THE FIRST THREE CHAPTERS OF THE *GRAHALĀGHAVA* OF GAṆEŚA DAIVAJÑA: EDITION, TRANSLATION, AND MATHEMATICAL AND HISTORICAL ANALYSIS

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A thesis
submitted in partial fulfilment
of the requirements for the degree
of
Master of Philosophy
in
Mathematics

Department of Mathematics and Statistics
University of Canterbury
New Zealand
2022

Abstract

The field of history of mathematics allows mathematicians the ability to trace patterns of mathematical development throughout human history, and often reconnect them to the inspirations and motivations that led to mathematical progress [War10, p. vii]. The narrative of mathematical development in India, specifically the second millennium Indian exact sciences, has only been a topic of close study in recent years [Plo09a, p.280]. This classical era of mathematical astronomy in India featured a diverse expanse of calculatory works devoted to predictive astronomy, or finding the timings, locations, and appearances of celestial phenomena. The choice of parameters for time divisions and number of cyclic planetary revolutions for these calculations primarily differentiated the different schools during this era.

One such school, the *Gaṇeśapakṣa*, was founded by Gaṇeśa Daivajña (b. 1507 CE) of Nandigrāma, India with the composition of his text, the *Grahalāghava* ("brevity [in calculations] of the planets") (1520 CE). The text featured innovative modifications to the previously established time divisions and mean longitude calculations, as well as the ambitious removal of trigonometric computations in all stated formulas. These parameters and procedures inspired a sizable proliferation of commentaries and astronomical tables through much of the second half of the second millennium, with many extant manuscripts and much relevance to astrology in India today.

In this thesis, we use critical editions and manuscripts of the *Grahalāghava* and earlier astronomical works to study the first three chapters of the *Grahalāghava* with the aim of exploring Gaṇeśa's trigonometry-approximating techniques and tracing potential influences of earlier works to contextualize the *Grahalāghava* in the larger tradition.

Acknowledgments

श्रीगुरुभ्यो नमः ।

I am forever indebted to Clemency Montelle and Kim Plofker for treating me to as close to a "*gurukula*" immersive educational environment I could get. That first trip to Schenectady to visit Kim, kicked off one of the most fruitful and challenging journeys in my life of yet. We sat in your office to discuss more about this niche field of history of exact sciences in India that somehow was an amalgamation of my Sanskrit, linguistic, and mathematical interests. That first Skype call to Clemency during which we read parts of *Jyācāpa*, was so filled with enthusiasm and energy that I was hooked– the same infectious passion that fueled me throughout this thesis journey. I did not quite understand the power of the Te Reo Māori word *whānau* until you both had come to receive me, bags and all, from Christchurch Airport. The entire Montelle/Williams clan welcomed me with open arms and will always hold a special place in my heart.

Over the course of this thesis journey, I have experienced this sense of community and comfort in every environment. Thank you dearly to my "academic brothers" AJ, Bhante, and Adi for all the insight and guidance throughout the way. Thank you to Ram Anna for showing me how much fun a one-week bootcamp in Sanskrit mathematical astronomy could be, and thank you to Glen and Kailyn for fascinating meal-time and hike chats about the history of mathematics outside of Sanskrit sources.

The Erskine building was filled with people I admire and care deeply for. I pestered Paul, Allen, Steve, and Kiel constantly about the inevitable and never-ending technology issues that plagued me even today. Leigh, Lucia, Matt, and Helen always made sure that all the administrative details and logistics were taken care of. Postgraduate coordinators Phil and Jeanette, and previously Daniel, as well as the other members of the Erskine Breakfast Club provided me with invaluable advice and numerous laughs over coffee. Especially Chris who so enthusiastically shouted me down to grab available Timtams in the tea room.

Outside of Erskine, the UC library Interloans team and the EPS Library team, especially Joanna Frampton and Fiona, somehow always found and promptly scanned the arcane articles I needed to consult. Thank you to the Rutherford Fellowship and the UC College of Engineering for funding my research and making this thesis and the start of my academic journey possible.

I would like to express immense gratitude to everyone I've had the pleasure of calling friend and even those who evolved into my Christchurch *kuṭumbam* these years. Surviving during a global pandemic was no easy task, and somehow you all made it tolerable and at times extremely joyful!

And finally, my heartiest thanks to Amma, Appa, and Surya for all your love and support and faith in me. I love you.

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Chapter 1

Introduction

1.1 Overview on the formation of classical Sanskrit astronomy

In every recorded civilization of antiquity, mathematics originated as a practical tool predominately for administrative duties such as tax collection and trade, as well as measurement and architectural design [Kat09, p.2]. Naturally, as people noticed patterns in their natural world, such as seasonally variable lengths of daylight, calendar-making as the earliest form of astronomy soon joined the numerous applications of the field. Curious scholars quickly abstracted mathematics from its pure utilitarian contexts to become a discipline worthy of study in its own right. Rudimentary calendrics too evolved to include observations and predictions of celestial phenomena using mathematical models, and thus mathematical astronomy was born [Neu75, p.1].

The well-established classical Sanskrit astronomy tradition of the second millennium traces much of its foundations back to the ritualistic and philosophical Vedas. Also referred to as *śrūti* ("that which was heard [in the beginning by holy sages]"), the orally transmitted Vedas (*Ṛgveda*, *Yajurveda*, *Sāmaveda*, *Atharvaveda*) had four subdivisions: *Saṃhitās* (benedictions), *Āraṇyakas* (texts on rituals and practices), *Brāhmaṇas* (commentaries on rituals and practices, and *Upaniṣads* [Bha06]. Astronomical knowledge lay primarily in cosmology and time-keeping, with the focus on the position and motions of the Sun (with importance placed on its path, the ecliptic) and Moon. Crucial astronomical innovations found in the Vedas include: nomenclature of seasons and lunar/solar months, the *amānta*- (starting with new moon) and *pūrṇimānta*- (starting with full moon) synodic month reckoning systems, the 15 *tithis* (lunar days) that comprised each *pakṣa* (half of a lunar month), intercalation, the days of the 6 (and later 7) day week, and the 27/28 *nakṣatras* (constellations) that make up the Moon's path.[1]

Scholars identified prerequisite lines of inquiry to fully understand the Vedas called the *Vedāṅgas* ("components of the *Vedas*"): *śikṣā* ("teaching [of phonetics]"), *chandas* ("meter"), *vyākaraṇa* ("grammar"), *nirukta* ("explained [etymology]"), *jyotiṣa* ("astronomy"), and *kalpa* ("ritual [practice]"). Due to the importance of speech and articulation in the Sanskrit oral tradition[2] four out of the six components were focused on perfecting scholars' command of Sanskrit [Yan87, p.51]. The two components *jyotiṣa* and *kalpa* involved the reckoning of planetary configurations and ritual practices to be performed respectively. These ritual practices and sacrifices were to be performed at certain auspicious times, with fire altars of certain geometric configurations, and are thus some of the earliest detailed examples of mathematical thought in the Sanskrit tradition [Plo09b, p. 17] [Pin81b, p. 3]. Large time period divisions retained in the later classical Sanskrit astronomy were also products of this post-Vedic era. For instance, a *kalpa* was defined as a single day in

[1]See for example [SS85, p.xx-xxi] [Shu87] [DS62] [Plo18] [Kak98] [Oha93] for more comprehensive discussions of Vedic astronomical knowledge.

[2]So much so that speech was personified to be the goddess Vāc, sometimes identified with the goddess Sarasvatī, and the chief teacher of the gods *Bṛhaspati* became known as Vācaspati .

the life of *Brāhma*, the Creator; equivalent to 4,320,000,000 Earth years. A *kalpa* was made up of 1,000 *mahāyugas*, each comprising the four *yugas*: *Kṛtayuga* (1,728,000 Earth years), *Tretāyuga* (1,296,000 Earth years), *Dvāparayuga* (864,000 Earth years), and Kaliyuga (432,000 Earth years).

The primary text expounding *Vedāṅga* astronomy was the *Vedāṅgajyotiṣa*, a Sanskrit text in metric verse. The *Vedāṅgajyotiṣa* occurred in both the *Ṛgveda* and *Yajurveda* recensions, with the former appearing to be a consolidation of astronomical thought of the time by Lagadha. The *Vedāṅgajyotiṣa* calendrical system included the definition of a time-unit called a *yuga* of 5 years with 2 intercalary months (the difference between solar and lunar months), with 366 civil days (time between sunrise to sunrise) per year [Oha02]. Other time units were updated as well to produce more whole-numbered time divisions [SS85, p.xxi]. Additionally, both recensions of the *Vedāṅgajyotiṣa* included linear formulae for the length of daylight in the Vedic time-unit of *muhūrtas* (1/30th of a day); products possibly of extrapolating observational data about seasonal changes of the length of daytime especially around equinoxes [Oha93, p.205] [Mak13b].

The oldest surviving post-*Vedāṅgajyotiṣa* Sanskrit astronomy texts are from around the start of the common era, currently preserved only in early 6th century compendium.[3] The earliest text in this period, the *Vasiṣṭhasiddhānta* (presumably by the aforementioned Vasiṣṭha) improved upon the *Vedāṅgajyotiṣa* system, with modifications made for accuracy. For instance, in addition to the sole focus of the Sun and Moon, the mean motions and longitudes of the other five planets were found using crude formulae from the zero point of the zodiacal sign Meṣa, or Aries [BSS71, p.83]. Along with the *Vasiṣṭhasiddhānta* four texts are represented in Varāhamihira's summary text *Pañcasiddhāntikā* (sixth-century CE, epoch unless specified assumed to be 21 March 505 CE) are: the *Paitāmahasiddhānta* (epoch 80 CE), *Pauliśasiddhānta*, *Romaka-siddhānta*, and *Sūryasiddhānta* [NP70] [SS93]. The post-Vedic period witnessed the earliest textual evidence of Greco-Babylonian or Greek thought in the Sanskrit tradition. Other than the *Paitāmahasiddhānta* which was still reliant on the *Vedāṅgajyotiṣa* system, the other texts show an adoption of Hellenistic ideas, using a mixture of systems that were not fully Ptolemaic [Pin71b]. The text *Yavanajātaka* (dated sometime after 22CE and as late as the early seventh century CE) was a text believed to be composed by Sphujidhvaja that combined Sanskrit and Greek astronomy. Its last chapter showcased that the text followed the same tradition of mathematical astronomy as the *Vedāṅgajyotiṣa*, and texts featured in the *Pañcasiddhāntikā* [Mak13b]. The text contained elements of astrology such as dividing the ecliptic into the zodiac and the names of the zodiacal signs. Astronomical inquiry in this post-Vedic period had also evidently gone beyond calendrics for rituals and ceremonies but to investigating the mean and trigonometrically-corrected positions and motions of all seven planets (Sun, Moon, Mars, Mercury, Jupiter, Venus, and Saturn), and predicting solar and lunar eclipses and the visibility of lunar phases.

While Āryabhaṭa I (b. 476 CE) and other scholars used different systems for representing numbers, the *bhūtasaṃkhyā* ('word-numeral') system was one of the most popular in the Sanskrit tradition. The works of Varāhamihira such as the *Pañcasiddhāntikā* feature the earliest attested usage of the *bhūtasaṃkhyā* system [Mak13a, p. 8, footnote 19]. Physical, philosophical, or cultural entities connoting a numeral would then denote the numeral. For example, the number one is expressed by synonyms for words referring to the Moon and the Earth, the number two is expressed by synonyms for words referring to eyes or hands, and so on. Composers would then craft grammatical compounds expressing the place value of numbers [DS62, p. 54] [Plo09b, p. 47].

By just before 500CE, the classical Sanskrit astronomy age of *siddhānta* ("astronomical treatise") texts began with Āryabhaṭa I's composition of his *Āryabhaṭīya* (499 CE). Longer and more thorough explanations of models, theories, and formulae– all still in metric verse–became a general characteristic of *siddhānta* texts, as well as their choice of epoch at the beginning of a *kalpa*. The selection of epoch was the main difference between *siddhāntas* and *tantras*, which were treatises with epochs at the beginning of the current *yuga* [WK14, p.464]. Both types of astronomical treatises primarily discussed predictive astronomy, with

[3]Much of what is currently known about Sanskrit astronomy and its origins is retained in manuscript data, but the majority of available manuscripts themselves date back only to the second millennium CE [Wuj14]. Factors such as insufficient preservation given the delicate medium of palm-leaves in the hot and humid climate found in most parts of India surely impacted the number of manuscripts available to us today; the newer manuscripts of the last 300-500 years are more likely to still be preserved than those from earlier.

versified parameters and formulae necessary for computing celestial phenomena for a particular date and time. Soon after these texts developed the abbreviated *karaṇas* ("astronomical handbook"). While *siddhānta* texts included explanations of background astronomical theory, *karaṇa* texts simply presented the formulas and calculations. As a result, *karaṇa* texts were typically less dense and prioritized quicker computations to once again achieve practical purposes such as horoscope creation. Epochs of *karaṇa* texts were close to the time of their authors. An independent genre of *koṣṭhakas* ("astronomical computation tables") developed somewhat later, achieving prominence in the second millennium. The *koṣṭhaka* texts, which featured tables of precomputed data, were often self-contained works that optionally included explanatory text. While early *siddhāntas* such as the *Brāhmasphuṭasiddhānta* contained individual chapters devoted to explaining the astronomical instruments used for observation, in the second millennium separate *yantra* ("[astronomical] instrument") texts were created, either explaining one or multiple instruments and their uses. While *siddhāntas*, *tantras*, *karaṇas*, and *koṣṭhakas* were used to mathematically compute positions of astral bodies and predict phenomena, astronomers verified their data with observational data produced with *yantras*. The ideal of Indian astronomers was achieving agreement between the computed and observed positions of celestial bodies, which then motivated the selection of appropriate texts [Sar12, p.321].

Based on the adherence to certain parameters and methodology for astronomical computations, accomplished astronomers branched out into separate *pakṣas* ("schools"). These *pakṣas* were the Āryapakṣa, Ārdharātrikapakṣa, Brāhmapakṣa, and Saurapakṣa. Each *pakṣa* used slightly different values for the division of time and revolutions of the celestial bodies, as seen in Tables 1.1 and 1.2. The Āryapakṣa started around 500 by Āryabhaṭa also included Bhāskara I (early 7th century), Lalla (probably 8th century), and Vaṭeśvara (10th century). The Ārdharātrikapakṣa was also founded by Āryabhaṭa, and while no surviving *siddhāntas* follow its parameters, the *karaṇa* text *Khaṇḍakhādyaka* by Brahmagupta . Brahmagupta, Śrīpati (mid-11th century), and Bhāskara II (mid-12th century) were the most well known followers of the Brāhmapakṣa. The Saurapakṣa was founded with the text *Sūryasiddhānta* (around 800 CE) which was ascribed to the Hindu sun god Sūrya[Plo09b]. While it could have been the case that some astronomers produced texts in alignment of the *pakṣas* of predecessors in their professional lineages or patrilineages, this was not at all a requirement of the system. Brahmagupta serves as an important example of this, with his *Brāhmasphuṭasiddhānta* belonging to the Brāhmapakṣa and*Khaṇḍakhādyaka* belonging to the Ārdharātrikapakṣa.

1.2 The role of commentators in Sanskrit mathematical astronomy

Mathematical astronomy *mūla* "main" texts like *siddhāntas* and *karaṇas* were expressed in poetic metric verse to stimulate the student's memory [Pin81b, p.1]. But these verses were often so concise or complex in showcasing a text author's command of Sanskrit that they seem to require a commentator to explain and gloss their content [TB07, p.1]. To facilitate understanding and education, genres of technical commentaries on these *mūla* texts evolved and were called *ṭīkā*, *bhāṣya*, *vyākhyā*, etc. It is likely that the teacher's oral commentary accompanied the taught and memorized verses first [Fil04, p.148]. Scholars speculate that teachers would have used a wooden board covered with sand or dust to demonstrate procedures in class, given the names of arithmetic as *pāṭīgaṇita* ("board-calculation") and the process of working through calculations as *dhūlīkarma* ("dust-work") [Dha19, pg.3]. Later, this oral explanation was transcribed as a self-commentary or synthesized and expounded upon by the student as a new commentary.

The content of the *mūla* "main" *siddhānta* and *karaṇa* texts was restricted to just the theory or algorithms necessary for planetary computations, rarely with any explanation or rationale. Besides providing technical explanations of their *mūla* texts, commentaries contained crucial biographical information such as the professional and patrilineal relationships of the commentator and text author. They also provided useful contextual information such as evidence of the scholastic practices of the time. The post-Vedic framework of education seemed to have prevailed: conservation of the past body of knowledge to preserve an ongoing tradition, compilation of current working theories/methodologies, and criticism of others that seem out of date or illogical/erroneous [Moo89, p.72]. Authors frequently used commentaries to align themselves with the *mūla* text author, often as students demonstrating their understanding of their teacher's teachings, or

Āryapakṣa		
Length of period: *mahāyuga* = 4,320,000 years		
Civil days in period: 1,577,917,500		
Year length: 365;15,31,15 days		
Epoch: sunrise		
Planet	Revolutions	Mean motion
Sun	4,320,000	0°;59,8,10,13,3,31
Moon	57,753,336	13°;10,34,52,39,18,56
Lunar node	-232,226	-0°;3,10,44,7,49,44
Lunar apogee	488,219	0°;6,40,59,30,7,38
Mars	2,296,824	0°;31,26,27,48,54,22
Mercury's *śīghra*-apogee	17,937,020	4°;5,32,18,54,36,24
Jupiter	364,224	0°;4,59,9,038,51
Venus's *śīghra*-apogee	7,022,388	1°;36,7,44,17,4,45
Saturn	146,564	0°;2,0,22,41,41,32

Ārdharātrikapakṣa		
Length of period: *mahāyuga* = 4,320,000 years		
Civil days in period: 1,577,917,800		
Year length: 365;15,31,30 days		
Epoch: midnight		
Planet	Revolutions	Mean motion
Sun	4,320,000	0;59,8,10,10,37,48
Moon	57,753,336	13;10,34,52,6,50,56
Lunar node	-232,226	-0;3,10,44,7,41,54
Lunar apogee	488,219	0;6,40,59,29,51,10
Mars	2,296,824	0;31,26,27,47,36,54
Mercury's *śīghra*-apogee	17,937,000	4;5,32,17,45,23,13
Jupiter	364,220	0;4,59,8,48,36,56
Venus's *śīghra*-apogee	7,022,388	1;36,7,44,13,7,53
Saturn	146,564	0;2,0,22,41,36,36

Table 1.1: The period relation parameters of the Āryapakṣa and Ārdharātrikapakṣa, as given in Appendix B.2 of [MP18, pg. 276]. These are originally reconstructed from the *Āryabhaṭīya* and *Khaṇḍakhādyaka* respectively.

as a familiar framework upon which they may explore new concepts and methods [TB07, p. 3]. Likewise, commentaries yielded valuable information about the working notes of scholars, including methodology and applications of formulas in examples, or even possibly alternative methods to derive solutions to the same problems addressed in the *mūla* text. Manuscripts of commentaries such as Bhāskara I's commentary of the *Āryabhaṭīya* show evidence of space reserved for geometric figures used as visual aids [Kel05, pg.277]. But astronomers of different schools of thought also used commentaries as an opportunity to expose the inaccuracies of one school's parameters or an author's assumptions [Plo09b, p. 70].

Commentaries clearly were an integral member of the Sanskrit mathematical astronomy genre. In fact, scholars conservatively estimate that of 30 million total Indic manuscripts spanning all genres and topics, there are only 100,000 surviving *jyotiṣa* manuscripts, which discuss only 10,000 different astronomical works [Pin78, p. 64] [Sri19]. Even cursory glances of catalogs such as volumes of the Census of the Exact Sciences in Sanskrit (CESS) show that most *jyotiṣa mūla* texts surveyed have multiple extant commentaries. This popularity of prose commentary also testifies to the mathematical astronomy tradition's reliance on both writing and orality for textual transmission.

Brāhmapakṣa		
Kalpa length: 4,320,000,000 years		
Civil days in a *kalpa*: 1,577,916,450,000		
Year length: 365;15,30,22,30 days		
Epoch: sunrise		
Planet	Revolutions	Mean motion
Sun	4,320,000,000	0;59,8,10,21,33,30
Moon	57,753,300,000	13;10,34,52,46,30,13
Lunar node	-232,311,168	-0;3,10,48,20,6,41
Lunar apogee	488,105,858	0;6,40,53,56,32,54
Mars	2,296,828,522	0;31,26,28,6,47,45
Mercury's *śīghra*-apogee	17,936,998,984	4;5,32,18,27,45,31
Jupiter	364,226,455	0;4,59,9,8,37,23
Venus's *śīghra*-apogee	7,022,389,492	1;36,7,44,35,18,27
Saturn	146,567,298	0;2,0,22,51,43,56

Saurapakṣa		
Length of period: *mahāyuga* = 4,320,000 years		
Civil days in period: 1,577,917,828		
Year length: 365;15,31,31,24 days		
Epoch: midnight		
Planet	Revolutions	Mean motion
Sun	4,320,000	0;59,8,10,10,24,12
Moon	57,753,336	13;10,34,52,3,49,8
Lunar node	-232,238	-0;3,10,44,43,10,4
Lunar apogee	488,203	0;6,40,58,42,31,5
Mars	2,296,832	0;31,26,28,11,8,56
Mercury's *śīghra*-apogee	17,937,060	4;5,32,20,41,51,16
Jupiter	364,220	0;4,59,8,48,35,47
Venus's *śīghra*-apogee	7,022,376	1;36,7,43,37,16,52
Saturn	146,568	0;2,0,22,53,25,46

Table 1.2: The period relation parameters of the Brāhmapakṣa and Saurapakṣa, as given in Appendix B.2 of [MP18, pg. 276]. These are originally reconstructed from the *Paitāmahasiddhānta*/*Brāhmasphuṭasiddhānta* and *Sūryasiddhānta* respectively.

