

**USING COMPUTER ASSISTED INSTRUCTION TO BUILD FLUENCY IN
MULTIPLICATION: IMPLICATIONS FOR THE RELATIONSHIP BETWEEN
DIFFERENT CORE COMPETENCIES IN MATHEMATICS**

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Definition of Terms

Dyscalculia: A developmental disorder in understanding and learning mathematics, which is not due to low intelligence or lack of learning opportunities; also called mathematics learning disability.

Computer Assisted Instruction (CAI): Instructional software that presents the learner with a task, provides a means for the learner to respond to the task, and provides feedback on the given response (Rasanen, Salminen, Wilson, Aunio, & Dehaene, 2009).

Numerosity: The number of objects in a set.

Number sense: The capacity of humans and animals to be able to apprehend approximate numerosity, to discriminate between different numerosities, and to perform approximate calculations non-symbolically (Dehaene, 2001).

Symbolic / verbal representation: Representation of number using linguistic symbols, e.g. number words, or Arabic digits, or storage of number facts using linguistic codes (e.g. verbal storage of memorized addition and multiplication facts).

Non-symbolic / non-verbal representation: Representation of number using numerosity or magnitude, rather than linguistic symbols (e.g. external representation using a random dot pattern). Tasks which use internal non-symbolic representation include estimation, comparison, and approximate arithmetic.

Abstract

Dyscalculia is a specific learning disability that affects an individual's core skills in mathematics, including calculation, recall of number facts, and approximating/comparing number. Research into the origins and aetiology of dyscalculia have suggested the presence of two different networks in the brain used for mathematics; one for verbal (symbolic) tasks such as recalling number facts, and one for non-verbal (non-symbolic) tasks such as approximation and number comparison. While these networks are located in different brain areas, they are often used together on calculation tasks, they are known to impact each other over the course of development, and they both appear to be impacted in dyscalculia. The current study used entertaining computer assisted instruction software, "Timez Attack", to target the symbolic network, i.e. to improve the fluency of multiplication fact recall in three 9 and 10 year old children who were performing below the expected level on multiplication. An ABA (applied behaviour analysis) multiple-baseline across subject design was used to track participants' performance on multiplication, addition, and number comparison over the course of the intervention. Results showed improved fluency of multiplication fact recall in all three participants; however this improvement did not generalise to addition or number comparison. This finding suggests that the symbolic and non-symbolic brain networks involved in mathematics are largely independent from each other by middle childhood, and that training targeting one network does not affect the other.

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1. Introduction

1.1 Dyscalculia

1.1.1 What is dyscalculia?

Dyscalculia is defined as a difficulty in understanding and learning mathematics, which is not due to low intelligence or lack of learning opportunities (Landerl, Fussenegger, Moll, & Willburger, 2009). It is estimated to have a prevalence of approximately 6% in primary and intermediate age children (Landerl et al., 2009; Mussolin, Mejias & Noel, 2010). The term mathematical learning disabilities (MLD) is defined very similarly to dyscalculia (Geary, 2004) and here the two are used interchangeably.

The manifestations of difficulties associated with dyscalculia are age and grade related (Shalev, 2004). When children with dyscalculia first begin school they will appear to have delayed counting skills and will have difficulty learning basic arithmetic facts. As children with dyscalculia get older, they will present with difficulty learning mathematic procedures such as multiplication and division and will struggle to complete calculations with multiple steps, such as carrying over (Shalev, 2004; Butterworth, 2010; Geary, 2012). Development of the mental number line (or the ability to imagine numbers in relation to space) and the speed of processing numerals is also significantly delayed in dyscalculia. For example, when shown two digits and asked which one has the largest numerical value, children with dyscalculia respond significantly slower in comparison to their non-dyscalculic peers. (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Landerl, Bevan, & Butterworth, 2004).

Some difficulties seen in dyscalculia (e.g. recalling procedures) have been proposed to be the result of poor working memory, which several pieces of research have shown to be impaired in this population (Geary, 2011; Mabbot & Bisanz, 2008; Monuteaux, Faraone, Herzig, Navsaria, & Biederman, 2005; & Shalev, 2004). Geary (2011) outlines the role that working memory plays in mathematical computations and common difficulties seen in children with

working memory deficits (due to dyscalculia or other disorders), which include forgetting what step of a procedure comes next, forgetting how many fingers they have counted and poor attentional control (Geary, 2011).

One factor that inhibits working memory during the calculation of mathematic equations is mathematical anxiety (Ashcraft, 2002). Math anxiety is often a consequence of dyscalculia, and is characterised by a high level of avoidance of mathematical tasks, or a high level of distress whilst carrying out mathematical tasks (Ashcraft & Kirk, 2001). For individuals who experience math anxiety, the high levels of distress experienced when carrying out mathematical tasks are so distracting that they act as a secondary task, and therefore limit the capacity of the working memory, which in turn hinders their ability to accurately complete tasks. Individuals with high levels of math anxiety often avoid mathematical tasks or sacrifice accuracy for speed in order to complete the as quick as possible so as to stop the feelings of distress.

Dyscalculia affects children both academically and socially; not only does it limit their ability to learn mathematics, they also struggle with every day activities such as telling the time, giving change, and understanding space and directions (Geary, 2011). The ongoing effects of dyscalculia or low achievement in mathematics also include limited career and educational opportunities, in particular in individuals with math anxiety who will avoid certain career paths if the training or job itself involves mathematical tasks.

1.1.2 Associated difficulties and disorders.

Research is growing to support the neuropsychological theory that dyscalculia is due to an impairment in number sense, a skill that requires the use of cognitive skills such as language and working memory (von Aster, Schweiter & Zulauf, 2007). With this in mind, it is no wonder that dyscalculia has a high comorbidity rate with other developmental disorders,

particularly dyslexia and attention deficit disorders, both of which are disorders that are characterised by difficulties with working memory and/or language.

von Aster et al. (2007; as cited in von Aster & Shalev, 2007) conducted a longitudinal study with a representative population based sample of 378 children and found that 6% of their sample group had dyscalculia, and of these, 70% also had dyslexia. von Aster et al. (2007) separated the group with pure dyscalculia and the group with comorbid dyscalculia (dyscalculia and dyslexia) and conducted further analyses on each group. Their findings also indicated that those with comorbid dyscalculia/dyslexia had a higher prevalence of attention disorders and performed lower on tasks involving high working memory loads than those with pure dyscalculia whose attention disorder rates were comparable with the control group. This research emphasizes the extent to which learning difficulties overlap, and suggests that underlying neural systems may not be as independent as thought. In line with this, in earlier work both Kaufman (2002) and Ashcraft (1995) proposed that calculation does not occur in isolation but rather alongside other cognitive abilities, particularly working memory and attention control.

Although it is widely accepted and documented that ADHD (attention deficit hyperactivity disorder) and learning disabilities in general have a high comorbidity rate, there has been little research carried out specifically on the comorbidity of dyscalculia and ADHD. Other than von Aster et al. (2007), one of the few studies that looked specifically at ADHD and dyscalculia was a study by Monuteaux et al. (2007), who conducted research that found that within a sample composed of children who had ADHD but not RD (n=234) compared to control children (n=229), the presence of dyscalculia was significantly higher amongst those with ADHD (11%) than amongst those without (6%). However, a familial risk analysis conducted by the authors showed that the two disorders are etiologically discrete. This latter finding is also supported by von Aster et al. (2007), who found that irrespective of the

presence of ADHD, children with dyscalculia showed a similar impairment in spatial number representation (e.g. placing numbers on a number line).

In summary, the work on comorbidity suggests while that the cognitive impairments associated with different disorders are largely independent; however in some cases there may be some overlap, particularly for working memory. This work also highlights the importance of screening for both dyslexia and ADHD when dyscalculia is suspected.

1.1.3 Dyscalculia versus low achievement in mathematics.

In a recent article, Geary (2012) argues that due to the high cut-off criterion used for inclusion in many studies (often as high as the 25th or 35th percentile), children who have been considered to have mathematical learning disabilities (MLD) can actually be divided into two groups. Those with true dyscalculia (or MLD) have mathematics performance at or below the 10th percentile, whereas those who are “low achieving” (LA) have performance below the 25th percentile. Geary (2012) argues that these two groups are qualitatively different, and that MLD children show core numeracy difficulties, whereas LA children primarily show difficulties in fluency of number processing and retrieval of addition facts. However it remains to be seen whether other researchers are able to replicate this pattern, or whether differences between these two groups are more quantitative than qualitative in nature.

Furthermore, despite the various criteria for identifying learning difficulties, an important point to note is the variability in the presentation and severity of arithmetic learning difficulties over time. Research by Silver, Penner, Black, Fair, and Balise (1999) assessed and classified children with various subtypes of learning difficulties, and followed their progress through specific learning programmes. They found that upon retesting, approximately 50% of the children identified as having a math learning difficulty no longer displayed a deficit in

mathematics, approximately one third continued to show an isolated mathematics deficit with no other co-occurring concerns and the remaining participants continued to demonstrate difficulties with arithmetic but had also developed concerns with reading and spelling. These results support the researcher's hypothesis that classification of a learning difficulty at a sub-type level can be unreliable as children may move from one category to another over time. This is important to note when working with children who have been identified by criteria such as Geary's MLD vs LA as it is possible that those children will fluctuate between the two categories over time.

1.1.4 Aetiology of dyscalculia.

Several possible causes of dyscalculia have been identified, ranging from genetic to environmental. An at least partial genetic etiology for dyscalculia is now well established, although no specific genes have yet been identified (Butterworth, 2001). Evidence for this comes from family and twin studies, as well as genetic disorders. For instance, the prevalence of dyscalculia in families where one sibling has dyscalculia is almost tenfold the expected prevalence in the general population, indicating that dyscalculia has a strong genetic component (Shalev, Manor, Kerem, Avali, Badichi, Friedlander, et al. 2001). A twin study by Kovas et al. (2007) showed a high concordance of dyscalculia for identical twins versus non-identical twins (68% vs. 44%, respectively), again indicating a strong genetic component. Lastly, the prevalence of dyscalculia is higher than normal in some genetic syndromes, such as Williams' and Turner's (Bruandet, Molko, Cohen, & Dehaene, 2004).

One theory regarding the cause of dyscalculia is that it occurs as a result of a core cognitive deficit in the ability to understand and manipulate number non-verbally, or *number sense* (Butterworth, 2010; Landerl et al., 2009; Wilson & Dehaene, 2007). Number sense and core deficit theory are further discussed in the following section.

Alternatively, strictly environmental hypotheses regarding the etiology of dyscalculia are also possible. For instance strict behaviourists would likely argue that MLD is due to a lack of adequate practice opportunities or feedback. Another environmental hypothesis is that children with lower than average mathematics skills could consequently develop anxiety and avoidance, resulting in reduced practice opportunities and impaired learning (Shalev, 2004). A child with concerns or anxiety about their poor mathematics skills may avoid doing mathematics in order to avoid negative feedback, or may be so anxious about mathematics that their concentration is impaired. Thus, children with poor math skills may miss out on learning opportunities and fall further and further behind their peers. However, psychological interventions for anxiety (Cognitive Behaviour Therapy, Graduated Exposure and Systematic Desensitisation) have been unable to give full remediation of dyscalculia, suggesting that anxiety is perhaps not the cause of dyscalculia but rather a perpetuating and maintaining factor of its presence (Rubinsten & Tannock, 2010).

1.1.5 Mathematics and the brain: Number sense.

Number sense is a term used by Dehaene (2001) for our core knowledge capacity to represent the magnitude of numbers (i.e. their meaning), and to manipulate them. It is the capacity to be able to apprehend approximate numerosity (the number of objects in a set), to discriminate between different numerosities, and to perform approximate calculations non-verbally (Dehaene, 2001). For example, it is the ability to be able to look at two different sets of objects and automatically discriminate which of the two has a larger number of objects.

Research has shown that this ability is not specific to humans and that it is also exhibited in animals by their ability to visually or auditorally discriminate numerosity. For instance, pre-verbal children, adults, and animals are all able to discriminate which is the larger of two sets of objects, as long as the ratio between their numerosities is sufficiently large. This suggests

that number sense is an innate skill that operates independently of language (Libertus & Brannon, 2009; McCandliss, Cohen, & Dehaene, 2003).

Neuroimaging studies support this theory that number sense is independent from language by showing that this capacity is dependent on a specific brain network, separate to that used by language-based (symbolic) arithmetic. For example, a meta-analysis of brain imaging studies conducted by Dehaene, Piazza, Pinel & Cohen (2003) showed that tasks involving number sense (e.g. comparing numbers, approximation) tended to activate the bilateral intraparietal sulcus, whereas tasks involving recall of symbolic number facts from memory (e.g. multiplication) tended to activate the left angular gyrus, which is part of the perisylvian language circuit. Later research showed that neural populations in the intraparietal region were tuned to specific numerosities, both in humans and monkeys (Hubbard, Piazza, Pinel, Dehaene, 2005). In addition, cross-cultural studies have shown that number sense is located in the inferior parietal lobe in both English and Chinese speakers (Ansari, 2008), indicating that this phenomenon is not specific to the English language. These findings support the theory that number sense is a skill that is separate from language development, and that it is an innate skill, present in both humans and animals.

Consequently, a neurological hypothesis regarding the aetiology of dyscalculia is that individuals with dyscalculia have a core deficit in this inborn capacity to conceptualise and perceive non symbolic representations and quantities (number sense) in early stages of development which leads to poor understanding of symbolic representations later in development (Butterworth, 2010; Landerl et al., 2009; Wilson & Dehaene, 2007).

1.1.6 Verbal versus non-verbal number representations.

Verbal representation of number and arithmetic is another crucial aspect of human mathematical capability. While humans and animals have the same ability to represent non-

symbolic numerosity (Libertus & Brannon, 2009), human ability to use language allows us to have a precise representation of number, to carry out exact calculation, and to learn advanced mathematics such as algebra and calculus (Lemer, Dehaene, Spelke, & Cohen, 2003). Both studies with neurological patients and brain imaging data suggest that verbal representation of arithmetic facts such as multiplication involves a different area of the parietal lobe to that of number sense, the angular gyrus (Lemer et al., 2003; Dehaene et al. 2003). A recent brain imaging study by Holloway et al. (2010) suggests that the left angular gyrus and the superior temporal gyrus together are responsible for symbolic number processing and arithmetic fact retrieval. The angular gyrus shows an increase in activation as a result of drill training in multiplication (Ischebeck, Zamarian, Egger, Schocke, and Delazer, 2007); a result which is attributed to the use of verbal memory retrieval for multiplication.

Neuropsychological studies have found a double dissociation between verbal versus number sense mathematical tasks. For example, Lemer et al. (2003) report a patient with lesions to the inferior parietal lobe, who had difficulty with non symbolic mathematics skills such as approximations and comparisons, but was unaffected in symbolic mathematics skills such as multiplication as compared to a different patient with the opposite dissociation.

In summary, previous research on multiplication and number sense suggests that they are learnt in different ways and represented in separate parts of the brain (Dehaene, Piazza, Pinel & Cohen, 2003). Multiplication is highly symbolic, or linked to language, and is learnt with the acquisition of language and education. Contrastingly, number sense has its roots in our non-symbolic or non-linguistic ability to perceive and manipulate quantity; skills which are thought to be innate and are evident in children and animals (Gilmore, McCarthy, & Spelke, 2010; Ansari, 2008).

1.1.7 Brain imaging research on dyscalculia.

The research reviewed above has shown that in typically developing children and adults, the left and right intraparietal sulci are responsible for the completion of non-verbal approximation tasks such as number comparison, whereas the angular gyrus is responsible for verbal fact recall in exact calculation, such as multiplication. Given that children with dyscalculia experience difficulties with both types of task, one might expect that brain imaging studies would show impairments in both neural networks. Although brain imaging research on dyscalculia has only started to appear in the last 10-15 years, several studies have found decreased activation of the intraparietal region during approximation tasks compared to typically developing children, strengthening the number sense “core deficit” theory (Kucian et al., 2006; Ashkenazi, Rosenberg-Lee, Tenison, & Menon, 2012) However so far there is no evidence for impairment in the network involved in verbal mathematical tasks.

