposition (SVD), known to be a highly reliable and computationally stable mathematical tool, can be used to solve under-determined (partially observable) systems. It provides a particular solution and a null space vector for each singularity. Although it can be significantly slower than traditional methods in solving over-determined systems and requires more storage and computational efforts [26], it is less susceptible to built up round-off error
due to finite computation precision. Generally, prior to state estimation, Observability Analysis (OA) is used to check the solvability of the system [27]-[28], whether the set of available measurements is sufficient to calculate all the state variables of the system uniquely. This can be replaced by the use of SVD technique, unobservable islands within the network and the level of observability can be found using SVD as a by-product.

The SVD method represents an $m \times n$ matrix $[A]$ in the factored form

$$[A] = [U][W][V]^T$$

(2.18)

where $[U]$ and $[V]^T$ are orthogonal matrices. The columns of $[U]$ $m \times m$ are eigenvectors of $[A][A]^T$, and the columns of $[V]$ $(n \times n)$ are eigenvectors of $[A]^T[A]$ and $[W]$ is an $m \times n$ diagonal matrix with entries of singular values [29]. SVD constructs orthonormal bases in a special way. Not only are they orthonormal, but if $[A]$ is multiplied by the column $[V]$, it produces a multiple of a column of $[U]$ (2.19). Moreover, the columns of $[U]$ and $[V]$ give orthonormal bases for the four fundamental subspaces of $[A]$: column space, row space, left nullspace and nullspace. The columns of $[U]$ correspond to the non-zero singular values and the columns of $[V]$ correspond to the zero singular values are the column space and null space of $[A]$ respectively [29].

$$[A][V] = [U][W]$$

(2.19)

The SVD solution chosen is one that minimizes the length (norm) of $x$

$$\text{Minimise} \|x\| = \left\| x_p + \sum_{i=1}^{n-rank(A)} k_i x_{n_i} \right\|^2$$

(2.20)

To illustrate the effectiveness of SVD, consider the following system:
\[
\begin{bmatrix}
1 & -2 & 1 \\
2 & -3 & 4 \\
3 & -5 & 5 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
4 \\
-1 \\
3 \\
\end{bmatrix}
\]

As the rank of the measurement matrix is equal to two, there exists only two independent sets of measurements and therefore an infinite number of solutions can be found in the form of (2.17). The set of possible solutions is illustrated by the longest line in Fig. 2.31. The nullspace vector (-5, -2, 1) is in parallel with the solution set. Point A (0.6666, -3.1333, -2.9333) is the solution chosen by SVD and point B (-8.462, -6.7848, -1.1076) is an arbitrary solution. Although both solutions satisfy the system above, when looking at the norm of the solutions, SVD produces a least squares solution with a norm of 4.3436 for point A, in comparison to 10.903 for point B.

![Graph showing nullspace vector and a set of possible solutions](image)

Fig. 2.31 Minimum least squares solution

### 2.6.1 Unobservable variables

Consider the system of

\[
\begin{bmatrix}
1 & -2 & 1 \\
2 & -3 & 4 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
4 \\
-1 \\
\end{bmatrix}
\]
This is an undetermined system where there are more unknown variables than
equations. The SVD method represents the matrix $[A]$ in the form given by eqn. (2.18), where

$$
[U] = \begin{bmatrix}
-0.39249 & -0.91976 \\
-0.91976 & 0.39249
\end{bmatrix}
$$

$$
[W] = \begin{bmatrix}
5.8413 & 0 & 0 \\
0 & 0.93767 & 0 \\
-0.38211 & -0.14374 & -0.91287
\end{bmatrix}
$$

$$
[V] = \begin{bmatrix}
0.60676 & 0.70605 & -0.36515 \\
-0.69702 & 0.69342 & 0.18257 \\
-0.91287 & -0.36515 & 0.18257
\end{bmatrix}
$$

The nullspace of the matrix $[A]$ can be seen as the column of $[V]$ corresponding to

the zero singular values, i.e.

$$
\begin{bmatrix}
-0.91287 \\
-0.36515 \\
0.18257
\end{bmatrix}
$$

The non-zero entries in the nullspace vector indicate that there is no unique solution
for the corresponding variable $x$'s since altering the values of $k$ in eqn. (2.17) will
result in another set of valid solutions. The system is therefore not observable.

### 2.6.2 One observable variable

Consider another system given by

$$
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 2 \\
2 & 2 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
4 \\
7
\end{bmatrix}
$$

where the matrices arising from SVD are

$$
[U] = \begin{bmatrix}
-0.33378 & 0.74516 & -0.57735 \\
-0.47843 & -0.66164 & -0.57735 \\
-0.81221 & 0.083515 & 0.57735
\end{bmatrix}
$$

$$
[W] = \begin{bmatrix}
5.0761 & 0 & 0 \\
0 & 0.48255 & 0 \\
0 & 0 & 1.9596e-016
\end{bmatrix}
$$

$$
[V] = \begin{bmatrix}
-0.48002 & 0.51921 & 0.70711 \\
-0.48002 & 0.51921 & -0.70711 \\
-0.73428 & -0.67885 & -4.996e-016
\end{bmatrix}
$$

Matrix $[A]$ is of rank two since the third equation is a linear combination of the first
two equations. The SVD gives the singular values as 5.0761, 0.48255, 0 with the
computer precision of 1e-08. Hence, the nullspace vector is the third column of the matrix $[V]$ i.e. \[
\begin{bmatrix}
0.70711 \\
-0.70711 \\
0
\end{bmatrix}
\]. The system has one observable variable $x_3$ since any linear combination of the nullspace vector will only alter the values of $x_1$ and $x_2$ while the value of $x_3$ remains unchanged.

### 2.6.3 Multiple nullspace vectors

It is also possible to have more than one nullspace vector for the system $[A]{\vec{x}} = {\vec{b}}$. On the basis of mathematical topology, the dimension of the subspace $\mathbb{R}^n$ is expressed as

\[
N = \dim(\text{col}[A]) + \dim(\text{null}[A])
\]  

(2.21)

where $N$ is the number of columns of the matrix $[A]$, $\dim(\text{col}[A])$ and $\dim(\text{null}[A])$ are the dimensions of the column space and the nullspace of the matrix $[A]$ respectively.

Consider the simple system with multiple nullspace vectors:

\[
\begin{bmatrix}
1 & -2 & 1 & -1 \\
2 & -3 & 4 & -3 \\
3 & -5 & 5 & -4 \\
-1 & 1 & -3 & 2
\end{bmatrix}\begin{bmatrix}
{x_1} \\
{x_2} \\
{x_3} \\
{x_4}
\end{bmatrix} = \begin{bmatrix}
4 \\
-1 \\
3 \\
5
\end{bmatrix}
\]

where SVD gives the following

\[
[U] = \begin{bmatrix}
-0.21656 & 0.5352 & 0.80614 & 0.12963 \\
-0.5352 & -0.21656 & -0.12963 & 0.80614 \\
-0.75176 & 0.31863 & -0.33826 & -0.46788 \\
0.31863 & 0.75176 & -0.46788 & 0.33826
\end{bmatrix}
\]

\[
[W] = \begin{bmatrix}
11.498 & 0 & 0 & 0 \\
0 & 1.6706 & 0 & 0 \\
0 & 0 & 9.2916e-016 & 0 \\
0 & 0 & 0 & 1.1316e-016
\end{bmatrix}
\]

\[
[V] = \begin{bmatrix}
0.33578 & 0.18329 & -0.92179 & -0.062994 \\
0.53192 & -0.75546 & -0.3539 & 0.14524 \\
-0.61505 & -0.59449 & 0.13992 & -0.49871 \\
0.47542 & 0.2056 & -0.074061 & -0.85219
\end{bmatrix}
\]

The bases of the nullspace are shown as the third and fourth columns of the matrix $[V]$,

\[
\begin{bmatrix}
-0.33578 \\
0.53192 \\
-0.61505 \\
0.47542
\end{bmatrix}, \quad \begin{bmatrix}
0.18329 \\
-0.75546 \\
-0.59449 \\
0.2056
\end{bmatrix}
\]

i.e. $\begin{bmatrix}
0.33578 \\
0.53192 \\
-0.61505 \\
0.47542
\end{bmatrix}, \quad \begin{bmatrix}
0.18329 \\
-0.75546 \\
-0.59449 \\
0.2056
\end{bmatrix}$. There are two independent nullspace vectors and two
independent column space vectors for the matrix $[A]$ which satisfy (2.21). The dimensions are:

$$\text{dim}(\text{col}[A]) = \text{rank}[A] = 2$$
$$\text{dim}(\text{null}[A]) = 2$$
$$N = 4$$

The system contains unobservable variables since the non-zero entries in the nullspace vectors suggested that all the state variables $(x_i)$ are not uniquely defined. The system is therefore unobservable.

### 2.6.4 Case Study: HSE using the SVD technique

In the simulation, SVD technique is used to overcome the ill-conditioning of the measurement matrix when the system is not fully observable. Systems containing single and multiple unobservable island(s) are simulated to verify the performance of SVD under different levels of observability.

By selecting measurements at the appropriate locations, partially unobservable areas can be created. Figs. 2.32 and 2.33 show the Lower South Island of the N.Z system under different observability. The unobservable locations are indicated by the grey areas.
Fig. 2.32 Single unobservable island at Roxburgh-2 11kV
Fig. 2.33 Multiple unobservable islands at Roxburgh 220kV, Roxburgh-2 11kV and Invercargill 33kV

Simulation 1: Single unobservable island network

The test system (Fig. 2.32) is partially observable with a single unobservable island at Roxburgh-2 11kV busbar which is related to state variables 4,5,6. From the infinite number of solutions, an arbitrary solution is compared with the solution chosen by SVD and pseudo-inverse. The solutions are shown in Figs. 2.34-2.37. Although both solutions satisfy the measurement system, the minimum least squares solution produced by SVD is much closer to the actual values (in this case it matches with the actual values). The arbitrary solution, however, has greater estimation errors at the unobservable island at Roxburgh-2 11kV.
Fig. 2.34 Harmonic voltage magnitude (arbitrary solution where $k=0.3$)

Fig. 2.35 Harmonic voltage phase angle (arbitrary solution where $k=0.3$)
Fig. 2.36 Harmonic voltage magnitude (SVD solution)

Fig. 2.37 Harmonic voltage phase angle (SVD solution)
Simulation 2: Multiple unobservable islands network
The effectiveness of SVD and pseudo-inverse technique in HSE is further investigated in a multiple unobservable islands network. In this case, the unobservable islands are located at Roxburgh 220kV, Roxburgh 11kV and Invercargill 33kV as shown in Fig. 2.33. Thus, the complexity in determining the actual solution is increased due to an increase in the number of nullspace vectors and unknowns. The SVD solution shown in Figs 2.40 and 2.41 is the least squares solution of the HSE measurement system. The randomly picked arbitrary solution is shown in Figs 2.38 and 2.39. The SVD technique is capable of producing a solution with less error at the unobservable islands when compared with the arbitrary solution. The observable areas (nodes 1-3, 12-21 and 25-27) in both cases remain error free.

Fig. 2.38 Harmonic voltage magnitude (arbitrary solution where k=0.25)
Fig. 2.39 Harmonic voltage phase angle (arbitrary solution where k=0.25)

Fig. 2.40 Harmonic voltage magnitude (SVD solution)
The SVD method was able to solve the partially observable systems where traditional methods fail. SVD not only solved the network with a single unobservable island correctly but also suppressed estimate errors on a network with multiple unobservable islands. The solution produced by SVD was found to be more reliable in comparison to other arbitrary solutions. No observability analysis was needed to identify unobservable islands prior to the estimation. Unobservable islands are revealed as a by-product using SVD.

2.7 Dynamic Harmonic State Estimation

As previously mentioned there is a need to allow for the time-varying nature of the harmonic injection. The level of harmonic injection is changing with time and as a result the harmonic impact on the supply network is dynamic. In the literature, the use of Kalman Filter to track harmonic injection in dynamic HSE is the most popular. The main feature of Kalman filter is the time domain stochastic optimal estimator that provides an efficient recursive solution in the least square sense. Its process utilizes the previous \textit{a posteriori} estimate to predict the new \textit{a priori} estimate. Despite its success in many tracking applications, it should be noted that for the optimum filtering results, the exact knowledge of the process noise covariance matrix (Q) and noise measurement covariance matrix (R) are required. In the case of large time
varying systems where Q and R are usually unknown, the estimator can suffer from divergence problems caused by the ‘dropping off’ effect of the filter. Due to the nature of the filter, the Kalman gain is independent of the measurements. Thus, as the filter approaches steady-state, it becomes less sensitive to parameter variations and begin to lose its ability in tracking dynamic properties.

2.7.1 The Kalman Recursive Process
The Kalman filter is a recursive least squares process, of which the principle feature is the recursive processing of the noise measurement risk. It is therefore highly suitable for dynamic state estimation. In the Kalman filter algorithm, the discretised linear stochastic difference equation and measurement equation are

\[
x_{k+1} = \phi_k x_k + v_k \\
z_k = [H_k] x_k + w_k
\]  

(2.22)  
(2.23)

In the equations, \( x_k \) is the process state vector at time \( k \). Equation (2.22) is the process model which describes the dynamics of the state over time. The state at time \( k + \Delta k \) is obtained by multiplying the state transition matrix \( \phi_k \) by the previous state \( x_k \). Both the process model and measurement model are imperfect. The process noise and measurement noise vectors \( v_k \) and \( w_k \) are assumed to be independent of each other, white and with normal probability distribution:

\[
p(w_k) \sim N(0, Q_k) \\
p(v_k) \sim N(0, R_k)
\]

\[
E[w_k w_k^T] = Q_k \\
E[v_k v_k^T] = R_k
\]

The recursive Kalman filter computation described in Appendix III, is divided into two parts: “Time update” and “Measurement update”. The recursive steps are:

Time update
1. Project the state ahead

\[
\hat{x}_{k+1}^- = \phi_k \hat{x}_k
\]  

(2.24)

2. Project the error covariance ahead
\[ P_{k+1}^{-} = \phi_k^T P_k \phi_k + Q_k \] 

(2.25)

Measurement update

1. Kalman gain

\[ K_k = P_k^{-} H_k^T \left( H_k P_k^{-} H_k^T + R_k \right)^{-1} \] 

(2.26)

2. Update estimate with measurement \( z_k \)

\[ \hat{x}_k = \hat{x}_k^{-} + K_k \left( z_k - H_k \hat{x}_k^{-} \right) \] 

(2.27)

3. Update the \textit{a posteriori} estimate error covariance

\[ P_k = \left( I - K_k H_k \right) P_k^{-} \] 

(2.28)

To overcome the divergence problem of the filter, an adaptive Kalman filtering is developed. The Kalman recursive process switches between two simple covariance Q models depending on whether the system is in steady-state or transient. This approach overcomes the divergence problem of the filter and avoids the need to determine an explicit Q model.

\subsection*{2.7.2 Two Q models approach}

It has become apparent that the use of Kalman filtering technique requires accurate system modelling in order to produce acceptable results. In particular, the performance of the filter is very dependent on the covariance matrix (Q). The filter can suffer from “divergence” problems which can lead to erroneous results predicted by the equations of the filtering process when:

1. under an extended period of operation, the covariance matrix becomes unrealistically small and subsequent measurements are effectively ignored.
2. the optimal covariance matrix (Q) is not used.

For the latter case, the search of the optimum Q requires extensive knowledge and understanding of the system. The fact that the optimal Q value compromises the filter performance for both transient and steady-state condition makes it difficult to determine for a dynamic power system. To overcome these problems, a two Q models approach has been developed. The optimal covariance matrix (Q) model is replaced by the use of two basic Q models: Kalman constant model and Kalman random walk model, depending on whether the system is under transient or steady-state condition.
A. Kalman constant model

The noise covariance matrix (Q) is set to zero in this case. The process model assumes that the system is in steady-state condition and the effect of measurement noise is minimized. Fig. 2.42 depicts a simple case of tracking busbar harmonic voltage in a power system using the Kalman recursive equations (2.24)-(2.28). The filter shows a good tracking capability for the initial period, but as the filter approaches its steady-state (Fig 2.43), the filter becomes relatively insensitive to adapt to any new parameter variation which will no longer the step change satisfactory.

![Diagram of voltage magnitude over iterations](image)

Fig. 2.42 Estimation with Kalman constant model
B. Kalman random walk model

The noise covariance matrix (Q) is set to be an identity matrix in this case. The process model assumes that the system is under transient condition, which increases the Kalman gain value to adopt new changes during transient. Fig. 2.44 is the tracking response when the random walk model is used. There is a significantly improved latency in tracking the step-change, however the results are relatively noisy for the constant part of the estimation. This is due to the high Kalman gain as shown in Fig. 2.45.
Fig. 2.44 Estimation with Kalman random walk model

Fig. 2.45 Kalman gain $K_{4,4}$ for random walk model
2.7.3 The Adaptive Kalman Filter

The proposed adaptive Kalman filter applies two different Q models to achieve better overall performance for steady-state and transient estimation. The benefits of both Q models should yield a better overall performance. For the proposed linear frequency independent HSE model, it is expected that the measurement difference between each time-step $\Delta z_{k,k-1}$ is relatively small in the steady-state. Thus, a hypothesis test (student’s t statistical model with the level of significance $\alpha$) on the measurement difference $\Delta z_{k,k-1}$, can determine whether the system is in steady-state or transient condition. The null and alternative hypotheses are:

$$
H_0 : \mu = 0; \\
H_A : \mu \neq 0
$$

(2.29)

The test statistic $t$ is calculated using

$$
t = \frac{\bar{\Delta z}_{k,k-1} - \mu}{s / \sqrt{n}}
$$

(2.30)

where $\bar{\Delta z}_{k,k-1}$ is the mean measurement difference between time k and k-1

$\mu$ is the population mean

$n$ is the number of test sample

$s$ is the sample std. deviation

Table 2.8 is a decision summary for the hypothesis test. The Kalman Q model is determined by the decision of the null hypothesis based on the condition of the system. In general, if the null hypothesis is rejected, the system is assumed to be in transient condition and the identity matrix is used for matrix Q. Likewise, if the null hypothesis is accepted, the system is assumed to be in steady-state and the zero matrix is used instead.

<table>
<thead>
<tr>
<th>$t$ statistic</th>
<th>Decision</th>
<th>System condition</th>
<th>Kalman Q model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$=5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t &lt; -t_{\alpha/2}$</td>
<td>reject $H_0$</td>
<td>transient</td>
<td>$Q_k = I$</td>
</tr>
<tr>
<td>$t &gt; +t_{\alpha/2}$</td>
<td>reject $H_0$</td>
<td>transient</td>
<td>$Q_k = I$</td>
</tr>
<tr>
<td>$-t_{\alpha/2} &lt; t &lt; +t_{\alpha/2}$</td>
<td>accept $H_0$</td>
<td>steady-state</td>
<td>$Q_k = 0$</td>
</tr>
</tbody>
</table>
The computational sequence of the adaptive Kalman filter is shown in Fig. 2.46. The hypothesis test determines the appropriate covariance matrix $Q$ to apply to the Kalman filter equations. This allows resetting of the Kalman gain to prevent divergence. Fig. 2.47 shows the performance of the proposed adaptive Kalman filter on the previous example in Section 2.7.2. The estimation shows a significant improvement in tracking the actual waveform.
for harmonic order $h$

$k = 1$

read $z_k$ and $z_{k-1}$

compute $\Delta z_{k,k-1}$ and std. deviation of $\Delta z_{k,k-1}$

compute $t$-value
for 95% confidence interval

accept $H_0$

$H_0 : \mu = 0$

reject $H_0$

$H_1 : \mu \neq 0$

$Q_k = 0$

$Q_k = I$

Time update

$\hat{x}_{k|k} = A\hat{x}_k + Bu_k$

$P_{k|k} = AP_kA^T + Q_k$

Measurement update

$K_k = P_{k|k}H^T \left(HP_{k|k}H^T + R_k \right)^{-1}$

$\hat{x}_k = x_{k|k} + K_k \left( z_k - H\hat{x}_k \right)$

$P_k = (I - K_kH)P_{k|k}$

end of estimation?

next time step $k = k + 1$

no:

STOP

where $\Delta z_{k,k-1}$ is the change in measurement between $z_k$ and $z_{k-1}$

Fig. 2.46 Adaptive Kalman filter flow chart
Fig. 2.47 Estimation from the proposed adaptive Kalman filter

2.7.4 Simulation Results

The proposed dynamic HSE and adaptive Kalman filter described in Section 2.7 were applied to the N.Z Lower South Island test system. For simplicity, only one harmonic source is considered, thus the 12-pulse rectifier at Tiwai 220kV is disconnected and only the 6-pulse rectifier at Invercargill 33kV is used in the simulation. An assumed harmonic current injection profile of the 6-pulse converter at Invercargill 33kV (3 phase balanced) over a period of 24 hours is shown in Figs. 2.48-2.50.
Fig. 2.48 Harmonic current injection (Invercargill 33kV, phase A)

Fig. 2.49 Harmonic current injection (Invercargill 33kV, phase B)
Fig. 2.50 Harmonic current injection (Invercargill 33kV, phase C)

To demonstrate the adaptive Kalman filter technique, the measurement locations shown in Fig. 2.51 were selected to provide a fully observable system. Similar to previous simulations, the partially “measured” values at the measurement points were obtained from the corresponding results of harmonic penetration simulation. No information on the harmonic source is supplied to the HSE algorithm. Random noises of Gaussian distribution are also added to all measurements.
The main dynamic HSE results at instants 0600 hrs, 1200 hrs and 1800 hrs are shown in Figs. 2.52-2.54. There are no significant differences between the estimated and simulated results. This shows that the proposed dynamic HSE produces reliable results for the whole system. Figs. 2.55-2.59 show the simulated (solid line) and estimated (dotted line) node harmonic voltages over a period of 24 hours for the selected busbars. It can be seen that the adaptive Kalman gain ‘resets’ when there is a significant change in the measurements (Fig. 2.55), while in traditional Kalman filters, the gain is independent of the measurements. The sharp rises and fall during the peak hour 1700-2000 indicated by the arrows in Fig. 2.55 corresponds to the resets
of the Kalman gain shown in Fig. 2.56. The increase in the Kalman gain allows the step changes to be tracked more quickly. Moreover, the adaptive Kalman filter showed good tracking of the actual waveform with minimal oscillation when the level of harmonic injection is relatively low with little variation, e.g. during the time 0000-0700.