1.3 Scholarship in the *karaṇa* genre

Improvements to computational accuracy and ease, and creative approximation techniques mainly drove scholarship [Mis16, pg. 4]. Indeed, one of the defining features of the astronomical handbook tradition was the substitution of complex analytically exact formulae with simpler approximating procedures, without sacrificing accuracy. So, *karaṇa*-authors saw the the modification of trigonometric procedures as a natural opportunity to innovate. Performing computations with sines and arcs could be tedious since they required users to look up tabulated values and interpolate them. The process could also be inaccurate, depending on the number of tabulated values and the value of the non-unit radius.[4] Bhāskara I in the seventh century was one of the earliest authors to express a desire to avoid sine and arc computations, and prefaced his ingenious quadratic approximation to the sine as 'a rule/computation independent of *makhi* etc.', i.e., without tabulated numerical sine values.[5]

Many well-known handbooks of different *pakṣas* soon featured the abridgment and replacement of tabulated trigonometric function values. The crudest form of simplification was merely to reduce the number and precision of the tabulated values, sacrificing accuracy for ease of memorization and use. One of the first extant examples of the genre, the 665 *Khaṇḍakhādyaka* of Brahmagupta, includes a sine table with only six tabulated sines and a notably small radius value of 150. To compensate for the lack of entries, and the resulting rough quantities the user would produce via linear interpolation for intermediate values, Brahmagupta offered an insightful second-order interpolation formula based on second differences. Abridgement of sine tables was taken to the extreme by the tenth-century author Muñjāla in his work *Laghumānasa* ('Easy thinking') where he proposed a radically reduced sine table containing only three values. A later commentator on his work, Mallikārjuna Sūri, provided a higher-order interpolation rule based on difference factors, to ensure accuracy in the absence of more geometrically computed sines.[6]

Alternate rules replacing trigonometric relations by precomputed versified data and/or algebraic approximations were also employed. For instance, Brahmagupta's *Khaṇḍakhādyaka* includes a rule for ascensional differences which replaces tangent and arc sine operations with a set of ingeniously chosen constant coefficients, among many other clever substitutions.[7] His successor Vaṭeśvara in the tenth century celebrated as accomplished the astronomer who could 'compute the sine (*jīvā*) of the upright arc (*bhuja*) and its complement (*koṭi*), and also the arc (*cāpa*), without making use of the tabulated sine values (*jyā*)'.[8] The renowned astronomer-mathematician Bhāskara II likewise embraced algebraic approximations throughout his 1183 handbook *Karaṇakutūhala*. In particular, its chapter on problems of local direction, place and time is replete with such substitutions, including algebraic alternatives to determine solar declination, shadow lengths and terrestrial latitude, all of which are underpinned by sines and related trigonometric functions.

But while these earlier authors formulated inventive substitutions sporadically throughout their works, it was not until the mid-second millennium that a uniquely innovative astronomer would fully implement the *karaṇa* ideal of dispensing with trigonometric tables entirely.

[4] The choice of non-unit radius measures differed from text author to text author as they saw fit, with the hope of achieving evenly divisible numbers in their other parameters and formulas, as seen in [Gup78].

[5] *'makhyadirahitaṃ karma'*: *Mahābhāskarīya* 7.17 [Sas57, pp. 377–378]. The term *makhi* is an artificial word representing the number 225, the first in the list of twenty-four sine differences encoded by Āryabhaṭa (b. 476) in his own idiosyncratic alphanumeric system; see *Āryabhaṭīya Gītikā* 12 [Shu76, pp. 29–30]. Bhāskara I's quadratic approximation was a convenient and highly accurate alternative to the sine function and was adopted by many of his successors. See [Gup67], [Hay91], [Plo09a, pp. 81–82].

[6] See [Plo09a, pp. 105–106].

[7] See, for instance, [MP18, p. 49, fn. 7].

[8] *'jyābhir vinaiva kurute bhujakoṭijīve cāpaṃ ca yaḥ'*: *Vaṭeśvarasiddhānta* 2.7.10 [Shu86, vol. 1, p. 139].

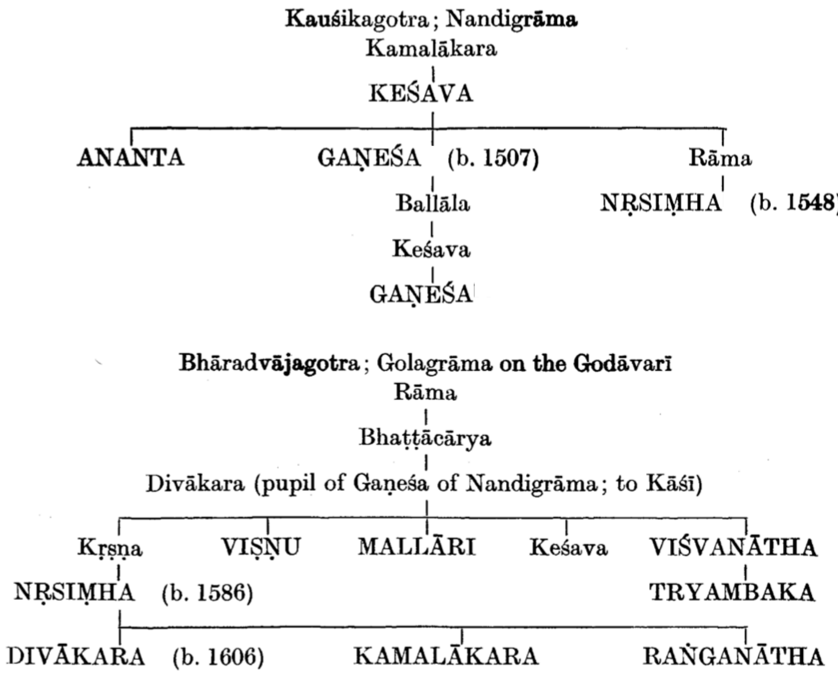

Figure 1.1: The *Gaṇeśapakṣa* was predominantly comprised of Gaṇeśa's patrilineage (above) and several members of his pupil Divākara's patrilineage (shown below). Names that are fully capitalized have works that are known to us (cf. Tables 11 and 12 in [Pin81b, pp.125–126]).

1.4 Gaṇeśa, the *Gaṇeśapakṣa*, and the *Grahalāghava*

While biographic information about Sanskrit astronomers is more readily available than that of Sanskrit text authors in other disciplines [Sar10, p.1], rarely are patrilineages/professional lineages as well-known as that of Gaṇeśa Daivajña (b.1507) of Nandigrāma, Maharashtra.[9] The son of Lakṣmī and Keśava (fl. 1496/1507), Gaṇeśa belonged to the prestigious Kauśika *gotra* lineage of Brahmins in Maharashtra. His father and guru Keśava was a well-respected observational astronomer who composed an astronomical treatise called *Grahakautuka*.[10] Keśava's teacher was Vaidyanātha, and his father was Kamalākara, another accomplished astronomer. Along with Gaṇeśa, Keśava's other two sons Ananta and Rāma were famous astronomers (CESSa2,65colb dvivedin 1892 53-55 sb dikshit 1896 258-259). Ananta's son, Nṛsiṃha (b.1548) was Gaṇeśa's first pupil, and Gaṇeśa's professional lineage further expanded when Gaṇeśa taught Divākara of Golagrāma, Maharashtra. Once his training was complete, Divākara relocated to Kāśī (Varanasi/Benares) and passed on his craft to his five sons, who in turn transmitted the profession to succeeding generations. From the early 1500s to 1700 or after, an exceptional corpus of works blossomed from Gaṇeśa's teachings, including handbooks, almanacs, astrological treatises, commentaries and astronomical tables. This corpus was so well-populated it attained the status of a new astronomical *pakṣa* named after Gaṇeśa: the *Gaṇeśapakṣa* ("school of Gaṇeśa"). Figure 1.4 shows both Gaṇeśa's and Divākara's patrilineages; several members of which contributed greatly to the popularity of the *Gaṇeśapakṣa*.

Two well-known students of Gaṇeśa's academic line were Mallāri and Viśvanātha. Mallāri (b. 1571) [Dvi33, p.77]; [Pin81a, pp.365-366] and Viśvanātha were both sons of Gaṇeśa's pupil Divākara. Mallāri was the uncle and teacher of Nṛsiṃha (b. 1586) and the great uncle of Divākara (b. 1606). While it is clear

[9] Previously sometimes identified with Nandod in Gujarat, Nandigrāma is actually the modern Nandgaon about 64 km south of Mumbai on the Konkan coast [Sar10].

[10] Gaṇeśa claims to base his *Grahalāghava* on his father's *Grahakautuka* [Pin70, p. 24]. This claim has yet to be substantiated, as there are no critical editions and analyses of the *Grahakautuka* thus far[Pin81b, p. 36]; [Min02, p. 498]; [Dvi33, p. 53].

that he wrote a commentary on the *Grahalāghava*, *Grahalāghavaṭīkā* ("commentary on the *Grahalāghava*"), it is unclear if he is the author of any other texts.[11] Mallāri's commentary, the *Grahalāghavaṭīkā* seems to explain the foundational information that Gaṇeśa assumes of his readers, and attempts derivations of Gaṇeśa's algebraic approximations of trigonometric formulas.

Meanwhile, the commentary *Grahalāghavodāharaṇa* ("example of the *Grahalāghava*") (1612) of Viśvanātha (b. 1578) [Dvi33, p. 82] features worked-through examples to illustrate to handbook users how Gaṇeśa intended the formulas to be used. Viśvanātha also wrote commentaries on works such as Bhāskara II's *Karaṇakutūhala* called the *Brahmatulyodāharaṇa* (1612), Gaṅgādhara's Candramānatantra, Keśava's *Jātakapaddhati* (1618), Nīlakaṇṭha's *Tājikanīlakaṇṭhī* (1629), Gaṇeśa's *Laghutithicintāmaṇi* (the example is dated from 1644), Gaṇeśa's *Pātasāraṇī* (the example is dated from 1631), Makaranda's Makaranda (examples ranging from 1610-1634), Rāma's *Rāmavinoda* (examples between 1600-1602), Viḍḍaṇa's *Vārṣikatantra* (1647), the *Sūryasiddhānta* (1620), Viṣṇu's *Sūryapakṣaśaraṇa* or *Saurapakṣagaṇita* (1623), Keśava's *Varṣaphalapaddhati* (example dated 1579) and possibly commentaries on Bhāskara II's *Siddhāntaśiromaṇi*, the *Somasiddhānta*, Śatānanda's *Bhāsvatī*, Varāhamihira's *Bṛhajjātaka* and *Bṛhatsaṃhitā*,Keśava's *Muhurtatattva*, and Keśava's *Grahakautuka*.[12] His other works include a *karaṇa* based on the *Sūryasiddhānta* called the *Mitāṅkagaṇita* (epoch 1612), the *Janmapatralekhanapaddhati* (featuring the horoscope of Khurram Shāh born 14 January 1592), and possibly a *Śrīpatyudāharaṇa* [Pin94, pp.686-687].

Sixteen texts have been clearly attributed to Gaṇeśa. Thirteen of these works are mentioned at the beginning of the commentary text *Harṣakaumudī* by Gaṇeśa's nephew Nṛsiṃha (as quoted by Viśvanātha at the beginning of his commentary *Grahalāghavodāharaṇa* [Jos81, lines 22-27, p.3]):

> कृत्वादौ ग्रहलाघवं लघुबृहत्तिथ्यादिचिन्तामणिं
> सत्सिद्धान्तशिरोमणेश्च विवृतिं लीलावतीव्याकृतिम् ।
> श्रीबृन्दावनटीकिकां च विवृतिं मौहूर्ततत्त्वस्य वै
> सच्छ्राद्धादिविनिर्णयं सुविवृतिं छन्दोऽर्णवाख्यस्य वै ॥
>
> सुधीरञ्जनं तर्जनीयन्त्रकं च सुकृष्णाष्टमीनिर्णयं होलिकायाः ।
> लघूपाययातस्तथान्यानपूर्वान् गणेशो गुरुर्ब्रह्मनिर्वाणमापत् ॥
>
> *kṛtvādau grahalāghavaṃ laghubṛhattithyādicintamaṇiṃ*
> *satsiddhāntaśiromaṇeś ca vivṛtiṃ līlāvatīvyākṛtim* |
> *śrībṛndāvanaṭīkikāṃ ca vivṛtiṃ mauhūrtatattvasya vai*
> *sacchrāddhādivinirṇayaṃ suvivṛtiṃ chando' rṇavākhyasya vai* |
>
> *sudhīrañjanaṃ tarjanīyantrakaṃ ca sukṛṣṇāṣṭamīnirṇayaṃ holikāyāḥ* |
> *laghūpāyayātas tathānyan apūrvān gaṇeśo gurur brahmanirvāṇam āpat* ||
>
> Having first made the *Grahalāghava*, the *Tithicintāmaṇi* beginning with both *Laghu*- and *Bṛhat*, a commentary of the excellent *Siddhāntaśiromaṇi*, and a commentary of *Līlāvatī*, a commentary on the [*Vivāha*-]*vṛndāvana*, and a commentary of the *Muhurtatattva*, a true determination of *Śrāddha* and so on, the excellent commentary on the *Chandorṇava*, the *Sudhīrañjana*, the *Tarjanīyantraka*, the excellent *Kṛṣṇāṣṭaminirṇaya*, and [*nirṇaya* (determination)] of Holikā by means of easy methods, [and] other unprecedented [works] likewise, the teacher Gaṇeśa attained unity with the Supreme Being.

As stated in Nṛsiṃha's verse, these thirteen works are as follows:

[11] He is possibly attributed to authoring an eclipse text *Parvadvayasādhana* (1588) [Pin81b, p.55] and a commentary on Gaṇeśa's father's text, *Varṣaphalapaddhati*[Pin94, pp.284-285].

[12] The manuscripts of his *Grahakautuka* commentary must especially be verified because these commentaries may just be other commentaries of the *Grahalāghava* [Pin94, p.669]. Additionally, Viśvanātha may have authored a commentary of Brahmadeva's *Karaṇaprakāśa* [Pin81b, p.125].

Grahalāghava (1520) a *karaṇa* text which Gaṇeśa is believed to have composed at the age of 13 [Dvi33, p.58].[13]

Laghutithicintāmaṇi (1525 CE) a table-text which consists of tables for determining *tithis*, *nakṣatras*, and *yogas* and a short introductory exposition

Bṛhattithicintāmaṇi (1552 CE) the subsequent but less popular table-text which also contains *tithi*, *nakṣatra*, and *yoga* computing tables with a longer accompanying text

unnamed *Siddhāntaśiromaṇi* commentary a commentary (date unknown) on Bhāskara II's *Siddhāntaśiromaṇi* of which there are no known manuscripts [Pin81a, p.312]

Buddhivilāsinī (1545 CE) a commentary on Bhāskara II's *Līlāvatī*

Vṛndāvanatīkikā (1554 CE) a commentary on the *Vivāhavṛndāvana* of Keśavārka, which discusses astrology used in the context of marriage

Muhurtadīpikā (before 1554 CE) a commentary on his father's *Muhurtatattva*, which deals with astrologically auspicious timings

Śrāddhanirṇaya which is a work describing offerings to ancestors

Chandorṇava a Sanskrit prosody text on different meters, which also has little information about it and no known manuscripts

Sudhīrañjana a text about the astronomical instrument of the same name

Tarjanīyantraka another text about the astronomical instrument of the same name

Kṛṣṇāṣṭaminirṇaya a work detailing the festival of Lord Kṛṣṇa's birthday which has no known manuscripts or associated publications

Holikānirṇaya a work on the Holi festival which also has no known manuscripts or associated publications

Three other texts have also been ascribed to Gaṇeśa [Sub08] [Pin71a, p.94]:

Pātasāraṇī (1522) a table-text with sets of tables to compute the *pātas* of the Sun and Moon along with a short explanation

Cābukayantra a text about the astronomical instrument of the same name

Pratodayantra another text about an astronomical instrument

Apart from these texts, Gaṇeśa is said to be the author of the *Dhruvabhramanayantravyākhyā*, a commentary on Padmanābha's *Yantraratnāvalī* describing an instrument used to view the North Pole star [Sub08]. There are no known manuscripts to confirm this, so we do not include this work in Gaṇeśa's bibliography.

Among his prolific scholarship, Gaṇeśa's *Grahalāghava* can be considered his magnum opus. This astronomical handbook quickly became popular throughout northern and western India [Dvi33, p.58]. In fact, the *Grahalāghava* was the most influential astronomical handbook since Bhāskara II's *Karaṇakutūhala* [Min02]. It inspired in the ensuing centuries a proliferation of astronomical tables based on its parameters

[13] Dvivedi mentions that Gaṇeśa composing the text at age 13 is folklore–चिरात् जनश्रुतिः (*cirāt janaśrutiḥ*, "heard by people, a long time ago") [Dvi33, line 6, p.58]. Dvivedi's previous statement in his biographical description of Gaṇeśa discusses how people say that Gaṇeśa was named so because it was as though he were an incarnation of the Hindu God, Gaṇeśa: भारतवर्षेऽयं गणेशो गणेशावतार इति कथ्यते (*bhāratavarṣe 'yaṃ gaṇeśo gaṇeśāvatāra iti kathyate*, "in India, this Gaṇeśa [Daivajña] is said to be an incarnation of [Lord] Gaṇeśa") [Dvi33, lines 4–5, p.58]. We too humorously assert that if a boy at the mere age of 13 had such strong command over astronomy and was well-versed in the works of his successors to compose the *Grahalāghava*, he certainly would have to be an incarnation of a deity of knowledge and would be the "talk of the town"!

Commentator	Commentary	Reference
Mallāri (fl.ca. 1575-1600)	*Grahalāghavaṭīkā*	CESS A4, p.366a-367b; CESS A5 p.285a
Nṛsiṃha (b. 1548)	*Harṣakaumudī*	CESS A3 p.202b-203a, p.203b; CESS A4 p.162b; CESS A5 p.202a
Dinakara (fl. 1578-1583)	Sanskrit commentary	CESS A3 p.104b; CESS A5 p.139b
Gaṅgādhara (fl. 1586)	*Manoramā* (1586)	CESS A2 p.82a; CESS A5 p.64b
Kamalākara (fl.ca. 1550-1650)	*Manoramā*	CESS A2 p.21a
Viśvanātha (fl.ca. 1600/1650)	*Grahalāghavodāharaṇa* (1612)	CESS A5 p.669b-674b
Nārāyaṇa (fl.ca. 1635/1678)	*Grahalāghavodāhṛti/ Siddhāntarahasyodāhṛti*	CESS A3 p.165b; CESS A4 p.139a, p.165b-166a
Yaśavatsāgara (fl. 1664/1705)	*Grahalāghavavartikā* (1703)	CESS A5 p.331a
Bālagovinda cf. Govindasūnu (fl. 1773)	commentary	CESS A4 p.246a
Keśava Daivajña	*Grahalāghavavyākhyāna*	[Sen66, p. 109]
Mayādānava	alleged commentary *Grahalāghavabhāṣya*	CESS A4 p.358b
Nekanātha (=Ekanātha?)	commentary	CESS A5 p.203b
Caturvijaya (fl. 1781)	Rajasthani commentary (1781)	CESS A5 p.106b
Raghunātha	Marathi commentary	CESS A5 p.369a

Table 1.3: Commentators of the *Grahalāghava* and their commentaries

and procedures, as well as a number of substantial commentaries. In testament to its enduring popularity, many hundreds of manuscript copies of the work are still extant and Gaṇeśa's system is still considered relevant by many practitioners of astrology in India today. The professional circumstances of this lineage and their scientific achievements are thus of considerable importance in the history of Indian astronomy in the second millennium. Table 1.3 shows the numerous known commentators of the *Grahalāghava* and their commentaries, ranging from members of Gaṇeśa's lineage to modern scholars. Table 1.4 shows the various critical editions. While editors prioritized editing the *Grahalāghava* with the commentaries of Mallāri and/or Viśvanātha, some editors also included their own commentary.

The *Grahalāghava* follows the traditional format of a *karaṇa* or Sanskrit astronomical handbook: over the course of sixteen chapters it covers planetary positions and velocities, timekeeping and calendar construction, eclipses, heliacal rising and settings, the orientation of the lunar crescent, planetary conjunctions, and the so-called *mahāpātas*, ominous configurations of the sun and the moon. However, Gaṇeśa's handbook is original and innovative in many senses. Its distinctive features include revised base parameters for planetary motion and procedures that avoid the use of tables of arc and sine values. These procedures are a rather remarkable culmination of various strategies in previous *karaṇas* to simplify, or approximate algebraically, the mechanisms of trigonometry. Given that geometric models of planetary motion are founded upon the application of orbital inequalities requiring angular measurement and chord-arc conversion, trigonometry of course cannot be totally eliminated from these models. But Gaṇeśa has succeeded in concealing it, as far as the user is concerned, within versified tables of astronomical function values and ingenious algebraic approximations, thus freeing the user from frequent consultation and manipulation of complex tables and interpolation techniques.