A study by Kucian et al. (2006) carried out fMRI on 18 children age 9-12 years with dyscalculia and 18 age-matched children without dyscalculia to determine the pattern of brain activation during exact and approximate arithmetic tasks. The study involved presenting a series of approximate and exact arithmetic tasks to the subjects using video goggles whilst they were in the fMRI scanner. The results showed that the children with dyscalculia had significantly less activation of the intraparietal sulcus during approximation tasks. In contrast, there were no significant differences in brain area activation for exact calculation tasks between the dyscalculic and control group, i.e. both groups had a similar level of angular gyrus activation when carrying out an exact calculation task (addition of one-digit numbers). This lack of a neural correlate for the difficulties children with dyscalculia have in number fact recall is puzzling, however one possible explanation is that addition is a task that is only sometimes involves recalling number facts. For instance children do not routinely memorise facts such as $7 + 2 = ?$, in the way that they do multiplication facts, and research has found

that even skilled adults use a range of strategies to complete such problems (LeFevre, Smith-Chant, Hiscock, Daley & Morris, 2003).

A recent fMRI study by Ashkenazi et al. (2012) found similar brain activation patterns to Kucian et al. (2006) when comparing brain images of children with and without dyscalculia. Ashkenazi et al. (2012) presented 17 children with dyscalculia, and 17 age-matched typically developing children without dyscalculia, a series of increasingly difficult arithmetic tasks whilst brain imaging was occurring. Their results found that the children with dyscalculia had less activation of the intraparietal sulcus than the typically developing children, along with decreased activation of other areas of the arithmetic network such as the superior parietal lobule.

In conclusion, thus far both neuro-imaging and behavioural studies support the core deficit theory of dyscalculia.

1.1.8 Dyscalculia and number sense: Core deficit or access deficit?

A variation on the core deficit theory is that the deficit in dyscalculia may be in number sense *access*, rather than in number sense per se (Wilson, Dehaene, Dubois, & Fayol, 2009; Rousselle & Noel, 2007). This theory suggests that children with dyscalculia do not have difficulty in representing number non-symbolically, but have difficulty in accessing this representation, because of a difficulty associating it to the verbal symbols that accurately represent that number (Meijjas, Mussolin, Rousselle, Gregoire, & Noel 2012). This deficit would therefore impair an individual's ability to rapidly and precisely access number magnitude, an impairment which could easily be mistaken for a difficulty with number sense itself, without an experimental methodology which distinguished the two.

Even if dyscalculia itself is not due to such an access difficulty, it may be a factor in the more general low achievement in mathematics group proposed by Geary (2012) and discussed

earlier. For instance Jordan and Levine (2009) review research on low mathematical achievement associated with socio-economic status (SES), and report that low SES children score lower than high SES children on mathematical tasks presented symbolically, but similarly on the same tasks when they are presented non-symbolically. This finding indicates that at least in the case of low mathematical achievement seen in children from low SES families, difficulties may be attributable to their lack of experience with and exposure to symbolic representations of number which is impacting their ability to access their number sense, as opposed to any deficit in number sense itself.

1.1.9 Dyscalculia and multiplication.

If the core deficit in dyscalculia is primarily in number sense, and number sense has a neural representation separate to that of verbal components of mathematics such as multiplication, why then is difficulty recalling multiplication facts a common symptom seen in dyscalculia? One possibility is the non-verbal and verbal systems are not as separated as the above research might lead us to believe, particularly during development. For instance, Butterworth (2005) proposes that children with dyscalculia find multiplication facts difficult to learn because they have little meaning for them. This hypothesis conversely suggests that children without dyscalculia form semantic links between number sense and verbal storage of arithmetic facts, at least during the establishment of the latter. If this is the case, training of multiplication might benefit number sense, or vice versa, via these semantic links.

Another possible explanation for why children with dyscalculia experience difficulties recalling multiplication facts is that they may have an impairment in executive function (an individual's cognitive processes used to organize thought, including planning, monitoring and evaluating one's own thought; for example when problem-solving). Swanson and Sachse-Lee (2001) carried out a study in which children with learning disabilities were compared to chronologically age-matched peers and curriculum-matched peers on a series of

computational and comprehension based tasks measuring aspects of executive functioning including verbal and visual working memory, problem solving, and phonological processing. The findings from this research indicated that not only were phonological processes, verbal processes and visual-spatial working memory all core components required to accurately solve mathematics problems, but also that these executive functions were all impaired in children with learning disabilities. In addition a later study by Swanson and Beebe-Frankenberger (2004) conducted a battery of assessments on 353 second, third and fourth graders who were considered either at risk, or not at risk, of serious math difficulties. Like Swanson and Sachse-Lee (2001), they also found that children's working memory was a predictor of mathematical solution accuracy and that children with learning disabilities (including MLD) have deficits in working memory and other areas of executive functioning. These studies suggest that children with learning difficulties may have difficulty solving multiplication problems because to do so they require executive functioning skills to operate simultaneously with or to regulate verbal memory recall, and that these skills are in some way compromised in children with learning disabilities / MLD.

As mentioned earlier, an alternative, strictly behavioural, explanation for the difficulties that children with dyscalculia have with multiplication is these are due to reduced practice opportunities. This could be because their core number difficulties result in them completing less work, progressing more slowly, or avoiding mathematics. This latter theory would predict that multiplication deficits in dyscalculia could be simply addressed by fluency training, but that number sense should remain unaffected by this training. It would also predict that training in number sense should not immediately benefit multiplication, but that a benefit might be seen at a later stage due to increased practice opportunities.

1.2 Number fact fluency

1.2.1 Fluency and mathematics.

Number fact fluency, or fast and automatic recall of number facts, is of great importance in learning mathematics. It has been found that fluency is effective in maintaining interest and motivation in mathematics, increasing learning and response opportunities, and aiding in the development of more advanced skills (Bliss, Skinner, McCallum, Saecker, Rowland-Bryant, & Brown, 2010). Miller and Heward (1992) highlight the importance of fluency training because it fosters the development of component skills (basic or key skills), enhances the performance of composite skills (complex skills that build on component skills), and therefore increases the acquisition of new skills. Other theories supporting the use of fluency training in mathematics describe the role of fluency as being motivational in the sense that increased fluency generates more reinforcement, and thereby acts as a protective factor towards mathematics related anxiety (Chiesa & Roberston, 2000). Research suggests that students exhibiting low levels of fluency are less likely to engage with mathematics problems, therefore limiting the amount of practice and reinforcement opportunities available to them (Miller & Heward, 1992). Furthermore, low fluency levels require students to expend more effort and time when answering mathematics problems, therefore also limiting the amount of practice opportunities and reinforcement that is available (Bliss et al., 2010). Lastly, with poor fluency, working memory is occupied with backup strategies such as “count by” and “skip counting”, leaving little room for other material related to the problem at hand.

1.2.2 Fluency and learning disabilities.

Students with learning disabilities are less likely than other students to be given opportunities to increase fluency in basic number facts, as teachers have a tendency to give them more time to complete exercises (Miller, Hall, & Heward, 1995). This likely prevents these students from benefitting from the above mentioned benefits of fluency, and means that they are likely to fall further behind their peers in mathematics. Effective fluency interventions are therefore

particularly crucial for students with dyscalculia. Research suggests that an effective way to achieve this may be with computer assisted instruction (CAI).

1.2.3 Non CAI interventions for fluency in learning disabilities

Although there is research to suggest that CAI is an effective intervention for improving fluency of arithmetic tasks in children with learning disabilities, there are also a number of non-computerised strategies that have been used to improve multiplication fluency with this population. One such strategy is the *count-by* technique which involves teaching children to translate a multiplication problem (e.g. 6×4) into counting by the multiplicand as many times as specified by the multiplier (e.g. count by 6, four times). The effectiveness of the count by technique for improving the fluency of math facts in children with learning disabilities was evaluated by McIntyre et al. (1991). Their case study in an 11-year-old with MLD used a multiple-probe design across three different fact sets, and found that the introduction of the count by technique resulted in an increase in the number of correct responses per minute on targeted fact sets and that these improvements were maintained over time. McIntyre et al. (1991) concluded that the count by technique is likely to be effective at improving the fluency of single digit multiplication fact recall in children with learning disabilities, but that further research was needed with a larger sample size.

Wong and Evans (2007) conducted a study which determined that systematic practice of multiplication facts using pencil and paper was an effective method of increasing multiplication fact recall in 27 nine- and ten-year-old children. Participants were given worksheets of multiplication fact sets to learn, and as they mastered each set, it was replaced with a new set to be learnt. This systematic practice also included interspersing previously learnt facts with to be learnt facts in order to ensure fact retention. Before and after the intervention, the students were given a 60 item multiplication test to complete in one minute, which acted as a pre and post measure of multiplication fact recall. The findings were that

systematic practice was an effective method of increasing multiplication fact recall, as shown by an average 63% improvement in fact recall from pre to post intervention.

1.3 Computer Assisted Instruction (CAI)

1.3.1 Overview and history of CAI.

With the improvements in technology and software design in recent decades, the use of Computer Assisted Instruction (CAI) has increased dramatically. This is evident in the increasing number of evidence based CAI programs that have been developed to help students with learning disabilities in mathematics, reading and language. However, research findings into the effectiveness of CAI versus traditional instruction over the past 40 years are mixed, with findings showing support for both teaching mediums. In addition to identifying the effectiveness of CAI at increasing mathematical skills, research has also evaluated the motivational advantages of using CAI as a teaching medium for children with learning disabilities.

A meta-analysis by Kroesbergen and Van Luit (2003) evaluated the effectiveness of a range of interventions intended to increase mathematic skills in children with special education needs. The meta-analysis included 58 studies, 12 of which used CAI as the main teaching medium. The authors found that all 12 of the interventions using CAI were effective (median effect size = 0.64) in enhancing the mathematic skills of children, but not as effective as the interventions that used traditional teaching methods (median effect size = 1.32). However, the authors did note that the interventions using CAI were more likely to increase motivation in mathematics, especially in children with learning difficulties. This last hypothesis reflects earlier research (Okolo, 1992) that reported an increase in motivation in children with learning disabilities when using CAI.

A more recent meta-analysis by Slavin and Lake (2008) reviewed the effectiveness of three different approaches to improving mathematics achievement in primary aged children; altering curricula, altering teaching practices and introducing CAI. The review evaluated the effectiveness of 87 studies across the three approaches, including the median effect size for each approach. Results from the 38 CAI studies evaluated in the review showed that CAI interventions in general were an effective intervention for improving mathematical achievement, and that they were more effective (effect size; $ES = 0.19$) than interventions altering curricula ($ES = 0.10$). However CAI interventions were less effective than interventions altering teaching practices ($ES = 0.33$).

A meta-analysis by Seo and Bryant (2009) assessed the effectiveness of CAI interventions for improving a range of mathematical skills in students with dyscalculia. The analysis reviewed 11 studies that fell into one of the following three categories; CAI vs. teacher-directed instruction (6 studies), comparison of CAI types (3 studies), and enhanced CAI (3 studies) which examined the effectiveness of particular instructional variables within the CAI interventions. Of the six studies in the CAI vs. teacher-directed instruction category, three group design studies showed that the subjects in the CAI group performed better on post intervention measures than the subjects in the teacher-directed group; however large effect sizes were not found in any of these studies. Similarly, two single-subject design studies also reported children with learning difficulties showing improvements in achievement with the introduction of a CAI, but again failed to show large effects. Overall, of the eleven studies in the review, nine yielded improvements in mathematical skills following the use of CAI, however none of these improvements were found to be statistically significant. Seo and Bryant (2009) attributed that result to methodological constraints within individual studies (as opposed to ineffectiveness of CAI) and suggested that further studies using CAI with mathematical difficulties should be carried out with better control conditions. The

methodological shortfalls identified by Seo and Bryant included unsatisfactory descriptions of instructional variables, unsatisfactory control of instructional variables (e.g. number of practice opportunities), insufficient CAI sessions, failure to report on technical aspects of measures and a failure to assess generalisation of targeted skills.

1.3.2 CAI Multiplication Interventions.

In addition to the use of CAI to assist with the learning of multiplication facts in children with dyscalculia, CAI interventions also have a long history of being used as a teaching method for multiplication facts with non learning disabled learners as a means of increasing fluency and accuracy. The majority of the research carried out in this area has compared the effectiveness of CAI with traditional instruction, however some research has also considered which specific components of a CAI are effective for increasing multiplication fact recall.

Wong and Evans (2007; discussed earlier) compared the effectiveness of using pen and paper to complete drill practice of multiplication facts to using a CAI. Both conditions were found to be effective for increasing multiplication fact recall in 64 children aged nine and ten from regular classrooms. However, pen and paper was more effective than the CAI condition, showing an average multiplication fact recall score of 37 facts correct per minute compared to 30 correct per minute for the CAI group, on pen and paper post-tests. The authors noted that there were a number of factors that may have influenced this outcome, including the use of pen and paper as the pre and post intervention measure of multiplication fact recall, possibly giving children who had this type of intervention an advantage.

Chang, Sung, Chen and Huang (2008) completed a study which evaluated the effectiveness of CAI intervention on the development of multiplication ability in 42 second grade children in regular classrooms. The software used in their study emphasised not only contained repeated practice of multiplication tasks, but also emphasized the comprehension of

multiplication concepts, because it has been identified that an understanding of the underlying concept of multiplication aids in learning multiplication (Dempsey and Marshall, 2001). Chang et al. (2008) found that the CAI was more effective for children who scored lower on the pre-test scores and had the least prior knowledge of multiplication. In addition, qualitative data from the study also supported the idea that the novelty of the CAI significantly increased the motivation of low scoring learners to engage in the intervention. Chang et al. (2008) therefore attributed the success of the CAI to an increase in motivation, which resulted in increased levels of engagement in the intervention. The authors concluded that low-scoring, poorly motivated learners are most likely to benefit from the use of CAI interventions over traditional instruction.

The aforementioned CAI interventions for multiplication fluency were with children without learning disabilities. Some previous studies have targeted children with specific learning needs, however only children with emotional and behavioural disabilities (EBDs). Hodge, Riccomini, Buford and Herbst (2006) conducted a review of studies in primary school aged children with EBDs focused on the acquisition of basic math skills, including multiplication. One component of the review included analysing the effectiveness of CAI interventions with this population. The authors found that CAI was no more effective at improving math performance in children with EBDs than was pen and paper drill. Two potential factors were identified as being likely to have impacted these findings; the computer tasks did not include any motivating factors or reinforcement, and the extra skills required to carry out the computers tasks may have hindered or distracted the students more than the straightforward pen and paper tasks. In addition, Hodge et al. (2006) suggested that CAI interventions may be more effective in children with learning disabilities compared to those with EBDs. The authors hypothesised that the decrease in emotion control and increased levels of frustration that are evident in children with EBDs may play some role in the reduced effectiveness of

CAI interventions in this population, because CAI may require more cognitive effort than pen and paper tasks

1.4 Current Study

The current study aimed to extend the literature base on the efficacy of CAI interventions for improving multiplication in children at risk for dyscalculia using “Timez Attack”, a recently produced, engaging, and interactive game which is designed to reinforce speed and accuracy, as well as provide both symbolic and nonsymbolic representations of the concept of multiplication. Both of these are factors which have not been specifically examined in relation to remediation for dyscalculia or low mathematical achievement. “Timez Attack” has yet to be tested for efficacy; a step which is essential for any intervention to be considered evidence-based for use in an educational setting.

“Timez Attack” differs from traditional CAI in that it looks and operates like a modern interactive video game. As well as mastering the recall of times tables through fluency exercises, the software is designed to motivate students with a 3-D graphical environment, and requirements to find keys and pass enemies in order to gain access to new levels within the game. Although “Timez Attack” looks and feels like a video game, it is still primarily educational, and adapts to the level of each individual student. A pre-test at the beginning of the game assesses the student’s current achievement level and that information is then used to give practice in identified areas of difficulty. Teacher reviews of the software have been positive and the publisher (Big Brainz, 2004) agreed for it to be used free of charge in this study.

The current study tested the efficacy of “Timez Attack” in improving multiplication fluency levels in three 8-10 year-old children with difficulties in mathematics, using a traditional applied behaviour analysis design (within subjects multiple baseline AB design). Children’s

progress in multiplication, addition, and number sense was monitored using cognitive-style computerised tasks which measured accuracy and reaction time. The multiple baseline design allows the researcher to use each subject as its own control (comparing baseline to intervention measures), but still have the ability to generalise findings to specific populations (Cooper, 2001). The staggered intervention start point allows the researcher to assess whether behaviour change occurs only after the introduction of the intervention for each subject and therefore conclude that any changes were the result of the intervention and not other time-related variables, such as test-retest effects.

1.5 Research Aims and Hypotheses

The primary aim of the current study was to determine the effectiveness of the computer assisted intervention “Timez Attack” in building fluency of multiplication in children struggling to learn multiplication facts (and therefore at risk for dyscalculia, or at the least, of Geary’s 2011 “low achievement” category). Ideally, screening for dyscalculia would have been preferable, but this was not possible, due to the practical constraints of the immediate post-earthquake environment in Christchurch in 2011.