![Diagram](image)

Fig. 2.52 Nodal harmonic voltages at time 0600
Fig. 2.53 Nodal harmonic voltages at time 1200

Fig. 2.54 Nodal harmonic voltages at time 1800
Fig. 2.55 5th Harmonic voltage at Tiwai 220kV phase A

Fig. 2.56 Kalman gain for 5th harmonic voltage at Tiwai 220kV phase A
Fig. 2.57 5th Harmonic Voltage at Tiwai 220kV phase B

Fig. 2.58 5th Harmonic Voltage at Roxburgh 11kV phase A
Snapshots of the nodal harmonic current injection at times 0600, 1200 and 1800 are shown in Figs. 2.60-2.62. The results indicate the presence of harmonic sources at busbar #22, 23 and 24 (i.e. Invercargill 33kV) by the positive current injection. The tracking of the harmonic injection up to the 25th harmonic is shown in Figs. 2.63-2.65. The proposed adaptive Kalman filter is capable of tracking the actual harmonic voltage magnitude even when measurement noise is included.
Fig. 2.60 Nodal Harmonic Current Injection at Time 0600

Fig. 2.61 Nodal Harmonic Current Injection at Time 1200
Fig. 2.62 Nodal Harmonic Current Injection at Time 1800

Fig. 2.63 Harmonic Current Injection at Invercargill 33kV phase A
Fig. 2.64 Harmonic Current Injection at Invercargill 33kV phase B

Fig. 2.65 Harmonic Current Injection at Invercargill 33kV phase C
Chapter 3  Transient State Estimation

3.1  Introduction to Transient State Estimation

Accurate knowledge of the system states is required to ensure that the power system operates correctly and that the disturbance sources are correctly identified when the system experiences a fault. Since all power system state variables are not directly measurable, due to the cost of instrumentations, the unmeasured state variables need to be obtained through a state estimation technique. Extensive work has been carried out on the state estimation algorithms, observability analysis, dynamic system modelling, bad data analysis, measurement system and topological error analysis for both fundamental and harmonic frequencies. During normal power system operation, under constant load and topology, the continuous electromechanical and electromagnetic distribution of energy among the system components is not modelled explicitly and the system behaviour can be represented by voltage and current phasors in the frequency domain. However, switching events and system disturbances resulting transients are not accounted for in traditional state estimation.

This chapter introduces a Transient State Estimation methodology which is a reverse process of transient simulation (Fig. 3.1), to provide the best estimate of the transient voltages and currents at the unmeasured locations. Similar to HSE, an electrical power system can be described by a set of linear state equations, using both lumped and distributed parameter models to represent the network components. The measurement equations are then formulated based on the network topology and the set of system state equations describing the measurements to the state variables. Each measurement adds a corresponding measurement equation to build up the measurement system.

![Diagram](image.png)

Fig. 3.1 Relationship between transient simulation and TSE.

If the estimation procedure is of sufficient accuracy, it is even possible to identify the
disturbance location by inspecting the current mismatch within the network, as illustrated in Fig. 3.2.

At present, detailed transient simulations are required of potential fault scenarios to identify the most likely source of disturbance. This method however is time consuming and impractical for use in a large power system. The lack of efficient fault identification procedures significantly prolongs the network outage and disruption, causing interruption of the network’s normal operation. The use of TSE provides an effective approach to identify the possible location so that remedial actions can be taken quickly. Fig. 3.3 shows the framework of TSE.

![Electric Power System](image)

**Fig. 3.2 Current mismatch due to fault**

![Flowchart of TSE](image)

**Fig. 3.3 The framework of TSE**
3.2 Previous work on Transient State Estimation

Although state estimation techniques have been applied to power systems since early 1960s, the concept of transient state estimation has yet to be explored in detail. There are only a few contributions on this topic.

Ueda et.al [30] proposed the use of a non-linear observer to estimate the transient state of an infinite bus system. The transient state of the power system is represented continuously as a set of non-linear differential equations and observation of the output is obtained as a discretized value at every cycle of the system frequency by using a mean value detecting circuit device. The non-linear differential equation of state is discretized at each observation instant by Taylor expansion and then the discrete linear observer is applied to the discretized system. The observer provides a basis for transient power system state estimation.

Ueda et.al in [31] later improved the estimation technique and applied it to a multi-machine system. A Kalman filter type gain was used to replace the gain adopted in [30] to improve the estimation performance. The proposed method showed good estimation performance and tracking when applied to a three machine system. However, if this technique is to be applied to a large power system, the following problems need to be addressed: (i) the numerical method constituting the approximate discretized difference equation to the non-linear differential equations and; (ii) the selection of the observer gain.

3.3 Transient State Estimation Modelling

In the TSE algorithm, the power system is modelled using state-space theory, represented by its equivalent lumped circuit. With solution efficiency in mind, the different state-space models for the components have undergone significant, and sometimes restricting, simplifications aimed at a particular study. In the present investigation the relevant system components are modelled as follows:

- Generators are modelled by a voltage source and an equivalent R-L impedance.

- Transformers are represented using two winding models taking into account the type of magnetic circuit and the connections of the terminals and the neutrals, i.e. delta or wye. Core losses are represented internally with an equivalent shunt resistance across each winding in the transformer.
- Transmission lines are modelled by the three phase nominal PI model and hence are capable of incorporating non symmetric condition.

- The real and reactive power components of the loads are represented by their equivalent resistance and inductance.

The complete electrical power system can be described by a set of first order differential equations. Each branch element is described either as a current or voltage equation. Two approaches in developing the set of system equations are presented here.

### 3.3.1 A Systematic Formulation

Difference equations are widely used in the modelling of power system elements and these are then combined to form a difference equation system that describes the complete electrical power system. The set of network equations can be derived systematically using Kirchhoff’s current and voltage laws, which involve combining the state equations for each branch element and taking the node voltages and branch currents as state variables. This method allows the complete system equations to be built without the necessity to analyze the topology of the network, and without the need to identify the set of independent state variables. This is particularly advantageous for large systems where the set of independent state variable is not obviously apparent. However, in some cases, it is important to ensure that a minimum set of linear independent state variables is used to form the state-space description of the dynamic system [32].

### A. Generator Model

The impedance model of a three-phase generator is modelled by three independent RL branches with no interphase mutual coupling. Fig. 3.4 shows the basic RLC branch used to model a generator. The formulation is based on the single-phase RLC circuit state equations shown in (3.1) and (3.2), where \( L \) is the transient reactance, \( V_c \) being zero and the row \( \frac{dV_c}{dt} \) eliminated when there is no capacitance in the circuit.
\[ V_g - V_j = RI_s + L \frac{dI_s}{dt} + V_c \]  \hspace{1cm} (3.1)

\[ C \frac{dV_c}{dt} = I_s \]  \hspace{1cm} (3.2)

The state equations for a generator are therefore

\[
\begin{bmatrix}
\frac{dI_s}{dt} \\
\frac{dV_c}{dt}
\end{bmatrix} = \begin{bmatrix}
-L^{-1}R & -L^{-1} \\
C^{-1} & 0
\end{bmatrix} \begin{bmatrix}
I_s \\
V_c
\end{bmatrix} + \begin{bmatrix}
L^{-1} & -L^{-1} \\
0 & 0
\end{bmatrix} \begin{bmatrix}
V_g \\
V_j
\end{bmatrix}
\]  \hspace{1cm} (3.3)

**B. Transformer Model**

The transformer is represented using a coupled coil model that offers different impedance to the different components of current depending on the type of magnetic circuit as well as the connections of the main terminals and neutral. It is possible to simplify the three-phase transformer model by ignoring the interphase mutual coupling and reduce the equations to three independent phase sets. This is accurate when banks of single phase units are used. Effects such as phase shifts, zero sequence circulating currents are catered for by the terminal connections. Off-nominal taps on either primary or secondary winding can be represented by scaling the self and mutual elements accordingly.

A single-phase transformer can be modelled in terms of self inductance \( L_{ij} \), mutual inductance \( L_{ij}, L_{ji} \) and core losses \( R_i \) as in (3.4).

\[
\begin{bmatrix}
\frac{dI_i}{dt} \\
\frac{dI_j}{dt}
\end{bmatrix} = \begin{bmatrix}
L_{ii} & L_{ij} \\
L_{ji} & L_{jj}
\end{bmatrix}^{-1} \begin{bmatrix}
R_i & 0 \\
0 & R_j
\end{bmatrix} \begin{bmatrix}
I_i \\
I_j
\end{bmatrix} + \begin{bmatrix}
L_{ii} & L_{ij} \\
L_{ji} & L_{jj}
\end{bmatrix}^{-1} \begin{bmatrix}
V_i \\
V_j
\end{bmatrix}
\]  \hspace{1cm} (3.4)
where $V_i$ and $V_j$ are the voltages across the first and second winding respectively.

$I_i$ and $I_j$ are the current flows into the first and second winding respectively.

Similarly, for the three phase star-g/star-g connected transformer shown in Fig. 3.5, neglecting core losses, the single-phase state-space model can be expanded in terms of HV and LV branch currents as

$$
\begin{align*}
\frac{d}{dt} \begin{bmatrix} I_i \\ I_{i+1} \\ I_{i+2} \\ I_j \\ I_{j+1} \\ I_{j+2} \end{bmatrix} &= \left[K_{ia}\right]^{-1} \begin{bmatrix} L_{i,j} \\ L_{i,j} \\ L_{i,j} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} V_i \\ V_{i+1} \\ V_{i+2} \\ V_j \\ V_{j+1} \\ V_{j+2} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_i \\ V_{i+1} \\ V_{i+2} \\ V_j \\ V_{j+1} \\ V_{j+2} \end{bmatrix}
\end{align*}
$$

for Y-G/Y-G connection: $\left[K_{ia}\right] = \left[K_{ia}\right]$

where

$K_{ia}$ and $K_{ia}$ are the branch-node incidence (connection) for voltages and currents respectively.

$L_i, V_i$ are the primary currents and voltages at node i.
\( I_j, V_j \) are the secondary currents and voltages at node j.

C. Transmission Line Model
The transmission line nominal coupled PI model is used for short to medium length transmission lines. The capacitive and inductive coupling between the phases are included. The lumped three-phase representation of a transmission line is illustrated in Fig. 3.6. By summing up the nodal currents and loop voltages the following set of differential equations can be derived for the transmission line state-space model.

\[
\begin{align*}
L_x &= L_x + \frac{L_{sh}}{2} \\
L_r &= L_r - \frac{L_{sh}}{2} \\
V_x &= [R_x]L_x + [L_x]\frac{dI_x}{dt} + V_x \\
\frac{dV_x}{dt} &= \left[ \frac{C_{sh}}{2} \right]^{-1} \left( L_x - L_r - \left[ \frac{2}{R_{sh}} \right] V_x \right) \\
\frac{dV_r}{dt} &= \left[ \frac{C_{sh}}{2} \right]^{-1} \left( L_r - L_x - \left[ \frac{2}{R_{sh}} \right] V_r \right) \\
\frac{dI_x}{dt} &= [L_x]^{-1} \left( V_x - V_r - \left[ R_x \right] I_x \right)
\end{align*}
\]  

(3.6)  

(3.7)  

(3.8)  

(3.9)

\[
\begin{bmatrix}
\frac{dI_x}{dt} \\
\frac{dV_x}{dt} \\
\frac{dV_r}{dt} \\
\end{bmatrix} =
\begin{bmatrix}
0 & \left[ L_x \right]^{-1} [R_x] & 0 & \left[ L_x \right]^{-1} & \left[ L_x \right]^{-1} \\
& \ldots & \ldots & \ldots & \ldots & \ldots \\
& \left[ C_{sh} \right]^{-1} & \left[ C_{sh} \right]^{-1} & 0 & \left[ C_{sh} \right]^{-1} \frac{2}{R_{sh}} & 0 \\
& \ldots & \ldots & \ldots & \ldots & \ldots \\
& 0 & \left[ C_{sh} \right]^{-1} & \left[ C_{sh} \right]^{-1} & 0 & \left[ C_{sh} \right]^{-1} \frac{2}{R_{sh}} & [V_x] \\
\end{bmatrix}
\begin{bmatrix}
I_x \\
L_x \\
L_r \\
V_x \\
V_r \\
\end{bmatrix}
\]  

(3.10)

where \( I_s, I_r, I_L, I_{sh}, V_s, V_r \) are sending end and receiving end current, series line current, shunt current, sending end and receiving end busbar voltages respectively. \( [R_x], [L_x], [C_{sh}], [R_{sh}] \) are series resistance, series inductance, shunt capacitance and
shunt resistance matrices respectively.

Fig. 3.6 Equivalent coupled PI model

D. Load Model
The real and reactive power components of the system load are represented by its equivalent resistance and inductance. Fig. 3.7 depicts the equivalent load model and its state-space equations are given by eqn. (3.11).

Fig. 3.7 Equivalent load model
\[
\begin{bmatrix}
\frac{dI_{i_0}}{dt} \\
\frac{dI_{i_1}}{dt} \\
\frac{dI_{i_2}}{dt} \\
I_k \\
I_{k+1} \\
I_{k+2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & L_{k-1}^{-1} & 0 & 0 \\
0 & 0 & 0 & L_{k+1}^{-1} & 0 \\
0 & 0 & 0 & 0 & L_{k+2}^{-1} \\
1 & 0 & 0 & R_{k}^{-1} & 0 \\
0 & 1 & 0 & 0 & R_{k+1}^{-1} \\
0 & 0 & 1 & 0 & R_{k+2}^{-1}
\end{bmatrix}
\begin{bmatrix}
I_k \\
I_{k+1} \\
I_{k+2} \\
V_k \\
V_{k+1} \\
V_{k+2}
\end{bmatrix}
\] (3.11)

Since state-variable analysis is usually performed recursively for each time-step, discretisation of the continuous time differential equations is necessary. The operator \( \frac{d}{dt} \) can be approximated using Euler's formula (3.12).

\[
y_{i+\Delta t} = y_i + \Delta t f_i
\] (3.12)

Equations (3.5), (3.10) and (3.11) can then be expressed as:

\[
\begin{bmatrix}
I_i \\
I_{i+1} \\
I_{i+2} \\
I_j \\
I_{j+1} \\
I_{j+2}
\end{bmatrix}
= 
\begin{bmatrix}
I_i \\
I_{i+1} \\
I_{i+2} \\
I_j \\
I_{j+1} \\
I_{j+2}
\end{bmatrix} - \Delta t \left( \begin{bmatrix}
K_{i_0}^{-1} & [L]^{-1} & K_{i_0}^{-1}
\end{bmatrix}
\right)
\begin{bmatrix}
V_i \\
V_{i+1} \\
V_{i+2} \\
V_j \\
V_{j+1} \\
V_{j+2}
\end{bmatrix}
\]

\[
(3.13)
\]

where \([L] = \begin{bmatrix}
L_{i,i} & 0 & 0 & 0 & 0 \\
0 & L_{i,i} & 0 & 0 & 0 \\
0 & 0 & L_{i+1,i+1} & 0 & 0 \\
0 & 0 & 0 & L_{i+2,i+2} & 0 \\
0 & 0 & 0 & 0 & L_{i+2,i+2}
\end{bmatrix}\]

\[
\begin{bmatrix}
L_i \\
\vdots \\
L_{i+N}
\end{bmatrix} = \Delta t \begin{bmatrix}
\frac{C_{\phi}}{2} & 0 \\
0 & \frac{C_{\phi}}{3}
\end{bmatrix}
\begin{bmatrix}
\frac{2}{R_{\omega}} + [1] \\
\frac{2}{R_{\omega}} + [1]
\end{bmatrix}
\begin{bmatrix}
L_i \\
L_i \\
L_i \\
V_i \\
V_i
\end{bmatrix}
\]

\[
(3.14)
\]
When the discretised state equations are formed in terms of state variables at previous time step, it allows the history values \((t-\Delta t)\) to be used as virtual measurements (i.e. measurements that do not need metering) as well as real-time measurements.

Inspection of (3.14) shows that the transmission series currents can be used as measurement to form the TSE measurement system, however, given that they are usually not well defined variables (i.e. difficult to measure) in a practical system, the sending end and receiving branch currents are used instead. Therefore, the state equations become:

\[
\begin{bmatrix}
I_t \\
I_{t+1} \\
I_{t+2} \\
I_{k+1} \\
I_{k+2}
\end{bmatrix}_f =
\begin{bmatrix}
1 & 0 & 0 & \Delta tL_a^{-1} & 0 & 0 \\
0 & 1 & 0 & 0 & \Delta tL_b^{-1} & 0 \\
0 & 0 & 1 & 0 & 0 & \Delta tL_c^{-1} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
I_t \\
I_{t+1} \\
I_{t+2} \\
V_k \\
V_{k+1}
\end{bmatrix}_f
\]

(3.15)

\[
\begin{bmatrix}
I_t \\
I_{t+2} \\
I_{t+2} \\
I_{t+1} \\
I_{t+1}
\end{bmatrix}_f =
\begin{bmatrix}
1 & 0 & 0 & 2 & \frac{C_{sh,x}}{2\Delta t} & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} \\
0 & 1 & 0 & 2 & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} \\
0 & 0 & 1 & 2 & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} & \frac{C_{sh,x+4}}{2\Delta t} \\
2 & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} & \frac{C_{sh,x+4}}{2\Delta t} & \frac{C_{sh,x+5}}{2\Delta t} \\
0 & 0 & 0 & 2 & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} \\
0 & 0 & 0 & 0 & 2 & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t}
\end{bmatrix}
\begin{bmatrix}
I_t \\
I_{t+1} \\
I_{t+2} \\
V_k \\
V_{k+1}
\end{bmatrix}_f
\]

(3.16)

where

\[
\begin{bmatrix}
I_t \\
I_{t+1} \\
I_{t+2} \\
I_{t+1} \\
I_{t+1}
\end{bmatrix}_f =
\begin{bmatrix}
I_t \\
\dot{I}_t \\
I_{t+1} \\
\dot{I}_{t+1} \\
I_{t+2} \\
\dot{I}_{t+2}
\end{bmatrix}_f +
\begin{bmatrix}
C_{sh,x} & \frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} \\
\frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} \\
\frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} & \frac{C_{sh,x+4}}{2\Delta t} \\
\frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} \\
\frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t} \\
\frac{C_{sh,x+1}}{2\Delta t} & \frac{C_{sh,x+2}}{2\Delta t} & \frac{C_{sh,x+3}}{2\Delta t}
\end{bmatrix}
\begin{bmatrix}
V_t \\
V_{t+1} \\
V_{t+2} \\
V_{t+1} \\
V_{t+1}
\end{bmatrix}_f - \Delta t
\]
\[
\begin{bmatrix}
I_t \\
I_{rs, t} \\
I_{rs+1, t} \\
I_{rs+2, t}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & -\frac{C_{sh, 2}}{2\Delta t} & -\frac{C_{sh, 21}}{2\Delta t} & -\frac{C_{sh, 22}}{2\Delta t} \\
0 & 1 & 0 & -\frac{C_{sh, 11}}{2\Delta t} & -\frac{C_{sh, 12}}{2\Delta t} & -\frac{C_{sh, 13}}{2\Delta t} \\
0 & 0 & 1 & -\frac{C_{sh, 21}}{2\Delta t} & -\frac{C_{sh, 22}}{2\Delta t} & -\frac{C_{sh, 23}}{2\Delta t} \\
0 & 0 & 0 & -\frac{C_{sh, 31}}{2\Delta t} & -\frac{C_{sh, 32}}{2\Delta t} & -\frac{C_{sh, 33}}{2\Delta t}
\end{bmatrix}
\begin{bmatrix}
I_L \\
I_{rs+1} \\
I_{rs+2} \\
V_r \\
V_{rs+1} \\
V_{rs+2}
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
I_t \\
I_{rs, t} \\
I_{rs+1, t} \\
I_{rs+2, t}
\end{bmatrix} =
\begin{bmatrix}
\frac{C_{sh, 2}}{2\Delta t} & \frac{C_{sh, 21}}{2\Delta t} & \frac{C_{sh, 22}}{2\Delta t} \\
\frac{C_{sh, 11}}{2\Delta t} & \frac{C_{sh, 12}}{2\Delta t} & \frac{C_{sh, 13}}{2\Delta t} \\
\frac{C_{sh, 21}}{2\Delta t} & \frac{C_{sh, 22}}{2\Delta t} & \frac{C_{sh, 23}}{2\Delta t} \\
\frac{C_{sh, 31}}{2\Delta t} & \frac{C_{sh, 32}}{2\Delta t} & \frac{C_{sh, 33}}{2\Delta t}
\end{bmatrix}
\begin{bmatrix}
V_r \\
V_{rs+1} \\
V_{rs+2}
\end{bmatrix}
\]

3.3.2 Diakoptical Formulation

The general state variable approach requires the identification of independent state variables and formulation of the appropriate equations [33]. Arbiy trary use of inductor current and capacitor voltage is not sufficient due to the possible presence of inductor cut-sets or loops of capacitors and voltage sources. These occur in component models and also when components are interconnected. For example capacitive loops exist in transmission lines represented by a PI model. Use of graph theory or linear matrix methods can find the appropriate state variables. However, when the system is undergoing topological changes due to frequent switchings, the computational effort needed to identify the new set of independent state variables and form the state equations is impractical. Nodal formulation with a diakoptical technique avoids the need to identify the independent state variables. The technique diakoptically segregates the system nodes depending on the type of branches that are connected to them, so only the relevant parts of the network equations are considered when the system undergoes topological changes. This significantly reduces the computation needed when forming the state equations for systems with periodically varying topology, such as an ac-dc converter. The concept is the basis of Kron’s tensor technique [34], which forms the basis of Transient Converter Simulation (TCS), This work started at UMIST and continued at the University of Canterbury [35]. Further contributions to the TCS algorithm made over the years include, the use of different ac system models, a more realistic dc fault representation, transformer saturation and hysteresis effects [36], detailed frequency dependent ac system equivalent model [37] and enhanced control system modelling using basic building blocks [38]. Its logical procedure for the automatic assembly and solution of network equations should
provide an efficient approach to formulate the measurement equations if implemented in the TSE technique.