1.5 Thesis objectives and methodology

In this thesis, we examine the first three chapters of the *Grahalāghava* with the following aims:

Place/Date	Editor	Edition Details	Publisher/Pingree Collection Citation
Calcutta 1843	Lancelot Wilkinson	w. comm. of Mallāri	Agra School-Book Society
1854			AS Bombay (Indraji) 105
Kāśī 1865	Bhalacandra	w. comm. of Mallāri and Viśvanātha	BM
Bombay 1873	Kṛṣṇa Sastri Godabole and Vamana Kṛṣṇa Josi Gadre	comm. of Viśvanātha and Marathi translation	(IO 8.G.4)
	Nilambara Jha (b.1823)	*Grahalāghavopapatti*	
Mumbai 1875		w. comm. of Mallāri	BM
Benares 1877		w. comm. of Mallāri	Phauka Press (IO 8.I.10)
Delhi 1877		w. comm. of Mallāri	BM
Mumbai 1882		w. comm. of Mallāri	Srivardhana Press (IO 13.E.15)
Mumbai 1883		w. comm. of Mallāri	BM
Kolkata 1886		w. comm. of Mallāri	NL Calcutta 180.Kb.88.3
Kolkata 1887	Rasikamohana Cattopadhyaya	w. comm. of Viśvanātha and Bengali translation	
Kalyan-Mumbai 1899	Ramesvara Bhatta	w. Hindi translation by Jiyarama Sastri	BM 14053.ccc.26
Mumbai 1901	Hariprasada Sarman	w. comm. of Mallāri	
Benares 1904	Sudhakara Dvivedin	w. comm. of Mallāri and Viśvanātha and Sanskrit comm. *Sadvāsanā*	The Chandraprabha Press (IO 26.I.12)
Masulipatam 1915	Mangipudi Virayya Siddhantigar	w. comm. of Mallāri and Telugu *Āndhrāṭīkā*	IO 12.L.19
Benares 1932, Benares 1941, reprinted Delhi 1975	Sītārāma Jhā	w. Sanskrit comm. *Sudhāmañjarīvāsanā* and Hindi comm.	MM 142, NL Calcutta 180.Kc.94.1
Benares 1946	Kapilesvara Sastri	w. comm of Viśvanātha, Sanskrit comm. *Mādhurī* of Yugeśvara Jhā, and Hindi comm.	Chowkhamba Sanskrit Series Office
Jammu 1976	Ramachandra Pandeya	w. comm. of Mallāri and Hindi comm. *Maṅgala*	
Delhi-Varanasi-Patna 1983	Kedaradatta Josi	w. comm. of Mallāri and Viśvanātha, and Hindi comm.	[Jos81]
Mumbai 1987	Ramasvarupa	w. Sanskrit and Hindi comm.	
Varanasi 1987	Brahmananda Tripathin	w. Sanskrit comm. *Tārā* and Hindi comm.	
New Delhi 2007	S. Balachandra Rao and SK Uma	English exposition w. examples and explanations	Indian National Science Academy

Table 1.4: The numerous editions of the *Grahalāghava* and its commentaries.

Text	Manuscript Description
Grahalāghava	Poleman 4685 (UPenn 699). 17ff. Devanāgarī. Incomplete. 9.25 x 4.5 in. 11 lines. Poleman 4686 (UPenn 1785). 7ff. Devanāgarī. Incomplete. 12.1 x 5 in. 11 lines. Poleman 4689 (UPenn 1822). 34ff. 9.1 x 4 in. 7 lines. Devanāgarī. Copied in Śaka 1738 = AD 1816.
Grahalāghavaṭīkā	VVRI 1416. 17ff. Devanāgarī. Incomplete VVRI 2502. 30ff. Devanāgarī. Incomplete. VVRI 2589. 14ff. Devanāgarī. Incomplete. VVRI 2595. 11ff. Devanāgarī. Incomplete.
Grahalāghavodāharaṇa	VVRI 1274. 27ff. Devanāgarī. Copied in Sam. 1905 = AD 1848. VVRI 2359. 58ff. Devanāgarī. Incomplete.

Table 1.5: Selected manuscripts of the *Grahalāghava* and its commentaries (*Grahalāghavaṭīkā* by Mallāri and *Grahalāghavodāharaṇa* by Viśvanātha) from the Kislak Center for Special Collections within the Rare Books and Manuscripts Collection at the University of Pennsylvania (UPenn) [Pol38, p. 232], and the Lalchand Research Library Ancient India Manuscript Collection, DAV College, Chandigarh supported by the SP Lohia Collection (also known as Vishveshvaranand Vedic Research Institute (VVRI)) [Ban59, p. 76].

1. Providing a faithful English translation for each Sanskrit verse
2. Explaining the technical procedures stated in the verses in accordance with the basic astronomical models
3. Tracing influences of specific earlier works in some of the verses
4. Investigating some of the results and derivation of Gaṇeśa's trigonometry-approximating techniques, drawing on commentarial texts when appropriate

For each of the three chapters, we present the Sanskrit text of the verses of Gaṇeśa's chapter with its transliteration and translation, followed by our explanation of its technical import. We use Dvivedi's edition of the *Grahalāghava* as our version of the Sanskrit text [Dvi25] due to his being a reputable and prolific authority in Sanskrit mathematical astronomy, and consult Joshi's edition where required [Jos81]. In the occasional instance where the edition seemed problematic, we have consulted original manuscripts for alternate readings (see Table 1.5). These manuscripts (courtesy of the "Penn in Hand" site, hosted by University of Pennsylvania and the Lalchand Research Library Ancient Indian Manuscript Collection supported by the SP Lohia Collection, hosted by DAV College, Chandigarh)) are graciously digitally available to be used for research purposes, and were thus chosen for consultation. We appeal occasionally to excerpts from the commentaries of Mallāri and Viśvanātha for clarifications or elaborations of Gaṇeśa's meaning.

Chapter 2

Madhyamādhikāra
chapter on mean motions

2.1 Introduction

The first chapter of the *Grahalāghava* is the *madhyama* or 'mean [motion]' chapter. As in all pre-heliocentric spherical astronomy traditions, the geometric models of Indian astronomy start from the simplifying assumption of uniform circular motion of celestial bodies revolving about a stationary spherical earth. Each body is assigned a period-relation parameter expressing the number of 360° revolutions it completes in a given number of years, as shown in Tables 1.1 and 1.2. Longitudes are expressed in signs, degrees, minutes, seconds, etc. and are restricted between 0 and 12 signs, each of 30 degrees, or 360 degrees (so mean longitudes are values modulo 360). After the degrees place, each consecutive place is a sexagesimal (base 60) unit. Given its known starting position (celestial longitude) at a known past time, its mean longitude at any later time can be computed using the given parameter. Mean longitudes of the planets are computed for mean sunrise on the standard meridian, which passes through the ancient city of Ujjain, India and intersects the equator at Laṅkā, the terrestrial latitude and longitude zero point. The mean longitudes are subsequently corrected to true longitudes for mean sunrise at Laṅkā using more complicated orbital models, with additional circles called *manda* and *śīghra*, not centered upon the earth. These orbitally-corrected longitudes have additional time and place corrections applied to yield the true longitudes of the planets at a desired time for an observer's locale. These calculations are typically described in the second chapter or *spaṣṭādhikāra* ('true [motion] chapter'). This standard assignment of second chapter to *spaṣṭa* ("true") procedures is modified by Gaṇeśa as two separate chapters, one for the Sun and Moon and one for the five star-planets, the *Grahalāghava*'s second and third chapters respectively.

Gaṇeśa introduces his text with a conventional *maṅgalācaraṇa* ('auspicious practice') invoking the divine and supplying some contextual information about the work. The technical content of the chapter comprises the following:

- Computation of the *ahargaṇa* or accumulated days since the end of the most recent completed cycle (*cakra*) of 11 years elapsed since the epoch date.
- The epoch mean longitude (*kṣepaka*) and residual mean longitude (*dhruva*) at the end of a *cakra* for each planet. These numerical values are expressed in standard Sanskrit *bhūtasaṃkhyā* or 'word-numeral' format, i.e., with numerical digits represented by nouns traditionally associated with the number in question.

  The nine 'planets' in this context are the Sun, the Moon, Mars, Jupiter, Saturn, the apogee and ascending node of the lunar orbit, and the quantities called *śīghra-kendra* or *śīghra*-anomaly associated

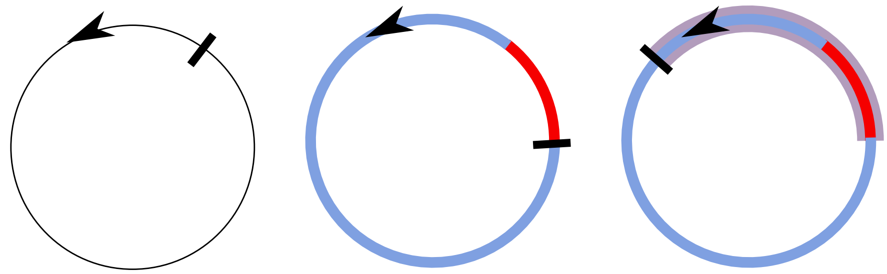

Figure 2.1: Illustration of the components of planetary mean longitude in the *Grahalāghava*. The circle represents the mean planet's geocentric orbit and the tickmark on the circumference represents its position in longitude, increasing counterclockwise. The desired date is here assumed to be less than two *cakras* since epoch.
Left: The planet's starting mean longitude at the epoch date.
Center: Mean longitude after the lapse of one *cakra* since epoch. Adding to the epoch mean longitude the planet's mean longitudinal increment in a *cakra* (blue) is equivalent to subtracting from it the planet's specified *dhruva* (red).
Right: Adding to the previous mean longitude the planet's mean longitudinal increment during the *ahargaṇa* (gray) produces the mean longitude for the desired date.

with Mercury and Venus. Since the (mean) positions of the actual bodies of Mercury and Venus are considered to coincide with the mean solar longitude, their distinctive mean-motion parameters are assigned to their *śīghra*-anomalies instead. The *śīghra*-anomaly is the longitudinal elongation between the planet's mean position and the apogee of its *śīghra*-circle.

- Algorithms to determine for each planet its net increment in mean longitude during the *ahargaṇa*, i.e., from the end of the most recently completed *cakra* to the start of the desired day. The planet's mean longitude at the start of the desired day is thus the sum of its epoch mean longitude, the (negative) product of its *dhruva* with the number of completed *cakras* since epoch, and its *ahargaṇa*-increment, as illustrated in figure 2.1.

- Approximate values of the planetary mean-motion parameters in the form of mean daily longitudinal increments in degrees and/or arcminutes per day.

- Derivation of different planetary mean-motion parameters from different astronomical schools or *pakṣas*.

2.2 Verses 1.1–1.3: Benediction (*Maṅgalācaraṇa*)

By Gaṇeśa's time, authors of treatises in the Sanskrit *śāstras* or technical disciplines customarily opened their works with a *maṅgalācaraṇa* consisting of one or more highly embellished verses.[1] These embellishments (Skt. *alaṅkāra*), imitating the literary ornamentation employed in poetry (*kāvya*) and other Sanskrit belles-lettres, play with the meaning of the words (*artha-alaṅkāra*) or with their sound (*śabda-alaṅkāra*).

Gaṇeśa has composed three *maṅgalācaraṇa* verses, of which the first two employ an *arthālaṅkāra* called *śleṣa* (lit. 'union"), i.e., double meaning or pun. This form of *śleṣa* is a complex double narrative (*dvi-saṃdhāna*) in which entire verses or verse sequences can be read with two distinctly different meanings. In the first verse, one of Gaṇeśa's meanings praises the deity Viṣṇu under his epithet Keśava, together with his speech (*vāc*), embodied as the sacred Vedic text (*śrūti*). The other meaning praises Gaṇeśa's father Keśava and his composed 'speech' in the form of astronomical texts, as discussed in Section 2.7. The second verse

[1]The history of the *maṅgalācaraṇa* and its characteristics is surveyed in [Min06].

also can be read in two ways: praising Rāma as an incarnation of Viṣṇu, or introducing the reader to a *karaṇa* text.

ज्योतिःप्रबोधजननी परिशोध्य चित्तं
तत्सूक्तकर्मचरणैर्गहनार्थपूर्णा।
स्वल्पाक्षरापि च तदंशकृतैरुपायै-
र्व्यक्तीकृता जयति केशववाक् श्रुतिश्च ॥ १ ॥ ॥ वसन्ततिलका ॥

jyotiḥprabodhajananī pariśodhya cittaṃ
tatsūktakarmacaraṇair gahanārthapūrṇā |
svalpākṣarāpi ca tadaṃśakṛtair upāyair
vyaktīkṛtā jayati keśavavāk śrutiś ca || 1 || || *vasantatilakā* ||

Reading 1: The generator (*jananī*) of the awakening (*prabodha*) of enlightenment (*jyotiḥ*), purifying the mind by the performance (*caraṇa*) of the well-proclaimed (*sūkta*) Vedic rituals (*karma*), full of explanation of the inexplicable (*gahanārtha*) yet few in syllables (*svalpākṣarā*), made clear by aids (*upāya*) composed by his devotees: long live the Vedas (*śruti*), the word of Keśava [Viṣṇu]!

Reading 2: The generator (*jananī*) of understanding (*prabodha*) in astronomy (*jyotiḥ*), clarifying [doubtful] thinking by the practice of its well-articulated rules (*tatsūktakarma*), full of deep meaning (*gahanārtha*) yet few in syllables (*svalpākṣarā*), made clear by supplements (*upāya*) composed by his adherents: long live the words and speech of [the astronomer] Keśava!

परिभग्नसमौर्विकेशचापं दृढगुणहारलसत् सुवृत्तबाहुम् ।
सुफलप्रदमात्तनृप्रभं तत् स्मर रामं करणं च विष्णुरूपम् ॥ २ ॥ ॥ औपच्छन्दसिक ॥

paribhagnasamaurvikeśacāpaṃ dṛḍhaguṇahāralasat suvṛttabāhum
suphalapradam āttanṛprabhaṃ tat smara rāmaṃ karaṇaṃ ca viṣṇurūpam ||2|| *aupacchandasika* ||

Reading 1: And bear in mind, [student,] that skillful (*karaṇa*) Rāma, the form of Viṣṇu, [by whom] the bow of Śiva (*īśacāpa*) with the bowstring (*maurvikā*) [was] broken, gleaming with a garland (*hāra*) of [many] bound strands (*dṛḍha-guṇa*) [and/or: with a garland of the quality (*guṇa*) of steadfastness (*dṛḍha*)], with well-shaped arms (*suvṛtta-bāhu*), bestowing good outcomes (*su-phala-prada*), [having] taken (*ātta*) the form of man (*nṛ-prabhā*).

Reading 2: And bear in mind, [student,] that lovely (*rāma*) astronomical handbook (*karaṇa*), as beautiful as Viṣṇu, devoid of arcs with their sines (*maurvikā-īśacāpa*), adorned with reduced (*dṛḍha*) multipliers (*guṇa*) and divisors (*hāra*), [with] excellent circles and arcs (*suvṛtta-bāhu*), bestowing excellent results (*su-phala-prada*), [with] the gnomon-shadow (*nṛ-prabhā*) obtained (*ātta*).

Commentary: In the 'Vedic' interpretation of the first verse, the ancient divinely proclaimed (*śrūti*) sacred texts are expounded by sages, and thus the ritual acts (*karma*) they prescribe are correctly performed and the practitioner attains enlightenment. Alternatively, in the astronomical interpretation, the statements of Gaṇeśa's father are elucidated by commentaries and thus the reader attains understanding of *jyotiṣa* 'light, luminary, astronomy'.

The second verse extols the hero Rāma, an incarnation of the god Viṣṇu, and the well-known Hindu legend of his unsurpassed strength breaking the great bow of the god Śiva, which none of Rāma's rivals had even managed to string. Its astronomical interpretation puns on the technical meanings of many of the deity's epithets, and especially on the arc and sine concepts represented by 'bow' and 'bowstring', which Gaṇeśa prides himself on having eliminated from the *Grahalāghava* (see section 1.4). The other features singled out for praise in this verse will be discussed as they appear in subsequent sections of the handbook.

Mallāri and Viśvanātha, in addition to providing *maṅgalācaraṇa* verses of their own at the start of their commentaries, painstakingly explain most of the *śleṣa* meanings in Gaṇeśa's invocatory verses. The following excerpts will give an idea of their exposition.

Mallāri, invoking the supernatural beings Rāvaṇa and Vāc:[2]

> [...] अर्थावबोधं विना श्रुत्युक्तकर्माचरणं कथं स्यातत एवाह । तदंशकृतैस्तस्य परमेश्वरस्य येंऽशा रावणाद्यास्तैः कृता ये उपाया भाष्यादयस्तैर्व्यक्तीकृता प्रकटीकृता रावणभाष्याद्यवलोकनेन तदुक्तकर्माचरणं सम्यगेव स्यादिति विष्णुपक्षे । अथ पितृपक्षे । केशवस्य पितुर्वाक् ग्रहकौतुकादिग्रन्थरूपा जयतीति । [...]
>
> [...] *arthāvabodhaṃ vinā śrutyuktakarmācaraṇaṃ kathaṃ syāt ata evāha | tadaṃśakṛtais tasya parameśvarasya ye 'ṃśā rāvaṇādyās taiḥ kṛtā ye upāyā bhāṣyādayas tair vyaktīkṛtā prakaṭīkṛtā rāvaṇabhāṣyādyavalokanena taduktakarmācaraṇaṃ samyag eva syād iti viṣṇupakṣe | atha pitṛpakṣe | keśavasya pitur vāk grahakautukādigrantharūpā jayatīti* | [...]
>
> [...] Without perception of the meaning, how [can] the performance of the acts proclaimed in *śrūti* be [carried out]? Here he states that. 'By his devotees': By those who [are] the devotees of that Supreme Lord, Rāvaṇa etc., [are] made the 'aids' (vernacular commentary etc.) by which [*śrūti* is] 'made clear', made manifest; through studying the *Rāvaṇabhāṣya* etc., the performance of the acts proclaimed by that [*śrūti*] indeed will be correct. Thus the Viṣṇu-interpretation.
>
> Now the father-interpretation: 'Long live' the 'speech' of [his] father Keśava, in the form of the books *Grahakautuka* etc. [...]

Viśvanātha:

> [...] ज्योतिःप्रबोधजननी । ज्योतिषां ग्रहनक्षत्रतारादीनां प्रबोधं ज्ञानं जनयतीति सा । अन्यत्र ज्योतिषस्तेजसः परब्रह्माख्यस्य प्रबोधो ज्ञानं तज्ज्ञनयतीति सा । [...]
>
> [...] *jyotiḥprabodhajananī* | *jyotiṣāṃ grahanakṣatratārādīnāṃ prabodhaṃ jñānaṃ janayatīti sā* | *anyatra jyotiṣas tejasaḥ parabrahmākhyasya prabodho jñānaṃ tajjanayatīti sā* | [...]
>
> [...] 'Generator of the awakening of enlightenment': [It] causes to be born the 'awakening', knowledge, of the 'lights', planets [and] lunar mansions [and] stars etc; in this way it [is Keśava's speech].
>
> In another [interpretation]: The 'awakening', knowledge, of enlightenment, spirit, [what is] called Brahman—[it] causes that to be born; in this way it [is *śrūti*]. [...]

The third verse of the *maṅgalācaraṇa* features Gaṇeśa's statement of purpose for the *Grahalāghava*.

> यद्यप्यकार्षुरुरवः करणानि धीरा-
> स्तेषु ज्यकाधनुरपास्य न सिद्धिरस्मात्।
> ज्याचापकर्मरहितं सुलघुप्रकारं
> कर्तुं ग्रहप्रकरणं स्फुटमुद्यतोऽस्मि ॥ ३ ॥ ॥ वसन्ततिलका ॥
>
> *yady apy akārṣur uravaḥ karaṇāni dhīrās*
> *teṣu jyakādhanurapāsya na siddhir asmāt* |
> *jyācāpakarmarahitaṃ sulaghuprakāraṃ*
> *kartuṃ grahaprakaraṇaṃ sphuṭam udyato'smi* || 3 || || *vasantatilakā* ||

[2]To Rāvaṇa, the demon antagonist of Rāma in the great epic *Rāmāyana*, is ascribed the *Ṛgveda* commentary *Rāvaṇabhāṣya* (FitzEdward Hall, 1862). We have explained the prestige of Vāc in footnote 2 of section 1.1.

Although great scholars made astronomical handbooks, in them accuracy is not obtained in the omission of sines and arcs. Therefore, I am undertaking to make a correct treatise on the planets, of an excellent and easy kind, excluding operations with sines and arcs.

Commentary: Gaṇeśa concludes these opening invocatory verses with his aspirations for this work, most importantly the complete elimination of trigonometry from the rules and procedures he presents. As seen briefly in section 1.3, many of Gaṇeśa's predecessors came up with methods of easing computation. He acknowledges that many of his 'great scholar' predecessors had similar ambitions, but he is critical of their attempts, dismissing them as inaccurate.