A secondary aim was to determine if any effect was specific to multiplication or if it generalised to other mathematical skills, and whether any transfer seen was a function of the degree to which these skills are known to draw on verbal or non-verbal brain networks involved in mathematics. Thus the two other tasks tested were addition (which as mentioned earlier is thought to involve both verbal and non-verbal networks), and number comparison, which is thought to involve predominantly the non-verbal number sense network. If transfer to both addition and number comparison is seen, this would suggest that verbal and non-verbal mathematical networks are semantically linked during the learning of multiplication facts. If transfer to only addition is seen, this would suggest that benefits of multiplication training can spread to other tasks which involve the verbal mathematical network, but not to

tasks involving the non-verbal network. If transfer to neither task is seen, this would suggest that training of mathematical tasks is highly specific within the verbal network.

Overall determining whether “Timez Attack” is effective in improving multiplication fluency will contribute to the search for evidence-based intervention methods for children struggling with mathematics, whereas measuring the generalisation to number sense will address the theoretical question of the extent to which verbal and non-verbal mathematical competencies are linked in development and may interact in dyscalculia.

We hypothesised that use of the CAI intervention would result in an improvement in multiplication fact recall, but that these improvements would not generalise strongly to number sense, because of the different neural bases of these two competencies. We expected that there would be moderate transfer to addition, which involves both number sense and verbal fact recall.

2. Method

2.1 Participants

Participants were n=3 children living in and attending school in Canterbury, New Zealand; a boy and a girl aged 10 years and a 9-year-old boy. All three participants lived at home with their families and were described by their parents as struggling to learn their times tables but having no learning difficulties in subjects other than mathematics. All parents and children provided informed consent. This study was granted ethical approval by the Educational Research Human Ethics Committee at the University of Canterbury (See Appendices 1a-1d for Information sheets and Consent forms.)

2.1.1 Criteria.

The age range for potential participants was 9-10 years old, the age at which the NZ curriculum states rapid recall of multiplication facts is to be taught (Ministry of Education, 2009). Potential participants had to show difficulties in learning multiplication facts as identified by a teacher and/or parent, and have an estimated IQ ≥ 85 . A further exclusion criterion was any known learning or developmental disabilities other than dyslexia (e.g. autism, cerebral palsy, epilepsy, ADHD etc.). Dyslexia was not used as an exclusion criterion due to the high overlap between dyscalculia and dyslexia (Landerl et al., 2009). To run the software, participants were required to have a PC computer with internet access and a minimum configuration of Windows XP OS, 1.0 GHz processor, 256 MB RAM. A sample of six was originally planned, however as a result of disruption from the Canterbury earthquakes, only three participants were able to be included.

2.1.2 Recruitment.

Nine schools in both the immediate and wider vicinity of the University of Canterbury were contacted with information about the current study to pass on to parents of any students who

were likely to fit the above criteria. Three schools contacted failed to reply to emails or answer phone messages from the researcher. Of the remaining six schools, five responded stating that they were too busy to take on further commitments, and one stated that they would discuss the study at a staff meeting and notify us of any interest. There was no further communication from that school, and a follow up email from the researcher did not receive a reply. The researcher then contacted a supermarket near the University and was given permission to place an advertisement for the study on the staff notice board. This advertisement yielded three responses, two from supermarket staff and one from a friend of a staff member.

2.2 Materials and Procedure

2.2.1 Design.

This study was an intervention study using a common design in applied behaviour analysis; the multiple baseline design, with an AB intervention pattern (A = baseline, B = intervention). This design was selected because it is efficient, appropriate for a small sample, and provides replication across subjects.

2.2.2 Participant screening.

Participants were screened with psychometric measurements to determine eligibility. These included the short form of the Wechsler Intelligence Scale for Children-Fourth Edition (WISC-IV) to estimate Global Ability Index (or estimated IQ), the Woodcock Johnson reading subtests Word Attack and Word Identification, the math subtest of the Wide Range Achievement Test-Fourth Edition (WRAT-4), and a parent interview. The inattention and hyperactivity subscales from the Child Behaviour Checklist (CBCL) were also included to screen for possible attention difficulties (ADHD). The parent interview (attached in Appendix 2) gathered information related to the child's development and difficulties with mathematics, their suitability to participate in the study (e.g. temperament, other commitments, anxiety),

and socio-economic indicators for the parents. Screening was carried out by BM (who has training and experience in psychometric testing) in a quiet room in participants' homes.

Administration took approximately two hours per participant.

2.2.3 Results of Screening Measures.

Participant One (P1) was a ten year old girl living at home with her parents. Her performance on the WISC-IV short form indicated she has a Global Ability Index Score of 88 (an estimated measure of IQ based on verbal comprehension and perceptual reasoning skills) placing her in the 21st percentile of intelligence for her age group. Her score on the WRAT-4 Math Subtest placed her math abilities at the 25th percentile for her age group, and her scores on the WJ Word Identification and Word Attack subtests indicate she is performing at approximately the 40th percentile for reading.

Participant Two (P2) was a ten year old boy living at home with his parents, older brother and older sister. His performance on the WISC-IV short form indicated he has a Global Ability Index Score of 88 placing him in the 21st percentile of intelligence for his age group. His score on the WRAT-4 Math Subtest placed him in the 32nd percentile among his peers and his score on the WJ Word Identification and Word Attack subtests indicates he is performing at a approximately the 25th percentile for reading.

Participant Three (P3) was a nine year old boy living at home with his parents, older brother and two older sisters. His performance on the WISC-IV short form indicated he has a Global Ability Index Score of 100 placing him in the average range of intelligence for his age group. His score on the WRAT-4 Math Subtest placed his math abilities in the average range for his peers at the 25th percentile and his score on the WJ Word Identification and Word Attack subtests indicates he is performing at approximately the 65th percentile for reading.

The parent interviews conducted did not yield any information which would result in any of

the children meeting exclusion criteria for the study. On the Inattentive/Hyperactive subscale of the CBCL all of the parent ratings indicated that their child was within normal limits, indicating that they were unlikely to have ADHD.

On the basis of these results the participants were all deemed eligible for inclusion in the study as they all met the criteria specified in section 2.1.1 above.

2.2.4 Dependent measures.

Childrens' progress was measured pre, post, and during the study using five-minute computerised tests for multiplication, addition, and symbolic number sense. These were administered by the researcher approximately twice a week in a 15-min session in participants' homes. The tests were programmed in E-Prime (Schneider, 2002), and run on a dedicated laptop computer controlled by the researcher. Responses were recorded using an E-Prime serial response box and microphone, providing close to millisecond accuracy in reaction times.

Previous studies have shown that this methodology can be successfully adapted into short, colourful and entertaining tests for children which give good quality data (see, e.g. Wilson, Revkin, Cohen, Cohen, & Dehaene; 2006). The advantage of using this methodology over traditional pen and paper is that it is able to collect response times to each stimulus, does not require children to write (and thus is a more pure measure of recall speed), and can be used to rapidly produce summaries of large amounts of data, including of sub-categories of multiplication problems.

In each test, children were asked to answer each question as quickly as possible when it appeared on the screen. Regardless of whether or not the child gave the correct response, a colourful design appeared on the screen for a few seconds, providing reinforcement for responding quickly. For tests requiring a vocal response, the researcher then entered the

child's response into the computer. The researcher also recorded any input errors or problems with the microphone so that any corrections could be made to the data at a later point.

In the *symbolic number sense* test, two Arabic digits appeared side by side on the computer screen and children had to click the left or the right mouse button to indicate which had the highest numerical value. If the child did not respond within 10 seconds, the trial timed out. The same stimulus set of 36 items (see Appendix 3a) was used at each testing session, however the order of presentation was varied randomly by the computer.

In the *addition* test, participants attempted 32 single digit addition problems (see Appendix 3b) using a microphone to record vocal response time. Each problem appeared on screen and the child responded by giving an answer into the microphone. Children were told that if they did not know the answer they should try and work it out. If the child did not respond within 10 seconds, the trial timed out. This stimulus set was coded into two categories, large and small. Large problems were those that had a sum of 11 or higher whereas small problems were those with a sum of ten or less.

In the *multiplication* participants attempted 36 multiplication problems (see Appendix 3c), again using a microphone to record vocal response time. The stimulus set was a subset of that used in "Timez Attack", with rules (e.g. 10 x table) and ties (e.g. 5 x 5) excluded. One other item (2x3) was also removed from the stimulus set so that there was an even number of stimuli in large and small categories. Large problems had an answer of 36 or greater, whereas small problems had an answer of less 36.

2.2.5 Baseline

After the screening phase, a schedule was then discussed with each participant and their parents and the baseline phase began. Measurement sessions consisted of BM visiting each participant in their home and administering each of the three computerised tests and

recording any errors along with any possible variables that occurred either in the session or since the previous session on the Measurement Log (see Appendix 5). In accordance with Multiple Baseline design, the endpoint of the baseline phase was staggered across participants. P1's baseline consisted of three measures, P2's consisted of five measures, and P3's consisted of four measures. The same measurement process then continued on throughout the intervention phase to measure and track changes in the participants' learning.

2.2.6 Intervention software: "Timez Attack".

At the completion of the Baseline phase the researcher downloaded and installed the "Timez Attack" intervention software on each of the participants' home computers. Participants were given a log on name and password and given a short demonstration of how to play. They were asked to log on and play "Timez Attack" three to four times a week for 15 to 20 minutes at a time and to keep track of how much they played on the Playing Log (see Appendix 4). Access to the game was secured using a username and password and participants' data and progress was saved at the end of each session so that they could begin the next session at the same stage.

The "Timez Attack" game is set in a 3-D dungeon with multiple levels, each with a simple floor layout which children travel through in order to pass to the next level. There are up to eleven doors on each level, each of which teaches a new multiplication fact (operands from 2-12), and eleven levels.

New multiplication facts are taught to children using concrete representations. As the child approaches a door, a multiplication fact question (e.g. $5 \times 4 = ?$) appears on the door beside an outline of multiple dice (see Figure 1). At the same time, a corresponding number of snails emerge from the door and move around the chamber, each representing one multiple of the

equation to be solved. For example for the problem 5×4 , five snails emerge from the door each representing one multiple of four. The child catches each snail by moving up to it, at which point it is thrown at the door and fills in one of the dice. Once all snails (or multiples) are collected (counted) the child enters the answer (15) using the keypad, without a time limit.



Figure 2 An example of a question in “Timez Attack”

Once the new fact is learned, children practice fluency on all of the learned facts for that level, by using fast recall to beat a monster and collect an access key (see Figure 2). During these recall tasks the player has to correctly answer all the facts learned since the previous checkpoint (3-6 facts depending on the level) in a time limit of 5 secs per question. If the player does not answer the question within the time limit or correctly, they are knocked on the head by the monster and the clock is restarted. For each incorrect or late answer given, the player has to give the same amount of correct answers in order to get past the monster. For example, if a player failed to answer the question 2×5 correctly or within the 5 second limit twice, then in order to collect a key and move on the player has to answer that fact correctly 3

times; once to show they can answer it, and twice more to make up for the incorrect or late answers. During these recall task the player is given as many opportunities as it takes to answer all the questions fluently.



Figure 3: An example of a fluency exercise in “Timez Attack”

Throughout the game, at the completion of each level, participants are required to complete retention tests to ensure facts that are mastered earlier in the game are still retained. Retention tests require players to correctly and fluently (within 5secs) answer all facts from the previous levels, before they can continue on with the game. Facts that are not answered correctly during a retention test are given additional practice before the retention test is re-administered.

On the first use, the software gives children a pre-test, to ensure that only unknown multiplication facts are targeted during the intervention. The stimulus set used by the software includes 80 single digit and 30 double digit multiplication problems, displayed using Arabic digits. Completion of the game requires fluent recall of all 110 multiplication facts in the stimulus set.

3. Results

The following section presents results organised by task and participant. For each task, results from accuracy and response time are reported separately, then inverse efficiency (IE; vocal RT in msec / proportion correct) results are reported in order to check for the presence of speed / accuracy tradeoffs which could negate intervention effects.

3.1 Data Analysis

Participants' median RTs and average accuracy were calculated E-Prime, broken down by condition, task, and session. This data was then analysed using Microsoft Excel and SPSS. Trials with a response time of less than 200 msec were considered anticipations, and were excluded from RT analyses, as were those with any error noted in the collection of vocal RT. Learning rates were calculated using the least-squares line of best fit for the observations in each condition.

3.2 Measurement fidelity

The number and frequency of measurements that were taken during each phase of the study differed between participants for various reasons. A programming error in one task (multiplication) that was outside of BM's responsibilities resulted in only partial RT baseline data for P2 and P3, and no RT baseline data for P1. Secondly, BM was unavailable to take measurements for two weeks, due to multiple deaths of close family/friends. These events coincided with participant absences (holidays, illness), resulting in an unplanned between-measurement interval of 2-4 weeks for all participants during the intervention phase.

Additional difficulties were encountered with P3, whose parents had anticipated being able to accommodate testing visits two times a week, but decided this was no longer possible, resulting in measurements only once per week. Furthermore, following his baseline phase,

P3's family computer broke down, causing a three week delay prior to starting the intervention phase.

Table 1: Participant characteristics and intervention usage

Participant	P1	P2	P3
Age	10	10	9
Gender	Female	Male	Male
Estimated IQ (GAI)	Low average (88)	Low average (88)	Average (100)
Woodcock Johnson Reading	40 th percentile	25 th percentile	65 th percentile
WRAT-4 Math Subtest	25 th percentile	32 nd percentile	25 th percentile
Sessions using software (#)			
Time spent using software (min)	308	376	148
Baseline measurements (#)	3	5	4
Intervention measurements (#)	12	12	6

3.3 Multiplication results

During the baseline phase P1's average multiplication accuracy was 40 - 45%, with an average of 42% (see Figure 3a). Following the onset of the intervention, P1's accuracy rate began to rise, and from day 10 onwards it remained above 50%. On the second and tenth intervention measures, P1 had high outlying accuracy scores. Observational data suggests that on those days, P1 was more motivated and focussed and it is therefore likely that those outliers can be accounted for by increased mental effort. Regardless of the days in which more effort was applied, P1 shows a clear improvement in accuracy as the intervention increased. The average accuracy for the intervention phase was 55%; an increase of 13% from the baseline phase, suggesting that the intervention had a positive effect on accuracy. P1's accuracy learning rate increased from -0.5% per week during the baseline phase to 2% per week during the intervention phase, an increase of 2.5% per week.

P1's vocal response time (RT) is also shown in Figure 3a, on the secondary y-axis. P1's RT average and learning rate could not be calculated for the baseline phase, however during the intervention phase average RT was 2,981 msec, and the RT learning rate was -70 msec per week, indicating that response times were becoming faster over time. There was thus no evidence that the improvement in accuracy associated with the intervention period was due to a speed/accuracy trade-off.

To further verify that no speed/accuracy trade-off occurred, Figure 3b shows P1's inverse efficiency (IE). The IE learning rate could not be calculated for the baseline phase for P1 due to the missing RT data. However over the course of the intervention phase, P1 became more efficient with a learning rate of -112 msec per week. The outlying point on the second day of intervention measurements seen in the accuracy data remains in the IE data, however a clear improvement trend is still present without this point.

P2's multiplication data is shown in Figure 4a. During the baseline phase, P2's multiplication accuracy was variable, ranging from 45 to 68%, with a mean of 55%. During the intervention phase P2's accuracy rose steadily, never dipping below 55% and reaching as high as 78%.

The average accuracy for the intervention phase was 67%, an increase of 12% from the baseline phase. This suggests that the intervention had a positive effect on multiplication accuracy. The accuracy learning rate during the baseline phase was 0.1% per week, compared to a rate of 1.5% per week during the intervention phase, an increase of 1.3% from baseline to intervention.

P2's vocal RT during the baseline phase was an average of 3,198 msec. In comparison, the average RT for the intervention phase was 2,739 msec, an average improvement of 459 msec, almost half a second. Based on the measurements available, P2's RT learning rate was faster during the baseline phase (-483 msec per week) than during the intervention phase (-56 msec per week).