Consider a system with $n$ nodes systematically expanded into its elementary resistive $r$, inductive $l$, capacitive $c$ and current source $s$ branches. Diakoptics segregation of the network, subdivides the system nodes into three parts according to the type of branches connected to them:

- $\alpha$ nodes: At least one capacitive branch.
- $\beta$ nodes: At least one resistive branch but no capacitive branches.
- $\gamma$ nodes: Only inductive branches.

The resulting branch-node incidence (connection) matrices for the $r$, $l$ and $c$ branches are $K^T_{rn}$, $K^T_{ln}$ and $K^T_{cn}$ respectively. The elements in the branch-node incidence matrices are determined as follows:

$$K^T_{bn} = 1; \text{ If node n is the sending end of branch b}$$

$$K^T_{bn} = -1; \text{ If node n is the receiving end of branch b}$$

$$K^T_{bn} = 0; \text{ If branch b is not connected to node n}$$

The network is subjected to some restrictions on the type of circuit topology that can be analysed. By restricting the number of possible network configuration to those commonly encountered in practical systems, the efficiency of the solution can be improved significantly. The restrictions are [39]:

1. capacitive branches have no series voltage sources
2. resistive branches have no series voltage sources
3. capacitive branches are constant valued ($dC_{cc}/dt = 0$)
4. every capacitive branch sub-network has at least one connection to the system reference (ground node)
5. resistive branch sub-networks have at least one connection to either the system reference or an $\alpha$ node.
6. inductive branch sub-networks have at least one connection to the system reference or an $\alpha$ or $\beta$ node.

Neglecting current sources, the fundamental branches that result from the restrictions and their diakoptical network equations are:
Resistive branch

\[
I_r = R_{rr}^{-1} \left( K_{ra}^T \alpha + K_{r \beta}^T \beta \right)
\]  
(3.18)

Inductive branch

\[
E_i \frac{d}{dt} \left( L_i \dot{I}_i \right) - R_i I_i + K_{ia}^T \alpha + K_{i \beta}^T \beta + K_{i \gamma}^T \gamma = 0
\]  
(3.19)

Capacitive branch

\[
I_c = C_{cc} \frac{d}{dt} \left( K_{ca}^T \alpha \right)
\]  
(3.20)

Applying the node type definitions, the nodal equations for each node type become:

\[
K_{aa} I_c + K_{ar} I_r + K_{ai} I_i = 0
\]

\[
K_{ba} I_r + K_{b \beta} I_i = 0
\]

\[
K_{i \gamma} I_i = 0
\]
(3.21)

Combining (3.18)-(3.21) and taking capacitive node voltage \( V_\alpha \) and inductive branch current \( I_i \) as the state variables, a set of diakoptical network equations can be formulated in the form of

\[
\dot{x} = [A]x + [B]u
\]  
(3.22)

\[
y = [C]x + [D]u
\]  
(3.23)

The state equations and dependent variable equations, derived in Appendix IV, are expressed as:

State equations
\[
\begin{bmatrix}
\frac{dl_l}{dt} \\
\frac{dl_r}{dt} \\
\frac{dV_a}{dt}
\end{bmatrix}
= \begin{bmatrix}
-L_l^3 M_u R_u^T & L_q M_u A_{ta}^T & I_i \\
-C_{ta}^\r A_{ta} & -C_{ta}^\r G_{ta} K_{ra}^T & V_a \\
0 & 0 & [E_i]
\end{bmatrix}
\] (3.24)

Dependent variables

\begin{align}
V_\beta &= -R_{\beta\beta} \left( K_{\beta\beta} I_i + K_{\beta\gamma} R_{\gamma\gamma}^{-1} K_{\gamma\alpha}^T V_\alpha \right) \\
V_\gamma &= -L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} \left( -R_{\gamma\gamma} - K_{\gamma\beta}^T R_{\beta\beta} K_{\beta\beta} \right) I_i + ... \\
&\quad - L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} \left( K_{\gamma\alpha}^T - K_{\gamma\beta}^T R_{\beta\beta} K_{\beta\beta} R_{\gamma\gamma}^{-1} K_{\gamma\alpha}^T \right) V_a - L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} E_i \\
I_r &= -R_{\gamma\gamma} K_{\gamma\gamma}^T R_{\beta\beta} K_{\beta\beta} I_i + ... \\
&\quad - R_{\gamma\gamma}^{-1} \left( K_{\gamma\alpha}^T - K_{\gamma\beta}^T R_{\beta\beta} K_{\beta\beta} R_{\gamma\gamma}^{-1} K_{\gamma\alpha}^T \right) V_a \\
I_c &= -K_{\gamma\alpha}^{-1} \left( A_{\alpha\gamma} I_i + G_{\alpha\gamma} K_{\alpha\alpha}^T V_\alpha \right)
\end{align} (3.25-3.28)

where

\[
M_u = U_u - K_{\gamma\gamma}^T L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} \\
A_{\alpha\gamma} = K_{\gamma\alpha} - K_{\gamma\alpha} R_{\alpha\alpha}^{-1} K_{\gamma\beta}^T R_{\beta\beta} K_{\beta\beta} \\
G_{\alpha\gamma} = K_{\gamma\gamma} R_{\gamma\gamma}^{-1} \left( U_{\alpha\gamma} - K_{\gamma\beta} R_{\beta\beta} K_{\beta\beta} R_{\gamma\gamma}^{-1} \right) \\
C_{\alpha\alpha} = K_{\gamma\gamma} C_{\alpha\alpha} K_{\gamma\gamma}^T \\
L_{\gamma\gamma} = \left( K_{\gamma\gamma}^T L_{\gamma\gamma} K_{\gamma\gamma} \right)^{-1} \\
R_{\beta\beta} = \left( K_{\beta\beta} R_{\beta\beta}^{-1} K_{\beta\beta}^T \right)^{-1} \\
I_u &= R_{\gamma\gamma} + K_{\gamma\beta} R_{\beta\beta} K_{\beta\beta}
\]

\(R, L\) and \(C\) are the resistance, inductance and capacitance matrices respectively. \(U_r\) and \(U_u\) are unit matrices of order \(r\) and \(l\) respectively.

**A. Generator Model**

In diakoptic formulation, generators are modelled by series RL branches as shown in Fig. 3.8.
It is assumed that there is no mutual coupling between the phases, therefore the branches can be treated independently and only the diagonal elements of the diakoptics matrices contain non-zero entries. The relevant diakoptics matrices are:

\[
L_{ij} = \begin{bmatrix}
L_g & 0 & 0 \\
0 & L_{g+1} & 0 \\
0 & 0 & L_{g+2}
\end{bmatrix}
\]

\[
R_{ij} = \begin{bmatrix}
R_g & 0 & 0 \\
0 & R_{g+1} & 0 \\
0 & 0 & R_{g+2}
\end{bmatrix}
\]

\[
K_m = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[\]

**B. Transformer Model**

Similar to the transformer model in Section 3.3.1, a coupled coil transformer model is used in the TSE algorithm. The model offers different impedance to the different components of current depending on the type of magnetic circuit as well as the connections of the terminals and neutral. Fig. 3.9 shows the Delta/Y-G connected transformer model. Off-nominal taps on either primary or secondary windings can be represented by scaling the self and mutual elements accordingly.
Fig. 3.9 Three-phase delta/star-g connected transformer

Neglecting core losses, the relevant diakoptics matrices for a three phase transformer are:

$$\begin{bmatrix}
L_{ii} & L_{ij} & 0 & 0 & 0 & 0 \\
L_{ji} & L_{jj} & 0 & 0 & 0 & 0 \\
0 & 0 & L_{vi,i+1} & L_{vi,j+1} & 0 & 0 \\
0 & 0 & L_{vj,i+1} & L_{vj,j+1} & 0 & 0 \\
0 & 0 & 0 & 0 & L_{vi+2,i+2} & L_{vi+2,j+2} \\
0 & 0 & 0 & 0 & L_{vj+2,i+2} & L_{vj+2,j+2}
\end{bmatrix}$$

$$\begin{bmatrix}
R_{ii} & 0 & 0 & 0 & 0 & 0 \\
0 & R_{ij} & 0 & 0 & 0 & 0 \\
0 & 0 & R_{ji,i+1} & 0 & 0 & 0 \\
0 & 0 & 0 & R_{ji,j+1} & 0 & 0 \\
0 & 0 & 0 & 0 & R_{ji+2,i+2} & 0 \\
0 & 0 & 0 & 0 & 0 & R_{ji+2,j+2}
\end{bmatrix}$$

for ΔY-G configuration,

$$K_{ni} = \begin{bmatrix}
1 & 0 & 0 & 0 & -1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

where

$K_{ni}$ is the branch-node incidence (connection) matrix

$L_{ii}$ is the self inductance.

$L_{ij}, L_{ji}$ are the mutual inductance.
C. Transmission Line Model

A three phase PI nominal model is used to represent short to medium length transmission line in the diakoptical formulation. Fig. 3.10 shows the transmission line PI model. The model includes transmission line coupling effects. These are represented by the off-diagonal non-zero entries in the RLC matrices. The relevant RLC and connection matrices K are:

\[
R_h = \begin{bmatrix}
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}} \\
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}} \\
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}}
\end{bmatrix}
= \begin{bmatrix}
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}} \\
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}} \\
R_{sh_{12}} & R_{sh_{13}} & R_{sh_{23}}
\end{bmatrix}
\]

\[
C_h = \begin{bmatrix}
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}} \\
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}} \\
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}}
\end{bmatrix}
= \begin{bmatrix}
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}} \\
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}} \\
C_{sh_{12}} & C_{sh_{13}} & C_{sh_{23}}
\end{bmatrix}
\]

\[
L_s = \begin{bmatrix}
L_{s11} & L_{s12} & L_{s13} \\
L_{s21} & L_{s22} & L_{s23} \\
L_{s31} & L_{s32} & L_{s33}
\end{bmatrix}
\]

\[
R_y = \begin{bmatrix}
R_{y11} & R_{y12} & R_{y13} \\
R_{y21} & R_{y22} & R_{y23} \\
R_{y31} & R_{y32} & R_{y33}
\end{bmatrix}
\]

\[
R_{ur} = \begin{bmatrix}
[LR_{ur}] & 0 \\
0 & [LR_{ur}]
\end{bmatrix}
\]

\[
C_{ur} = \begin{bmatrix}
[UC_{ur}] & 0 \\
0 & [UC_{ur}]
\end{bmatrix}
\]

\[
K_{al} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad K_{nr} = K_{nr} \begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & -1 & 0 & -1 & 1
\end{bmatrix}
\]

where

\(K_{al}, K_{nr} \) and \(K_{nc}\) are the branch (RLC)-node incidence (connection) matrix,
\(R_{sh}\) and \(C_{sh}\) are shunt resistance and capacitance matrix respectively.
\(L_s\) and \(R_s\) are series inductance and resistance matrix respectively.

\(UR_{sh}, UR_{sh}, UC_{sh}, UC_{sh}\) are diagonal matrices with entries from the
upper-triangle $R_{sk}$, $R_{sh}$, $C_{sk}$, and $C_{sh}$ respectively.

![Fig. 3.10 Three-phase transmission line PI model](image)

**D. Load Model**

Fig. 3.11 shows an equivalent model of a system load represented by its equivalent resistance and reactance. The relevant diakoptics matrices are as follows:

![Fig. 3.11 Three-phase load model](image)

$$L_{ij} = \begin{bmatrix} L_k & 0 & 0 \\ 0 & L_{k+1} & 0 \\ 0 & 0 & L_{k+2} \end{bmatrix}$$

$$R_{ij} = \begin{bmatrix} R_k & 0 & 0 \\ 0 & R_{k+1} & 0 \\ 0 & 0 & R_{k+2} \end{bmatrix}$$

$$K_{il} = K_{ir} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
3.4 Diakoptical State Variable Solution

Transient simulation based on state space theory [40]-[42] was once popular, but the interest has diminished despite its advantages [43] and [44] due to high computation cost. Diakoptical segregation formulation is computation efficient if sparsity of coefficient matrices is used, especially when the system contains transmission lines with mutual inductive effects and a high degree of capacitive branch interconnections. Furthermore, the data input burden is significantly reduced if the diakoptic formulation is performed by an automatic procedure. The automatic procedure expands each component into its equivalent lumped branches and assigns them with appropriate branch numbers. For example, for an inductive branch, the general structure of $L_u$ is:

$$L_u = \begin{bmatrix}
\begin{array}{c}
\text{component 1} \\
\text{component 2} \\
\vdots \\
\text{component n}
\end{array}
\end{bmatrix}
\begin{bmatrix}
L_{u1} \\
L_{u2} \\
\vdots \\
L_{un}
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\vdots & \ddots & \vdots \\
0 & 0 & 0
\end{bmatrix}
$$

The diagonal submatrices represent the inductance matrix $L_u$ of the individual components. Identification of the independent system state variables is no longer required when the nodal equation is used. The nodal inductance matrix for the $\gamma$ nodes is expressed as:

$$L_{\gamma\gamma} = \left(K_{\gamma1}L_u^{-1}K_{1\gamma}^T\right)^{-1} = \begin{bmatrix}
L_{\gamma1,1} & L_{\gamma1,2} & \cdots & L_{\gamma1,n} \\
L_{\gamma2,1} & \ddots & & \vdots \\
\vdots & & \ddots & \vdots \\
L_{\gamma n,1} & \cdots & & L_{\gamma n,n}
\end{bmatrix}
$$

Cross-referencing information is created relating the system busbars to those node numbers. When the process is repeated for all the components, the diakoptical matrices containing the indexing and component information are generated.

The set of diakoptical state variable equations can be solved iteratively using implicit trapezoidal integration for each time-step with no restriction imposed on the selection
of the integration step length [45]. This property allows the step length to be varied and to fall exactly on the time of the changes to represent the new condition, thus eliminating the problem of numerical oscillation due to step changes or switching.

The state equations can be written as in (3.29) and the change in the state variable $\Delta x$ is defined in (3.30).

$$\dot{x} = f(I_t, V_u)$$  \hspace{1cm} (3.29)

$$\Delta x = \frac{h}{2} (\dot{x}_t + \dot{x}_{t+h})$$  \hspace{1cm} (3.30)

The diakoptical solution can be obtained through an iterative procedure.

1) For an initial estimate, it is assumed $\dot{x}_{t+h} = \dot{x}_t$

2) An intermediate $x_{t+h}$ based on the $\dot{x}_{t+h}$ estimate is then calculated.

$$x_{t+h} = x_t + \frac{h}{2} (\dot{x}_t + \dot{x}_{t+h})$$

3) An update of the $\dot{x}_{t+h}$ can be calculated from the intermediate $x_{t+h}$ value from the state equation.

$$\dot{x}_{t+h} = [A] x_{t+h} + [B] u_{t+h}$$

Steps 2) and 3) are performed iteratively until convergence is reached. In the case that convergence fails, the step length is halved and the iterative procedure is restarted. Dependent state variables can be calculated at the end of each time-step ($h$) or at the end of the simulation.

### 3.4.1 Transient simulation using State-Space Diakoptical Segregation Methodology

The test system is taken at the 220kV grid below Roxburgh in the South Island of New Zealand. The harmonic sources at the three locations, Roxburgh 11kV, Tiwai 220kV and Invercargill 33kV modelled previously in Chapter 2 are replaced by three linear loads (90MW, 54MVAR), (135MW, 36MVAR) and (480MW, 130MVAR) respectively.

Two different fault scenarios (symmetrical line-to-ground and asymmetrical line-to-ground) are simulated using diakoptic state-space equations and PSCAD/EMTDC. Both simulations model each system component as described in Section 3.3.2, e.g. transmission lines are represented using PI model. The fault location is shown in Fig. 3.12.
The state variables are taken as the capacitive node voltages ($V_a$) and inductive branch currents ($I_r$). There are 12 capacitive nodes and 72 inductive branches, therefore, giving a total of 84 state variables. The dimensions of the matrices $L_a$, $R_a$, $R_r$, and $C_{cc}$ depend on the number of corresponding individual elements connected in the physical network. Matrices $L_a$ and $R_a$ of dimension 72x72 correspond to the number of inductive branches and is thus of the same row size as $I_r$. $C_{cc}$ of dimension 150x150 corresponds to the transmission line shunt capacitance which consist of three double circuit transmission lines (3x42) and two single circuit transmission lines (2x12). The dimension of $R_r$ is 159x159 and 165x165 respectively under normal network operation (which consist of the transmission line shunt resistance and the three linear loads resistance) and fault condition operation. The extra six resistances correspond to the line-ground fault modelled by a small resistance connected to ground.
A. A symmetrical fault
A three-phase line-to-ground fault of 2.5 cycle duration on the Invercargill-Tiwai transmission line is simulated. The symmetrical fault is modelled by a small resistance connected to ground at each phase which causes a complete collapse of the three phase receiving end voltages as well as over-currents at the corresponding sending and receiving end branches.

Figs. 3.13-3.21 show the simulation results for Tiwai 220kV busbar voltages,
Invercargill-Tiwai sending end branch currents and the fault currents. The Diakoptical State-Variable (DSV) and PSCAD/EMTDC simulations are plotted in solid and dotted lines respectively and their differences are shown in the 2\textsuperscript{nd} graph. As shown, the difference is relatively small during steady state, but becomes larger when there is a step change.

![Graph showing busbar voltages at Tiwai 220kV phase A]

Fig. 3.13 Busbar voltage at Tiwai 220kV phase A
Fig. 3.14 Busbar voltage at Tiwai 220kV phase A

Fig. 3.15 Busbar voltage at Tiwai 220kV phase C
Fig. 3.16 Sending end Invercargill-Tiwi branch current, phase A

Fig. 3.17 Sending end Invercargill-Tiwi branch current, phase B
Fig. 3.18 Sending end Invercargill-Tiwi branch current, phase C

Fig. 3.19 Fault current, phase A
B. Asymmetrical fault

While the results obtained under symmetric fault condition showed a good match with the simulation, it is often necessary to look at the performance under unbalanced conditions as well before any conclusions can be made. A single phase line-to-ground
fault of a 2.5 cycles duration at the Invercargill-Tiwi transmission line is simulated in this case. The asymmetrical fault is modelled by a small resistance connected to ground at phase A and a high resistance connected to the other two phases. Figs. 3.22-3.27 show the Tiwi 220kV busbar voltages and the Invercargill-Tiwi sending end currents. The three phase fault currents are depicted in Figs. 3.28-3.30. The results are again in good agreement with those of the PSCAD/EMTDC simulation, the differences were high at the inception of the fault due to the topological changes, but decayed to a negligible level in less than half a cycle. The ability to predict the asymmetrical transient response justifies the use of the proposed diakoptical state-variable algorithm for transient simulation. Based on the two test scenarios, the agreement between the two fundamentally different algorithms verifies that the proposed algorithm can be used with confidence in a.c. systems.

![Graph of Tiwi 220kV phase A](image)

**Fig. 3.22 Busbar voltage at Tiwi 220kV phase A**
Fig. 3.23 Busbar voltage at Tiwai 220kV phase B

Fig. 3.24 Busbar voltage at Tiwai 220kV phase C
Inv-Tiw 220kV, phase A, sending end

Fig. 3.25 Sending end Invercargill-Tiwi branch current, phase A

Inv-Tiw 220kV, phase B, sending end

Fig. 3.26 Sending end Invercargill-Tiwi branch current, phase B
Fig. 3.27 Sending end Invercargill-Tiwi branch current, phase C

Fig. 3.28 Fault current, phase A
Fig. 3.29 Fault current, phase B

Fig. 3.30 Fault current, phase C
3.5 Transient State Estimation Algorithm

In TSE, the estimation of the state variables is achieved by solving an estimator consisting of a set of measurement equations. The measurement equations are linear when using the two formulation methods discussed in Section 3.2. The measurement system is expressed as:

\[ z = [H]x + \epsilon \]  

(3.31)

where \( \epsilon \) is the error vector and \( [H] \) is the measurement matrix.