2.3 Verses 1.4–1.5: Calculating the *ahargaṇa*, 'accumulated days' between the epoch and the desired time

In the following two verses, Gaṇeśa outlines the process to determine the *ahargaṇa* or accumulated civil days between a known point in the past and the user's desired date. This involves computing the number of elapsed lunar (synodic) months in that interval, converting this to an associated amount of lunar days or *tithis* (i.e., thirtieths of a mean synodic month), then converting this quantity of *tithis* into civil days. An accompanying rule determines the current week-day from the computed *ahargaṇa*.

द्व्यब्धीन्द्रोनितशक ईशहृत् फलं स्या-
च्चक्राख्यं रविहतशेषकं तु युक्तम्।
चैत्राद्यैः पृथगमुतः सदृग्घ्नचक्राद्-
दिग्युक्तादमरफलाधिमासयुक्तम् ॥ ४ ॥ ॥ प्रहर्षिणी ॥

खत्रिघ्नं गततिथियुङ्निरग्रचक्रा-
ङ्गांशाढ्यं पृथगमुतोऽब्धिषट्कलब्धैः।
ऊनाहैर्वियुतमहर्गणो भवेद्वै
वारः स्याच्छरहतचक्रयुग्गणोऽब्जात् ॥ ५ ॥ ॥ प्रहर्षिणी ॥

dvyabdhīndronitaśaka īśahṛt phalaṃ syāt
cakrākhyaṃ ravihataśeṣakaṃ tu yuktam |
caitrādyaiḥ pṛthagamutaḥ sadṛgghnacakrāt
digyuktād amaraphalādhimāsayuktam || 4 || || *praharṣiṇī* ||

khatrighnaṃ gatatithiyuṅniragracakrā-
ṅgāṃśāḍhyaṃ pṛthagamuto'bdhiṣaṭkalabdhaiḥ |
ūnāhair viyutam ahargaṇo bhavedvai
vāraḥ syāc charahatacakrayuggaṇo'bjāt || 5 || || *praharṣiṇī* ||

When the present Śaka year is decreased by 1442 [and] divided by 11 (*īśa*), the quotient is referred to as the *cakra* ("cycle"). The remainder is multiplied by 12 (*ravi*) [and] increased by the [number of elapsed months of the present year] starting from Caitra. [It is] increased by the *adhikamāsa* ('[number of] intercalary months") [derived] from that [previous result placed] separately [and] increased by the *cakra* multiplied by 2 (*dṛk*), increased by 10 (*dik*), [and] divided by 33 (*amara*).

[That result is] multiplied by 30 (*kha-tri*) and increased by [the number of] elapsed *tithis* [of the present month] and a sixth (*aṅga*) part, without remainder, of the *cakra*. [That result] decreased by the omitted days, [i.e., a] sixty-fourth (*abdhi-ṣaṭka*) part of that [same result placed] separately, is certainly the day-accumulation (*ahargaṇa*). The day of the week is [that *ahargaṇa*] increased by the *cakra* times 5 (*hara*), [counted modulo 7] from Monday.

Commentary: The user is required to find the number of accumulated civil days or *ahargaṇa* from a given point in the past to the desired date in the present year y_0. This given past moment is found by counting from the *Grahalāghava*'s epoch date—which is the start of Śaka year[3] 1442 (mostly corresponding to 1520 CE)—successive integer cycles of 11 years, which Gaṇeśa calls *cakras.* The beginning of the present incomplete *cakra* is the moment from which the *ahargaṇa* is reckoned.

Casting out the *cakras* yields an integer number c of elapsed *cakras* since epoch and an integer number $y_c < 11$ of elapsed years within the current *cakra*:

$$c = (y_0 - 1442) \quad \text{div } 11, \qquad y_c = (y_0 - 1442) \quad \text{mod } 11.$$

The y_c years are converted into $12y_c$ solar months or twelfths of a year and added to the m elapsed calendar months of the present year, starting from the month Caitra.

In Gaṇeśa's algorithm, this accumulated solar-month count $12y_c + m$ is augmented by a corresponding number of intercalary months (*adhimāsas*) to produce a sum m_c of lunar months elapsed in the current *cakra*:

$$m_c = (12y_c + m) + \frac{(12y_c + m) + (2c + 10)}{33}.$$

The *adhimāsa* term, according to Mallāri's commentary [Jos81, p. 10, lines 29–33], is derived from the Brāhmapakṣa parameters for the lifetime of the universe or *kalpa*:

> यदि कल्पसौरमासैः ५१८४०००००००० कल्पाधिमासा १५९३३०००००० लभ्यते तदेष्टसौरमासैः किमिति । अत्र कल्पाधिमासैः कल्पसौरमासेषु भक्तेषु लब्धम् ३२ । १६ । ४ एभिर्मासैरेकोऽधिमासः ॥
>
> *yadi kalpasauramāsaiḥ 51840000000 kalpādhimāsā 1593300000 labhyate tadeṣṭasauramāsaiḥ kim iti | atra kalpādhimāsaiḥ kalpasauramāseṣu bhakteṣu labdham 32 | 16 | 4 ebhir māsair eko 'dhimāsaḥ | |*
>
> If 1593300000 intercalary months in a *kalpa* are obtained with 51840000000 solar months in a *kalpa*, then what [is obtained] with the desired [number of] solar months? Here, when the *kalpa* solar months are divided by the *kalpa* intercalary months, the quotient is 32|16|4: one intercalary month in these months.

Given that Gaṇeśa has stated that his solar and lunar calculations are derived from Saurapakṣa parameters, not Brāhmapakṣa (see section 2.7), it is a little odd that Mallāri has chosen Brāhmapakṣa parameters for his demonstration of the formula. In fact, the Saurapakṣa parameters for a *mahāyuga* of 4320000 years or $12 \cdot 4320000 = 51840000$ solar months are a slightly better fit to the intercalation period stated by Mallāri:[4]

$$\frac{\text{Number of solar months}}{\text{Number of } \textit{adhimāsas}} = \frac{51840000}{1593336} \approx 32.53551 \approx 32 \text{ months, } 16 \text{ days, } 4 \ \textit{ghaṭīs},$$

assuming 30 days per solar month and 60 *ghaṭīs* per day. This quantity rounded up to 33 produces the divisor in the *adhimāsa* term.

[3] Two main calendars were used in the Sanskrit astronomy tradition– the Śaka era beginning at 78CE and the Saṃvat era beginning at 57CE.

[4] The corresponding Brāhmapakṣa ratio $\frac{51840000000}{1593300000} \approx 32.53625$ converts to 32 integer solar months, 16 days and a little over 5 *ghaṭīs*. But Mallāri's value is corroborated by a verse that he quotes immediately after the above passage:

> उक्तं च ब्रह्मसिद्धान्ते ।
> द्वात्रिंशद्भिर्गतैर्मासैर्दिनैः षोडशभिस्तथा ।
> घटिकानां चतुष्केण पतति ह्यधिमासक इति ।
>
> *uktaṃ ca brahmasiddhānte |*
> *dvātriṃśadbhir gatair māsair dinaiḥ ṣoḍaśabhis tathā |*
> *ghaṭikānāṃ catuṣkeṇa patati hy adhimāsaka iti |*
>
> And it is stated in the Brahma *siddhānta*: 'An intercalary month occurs with [the period of] thirty-two elapsed [solar] months, sixteen days, and four *ghaṭīs*'.

We have not found this verse in the *Brāhmasphuṭasiddhānta*, and are not sure what Mallāri's source for it is.

In an 11-year *cakra* of 132 solar months, there will consequently be four complete intercalation periods plus an excess of a little less than 2 months:

$$132 - 4 \cdot \frac{51840000}{1593336} \approx 1.85796 \quad \approx 1 \text{ month, 26 days.}$$

This remainder is rounded up to 2 excess solar months per *cakra* in the formula for m_c. Gaṇeśa has reckoned that 10 of these excess solar months had been accumulated at his epoch date Śaka 1442. And since two additional excess months are accumulated in each of the c *cakras* since epoch, the total number of required intercalary months at the desired date will be 1/33 of the sum $(12y_c + m) + (2c + 10)$.

The m_c synodic months are converted into $30m_c$ *tithis* and added to the τ elapsed *tithis* of the present month, along with an additional specified term $\frac{c}{6}$, to give the number τ_c of *tithis* in the current *cakra*:

$$\tau_c = (30m_c + \tau) + \frac{c}{6}.$$

Finally, the corresponding number d_c of civil days or *ahargaṇa* in the current *cakra* is declared to be

$$d_c = \tau_c - \frac{\tau_c}{64}.$$

These conversion rules are likewise derivable from *pakṣa* period relations. The number of *tithis* per year based on a Brāhmapakṣa *kalpa* is

$$\frac{\text{Number of } \textit{tithis}}{\text{Number of years}} = \frac{30 \cdot \text{Number of synodic months}}{\text{Number of years}} = \frac{1602999000000}{4320000000} \approx 371.06458,$$

and based on a Saurapakṣa *mahāyuga*,

$$\frac{\text{Number of } \textit{tithis}}{\text{Number of years}} = \frac{30 \cdot \text{Number of synodic months}}{\text{Number of years}} = \frac{1603000080}{4320000} \approx 371.06483.$$

The corresponding number of civil days per *tithi* is, per the Brāhmapakṣa,

$$\frac{\text{Number of days}}{\text{Number of } \textit{tithis}} = \frac{1577916450000}{1602999000000} = 1 - \frac{25082550000}{1602999000000} \approx 1 - \frac{1}{63.9089} \approx 1 - \frac{1}{63;54,32,9},$$

or per the Saurapakṣa,

$$\frac{\text{Number of days}}{\text{Number of } \textit{tithis}} = \frac{1577917828}{1603000080} = 1 - \frac{25082252}{1603000080} \approx 1 - \frac{1}{63.9097} \approx 1 - \frac{1}{63;54,34,55}.$$

Thus the conversion factor from *tithis* to days is approximately $1 - \frac{1}{64}$.

But as Mallāri points out [Jos81, p. 11, lines 15–19], the *tithi*-number τ_c requires adjusting by the $\frac{c}{6}$ term before converting to days:

> [...] चक्रषट्के वर्षाणि ६६ एषां दिनानि २४४८६ एकत्र ६३ । ५४ । ३२ एभिरेकत्र च ६४ एभिर्भक्तं लब्धे फले ३८३ । ३८२ अवयवस्य त्यागः । फलान्तरम् १ । तेनानुपातः । यदि षड्भिश्चक्रैरेकदिनतुल्यमन्तरं तदेष्टचक्रैः किमित्यतो [...]
>
> [...] *cakraṣaṭke varṣāṇi 66 eṣāṃ dināni 24486 ekatra 63 | 54 | 32 ebhir ekatra ca 64 ebhir bhaktaṃ labdhe phale 383 | 382 avayavasya tyāgaḥ | phalntaram 1 | tennuptaḥ | yadi ṣaḍbhiś cakrair ekadinatulyam antaraṃ tadeṣṭacakraiḥ kim ity* [...]
>
> [...] In 6 *cakras* [there are] 66 years; [when] their 24486 [lunar] days [are] divided in one place by 63;54,32, and in [another] place by 64 the two quotient results [are] 383 [and] 382, leaving out

> the fractions. The difference of the results [is] 1. The proportion using that: If with six *cakras* the difference [of the two results] is equivalent to 1 [lunar] day, then what [is the difference] with the desired [number of] *cakras*? [...]

That is, there are about $371 \cdot 66 = 24486$ *tithis* in 66 years or 6 *cakras*. Dividing this number by 64 to get the day-conversion term yields a quotient of approximately 382, whereas dividing it by the more precise $63;54,32$ (note that this number too indicates Mallāri's reliance on Brāhmapakṣa parameters) yields approximately 383. Consequently, for every six *cakras* in the time since epoch, one extra *tithi* has to be thrown into the τ_c sum, to compensate for the too-large divisor 64 making the quotient $\frac{\tau_c}{64}$ too small.

We have made a few additional approximations in the calculations so far. For instance, we added the elapsed synodic months in the current year to the number of solar months elapsed in the current *cakra*, despite synodic months and solar months having unequal time duration. Moreover, rounding errors have not been accounted for systematically. As a result, we may need to empirically adjust the number of civil days in the *ahargaṇa* to sync with the current weekday at the end of it.

Gaṇeśa prescribes determining that weekday from the assumptions that the epoch weekday was a Monday and that the number of days in an 11-year *cakra* is 4016. Then the increment to the weekday per *cakra* is $4016 \mod 7 = 5$. So the current weekday number, counting from Monday, will be $(d_c + 5c) \mod 7$. Mallāri explains how to deal with any resulting off-by-one errors by invoking Bhāskara II's *Siddhāntaśiromaṇi* 1.6.1 [Jos81, 10, line 6], [Śās89, p. 32]:

> अभीष्टवारार्थमहर्गणश्चेत् सैको निरेकस्तिथयोऽपि तद्वदिति ।
>
> *abhīṣṭavārārtham ahargaṇaś cet saiko nirekas tithayo'pi tadvad iti.*
>
> For the purpose of [finding civil days corresponding to] the desired day, the number of civil days may be one day less or greater; and the [number of] *tithis* likewise.

The choice of the 11-year *cakra* cycle and its equation to 4016 civil days remain somewhat obscure. A realistic value of year-length—in fact, any value between 365.2 and 365.3 days—yields instead around 4017 to 4018 days in 11 years. But we can reproduce Gaṇeśa's slightly smaller value by estimating that the four complete intercalation cycles in an 11-year *cakra* produce a total of $12 \cdot 11 + 4 = 136$ synodic months or $30 \cdot 136 = 4080$ *tithis*, which converts to $4080 \cdot \left(1 - \frac{1}{64}\right) = 4016.25 \approx 4016$ days.

Mallāri explains the *cakra* concept itself as a technique for reducing the size of computations [Jos81, p. 10, lines 17–19]:

> एवं सत्याचार्येण बहुषु वर्षेष्वहर्गणबाहुल्यं स्यादतो लाघवार्थं शिष्यक्लेषभयार्थं च प्रथमं वर्षाणि यानि तान्येवैकादशतष्टानि कृतानि यल्लब्धं तस्य चक्रसंज्ञा कृता यच्चेषं तद्द्वादशगुणितं सन्मासाः कृतास्ते सौरमासाः ।
>
> *evaṃ saty ācāryeṇa bahuṣu varṣeṣv ahargaṇabāhulyaṃ syād ato lāghavārthaṃ śiṣyakleṣabhayārthaṃ ca prathamaṃ varṣāṇi yāni tāny evaikādaśataṣṭāni kṛtāni yallabdhaṃ tasya cakrasaṃjñā kṛtā yac-ceṣaṃ taddvādaśaguṇitaṃ san māsāḥ kṛtās te sauramāsāḥ.*
>
> With this [established] by the teacher: In many years there is a proliferation of civil days. So for the purpose of brevity and for fear of student mistakes, first, the very years that [have elapsed since epoch] are divided by 11. Whatever is the quotient, its name is the *cakra*; whatever is the remainder, it is multiplied by 12; the months produced are the solar months.

2.4 Verses 1.6–1.9: Mean longitude calculation parameters for the planets: *dhruvas* and *kṣepakas*

In verses 4–5, Gaṇeśa broke down the elapsed time between the *Grahalāghava*'s epoch and the user's desired day into two parts: the integer number c of completed 11-year *cakras*, and the *ahargaṇa* or integer number

$0 \leq d_c \leq 4016$ of civil days elapsed within the current *cakra*. In verses 6–8, Gaṇeśa states components of planetary mean longitudes that correspond to these time intervals.

The so-called *kṣepaka* is the planet's initial mean longitude $\bar{\lambda}_0$ at the epoch. In the course of one *cakra* after the epoch, the mean planet completes an integer number of revolutions plus an excess fractional revolution, or residual mean longitude. The *dhruva* $\bar{\lambda}_D$ is the explement or 360-complement of this residual mean longitude.

Subsequently, in verse 9, Gaṇeśa explains how these numbers are to be combined with the mean longitudinal increment accumulated in the elapsed *ahargaṇa* within the current *cakra*. That is, a planet's *dhruva* $\bar{\lambda}_D$ multiplied by the number c of elapsed *cakras* since epoch is subtracted from the planet's *kṣepaka* $\bar{\lambda}_0$ to produce its mean longitude at the end of the last completed *cakra*. The *ahargaṇa* d_c times the daily mean longitudinal increment of the planet is added to this result to obtain the planet's mean longitude on the desired day. This can be seen clearly in Figure 2.1.

2.4.1 Verses 1.6–1.8: *Dhruvas* and *kṣepakas*

खविधुतानभवास्तरणेर्ध्रुवः खमनला रसवार्धय ईश्वराः।
सितरुचो भमुखोऽथ खगा यमौ शरकृता गदितो विधुतुङ्गजः ॥ ६ ॥ ॥ द्रुतविलम्बित ॥

शैला द्वौ खशरा अगोः क्षितिभुवो भूतत्त्वदन्ता विदः।
केन्द्रस्याब्धिगुणोडवः सुरगुरोः खं षड्यमा वस्विलाः ॥
द्राक्केन्द्रस्य भृगोः कुशक्रयमला राश्यादिकोऽथो शनेः ।
शैलाः पञ्चभुवो यमाब्धय इमेऽथ क्षेपकः कथ्यते ॥ ७ ॥ ॥ शार्दूलविक्रीडित ॥

रुद्रा गोब्जाः कुवेदास्तपन इह विधौ शूलिनो गोभुवः षट्।
तुंगेऽक्षात्यष्टिदेवास्तमसि खमुडवोऽष्टाग्नयोऽथो महीजे ॥
दिक् शैलाष्टौ ज्ञकेन्द्रे विभकलनवभं पूजितेऽद्र्यश्विभूपाः ।
शौक्र केन्द्रे गृहाद्योऽद्रिनखनव शनौ गोतिथिस्वर्गतुल्यः॥ ८ ॥ ॥ स्रग्धरा ॥

khavidhutānabhavās taraṇer dhruvaḥ kham analā rasavārdhaya īśvarāḥ
sitaruco bhamukho'tha khagā yamau śarakṛtā gadito vidhutuṅgajaḥ || 6 ||
|| *drutavilambita* ||

śailā dvau khaśarā agoḥ kṣitibhuvo bhūtattvadantā vidaḥ
kendrasyābdhiguṇoḍavaḥ suraguroḥ khaṃ ṣaḍyamā vasvilāḥ |
drākkendrasya bhṛgoḥ kuśakrayamalā rāśyādiko'tho śaneḥ
śailāḥ pañcabhuvo yamābdhaya ime'tha kṣepakaḥ kathyate || 7 ||
|| *śārdūlavikrīḍita* ||

rudrā gobjāḥ kuvedās tapana iha vidhau śūlino gobhuvaḥ ṣaṭ
tuṃge'kṣātyaṣṭidevās tamasi khamuḍavo'ṣṭāgnayo'tho mahīje
dik śailāṣṭau jñakendre vibhakalanavabhaṃ pūjite'dryaśvibhūpāḥ
śaukra kendre gṛhādyo'drinakhanava śanau gotithisvargatulyaḥ || 8 ||
|| *sragdharā* ||

The *dhruva* of the sun is 0;1,49,11 (*kha-vidhu-tāna-bhava*) [and] the *dhruva* (*bhamukho*) of the moon is 0;3,46,11 (*kha-analā-rasa-vārdhi-īśvara*). Now 9,2,45 (*khaga-yama-śara-kṛtā*) are called [the *dhruva*] produced of the apogee of the moon.

[The *dhruva*] of the lunar ascending node is 7,2,50 (*śaila-dvau-kha-śara*), [the *dhruva* of] Mars is 1,25,32 (*bhū-tattva-danta*), [the *dhruva*] of the anomaly (*kendra*) of Mercury is 4,3,27 (*abdhi-guṇa-uḍava*), [the *dhruva*] of Jupiter is 0,26,18 (*kha-ṣaḍ-yama-vasu-ila*), the [*dhruva*] of the *śīghra*-anomaly of Venus is 1,14,2 (*ku-śakra-yamala*) beginning with [units] *rāśi*, now [the *dhruva*] of Saturn is 7,15,42 (*śaila-pañca-bhuva-yama-abdhi*). Now, the *kṣepaka* is described.

The *kṣepaka* in [the case of] the sun is 11,19,41 (*rudra-gobja-kuveda*), here in [the case of] the moon 11,19,6 (*śūlin-gobhuva-ṣaṭ*). In [the case of] the lunar apogee the *kṣepaka* is 5,17,33 (*akṣa-atyaṣṭi-deva*), the *kṣepaka* in [the case of] the lunar ascending node is 0,27,38 (*kha-uḍu-aṣṭa-agni*), and hereafter the *kṣepaka* in [the case of] Mars is 10,7,8 (*dik-śaila-aṣṭa*). The *kṣepaka* in [the case of] the [*śīghra*-]anomaly of Mercury is 8,29,33 (*vibhakalanavabha*), the *kṣepaka* in [the case of] Jupiter is 7,2,16 (*adri-aśvi-bhūpa*), the *kṣepaka* in [the case of] the [*śīghra*-]anomaly of Venus is 7,20,9 (*adri-nakha-nava*) [in] signs and so on, and the *kṣepaka* in [the case of] Saturn is equal to 9,15,21 (*go-tithi-svarga*).

Commentary: These versified tables of the *dhruvas* and *kṣepakas* (see Table 2.1) are expressed in *bhūta-saṃkhyā* units of signs, degrees, arcminutes, and arcseconds (*rāśis*, *aṃśas*, *kalās*, and *vikalās*).