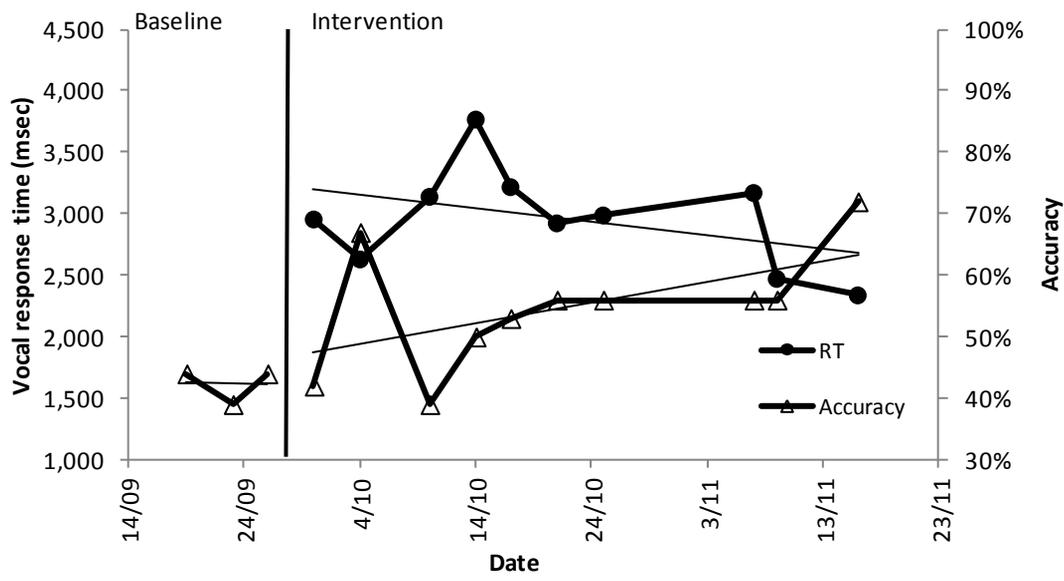


Figure 4a: P1's response time and accuracy for multiplication.

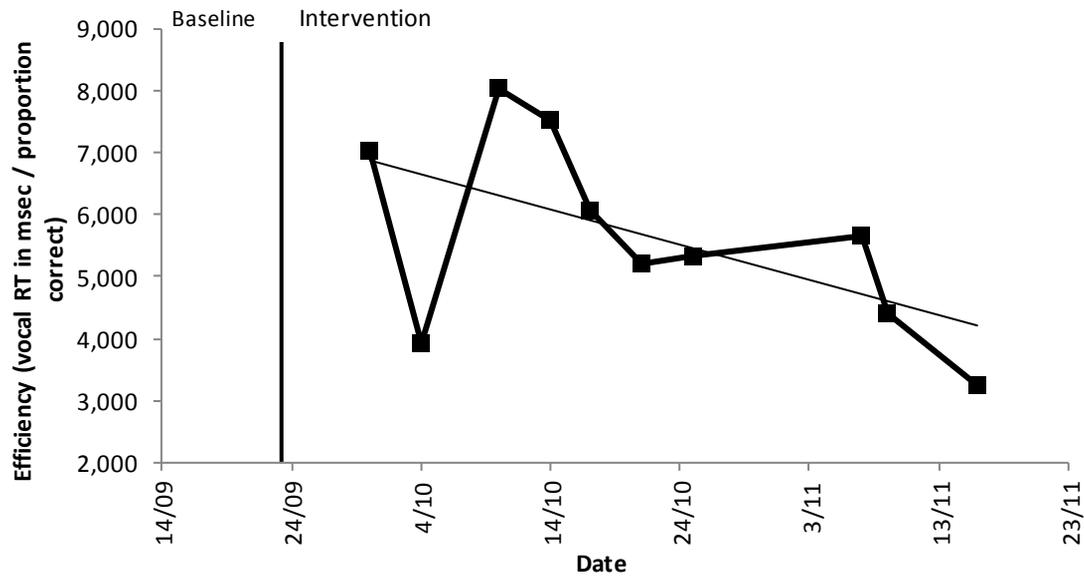


Figure 3b: P1's inverse efficiency for multiplication.

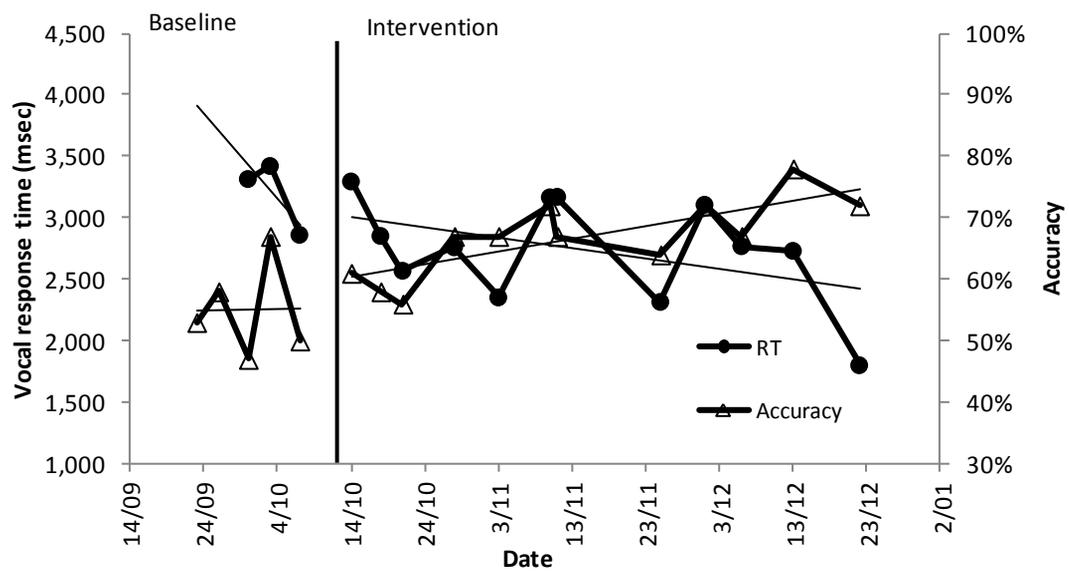


Figure 4a: P2's response time and accuracy for multiplication.

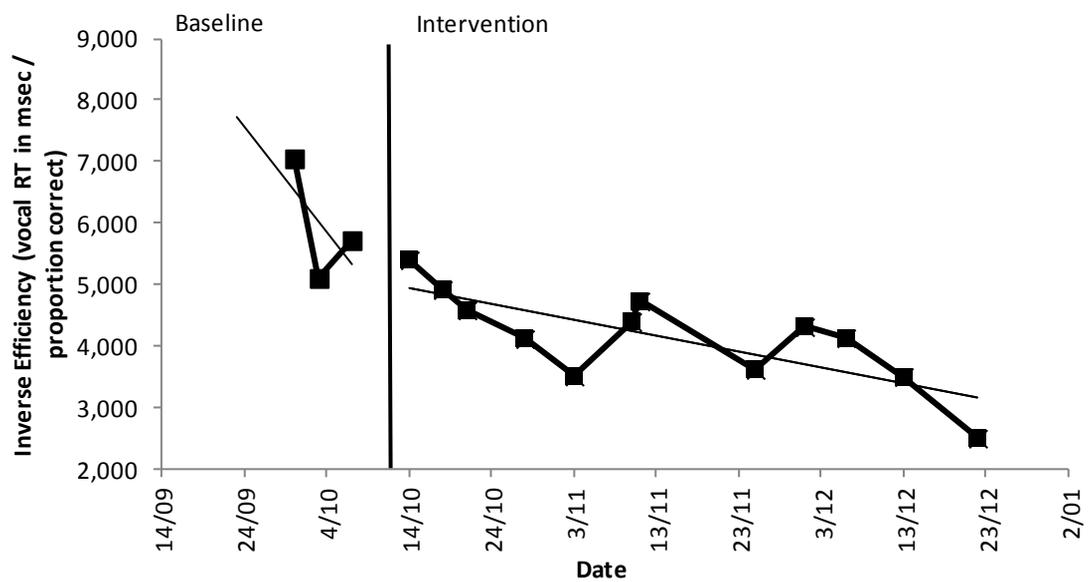


Figure 4b: P2's inverse efficiency for multiplication.

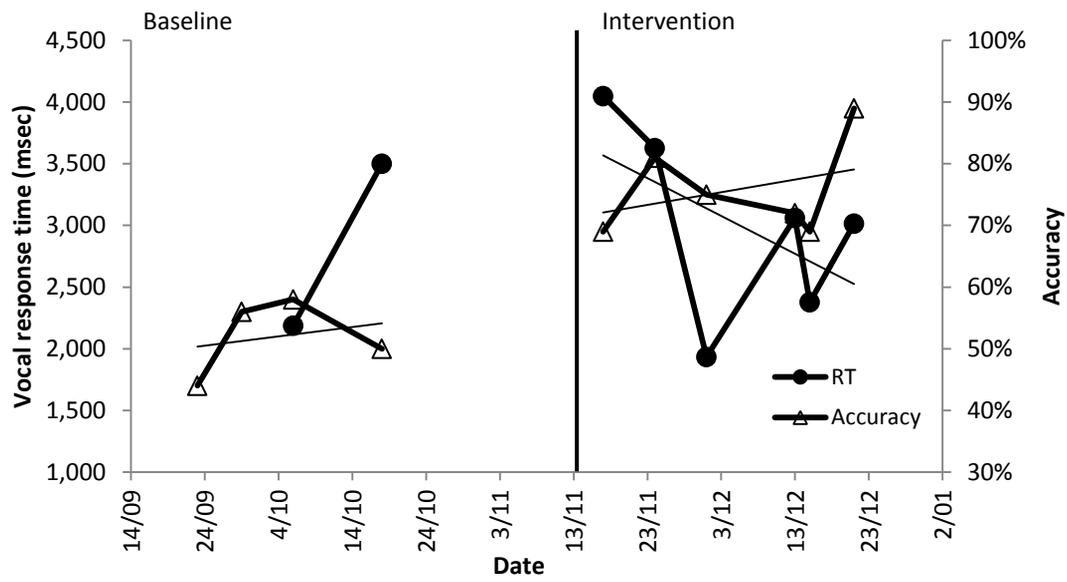


Figure 5a: P3's response time and accuracy for multiplication.

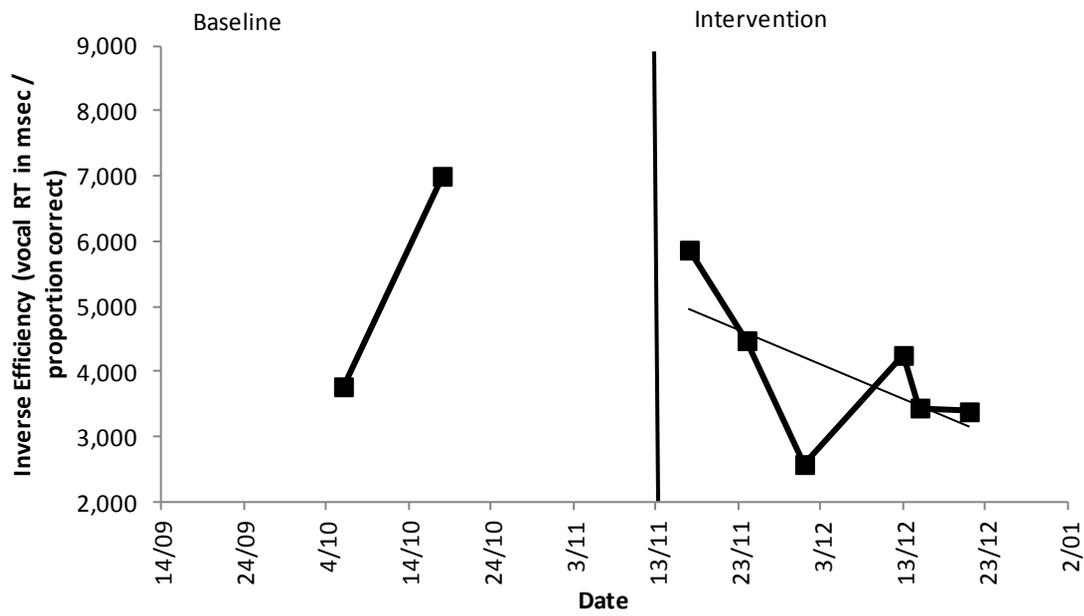


Figure 5b: P3's inverse efficiency for multiplication.

Figure 4b shows P2's inverse efficiency. Consistent with an effect of the intervention, average IE was greater in the baseline (5,959 msec) than the intervention phase (4,292msec). Unexpectedly, the learning rate during the baseline phase (-1,197 msec / week) was in fact higher than that in the intervention phase (-175 msec / week), however this finding should be interpreted with caution, as the learning rate in the baseline was calculated using only three data points, compared to twelve in the intervention phase and is driven mainly by the sharp improvement that was seen between sessions 3 and 4. Furthermore, an improvement of this size was not replicated again in P2's data during either the baseline or intervention phase therefore it is likely that this improvement was due to an intrinsic factor such as effort or motivation as opposed to an actual learning effect otherwise we would have expected to see a similar rate of improvement over subsequent sessions.

P3's data (shown in Figure 5a) is more difficult to interpret. Multiplication accuracy was 52% during the baseline phase, increasing to 76% during the intervention phase (a 24% improvement). However, due to the gap in measurements coinciding with the implementation of intervention, it is more difficult to infer that this was due to the introduction of the software, as opposed to some other event which occurred during the measurement gap. Interpretation is further clouded by the fact that P3's accuracy learning rate was similar in both phases (1.1% per week baseline; 1.5% per week intervention).

P3's vocal RT baseline data had only two measurements, and therefore must be interpreted with even more caution. Average RT during baseline was 2,844 msec, compared to 3,010 msec in the intervention phase. Thus, P3's average RT increased by 167 msec following the introduction of the intervention, although again the gap in measurements makes inference of causality problematic. P3's RT learning rate was not able to be calculated during the baseline phase, however for the intervention phase it was -210 msec / week, indicating there was learning during intervention. The third intervention measurement is an outlying point. P3 was

observed to be more excitable than usual on this occasion, tending to rush to give an answer, although this did not affect accuracy. The improvement trend in RT across the intervention period is remains without this data point.

The inverse efficiency data for P3 is shown in Figure 5b. The average IE during the baseline phase was 5,386 msec, compared to 4,001 msec in the intervention phase, an improvement of 1,385 msec from baseline to intervention. The rate of learning during the intervention phase was -364 msec / week. This shows that once speed / accuracy trade-offs are taken into account, P3 did show an ongoing increase in multiplication performance across the period of the intervention.

Multiplication: Small versus large items.

Given the positive overall results for the multiplication task, analyses were conducted to determine if the intervention had a differential effect on participants' performance on small (product < 36) compared to large (product \geq 36) multiplication problems. The inverse efficiency data was used for these analyses.

Analysis of P1's inverse efficiency data found that P1 showed no significant difference between the rate of learning for large versus small multiplication problems. This indicates that for P1 the effect of the intervention on small versus large problems was similar.

P2's inverse efficiency for small versus large multiplication problems is shown in Figure 6a. For small problems there was little change from an average of 3562 msec in the baseline phase to 2847 msec in the intervention phase, and the rate of learning was faster in the baseline phase (-498 msec / week) than the intervention phase (-35 msec / week). In contrast, for large problems, there was a large improvement in average efficiency from the baseline phase (10593msec) to the intervention phase (7186 msec; a difference of 3407 msec). The baseline data was too variable to calculate a reliable rate of learning, however there was a

clear learning trend in the intervention phase (-842 msec / week). Overall this data shows that P2's improvement in multiplication performance during intervention was predominantly due to improved recall of large multiplication problems.

P3's inverse efficiency for small versus large multiplication problems is shown in Figure 6b. The average IE for large problems during baseline was 10,854 msec, compared to 5,845 msec during the intervention phase, suggesting a strong increase in performance. However this must be interpreted with caution due to having only two measurements at baseline. Due to the latter, the rate of learning prior to the introduction of the intervention was not calculated. During the intervention phase, the learning rate for small problems showed consistent improvement (-527 msec / week. However for large problems, performance was highly variable, and thus the calculated trend (-995 msec / week) may not be reliable.