The error vector represents the difference between the actual measurements and their true values due to the presence of measurement noise or bad data. Measurement bad data degrades the quality of TSE and its effect needs to be considered, however, as an introduction to TSE which focuses on the verification of the proposed concept and methodology, measurement noise is neglected in this study. The measurement set \( z \) is formed from a combination of bus voltage and line current measurements, as well as their derivatives \( \frac{dV}{dt}, \frac{dI}{dt} \) to provide extra information. In the systematic formulation method, if the state variables are taken as the line currents and busbar voltages, then equations (3.13)-(3.17) are defined as the measurement equations. Each measurement adds a row of the corresponding measurement equation to the measurement matrix \([H]\). The estimator is formed when all the measurement equations are added. For example, the following row from Eqn. (3.13) is used for the phase C current measurement taken at the primary side of the transformer.

\[
\begin{bmatrix}
I_i \\
I_{i+1} \\
I_{i+2} \\
I_j \\
I_{j+1} \\
I_{j+2} 
\end{bmatrix}_{t-\Delta t} =
-\Delta t
\begin{bmatrix}
K_{v_{bh}}^{-1} & L_j^{-1} & K_{v_{nh}}^{-1} 
\end{bmatrix}
\begin{bmatrix}
V_i \\
V_{i+1} \\
V_{i+2} \\
V_j \\
V_{j+1} \\
V_{j+2} 
\end{bmatrix}_t
\]

(3.13)

where \([L]=\)

\[
\begin{bmatrix}
L_{i,i} & L_{i,i} & 0 & 0 & 0 & 0 \\
L_{j,i} & L_{j,i} & 0 & 0 & 0 & 0 \\
0 & 0 & L_{i+1,i+1} & L_{i+1,i+1} & 0 & 0 \\
0 & 0 & L_{j+1,j+1} & L_{j+1,j+1} & 0 & 0 \\
0 & 0 & 0 & 0 & L_{i+2,i+2} & L_{i+2,i+2} \\
0 & 0 & 0 & 0 & L_{j+2,j+2} & L_{j+2,j+2} 
\end{bmatrix}
\]
The derivative of the measurements can also be utilized. The following row from (3.10) is used for the derivative measurements of the sending end busbar voltage.

\[
\begin{bmatrix}
\frac{dV_e}{dt} \\
\frac{dV_i}{dt} \\
\vdots \\
\frac{dV_i}{dt}
\end{bmatrix}
= \begin{bmatrix}
0 & -[L]^{-1}[R] & 0 & [L]^{-1} & -[L]^{-1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & -[C_{de}]^{-1} & \frac{2}{R_{eb}} & 0 & \frac{2}{R_{eb}}
\end{bmatrix}
\begin{bmatrix}
I_e \\
I_i \\
\vdots \\
I_i
\end{bmatrix}
\] (3.10)

Alternatively, for the diakoptical formulation method, the measurements in relation to the state variables are:

State variable measurement

\[ \bar{z}_{\text{measured}} = [1] \bar{x} \] (3.32)

Derivative of state variable measurement

\[ \dot{\bar{z}}_{\text{measured}} = [A] \bar{x} + [B] \bar{u} \] (3.33)

Dependent variable measurement

\[ \bar{y}_{\text{measured}} = [C] \bar{x} + [D] \bar{u} \] (3.34)

Derivative of dependent variable measurement

\[ \dot{\bar{y}}_{\text{measured}} = [C] \dot{x} + [D] \dot{u} \]
\[ \ddot{\bar{y}}_{\text{measured}} = [CA] \dot{x} + [B] \ddot{u} + [D] \ddot{u} \] (3.35)

Rows of measurement equations corresponding to the selected measurement points are used to build up the measurement system. Based on the two different formulation methods developed in Section 3.2, two different measurement matrices namely \([H_{\text{diak}}]\) and \([H_{\text{sys}}]\) are formed. The latter, is an approximate measurement model where Euler's formula is used to replace the operator \(\frac{d}{dt}\). An aspect to note is that this approximation may lose its accuracy if the selected time-step \(\Delta t\) is inadequate. This means that for a very fast transient, the time step should be sufficiently small to accurately approximate the derivative values.

The measurement system is then solved using the weighted least squares.
The least-squares solution \( \hat{x} \) for the measurement system in (3.31) is such that

\[
J(\hat{x}) = \min_{\hat{x}} \quad r^T r
\]  
(3.36)

where the index \( J(x) \) is defined by

\[
J(x) = \frac{1}{2} (\bar{z} - H\hat{x})^T (\bar{z} - H\hat{x})
\]  
(3.37)

and the residual vector \( r \)

\[
r = \bar{z} - H\hat{x}
\]  
(3.38)

The performance index \( J(x) \) is differentiated to obtain the first order optimal conditions

\[
\frac{dJ(x)}{dx} \bigg|_{x=\hat{x}} = H^T H\hat{x} - H^T \bar{z} = 0
\]  
(3.39)

This yields the normal equation

\[
H^T H\hat{x} = H^T \bar{z}
\]  
(3.40)

Rearranging gives

\[
\hat{x} = \left( H^T H \right)^{-1} H^T \bar{z}
\]  
(3.41)

Once the state variables are known, dependent variables such as branch currents can be calculated, and the complete knowledge of the system power flow can be determined.

Fig. 3.31 summarizes the TSE implementation using the two formulation methods.
Fig. 3.31 Flowchart of TSE based on the two different system formulation methods
3.6 Simulation Results

The test system described in Section 3.4.1 (Fig. 3.12) is used in the TSE simulation. In Sections 3.6.1 and 3.6.2, the set of measurements is chosen arbitrarily. Once the measurements are chosen, the observability of the system is checked by inspecting the null-space vector of the measurement system using Singular Value Decomposition. SVD produces a nullspace vector for each singularity. The non-zero entries of the nullspace vector indicate the corresponding state variables that are unobservable [46].

3.6.1 An Approximate method for Transient State Estimation

The test system is the reduced three-phase 220kV network below Roxburgh in the South Island of New Zealand. The voltage and current measurement placements are shown by the dots and diamonds respectively in Fig. 3.32. To look at the transient state estimation performance, over-voltages caused by a sudden loss of a Tiwai load are simulated in PSCAD/EMTDC. Half of the Tiwai load is disconnected for 100ms at simulation time 0.8s. The measurements supplied to the state estimation algorithm are taken from the PSCAD/EMTDC simulation. No information is given about the disturbance location, thus, all three load locations are treated as suspicious sources and are separated from the backbone network. Furthermore, no measurements are taken at the disturbance location i.e. Tiwai 220kV.

The estimated and simulated three-phase load voltages are shown in Figs. 3.33, 3.35 and 3.36 as dotted and solid lines respectively. Figs. 3.37 and 3.38 show the estimation results of the Invercargill transformer line currents and the Invercargill-Tiwai transmission line currents. Fig. 3.34 takes a closer look at the voltage difference between the actual and estimated waveform for the Tiwai 220kV load. The figure showed that some discrepancies occurred but decayed very quickly, during the disconnection and reconnection of the Tiwai load. This is due to the very fast transient when the system experiences a sudden step change, during which the time-step used in the Euler’s approximation (3.12) was inadequate. The error can be reduced if a smaller estimation time step is used. Despite this, the estimates are a good approximation to the PSCAD/EMTDC simulation. The fact that both waveforms overlapped each other justifies this.

In this simulation, it is possible to determine the cause of the transient from the estimation results. The load currents at the three suspicious load locations, Tiwai 220kV, Roxburgh 11kV and Invercargill 33kV are provided by the estimator at the end of TSE. The results clearly verify that there is a significant drop of load current
indicating a lost of load at Tiwai 220kV (Figs. 3.39-3.41). This demonstrates that TSE provided a good estimate of the system states and was capable of correctly identifying the origin and duration of the lost of load, even when no measurements were made at or near Tiwai 220kV.

Fig. 3.32 Measurement placements for the test system
Fig. 3.33 Three phase busbar voltages at Tiwai 220kV

Fig. 3.34 Voltage difference at Tiwai 220kV
Fig. 3.35 Three phase busbar voltages at Roxburgh 11kV

Fig. 3.36 Three phase busbar voltages at Invercargill 33kV
Fig. 3.37 Invercargill 220kV/33kV primary side currents (Both actual and estimated are plotted)

Fig. 3.38 Invercargill-Tiwi double circuit branch currents
Fig. 3.39 Three phase load currents at Tiwai 220kV

Fig. 3.40 Three phase load currents at Roxburgh 11kV
In the simulation, the proposed approximate TSE generated a good estimate of the system states. The power flow of the network loads can be determined even when no information of the load is supplied to the TSE (i.e. the loads are separated from the network during TSE). This is particularly useful when there are unexpected hidden disturbances within the network. The simulation was performed under a noise-free environment. In practice, however, the measurements are different from their true values due to corruption caused by noise and bad data. The effect of noise/bad data will affect the accuracy of TSE and needs to be investigated, however it does not affect the solvability and validity of the proposed algorithm, which is the main purpose of this study. It is important to know that the Euler approximation assumes minimal magnitude variations within the time step, thus for a system experiencing very fast transients, the estimation time step needs to be sufficiently small enough for accurate estimation. At present the algorithm is too slow to run in real-time applications. The simulation took 923secs of CPU (AMD 2000+) time to complete 1sec of simulation. The computation time per time-step is 46.15ms. With greater computer processing power and an optimized algorithm using compiled code, the computation time is expected to reduce significantly.
3.6.2 Transient State Estimation using diakoptical segregation methodology

In this simulation, the same test system described in Section 3.4.1 is used. The measurement placements are indicated in Fig. 3.42. There are 108 measurement data sets and half of them are the measurements of derivative values. This significantly reduces the measurement points required for an over-determined estimation. A large portion of the measurements are taken at the HV and LV side of the transformer utilizing transducers that are already available to record the readings. The data used is obtained from a TCS based state-variable simulation [47] and they are considered as the true values. A 2.5 cycle duration line-to-ground fault is simulated near the Tiwai 220kV node as shown by the black crosses in Fig. 3.42.

Fig. 3.42 Test system from the Lower South Island of New Zealand
The estimated line currents and node voltages are shown in Figs 3.43 and 3.44. A closer look of the Manapouri-Invercargill sending end branch currents and Tiwai 220kV nodal voltages are shown in Figs. 3.45-3.50. The bottom of each figure shows the difference between the estimated and actual values (both results have been plotted, but are difficult to distinguish due to similarity). To represent the unexpected fault event, the fault is not modelled in the TSE algorithm. When the network topology is modelled incorrectly, the estimation results will be in error. This leads to current mismatch and inconsistent results, which can be used to give an insight to the possible fault location. Inspection of the Tiwai load current showed that the estimated and calculated load currents were inconsistent with each other. The calculated load current is obtained by dividing the estimated Tiwai node voltage by the equivalent load impedance. Fig. 3.51 is a comparison of the estimated and calculated Tiwai load currents. The inconsistency is due to the absence of the fault path in the TSE algorithm (i.e. the fault was not modelled explicitly) and therefore, the fault current must flow to the Tiwai load. The node voltage however, is estimated correctly since it is dependent on the sending end node voltages (Manapourui 220kV and Invercargill 220kV) and not by the fault current. The nodal current mismatch is then checked and shows a significant current mismatch at node 1-3 (i.e. Tiwai220kV), indicating that Tiwai 220kV is likely to be the fault location (Fig. 3.52).

The suspicious fault location is easily identified using TSE, but since there are two transmission lines and a load connected to the suspicious nodes, further testing is required to pinpoint the source of the fault. Despite this, the characteristics of the fault can still be determined by the current mismatch. The large symmetrical fault currents indicate that the fault is symmetrical and have a low fault resistance. Moreover, since the current mismatch is due to the fault not being modelled in the TSE, it is essentially equivalent to the fault current. This is justified in Figs 3.53-3.55, where the actual fault current is a close match with the current mismatch. When the fault model is included in the TSE algorithm, TSE is capable of producing a good estimate of the actual system states. Figs. 3.56 and 3.57 show the new TSE results when the fault source is modelled.
Fig. 3.43 TSE of line currents

Fig. 3.44 TSE of node voltages
Fig. 3.45 Manapouri-Invercargill 220kV circuit 1 branch current phase A

Fig. 3.46 Manapouri-Invercargill 220kV circuit 1 branch current phase B
Fig. 3.47 Manapouri-Invervargill 220kV circuit 1 branch current phase C

Fig. 3.48 Tiwai 220kV phase A voltage
Fig. 3.49 Tiwai 220kV phase B voltage

Fig. 3.50 Tiwai 220kV phase C voltage
Fig. 3.51 Estimation inconsistency

Fig. 3.52 Nodal current mismatch
Fig. 3.53 Phase A, circuit 1 fault current

Fig. 3.54 Phase B, circuit 1 fault current
Fig. 3.55 Phase C, circuit 1 fault current

Fig. 3.56 TSE of line currents with correct fault model
Fig. 3.57 TSE of node voltages with correct fault model
Chapter 4  Conclusions and Further Work

4.1 Conclusions

To promptly address the problems caused by electromagnetic disturbances, remedial actions require the fault type and location to be identified. Monitoring of the complete system is prohibitive due to the current cost of suitable monitoring equipment, and traditional ad hoc fault identification methods through ElectroMagnetic Transient (EMT) type programs are time consuming and inefficient. As an alternative, power quality state estimation technique using limited amount of measurement data has been used recently, but only with reference to harmonics. A review on the development of the harmonic state estimation algorithms as well as various performance tests were presented in Chapter 2, and the state estimation technique was further extended to transient state estimation in Chapter 3.

HSE combines the advantages of complete simulation and measurement techniques to give the best estimate of harmonic flow information throughout a power system using partial measurements. This information is particularly important for the design of harmonic filters. Generally, HSE is carried out with a restricted number of measurements and relies on observability analysis to search for the optimum measurement placement and gives information on the observable and unobservable island. Observability analysis is essential for earlier HSE based on normal equation approach.

This thesis introduced the use of SVD as an alternative to observability analysis. SVD provides an effective method to identify observable and unobservable islands. Therefore, it gives information on whether the selected measurements are adequate to solve the system and whether additional measurements are required. By finding the independent measurement transducer placement for each type of a.c. component, a near optimum measurement placement can be obtained. As a first step, the inter-dependency of current measurements on three-phase transformers was investigated. The preferred transformer measurements for the three most common types of transformer configurations have been discussed. As a rule of thumb, the zero sequence current must be known in order for the transformer circuit to be fully observable.

Under-determined estimation is subject to the accuracy of system modelling and the quality of the measurements. Measurements, in practice, are corrupted with noise
even though sophisticated pre-filtering stages are adopted. In this thesis, testing of HSE error in the presence of measurement noise and bad data via Monte Carlo simulation was performed. Cumulative probability density curves of HSE error were developed to show the expected error for a particular level of noise injected and hence the confidence in the results assessed. Improvement made by adding redundant measurements was also investigated.

The research on HSE was later extended to include the estimation and tracking of harmonic injection which varies with time. In much of the literature, the use of traditional Kalman filter technique has been proposed. However, it requires an optimal covariance $Q$ matrix so that the filter is optimized for both transient and steady state condition. An adaptive Kalman filter has been proposed which replaces the optimal $Q$ model using two simple $Q$ models. A hypothesis test condition was used to determine the condition of the states and applies the appropriate $Q$ model. This approach combines the best features in both the “constant” and “random walk” $Q$ models, thus allowing a significant improvement in the tracking latency. Moreover, an optimal covariance matrix is no longer required and the divergence problem can be avoided by resetting the Kalman gain when a transient condition is detected.

In Chapter 3, the state estimation technique was extended to Transient State Estimation. A new algorithm for TSE was developed which included the continuous energy exchanges among the system component during switching/disturbance events. It allows a system-wide estimation of the system states using limited measurements and therefore reduces the number of locations that needs to be monitored. Two formulation methods were developed to formulate the system equations. The systematic formulation describes the power system using a set of first order differential equations. When the state equations are discretised using Euler’s integration formula, history measurement at the previous time-step as well as the present measurements can be used to form the measurement system. The extra information increases the rank of an under-determined system and hence its solvability. The diakoptical formulation diakoptically segregates the sub-networks and forms the state variable equations without the need to identify the independent state variables. Its strength lies on the ability to use variable step-length integration to improve waveform solution and prevent numerical oscillation. With this possibility, TSE can be used to estimate switching transients in an electrical power system. Both methods result in a state estimator that can be solved linearly for each time-step.

With reference to the New Zealand Lower South Island system, the proposed TSE
algorithm was used to estimate the transient voltage and current at the unmeasured locations. The results were compared with the PSCAD/EMTDC simulation to verify their accuracy. TSE was able to obtain the complete knowledge of the system states and locate mismatches within the system. This information gives valuable information on the fault location and limits the number of such possible locations that require investigation.

In summary, the research has successfully achieved the following:

- Investigated the effect of measurement noise and gross error on the HSE performance and accuracy.
- Investigated the inter-dependency of current measurements on the three common types of transformer configuration and hence identified the optimal transformer current measurement placements.
- Investigated the use of SVD to solve for under-determined (partially observable) systems and identify observable and unobservable islands within the power system.
- Developed a new dynamic HSE algorithm which is capable of estimating and tracking time-varying harmonic injections.
- Extended the state estimation technique to transient state estimation. Two different formulation methods were developed.
- Investigated the use of TSE to give information on the disturbance location.

### 4.2 Further Work

The HSE and TSE algorithms were developed in the MATLAB environment to allow testing of ideas at the early stages. At present, the algorithms based on interpretive scripting language of MATLAB are too slow for real-time applications. An optimized algorithm using compiled code is needed if it is to be implemented for real-time estimation. Despite this fact, fault/harmonic disturbance monitors record pre-fault period of the voltage and current during the disturbance. This data can, in principle, be used by the TSE and HSE to determine the source of the event and effect on the whole system.

Several topics worth studying in future work related to HSE can include:

- Identification of combined effect of background harmonics due to numerous small sources at various sites.
- Implementation of bad data filtering stage for incoming measurements.
- HSE modelling giving consideration to non-linear network components.
- Investigate the use of extended Kalman filtering for dynamic HSE.

As an introduction to TSE, a.c. components are modelled using simple equivalent lumped circuits. Future work should include detailed component representation for further in-depth studies, for example: using the Bergeron model to represent transmission line to account for distributed parameters, transformer model with consideration of saturation characteristics, etc. The inclusion of measurement noise and bad data is also important to the TSE development, its effects on the TSE accuracy should be considered in the future work.
References

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Appendix I

Line current measurements

The measurement for a line connected between busbars $s$ and $r$ has two possible non-zero-entries. If the sending end harmonic current is measured, then the entries in column $s$ and $r$ are

$$ h(x,s) = y_{series} + y_{shunt_s} $$
$$ h(x,r) = -y_{series} $$  \hspace{1cm} (1.1)

If the receiving end harmonic current is measured, then the entries in column $s$ and $r$ are

$$ h(x,s) = y_{series} $$
$$ h(x,r) = -y_{series} - y_{shunt_r} $$  \hspace{1cm} (1.2)

Typically, $y_{shunt_s} = y_{shunt_r}$

This gives the line-node matrix $[Y_{ln}]$ in the following form

$$ [I_t] = \begin{bmatrix} V_s \\ V_r \end{bmatrix} = [Y_{ln}] [V_n] $$  \hspace{1cm} (1.3)
Appendix II

Transformer measurement matrix

Three types of transformer connections are considered. Consider that a transformer is connected between busbars $i$ and $j$ and the impedance is in the form of $z_s = R + jhX$ at harmonic order $h$. Then the system matrices for the three transformer connections are:

1. Star-G/Star-G connection

The system matrix $Y_{np}$ is

$$Y_{np} = \begin{bmatrix} Y_s & -Y_s \\ -Y_s & -Y_s \\ Y_s & Y_s \end{bmatrix}$$  \hspace{1cm} (II.1)

2. Star-G/Star connection

The system matrix $Y_{ns}$ is
3. Star-G/Delta connection

The system matrix $Y_{\text{sy}}$ is

$$
Y_{\text{sy}} = \begin{bmatrix}
\frac{-y_s}{\sqrt{3}} & 0 & \frac{y_s}{\sqrt{3}} \\
\frac{y_s}{\sqrt{3}} & \frac{-y_s}{\sqrt{3}} & 0 \\
0 & \frac{y_s}{\sqrt{3}} & \frac{-y_s}{\sqrt{3}}
\end{bmatrix}
$$

(II.3)
Appendix III

Kalman filtering

The concept of Kalman Filtering is essentially a set of mathematical equations that implements a predictor-corrector type estimator that minimises the estimated error covariance $P_k$. Consider that $\hat{x}_k^- \in \mathbb{R}^n$ to be the a priori state estimate at step $k$ given knowledge of the process prior to step $k$, and $\hat{x}_k \in \mathbb{R}^n$ to be the a posteriori state estimate at step $k$ given measurement $z_k$. The a priori and a posteriori estimate errors can then be defined as

$$
e_k^- = x_k - \hat{x}_k^- \quad e_k = x_k - \hat{x}_k \quad \text{(III.1)}$$

The a priori estimate error covariance is

$$P_k^- = E[e_k^- e_k^{\top}] = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^\top] \quad \text{(III.2)}$$

and the a posteriori estimate error covariance is

$$P_k = E[e_k e_k^\top] = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^\top] \quad \text{(III.3)}$$

At each time-step, Kalman filter updates the a priori estimate $\hat{x}_k^-$ using the new measurement $z_k$ to give an improved estimate denoted as $\hat{x}_k$ using the following equation.