Name of Planet	*dhruva*	*kṣepaka*
Sun	$0^s, 1°; 49, 11$	$11^s, 19°; 41$
Moon	$0^s, 3°; 46, 11$	$11^s, 19°; 6$
Lunar apogee	$9^s, 2°; 45$	$5^s, 17°; 33$
Lunar (ascending) node	$7^s, 2°; 50$	$0^s, 27°; 38$
Mars	$1^s, 25°; 32$	$10^s, 7°; 8$
Mercury's *śīghra*-anomaly	$4^s, 3°; 27$	$8^s, 29°; 33$
Jupiter	$0^s, 26°; 18$	$7^s, 2°; 16$
Venus's *śīghra*-anomaly	$1^s, 14°; 2$	$7^s, 20°; 9$
Saturn	$7^s, 15°; 42$	$9^s, 15°; 21$

Table 2.1: The planetary *dhruvas* or explements of residual mean longitude, and *kṣepakas* or epoch mean longitudes as stated by Gaṇeśa in verses 6–8.

The computations underlying these values for the *dhruvas* and *kṣepakas* are not explained in Gaṇeśa's verses. Mallāri's commentary, however, offers an exegesis of them after glossing Verses 6–7, emphasizing the *dhruva*'s transformation from a residual longitude to its 360-complement [Jos81, p. 18-19, lines 29–30, 1–7]:

> अत्रोपपत्तिः । अत्राचार्येण एकादशतष्टानि वर्षाणि कृत्वाहर्गणानयनं कृतम् । एवं योऽहर्गणः स एकादश-वर्षमध्यस्थ एव । तदुत्पन्ना ये ग्रहास्ते एकादशवर्षमध्य एव भवन्ति । अतो यावन्ति चक्राणि भुक्तानि तेषां ग्रहानानीय एतेषु प्रक्षिप्य ग्रन्थशकादिमारभ्यः ग्रहाः स्युरिति । एवमाचार्येण एकमितचक्रादेकादशवर्षात्मकात् ग्रहाः साधितास्ते यथा कल्पसौरवर्षैः कल्पग्रहभगणास्तदैकादशवर्षैः कतीति । अत्रागतानां भगणानां प्रयोज-नाभावाद्राश्याद्या एव गृहीतास्तेषां ध्रुवसंज्ञा कृता स्थिरत्वात् । अथवैकादशवर्षाणामहर्गणं प्रसाध्य पूर्वकरणोक्त-रीत्या ग्रहाः साधितास्ते ग्रहेषु योज्याः । अत्राचार्येण द्वादशराशिशुद्धान् कृत्वा ध्रुवसंज्ञा कृता । अतो दिनगणागतग्रहेषु ध्रुवा वियोज्या इत्यग्रे उक्तमस्ति चक्रशुद्धत्वात् ।
>
> *atropapatti | atrācāryeṇa ekādaśataṣṭāni varṣāṇi kṛtvāhargaṇānayanaṃ kṛtam | evaṃ yo'hargaṇaḥ sa ekādaśavarṣamadhyastha eva | tadutpannā ye grahās te ekādaśavarṣamadhya eva bhavanti | ato yāvanti cakrāṇi bhuktāni teṣāṃ grahān ānīya eteṣu prakṣipya granthaśakādim ārabhyaḥ grahāḥ syur iti | evam ācāryeṇa ekamitacakrād ekādaśavarṣātmakāt grahāḥ sādhitās te yathā kalpasaura-varṣaiḥ kalpagrahabhagaṇās tadaikādaśavarṣaiḥ katīti | atrāgatānāṃ bhagaṇānāṃ prayojanāb-hāvād rāśyādyā eva gṛhītās teṣāṃ dhruvasaṃjñā kṛtā sthiratvāt | athavaikādaśavarṣāṇām ahar-gaṇaṃ prasādhya pūrvakaraṇoktarītyā grahāḥ sādhitās te graheṣu yojyāḥ | atrācāryeṇa dvādaśa-rāśiśuddhān kṛtvā dhruvasaṃjñā kṛtā |ato dinagaṇāgatagraheṣu dhruvā viyojyā ity agre uktam asti cakraśuddhatvāt |*
>
> Here is the explanation. Here, having grouped the [elapsed] years in elevens, the teacher makes the determination of the *ahargaṇa*. The *ahargaṇa* thus [determined] falls within an eleven year [period]. [The mean longitudinal displacements of] the planets produced with that [*ahargaṇa*] fall within [an] eleven-year [period]. So, having found [the mean longitudinal displacements of]

> the planets [in] however many *cakras* have elapsed, and adding to them [the mean longitudinal displacements of the planets found with the *ahargaṇa*], [the mean longitudes of] the planets beginning from the the text's *śaka* [epoch] are produced.
>
> [The mean longitudinal increments of] the planets are found in that way by the teacher from one *cakra* consisting of 11 years; they are as [follows]: [if] in [the number of] solar years of a *kalpa* there are [a number of] revolutions of planets in a *kalpa*, then in eleven years, how many [revolutions are there]? Here, due to not using the resulting [completed integer] revolutions, only [the partial revolutions] in *rāśis* and so on are taken; the name *dhruva* is given to them because of the fixedness [of the mean longitudinal displacement over 11 years]. Next, having found the *ahargaṇa* of eleven years using the ways previously stated in [this] *karaṇa*, [the mean longitudinal displacements of] the planets are found; they are added to [the longitudes of] the planets.
>
> Here, making a subtraction from 12 signs, the teacher makes the definition of the *dhruva*. So, the *dhruvas* are subtracted from the [mean longitudinal increments of the] planets produced from the *ahargaṇa*: thus it is said below because of the subtraction from 360.

In this commentary section, Mallāri broadly explains the general process of finding mean longitudes of the planets. Their epoch positions are found by adding the mean displacements of the planets over elapsed *cakras* and mean displacement in the current *cakra* over elapsed *ahargaṇa*. He first explains how the *ahargaṇa* as computed in verses 4–5 is restricted to Gaṇeśa's 11-year or 4016 civil day range. So, the *dhruva* of planets in its conventional sense comes from the mean displacements of the planets in the multi-year period of one *cakra*. Thus, Mallāri states a proportion using period relations to find the mean longitudinal displacement of a planet in a *cakra*. The number of revolutions a planet completes in an 11-year cycle or *cakra* is proportional to the number of its revolutions in an entire *kalpa*:

$$\frac{\text{Revolutions of a planet in a } \textit{kalpa}}{\text{Number of solar years in a } \textit{kalpa}} = \frac{\text{Revolutions of a planet in a } \textit{cakra}}{\text{11 solar years in a } \textit{cakra}}$$

Multiplying both sides through by 11, the result is the total integer and fractional revolutions completed by the planet in a *cakra*. The integer part of this amount is discarded, and the fractional part represents the mean longitudinal displacement of a planet in a *cakra*. This displacement is then multiplied by the number of elapsed *cakras* to yield the mean displacement of a planet in the duration of elapsed *cakras*, which is alluded to but not explicitly stated in Mallāri's commentary here. Then, that displacement is added to the mean longitudinal displacement of a planet calculated for the *ahargaṇa* (as described in the last two verses) elapsed in the current *cakra*, and added to epochal mean longitudes to yield the mean longitudes of each planet. So, the canonical definition of a *dhruva* is an additive quantity involving the fractional displacements of planets in a *cakra*.

After explaining this canonical definition, Mallāri highlights the novelty of Gaṇeśa's mean longitude calculation in redefining the *dhruva* to refer to the 360-degree explement of the mean displacement of a planet in a *cakra*, instead of the mean displacement itself. After finding the mean longitudinal displacements of each planet per *cakra*, Gaṇeśa subtracts them from 12 *rāśis* (= 360°) to yield what he calls *dhruvas*, which are eventually multiplied by the number of elapsed *cakras* and are subtractive.

Mallāri subsequently provides an example of a *dhruva* computation [Jos81, p. 19, lines 8–10]. He computes the *dhruva* for the Sun, but does not explain the *dhruva* calculations for any other planet.

> अत्र बालावबोधार्थं धूलीकर्मणा एकादशवर्षाणामयमहर्गणः ४०१६ । अतोऽयमहर्गणो विश्वगुणस्त्रिखाङ्कैर्भक्त इत्यादिना जातो मध्यमो रविः ११ । २८ । १० । ४९ अयं द्वादशशुद्धो जातो रविध्रुवः ० । १ । ४९ । ११ ।
>
> *atra bālāvabodhārthaṃ dhūlīkarmaṇā ekādaśavarṣāṇām ayam ahargaṇaḥ 4016 | ato'yam ahargaṇo viśvaguṇas trikhāṅkair bhakta ityādinā jāto madhyamo raviḥ 11 | 28 | 10 | 49 ayaṃ dvādaśaśuddho jāto ravidhruvaḥ 0 | 1 | 49 | 11 |*
>
> Here, in order for a child to [easily] understand [the *dhruva*-computation], the *ahargaṇa* of eleven years [is] this 4016, [found] using means of computation [carried out] on dust [spread on the

ground or a board]. So, with "the *ahargaṇa* multiplied by 13 [and] divided by 903…" and so on the mean [longitude of the] Sun: $11^s, 28°; 10, 49$; This is subtracted from 12 signs to yield the *dhruva* of the Sun: $0^s, 1°; 49, 11$.

Mallāri explicitly notes here that there are 4016 civil days in one *cakra* and that this quantity has already been previously computed by a teacher (perhaps by Mallāri or Gaṇeśa himself) using the full process of calculating *ahargaṇa* for the time duration of 11 solar years. Here, with his mention of "*ahargaṇo viśvaguṇas trikhāṅkair bhakta…*" Mallāri refers to the mean longitudinal increment formula for the Sun in verse 1.7 of Bhāskara II's *Karaṇakutūhala*.[5] We now attempt to reconstruct the computation of the Sun's *dhruva* using the *Karaṇakutūhala*'s mean longitude calculation, since Mallāri has directly quoted it. The *Karaṇakutūhala* mean longitude formula for the Sun has three main parts: the epoch mean longitude (retained as the variable $\bar{\lambda}_0$ here), a mean longitudinal increment approximation roughly equivalent to the Sun's mean daily motion multiplied by *ahargaṇa*, and a correction term for accuracy (subtracting a 64th part of the number of years elapsed since epoch). So the Sun's mean longitude $\bar{\lambda}_0$ at the end of one 11-year *cakra* (equivalent to 4016 civil days) elapsed since the *Karaṇakutūhala*'s epoch is:

$$\begin{aligned}\bar{\lambda} &= \bar{\lambda}_0 + \left(\textit{ahargaṇa} - \frac{13 \cdot \ \textit{ahargaṇa}}{903}\right)^{\circ} - \left(\frac{\text{years elapsed since epoch}}{64}\right)' \\ &= \bar{\lambda}_0 + \left(4016 - \frac{13 \cdot 4016}{903}\right)^{\circ} - \frac{11'}{64} \\ &= \bar{\lambda}_0 + \left(10 + \frac{8086}{8127}\right) \text{ revolutions} - \frac{11'}{64}\end{aligned}$$

and ignoring the 10 complete revolutions as longitudes are signs, degrees, etc. mod 360, we get:

$$\begin{aligned}\bar{\lambda} &= \bar{\lambda}_0 + \frac{8086}{8127} \text{ revolutions} - \frac{11'}{64} \\ &\approx \bar{\lambda}_0 + 11^s, 28°; 11, 1, 47, 38 - 0^s, 0°; 0, 10, 18, 45 \\ &= \bar{\lambda}_0 + 11^s, 28°; 10, 51, 28, 53\end{aligned}$$

So, as per the *Karaṇakutūhala* formula, the mean longitudinal displacement of the Sun is $11^s, 28°; 10, 51$ (read 11 signs, 28 degrees, 10 minutes, 51 seconds) with the corrective term, or $11^s, 28°; 11, 2$ without. Subtracting both of these values from 12 signs yields either $1°; 49, 9$ or $1°; 48, 58$ respectively as *dhruvas*, neither of which are exactly Gaṇeśa's stated $1°; 49, 11$. Using the Brāhmapakṣa period relation of 4320000000 revolutions of the Sun completed in 1577916450000 civil days also yields its *dhruva* as $1°; 49, 9$ rounded to Gaṇeśa's stated significant figures. Interestingly, using the Saurapakṣa period relation of 4320000 revolutions of the Sun completed in 1577917828 civil days yields 10 complete revolutions + $11^s, 28°; 10', 49'$, which is Mallāri's stated mean longitudinal displacement, and yields Gaṇeśa's *dhruva*: $1°; 49, 11$.

Since recomputation using the *Karaṇakutūhala* algorithm does not accurately reproduce Gaṇeśa's stated value, maybe Mallāri had another reason for quoting the *Karaṇakutūhala* verse. Perhaps it simply indicated that a variety of different parameters and algorithms were viewed as acceptable sources of *dhruva* data. If so, this may shed some light on Gaṇeśa's intentional selection and rejection/modification of the parameters of other *pakṣas* as discussed in verse 16 and section 2.7.

[5] Mallāri is quoting the first part of verse 1.7 of the *Karaṇakutūhala* [MMP20, pp. 180–181]:

ahargaṇo viśvaguṇas trikhāṅkair
bhaktaḥ phalono dyugaṇo lavādyāḥ |
ravijñaśukrāḥ syur athābdavṛndād
vedāṃgalabdhena kalādinonāḥ ||

The *ahargaṇa* is multiplied by 13 [and] divided by 903. The *ahargaṇa* diminished by the result is [the mean longitude increment of] the Sun, Mercury, and Venus, beginning with degrees. These should be diminished by the minutes etc. of a 64th part of the accumulated years.

The next section of Mallāri's commentary includes his discussion of the *kṣepakas*. After glossing verse 8, Mallāri explains that these *kṣepakas* are epoch mean longitudes accounting for the mean longitudinal displacement of the planets from the start of the *kalpa* to the epoch of the *Grahalāghava*. He also alludes to their manner of computation, before remarking that they are by definition additive quantities [Jos81, p. 19, lines 26–30]:

> येऽत्र ग्रहास्ते ग्रन्थारम्भमारभ्य जाता अतो ग्रन्थारम्भग्रहा अत्र योज्यास्ते कल्पादितः स्युरिति । तत्साधनं यथा । द्व्यब्धीन्द्रतुल्यं १४४२ शकं प्रकल्प्य चैत्रशुक्लप्रतिपदि सूर्योदयिका मध्यमा ग्रहा यस्माद्यस्मात् पक्षाद्ये ये घटन्ते तत्तत्पक्षेभ्यस्ते ते साधितास्तेषां क्षेपसंज्ञा कृता यतः क्षिप्यतेऽसौ क्षेपः । अस्य ग्रहेषु क्षेप्यत्वात् क्षेपत्वम् ।
>
> *ye'tra grahās te granthārambham ārabhya jātā ato granthārambhagrahā atra yojyās te kalpāditaḥ syur iti |tatsādhanaṃ yathā |dvyabdhīndratulyaṃ 1442 śakaṃ prakalpya caitraśuklapratipadi sūryodayikā madhyamā grahā yasmād yasmāt pakṣād ye ye ghaṭante tattatpakṣebhyas te te sādhitās teṣāṃ kṣepasaṃjñā kṛtā yataḥ kṣipyate'sau kṣepaḥ |asya graheṣu kṣepyatvāt kṣepatvam |*
>
> [The *kṣepakas* of] the planets here are those produced from the start of text, so [the mean longitudes of] the planets at the start of the text that are added here are [computed] from the start of the *kalpa*. Their determination is thus. Fixing the *śaka* year equivalent to 1442, the mean [longitudes of] the planets [at] sunrise on the first day of the bright fortnight of the month of Caitra, which are those computed from whatever *pakṣa* [are] known as *kṣepa*, since the [entity] being added is the *kṣepa*. [It is] an addendum due to its needing to be added to [the mean longitudinal displacements of] the planets.

So, Mallāri states that the *kṣepakas* can be found by calculating the mean longitudinal displacement of each of the planets from the start of the *kalpa* until epoch (first day of the bright fortnight of the month of Caitra in *śaka* year 1442). These *kṣepakas* are also added to the mean longitudinal displacements produced from *ahargaṇa* calculation and so on, which is why they are named "*kṣepaka*" (lit. "entity which is added"). Most notable in this commentary section, however, is Mallāri's mention of using parameters of other *pakṣas* (*tattatpakṣebhya*) in the method of calculating *kṣepakas*. This notion of selecting *pakṣa* period relations is explicitly addressed in Gaṇeśa's verse 16. Thus in our commentary of that verse in Section 2.7 we explore the reconstruction of the *dhruvas* and *kṣepakas* using period relations from *siddhāntas* of each *pakṣa*.

While Mallāri's commentary only explicitly confirms that sourcing parameters from different *pakṣas* applies to the computation of *kṣepakas*, Viśvanātha's commentary seems to indicate that this is the case for *dhruva* computation as well. In his commentary, Viśvanātha provides three verses with updated *dhruvas* and *kṣepakas* users he had computed using Āryapakṣa parameters. He states that these should instead be used for mean longitudinal calculations of the Sun, Moon, and lunar apogee. We discuss his commentary in more depth in Section 2.7.

2.4.2 Verse 1.9: Determining the mean longitudes of the planets at a desired time, with the *deśāntara*-correction for the Moon

Gaṇeśa now states the mean longitude formula in verse 9. The *kṣepakas* and *dhruvas* mentioned in the previous three verses are used to find the mean displacements of the planet from the *Grahalāghava*'s epoch until the beginning of the current *cakra*. The formulae supplying the displacements in the current *cakra* corresponding to elapsed *ahargaṇa* are deferred until verses 10–14ab. The second half of this verse describes the Moon's *deśāntara* or "difference in place" correction, representing the longitudinal distance between an observer's locale and mean sunrise at Laṅkā.

> दिनगणभवखेटश्चक्रनिघ्नध्रुवोनो
> दिवसकृदुदये स्वक्षेपयुङ्मध्यमः स्यात् ।

निजनिजपुररेखान्तःस्थिताद्योजनौघा-
द्रसलवमितलिप्ताः स्वर्णमिन्दौ परे प्राक् ॥ ९ ॥ ॥ मालिनी ॥

dinagaṇabhavakheṭaś cakranighnadhruvono
divasakṛdudaye svakṣepayuṅmadhyamaḥ syāt |
nijanijapurarekhāntaḥsthitād yojanaughād
rasalavamitaliptāḥ svarṇam indau pare prāk || 9 || || *mālinī* ||

The [mean longitudinal displacement of] a planet corresponding to the *ahargaṇa*, diminished by the *dhruvas* multiplied by the *cakras*, increased by [the planet's] own *kṣepaka*, is the mean longitude [of the body] at sunrise.
The minutes of arc commensurate with a sixth (*rasa*) part of the number of *yojanas* situated between one's own place and the standard meridian [are applied] to [the mean longitude of] the Moon negatively or positively [when the location is] westwards [or] eastwards [of the meridian].

Commentary: Given a planet's *kṣepaka* $\bar{\lambda}_0$, *dhruva* $\bar{\lambda}_D$, number of elapsed *cakras* c, and mean longitudinal displacement in the current *cakra* $\bar{\lambda}_c$, its mean longitude $\bar{\lambda}$ is

$$\bar{\lambda} = \bar{\lambda}_0 - c \cdot \bar{\lambda}_D + \bar{\lambda}_c.$$

This mean longitude formula yields the mean longitudes of the planets on a desired date at mean sunrise at Laṅkā, the terrestrial latitude and longitude zero point. But at a nonzero longitude, mean sunrise may be earlier or later depending on if the observer is east or west of the standard meridian. Consequently, planets may have completed more or less of their mean revolutions about the Earth. This difference in mean sunrise times as a result of the distance in terrestrial longitude between the Laṅkā zero point and an observer's locale is accounted for by the *deśāntara*-correction. The *deśāntara*-correction $\Delta\bar{\lambda}$, which Gaṇeśa only specifies for the Moon's mean longitude, given the distance D in *yojanas* between the standard meridian and an observer's own longitude, is

$$\Delta\bar{\lambda} = \left(\frac{D}{6}\right)'$$

which is applied in arcminutes to the previously found mean longitude for the Moon. If the local terrestrial longitude of an observer is east of the standard meridian, the *deśāntara*-correction is a subtractive quantity, and if local terrestrial longitude is west of the standard meridian, the correction is additive.