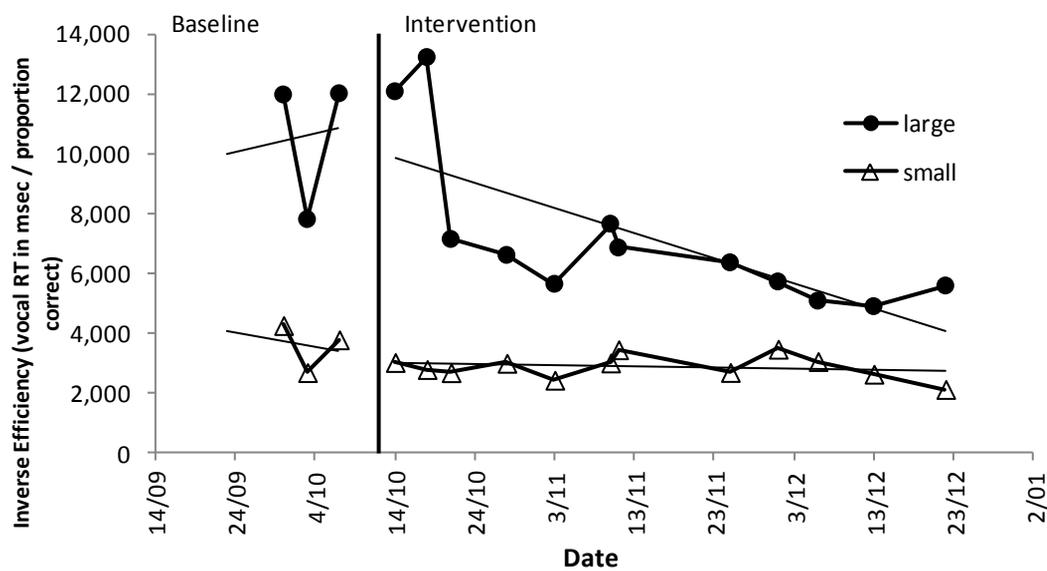


Figure 6a: P2's inverse efficiency for multiplication; Large vs. small items

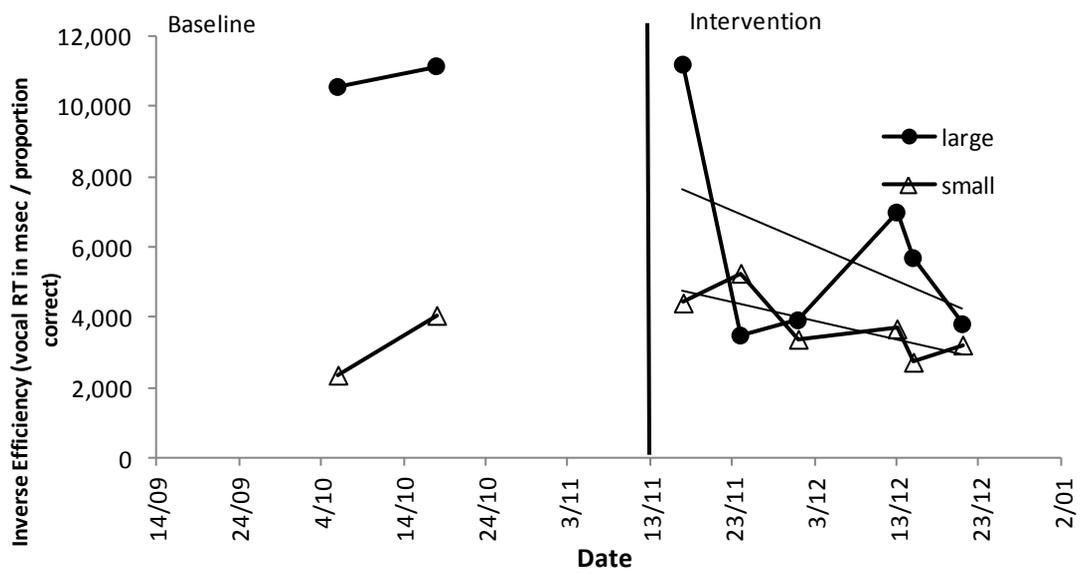


Figure 6b: P3's inverse efficiency for multiplication; Large vs. small items

3.4 Addition results

Addition results for P1 are shown in Figure 7a. During the baseline phase P1's addition accuracy ranged between 79 - 86% with a mean of 81%. Throughout the intervention P1's accuracy improved to between 86 - 96% with a mean of 89%, a small increase of 8% from the baseline phase. The accuracy learning rate during the baseline phase was -7% per week. A negative rate was not expected at baseline, however it appears to be driven by the first accuracy data point. It is possible that this was due to a "novelty effect" with P1 applying more effort and attention to the task the first time she completed it, a theory which is supported by observational data which noted a decline in interest as the intervention progressed. P1's accuracy learning rate was 0.5% per week in the intervention phase, showing a small but steady upward trend. This suggested that for P1 the intervention might have had a small positive effect on addition accuracy as well as on multiplication.

P1's addition vocal RT showed little movement throughout the baseline phase, ranging from 2,128 to 2,242 msec with a mean of 2,179 msec. During the intervention phase, P1 showed much more variability in her response times which ranged from 1,599 msec to 2,762 msec with a mean of 2,224 msec. P1's vocal RT showed a trend to *became slower* during both the baseline and intervention periods (105 vs. 56 msec / week respectively), which suggested that the small improvements seen in accuracy might be due to a speed / accuracy tradeoff.

Examination of P1's inverse efficiency data confirmed that there was indeed a trade-off between speed and accuracy (see Figure 7b). The trendlines for both the baseline phase and the intervention phase had positive slopes (280 versus 35 msec / week respectively). Thus, once weighted by accuracy, P1 still showed an overall slowing in addition response time. This suggests that the slightly increasing accuracy seen during the intervention period may have been achieved by using strategies that were more costly in time (or possibly by being less willing to give up on calculating or recalling an answer).

P2's addition results (see Figures 8a,b) showed a different pattern to those of P1. During the baseline phase P2's addition accuracy was close to ceiling, ranging from 93 to 96% with a mean of 94%. During the intervention phase P2's accuracy ranged from 89 - 100% with a mean of 96%, a small improvement of 2% from the baseline phase. The accuracy learning rate was slightly positive in the baseline phase (1.2% per week, but changed to close to zero in the intervention phase (-0.2% per week).

P2's vocal RT for addition ranged from 3,431 msec to 2,049 msec in the baseline phase, with a mean response time of 2,552 msec. A large improvement was shown during this phase, with a learning rate of -609 msec / week. During the intervention phase, vocal RT ranged from 2,475 to 1,736 msec with an average RT of 2,136 msec. This was an average improvement of 416 msec compared to the baseline phase, however the improvement seen during the baseline phase was still greater than that seen across phases. The RT learning rate was -35 msec / week in the intervention phase, slower than that seen in the baseline phase.

Overall these results suggest that the improvement in addition RT seen in P2 was primarily due to practice on the testing task, rather than to the intervention. This interpretation was supported by the inverse efficiency analyses, which showed that the bulk of the improvement in addition performance occurred during the baseline phase (-679 msec / week) compared to the intervention phase (-35 msec / week).

P3's addition results showed a different pattern again (see Figures 9a, b). P3's accuracy was high at baseline, and very stable, ranging from 93% to 96% with an average of 94%. During the intervention phase P3's accuracy became less consistent, ranging from 86% to 100% with an average accuracy of 95%. This nevertheless represented a slight average improvement of 1% from baseline. P3's accuracy learning rate during the baseline phase was negative (-0.7% / week), compared to a slight positive trend that was shown in the intervention phase (1.3% / week).

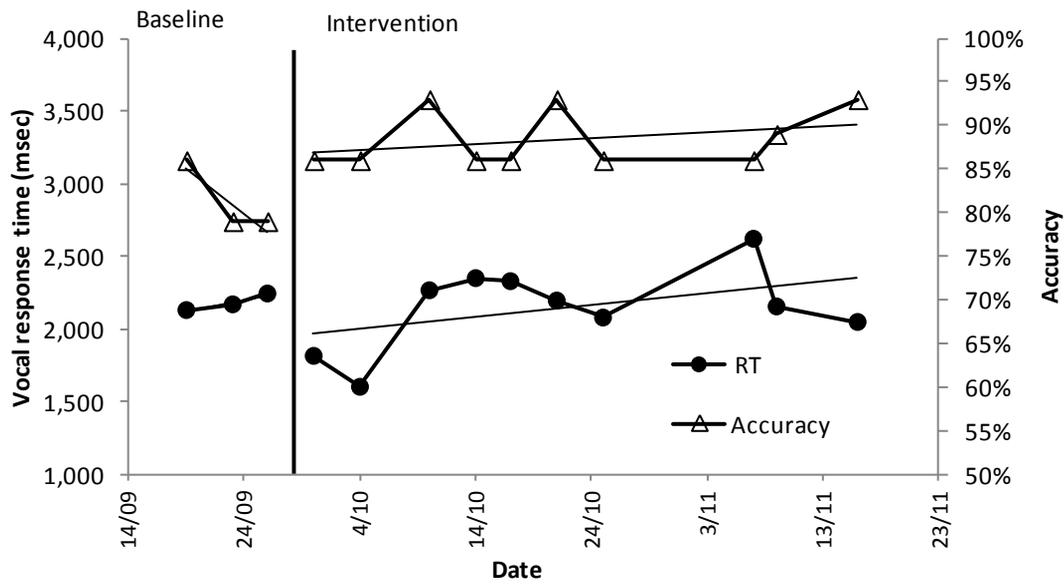


Figure 7a: P1's response time and accuracy for addition

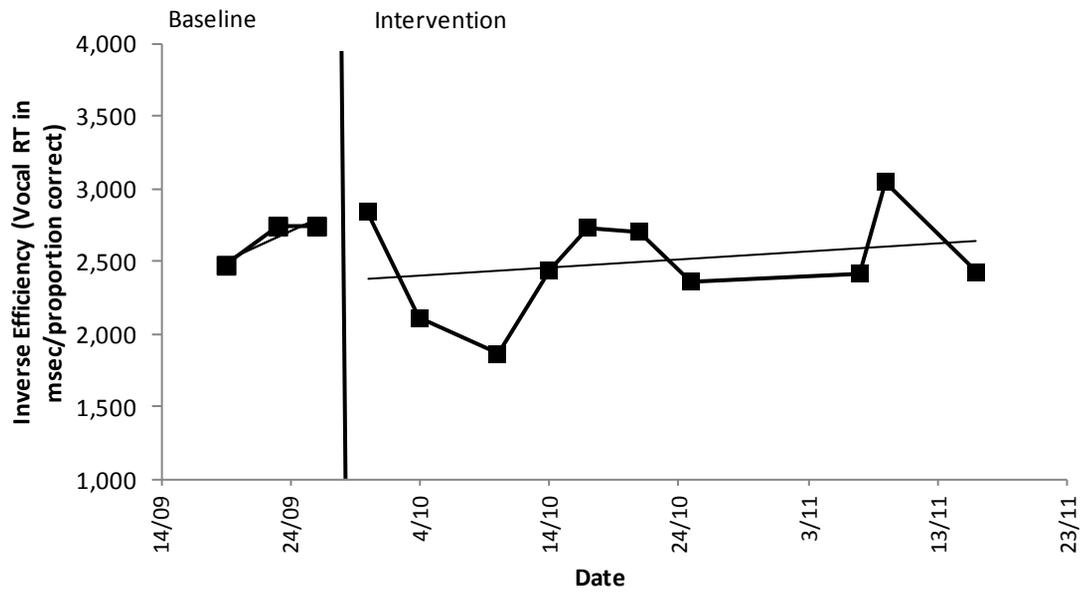


Figure 7b: P1's inverse efficiency for addition.

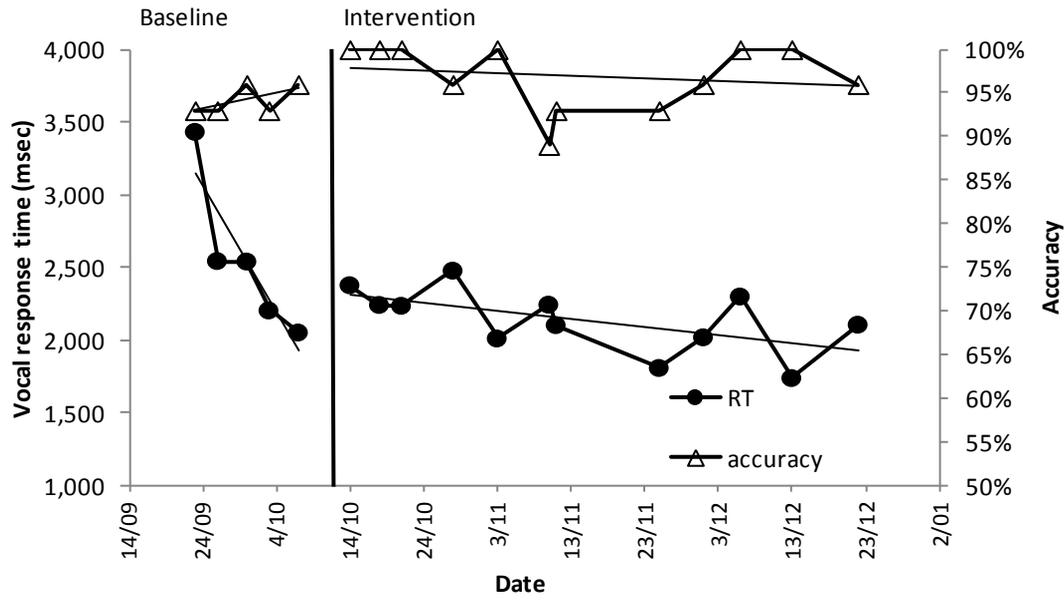


Figure 8a: P2's response time and accuracy for addition

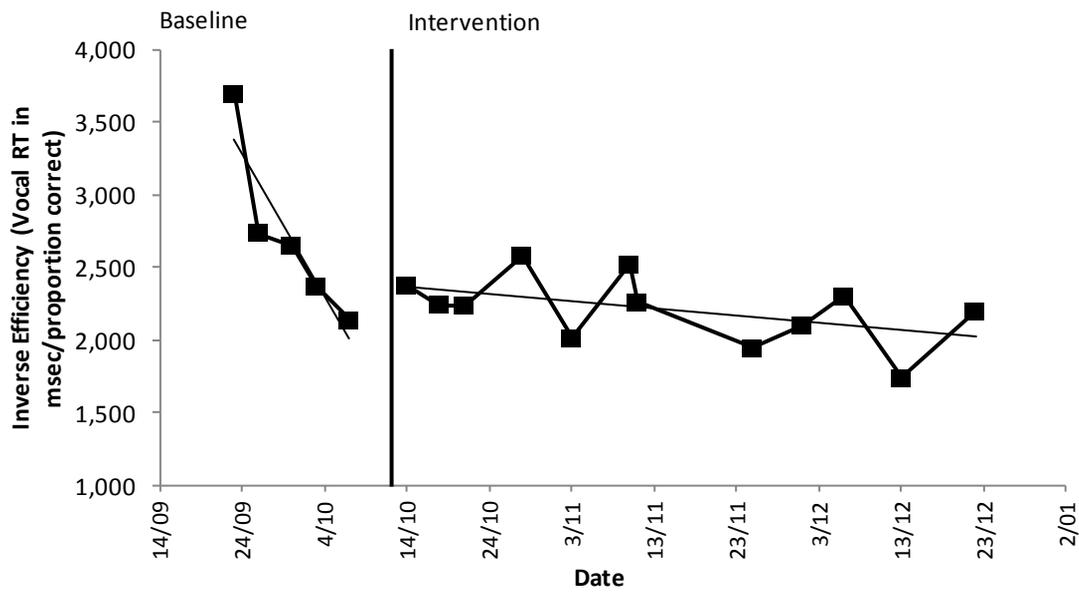


Figure 8b: P2's inverse efficiency for addition.