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k\hat{x}_k^-) \quad \text{(III.4)}$$

The design criteria of the Kalman gain matrix $K$ is that it minimises the estimated error covariance $P_k$. This leads to the search for the particular $y$ that yeilds an optimal estimate using the minimum mean-square error as the performance criterion.

$$P_k(y) = E[(x_k - \hat{x}_k(y))(x_k - \hat{x}_k(y))^\top] \quad \text{(III.5)}$$

To find the particular $y$ that minimizes the estimation error covariances of the state variables, the trace of $P_k(y)$ is differentiated and set to equal to zero.
\[
\frac{d\{\text{trace}(P(x))\}}{dy} = 0 \quad (\text{III.6})
\]

After many mathematical manipulations, one form of the Kalman gain resulted is

\[
K_k = P_k^{-1}H_k^T\left(H_kP_k^{-1}H_k^T + R_k\right)^{-1} \quad (\text{III.7})
\]

It is important to note that when the measurement error covariance \(R\) approaches to zero, the Kalman gain \(K\) weights the residual more heavily.

\[
\lim_{R_k \to 0} K_k = H^{-1} \quad (\text{III.8})
\]

Similarly, as the \textit{a priori} estimate error covariance \(P_k^-\) approaches to zero, the Kalman gain \(K\) weights the residual less heavily.

\[
\lim_{P_k^- \to 0} K_k = 0 \quad (\text{III.9})
\]
Appendix IV

Diakoptical formulation

The matrix equations for resistive, inductive and capacitive branches can be written as:

\[ R_r I_r = K_m^T V_n \tag{IV.1} \]

where \( R_r \) is the branch resistance matrix
\( I_r \) is the vector of resistor currents
\( V_n \) is the vector of node voltages

\[ \frac{d}{dt} (L_l I_l) = E_l - R_l I_l + K_m^T V_n \tag{IV.2} \]

where \( L_l \) is the branch inductance matrix
\( E_l \) is the vector of e.m.f sources in inductive branches
\( R_l \) is the inductive branch resistance matrix
\( I_l \) is the vector of inductor currents

\[ I_c = C_c \frac{d}{dt} (K_m^T V_n) \tag{IV.3} \]

where \( C_c \) is the branch capacitance matrix
\( I_c \) is the vector of capacitor currents
\( V_n \) is the vector of node voltages

In the absence of current source, Kirchoff's current law gives:

\[ K_{nc} I_c = -K_m I_r - K_n I_l \tag{IV.4} \]

To obtain expressions for the voltage vectors \( V_{\alpha}, V_{\beta} \) and \( V_{\gamma} \), (IV.3) is premultiplied by \( K_{nc} \) and substitute to (IV.4), this yields:

\[ K_{nc} C_c K_m^T \frac{d(V_n)}{dt} = -K_m I_r - K_n I_l \tag{IV.5} \]

Given that
\[ K_{\beta c} = 0 \]
\[ K_{\gamma c} = 0 \] (IV.6)
\[ K_{\gamma r} = 0 \]
due to the node definitions, (IV.5) can be partitioned to

\[
\begin{bmatrix}
K_{ac} C_{cc} K_{ca}^T \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} (V_a) \\
\frac{d}{dt} (V_\beta) \\
\frac{d}{dt} (V_\gamma)
\end{bmatrix}
= -
\begin{bmatrix}
K_{ar} I_r + K_{ao} I_i \\
K_{br} I_r + K_{bo} I_i \\
K_{\gamma l} I_l
\end{bmatrix} \tag{IV.7}
\]

Defining

\[ C_{ao} = K_{ac} C_{cc} K_{ca}^T \] (IV.8)

then the followings can be written based on (IV.7)

\[
\frac{d}{dt} (C_{ao} V_a) = -(K_{ar} I_r + K_{ao} I_i) \tag{IV.9}
\]

\[ K_{br} I_r = -K_{br} I_i \tag{IV.10} \]

\[ -K_{\gamma l} I_l = 0 \tag{IV.11} \]

\[ K_{\gamma l} \frac{d}{dt} (I_i) = 0 \tag{IV.12} \]

For the voltage vector of \( \beta \) nodes \( V_\beta \), (IV.1) is expanded to

\[ I_r = R_{rr}^{-1} \left( K_{ra}^T V_a + K_{r\beta}^T V_\beta + K_{r\gamma}^T V_\gamma \right) \tag{IV.13} \]

Premultiplying (IV.13) by \( K_{br} \), substituting (IV.6) and (IV.10) leads to the following expression:

\[ -K_{br} I_i = K_{br} R_{rr}^{-1} K_{ra}^T V_a + K_{br} R_{rr}^{-1} K_{r\gamma}^T V_\gamma \tag{IV.14} \]

rearranging gives

\[ V_\beta = -R_{\beta \beta} \left( K_{br} I_i + K_{br} R_{rr}^{-1} K_{ra}^T V_a \right) \tag{IV.15} \]

where \( R_{\beta \beta} = (K_{br} R_{rr}^{-1} K_{ra})^{-1} \)

Similarly for the voltage vector of \( \gamma \) nodes \( V_\gamma \), (IV.2) is premultiplied by \( K_{\gamma l} I_l^{-1} \) and
partitioning $K_m^T V_n$,

$$K_m L_{ii}^{-1} \frac{d}{dt} (L_{ii} I_i) = K_{i\alpha} L_{ii}^{-1} \left( E_i - R_u I_i + K_{i\alpha}^T V_\alpha + K_{i\beta}^T V_\beta + K_{i\gamma}^T V_\gamma \right)$$  \hspace{1cm} (IV.16)

Assuming for time invariant inductance, then using (IV.13) and rearranging gives

$$V_\gamma = -L_{i\gamma} K_{i\gamma} L_{ii}^{-1} \left( E_i + K_{i\alpha}^T V_\alpha + K_{i\beta}^T V_\beta - R_u I_i \right)$$  \hspace{1cm} (IV.17)

where $L_{i\gamma} = (K_m L_{ii}^{-1} K_{i\gamma})^{-1}$

Defining the ancillary variable $V_i$ as

$$V_i = E_i + K_{i\alpha}^T V_\alpha + K_{i\beta}^T V_\beta - R_u I_i$$  \hspace{1cm} (IV.18)

(IV.16) can be rewritten as

$$V_\gamma = -L_{i\gamma} K_{i\gamma} L_{ii}^{-1} V_i$$  \hspace{1cm} (IV.19)

For the state equation formulation, given that the rate of change of the state variables (capacitor voltages $V_\alpha$ and inductor currents $I_i$) can be expressed as

$$\frac{d}{dt} (L_{ii} I_i) = V_i + K_{i\gamma}^T V_\gamma$$  \hspace{1cm} (IV.20)

$$\frac{d}{dt} (C_{\alpha} V_\alpha) = -K_{\alpha r} I_r - K_{\alpha i} I_i$$  \hspace{1cm} (IV.21)

substituting for $V_i$ and $V_\gamma$, (IV.20) becomes

$$\frac{d}{dt} (L_{ii} I_i) = \begin{bmatrix} U_{ii} - K_{i\gamma}^T L_{i\gamma} K_{i\gamma} L_{ii}^{-1} \end{bmatrix} \left[ E_i + K_{i\alpha}^T V_\alpha + K_{i\beta}^T V_\beta - R_u I_i \right]$$  \hspace{1cm} (IV.22)

where $U_{ii}$ is a unit matrix of order $l$

Defining

$$M_{ii} = U_{ii} - K_{i\gamma}^T L_{i\gamma} K_{i\gamma} L_{ii}^{-1}$$  \hspace{1cm} (IV.23)

and

$$R_u = R_u + K_{i\beta}^T R_{\beta\beta} K_{\beta\beta}$$  \hspace{1cm} (IV.24)

substituting for $V_\beta$ and rearranging gives

$$\frac{d}{dt} (L_{ii} I_i) = M_{ii} \left[ E_i + K_{i\alpha}^T - K_{i\beta}^T R_{\beta\beta} K_{\beta\beta} R^{-1}_{\beta\beta} K_{\beta\gamma}^T V_\alpha - R_u I_i \right]$$  \hspace{1cm} (IV.25)
Similarly, substituting for $I_\alpha$, the rate of change of the state variable $V_\alpha$ can be expressed as:

$$\frac{d}{dt}(C_{c\alpha}V_\alpha) = -\left[K_{c\alpha}I_\alpha + K_{c\alpha}R_{\gamma\gamma}^{-1}\left(K_{c\alpha}^T V_\alpha + K_{c\alpha}^T V_\beta\right)\right]$$  \hspace{1cm} (IV.26)

Substituting for $V_\beta$ and rearranging yields

$$\frac{d}{dt}(C_{c\alpha}V_\alpha) = -\left(K_{c\alpha}R_{\gamma\gamma}^{-1}K_{c\alpha}^T - K_{c\alpha}R_{\gamma\gamma}^{-1}K_{c\alpha}^T R_{\gamma\gamma} R_{\gamma\gamma} K_{c\alpha} R_{\gamma\gamma}^{-1}K_{c\alpha}^T\right)V_\alpha + ...$$  \hspace{1cm} (IV.27)

$$-\left(K_{c\alpha} - K_{c\alpha} R_{\gamma\gamma}^{-1} K_{c\alpha}^T R_{\gamma\gamma} K_{c\alpha}\right)I_\alpha$$

Defining

$$A_{c\alpha} = K_{c\alpha} - K_{c\alpha} R_{\gamma\gamma}^{-1} K_{c\alpha}^T R_{\gamma\gamma} K_{c\alpha} \hspace{1cm} (IV.28)$$

$$A_{c\alpha}^T = K_{c\alpha}^T - K_{c\alpha}^T R_{\gamma\gamma} K_{c\alpha} R_{\gamma\gamma}^{-1} K_{c\alpha}^T \hspace{1cm} (IV.29)$$

$$G_{c\alpha} = K_{c\alpha} R_{\gamma\gamma}^{-1} \left(U_{\gamma\gamma} - K_{c\alpha} R_{\gamma\gamma} K_{c\alpha} R_{\gamma\gamma}^{-1}\right) \hspace{1cm} (IV.30)$$

then simplify (IV.26) using the equations defined in (IV.28) and (IV.30), (IV.26) can be rewritten as

$$\frac{d}{dt}(C_{c\alpha}V_\alpha) = -G_{c\alpha}K_{c\alpha}^T V_\alpha - A_{c\alpha}I_\alpha$$  \hspace{1cm} (IV.31)

(IV.25) can also be rewritten using the equation defined in (IV.29),

$$\frac{d}{dt}(L_\beta I_\beta) = M_\beta \left[E_\beta + A_{\beta\alpha}^T V_\alpha - R_{\beta\beta} I_\beta\right] \hspace{1cm} (IV.32)$$

Finally combining equations (IV.31) and (IV.32) in the form of $\dot{x} = Ax + Bu$, assuming that inductances and capacitances are time invariant, yields

$$\begin{bmatrix}
\frac{dl_\beta}{dt} \\
\frac{dV_\alpha}{dt}
\end{bmatrix} = 
\begin{bmatrix}
-L_{\alpha\alpha}^T M_\alpha R_{\alpha\alpha} & L_{\alpha\alpha}^T M_\alpha A_{\alpha\alpha}^T \\
-C_{\alpha\alpha}^{-1} A_{\alpha\alpha} & -C_{\alpha\alpha}^{-1} G_{\alpha\alpha} K_{\alpha\alpha}^T
\end{bmatrix}
\begin{bmatrix}
I_\beta \\
V_\alpha
\end{bmatrix} + 
\begin{bmatrix}
L_{\gamma\gamma}^T M_\gamma \\
0
\end{bmatrix}E_\gamma \hspace{1cm} (IV.33)$$

It is also possible to express the dependent variables in terms of the state variables $V_\alpha$ and $I_\alpha$,

$$V_\beta = -R_{\beta\beta}(K_{\beta\beta}I_\beta + K_{\beta\beta} R_{\gamma\gamma} K_{\gamma\gamma}^T V_\alpha)$$  \hspace{1cm} (IV.34)$$
For $V_\gamma$, substituting for $V_\beta$ in (IV.17) yields,

$$V_\gamma = -L_{\gamma\gamma}K_{\gamma\gamma}L_{\gamma\gamma}^{-1} \left( -R_\beta - K_{\gamma\beta}^T R_{\gamma\beta} K_{\gamma\beta} \right) I_i + ...$$

$$- L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} \left( K_{T\gamma}^T - K_{\gamma\beta}^T R_{\gamma\beta} K_{\gamma\beta} R_{\gamma\beta}^{-1} K_{\gamma\beta}^T \right) V_\alpha - L_{\gamma\gamma} K_{\gamma\gamma} L_{\gamma\gamma}^{-1} E_i$$  (IV.35)

For $I_r$, substituting for $V_\beta$ in (IV.13) yields,

$$I_r = -R_{rr} K_{rr}^T R_{\beta\beta} K_{\beta\beta} I_i + ...$$

$$R_{rr}^{-1} \left( K_{rr}^T - K_{rr}^T R_{\beta\beta} K_{\beta\beta} R_{\beta\beta}^{-1} K_{rr}^T \right) V_\alpha$$  (IV.36)

For $I_e$, substituting for $V_\beta$ and $I_r$ in (IV.4) yields,

$$I_e = -K_{cc}^{-1} \left( A_{\alpha i} I_i + G_{\alpha r} K_{rr}^T V_\alpha \right)$$  (IV.37)
Appendix V

Publications
An Adaptive Kalman Filter for Dynamic Harmonic State Estimation and Harmonic Injection Tracking

Kent K. C. Yu, N. R. Watson, and J. Arrillaga, Fellow, IEEE

Abstract—Knowledge of the process noise covariance matrix \( Q \) is essential for the application of Kalman filtering. However, it is usually a difficult task to obtain an explicit expression of \( Q \) for large time varying systems. This paper looks at an adaptive Kalman filter method for dynamic harmonic state estimation and harmonic injection tracking. The method models the system as a linear frequency independent state model and does not require an exact knowledge of the noise covariance matrix \( Q \). As an alternative, the proposed adaptive Kalman filter switches between the two basic \( Q \) models for steady-state and transient estimation. Its adaptive algorithm allows for the rescaling of the Kalman gains to avoid Kalman filter divergence problems under steady-state and allow fast tracking of system variations in transient conditions. Simulations results on the 220 kV network of the lower South Island of New Zealand are presented to validate this approach.

Index Terms—Harmonic analysis, Kalman filtering, state estimation.

I. INTRODUCTION

HARMONIC injection has been a growing power quality concern over the years. With the increase in the use of power electronic devices, the maintenance of the quality of power has become a major problem for the electric utility companies. The voltage distortion from interconnection of nonlinear loads through transmission and distribution lines degrades the quality of power to all users connected to the network. Harmonic state estimation technique has been presented in the past [1]–[5] for locating and identifying harmonics sources within the network. Its ability in evaluating the severity of harmonic distortions with limited measurements has been very effective. However, with the dynamic nature of harmonic injection, the harmonic level in the power system is rarely constant, thus it is essential to extend the harmonic state estimation technique to track system parameter variations for accurate dynamic assessments. In the earlier papers [6]–[10], the Kalman filter methodology has been introduced for dynamic harmonic state estimation. The Kalman filter itself is a time domain stochastic optimal estimator that provides an efficient recursive solution in the least square sense. Its process utilizes the previous a posteriori estimate to predict the new a priori estimate. Although the filter is not susceptible to measurement noise, but it can suffer from divergence problems caused by the “dropping off” effect of the filter [6]. Due to the nature of the filter, the Kalman gain is independent to the measurements, thus as the filter approaches steady-state, it becomes less sensitive to parameter variations and begin to lose its ability in tracking dynamic properties. For optimum filtering results, the exact knowledge of the process noise covariance matrix \( Q \) and noise measurement covariance matrix \( R \) are required. However, they are usually unknown in practice, particularly for large time varying systems. Saab [7] discussed the Kalman filter gain can be insensitive to scaling of the noise covariance matrix. The state estimates remain optimal even under incorrect noise covariance in special cases. Beides and Heydt [8] used the Kalman filter methodology in estimating harmonic busbar voltages using real and reactive power measurements, which could be misleading as reactive power is generally not well defined. Girisig and Ma [9] also used Kalman filter for tracking of harmonic sources. The model included system admittance matrix information that allows harmonic state estimation across the power network. Their proposed technique is frequency dependent and requires the fundamental frequency to be known. This is rarely the case in practice as the fundamental frequency is time varying in dynamic systems. Similarly, Girisig, Chang and Makram [10] also proposed a nonlinear fundamental frequency dependent state model using a Kalman recursive measurement scheme for harmonic tracking.

In this paper, the proposed state model is linear and frequency independent with harmonic busbar voltages as the state variables. An adaptive Kalman filter utilizing statistical rules to switch between the two basic \( Q \) models is presented for dynamic harmonic state estimation. The method replaces the optimal \( Q \) model with the use of two basic \( Q \) matrices, which essentially eliminates the need for searching for an optimal \( Q \) matrix. Moreover, harmonic injection tracking through dynamic HSE is also demonstrated.

The paper outlines the mathematical state model in Section II. The tracking response of the two proposed Kalman \( Q \) models is presented in Section III. Section IV describes the proposed adaptive Kalman filter implementation. Finally the simulation results are provided in Section V.

II. MATHEMATICAL STATE MODEL

Harmonic state estimation technique uses a few synchronize measurements to obtain the harmonic information of the whole network. The general state equation which relates the measurements \( z \) to the state variable \( x \) can be expressed as in (1).

\[
g = h(x) + \xi
\]

where \( \xi \) is the measurement error vector.
The three-phase transmission line model is shown in Fig. 1. If the harmonic busbar voltages, $V_b$, are chosen as the state variables, $y_j$, and harmonic busbar voltages $V_b$, harmonic current injection $I_{b,n}$ and harmonic line current $I_{b,m}$ are the measured quantities, then (1) becomes linear.

Building of the measurement matrix $[H]$ is based on the type of measurement. For a $n$-busbar three-phase power system, where transmission lines are modeled by the equivalent three phase PI models as shown in Fig. 1. The three phase transmission line admittance matrix between bus $i$ and $k$ are:

$$
\begin{bmatrix}
  y_{n,i} & y_{n,i+1} & y_{n,i+2} \\
  y_{n+1,i} & y_{n+1,i+1} & y_{n+1,i+2} \\
  y_{n+2,i} & y_{n+2,i+1} & y_{n+2,i+2} \\
  \end{bmatrix}
$$

$$
\begin{bmatrix}
  y_{s,i} & y_{s,i+1} & y_{s,i+2} \\
  y_{s+1,i} & y_{s+1,i+1} & y_{s+1,i+2} \\
  y_{s+2,i} & y_{s+2,i+1} & y_{s+2,i+2} \\
  \end{bmatrix}
$$

$$
\begin{bmatrix}
  y_{shunt,i} & y_{shunt,i+1} & y_{shunt,i+2} \\
  y_{shunt+1,i} & y_{shunt+1,i+1} & y_{shunt+1,i+2} \\
  y_{shunt+2,i} & y_{shunt+2,i+1} & y_{shunt+2,i+2} \\
  \end{bmatrix}
$$

$$
\begin{bmatrix}
  y_{shunt,k} & y_{shunt,k+1} & y_{shunt,k+2} \\
  \end{bmatrix}
$$

(2)

Each measurement adds one extra row to the measurement matrix. For example current injection measurement at bus $j$ ($I_{b,n}$) adds the corresponding row of the nodal admittance matrix $[y_{nodal}]$ to the measurement matrix $[H]$. Sending and receiving end line current measurements, $I_{b,n,a}$ and $I_{b,n,r}$, they are formulated using Kirchhoff's current law and transmission admittance matrices (2). Thus, (1) can be expressed as shown at the bottom of the page in (3) where $[y_{series} + y_{shunt,i}]$ is a $3 \times 3$ admittance matrix comprising of the lines series and shunt admittances [11].

III. KALMAN RECURSIVE PROCESS

The Kalman filter is a least squares estimate in which state estimation is added to allow its application to a dynamic system, where the parameters are varying. Its principle feature is the recursive processing of the noise measurement risk. In the Kalman filter algorithm, the discretised system state and measurement equation are:

$$
x_{k+1} = \phi_k x_k + v_k \\
x_k = \Phi_k x_k + w_k
$$

where the system covariance matrices for $w_k$ and $v_k$ are:

$$
E [w_k w_k^T] = [R_k] \\
E [v_k v_k^T] = [Q_k]
$$

The recursive Kalman filter computation is divided into two parts: "Time update" and "Measurement update." The recursive steps are:

Time update

1) Project the state ahead

$$
\hat{x}_{k+1} = \Phi_k \hat{x}_k
$$

2) Project the error covariance ahead

$$
P_{k+1} = \Phi_k P_k \Phi_k^T + Q_k
$$

Measurement update

1) Kalman gain

$$
K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1}
$$

2) Update estimate with measurement $z_k$

$$
\hat{x}_k = \hat{x}_k^+ + K_k (z_k - H_k \hat{x}_k)
$$

3) Update the error covariance

$$
P_k = (I - K_k H_k) P_k^+
$$

The proposed recursive Kalman process switches between 2 Q models depending whether the system is in steady-state or transient. The two Q models are:

A. Kalman Constant Model

The noise covariance matrix $Q$ is set to zero in this case, i.e., the process model assumes the system is in steady-state condition. Fig. 2 depicts a simple case of tracking busbar harmonic voltage in a power system using the Kalman recursive equations (6)-(10). The filter shows a good tracking capability for the initial period, but as the filter approaches to its steady-state as shown in Fig. 3, the filter is relatively insensitive to any new parameter variations and began to diverge when there is a step change.
IV. ADAPTIVE KALMAN FILTER

The adaptive Kalman filter applies two different Q models for steady-state and transient estimation as discussed in Section III. For the proposed linear frequency independent state model, it is expected that the measurement difference between each time step $\Delta z_{k,k-1}$ is relatively small in steady-state. Thus, the hypothesis test is focused on the measurements difference $\Delta z_{k,k-1}$ using student's $t$ statistical model with the level of significance $\alpha = 5\%$. The null and alternative hypotheses are

\[ H_0 : \mu = 0 \]
\[ H_A : \mu \neq 0 \]  

where $\Delta z_{k,k-1}$ is the mean of the measurement difference

\[ t = \frac{\Delta z_{k,k-1} - \mu}{\frac{S}{\sqrt{n}}} \]  

\[ \text{(12)} \]

The test statistic $t$ is calculated using (12).

Table I is a decision summary for the hypothesis test. The Kalman Q model is determined by the decision of the null hypothesis based on the condition of the system. In general, if the null hypothesis is rejected, the system is assumed to be in transient condition and the identity matrix is used for matrix Q. Likewise, if the null hypothesis is accepted, the system is assumed to be in steady state and the zero matrix is used instead.