Mallāri explains Gaṇeśa's formula using trigonometric proportions in [Jos81, p. 23, lines 11–17]:

अत्र रेखार्कोदयात् स्वार्कोदयः कदा भविष्यतीति ज्ञानार्थमुपायः । लङ्कायामुक्तः परमो भूपरिधिः सप्तारिनन्दाब्धि-तुल्यः ४९६७ । मेरौ परिधेरभावः । मध्ये ऽनुपातः । स यथा । लङ्कायामक्षज्याभावाल्लम्बज्या परमा त्रिज्यातुल्या । अतो यदि त्रिज्यातुल्यया लम्बज्ययायमुक्तो भूपरिधिस्तदेष्टलम्बज्यया किमिति लम्बज्यायाः सर्वत्र त्रिज्यातोऽल्पत्वादुक्तात् सर्वत्रोन एव भूपरिधिः स्यात् । अतः सुखार्थमष्टचत्वारिंशच्छतमितो गृहीतः ४८०० । ततो ऽनुपातः । यद्येभिः परिधियोजनै-४८०० ग्रहो गतिकलाः क्रामति तदेष्टैः रेखास्वदेशान्तरयोजनैः किमिति ।

atra rekhārkodayāt svārkodayaḥ kadā bhaviṣyatīti jñānārtham upāyaḥ | laṅkāyām uktaḥ paramo bhūparidhiḥ saptārinandābdhitulyaḥ 4967 | merau paridher abhāvaḥ | madhye 'nupātaḥ | sa yathā | laṅkāyām akṣajyābhāvāllambajyā paramā trijyātulyā | ato yadi trijyātulyayā lambajyayāyam ukto bhūparidhis tadeṣṭalambajyayā kim iti lambajyāyāḥ sarvatra trijyāto 'lpatvād uktāt sarvatrona eva bhūparidhiḥ syāt | ataḥ sukhārtham aṣṭacatvāriṃśacchatamito gṛhītaḥ 4800 | tato 'nupātaḥ | yady ebhiḥ paridhiyojanair 4800 graho gatikalāḥ krāmati tadeṣṭaiḥ rekhāsvadeśāntarayojanaiḥ kim iti |

Here, [is] a method for knowing when local sunrise will be from sunrise at the central meridian. At Laṅkā, the circumference of the Earth is said [to be] the maximal four thousand nine hundred

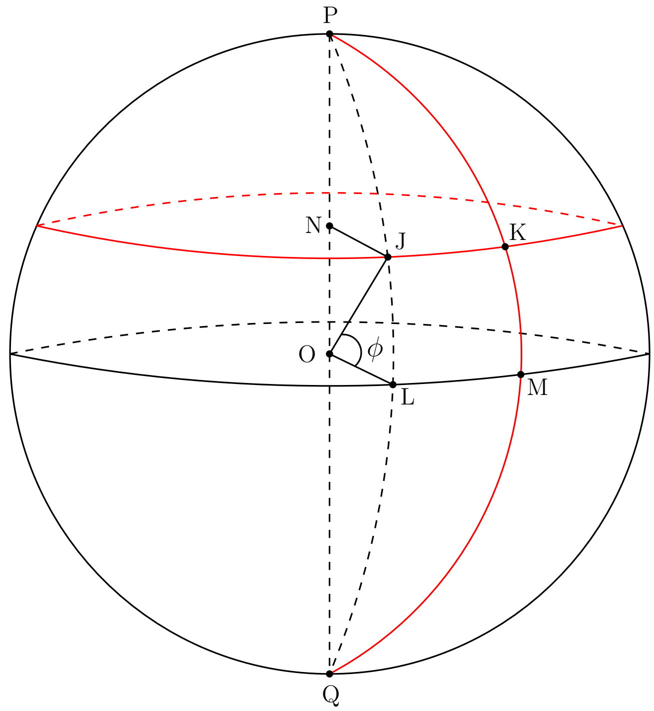

Figure 2.2: The *deśāntara*-correction is applied based on the terrestrial longitudinal distance in *yojanas* D from an observer's locale K at latitude ϕ on the meridian PKM and the zero point L at Laṅkā on the prime meridian PJL.

> and sixty seven 4967. At Meru, the circumference is nonexistent. In the middle, a proportion. That is thus: At Laṅkā, due to the nonexistence of the Rsine of latitude, the Rcosine of latitude is maximal [and] equivalent to three signs. So, if that stated circumference of the Earth [occurs] with the Rcosine of latitude equivalent to three signs, then what [occurs] with the Rcosine of the desired latitude? Due to the stated smallness of the Rcosine of latitude everywhere [as compared to] three signs, the circumference of the Earth will only be less everywhere [at a nonequatorial latitude]. So, for ease [of computation], [the circumference is] taken commensurate with four thousand eight hundred 4800. Then a proportion. If the planet traverses the arcminutes of [its mean daily] motion in these 4800 *yojanas* of circumference, then what is [its motion] in the desired *yojanas* of difference between one's own place and the central meridian?

Mallāri states that the circumference of the Earth is said to be 4967 *yojanas*, a value from the *Siddhāntaśiromaṇi* [Cha81b, p. 93, 1.7.1]. This is the circumference of the equator, which passes through through Laṅkā (point L on Figure 2.2), and has as its radius OL. At Mount Meru (point P), which is the North Pole, the latitudinal circle has no circumference. The latitudinal circle has its maximum circumference at the equator; every other latitudinal circle will have a smaller circumference. So, the circumference of a latitude of an observer's locale between Meru and Laṅkā can be found using trigonometry (with Figure 2.3 which shows the great circle through the North and South Poles and Laṅkā while passing through the intersection at J of an observer's parallel of latitude through JK with radius JN) and Mallāri's proportion.

An observer's latitude ϕ is the angle between the equator (the equatorial radius LO on the figure) and the observer's parallel of latitude (latitudinal radius JN). Point N is the perpendicular intersection of the latitudinal circle and the polar axis $PNOQ$. Then:

$$JN = JO \cdot \cos \angle OJN$$

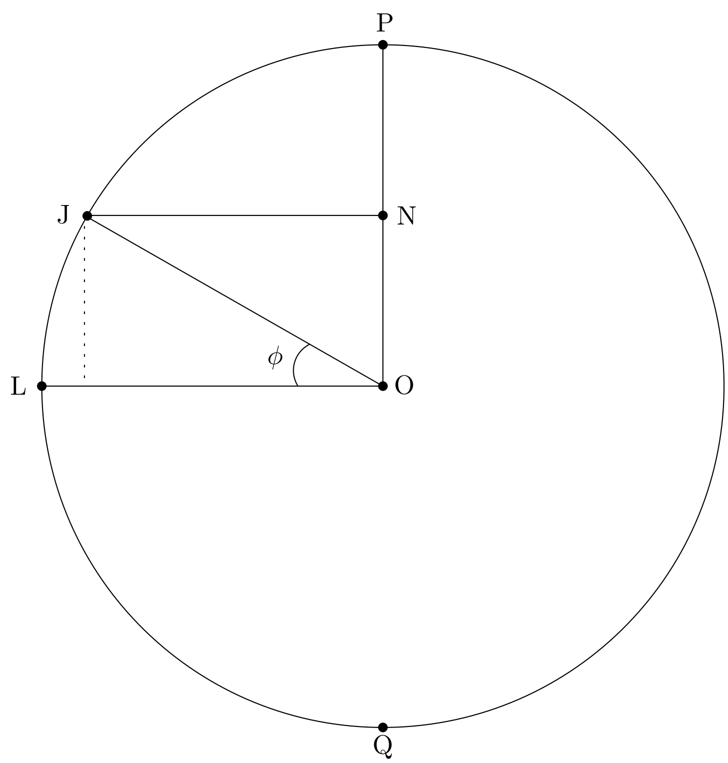

Figure 2.3: The great circle passing through the North Pole P, South Pole Q, and Laṅkā L with the center of the Earth O. The observer's parallel of latitude ϕ intersects this great circle at J, and N is the center of the small circle formed at the observer's latitude.

where $JO = LO = R$ which is the radius of the Earth. But $\angle OJN = \phi$ (which is the observer's latitude) since $JN \parallel LO$. In the case of an observer at Laṅkā, $\phi = 0$ and so the Rsine of latitude is also 0 and the Rcosine is maximally R.

Then Mallāri's proportion relates the circumferences of the latitudinal circles at local latitude and at the equator. We just found the radius of the locale latitudinal circle to be $JN = R\cos\phi$, and we use that the radius of the Earth is R, and the circumference of the Earth is 4967 *yojanas* to find that the circumference of the locale latitudinal circle l_c is:

$$\frac{4967 \textit{ yojanas}}{R} = \frac{l_c}{R\cos\phi}$$
$$l_c = \frac{4967}{R} \cdot R\cos\phi$$

Mallāri then states that this circumference is generalized to be 4800 for computational ease.[6] So the celestial sphere completes 1 rotation in 1 civil day, or covers these 4800 *yojanas* in 1 civil day. The Moon's mean daily motion (as stated in verses 14cd–15) is 790 arcminutes and 35 arcseconds. So, the Moon travels $790'35''$ in the same duration that the celestial sphere rotates through 4800 *yojanas*. Then for a distance D *yojanas* between the observer's locale and the central meridian through Laṅkā, the Moon travels $\Delta\bar{\lambda}$:

$$\frac{790'35'}{4800 \textit{ yojanas}} = \frac{\Delta\bar{\lambda}}{D}$$
$$\Delta\bar{\lambda} \approx \frac{800' \cdot D}{4800} = \left(\frac{D}{6}\right)'$$

which is indeed the *deśāntara*-correction Gaṇeśa states for the Moon's mean longitude.

Mallāri briefly comments [Jos81, p. 23, lines 17–18] as to why only the Moon's mean longitude has the *deśāntara*-correction applied:

> अत्रायं संस्कारश्चन्द्रस्यैव कृतः । अन्येषां गतेरल्पत्वान्न कृतः ।
>
> *atrāyaṃ saṃskāraś candrasyaiva kṛtaḥ |anyeṣāṃ gater alpatvān na kṛtaḥ |*
>
> Here, only the Moon's [*deśāntara*-]correction is found. It is not found [for the other planets due to the smallness of the [mean daily] motions of the other [planets].

As seen in Table 2.2, the mean daily motion of the Moon is by far the greatest at $790'35''$, thus the *deśāntara*-correction which is applied in arcminutes to mean longitudes otherwise in signs, degrees, minutes and so on would be too small in the cases of the other planets and is thus treated as negligible.

2.5 Verses 1.10–1.14ab: Mean Longitudinal Displacements of Planets

In this section, we examine Gaṇeśa's compactly versified algorithms for the planets' mean longitudinal increments over the course of the *ahargaṇa*, or $\bar{\lambda}_c$. We reconstruct their possible derivations from planetary parameters provided by various astronomical *pakṣas*, whose period relations can be reduced by continued-fraction approximations to convenient values of mean longitudinal increment for one day, $\bar{\lambda}_d$. In these reconstructions we are guided by resemblances between the *Grahalāghava*'s algorithms and their similarly framed counterparts in the earlier *Karaṇakutūhala* of Bhāskara, and by the continued-fraction analysis based

[6] 4800 *yojanas* is approximately the circumference for observers at around the 15th parallel. At Gaṇeśa's Nandigrāma on the 18th parallel, the circumference is approximately 4720 *yojanas*.

on period-relation parameters applied to both texts by Dvivedin [Mis91], [Dvi25]. The details of Bhāskara's algorithms and their reconstruction are available for comparison to Gaṇeśa's in [MMP20, pp. 175–191].

As discussed in section 2.7, Gaṇeśa claims at the end of this chapter to have used different *pakṣas* to obtain the mean-motion parameters of different planets. However, his assignments of planet to *pakṣa* do not always correspond to the parameters that provide the most accurate versions of the corresponding $\bar{\lambda}_c$ formulae. We provide for each formula a table comparing the reconstructed versions from different *pakṣa* parameters, along with a detailed sample reconstruction.

2.5.1 Verse 1.10: Mean longitudinal increments of the Sun/Mercury/Venus and the Moon

स्वखनगलवहीनो द्युव्रजोऽर्कज्ञशुक्राः
खतिथिहृतगणोनो लिप्तिकास्वंशकाद्याः ।
गणमनुहतिरिन्दुः स्वाद्रिभूभागहीनः
खमनुहृतगणोनो लिप्तिकास्वंशपूर्वः ॥ १० ॥ ॥ मालिनी ॥

svakhanagalavahīno dyuvrajo'rkajñaśukrāḥ
khatithihṛtagaṇono liptikāsv aṃśakādyāḥ |
gaṇamanuhatir induḥ svādribhūbhāgahīnaḥ
khamanuhṛtagaṇono liptikāsv aṃśapūrvaḥ || 10 || || *mālinī* ||

The *ahargaṇa* (*dyuvraja*) [is] diminished by a 70th (*kha-naga*) part of itself; [further] decreased by the *ahargaṇa* divided by 150 (*kha-tithi*) in arcminutes, [it becomes the mean longitudinal increment of] the Sun (*arka*), Mercury (*jña*), and Venus (*śukra*) in degrees and so on.

The product of the *ahargaṇa* and 14 (*manu*), diminished by its own 17th (*adri-bhū*) part, decreased by the *ahargaṇa* divided by 140 (*kha-manu*) in arcminutes, [is the mean longitudinal increment of] the Moon, beginning with degrees.

Commentary: The mean longitudinal increment $\bar{\lambda}_c$ accumulated by the Sun (which is taken to be identical with those of mean Venus and Mercury) over the d_c days of the *ahargaṇa* is declared to be

$$\bar{\lambda}_c = \left(d_c - \frac{d_c}{70}\right)^\circ - \left(\frac{d_c}{150}\right)'. \qquad \textbf{(Sun)}$$

The corresponding formula for the Moon is

$$\bar{\lambda}_c = \left(14d_c - \frac{14d_c}{17}\right)^\circ - \left(\frac{d_c}{140}\right)'. \qquad \textbf{(Moon)}$$

The assignment of units to the different terms in these formulae is somewhat ambiguous in Gaṇeśa's verse. But we can conclusively determine which terms are in degrees and which in arcminutes, either from the commentators' glosses or via the following reconstructions.

In the case of the Sun/Venus/Mercury, we manipulate parameters from the Brāhmapakṣa for a *kalpa* period of 4320000000 years to calculate the approximate mean longitudinal increment $\bar{\lambda}_d$ for one civil day, or mean daily motion in degrees per day:

$$\begin{aligned}
\bar{\lambda}_d &= \frac{\text{Number of revolutions} \cdot 360°}{\text{Number of days}} = \frac{4320000000 \cdot 360}{1577916450000} = \frac{384000}{389609} \\
&= \frac{1}{1 + \frac{5609}{384000}} = \frac{1}{1 + \frac{1}{68 + \frac{2588}{5609}}} \\
&\approx \frac{1}{1 + \frac{1}{69}} = \frac{69}{70} = 1 - \frac{1}{70}.
\end{aligned}$$

The difference between this approximate value and the exact fraction can be found using the same technique:

$$\frac{69}{70} - \frac{384000}{389609} = \frac{3021}{27272630}.$$

Converting this very small discrepancy from degrees to arcminutes for increased precision, we determine a simplified approximate value for it:

$$\frac{3021^{(\circ)}}{27272630} = \frac{181260^{(\prime)}}{27272630} = \frac{1}{150 + \frac{8363}{18126}} \approx \frac{1}{150}.$$

Combining these terms gives us Gaṇeśa's formula for the Sun's mean daily longitudinal increment $\bar{\lambda}_d$, shown below with its corresponding continued-fraction equivalents.

Sun	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{69}{70}^{\circ} - \frac{1'}{150}$	$\frac{69}{70}^{\circ} - \frac{1'}{150 + \frac{8363}{18126}}$	$\frac{69}{70}^{\circ} - \frac{1'}{149 + \frac{5669}{9846}}$	$\frac{69}{70}^{\circ} - \frac{1'}{149 + \frac{5571697}{18495198}}$

The reconstruction of the $\bar{\lambda}_d$ factor for the Moon can be derived from Saurapakṣa *mahāyuga* parameters in precisely the same way:

$$\begin{aligned}
\bar{\lambda}_d &= \frac{\text{Number of revolutions} \cdot 360°}{\text{Number of days}} = \frac{57753336 \cdot 360}{1577917828} = \frac{20791200960}{1577917828} \\
&= 13 + \frac{1}{5 + \frac{46642962}{69567299}} = 13 + \frac{1}{5 + \frac{1}{1 + \frac{22954337}{46642962}}} \\
&\approx 13 + \frac{1}{5 + \frac{1}{1 + \frac{1}{2}}} = 13 + \frac{3}{17} = 14 - \frac{14}{17}.
\end{aligned}$$

Again, this approximate value is a slight overestimate of the exact fraction, so we calculate their difference in arcminutes to give us the second of the two terms in Gaṇeśa's formula:

$$\begin{aligned}
&\left(13 + \frac{3}{17}\right) - \frac{20791200960}{1577917828} = \frac{3}{17} - \frac{69567299}{394479457} = \frac{794288}{6706150769}, \\
&\frac{794288^{(\circ)}}{6706150769} = \frac{47657280^{(\prime)}}{6706150769} = \frac{1}{140 + \frac{34131569}{47657280}} \approx \frac{1}{140}.
\end{aligned}$$

Moon	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{224}{17}^{\circ} - \frac{1'}{140}$	$\frac{224}{17}^{\circ} - \frac{1'}{144 + \frac{3083}{4160}}$	$\frac{224}{17}^{\circ} - \frac{1'}{144 + \frac{481}{10176}}$	$\frac{224}{17}^{\circ} - \frac{1'}{140 + \frac{34131569}{47657280}}$

Mallāri's commentary on all Gaṇeśa's $\bar{\lambda}_d$ formulae takes a different approach to 'explain' them. The following example [Jos81, p. 27, lines 1–13] will suffice to illustrate his reasoning.

एवमग्रे ऽपि भविष्यन्महागणकैर्नलिकाबन्धादिना ग्रहवेधं कृत्वान्तराणि लक्षयित्वा ग्रहकरणानि कार्याणीत्यग्रे ग्रन्थसमाप्तावाचार्येणाप्युक्तमस्ति । अतो ऽस्मिन् काले ऽत्रत्या एव ग्रहा घटन्ते । एवमनया वर्तमानघटनया ज्ञाता मध्यमा रविगतिर्भागाद्या ० । ५९ । ८ । ३४ । १७ । ९ तत्रानुपातः । यद्येकदिनेनैतावती गतिस्तदाहर्गणेन किमिति । अहर्गणस्य गतिर्गुणः । अत्र खण्डगुणनार्थं गतेरेकं खण्डं गत्यपेक्षयाधिकं गृहीतम् । रग ० । ५९ । ८ । ३४ । १७ । ९ अत्रैको धृतः । अन्तरम् ० । ० । ५१ । २५ । ४२ । ५१ अनेनाहर्गणो गुण्यः रूपगुणाहर्गणाच्चोध्यः । अत्र कर्मगौरवम् । लाघवार्थमिदम् ० । ० । ५१ । २५ । ४२ । ५१ यथैकसङ्ख्यं स्यात् तथा केनापि गुण्यम् । एवं सप्तति ७० गुणित ऊर्ध्वं रूपं निःशेषं भवति । अतो गणो रूपगुणः सप्ततिभक्तः फलेन रूपगुणो ऽहर्गणो हीनः कार्यो यतो ऽधिकं गृहीतम् । उभयत्र रूपतुल्यस्य गुणस्याविकृतत्वान्नाशः । एवं स्वखनगलवहीन इति । अथ गतेरपेक्षयाधिकं गृहीतं यत् खण्डम् ० । ० । ० । २४ । ० । ० अनेन गणो गुण्यः फलं रवौ हीनं कार्यमधिकत्वात् । अत्रापि लाघवार्थमिदं खतिथिभिः १५० सवर्णितं जातं कलास्थाने रूपम् ।

evam agre 'pi bhaviṣyanmahāgaṇakair nalikābandhādinā grahavedhaṃ kṛtvāntarāṇi lakṣayitvā grahakaraṇāni kāryāṇīty agre granthasamāptāv ācāryeṇāpy uktam asti | ato 'smin kāle 'tratyā eva grahā ghaṭante | evam anayā vartamānaghaṭanayā jñātā madhyamā ravigatir bhāgādyā 0/59/8/34/17/9 tatrānupātaḥ | yady ekadinenaitāvatī gatis tadāhargaṇena kim iti | ahargaṇasya gatir guṇaḥ | atra khaṇḍaguṇanārthaṃ gater ekaṃ khaṇḍaṃ gatyapekṣayādhikaṃ gṛhītam |

ra.ga. 0/59/8/34/17/9 atraiko dhṛtaḥ | antaram 0/0/51/25/42/51 anenāhargaṇo guṇyaḥ rūpaguṇāhargaṇāccodhyaḥ | atra karmagauravam | lāghavārtham idam 0/0/51/25/42/51 yathaikasaṅkhyaṃ syāt tathā kenāpi guṇyam | evaṃ saptati 70 guṇita ūrdhvaṃ rūpaṃ niḥśeṣaṃ bhavati | ato gaṇo rūpaguṇaḥ saptatibhaktaḥ phalena rūpaguṇo 'hargaṇo hīnaḥ kāryo yato 'dhikaṃ gṛhītaṃ | ubhayatra rūpatulyasya guṇasyāvikṛtatvān nāśaḥ evaṃ svakhanagalavahīna iti | atha gater apekṣayādhikaṃ gṛhītaṃ yat khaṇḍam 0/0/0/24/0/0 anena gaṇo guṇyaḥ phalaṃ ravau hīnaṃ kāryam adhikatvāt | atrāpi lāghavārtham idaṃ khatithibhiḥ 150 savarṇitaṃ jātaṃ kalāsthāne rūpam |

Thus even in the beginning, different procedures to be done for the planets [were] distinguished by the future great astronomers having made observation of the planets with a sighting-tube etc.; so subsequently at the accomplishment of the book, [that] is stated by the teacher [Gaṇeśa] too. Therefore at this time the local [positions of the] planets occur. Thus with this connection to the present, the mean solar motion in degrees etc. is known: 0|59|8|34|17|9. Then the proportion: if in one day the [mean] motion is so much, then what [is it] in the *ahargaṇa*? The [mean] motion is the multiplier of the *ahargaṇa*. Here for the sake of multiplication of parts, one part of the motion [is] taken [as] greater with respect to the [mean] motion.