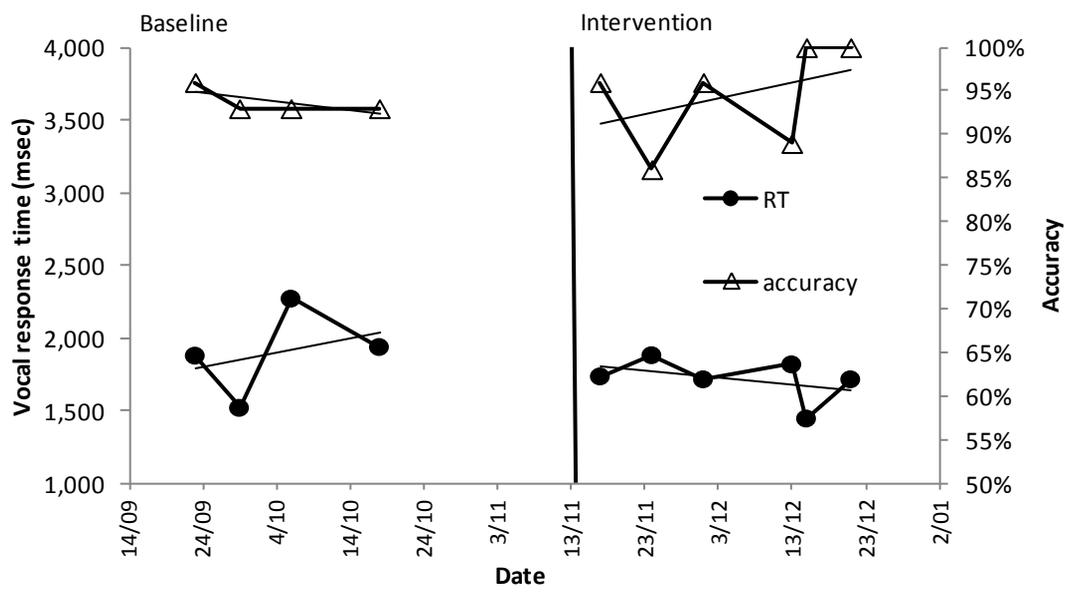


Figure 9a: P3's response time and accuracy for addition

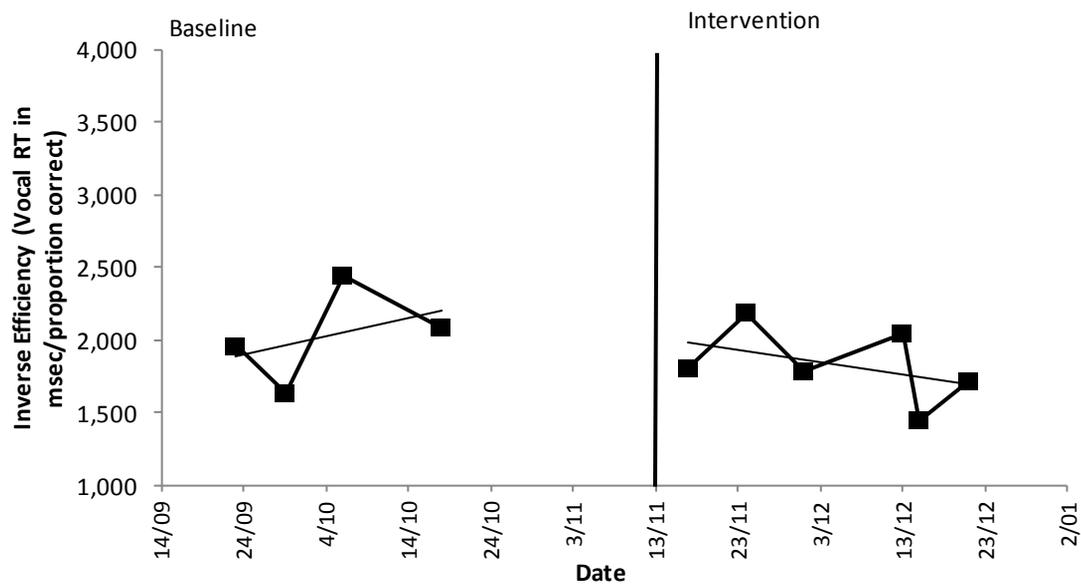


Figure 9b: P3's inverse efficiency for addition.

Conversely, P3's vocal RT for addition was more consistent during the intervention than the baseline phase. During the baseline phase P3's RT ranged from 1,516 to 2,268 msec with an average of 1,899 msec. During the intervention phase P3's RT ranged from 1,879 msec to 1,443 msec with an average response time of 1,716 msec, an improvement of 183 msec from the baseline phase. The RT slope during the baseline phase was 70 msec / week, showing an increase in RT. Contrastingly, the slope was -35 msec / week in the intervention phase, showing a slight decrease in RT over time.

Once these two measures were combined together to give inverse efficiency, however, P3 showed little change over the two periods of the study. The average IE during the baseline phase was 2,026 msec, compared to 1,828 msec in the intervention phase, an improvement of just 198 msec. The IE slope during the baseline phase was positive (84 msec / week), which was unexpected. During this period P3 was essentially becoming slower to respond, without increasing in accuracy. During the intervention phase the slope changed to -56 msec / week, reflecting a slight overall improvement in addition performance during the period of software use. This is consistent with a slight benefit of the intervention on addition as well as multiplication for P3, but must be interpreted with caution due to the unusual baseline pattern, and the gap between baseline and intervention.

3.5 Number Comparison Results

Effects in the number comparison task were expected to emerge only in reaction time, because at this age, accuracy can be expected to be at ceiling (Sekuler & Mierkiewitz, 1977). This was supported in the data which showed that all participants' accuracy rates remained between 89-100% throughout both baseline and intervention phases. For this reason only mean accuracy was compared between baseline and intervention (as opposed to learning rates, which were calculated for vocal RT only).

As can be seen in Figure 10a, P1's reaction time for number comparison ranged from 849 to 948 msec during the baseline phase, but showed a much greater fluctuation (707 - 1,109 msec) during the intervention phase. This fluctuation was sizeable (baseline $R^2 = 0.27$ vs. intervention $R^2 = 0.03$) and accounts for the small change in average reaction time from baseline (888msec) to intervention (873msec) which was only 15 msec. P1's RT learning rate during the baseline phase was 28 msec / week compared to -14 msec / week during the intervention, a very small difference which was far outweighed by the large fluctuation in RT during the intervention period.

P1's accuracy for number comparison changed only very slightly from baseline (96%) to intervention (95%), suggesting there was negligible change in accuracy with the introduction of the intervention. Efficiency analyses were nevertheless conducted (Figure 10b); firstly to verify that effects in RT remained once fluctuations in accuracy were accounted for, but also to see if fluctuations in accuracy could account for some of those seen in RT in the intervention period. The baseline efficiency slope was negligible (21 msec / week). The intervention efficiency slope showed very little learning (-7 msec / week), an change which was still dwarfed by the fluctuation in RT during intervention. Overall therefore, there was no strong evidence of an improvement in number comparison due to the intervention.

P2's number comparison results are shown in Figures 11a and b. P2's reaction times ranged from 775 to 619 msec during baseline (average 669 msec) and from 680 to 502 msec during intervention (average 588 msec). The RT learning rate was -28 msec / week during baseline compared to 2.8 msec / week during intervention. Thus the average improvement in RT of 81 msec from baseline to intervention was primarily due to learning which occurred during baseline, rather than during intervention.

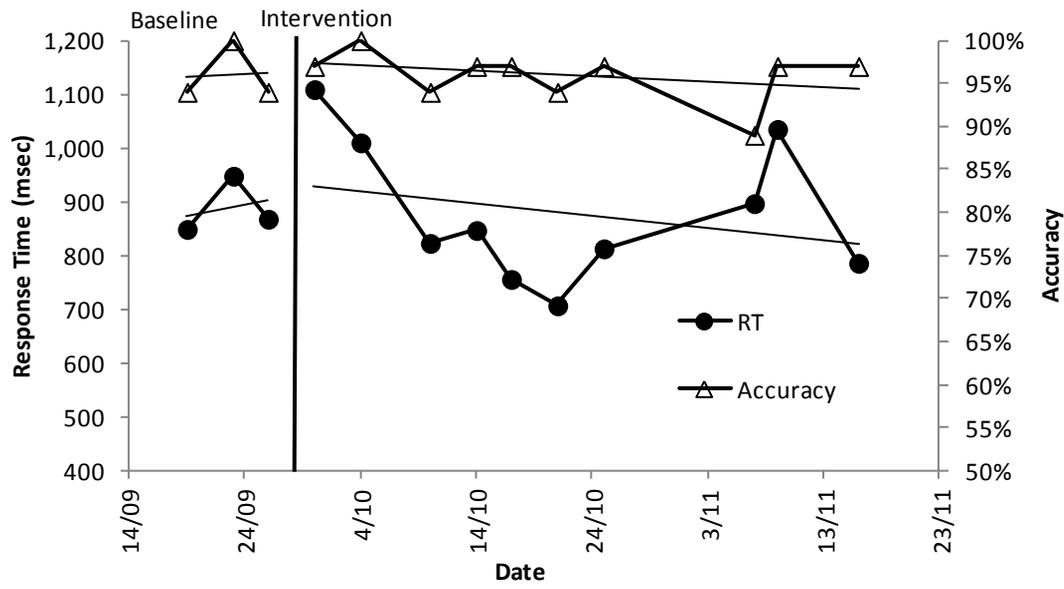


Figure 10a: P1's reaction time and accuracy for number comparison.

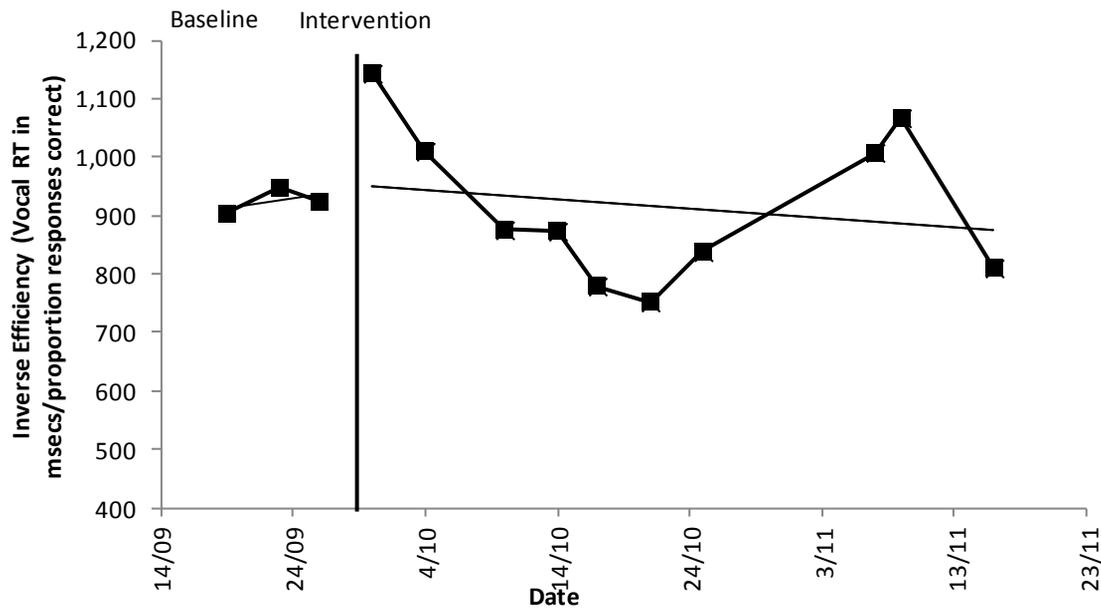


Figure 10b: P1's inverse efficiency for number comparison.

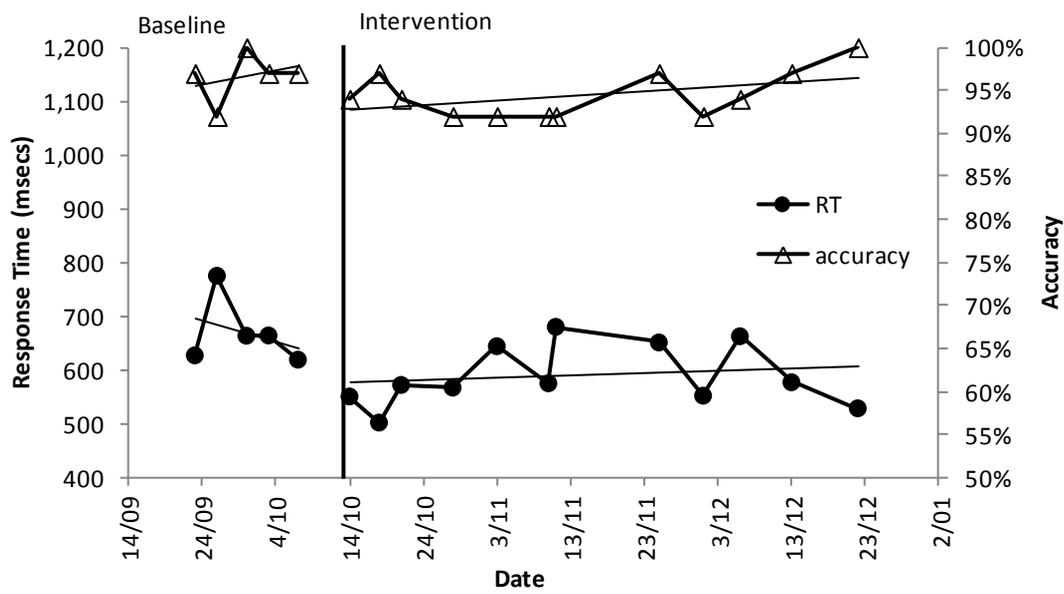


Figure 11a: P2's reaction time and accuracy for number comparison.

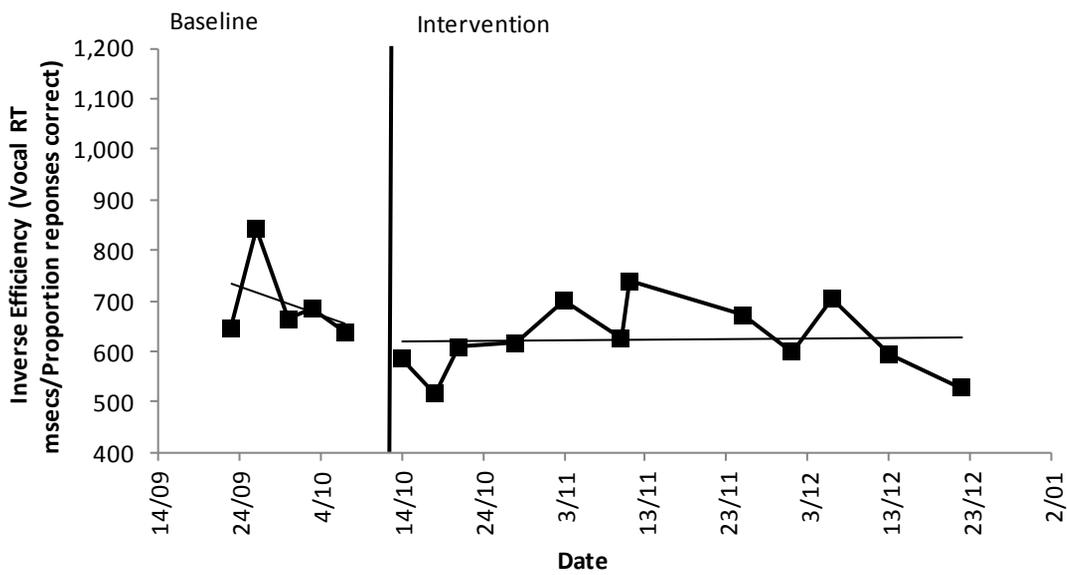


Figure 11b: P2's inverse efficiency for number comparison

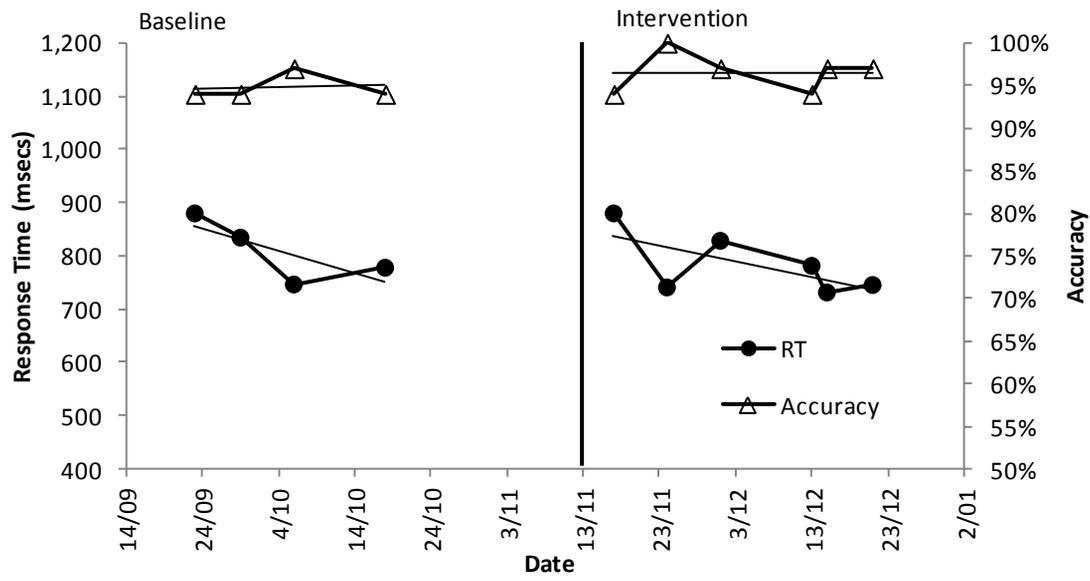


Figure 12a: P3's reaction time and accuracy for number comparison.