The computational sequence for the adaptive Kalman filter is shown in Fig. 6. With the adaptive technique, the Kalman estimator is benefited from both Q models. The adaptive technique determines whether the system is under dynamical changes through measurement hypothesis testing and applies the appropriate Kalman filter model for harmonic state estimation. This allows resetting of the Kalman gain to prevent the divergence problem as described in Section II. Fig. 7 is the performance of
TABLE I

<table>
<thead>
<tr>
<th>t statistic</th>
<th>Decision</th>
<th>System condition</th>
<th>Kalman Q model</th>
</tr>
</thead>
<tbody>
<tr>
<td>t ≤ -t_{0.05}</td>
<td>reject H₀</td>
<td>transient</td>
<td>Qₐ = 1</td>
</tr>
<tr>
<td>t &gt; t_{0.05}</td>
<td>reject H₀</td>
<td>transient</td>
<td>Q₂ = 1</td>
</tr>
<tr>
<td>-t_{0.05} &lt; t &lt; t_{0.05}</td>
<td>accept H₀</td>
<td>steady-state</td>
<td>Qₙ = 0</td>
</tr>
</tbody>
</table>

Fig. 6. Adaptive Kalman filter flow chart.

where Δzₙ₋₁ is the change in measurement between zₙ and zₙ₋₁

the proposed adaptive Kalman filter on the same example as in Section II. The estimation shows a significant improvement in tracking and detecting the step change.

V. TEST SYSTEM

The proposed adaptive Kalman filter is employed into harmonic state estimation for dynamic harmonic content analysis. The algorithm monitors and tracks busbar harmonic injection via the estimation of the busbar harmonic voltages as state variables. The test network (Fig. 8) is from the lower South Island of New Zealand. The modeling on the system components and estimation details are discussed in the papers [2], [3].

A typical harmonic current injection profile of a 6-pulse converter at Invercargill 33 kV (3 phase) is shown in Figs. 9–11.

In the test system, there are only one synchronized measurements points (indicated by the black squares in Fig. 8); they are placed at Manapouri 220 kV (M220), Roxburgh 11 kV (R11B) and Invercargill 33 kV (I33). For the purpose of demonstrating the adaptive Kalman filter technique, these measurements were selected to provide a fully observable system. In the absence of a complete set of field data, the partial "measured" values at the measurement points were obtained from the corresponding results of harmonic penetration simulation. However, no information on the harmonic source is supplied to the HSE part of the
algorithm. Random noises with Gaussian distribution are also added to all measurements.

The main results of the dynamic HSE at time interval 0600, 1200 and 1800 are shown in Figs. 12–14. There are no significant differences between the estimated and simulated results. This indicates that the dynamic HSE provides reliable results for the whole system. Figs. 15–19 shows the estimated and simulated node harmonic voltages in detail for selected busbars. It can be seen that the adaptive Kalman gain ‘resets’ when there is a significant change in the measurements (Fig. 16), while in traditional Kalman filters, the gain is independent to the measurements.

Snapshots of the nodal harmonic current injection at time 0600, 1200, and 1800 are shown in Figs. 20–22. The results indicate the presence of harmonic source at busbar #22, 23, and 24 (i.e., Invercargill 33 kV) by the positive current injection. Furthermore, the passive load at busbar # 4, 5, and 6 (i.e., Roxburgh 11 kV) is also identified. A more detailed harmonic injection tracking via the dynamic HSE is shown in Figs. 23–25. The results verify the effectiveness of the proposed adaptive
Kalman filter method in monitoring and tracking harmonic content within a power network.
random walk model depending on the state of the system. Unlike the traditional Kalman filter, the adaptive Kalman filter is measurement dependent, in which an adaptive Kalman gain is allowed for dynamic HSE. Furthermore, the exact knowledge of the process noise covariance Q is not required for this method. The results show good stability and ability in tracking harmonic content in a system by selecting the Q model as either the identity matrix or zero matrix.

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Kent K. C. Yiu received the B.E. degree (Hons.) in 2001 from the University of Canterbury, Christchurch, New Zealand, where he is currently pursuing the Ph.D. degree.

His area of research is state estimation techniques.

N. R. Watson received the B.E. (Hons.) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury, Christchurch, New Zealand, where he is now an Associate Professor.

His interests include power quality and the steady-state and dynamic analysis of ac/dc power systems.

J. Arrillaga (F’91) received the B.E. degree from Bilbao University, Spain, and the M.Sc., Ph.D., and D.Sc. in Manchester, where he led the power systems group of UMIST between 1970–74. He has been a Professor at the University of Canterbury, Christchurch, New Zealand, since 1975.

Dr. Arrillaga is a Fellow of the IEEE and of the Royal Society of New Zealand.
Error Analysis in Static Harmonic State Estimation: A Statistical Approach

Kent K. C. Yu, Neville R. Watson, Senior Member, IEEE, and Jos Arrillaga, Fellow, IEEE

Abstract—The effectiveness of harmonic state estimation (HSE) in identifying the location and magnitude of harmonic sources is largely dependent on the accuracy of the measurements. Measurement errors (or bad data) can be classified into two groups; measurement noise and gross error. This paper uses a statistical approach (cumulative probability density functions) obtained from five thousand Monte Carlo runs to investigate the impact of measurement noise and gross errors in harmonic state estimation. The Lower South Island of the New Zealand system is used as the test system and the results are probability curves containing the statistics of the estimation error. The effect of additional measurements on an over-determined system to filter noise is also discussed.

Index Terms—Error analysis, harmonic analysis, state estimation.

I. INTRODUCTION

T HE static harmonic state estimation has been extensively studied [1]–[6] and many efficient HSE algorithms have thus been developed. These allow measurement locations to be far away from the suspicious load without compromising the effectiveness of harmonic state estimation. One issue that needs addressing is the sensitivity of the algorithm to bad data, incorrect system parameters or incorrect network topology. These effects are inevitable and therefore the main concerns are the extent to which they affect the estimation accuracy and what can be done to reduce the estimation errors.

Extensive research has been performed in the past in the detection, identification and removal of bad data [7]–[10] to improve the accuracy of the fundamental frequency state estimation. However, this usually requires the system to be over-determined and is therefore of limited applicability to harmonic state estimation. Without redundant measurements, the estimation is very subject to the presence of bad data.

Gross errors, problems usually caused by equipment failure or data communication problems such as loss of communication, are usually large in magnitude, on the other hand measurement noise errors are relatively small but likely to occur in all measurements.

This paper looks at the performance of static harmonic state estimation in the presence of measurement noise and gross errors through Monte Carlo simulation. A statistical approach utilizing a cumulative probability density function resulting from Monte Carlo simulation is used to assess the effect of bad data on harmonic state estimation. The simulation results are presented in Section III. The ability of redundant measurements to filter measurement noise is also discussed.

II. STATISTICAL APPROACH

For a given measurement set and system topology, the relationship between the measurement and state vector is

\[ \begin{equation} \mathbf{z} = [H] \mathbf{x} + \mathbf{e} \end{equation} \]

where

- \( \mathbf{z} \) is the measurement vector \((m \times 1)\)
- \([H]\) is the measurement matrix corresponds to the measurement vector \( \mathbf{z} \)
- \( \mathbf{x} \) is the state vector \((n \times 1)\)
- \( \mathbf{e} \) is the Gaussian measurement error vector with a standard deviation \( \sigma \)

This equation has been linearised by choosing the busbar harmonic voltages \( \mathbf{V}_b \) as state variables and measuring busbar harmonic voltages \( \mathbf{V}_h \), harmonic branch currents \( \mathbf{I}_b \), and harmonic injection currents \( \mathbf{I}_i \) [11]. The measurement equations are solved using Singular Value Decomposition (SVD). In the case where the system is under-determined, SVD produces a minimum least square solution.

A statistical approach is used to investigate the effect of measurement noise on static HSE. Five thousand Monte Carlo runs have been simulated with Gaussian noise on all the measurements. The measurement noises are assumed to be uncorrelated, unbiased and normally distributed with a 5% standard deviation. Although, in practice, the level of noise is usually unknown, it can be determined from experience and historical data. For comparison, a detailed three-phase harmonic simulation of the system is performed to obtain the actual values. These actual values are then compared with the estimation from the HSE algorithm.

Prior to each Monte Carlo run, it is assumed that a new set of measurement \( \mathbf{z} \) with random noise is available and the actual data are known from the harmonic simulation. The sequence steps are as follows (Fig. 1):

1) Use a random number generator to select noise to be added to the actual data at the measurement points to simulate measurement noise. The random number generator can be weighted to give whatever distribution is most appropriate (e.g., noise that is uniform or normally distributed),

2) to Compute \( \hat{\mathbf{x}} \) using SVD and pseudo-inverse

\[ \mathbf{z} = [H] \mathbf{x} + \mathbf{e} \]

\[ [H] = [U][W][V]^T \]

\[ \hat{\mathbf{x}} = [V][W]^{-1}[U]\mathbf{z} \]
3) Compute the estimation error $\Delta z(n)$, and tally chart $\Delta z(n)$ into levels of estimation error such as: estimation error 0.1-0.2%, 0.3-0.4%, etc.

$$\Delta z(n) = z(n) - \hat{z}(n)$$

4) Repeat steps 1-2 for enough Monte Carlo runs. For each run, the measurements incorporate random measurement noise.

5) Calculate probability $P_{sa}$ for each range of estimation error.

$$P_{sa}(n) = \frac{\# \text{ of occurrence} \Delta z(n)}{\# \text{ of simulation runs}}$$

The cumulative probability density curves are calculated. These show the probability of an estimate error being below a certain value. This information is particularly useful in determining the range of state estimation error (busbar voltages in this case) under the influence of measurement noise and hence the confidence in the estimation.

III. SIMULATION RESULTS

The test system used is the Lower South Island of New Zealand (Fig. 2). The harmonic sources consist of a 6 pulse and a 12 pulse rectifier at Invercargill 33 kV and Tiwai 220 kV respectively. A linear load is also placed at Roxburgh 11 kV. No information is given about their locations at the start of the HSE solution. The measurement placements are indicated by the black diamonds in Fig. 2. They are symmetric branch current measurements taken at Manapouri 220 kV, Roxburgh 11 kV and Invercargill 33 kV. Note that the system is fully observable.

A. Measurement Noise

For a fully observable system where $m = n$, any measurement noise or error will affect the state estimates directly. The solution fits the measurement model in the weighted least squares sense, i.e., the residuals between the actual harmonic measurements and estimated harmonic levels are minimized.
The results from the Monte Carlo computation are shown in Figs. 3–7. The cumulative probability density curves depict the characteristics of estimation errors for the harmonic orders h = 6n ± 1. They provide information on the susceptibility of an estimate to noise, and hence the confidence that can be placed on the estimate. The y axis gives the probability of the harmonic estimate error that is below the corresponding x level. For example, with reference to the 5th harmonic voltage level at busbar 4 (Fig. 3), 80% of the time (probability of 0.8), the estimation error is less than 7.5%.

A summary of the expected mean harmonic voltage estimate error and harmonic current injection is shown in Tables I and II.
II. This helps to identify areas that are sensitive to measurement noise. The inf (infinite) entry in Table II indicates that there is no harmonic injection at this point in the system.

Despite the measurement noise, the estimator achieves a good estimate of the system. The voltage and current injection magnitudes shown in Figs. 8 and 9 clearly demonstrate that the effect of measurement noise in HSE is minimal. The differences between the actual and estimated values are small and the harmonic sources can still be correctly identified by their significant characteristic harmonics i.e., \( h = 6n \pm 1 \) for the 6-pulse and \( 12n \pm 1 \) for 12-pulse rectifier. Two harmonic injectors consisting of a 6-pulse and 12-pulse rectifier at loyercargill (Busbar 22–24) and Twai (Busbar 1–3) respectively are identified. Moreover, the suspicious source at Roxburgh (Busbar 4–6) has also been identified as a harmonic absorber (passive load).

B. Gross Error

The other type of bad data, gross error, is usually caused by equipment failure and data communication error. The impact of one or more gross errors in HSE is shown in Figs. 10–13. The harmonic current measurement taken at branch 77 (phase B) and 34 (phase A) are replaced with a gross error (300% of its original value).
The figures show that gross error causes estimation discrepancy on bushar voltages, branch currents and injection currents that are significant and will give false identification of harmonic sources. As expected the errors in bushar current injection estimated (Fig. 13 and Table IV) are most significant for the busbars closer to the measurement with gross errors. Furthermore, additional points of harmonic injection (Busbar 1-3) were erroneously identified. This is because the gross errors caused large errors in the corresponding state estimates when solving (1). Tables III and IV show the harmonic estimation error when the measurement system is corrupted with two gross errors in branches 77 and 34.

The estimation algorithm can be improved and become more robust to bad data by having bad data detection and utilizing any possible redundant measurements within the system. Bad data detection schemes, such as residual testing, indicate the presence of error and identify which measurements are bad. In the presence of bad data, especially gross errors, the residual values are expected to be large. A statistical hypothesis test can be applied on the residual values to identify the bad data. However this requires the system to be over determined (i.e., m > n). Additional measurements in an over-determined system can also
act like a noise filter to clean up any erroneous data in the measurement set. These measurements are redundant, since they provide information that is already known.

Figs. 14–17 show the improvement in state estimation accuracy under the influence of measurement noise by increasing the number of redundant measurements. The figures display the estimation error for harmonic orders $h = 6n \pm 1$ up to the 25th harmonic. A significant improvement is shown by the use of nine redundant measurements in the measurement system (gray colored area) and further improvement is achieved with the addition of nine redundant measurements i.e., a total of eighteen redundant measurements (white colored area).

IV. CONCLUSION

The effectiveness of HSE is largely dependent on the quality of the measurements, i.e., whether the measurements are corrupted with noise or gross errors. A statistical approach has been introduced to assess the effect of the measurement noise on HSE. The effect of random measurement noise has shown to be less significant in comparison to gross errors. The latter cause system wide discrepancies on the state estimate. However, the simulation results show that the injection locations remain correctly identified but are erroneous in their magnitude. Significant improvement can be achieved with redundant measurements (i.e., 9 extra for the test system). This however requires the system to be over-determined. When the system is under determined, gross errors have a large impact on the estimates which can not be easily mitigated. Hence, a judicious trade off between accuracy and measurement costs has to be made.

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Kent K. C. Yu received the B.E. (Hons.) degree from the University of Canterbury, Christchurch, New Zealand, in 2001. He is currently pursuing the Ph.D. degree.

His area of research is state estimation techniques.

Neville R. Watson (SM’95) received the B.E. (Hons.) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury, Christchurch, New Zealand, where he is now a Senior Lecturer.

His interests include power quality, steady-state and dynamic analysis of power systems.

Jos Arrillaga (F’91) received the B.E. degree from Bilbao University, Spain, and the M.Sc., Ph.D., and D.Sc. degrees from the University of Manchester Institute of Science and Technology (UMIST), Manchester, U.K., where he led the power systems group of UMIST between 1970-74.

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Dr. Arrillaga is a Fellow of the IEEE and of the Royal Society of New Zealand.
A Comparison of Transient Simulation with EMTDC and State Space Diakoptical Segregation Methodology

K. K.C. Yu, Student Member, IEEE and N. R. Watson, Senior Member, IEEE

Abstract—The use of diakoptical technique can considerably reduce the computational burden when formulating network equations. Nodal analysis with diakoptical segregation of the plant component can efficiently modifies the necessary equations when the system undergoes topological changes. It avoids changes of the whole network equations and only the relevant parts of the network equations are considered. This particular benefits simulations that undergo frequent switching. In this paper, a state-space diakoptical segregation method is applied to transient simulation of a.c. systems under different fault conditions.

For comparison, the transient response of the proposed technique is compared with the EMTDC simulation. Two test scenarios based on the Lower South Island of the New Zealand system were set up. The scenarios looked at the performance of the proposed method under symmetrical and asymmetrical line-to-ground faults.

Keywords: Electromagnetic transients, Transient simulation, State variable equations

I. INTRODUCTION

Digital simulations in the time domain via electromagnetic transient program (EMTP, EMTDC or ATP) has become the industrial standard. They play a critical part in design and operation of modern power systems where analytical solution is prohibitive. These simulations provide the basis for; system control testing, equipment protection design, performance testing under disturbance/fault conditions to name but a few. Transient simulation based on state space theory [1]-[3] were once popular, however the high computation cost has caused interest to diminish even though it has many advantages [4]-[5]. One such methodology specifically developed to analyze the dynamic behavior of HVDC systems is the Transient Converter Simulation (TCS) program [6]. A diakoptically based nodal approach provides an ideal environment for the analysis of system with frequently switching components such as converters. It avoids involving the whole network in unnecessary topological changes and

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The authors are with the Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand.

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only modifies the relevant network equations to represent the new system state. Furthermore, the nodal approach becomes more efficient if sparsity of coefficient matrices is used to improve computational efficiency.

The general state variable approach requires the identification of independent state variables and formulation of the appropriate equations [7]. Arbitrary use of inductor current and capacitor voltage is not sufficient due to the possible presence of inductor cut-sets or loops of capacitors and voltage sources. Use of graph theory or linear matrix methods can find the appropriate state variables. However, when the system is undergoing topological changes due to frequent switchings, this identifying of state variables and formation of state equations is not practical due to the computational effort to identify independent state variables and form the state equations. The diakoptically segregation of the sub-network presented avoids the need to identify independent state variables. In the present approach a diakoptical technique is used as an efficient method of forming the state equations for systems which may have frequently switching components in them. There is a slight loss of generality, for example every capacitive sub-network must have a connection to ground, but this is minor to the enormous computational benefits.

In the TCS algorithm, the power system is represented by an equivalent network of lumped inductive, resistive and capacitive. The set of network equations are formulated from diakoptical segregation of the sub-network where the nodes are partitioned into three possible groups depending on what type of branches are connected to them. This results in a set of first order differential equations being developed and the system matrices in compact form containing their indexing information, assigns node numbers for branch elements and interconnection information. The state variable solution can then be obtained using numerical integration technique, such as implicit trapezoidal integration. The state equations are solved at each time-step iteratively, with no restriction imposed on the selection of the integration time step length. Thus, when changes in the system topology occur, the network equations, the connection matrix and the step length can be modified to fall exactly on the time of the changes to represent the new conditions.

In this paper, a new implementation of the TCS algorithm is applied to an a.c. system for prediction of transient response under symmetric and asymmetric fault conditions. Minimal modification of the network equations and connection matrix to
II. TCS DKIAOPTICS FORMULATION

Consider a network with \( n \) nodes systematically expanded into its elementary resistive \( r \), inductive \( l \), capacitive \( c \) and current source \( s \) branches. Diakoptical segregation of the network, subdivides the network nodes into three parts according to the type of branches connected to them:

- \( \alpha \) nodes: At least one capacitive branch.
- \( \beta \) nodes: At least one inductive branch but no capacitive branches.
- \( \gamma \) nodes: Only inductive branches.

The resulting branch-node incidence (connection) matrices for the \( r \), \( l \) and \( c \) branches are \( K_{rn}^T \), \( K_{ln}^T \) and \( K_{cn}^T \) respectively. The elements in the branch-node incidence matrices are determined as follows:

- \( K_{rn}^T = 1 \); if node \( n \) is the sending end of branch \( b \)
- \( K_{ln}^T = 1 \); if node \( n \) is the receiving end of branch \( b \)
- \( K_{cn}^T = 0 \); if branch \( b \) is not connected to node \( n \)

By restricting the number of possible network configuration to those commonly encountered in practical systems, the efficiency of the solution can be improved significantly. The restrictions are [8]:

1. every capacitive branch sub-network has at least one connection to the system reference (ground node)
2. resistive branch sub-networks have at least one connection to either the system reference or an \( \alpha \) node.
3. inductive branch sub-networks have at least one connection to the system reference or an \( \alpha \) or \( \beta \) node.

Neglecting current sources, the fundamental branches that result from the restrictions and their diakoptical network equations can be written as:

Resistive branches

\[
I_r = R_n^{-1} \left( K_{nr}^T V_a + K_{nr}^T V_b \right) \tag{1}
\]

Inductive branches

\[
E_l - \frac{d}{dt} \left( I_n I_l \right) - R_l I_l + K_{ln}^T V_a + K_{ln}^T V_b + K_{cn}^T V_l = 0 \tag{2}
\]

Capacitive branches

\[
C_n \}

Applying the node type definitions, the nodal equations for each node type become:

\[
\begin{align*}
K_{rr} + K_{nr} + K_{rc} &= 0 \\
K_{rr} + K_{rl} &= 0 \\
K_{rr} &= 0
\end{align*} \tag{4}
\]

Combining eqns. 1-4, a set of diakoptical network equations in the form of eqns. 5 and 6 can be formulated by taking capacitive node voltage \( V_a \) and inductive branch current \( I_l \) as the state variables. The state equations and dependent variable equations are expressed as in eqns. 7-11.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} I_r \\
V_a \\
I_l 
\end{bmatrix} &= \begin{bmatrix} -L_n M_{nl} & L_n M_{nl} & K_{nr} \\
-C_n G_{nr} & -C_n G_{nr} & K_{nc} \\
I_r & 0 & C_n \end{bmatrix} \begin{bmatrix} I_r \\
V_a \\
I_l 
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
I_r 
\end{bmatrix} \tag{7}
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dt} V_a &= -L_n M_{nl} I_r + L_n M_{nl} I_l + \ldots \tag{8}
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dt} I_l &= -R_n K_{nl} I_l + \ldots \tag{9}
\end{align*}
\]

\[
\begin{align*}
I_r &= -R_n K_{nl} R_{nl} K_{nl} I_l + \ldots \tag{10}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} V_a \\
I_r \\
I_l 
\end{bmatrix} &= \begin{bmatrix} A_n \\
C_n A_n \\
C_n \end{bmatrix} \begin{bmatrix} I_r \\
I_l \\
V_a 
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
G_n \end{bmatrix} \begin{bmatrix} V_a 
\end{bmatrix} \tag{11}
\end{align*}
\]

where

\[
\begin{align*}
M_{nl} &= U_n - K_{nr}^T R_{nr} K_{nr} \\
A_{nl} &= K_{nl} - K_{nl} K_{nl}^T R_{nl} K_{nl} \\
G_{nl} &= K_{nl} R_{nl} \left( U_n - K_{nr}^T R_{nr} K_{nr} \right) \\
C_n &= \left( K_{nl} C_{nl} K_{nl}^T \right)^{-1} \\
L_n &= \left( K_{nl} L_{nl} K_{nl}^T \right)^{-1} \\
R_{nl} &= \left( K_{nl} R_{nl} K_{nl}^T \right)^{-1} \\
R_n &= R_n + K_{nr}^T R_{nr} K_{nr}
\end{align*}
\]

Dependent variables

\[
\begin{align*}
V_a &= -R_n I_r + K_{nr} R_{nr} K_{nr} V_a \\
I_l &= -R_n K_{nl} R_{nl} K_{nl} I_l + \ldots
\end{align*}
\]
$R, L$ and $C$ are the resistance, inductance and capacitance matrices respectively. $U_n$ and $U_f$ are unit matrices of order $r$ and $l$ respectively.