S[un] [mean] mo[tion]: 0|59|8|34|17|9; here one [degree] is assumed. The difference 0|0|51|25|42|51 is to be multiplied by that *ahargaṇa* and subtracted from the *ahargaṇa* multiplied by one. In this [there is] laborious work. For the sake of brevity, this 0|0|51|25|42|51 in [the form of] one number should be multiplied by something. Thus when it is multiplied by seventy, the highest [digit] is one, without remainder. So, the *ahargaṇa* multiplied by one [is] divided by seventy; the *ahargaṇa* times one is to be diminished by the result, since [it was] taken [as] greater. The subtraction is because of the invariability of the multiplier equal to one in both places, thus '*svakhanagalavahīna*'. Now the part taken [as] greater with respect to the [mean] motion, 0|0|0|24|0|0: the *ahargaṇa* is multiplied by that, [and] the result is to be subtracted from the [mean] Sun because of the excess. Here also for the sake of brevity, this constant is put into the same category with '*kha-tithi*' [meaning] '150' in the minutes place.[7]

[7] As seen in [CK02, p. 95], Bhāskara I in his commentary of the *Āryabhaṭīya* uses the operational phrase '*savarṇitaṃ jātaṃ*' to mean 'put into the same category', i.e., converting a mixed number into an improper fraction. In this case, Mallāri means converting a quantity into a fraction.

That is, Mallāri has back-calculated Gaṇeśa's solar $\bar{\lambda}_d$ formula by declaring the fraction $\frac{1}{70} \approx 0;0,51,25,42,51$ to be the difference between one degree and an arbitrary time-specific solar daily motion of $0;59,8,34,17,9$ degrees per day. This quantity is about $0;0,0,24$ greater than the standard solar mean daily motion of $0;59,8,10,\ldots$ degrees per day, and $0;0,0,24 = \frac{1}{150}$ in arcminutes. Therefore subtracting from the *ahargaṇa* its own $\frac{1}{70}$ part in degrees and its own $\frac{1}{150}$ part in arcminutes will effectively be the same as multiplying the *ahargaṇa* by the solar mean daily motion.

Mallāri's *ad hoc* justification of these numbers is interesting as a reassertion of the general dependence of numerical models upon observation, although it is not persuasive as a demonstration or reconstruction of the formula. The number $0;59,8,34,17,9$ $(= 1-0;0,51,25,42,51)$, for instance, has no plausible empirical origin except via conversion of the fraction $\frac{1}{70}$ into the sexagesimal $0;0,51,25,42,51$. Obtaining such fractions by 'Euclidean division' from planetary period relations with continued-fraction approximations seems far more likely than extracting them from observational data, especially considering how familiar these division techniques would have been from well-known *kuṭṭaka* problems in Indian mathematics and astronomy. Was Mallāri not privy to the details of Gaṇeśa's construction, or was he consciously concealing them as a sort of trade secret?

2.5.2 Verse 1.11: Mean longitudinal increments of the Moon's apogee and the Moon's ascending node

नवहृतदिनसंघश्चन्द्रतुङ्गं लवाद्यं
भवति खनगभक्तद्युव्रजोपेतलिप्तम् ।
नवकुभिरिषुवेदैर्घस्रसंघाद्द्विधाप्तात्
फललवकलिकैक्यं स्याद्गुश्चक्रशुद्धः ॥ ११ ॥ ॥ मालिनी ॥

navahṛtadinasaṃghaś candratuṅgaṃ lavādyaṃ
bhavati khanagabhaktadyuvrajopetaliptam |
navakubhir iṣuvedair ghasrasaṃghād dvidhāptāt
phalalavakalikaikyaṃ syād aguś cakraśuddhaḥ || 11 || || *mālinī* ||

The *ahargaṇa* (*dinasaṃgha*) divided by nine (*nava*) is [the mean longitudinal increment of] the moon's apogee in degrees and so on, [with additional] arcminutes of a seventieth (*kha-naga*) part of the *ahargaṇa* (*dyuvraja*).

The sum of the results in degrees and arcminutes [respectively produced] from the *ahargaṇa* (*ghasrasaṃgha*) divided in two ways, by 19 (*nava-ku*) [and] by 45 (*iṣu-vedā*), is the 360-complement [of the mean longitudinal increment] of the moon's ascending node (*agu*).

Commentary: For the lunar apogee, the $\bar{\lambda}_c$ equation is

$$\bar{\lambda}_c = \frac{d_c}{9}^{\circ} + \frac{d_c}{70}^{\prime}, \qquad \textbf{(Lunar apogee)}$$

and for the lunar node, which revolves 'backwards' from east to west,

$$\bar{\lambda}_c = -\left(\frac{d_c}{19}^{\circ} + \frac{d_c}{45}^{\prime}\right). \qquad \textbf{(Lunar node)}$$

Dvivedin shows [Dvi25, pp. 57–58] in his continued-fraction analysis that the Brāhmapakṣa period relations yield the best approximation to Gaṇeśa's lunar-apogee formula, although still not an entirely accurate one:

$$\bar{\lambda}_d = \frac{\text{Number of revolutions} \cdot 360°}{\text{Number of days}} = \frac{488105858 \cdot 360}{1577916450000} = \frac{244052929}{2191550625} = \frac{1}{8 + \frac{239127193}{244052929}}$$
$$\approx \frac{1}{9}.$$

Since this approximate value is a little smaller than the exact fraction, their discrepancy is given by:

$$\frac{244052929}{2191550625} - \frac{1}{9} = \frac{547304}{2191550625},$$

or in arcminutes:

$$\frac{547304^{(\circ)}}{2191550625} = \frac{32838240^{(\prime)}}{2191550625} = \frac{1}{66 + \frac{1615119}{2189216}}.$$

This fraction would round most nearly to $\frac{1}{67}$, rather than Gaṇeśa's term $\frac{1}{70}$.

Lunar apogee	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{1}{9}^\circ + \frac{1}{70}'$	$\frac{1}{9}^\circ + \frac{1'}{66 + \frac{1615119}{2189216}}$	$\frac{1}{9}^\circ + \frac{1'}{60 + \frac{130935}{260804}}$	$\frac{1}{9}^\circ + \frac{1'}{61 + \frac{6171311}{19299460}}$

We can employ a similar derivation from Brāhmapakṣa *kalpa* parameters for the lunar node's mean daily increment $\bar{\lambda}_d$:

$$\bar{\lambda}_d = \frac{\text{Number of revolutions} \cdot 360°}{\text{Number of days}} = \frac{232311168 \times 360}{1577916450000} = \frac{12906176}{243505625} = \frac{1}{18 + \frac{11194457}{12906176}}$$
$$\approx \frac{1}{19}.$$

Since this approximate value, again, is less than the exact fraction, we find an additive term corresponding to their discrepancy:

$$\frac{12906176}{243505625} - \frac{1}{19} = \frac{1711719}{4626606875},$$

or in arcminutes:

$$\frac{1711719^{(\circ)}}{4626606875} = \frac{102703140^{(\prime)}}{4626606875} = \frac{1}{45 + \frac{993115}{20540628}}.$$

Lunar node	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$-\bar{\lambda}_d$	$\frac{1}{19}^\circ + \frac{1}{45}'$	$\frac{1}{19}^\circ + \frac{1'}{45 + \frac{993115}{20540628}}$	$\frac{1}{19}^\circ + \frac{1'}{47 + \frac{1156379}{2101668}}$	$\frac{1}{19}^\circ + \frac{1'}{47 + \frac{29094823}{158851380}}$

2.5.3 Verse 1.12: Mean longitudinal increments of Mars and the *śīghra*-anomaly of Mercury

दिग्घ्नो द्विधा दिनगणोऽङ्ककुभिस्त्रिशैलै-
र्भक्तः फलांशककलाविवरं कुजः स्यात् ।
त्रिघ्नो गणः स्ववसुदृग्लवयुग्ज्ञशीघ्र-
केन्द्रं लवाद्यहिगुणाप्तगणोनलिप्तम् ॥ १२ ॥ ॥ वसन्ततिलका ॥

digghno dvidhā dinagaṇo'ṅkakubhis triśailair

bhaktaḥ phalāṃśakakalāvivaraṃ kujaḥ syāt |
trighno gaṇaḥ svavasudṛglavayug jñaśīghra-
kendraṃ lavādyahiguṇāptagaṇonaliptam || 12 || || *vasantatilakā* ||

The *ahargaṇa* (*dinagaṇa*) multiplied by 10 is [separately] divided in two ways by 19 (*aṅka-ku*) [and] by 73 (*tri-śaila*). The difference of the results in degrees and arcminutes [respectively] is [the mean longitudinal increment of] Mars.

The *ahargaṇa* (*gaṇa*) multiplied by 3 (*tri*) added to a 28th (*vasu-dṛk*) part of itself is [the mean longitudinal increment of] the *śīghra*-anomaly of Mercury, [with] subtracted arcminutes [equal to] the *ahargaṇa* divided by 38 (*ahi-guṇa*).

Commentary: As before, the verse states formulae for the mean longitudinal *ahargaṇa*-increments $\bar{\lambda}_c$, in this case for Mars and the *śīghra*-anomaly of Mercury:

$$\bar{\lambda}_c = \frac{10d_c}{19}^{\circ} - \frac{10d_c}{73}^{\prime}, \qquad \textbf{(Mars)}$$
$$\bar{\lambda}_c = \left(3d_c + \frac{3d_c}{28}\right)^{\circ} - \frac{d_c}{38}^{\prime}. \qquad \textbf{(Mercury's \textit{śīghra}-anomaly)}$$

Following the pattern in the previous verses for reconstructing the daily-increment $\bar{\lambda}_d$ formulas, we derive the Mars algorithm from Āryapakṣa *mahāyuga* parameters as follows:

$$\bar{\lambda}_d = \frac{\text{Number of revolutions} \cdot 360^{\circ}}{\text{Number of days}} = \frac{2296824 \times 360}{1577917500} = \frac{13780944}{26298625}$$
$$= \frac{1}{1 + \frac{12517681}{13780944}} = \frac{1}{1 + \frac{1}{1 + \frac{1263263}{12517681}}} = \frac{1}{1 + \frac{1}{1 + \frac{1}{9 + \frac{1148314}{1263263}}}}$$
$$\approx \frac{1}{1 + \frac{1}{1 + \frac{1}{9}}} = \frac{10}{19}.$$

The discrepancy between the too-large approximation and the exact fraction is

$$\frac{10}{19} - \frac{13780944}{26298625} = \frac{1148314^{(\circ)}}{499673875} = \frac{10 \cdot 6889884^{(\prime)}}{499673875} = \frac{10^{(\prime)}}{72 + \frac{3602227}{6889884}},$$

which is rounded to $\frac{10}{73}$ in the final approximation.

Mars	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{10}{19}^{\circ} - \frac{10}{73}^{\prime}$	$\frac{10}{19}^{\circ} - \frac{10}{72 + \frac{108338721}{191270582}}^{\prime}$	$\frac{10}{19}^{\circ} - \frac{10}{72 + \frac{3602227}{6889884}}^{\prime}$	$\frac{10}{19}^{\circ} - \frac{10}{72 + \frac{59590483}{103271100}}^{\prime}$

For the *śīghra*-anomaly of Mercury, the relevant Brāhmapakṣa *kalpa* parameters are the 17936998984 revolutions of Mercury's *śīghra*-apogee and the 4320000000 revolutions of the Sun, whose difference gives the cycles of the *śīghra*-anomaly:

$$\begin{aligned}\bar{\lambda}_d &= \frac{\text{Number of revolutions}\cdot 360^\circ}{\text{Number of days}} = \frac{(17936998984-4320000000)\times 360}{1577916450000} = \frac{6808499492}{2191550625}\\ &= 3+\frac{233847617}{2191550625} = 3+\frac{1}{9+\frac{1}{2+\frac{60003473}{86922072}}}\\ &\approx 3+\frac{1}{9+\frac{1}{3}} = 3+\frac{3}{28}.\end{aligned}$$

The discrepancy between the too-large approximation and the exact fraction is

$$\frac{3}{28}-\frac{233847617}{2191550625} = \frac{26918599^{(\circ)}}{61363417500} = \frac{26918599^{(\prime)}}{1022723625} = \frac{1^{(\prime)}}{37+\frac{26735462}{26918599}},$$

which is rounded to $\frac{1}{38}$ in the final approximation.

Mercury *śīghra*-anomaly	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{87}{28}^\circ - \frac{1}{38}'$	$\frac{87}{28}^\circ - \frac{1'}{37+\frac{26735462}{26918599}}$	$\frac{87}{28}^\circ - \frac{1'}{38+\frac{44473}{192609}}$	$\frac{87}{28}^\circ - \frac{1'}{38+\frac{70067569}{70823385}}$

2.5.4 Verse 1.13: Mean longitudinal increments of Jupiter and the *śīghra* anomaly of Venus

द्युपिण्डोऽर्कभक्तो लवाद्यो गुरुः स्यात्
द्युपिण्डात् खशैलाप्तलिप्ताविहीनः ।
त्रिनिघ्नाद् द्युपिण्डाद्विधाक्षैः क्विभाब्जै-
रवाप्तांशयोगो भृगोराशुकेन्द्रम् ॥ १३ ॥ ॥ भुजङ्गप्रयात ॥

dyupiṇḍo'rkabhakto lavādyo guruḥ syāt
dyupiṇḍāt khaśailāptaliptāvihīnaḥ |
trinighnād dyupiṇḍād dvidhākṣaiḥ kvibhābjair
avāptāṃśayogo bhṛgor āśukendram || 13 || || *bhujaṅgaprayāta* ||

The *ahargaṇa* (*dyupiṇḍa*) divided by 12 (*arka*) is [the mean longitudinal increment of] Jupiter [in] degrees and so on, [when] decreased by the quotient from [dividing] the *ahargaṇa* by 70 (*kha-śailā*), in arcminutes.

The sum in degrees of the [two] quotients from [separately dividing] the *ahargaṇa* multiplied by 3 in two ways, by 5 (*akṣa*) and by 181 (*ku-ibha-abja*), [is] [the mean longitudinal increment of] the *śīghra*-anomaly of Venus.

Commentary:

$$\bar{\lambda}_c = \frac{d_c}{12}^\circ - \frac{d_c}{70}', \qquad \textbf{(Jupiter)}$$
$$\bar{\lambda}_c = \left(\frac{3d_c}{5}+\frac{3d_c}{181}\right)^\circ. \qquad \textbf{(Venus's \textit{śīghra}-anomaly)}$$

We derive the Jupiter algorithm from Āryapakṣa *mahāyuga* parameters as follows:

$$\begin{aligned}\bar{\lambda}_d &= \frac{\text{Number of revolutions}\cdot 360^\circ}{\text{Number of days}} = \frac{364224\times 360}{1577917500} = \frac{2185344}{26298625} = \frac{1}{12+\frac{74497}{2185344}}\\ &\approx \frac{1}{12}.\end{aligned}$$

The discrepancy between the too-large approximation and the exact fraction is

$$\frac{1}{12} - \frac{2185344}{26298625} = \frac{74497^{(\circ)}}{315583500} = \frac{74497^{(')}}{5259725} = \frac{1^{(')}}{70+\frac{44935}{74497}}.$$

Jupiter	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{1}{12}^\circ - \frac{1}{70}'$	$\frac{1}{12}^\circ - \frac{1'}{70+\frac{325165}{412793}}$	$\frac{1}{12}^\circ - \frac{1'}{70+\frac{44935}{74497}}$	$\frac{1}{12}^\circ - \frac{1'}{70+\frac{1829507}{5609285}}$

For the *śīghra*-anomaly of Venus, we use Āryapakṣa *mahāyuga* parameters: 7022388 revolutions of Venus's *śīghra*-apogee and 4320000 revolutions of the Sun, whose difference gives the cycles of the *śīghra*-anomaly:

$$\begin{aligned}\bar{\lambda}_d &= \frac{\text{Number of revolutions}\cdot 360^\circ}{\text{Number of days}} = \frac{(7022388-4320000)\times 360}{1577917500} = \frac{16214328}{26298625}\\ &= \frac{1}{1+\frac{10084297}{16214328}} = \frac{1}{1+\frac{1}{1+\frac{6130031}{10084297}}}\\ &\approx \frac{1}{1+\frac{1}{1+\frac{1}{2}}} = \frac{3}{5}.\end{aligned}$$

The discrepancy between the too-small approximation and the exact fraction, in degrees, is

$$\frac{16214328}{26298625} - \frac{3}{5} = \frac{435153}{26298625} = \frac{3\cdot 435153}{78895875} = \frac{3}{181+\frac{44394}{145051}}.$$

Venus's *śīghra*-anomaly	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{3}{5}^\circ - \frac{3^\circ}{181}$	$\frac{3}{5}^\circ - \frac{3^\circ}{181+\frac{981884}{3296761}}$	$\frac{3}{5}^\circ - \frac{3^\circ}{181+\frac{44394}{145051}}$	$\frac{3}{5}^\circ - \frac{3^\circ}{181+\frac{3670602}{10876943}}$

2.5.5 Verse 1.14ab: Mean longitudinal increment of Saturn

खाग्न्युद्धृतो दिनगणोंऽशमुखः शनिः स्यात्
षट्पञ्चभूहृतगणात् फललिप्तिकाढ्य । १४ab ।   ॥ वसन्ततिलका ॥

khāgnyuddhṛto dinagaṇo'ṃśamukhaḥ śaniḥ syāt
ṣaṭpañcabhūhṛtagaṇāt phalaliptikāḍhyaḥ | 14ab | || *vasantatilakā* ||

The *ahargaṇa* (*dinagaṇa*) divided by 30 (*kha-agni*) is [the mean longitudinal increment of] Saturn in degrees and so on, [when] increased by the arcminutes of the result from [dividing] the *ahargaṇa* (*gaṇa* by 156 (*ṣaṭ-pañca-bhū*).

Commentary:

$$\bar{\lambda}_c = \frac{d_c}{30}^{\circ} + \frac{d_c}{156}'. \qquad \textbf{(Saturn)}$$

We derive the Saturn algorithm from Brāhmapakṣa *kalpa* parameters as follows:

$$\bar{\lambda}_d = \frac{\text{Number of revolutions} \cdot 360^\circ}{\text{Number of days}} = \frac{146567298 \times 360}{1577916450000} = \frac{24427883}{730516875} = \frac{1}{29 + \frac{22108268}{24427883}}$$
$$\approx \frac{1}{30}.$$

The discrepancy between the too-small approximation and the exact fraction is

$$\frac{24427883}{730516875} - \frac{1}{30} = \frac{51547^{(\circ)}}{487011250} = \frac{309282^{(\prime)}}{48701125} = \frac{1^{(\prime)}}{157 + \frac{143851}{309282}}.$$

None of the *pakṣa* parameters produce a version of this term that rounds accurately to Gaṇeśa's $\frac{1}{156}$.

Saturn	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
$\bar{\lambda}_d$	$\frac{1}{30}^{\circ} - \frac{1'}{156}$	$\frac{1}{30}^{\circ} - \frac{1'}{157 + \frac{143851}{309282}}$	$\frac{1}{12}^{\circ} - \frac{1'}{158 + \frac{20761}{33158}}$	$\frac{1}{12}^{\circ} - \frac{1'}{157 + \frac{678555}{2508286}}$

2.6 Verse 1.14cd–15: Mean daily motions

After explaining how the mean longitudes for the planets are found for mean sunrise on a certain date, Gaṇeśa then lists the mean daily motions of the planets, which are used to compute mean longitudes at a certain time on the desired date.

गोऽक्षा गजा रविगतिः शशिनोऽभ्रगोऽश्वाः
पञ्चाग्नयोऽथ षडिलाब्धय उच्चभुक्तिः ॥ १४cd ॥ ॥ वसन्ततिलका ॥

राहोस्त्रयं कुशशिनोऽसृज इन्दुरामा-
स्तर्काश्विनो ज्ञचलकेन्द्रजवोऽर्यहिक्ष्माः ।
लिप्ता जिना विकलिकाश्च गुरोः शराः खं
शुक्राशुकेन्द्रगतिरद्रिगुणाः शनेर्द्वे ॥ १५ ॥ ॥ वसन्ततिलका ॥

go'kṣā gajā ravigatiḥ śaśino'bhrago'śvāḥ
pañcāgnayo'tha ṣaḍilābdhaya uccabhuktiḥ || 14cd || || *vasantatilakā* ||

rāhos trayaṃ kuśaśino'sṛja indurāmās
tarkāśvino jñacalakendrajavo'ryahikṣmāḥ |
liptā jinā vikalikāś ca guroḥ śarāḥ khaṃ
śukrāśukendragatir adriguṇāḥ śaner dve || 15 || || *vasantatilakā* ||

The [mean] daily motion of the Sun (*ravi*): 59,8 (*go-akṣa gaja*); of the Moon (*śaśi*): 790,35 (*abhra-go-aśva pañca-agni*); now the [mean] daily motion of the [lunar] apogee: 6,41 (*ṣaḍ-ila-abdhi*).

[The mean daily motion] of the lunar node: 3,11 (*traya ku-śaśin*); of Mars: 31,26 (*indu-rāma tarka-aśvin*); the [mean] daily motion of the *śīghra*-anomaly of Mercury is 186 (*ari-ahi-kṣmā*) minutes and 24 (*jina*) seconds; of Jupiter: 5,0 (*śara kha*); the [mean] daily motion of the *śīghra*-anomaly of Venus is 37 (*adri-guṇa*), of Saturn 2 (*dve*).