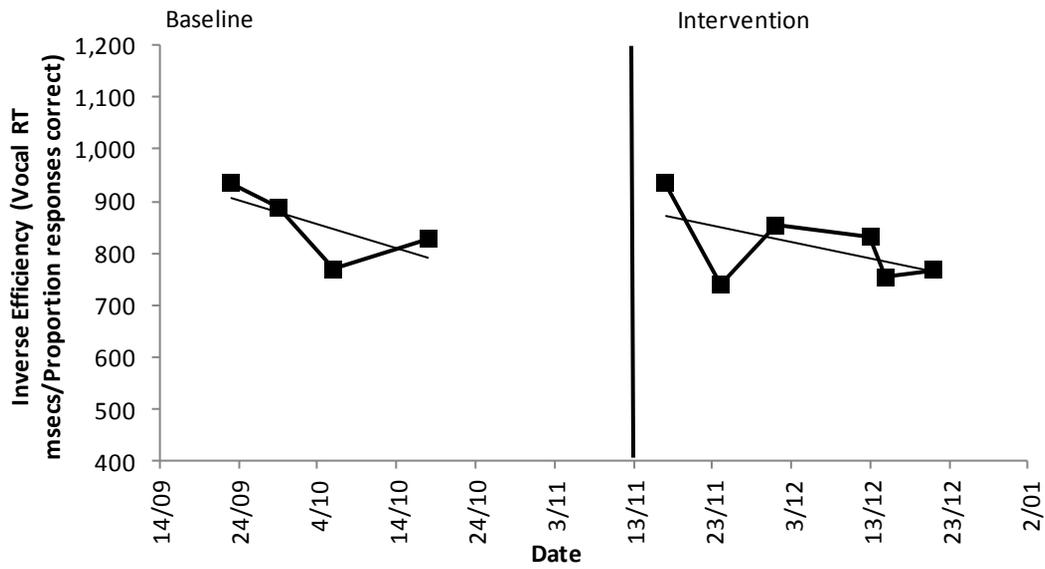


Figure 12b: P3's inverse efficiency for number comparison.

P2's number comparison accuracy was close to ceiling, as expected. Accuracy ranged from 92 to 100% (average 96%) during both the baseline and intervention phases with an average of 96% and 94% respectively. Inverse efficiency analyses confirmed the findings of the reaction time analyses. P2's IE during baseline ranged from 638 msec to 842 msec (average 695 msec) and from 518 msec to 739 msec (average 624 msec) during intervention. P2's IE learning rate during baseline was -35 msec / week, compared to 0.7 msec / week during the intervention phase. Again, therefore, there was little evidence for an effect of the intervention on accuracy.

Figures 12a and b show P3's number comparison results. P3's reaction time during the baseline phase ranged from 746 msec to 880 msec (average 809 msec) and from 731 msec to 880 msec (average 784 msec) during the intervention phase. The RT learning rate during the baseline was -28 msec / week compared to -14 msec / week during the intervention phase.

This showed that a similar rate of learning was occurring during both the baseline and intervention phases, suggesting that the intervention had little effect on number comparison.

P3's average accuracy was close to ceiling during both phases, as expected (Baseline: range 94 - 97%, average 94%; Intervention: range 97 - 100%, average 96%), although there was a slight improvement (2%) in average accuracy. P3's inverse efficiency data also indicated that the intervention had little effect on number comparison, with a baseline IE of -28 msec / week compared to -21 msec / week during intervention.

3.6 Number Comparison: Distance Effect

If an improvement in number sense had occurred due to the intervention, this might be observable in the distance effect rather than in the overall number comparison reaction times. (The distance effect describes the highly replicable observation in children and adults of longer RTs to compare two numbers with a small numerical distance between them, e.g. 5 vs.

6, and shorter RTs when there is a larger numerical distance between them, e.g. 5 vs. 9). The size of this RT difference decreases over childhood as children's mathematical performance improves (Sekuler & Mierkiewitz, 1977). Analyses were therefore conducted to examine whether there was any decrease seen in participants' distance effect associated with the intervention.

The distance effect was calculated for each participant and testing session, by dividing the stimulus set into four equal categories based on the log ratio of the distance for each trial, calculated using the following formula:

$$\text{log ratio} = \left| \log \frac{n}{m} \right|, \text{ where } n < m$$

The log ratio was used because previous research has shown that there is a logarithmic relationship between numerical distance and RT (Dehaene, 2001). The average log ratio for each stimulus category was then used to calculate a linear fit for median RT and accuracy, i.e. to find the slope of the distance effect for each participant. Finally, results were graphed (see Appendix 4) to determine if there was any improvement (reduction) in the distance effect as the intervention progressed. Visual inspection revealed no evidence for any decrease in the size of the distance effect associated with the intervention, therefore these data are not reported further.

4. Discussion

Findings from the current study provided modest initial evidence for the efficacy of “Timez Attack” in improving multiplication fluency in nine to ten year old children. Despite some difficulties with measurement fidelity, reasonably clear and ongoing improvement was seen in both accuracy and reaction time in all three participants following the introduction of the intervention (with the exception of RT for P1 due to missing baseline measurements).

Inverse efficiency analyses confirmed that this pattern could not be explained by speed / accuracy trade-offs. A break-down of the data into small versus large multiplication problems showed that for P2 and P3, the improvement was primarily due to learning large multiplication problems.

A concurrently run study also evaluated the effectiveness of using a CAI as an intervention for teaching math fact fluency with primary aged children. Gross and Duhon (2013), evaluated the effectiveness of a classroom based CAI at improving the accuracy and fluency of addition or multiplication facts with three primary aged children and reported an overall improvement in both fluency and accuracy across participants. Although there were differences in the implementation of the two studies and the measurement procedures differed, the overall conclusions were consistent in that the addition of a CAI as a math intervention was deemed effective at improving the accuracy and fluency of math fact recall.

The first conclusion that can be drawn from this research is that interactive CAI interventions using behavioural interventions can be an effective way of teaching multiplication fluency in 9 and 10 year old children who are struggling to learn their times tables. Prior research has suggested that CAI interventions which include numerous visual graphics and other potentially distracting stimuli may be an ineffective form of teaching, however the current results indicate the opposite. It is likely that the extra components of “Timez Attack” act as reinforcement for engagement with the intervention and that the key

aspect in terms of learning is the fluency component of the CAI which reinforces fast, accurate responses.

A second aim of the current study was to investigate whether the training in multiplication would generalise to other mathematical skills, namely addition and number comparison. It was hypothesized that some generalisation might be seen to addition (since it involves both verbal memory and number sense), but that no generalisation would be seen to number comparison (since it involves no verbal memory, and only number sense).

There appeared to be some generalisation to accuracy in adding two one-digit numbers, with small improvements seen in this (it should be noted that participants were all close to ceiling on addition accuracy before the intervention started and therefore had little room for improvement). However there was no evidence for improvement in addition response time, with conflicting results across participants (from baseline to intervention P1's increased, P2's decreased and P3's remained relatively stable). Inverse efficiency analyses for addition were similarly mixed, and in the cases where improvement was seen (e.g. P2), it occurred at a greater rate prior to the intervention than after. Overall, as expected, there was little change in addition performance for any of the participants with the introduction of the intervention. As with all null results, it is difficult to be sure that this is a lack of effect of intervention, or merely an inadequate test. For instance all three participants' performance on the addition task was already quite accurate prior to the study, and both P1 and P3 already responded relatively fast. P2 showed an improvement in reaction time, but this improvement appeared to be more due to practice on the task itself. Thus it is possible that the addition task was too easy, and that a generalisation effect of training might have been seen with a task that taxed verbal memory more, for instance with a bridging addition task (e.g. $8 + 5 = ?$). Another possible explanation for these results is that due to the ease of the task, the participant's ceiling performance was reached more quickly and as such a limited improvement was found.

Finally, as expected, children's comparison of one-digit numbers showed no changes associated with intervention, neither in response time, accuracy, nor inverse efficiency. This was true for overall reaction time, as well as for the size of the distance effect.

Overall then, it was found that the intervention benefitted multiplication greatly, addition only slightly, and number comparison not at all. This is consistent with what we know from the cognitive neuroscience literature about symbolic and non-symbolic mathematical skills being associated with separate neural networks in the brain. For instance, studies in adults have shown that training on verbal facts does not improve estimation, and vice versa (Spelke & Tsivkin, 2001), and neuro-imaging studies indicate that verbal and non-verbal training of facts activate different areas of the brain (Ischebeck et al. 2007). The results also show that within the verbal mathematical network, training on multiplication fact recall does not benefit training on addition fact recall. Thus we can speculate that the benefits of training may be in strengthening the individual memory traces associated with each fact, rather than recall of verbal facts in general.

The discovery that training effects did not generalise beyond the task trained is in fact a very common one in intervention studies targeting particular cognitive skills (Rabipour & Raz, 2012). Rabipour & Raz (2012) examined the literature and research in the area of "Brain Training" to provide a review of the effectiveness of the interventions marketed to enhance brain cognition, thought process, action and emotions. Many of the interventions available that claim to enhance specific cognitions have sufficient evidence to support these claims, however across the brain training market the evidence that supports the generalisation of these enhancements to other cognitions is either missing or contentious. For example, Rabipour and Raz (2012) highlight that there is no evidence to support suggestions that improvements in one area of cognition will generalise to other areas of cognition as a direct result of the initial intervention. Alternatively, they suggest any generalisations in

improvement are likely to be as a result of shared neural pathways between the targeted cognition and other improved cognitions, as opposed to the intervention itself having a direct impact. This notion is supported in the current study in that improvements in symbolic tasks were not generalised to improvements in non-symbolic tasks, which in this study are hypothesised to be on separate neural pathways.

4.1 Limitations

There are several limitations of the current study. The two most important were unplanned, and emerged as a result of unforeseen events mentioned earlier. The first of these is that the sample size was smaller than planned (ideally the study would have included six or more participants). This restricts generalisability, as the current results have only been replicated in three participants. The second is that the number and timing of measurements in each phase were not ideal. There should have been more baseline measurements, and no increase in measurement interval between baseline and intervention. In the future, more data should be collected with more and regular measurements in order to verify that the effects found do indeed replicate. In particular, the baseline phase should be continued for each participant until a stable baseline is established. This would allow for more conclusive comparisons to be drawn between the baseline and intervention phase, therefore improving the validity and reliability of the study. However, the multiple baseline design across subjects goes some way to counteract this limitation as comparisons can be drawn across subjects, therefore strengthening findings. The same could be said to the intervention phase, which did not show stable data in some instances (particularly for P3).

One aim of the study was to combine methodological strengths of both the cognitive and behavioural approaches to measuring learning. However an unforeseen limitation of the cognitive measures used is that while they have been previously shown to work well for group data (e.g. Wilson et al., 2006), at the individual level they showed quite large

variability between testing sessions. This compounded the measurement issues discussed above and in some cases clouded interpretation of the data. Such a problem could be avoided in the future by piloting the stability of measures adapted from group studies for ABA designs.

Both the current study and Gross and Duhon's (2013) research identified that the inclusion of auditory and visual feedback in the CAI was reported by the participant's as a motivating part of the CAI, however neither study accounted for the measurement of this and as such are unable to comment on the effectiveness. Furthermore, Gross and Duhon (2013) required their subjects to complete pen and paper tasks, whereas the participants in the current study completed their measurements using a response box with keys. It was identified by Gross and Duhon (2013) that the rehearsal on the computer may not generalise to pen and paper work therefore the measurements may not have captured the students true improvement rates, whereas in the current study, it is unknown if any of the progress made on the intervention will generalise to pen and paper work in the classroom. Gross and Duhon (2013) also acknowledged the limitations of their study at accounting for participant's proficiency with both pen and paper, and number keypad's as both may have impacted on their performance, this is also a flaw in the current study.

A final limitation is that it is not possible to prove a null hypothesis in a behavioural experiment; i.e. that variable A will have no effect on variable B. In this study the null hypothesis was that Timez Attack would have no effect on the participants' fluency levels on number sense or addition tasks. In the case of the addition task, whilst the results were consistent with this, we cannot rule out alternative explanations, such as ceiling effects in the addition task. In the case of the number comparison task, we know from the developmental literature that children will improve in speed as they get older, at least in group studies (Sekuler & Mierkiewitz, 1977). However individual variations in day-to-day performance

over the short time of the current study may have made the power to detect an effect quite low.

4.2 Future directions

Future research in this area would benefit from having larger sample sizes, more participants and longer baseline phases in order to create more reliable and robust data. In addition, a double-dissociation study would help support the current findings and render the null effect of the intervention on addition and number sense interpretable. For example, if an experiment was conducted that trained participants using a number sense intervention and improvements were found in number sense and addition but not multiplication, this finding would support the hypothesis that symbolic and non-symbolic arithmetic tasks draw on separate neural pathways, therefore supporting the current tentative conclusion that improvements in symbolic arithmetic do not lead to improvements in non-symbolic number sense.

Following on from the current study and the work of Gross and Duhon (2013), it would be useful to conduct a study which evaluated the effectiveness of both visual and auditory feedback, separate from CAI's in order to determine whether CAI's, auditory feedback or visual feedback are effective at improving math fact fluency either separately or in combination with one another.

Lastly, another possibility for improving future research in this area is to conduct a similar experiment with participants who meet criteria for developmental dyscalculia, as opposed to students with low mathematical achievement, who could only be considered "at risk" for dyscalculia. Such an experiment would identify for example whether children with dyscalculia respond to intervention in similar ways to those with low mathematical achievement, or whether different approaches may be required for teaching learners in the two groups.

4.3 Conclusion

Using a multiple baseline across subjects design, the fluency of multiplication fact recall was measured in three participants at risk for dyscalculia, both before and during use of the software “Timez Attack”, in order to determine the effectiveness of this computer-assisted instruction (CAI) intervention. As predicted, the fluency rates of all three participants’ multiplication fact recall improved with the introduction of the CAI, indicating that “Timez Attack” is an effective means of improving fluency in this population. Furthermore, these improvements were then compared against the participants’ fluency rates for number comparison and addition tasks which had been measured over the same time period. Results from this comparison showed that as multiplication fluency increased, there were no significant improvements in the rates of fluency for any of the participants across either number comparison or addition tasks. This lack of generalisation of improvement from multiplication to number comparison and addition tasks supports cognitive neuroscience findings that symbolic and non-symbolic arithmetic tasks are associated with separate neural pathways in the brain and therefore operate independently and are not influenced by one another. It is also consistent with the finding that cognitive training often has very specific effects which do not generalise (Rabipour & Raz, 2012).

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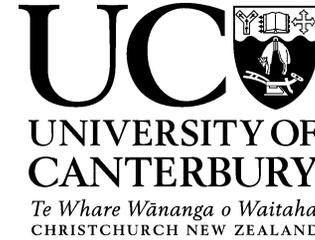
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6. Appendices

6.1 Appendix 1 – Consent and Information Forms

6.1.1 Appendix 1a – Child Information Sheet

Ph: +64 3 341 6556
Email: brm64@uclive.ac.nz
5a Kawaka St
Riccarton
Christchurch



Using a computer game to learn times tables

Information Sheet for Children

My name is Brinley McIntosh and I am doing a project at the University. If you agree, I am going to work with you and your parents to help you with your times tables.

First of all I will come to your house and do some math, reading, and thinking activities with you.

I will show you how to play a computer game that will help you learn your times tables. Then your parents will ask you to play it five times a week for half an hour I will also visit you two or three times a week and ask you to do some math exercises on my computer, over a period of six weeks. These exercises will be quick and will help me see how you're doing.

The information I collect from your tests will be used together with information from other childrens' tests, for a research project I am doing as part of my University work. When I have finished my project, I will type it up and give a copy to the University. Also, a copy will go in the University library for other students to look at if they are interested. If my project is really good it might be published in a journal for other researchers to read.

You will be given a code name so that no-one will know your name, your parents' names or the name of your school. Information about you will only be used by me and my boss, and will always be kept in a locked cabinet or office, so no one can take it. After five years, we will destroy it. Your parents will be given a small report of the project if we want to see it.

If the game or exercises ever make you feel worried about anything, you can tell me or your parents about it.