To solve for the state variables at each time step, implicit trapezoidal approximation is used owing to its good stability, accuracy and simplicity. The state equations can be written as in (12) and the change in the state variable $\Delta x$ is defined in (13).

$$\dot{x} = f(I, V_n)$$  \hspace{1cm} (12)

$$\Delta x = \frac{h}{2} (\dot{x} + \ddot{x})$$  \hspace{1cm} (13)

An iterative procedure can be used to determine $X_{rh}$ as follows:

1. For an initial estimates, it is assumed $\dot{x}_{rh} = \dot{x}$
2. An intermediate $x_{rh}$ based on the $\dot{x}_{rh}$ estimate is then calculated.
3. An update of the $x_{rh}$ can be calculated from the intermediate $x_{rh}$ value from the state equation.

Steps 2) and 3) are performed iteratively until convergence is reached. In the case that convergence fails, the step length is halved and the iterative procedure is restarted. Dependent state variables can be calculated at the end of each time step $h$ or at the end of the simulation.

III. SIMULATION MODEL AND RESULTS

Each component in an a.c. power system can be modelled by its equivalent circuit model defined in terms of passive elements. These elementary branches form the basis of the state equations and dependent variable equations (3)-(7).

1. Generators: these are modelled by an e.m.f. source voltage with its equivalent R-R/L/L impedance.
2. Transformers: they are represented using two windings model depending on the type of magnetic circuit and on the connections of the terminals and the neutrals. i.e. delta or wye. Core losses are represented internally with an equivalent sheet resistance across each winding in the transformer.
3. Transmission lines: these are modelled rigorously by the three phase PI models and hence capable of incorporating non symmetric condition.
4. Loads: the real and reactive power components are represented by its equivalent resistance and inductance respectively.

The test system (Fig. 1) is taken from the Lower South Island of the New Zealand system. It consists of three generators, five delta/star-g transformers, three double circuit transmission lines, two single circuit transmission lines and three passive loads. The system contains 84 state variables corresponding to the system's inductive branch currents and capacitive node voltages. For a credible validation of the proposed technique, the TCS based transient simulation is compared to those simulated using PSCAD/EMTDC. Two types of fault (three phase line-to-ground and single phase line-to-ground) are simulated at the fault location indicated by the black crosses in Fig. 1. The two crosses indicate the fault is simulated on a double ceot transmission line.

A. Symmetrical Fault

A duration of 2.5 cycles three-phase line-to-ground fault on the Invercargill-Tiwi transmission line is simulated. Complete collapse of all phase voltages at the receiving end and over-current at the corresponding sending and receiving end branches are expected.

Figs. 2-10 shows the Tiwi 220kV busbar voltages, Invercargill-Tiwi sending end branch currents and the fault currents. The TCS and PSCAD/EMTDC simulations are plotted in solid and dotted line respectively and their differences are shown at the bottom half of each figure. As indicated, the difference is relatively small during steady-state, but became larger when there is a step change. By comparing the simulation characteristics such as the dynamic response, indicates a close agreement between the TCS results and PSCAD/EMTDC simulations.
B. Asymmetrical Fault

While the results obtained under symmetric fault condition revealed good qualitative match, it is often necessary to look at the performance under unbalance condition as well before any conclusion can be made. A single phase line-to-ground fault of two and a half cycles duration at Invercargill-Tiwai transmission line is simulated in this case. Figs. 11-14 show the Tiwai 220kV busbar voltages and the Invercargill-Tiwai sending end currents. The three phase fault currents are depicted in Figs. 15-17. The results as shown are again in agreement with the PSCAD/EMTDC simulation despite small differences in the magnitude during step changes in the system topology. The ability in predicting asymmetrical transient response verifies the reliability of the proposed TCS algorithm in transient simulation.
IV. CONCLUSION

In this paper, a diakoptical state variable approach is used to provide an efficient method of forming the system state equations. It overcomes the need to identify the set of independent state variables by diakoptical segregation of the sub-networks and therefore is not affected by the presence of inductor cut-sets and loops of capacitors. Furthermore, for systems that undergo topological changes due to frequent switching, the diakoptical technique avoids changes of the whole network equations and only the relevant parts of the network equations are considered.

The performance of the proposed technique is compared with the PSCAD/EMTDC simulation based on the two test scenarios. The agreement between the two fundamentally different algorithms verifies that the new implementation of the TCS algorithm can be used with confidence in a.c. systems.

V. REFERENCES

Identification of Fault Locations using Transient State Estimation

K. K. C. Yu, Student Member, IEEE and N. R. Watson, Senior Member, IEEE

Abstract—In a large scale electrical power system, identification of system faults can be a tedious task. Unexpected fault events, which affect all customers, must be identified quickly so appropriate action can be taken out. Due to the unknown changes in the system topology during fault condition, accurate fault assessment through EMTP/ATP is prohibited. Furthermore, exhaustive search performed through traditional methods is time consuming and requires good knowledge of the fault event prior to the search. In this paper, an alternative methodology in fault identification via Transient State Estimation (TSE) is presented. The TSE algorithm models the ac system in its equivalent circuit where a set of current and voltage state equations can be developed to form the measurement system. Estimation of the complete system state from limited measurements is achieved by solving the measurement system equations and hence the fault positions and its magnitude can be determined by looking at the nodal current mismatch in the system. Simulation results from the Lower South Island of the New Zealand system are used to validate this approach.

Keywords: Fault identification, State estimation, Electromagnetic transients

I. INTRODUCTION

When a fault has occurred in a large power system, it is usually a difficult task to determine the fault position. Regardless of whether it occurred at transmission or load level, all customers that are connected to it will be affected to some degree. It is important the fault position is quickly identified so appropriate action can be carried out. Due to the measurement cost, complete monitoring of the system to determine the fault event is impractical. Therefore, when a transient event is recorded, the fault information available is often very limited. One method is to simulate the possible fault events through computer simulation EMTP/ATP [1]-[2] and then compares the simulation results with the actual recorded transient response. A good indication of the fault is achieved if the transient responses are closely matched. This exhaustive search method is arbitrary; the fact that it needs to simulate the possible fault events and compares with the actual response is time consuming and requires a good knowledge of all the possible fault events prior to the search. In this paper, a new fault identification methodology via transient state estimation technique is presented. The proposed method estimates the complete system state and power flow from partial measurements at each time-step. Once the complete system state is known, the fault position can be determined from the power flow accordingly.

An electrical power system expanded in its discrete equivalent RLC branches can be described by a set of current and voltage state equations. By combining these equations with appropriate state variables, the complete state model of the power system is achieved. The set of system equations can be formed without knowing the set of independent state variables, but are bounded by topological constraints (i.e., interconnection) and algebraic constraints. This is beneficial particularly in large power systems where dependency of the state variables is not obviously apparent due to the presence of network interwining.

Complete estimation of the system state requires the system to be fully observable. The system is said to be fully observable if the measurement system solves the state variables uniquely. By converting the system differential equations to discrete time equations using Euler's rule, history values at previous time-step can be used as additional measurements when forming the measurement system. These additional measurements act as measurement noise filter in over-determined system or provide extra measurement information in under-determined system.

The test system used to validate the proposed method is taken from the Lower South Island of the New Zealand.

II. FORMULATION OF THE STATE EQUATIONS

Power system modelled by its lumped RLC equivalents can be described as a set of first order differential equations. These equations describe the interconnection between the RLC branches and the state variables. The relevant system components are modelled as follows:

1. Generators: these are modelled by a voltage source with their equivalent R-R/L impedance.
2. Transformers: they are represented using two windings model depending on the type of magnetic circuit and on the connections of the terminals and the neutrals, i.e. delta or wye. Core losses are represented internally with an equivalent shaft resistance across each winding in the transformer.
3. Transmission lines: these are modelled by the three-phase
PI models and hence capable of incorporating non-symmetric condition.

4. Loads: The real and reactive power components are represented by its equivalent resistance and inductance respectively.

A Transformer Model
A two windings transformer model is used in the TSE algorithm. It is capable of modelling different impedance to the different components of current depending on the type of magnetic circuit and on the connections of the terminals & the neutral. Fig. 1 shows the two windings Y-G/Y-G connected transformer model. It is possible to simplify the three-phase transformers model by ignoring the interphase mutuals and reducing the equations to three independent sets of the three phases. This is accurate when banks of single-phase units are used. Effects like phase shifts, zero sequence circulating currents, etc are catered for by the terminal connections. Off-nominal taps on either primary or secondary winding can be represented by scaling the self and mutual elements accordingly.

![Fig. 1. Three phase Y-G/Y-G connected transformer](image)

Neglecting core losses, the differential equations for a three phase transformer can be expressed as:

$$\begin{bmatrix}
I_x \\
\dot{I}_x \\
K_{ix}
\end{bmatrix} = \begin{bmatrix}
L_x & -L_x & 0 \\
-L_x & L_x & 0 \\
0 & 0 & L_{ix}
\end{bmatrix} \begin{bmatrix}
V_x \\
\dot{V}_x \\
K_{ix}
\end{bmatrix}$$

B Transmission Line Model
A three phase PI model is used to represent short to medium length transmission line in the TSE algorithm. The model includes transmission line coupling effects. Fig. 2 shows the transmission line PI model.

Kirchhoff's laws are used to form the algebraic equations as shown in (2). These equations are used to derive the transmission line model expressed in (3).

$$L_x = L_x + \frac{L_{ab}}{2}$$
$$L_x = L_x - \frac{L_{ab}}{2}$$

$$V_x = [R_x] L_x + \frac{\frac{dL_x}{dt} + V_x}{L_x}$$

where

$I_x, I_{x1}, I_{x2}, I_{x3}$ are sending end currents, receiving end currents, inductor currents and shunt currents.

$V_x, V_r$ are sending and receiving end voltages.

![Fig. 2. Three phase transmission line PI model](image)

C Load Model
The algorithm models the system load by its equivalent resistance and reactance as shown in Fig. 3. Its state equations are described in (4).

$$\begin{bmatrix}
I_x \\
\dot{I}_x \\
K_{ix}
\end{bmatrix} = \begin{bmatrix}
L_x & -L_x & 0 \\
-L_x & L_x & 0 \\
0 & 0 & L_{ix}
\end{bmatrix} \begin{bmatrix}
V_x \\
\dot{V}_x \\
K_{ix}
\end{bmatrix}$$

where

$K_{ix}$ and $K_{ix}$ are the branch-node incidence (connection) for voltages and currents respectively.

$L_x$ is the self inductance.

$L_{ab}, L_{ba}$ are the mutual inductance.
III. DIFFERENCE EQUATIONS

The discretized state equations in terms of state variables at previous time step are advantageous in forming the measurement system. It allows history values (t−Δt) to be used as virtual measurements (i.e., measurements do not need metering) as well as real-time measurements.

By inspecting (7), it is shown that the transmission line inductor current can be used as measurements when forming the measurement system, however, given that it is usually not a well defined variable (i.e., difficult to measure) in a practical system, the sending end and receiving branch currents are used instead. The state equations become:

\[
\begin{bmatrix}
I_{s+1} \\
I_{s+2} \\
\vdots \\
I_{s+M-1}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
V_{s} \\
V_{s+1} \\
\vdots \\
V_{s+M-1}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{s} \\
\Delta I_{s+1} \\
\vdots \\
\Delta I_{s+M-1}
\end{bmatrix}
\]

\[\text{where } [L] = \begin{bmatrix}
L_{s} & 0 & \cdots & 0 \\
0 & L_{s+1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & L_{s+M-1}
\end{bmatrix} \]

\[\begin{bmatrix}
I_{s} \\
I_{s+1} \\
\vdots \\
I_{s+M-1}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
V_{s} \\
V_{s+1} \\
\vdots \\
V_{s+M-1}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{s} \\
\Delta I_{s+1} \\
\vdots \\
\Delta I_{s+M-1}
\end{bmatrix}
\]

\[\text{where } \Delta I_{s} = \begin{bmatrix}
\Delta I_{s,1} \\
\vdots \\
\Delta I_{s,M}
\end{bmatrix} \]

The discretized state equations in terms of state variables at previous time step are advantageous in forming the measurement system. It allows history values (t−Δt) to be used as virtual measurements (i.e., measurements do not need metering) as well as real-time measurements.

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\[
\begin{bmatrix}
I_{s+1} \\
I_{s+2} \\
\vdots \\
I_{s+M-1}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
V_{s} \\
V_{s+1} \\
\vdots \\
V_{s+M-1}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{s} \\
\Delta I_{s+1} \\
\vdots \\
\Delta I_{s+M-1}
\end{bmatrix}
\]

\[\text{where } [L] = \begin{bmatrix}
L_{s} & 0 & \cdots & 0 \\
0 & L_{s+1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & L_{s+M-1}
\end{bmatrix} \]

\[\begin{bmatrix}
I_{s} \\
I_{s+1} \\
\vdots \\
I_{s+M-1}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
V_{s} \\
V_{s+1} \\
\vdots \\
V_{s+M-1}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{s} \\
\Delta I_{s+1} \\
\vdots \\
\Delta I_{s+M-1}
\end{bmatrix}
\]

\[\text{where } \Delta I_{s} = \begin{bmatrix}
\Delta I_{s,1} \\
\vdots \\
\Delta I_{s,M}
\end{bmatrix} \]
IV. TRANSIENT STATE ESTIMATION

The main purpose of TSE is to estimate the power flow of the whole network from limited synchronized measurements. Ideally, an accurate TSE requires:
1. Accurate modelling of the system topology.
2. The system to be fully observable for the given set of measurements (i.e. over-determined system).
3. Absence of bad measurements.

Consider a general measurement system which relates the measurement vector \( \bar{x} \) to the state variables \( \bar{x} \) expressed in (11)

\[
\bar{x} = \bar{H} \bar{x} + \bar{e}
\]

where \( \bar{e} \) is the error vector and \( \bar{H} \) is the measurement matrix.

To build up the measurement system, rows are picked from the differential equations (6)-(10) for the corresponding measurements. In the case where field measurements data are absent, measurement data obtained from simulation can be used to supply to the TSE algorithm and are considered as the true measurements of the system states. The state variables to be estimated are the inductor currents and nodal voltages. Once they are obtained, the power flow of the whole network can be calculated accordingly. The procedure of the TSE can be described as follows:

1. Establish appropriate state model for linear and non-linear system components, such as transformers and transmission lines.
2. Equate complete differential and algebraical equations for the power system. Euler's formula is used to discretize these equations in this case.
3. Read off-line (history) and on-line measurements from selected measurement locations.
4. Build up the measurements equations by adding rows of dynamics equations which relate the measurements to the state variables.
5. Solve the measurement system for current time-step.
6. Calculate dependent variables such as branch currents.
7. Record estimation and use as offline measurements (\( \bar{x}_{\text{off}} \)) for the next time-step.
8. Repeat 2-7 for the next time-step.

V. TEST SYSTEM AND SIMULATION RESULTS

The test system chosen is the Lower South Island of the New Zealand system. The system consists of 27 nodes and 87 branches. Three linear loads are located at Tiwai 220kV, Invercargill 33kV and Roxburgh 11kV. The symmetric and asymmetric measurement placements are shown in Fig. 4. Due to the lack of field measurements, the measurement data used are obtained from the Transient Converter Simulation (TCS) [3] based simulation. TCS is a nodal approach with diakoptical segregation of the plant components, specially developed to analyse the dynamic behavior of HVDC systems. Method developed A 2.5 cycle duration symmetrical line-to-ground fault is simulated in the test system. The fault location is shown by the black crosses (doubt cct. transmission line) in Fig. 4.

Figs. 5-9 show the estimated voltages at Tiwai 220kV and receiving end line currents at Manapouri-Tiwai 220kV transmission line. The actual and estimated values are shown as dotted and solid line respectively. The difference is shown at the bottom half of the figure. Despite the difference occurred during the fault transition period (switched in/out), which is caused by an increase error in the Euler’s approximation for very fast transients, the results show TSE is capable of estimating the network’s behavior from the partial measurements.

![Fig. 4. Lower South Island of the New Zealand System](image-url)
To represent the unexpected fault event in the test system, the system topology supplied to the TSE during the fault period remained unchanged i.e. the fault is not modelled in the TSE. This causes inconsistent estimation results and hence current mismatch at the fault location during the fault period. Fig. 10 shows the nodal current mismatch of the test system. As indicated by the significant current mismatch positioned at nodes 1-3 (Tiwall 220kV), the followings are deduced:

1. The fault position is located at Tiwall 220kV (nodes 1-3)
2. The current mismatch at the Tiwall 220kV nodes represents the fault currents. Figs. 11-13 show the comparison of the current mismatch with the simulated fault current at each phase.
3. The fault is symmetrical as the fault currents are balanced.
4. Low fault resistance (estimated by the nodal voltage/fault current) suggests the type of fault is a line-to-ground fault.
VI. CONCLUSION

At present, when a fault has occurred, the fault information available to identify its position is usually insufficient. In this paper, the use of TSE for fault identification, by estimating the complete system states from partial measurements, is presented. The discrete time state-space formulation allows historical system states to be used as measurements to provide additional measurement information. However, the time-step must be adequate to cater for fast transients when the operator $d/dt$ is approximated using Euler's formula.

From the test results, the TSE solution showed agreement with the TCS based simulation. Three important fault characteristics: 1) the fault position, 2) fault type and 3) the magnitude of the fault currents, are obtained by inspecting the nodal current mismatch of the system. Fault identification via TSE technique is accurate, efficient and easy to implement.

VII. REFERENCES


VIII. BIOGRAPHIES

Kent Yu completed his B.E. (Hon) in 2001 and is at present a Ph.D. candidate. His area of research is state estimation techniques.

Neville Watson received his B.E. (Hon) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury (New Zealand) where he is now a Senior Lecturer. His interests include power quality, steady-state and dynamic analysis of ac/dc power systems.
Influence of Transformer Configuration on Measurement Placement in Harmonic State Estimation

K. K.C. Yu, Student Member, IEEE and N. R. Watson, Senior Member, IEEE

Abstract—Identification of the location and magnitude of harmonic sources in an electric power system through the use of limited number of measurement can be achieved via Harmonic State Estimation. Measurement location greatly influences the range of observability and measurement cost. However, finding the optimal measurement location via exhaustive search is difficult due to the size of the problem. This paper focuses on the effect of transformer winding configuration on measurement placement and its influence towards the overall system observability. Singular value decomposition technique is used for the harmonic state estimation and the observable region of a partially observable system is determined by looking at the position of non-zero entries in the null space vector of the measurement matrix.

Index Terms—Harmonic State Estimation, Transformer configuration, Measurement placement

I. INTRODUCTION

The design of a measurement system to perform Harmonic State Estimation (HSE) [1] is a very complex problem due to the size of the system and the conflicting requirements of estimator accuracy, reliability in the presence of transducer and data communication noise and failures, adaptability to changes in the network topology and cost minimization. In fundamental frequency state estimation, power is the measured quantity due to the abundance of revenue meters. The over-determined system then allows bad-data detection techniques to be utilized. In HSE no pre-existing metering is available that harmonic voltage and harmonic current measurements are utilized, as this is what the instrumentation measures. Moreover there is no generally accepted definition of harmonic power. The number of harmonic instruments available is always limited due to cost, and the quality of the estimate is a function of the number and location of the measurement points.

Traditionally, the measurement placements have been symmetrical i.e. either three or no phases of a busbars or branch were measured. This requirement restricts the search for the optimal placement of measurement points in a three-phase asymmetrical power system. Furthermore, the system observability plays an important role in measurement system design i.e. the extent of the system that is observable for a given selection of measurement points. Normally, an increase in measurement points around the system can increase the accuracy and region of observability of the estimate, the issue is whether the measurement cost can be justified. However, redundant measurements can also exist which leads to the unnecessary increase in measurement cost. The main goal in optimal measurement placements is to minimize the number of measurement points for a given network observability, thus minimizing the measurement cost.

Earlier work on optimal measurement placement for HSE used an exhaustive search [2]. However the transformer winding configuration is expected to influence the optimal measurement location and therefore reduce the number of possible placement arrangements needing investigation if a search technique is used. So far no rules have been given as to whether measurements are preferred on HV side, LV side, based on winding configuration, or asymmetrical placements.

II. HSE FORMULATION

For a given measurement set and system topology, the basic circuit laws lead to the following measurement equation:

\[ Z = h(x) + \epsilon \]  

(1)

where \( Z \) and \( x \) are the vectors of measurements and state variables, respectively, and the measurement error vector \( \epsilon \) which is assumed to be made of independent random variables with Gaussian distribution. Although in general the measurement equation can be non-linear and must be solved by an iterative method, but by choosing harmonic voltages as state variable and measuring harmonic voltages, branch harmonic currents and injected harmonic currents result in a linear equation (2). In this case, the task of estimation \( x \) given \( z \) measurements in the presence of noise \( \epsilon \) is expressed as

\[ z = [h]x + \epsilon \]  

(2)

where \([h]\) is the measurement matrix.

As a first step, this paper looks at the independence of the measurement equations that would be incorporated in the measurement matrix \([h]\), from current measurements made on
a transformer with different winding configurations. Three transformer configurations are investigated (star-G/star-G, star-G/star, star-G/delta). After investigating the independence of measurement around a transformer in isolation, the lower South Island of New Zealand test system is used to demonstrate the effect on a practical system. The test system consists of nine buses and five delta/star-G connected two winding transformers, is used in section 3 to verify the effects of transformer winding configuration on system observability and hence an optimal measurement placement.