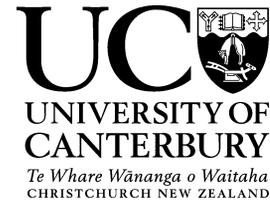
Your parents have also been asked to help. If you have any questions, you can talk to your parents or me. If you want to, you are allowed to change your mind about doing this at any time and that's fine, no one will be upset with you. All you have to do is to tell your parents or me.

Thank you for reading this, and I hope you would like to help with my project!

Brinley McIntosh

6.1.2 Appendix 1b – Parent Information Sheet

Telephone: +64 3 341 6556
Email: brm64@uclive.ac.nz



Using a computer game to learn times tables

Information Sheet for Parents/Guardians

I am a fifth year Child and Family Psychology student and I am currently carrying out research as part of my Master's degree. My research is looking into using a computer game to improve learning of times tables for children who are having difficulties with this.

I would like to invite you and your child to participate in this research. Participation would first involve me visiting your home for 1½ hours to test how your child is doing in math, reading, attention and general intellectual ability. I will give you a copy of the results from these tests. If your child is eligible, he/she would be asked to play an interactive, computer game called "Timez Attack", five times a week for half an hour at a time, for a period of four weeks. "Timez Attack" helps to teach and practice times table skills, and is fun to play.

As well as this I would also need to visit your home 2-3 times a week to carry out a 15 minute computerized test with your child, as measurement of their progress. These measurements are entertaining for children and would be needed over a period of six weeks altogether (while your child is playing the game, and just before and afterwards).

Participation in this research is completely voluntary. If you decide to agree for your child to participate, you have the right to withdraw your child from the study at any time without penalty. If you withdraw your child I will do my best to remove any information relating to you or your child, providing this is practically achievable.

Participation in this study requires you to have access to a PC computer or laptop with internet access. Minimum system requirements are: Operating system XP, Vista or Windows 7, processing speed 1.0 GHz, RAM 256 MB.

All information and data collected throughout the study will be available only to myself and my supervisor. The confidentiality and anonymity of you and your child will be assured by using numbers instead of names, and all information will be stored in a locked office or filing cabinet. The data from this project will be destroyed after five years.

There is a small risk that doing the tests in the study could make your child feel anxious. If this occurs we will stop the study, have a discussion with you and your child about how your child is feeling, and let you both decide whether you would like to continue. There is also a risk that the tests at the beginning of the study might suggest that your child could have difficulties in areas other than mathematics (e.g. reading, attention or general intellectual ability). If this does occur, we will suggest somewhere where you could go for more in-depth testing.

The results from the data collected from your child will be used in conjunction with data from other participants to answer my research questions. These results will be reported as part of my Master's Thesis which will be submitted to the University of Canterbury as a piece of academic work to be marked. A copy of my thesis will be made available to the university library and could potentially be published in academic journals. None of these reports will identify yourself or your child.

As participants, a copy of the finished study or a report on the findings can be made available to you or your child on request.

I encourage you to make contact with myself or my supervisor Dr Anna Wilson if you have questions about the study at any stage.

This study has been granted ethical approval by the University of Canterbury Educational Research Human Ethics Committee and any complaints should be addressed to The Chair, Educational Research Human Ethics Committee, University of Canterbury, Private Bag 4800, Christchurch or human-ethics@canterbury.ac.nz

If you are interested in participating in this study, please complete the enclosed consent form and give it to myself at the beginning of my first visit.

Brinley McIntosh

5a Kawaka St
Riccarton
Christchurch 8140

Research Supervisor:

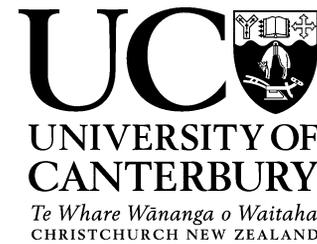
Dr Anna Wilson

College of Education
University of Canterbury
Private Bag 4800
Christchurch 8140

Email: anna.wilson@canterbury.ac.nz
Ph: +64 021 059-7036

6.1.3 Appendix 1c – Child Consent Form

Ph: +64 3 341 6556
Email: brm64@uclive.ac.nz
5a Kawaka St
Riccarton
Christchurch



Using a computer game to learn times tables

Consent Form for Children

My parent has told me about your project.

I am happy for you to help me with my times tables.

I understand that this means you will come to my house and give me some tests. After that you will give me a fun computer game to play, which will help me with my times tables. I will play this game for half an hour only, five times a week, for a month altogether. I will also do some quick computer exercises two-three times per week during that time.

I know that any information about me will not be told to anyone else and will be stored in a locked cabinet of office.

I understand that no-one will know my name, my parents' names or the name of my school, and that information about me will always be kept in a locked cabinet or office, so no one can take it.

I know that if anything makes me feel worried, I can tell you or my parents about it.

I understand that I can change my mind about being in this project anytime and no-one will mind.

I know that if I have any questions I can ask my parents or Brinley.

Child's name: _____

Signed by child (or on behalf of child): _____

Date: _____

6.1.4 Appendix 1d – Parent Consent Form

Ph: +64 3 341 6556
Email: brm64@uclive.ac.nz
5a Kawaka St
Riccarton
Christchurch



Using a computer game to learn times tables

Consent Form for Parents and Guardians

I have been given a full explanation of this research project and have been given an opportunity to ask questions.

I understand what will be required of me and my child if I agree to take part in this project.

I understand that my participation is voluntary and that I or my child may choose to withdraw at any stage without penalty.

I understand that any information I or my child provide will be kept confidential to the researcher and her supervisor and that any published or reported results will not identify me or my child.

I understand that all data collected for this study will be kept in locked and secure facilities and will be destroyed after five years.

I understand that there is a small risk that the tests in the study might make my child feel anxious, or might reveal difficulties in areas other than mathematics, and that if this occurs the researchers are happy to advise me on a course of action.

I understand that if I require further information, or that if I would like a copy of the findings of the study, I can contact either the researcher, Brinley McIntosh, or her supervisor, Dr Anna Wilson.

If I have any complaints I can contact the Chair of the University of Canterbury Educational Research Human Ethics Committee.

By signing below I agree for my child to participate in this research project.

Name: _____

Date: _____

Signature: _____

Email address: _____

6.2 Appendix 2 – Screening Interview

Screening Interview

- Name:
- Gender:
- Handedness:
- Age:
- School Year:

“What is your occupation?”

Mum:

Dad:

“What is your highest education level?”

Mum:

Dad:

“Has (child’s name) experienced difficulties with maths or reading either in the past or currently?”

Yes No

If yes, ask either maths and/or reading and past and/or present.

“Has (child’s name) ever had any assessments around these difficulties?”

Yes No

If yes, what types of assessments and what were the results?

“Has (child’s name) ever received extra help or tutoring for these difficulties or their school work in general (eg, SPELD, Reading Recovery, RTLB, tutoring)?”

Yes No

If yes please specify

Does (child’s name) experience any anxiety around doing maths?

“Is English (child’s name) first language?”

Yes No

If No, what is, and when did (child’s name) first start speaking English?

“Has (child’s name) ever had any problems with:

- Time (telling the time/thinking about days or months)?
- Understanding what people are saying?
- Motor coordination (riding a bike/gymnastics/balancing)?
- Spelling?
- Space (reading maps/following directions)?”

If Yes, specify which:

The following are a list of general medical questions:

“Do/Have any of the following medical conditions apply/applied to (child’s name)?”

- had a head injury?
- Premature birth?
- Seizures?
- Other neurological disorder?
- Normal vision/hearing?
- History of major illnesses?
- Genetic disorder?

Specify where appropriate

Has (child’s name) ever received or taken medication for a neurological or psychological disorder?

Was (child’s name) born close to his/her due date?

What was their birth weight?

Does/has (child’s name) ever experienced seizures or are they epileptic?

6.3 Appendix 3 – Stimulus Sets

Table A1: Stimulus Sets for multiplication, addition and number comparison tasks

Multiplication			Addition			Number Comparison	
Item	Answer	Size	Item	Answer	Size	Item	Answer
2 x 4	8	S	6 + 5	11	L	1 vs. 2	2
2 x 5	10	S	7 + 5	12	L	2 vs. 3	3
2 x 6	12	S	7 + 6	13	L	3 vs. 4	4
2 x 7	14	S	8 + 5	13	L	4 vs. 5	5
2 x 8	16	S	9 + 5	14	L	5 vs. 6	6
2 x 9	18	S	8 + 6	14	L	6 vs. 7	7
2 x 12	24	S	8 + 7	15	L	7 vs. 8	8
3 x 4	12	S	5 + 5	10	S	8 vs. 9	9
3 x 5	15	S	9 + 6	15	L	1 vs. 3	3
3 x 6	18	S	3 + 2	5	S	2 vs. 4	4
3 x 7	21	S	4 + 2	6	S	3 vs. 5	5
3 x 8	24	S	4 + 3	7	S	4 vs. 6	6
3 x 9	27	S	5 + 2	7	S	5 vs. 7	7
3 x 12	36	L	6 + 2	8	S	6 vs. 8	8
4 x 5	20	S	5 + 3	8	S	7 vs. 9	9
4 x 6	24	S	6 + 3	9	S	1 vs. 4	4
4 x 7	28	S	5 + 4	9	S	2 vs. 5	5
4 x 8	32	S	2 + 1	3	S	3 vs. 6	6
4 x 9	36	L	3 + 0	3	S	4 vs. 7	7
4 x 12	48	L	3 + 1	4	S	5 vs. 8	8
5 x 6	30	S	4 + 0	4	S	6 vs. 9	9
5 x 7	35	S	7 + 0	7	S	1 vs. 5	5
5 x 8	40	L	6 + 1	7	S	2 vs. 6	6
5 x 9	45	L	8 + 0	8	S	3 vs. 7	7
5 x 12	60	L	6 + 6	12	L	4 vs. 8	8
6 x 7	42	L	7 + 7	14	L	5 vs. 9	9
6 x 8	48	L	8 + 8	16	L	1 vs. 6	6
6 x 9	54	L	9 + 9	18	L	2 vs. 7	7
6 x 12	72	L	2 + 2	4	S	3 vs. 8	8
7 x 8	56	L	3 + 3	6	S	4 vs. 9	9
7 x 9	63	L	4 + 4	8	S	1 vs. 7	7
7 x 12	84	L				2 vs. 8	8
8 x 9	72	L				3 vs. 9	9
8 x 12	96	L				1 vs. 8	8
9 x 12	108	L				2 vs. 9	9
11 x 12	122	L				1 vs. 9	9

6.4 Appendix 4: Numerical comparison distance effect

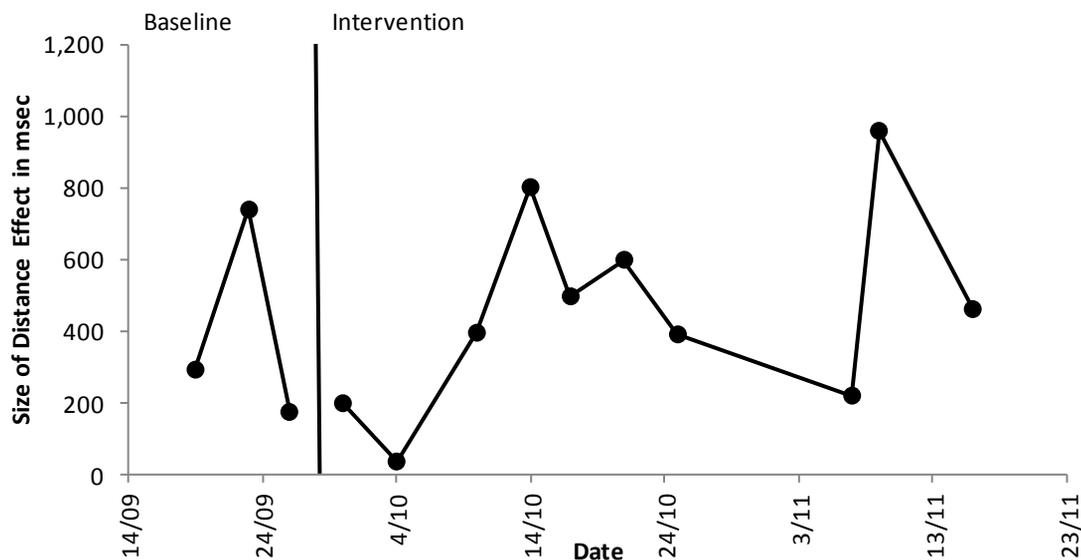


Figure 13a: P1's numerical comparison distance effect across baseline and intervention periods

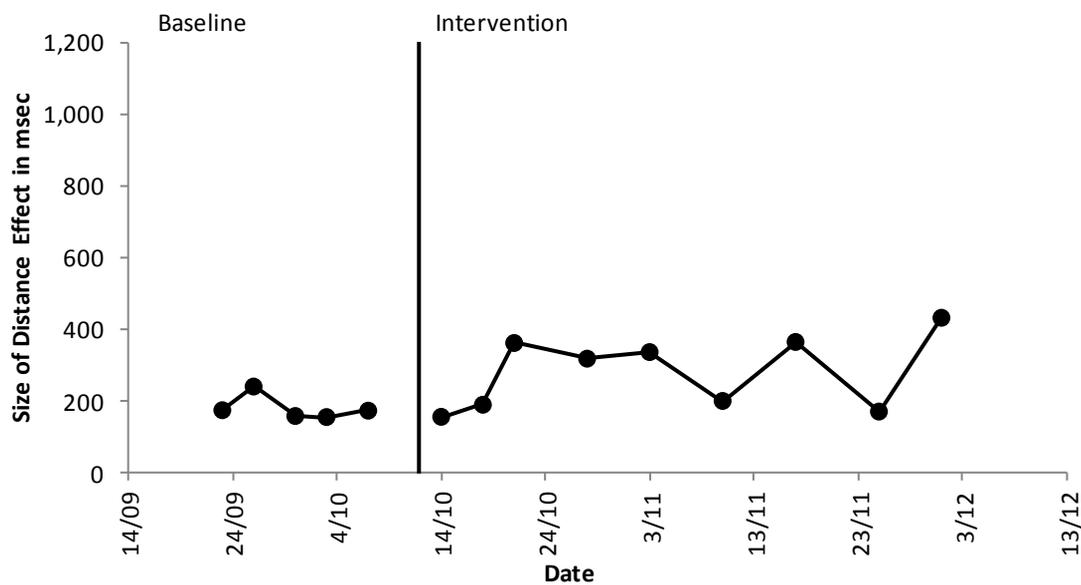


Figure 13b: P2's numerical comparison distance effect across baseline and intervention periods

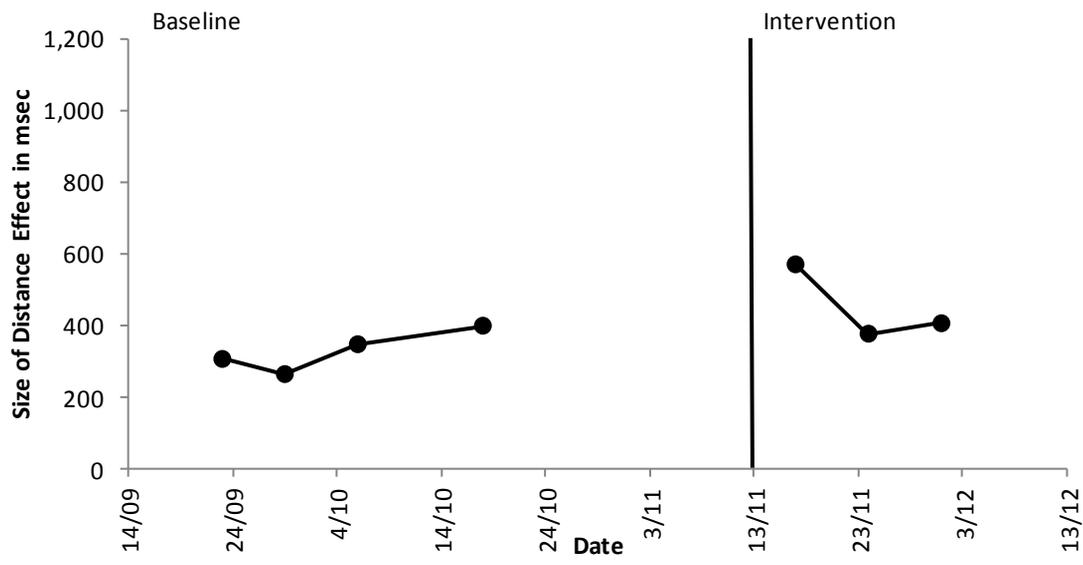


Figure 13c: P3's numerical comparison distance effect across baseline and intervention periods