A. The effect of measurement placement on transformer

A π model is used for the transformer in the HSR algorithm, with the shunt components representing the off-nominal turns ratio. The series impedance is scaled for skin effect. For each measurement set the measurement equations that would be incorporated in the measurement matrix \([B]\) are used to produce \([\text{meas}\,\text{mat}]\). To test the independence of the measurement equations the rank of \([\text{meas}\,\text{mat}]\) is checked.

The coupling between the different phases is dependent on the transformer winding configuration. In this section, three transformer winding configurations are studied to show the effect of the winding connection on measurement placements.

A three-phase two-winding transformer has six possible current measurement sites, thus the measurement locations are numbered from one to six respectively, i.e. measurement 1,2,3 and 4,5,6 are phase A,B and C measurements on the LV and HV sides respectively.

Case 1: Star-G/Star-G

Testing the rank of the transformer measurement matrix \([\text{meas}\,\text{mat}]\) showed that there existed only three independent measurements for star-G/star-G connected transformers out of six possible measurements (3 HV and 3 LV). Table 1 shows equation independence and observability for various combinations of measurement points. The first Yes or No in a cell shows whether the equations are independent, and the second indicates whether all the transformer currents are observable. For example although the two measurement points 1,2 are independent (row 1 column 1 of Table 1), the transformer circuits are not observable due to rank deficient. On the other hand, a rank of three will allow the transformer circuits to be observable. For example, measurement point 1,2,3 are independent and the transformer is observable under these measurements (thus the Y,Y entry).

Symmetric line current measurements on either the LV side (1,2,3) or HV side (4,5,6) are adequate, since the zero sequence current flowing on either side will be evident from symmetric measurements made on either the LV or HV side. However, asymmetric line current measurements, e.g. 1,2,5, are inadequate since there are an infinite number of possible solutions for the currents in the unmeasured branches to match the observed measurements. This is because the phase C current is unknown and hence zero sequence.

Using the same approach as in case 1, the results are shown in Table 2 for star-G/star connected transformers. The transformer currents are now observable with only two appropriate current measurements either in LV or HV side. This is due to the absence of zero sequence current flow in the transformer.

Case 3: Star-G/Delta

The observability of circuits with a Star-G/Delta transformere requires special consideration. Measurement of line currents on the delta side are inadequate since there are an infinite number of possible solutions for the currents in the Star-G winding to match the observed measurements. This is because the zero sequence current can flow into the Star-G winding and couple to a circulating current in the delta. Table 3 summarises the measurement properties. At least one measurement on the Star-G side is required for the system to be observable, but if sufficient measurements are made on the Star-G side, which is normally the LV side and hence closer to the load, then delta side measurements can be eliminated completely.

III. TEST SYSTEM AND RESULTS

The test system used for simulation is taken from the 220kV network below Roxburgh in the South Island of New Zealand. The system contains nine buses, three generators at Manapouri and Roxburgh, two single transmission lines between Roxburgh and Invercargill, three double transmission lines between Manapouri, Invercargill and Taw1 and five star-delta connected two-winding transformers. However, for the purpose of demonstrating the influence of transformer configuration on measurement placement, the transformer at Roxburgh 220kV/11kV was simulated as a test transformer with three different transformer winding configurations. Various combinations of current measurements were simulated for testing system observability. There are 27 state variables for the test system network as shown in Figure 1. These state variables are taken as phase harmonic voltages at the nine buses. A minimum measurement matrix \([\text{meas}\,\text{mat}]\) size of 27 x 27 and minimum measurement vector \([\text{meas}\,\text{mat}]\) size of 27 x 1 are required for solving (2) uniquely.

Furthermore, with the use of virtual measurements, the numbers of measurements are reduced, only nine which are needed on the test system as the first eighteen rows of the matrix \([\text{meas}\,\text{mat}]\) are build up using virtual measurements with zero injection. The additional nine measurements are taken at Manapouri 220kV, Roxburgh 11kV and Tiwai 33kV, as only these have loads connected directly.
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Table 2. Measurement properties for Star-G/Star transformation

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The actual values are determined by performing a detailed three phase harmonic penetration study of the system. Results from the simulation were used at the measurement locations and supplied to the HSE algorithm. The algorithm solves the system using singular value decomposition [5], providing a linear least squares solution. Furthermore, SVD also provides the nullspace vector(s) of the measurement matrix \( \mathbf{H}_{\text{obs}} \), allowing to identify any unobservable islands within the system. The unobservable islands are identified by non-zero entries in the nullspace vector(s). Three transformer winding configurations (star-G/star-G, star-G/Δ, and star-Δ/Δ) are simulated.

**A. Simulation 1: Star-G/Star-G**

In the first simulation the test transformer is simulated with a star-G/star-G winding configuration. The measurements taken at the test transformer are shown in Fig. 2. One possible way to identify unobservable islands within the system is to inspect the null space vector of the measurement matrix. Fig. 3 shows the entries of the nullspace vector \( (27 \times 1) \) of the measurement matrix \( \mathbf{H}_{\text{obs}} \) for two different sets of measurement placement shown in Fig. 2. The non-zero entries in the nullspace vector suggest that there are infinite possible solutions at the respective busbar, thus with the asymmetric current measurements, the system is unobservable. Harmonic voltage magnitude in Fig. 4 verifies the unobservable areas.
B. Simulation 2: Star-G/Star

In the second simulation, a star-g/star transformer winding configuration is used as shown in Fig. 6. The system network is now partially observable for both cases with an unobservable island at busbar 4,5,6 (Roxburgh 11kV) indicated by the non-zero entries in the nullspace vector of the measurement matrix \([\mathbf{h}_{\text{null}}]\) as shown in Fig. 7. With the above measurement placement, although the transformer circuits are observable when the transformer is isolated (as shown in Table 2), however, the measurements are dependent, thus resulting in a rank-deficient measurement matrix \([\mathbf{h}_{\text{null}}]\). The system cannot be solved with normal state estimation equations, but the algorithm utilizes the SVD technique to produce a least square solution that matches with the actual values (Fig. 8) even though the system is under-determined.

C. Simulation 3: Star-G/Delta

Simulation 3 was performed on a star-G/delta transformer, which is the normal connection of the Roxburgh 230kV/11kV transformer in the test scenario. The measurement points taken in this case are symmetric, being either the HV measurements or the LV measurements as shown in Fig. 9. Current measurements taken on the LV side (star-G) allow the test system to be fully observable; however, only partial observability is achieved when measurements are taken at the HV side (delta). The unobservable area in this case is the LV side of the test transformer, Roxburgh 11kV, nodes 4,5,6 as shown in Fig. 10. The non-zero entries in the nullspace vector indicate that the system is under determined, but once again, the SVD solution matches with the actual values as shown in Fig. 11. From these observations, symmetric measurements on the Star-G side are preferred on Star-G/Delta transformers. This is because as well as the transformer circuits being observable, all the measurements are taken at one site, thus, reducing the number of base units which are usually the predominant cost.
V. REFERENCES


VI. BIOGRAPHIES

Karen Yu completed her B.E. (Hons) in 2001 and is at present a Ph.D. candidate. His area of research is state estimation techniques.

Neville Watson received his B.E. (Hons) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury (New Zealand), where he is now a Senior Lecturer. His interests include power quality, stability and dynamic analysis of multi-power systems.

IV. CONCLUSION

Harmonic state estimation algorithm using SVD has the benefit of obtaining an estimate even for an under-determined system. Unobservable islands are revealed by looking at the position of the non-zero entries in the nullspace vector of the measurement matrix. The influence of transformer configuration on measurement placement in Harmonic State Estimation has also been presented. This development can be use to limit the possible combinations when searching for the optimum measurement placement in harmonic state estimation. The method has been applied to the reduced network below Rotorweg in the South Island of New Zealand. The simulation results showed the choice of measurement locations on transformers can highly degrade the overall system observability, hence the estimation of the system. Using independent measurement locations is beneficial, especially in the case when the numbers of measurements are limited. The measurement cost can also be reduced without affecting the system's observability by ensuring the best measurements locations around transformers are used.
Three-Phase Harmonic State Estimation using SVD for Partially Observable Systems

K. K. C. Yu, Student Member, IEEE, and N. R. Watson, Senior Member, IEEE

Abstract—This paper looks at a three phase Harmonic State Estimation algorithm that utilizes singular value decomposition (SVD) to increase availability for partially observable systems and eliminate the need of observability analysis prior to state estimation. For comparison, a non least squares solution i.e. an arbitrary solution is also supplied to the algorithm and the results are compared with the SVD approach.

The comparison shows a noticeable improvement with the use of SVD and pseudo-inverse; however its computation is much more complex than estimation using normal equations.

Index Terms—Singular Value Decomposition, Harmonic state estimation, Least square, Pseudo-inverse

I. INTRODUCTION

HARMONIC State Estimation (HSE) is an effective technique for identifying the level of distortion and the source location from limited measurements. Due to the size of electric power systems and to the deregulation of utility networks, it is impractical to obtain full measurement of the system states. With only partial measurement, the system observability is greatly affected. Traditionally, Observability Analysis (OA) [1,2] is performed to find the optimal measurement placement to obtain a system wide estimation. However, the singular value decomposition technique; a highly reliable and computationally stable mathematical tool, can be used to provide a particular solution and a null space vector for each singularity. SVD also reveals the observable and unobservable islands within the system, and thus essentially replaces the function of observability analysis. Furthermore, analysis of a partially observable system is made possible while traditional methods using normal equations seem to fail, due to the ill-conditioning of the gain matrix.

The performance of SVD in solving partially observable systems under different observability levels is demonstrated in this paper with reference to the lower South Island of New Zealand system.

II. PRINCIPLES OF THE SVD

The SVD method represents an \((m \times n)\) matrix \([A]\) in the factored form

\[
[A] = [U][\Sigma][V]^T
\]

where \([U]\) and \([V]^T\) are orthogonal matrices. The columns of \([U]\) \((m \times n)\) are eigenvectors of \([A][A]^T\), and the columns of \([V]^T\) \((n \times n)\) are eigenvectors of \([A]^T[A]\).

And \([\Sigma]\) is a diagonal matrix \((m \times n)\) with entries of singular values. [3]

SVD constructs orthonormal bases in a special way. Not only are they orthonormal, but if \([A]\) is multiplied by the column \([V]\), it produces a multiple of a column of \([U]\) (2).

Moreover, the columns of \([U]\) and \([V]\) give orthonormal bases for the four fundamental subspaces of \([A]\): column space, row space, left nullspace and nullspace. The columns of \([U]\) correspond to the non-zero singular values and the columns of \([V]\) correspond to the zero singular values are the column space and null space of \([A]\) respectively.

\[
[A][\Sigma][V]^T = [U][\Sigma][V]^T
\]

III. SOLVING AN UNDERDETERMINED SYSTEM USING SVD

The standard techniques used in the solution of over-determined systems are LU decomposition and back substitution, Cholesky decomposition and Gauss-Jordan elimination. Over-determined systems allow for bad data detection by acting like a noise filter to suppress any erroneous data in the redundant measurements. In contrast, obtaining sufficient harmonic measurements in practical power systems is not possible due to issues such as measurement cost and different ownership of different parts of the system, thus the system is normally underdetermined.

In such cases, while most traditional techniques fail, SVD is able to provide a particular solution and a null space vector for
each singularity.

The number of infinite solution of such system are expressed by

\[ x = [x_p] + \sum_{i=1}^{n_{infl}} k_i [x_n] \quad (3) \]

where \([x_p]\) is the particular solution, \(k_i\) is a constant and \([x_n]\) is the null space vector. If all the nullspace vectors have zero entries in a particular position, the corresponding state variable will be observable as any linear combination of nullspace vectors will not alter its value. SVD seeks for an optimal least squares solution i.e.

\[ [x]^T = [x_p] + \sum_{i=1}^{n_{infl}} k_i [x_n] \quad (4) \]

where \(A^{+} = [V][W]^{-1}[U]\) is the pseudo inverse, which is the same as the inverse, when \([A]\) is invertible \(\text{(4)}\). To illustrate the effectiveness of SVD, the solution of

\[
\begin{bmatrix}
1 & -2 & 4 \\
2 & -3 & 4 \\
3 & -5 & 5
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
4 \\
-1 \\
3
\end{bmatrix}
\]

is shown in Fig. 1. As the rank of the measurement matrix is equal to two, there exists only two independent sets of measurements and therefore an infinite number of solutions can be found in the form of \((3)\). The set of possible solutions is illustrated by the longest line shown in Fig. 1. The nullspace vector \((-5, -2, 1)\) is in parallel with the solution set. Point A \((0.6666, -3.1333, -2.9233)\) is the solution chosen by SVD and point B \((-8.662, -6.7848, -1.16756)\) is an arbitrary solution. Although both solutions satisfy the system above, when looking at the norm of the solutions, SVD produces a least squares solution with a norm of 4.2436, in comparison to 10.503 for point B.

### A. Nullspace Vector \([x_n]\)

SVD not only produces the least squares solution, but it can also provide a nullspace vector for every singularity. This nullspace vector can be used to determine the observable and unobservable infinite within the system. To illustrate this, consider the four cases listed below.

**Case 1. No observable variables**

Consider the system of

\[
\begin{bmatrix}
1 & -2 & 1 \\
2 & -3 & 4 \\
3 & -5 & 5
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
4 \\
-1 \\
3
\end{bmatrix}
\]

This is an underdetermined system since there are more unknowns than equations. The SVD method represents the matrix \([A]\) as a product of \((1)\), where

\[
[U] = \begin{bmatrix}
-0.39249 & -0.19176 \\
-0.91976 & 0.39249 \\
-0.8413 & 0.70665 \\
0.69702 & 0.69342
\end{bmatrix}
\]

\[
[V] = \begin{bmatrix}
0.60676 & -0.14374 & -0.91287 \\
0.70665 & 0.93767 & 0.18257 \\
-0.3821 & 0.4315 & 0.36515
\end{bmatrix}
\]

The nullspace of the matrix \([A]\) can be seen as the column of \([V]\) corresponding to the zero singular value, i.e.

\[
\begin{bmatrix}
0.60676 \\
0.70665 \\
-0.3821
\end{bmatrix}
\]

The non-zero entries in the nullspace vector indicate that there is no unique solution for the corresponding variable \(x_1\) since altering the values of \(k\) in \((2)\) will result in another set of valid solutions. The system is not observable.

**Case 2. One observable variable**

Consider another system of

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 2 & 1 \\
2 & 2 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
3 \\
4 \\
7
\end{bmatrix}
\]

where the product of \([U][W][V]^T\) are

\[
[U] = \begin{bmatrix}
-0.33378 & 0.74516 & -0.57735 \\
-0.47843 & -0.66164 & -0.57735 \\
-0.81221 & 0.083515 & 0.57735
\end{bmatrix}
\]

\[
[W] = \begin{bmatrix}
5.0761 & 0 & 0 \\
0 & 0.48255 & 0 \\
0 & 0 & 1.95964-016
\end{bmatrix}
\]

![Fig. 1: Minimum least squares solution](image-url)
Matrix $[A]$ is of rank two since the third equation is a linear combination of the first two equations. The SVD gives the singular values as $0.50761$, $0.48255$, $0$ with the corresponding precision of $1e-08$. Hence, the nullspace vector is

$$[v] = \begin{bmatrix} -0.48002 & 0.51921 & 0.70711 \\ -0.48002 & 0.51921 & 0.70711 \\ -0.73428 & -0.67885 & -0.9966-0.016 \end{bmatrix}$$

The system has one observable variable $x_2$ since any linear combination of nullspace vectors will only alter the values of $x_1$ and $x_3$ while the value of $x_2$ remains unchanged.

Case 3. Multiple nullspace vectors

It is also possible to have more than one nullspace vector for the system $[A]x = b$. On the basis of mathematical topology, the dimension of the subspace $R^n$ can be expressed as

$$N = \dim(\text{col}[A]) + \dim(\text{null}[A])$$

where $N$ is the number of columns of the matrix $[A]$, $\dim(\text{col}[A])$ and $\dim(\text{null}[A])$ are the dimension of the column space and the nullspace of the matrix $[A]$ respectively.

Consider the simple system with multiple nullspace vectors

$$[w] = \begin{bmatrix} 1 & -2 & 1 & -1 & x_1 \\ 2 & -3 & 4 & -3 & x_1 \\ 3 & -5 & 5 & -4 & x_1 \\ -1 & 1 & -3 & 2 & x_2 \end{bmatrix}$$

where SVD gives the followings

$$[U] = \begin{bmatrix} -0.21656 & 0.5352 & 0.80614 & 0.12963 \\ -0.5352 & -0.21656 & -0.12963 & 0.80614 \\ -0.75176 & 0.31863 & -0.33826 & -0.46788 \\ 0.31863 & 0.75176 & -0.46788 & 0.33826 \end{bmatrix}$$

$$[\Sigma] = \begin{bmatrix} 11.498 & 0 & 0 & 0 \\ 0 & 1.8706 & 0 & 0 \\ 0 & 0 & 9.29164 & 0 \\ 0 & 0 & 0 & 1.31661 \end{bmatrix}$$

$$[V] = \begin{bmatrix} -0.33578 & 0.18329 & -0.92179 & -0.062594 \\ 0.53192 & -0.75566 & -0.3539 & 0.14524 \\ -0.61305 & -0.59449 & 0.13992 & -0.49871 \\ 0.47542 & 0.2058 & -0.074061 & -0.85219 \end{bmatrix}$$

The basis of the nullspace are shown as the third and forth column of the matrix $[V]$, i.e.

$$\begin{bmatrix} -0.33578 & 0.18329 & -0.92179 & -0.062594 \\ 0.53192 & -0.75566 & -0.3539 & 0.14524 \\ -0.61305 & -0.59449 & 0.13992 & -0.49871 \\ 0.47542 & 0.2058 & -0.074061 & -0.85219 \end{bmatrix}$$

There are two independent nullspace vectors and two independent column space vectors for the matrix $[A]$, where satisfy (6) with the following values

$$\dim(\text{col}[A]) = \text{rank}[A] = 2$$

$$\dim(\text{null}[A]) = 2$$

$$N = 4$$

The system contains no observable variables as there are an infinite number of solutions for all state estimates that can be expressed in the form of (3).

IV. TEST SYSTEM AND SIMULATION RESULTS

The test system is taken from the 220kV network below Roxburgh in the South Island of New Zealand. To look at the ability of SVD in harmonic state estimation, systems containing multiple and single unobservable island(s) are chosen (Figs. 2 & 3) where the unobservable areas are indicated with the shaded areas. The tests will verify the performance of SVD under different levels of observability.
A. Single Unobservable Island Network

The first test system (Fig. 2) is partially observable with a single unobservable island at Roxburgh 11kV (Busbar 4, 5, 6). It cannot be solved using traditional methods due to the measurement matrix being rank deficient. From the infinite number of solutions, an arbitrary solution is compared with the solution chosen by SVD and pseudo-inverse. Although both solutions satisfy the measurement system, the minimum least squares solution produced by SVD is much closer to the actual values (in fact it matches the actual values.) The arbitrary solution in the form of (3), however, has greater estimate errors at the unobservable island (Busbar 4, 5, 6 Roxburgh 11kV) as shown in Figs. 4-7. Inspection of the nullspace of the measurement matrix \( [H] \) of this network shows that there are three non-zero entries corresponding to busbars 4, 5, 6 in the nullspace vector, hence the state estimates will be altered with any linear combination of the nullspace vector and are treated as unobservable.

B. Multiple Unobservable Islands Network

The effectiveness of three phase harmonic state estimation using SVD and pseudo-inverse is further investigated in a multiple unobservable islands network. In this case, the unobservable islands are much larger in area and they are
located at Roxburgh 220kV, Roxburgh 11kV and Inveraray 33kV as shown in Fig. 3. Thus, the complexity in determining the actual solution is increased due to an increase in the number of nullspace vectors and unknowns. The SVD solution as shown in Figs 8 & 9 is the least squares solution for the HSE measurement system. When compared with a randomly picked arbitrary solution (Figs. 10 & 11), the SVD technique produces a much more reliable solution with smaller errors induced at the unobservable islands even though about half of the network is unobservable. The observable areas (nodes 1, 2, 3, 12-21 and 25-27) in both cases remain error free since the corresponding state variables are uniquely defined.

The test simulation showed the superior of SVD in solving partially observable system with a large unobservable area. In comparison to an arbitrary solution, the SVD solution is much closer to the actual values where the estimation errors can be smaller or even eliminated depending on whether the system network contains single and multiple unobservable islands.

Fig. 8: Harmonic voltage magnitude (arbitrary solution in the form of (5)) where k=0.25

Fig. 9: Harmonic voltage phase angle (arbitrary solution in the form of (3)) where k=0.125

Fig. 10: Harmonic voltage magnitude (SVD solution)

Fig. 11: Harmonic voltage phase angle (SVD solution)

V. CONCLUSION

The use of SVD and pseudoinverse for three-phase Harmonic State Estimation has been presented. The SVD method has the ability to solve partially observable systems where traditional methods fail. SVD is a highly reliable and computationally stable technique. Although it can be significantly slower than traditional methods for over-determined systems and requires more storage and computational efforts [5], it is less susceptible to built-up round-off error due to finite computation precision. In SVD, the selection of the particular solution is based on minimum least squares. The SVD technique not only solves the single unobservable island network correctly but also improves estimate errors on a multiple unobservable islands network. Furthermore observability analysis is no longer required prior to harmonic state estimation since SVD is able to identify unobservable islands within the network and gives information on the observability as a by-product.

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VI. REFERENCES


VII. BIOGRAPHIES

Kent Yu completed his BE (Hons) in 2001 and is at present a Ph.D. candidate. His area of research is state estimator techniques.

Neville Watson received his B.E. (Hons) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury (New Zealand) where he is now a Senior Lecturer. His interests include power quality, steady-state and dynamic analysis of bulk power systems.