Planet	Mean daily motion
Sun	59′ 8″
Moon	790′ 35″
Lunar apogee	6′ 41″
Lunar node	3′ 11″
Mars	31′ 26″
Mercury's *śīghra* anomaly	186′ 24″
Jupiter	5′ 0″
Venus' *śīghra* anomaly	37′ [0]″
Saturn	2′ [0]″

Table 2.2: The mean daily motions of the planets listed in verses 14cd–15.

Commentary: The last step in determining mean planetary positions for a desired instant is to incorporate the mean longitudinal displacement the planet accumulates in the remaining fraction of the day. For this purpose, Gaṇeśa lists the mean daily motions of the planets in weekday order, namely: sun, moon, Mars, Mercury, Jupiter, Venus, and Saturn (see table 2.2). Notably, in the case of Mercury and Venus, Gaṇeśa has given the mean daily motions of the *śīghra*-anomaly (for inferior planets, the difference between the mean daily motions of the *śīghra*-apogee and the sun), rather than the more usual motion of the *śīghra*-apogee itself.

The parameters Gaṇeśa provides are precise at most to seconds. If the mean positions up to any given *ahargaṇa* are computed with the more precise formulas given in section 2.5, then these approximate mean daily motions will be adequate for finding the displacement in the course of the day.

2.7 Verse 1.16: Gaṇeśa's attribution of planetary parameters to different *pakṣas*

In this last verse of the *Grahalāghava*'s first chapter, Gaṇeśa states the *pakṣas* from which his parameters are derived, as well as modifications he had made.

सौरोऽर्कोऽपि विधूच्चमङ्ककलिको नाब्जो गुरुस्त्वार्य-
जोऽसृग्राहू च कजं ज्ञकेन्द्रकमथार्ये सेषुभागः शनिः ।
शौक्रं केन्द्रमजार्यमध्यगमितीमे यान्ति दृक्तुल्यतां
सिद्धैस्तैरिह पर्वधर्मनयसत्कार्यादिकं त्वादिशेत् ॥ १६ ॥ ॥ शार्दूलविक्रीडित ॥

sauro'rko'pi vidhūccam aṅkakaliko nābjo gurus tv āryajo
'sṛgrāhū ca kajaṃ jñakendrakam athārye seṣubhāgaḥ śaniḥ |
śaukraṃ kendram ajāryamadhyagam itīme yānti dṛktulyatāṃ
siddhais tair iha parvadharmanayasatkāryādikaṃ tvādiśet || 16 ||
|| *śārdūlavikrīḍita* ||

The Sun and lunar apogee are Saurapakṣa, as well as the Moon [but] less nine minutes. But Jupiter is derived from the Āryapakṣa, as well as Mars and the lunar node. The [*śīghra*-]anomaly of Mercury is Brāhmapakṣa. Now, an additional five degrees [applied] to the Āryapakṣa [value] is Saturn. The [*śīghra*-]anomaly of Venus is half [of the sum of its values] in the Āryapakṣa and Brāhmapakṣa. Thus these [values] come out in accordance with observation. Here, using these

Planet	Attribution
Sun	Saurapakṣa
Moon	Saurapakṣa - $0°;9$
Lunar apogee	Saurapakṣa
Lunar node	Āryapakṣa
Mars	Āryapakṣa
Mercury's *śīghra*-anomaly	Brāhmapakṣa
Jupiter	Āryapakṣa
Venus's *śīghra*-anomaly	(Āryapakṣa + Brāhmapakṣa)/2
Saturn	Āryapakṣa + 5°

Table 2.3: Gaṇeśa's *pakṣa* attributions and parameter modifications in verse 16.

determined [results] the moon phase (*parva*) [sacrifices], duty, conduct, good actions, and so on may be determined.

Commentary: Here Gaṇeśa attributes (some unspecified characteristic of) each of these nine celestial entities to one of the three main astronomical *pakṣas*, in some cases with additional modifications.[8] Table 2.3 lists these attributions to the Saurapakṣa, Brāhmapakṣa, and Āryapakṣa.

In order to identify exactly which parameter(s) Gaṇeśa's attributions in this verse are meant to indicate, we reconstruct both *dhruvas* and *kṣepakas* using period relations from *siddhāntas* of each of the three *pakṣas* Gaṇeśa mentions: the *Āryabhaṭīya* (Āryapakṣa), the *Siddhāntaśiromaṇi* (Brāhmapakṣa), and the *Sūryasiddhānta* (Saurapakṣa). These reconstructed *dhruvas* and *kṣepakas* are presented in Tables 2.5 and 2.6 respectively. The *dhruvas* were found by first calculating the mean longitudinal displacement of each planet in 4016 civil days per the period relations of each text, then subtracting this displacement from 360 degrees. The *kṣepakas* were calculated by using the methodology presented in each text: computing elapsed *ahargaṇa* from the epoch of each text until the epoch of the *Grahalāghava* (see Table 2.4), then using the stated period relations with small corrections to find mean epoch longitude. As per the *Āryabhaṭīya* and the *Siddhāntaśiromaṇi*, small yearly corrections to mean longitudes are to be made and are thus applied to both the reconstructed *dhruvas* and *kṣepakas*. Also, since the *Sūryasiddhānta*'s epoch is mean midnight while Gaṇeśa's epoch is mean sunrise, an epoch correction is applied to the *kṣepakas*. The rounded tabulated values including these prescribed corrections for each of the three *pakṣas* do not include Gaṇeśa's stated modifications so we may see the circumstances for which Gaṇeśa has stated a modification.

In the case of the *dhruvas*, Gaṇeśa's attributions hold up moderately well. The reconstructed Saurapakṣa *dhruvas* of the Sun and Moon, as well as the reconstructed Āryapakṣa lunar node *dhruva*, indeed agree with Gaṇeśa's stated *dhruvas*. The reconstructed Āryapakṣa *dhruva* of Saturn is the closest to Gaṇeśa's *dhruva* (difference of 12 arcseconds), and indeed the Brāhmapakṣa reconstructed *dhruva* of Mercury's *śīghra*-anomaly is closest to Gaṇeśa's *dhruva* (differing by almost 1 arcminute), while the Āryapakṣa reconstructed Jupiter *dhruva* is close (57 arcseconds different) but not exactly the closest to Gaṇeśa's stated *dhruva* (the Saurapakṣa *dhruva* is only 54 arcseconds different). The reconstructed Saurapakṣa lunar apogee *dhruva* is not at all the most appropriate– the Āryapakṣa reconstruction is much more suitable. Additionally, while Gaṇeśa attributes his stated Mars *dhruva* to be from the Āryapakṣa, his stated value more closely matches the reconstructed *dhruva* values of the Brāhmapakṣa and Saurapakṣa. As it stands, the reconstructed *dhruva* of Venus's *śīghra*-anomaly per the Āryapakṣa is closest to Gaṇeśa's stated value; even taking the average of the Brāhmapakṣa and Āryapakṣa values yields a different value.

[8] The first two lines of this verse are identical to those included in verse 18 of Gaṇeśa's *Tithicintāmaṇi*. See Ikeyama and Plofker [IP01, pp. 275–6].

	elapsed solar years until GL Epoch	elapsed *ahargaṇa* until GL Epoch
Sūryasiddhānta	$1,955,884,621$	$714,403,984,477$
Āryabhaṭīya	$3,244,621$	$1,185,125,975$
Siddhāntaśiromaṇi	$1,972,948,621$	$720,636,130,565$

Table 2.4: The number of years elapsed from each text's epoch until the epoch of the *Grahalāghava* (1520 CE = Śaka 1442), along with the corresponding number of *ahargaṇa*, calculated using procedures detailed in each text. The epoch for the *Sūryasiddhānta* is $1,955,880,000$ years before the start of the Kali epoch, which is 3179 years from the beginning of the Śaka era [Bag01]. The epoch for the *Āryabhaṭīya* is $3,243,600$ years after the start of the *mahāyuga*, which is 499 CE or Śaka 421 [Shu76, p.95]. According to Bhāskara II, the number of solar years from the beginning of the *kalpa* to the start of the Śaka calendar is 1,972,947,179 [Ark80, p. 13].

Planet	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
Sun	$1°;49,11$	$1°;49,8,31$	$1°;49,8,6$	$1°;49,11,4$
Moon	$3°;46,11$	$3°;45,40,5$	$3°;46,37,36$	$3°;46,11,12$
Lunar apogee	$9^s,2°;45$	$9^s,2°;46,36$	$9^s,2°;45,18$	$9^s,2°;41,10$
Lunar node	$7^s,2°;50$	$7^s,2°;51,9$	$7^s,2°;50,47$	$7^s,2°;47,13$
Mars	$1^s,25°;32$	$1^s,25°;32,19$	$1^s,25°;30,36$	$1^s,25°;32,17$
Mercury's *śīghra*-anomaly	$4^s,3°;27$	$4^s,3°;25,52$	$4^s,3°;9,25$	$4^s,3°;25,52$
Jupiter	$26°;18$	$26°;17,0$	$26°;18,57$	$26°;17,6$
Venus's *śīghra*-anomaly	$1^s,14°;2$	$1^s,13°;57,24$	$1^s,14°;3,40$	$1^s,13°;57,37$
Saturn	$7^s,15°;42$	$7^s,15°;42,17$	$7^s,15°;41,48$	$7^s,15°;42,28$

Table 2.5: The *dhruvas* as reconstructed with Mallāri's *siddhānta* period relation calculation method using each *pakṣa*'s parameters along with *Āryabhaṭīya* and *Siddhāntaśiromaṇi* corrections, as compared to the *dhruva* values stated by Gaṇeśa in verses 6–8. Note that these are all *dhruvas* as defined by Gaṇeśa to be the 360° explement.

Planet	Gaṇeśa	Brāhmapakṣa	Āryapakṣa	Saurapakṣa
Sun	$11^s,19°;41$	$11^s,19°;44,17$	$11^s,19°;47,11$	$11^s,19°;41,13$
Moon	$11^s,19°;6$	$11^s,19°;36,14$	$11^s,18°;53,24$	$11^s,19°;15,53$
Lunar apogee	$5^s,17°;33$	$5^s,15°;7,39$	$5^s,16°;4,42$	$5^s,17°;40,23$
Lunar node	$0^s,27°;38$	$0^s,28°;53,6$	$0^s,27°;38,46$	$0^s,29°;34,16$
Mars	$10^s,7°;8$	$10^s,4°;59,26$	$10^s,6°;28,55$	$10^s,6°;14,23$
Mercury's *śīghra*-anomaly	$8^s,29°;33$	$8^s,29°;14,29$	$9^s,13°;52,14$	$9^s,0°;20,50$
Jupiter	$7^s,2°;16$	$7^s,4°;18,15$	$7^s,2°;31,43$	$7^s,4°;10,56$
Venus's *śīghra*-anomaly	$7^s,20°;9$	$7^s,23°;32,23$	$7^s,17°;45,55$	$7^s,23°;30,24$
Saturn	$9^s,15°;21$	$9^s,10°;37,52$	$9^s,10°;22,12$	$9^s,10°;32,39$

Table 2.6: The *kṣepaka* values as stated by Gaṇeśa in verses 6–8, compared to the *kṣepakas* as reconstructed using each *pakṣa*'s parameters with the corrections as stated in the *Āryabhaṭīya*, *Siddhāntaśiromaṇi*, and *Sūryasiddhānta*.

Planet	Reconstructed *kṣepakas*	Gaṇeśa's *kṣepakas*	Difference	Stated *pakṣa* of origin (w. modification)
Sun	$11^s, 19°; 41$	$11^s, 19°; 41$	0	Saurapakṣa
Moon	$11^s, 19°; 6$	$11^s, 19°; 6$	0	Saurapakṣa - $0°; 9$
Lunar apogee	$5^s, 17°; 40$	$5^s, 17°; 33$	$0°; 7$	Saurapakṣa
Lunar node	$0^s, 27°; 38$	$0^s, 27°; 38$	0	Āryapakṣa
Mars	$10^s, 6°; 28$	$10^s, 7°; 8$	$0°; 40$	Āryapakṣa
Mercury's *śīghra*-anomaly	$8^s, 29°; 14$	$8^s, 29°; 33$	$0°; 19$	Brāhmapakṣa
Jupiter	$7^s, 2°; 31$	$7^s, 2°; 16$	$0°; 15$	Āryapakṣa
Venus's *śīghra*-anomaly	$7^s, 20°; 39$	$7^s, 20°; 9$	$0°; 30$	(Āryapakṣa + Brāhmapakṣa)/2
Saturn	$9^s, 15°; 22$	$9^s, 15°; 21$	$<0°; 1$	Āryapakṣa + $5°$

Table 2.7: In verse 16 Gaṇeśa attributes values with additional modifications of each planet to one of the three *pakṣas*: Saurapakṣa, Āryapakṣa, and Brāhmapakṣa. The *kṣepakas* of each planet were computed using the period relation from a *siddhānta* of the attributed *pakṣa*, and Gaṇeśa's stated modifications are applied to those values to yield the reconstructed *kṣepaka* values found in this table. These can be compared to Gaṇeśa's stated *kṣepaka* values, with a subsequent column observing any difference between the reconstructed *kṣepakas* and his stated ones. The final column shows the stated *pakṣa* of origin along with modifications.

However, the recomputed *kṣepakas* of the attributed *pakṣa* were generally in agreement with Gaṇeśa's stated values, as seen in Table 2.7, which shows the reconstructed *kṣepakas* compared to Gaṇeśa's stated *kṣepakas* and their differences. In the cases when the reconstructed *kṣepaka* of the attributed *pakṣa* didn't exactly match Gaṇeśa's stated value, the attributed *pakṣa* was consistently the closest in value. Through reconstruction, we were also able to deduce that the modifications stated in Gaṇeśa's verse are "one-off" modifications applied only to the *kṣepakas* found using the stated *pakṣa*'s period relation (especially evident in the case of Saturn– adding 5 degrees as displacement per elapsed *cakra* would produce too great an error in its mean longitude).

Based on preliminary reconstructions, we are most inclined to believe that Gaṇeśa attributes the computation of only his *kṣepakas* and not his *dhruvas*, especially given his inclusion of the modifications. We chose three likely *siddhāntas* and their period relations and specified additional corrections as the basis of our reconstructions. Further examination of other *siddhāntas* or alternative *karaṇa* methodologies may instead yield more exact *dhruva* and *kṣepaka* values (discounting variations due to rounding) in support of the attributions applying to both parameters.

Mallāri's commentary supports the notion that these attributions and modifications only apply to *kṣepakas*, as we discuss in Section 2.4.1. Mallāri only invokes attributions to the other *pakṣas* when explaining how Gaṇeśa's *kṣepakas* were computed. In his *dhruva* computation example, he referred to using various algorithms such as that of the *Karaṇakutūhala* for finding mean longitudinal displacement. Since Mallāri does not explicitly call attention to *pakṣa* attributions in his *dhruva* explanation, he may not have believed the attributions to be relevant.

Meanwhile, Viśvanātha's commentary mentions that he found the *dhruvas* and *kṣepakas* of the Sun, Moon, and the lunar apogee used in his lunar and solar eclipse observations to better match those produced using Āryapakṣa period relations. He then states three verses in which he provides these updated *dhruvas* and *kṣepakas* [Dvi25, lines 27-34, 1–11, p.16-17]:

अत्रेदानीं चन्द्रसूर्ययोर्ग्रहणे स्पर्शमोक्षावार्यपक्षेण भवत इति दृश्यत इति कारणादार्यपक्षस्थतिथिसाधनार्थं सूर्यचन्द्रतुङ्गानां ध्रुवकक्षेपानाह ।
यातेऽब्दे ग्रहलाघवस्य धरणीक्षोणीक्षपेशोन्मिते
संवीक्ष्य क्षणदाकरोष्णकरयोः पर्वार्यपक्षाश्रितम् ।
क्षेपान् सध्रुवकान् रवीन्दुशशभृत्तुङ्गोद्भवान् भादिकान्
दृष्टिप्रत्ययकारकान् गणितविच्छ्रीविश्वनाथो ब्रुवे ।। १ ।।
खविधुतानगजास्तरणेर्ध्रुवः ० । १ । ४९ । ८ ।
खमनला रसवारिधिसमिताः

नगगुणाः शशिनो ० । ३ । ४६ । ३७ ऽथ खगा यमौ
शरकृतः खयमा ९ । २ । ४५ । २० विधुतुङ्गजाः ।। २ ।।
क्षेपो भवा नन्दभुवोऽद्रिवेदा
विश्वे ११ । १९ । ४७ । १३ ऽर्के इन्दौ कुभुवो गजाब्जाः
रामेषवो बाणयमा ११ । १८ । ५३ । २५ स्तदुच्चे
बाणाः षडब्जाः श्रुतयः कुवेदाः ५ । १६ । ४ । ४१ ।। ३ ।।
अथ वा सिद्धानां सूर्यचन्द्रतुङ्गानां बीजसंस्कारमाह । यद्वा श्रीग्रहलाघवोत्थरणौ लिप्तादि बीजं धनं षड्विश्वे ६ । १३ ऽथ विधावृणं यमभुवः पञ्चग्नय १२ । ३५ स्तुङ्गके नागेभा नवभूमयः ८८ । १९ स्वमनला ३ स्तर्काश्विनः २६ खाश्विन २० श्चक्रघ्ना विकला रवीन्दुशशभृत्तुङ्गे स्वमस्वं त्वृणम् ।

atredānīṃ candrasūryayor grahaṇe sparśamokṣāv āryapakṣeṇa bhavata iti dṛśyata iti kāraṇād āryapakṣasthatithisādhanārthaṃ sūryacandratuṅgānāṃ dhruvakakṣepān āha |
yāte 'bde grahalāghavasya dharaṇīkṣoṇīkṣapeśonmite
saṃvīkṣya kṣaṇadākaroṣṇakarayoḥ parvāryapakṣāśritam |
kṣepān sadhruvakān ravīnduśaśabhṛttuṅgodbhavān bhādikān
dṛṣṭipratyayakārakān gaṇitavicchrīviśvanātho bruve || 1 ||
khavidhutānagajās taraṇer dhruvaḥ 0/1/49/8 |
khamanalā rasavāridhisamitāḥ
nagaguṇāḥ śaśino 0/3/46/37 'tha khagā yamau
śarakṛtaḥ khayamā 9/2/45/20 vidyutuṅgajāḥ ||2||
kṣepo bhavā nandabhuvo' drivedā
viśve 11/19/47/13 'rka indau kubhuvo gajābjāḥ ||
rāmeṣavo bāṇayamās 11/18/53/25 taducce
bāṇāḥ ṣaḍabjāḥ śrutayaḥ kuvedāḥ 5/16/4/41 ||3||
atha vā siddhānāṃ sūryacandratuṅgānāṃ bījasaṃskāram āha | yadvā śrīgrahalāghavottharaṇau liptādi bījaṃ dhanaṃ ṣaḍviśve 6/13 'tha vidhāvṛṇaṃ yamabhuvaḥ pañcagnayas 12/35 tuṅgake nāgebhā navabhūmayaḥ 88/19 svamanalās 3 tarkāśvinaḥ 26 khāśvinaś 20 cakraghnā vikalā ravīnduśaśabhṛttuṅge svam asvaṃ tv ṛṇam |

Now here it is seen, during the eclipse of the Sun and the Moon, the points of first and last contact occur with the Āryapakṣa; because of this, I state the *dhruvas* and *kṣepakas* of the Sun, Moon, and lunar apogee for the purpose of finding the *tithi* in [accordance with] the Āryapakṣa. Observing when the elapsed years [since the epoch] of the *Grahalāghava* are commensurate with 111 (*dharaṇī-kṣoṇī-kṣapeśa*), the eclipse of the Sun and Moon depends on the Āryapakṣa; I, the knower of mathematics, venerable Viśvanātha, state the *kṣepakas* along with the *dhruvakas* produced by the Sun, Moon, and lunar apogee in signs and so on, made on the basis of observation. The *dhruva* of the Sun: 0|1|49|8. [The *dhruva*] of the Moon: 0|3|46|37; now, those [numbers] of the lunar apogee: 9|2|45|20. The *kṣepaka*: 11|19|47|13 in [the case of] the Sun; in [the case of] the Moon: 11|18|53|25; its apogee: 5|16|4|41. Alternatively, I state the *bīja*-correction of the resulting [*kṣepakas*] of the Sun, Moon, and lunar apogee. Whatever [correction to be applied] at sunrise [at epoch] of the *Grahalāghava*, in arcminutes and so on is the positive *bīja*-[correction]: 6|13. Now in [the case of] the Moon is the negative 12|35. In [the case of] the lunar apogee is positive 88|19. 3, 26, and 20 multiplied by [the number of] *cakras* in arcseconds is applied to the Sun, Moon, or lunar apogee positively, negatively, and negatively (respectively).

In his commentary, Viśvanātha seems to have observed an eclipse, which was most likely the lunar eclipse on May 15, 1631 (111 years after the *Grahalāghava*'s epoch, and the only eclipse that year which would have been almost totally visible to Viśvanātha, as seen in Figure 2.4). We postulate that Viśvanātha recognized that the mean longitudes produced using Āryapakṣa period relations were most in agreement with his eclipse observations. So, it may be possible that Viśvanātha then reconstructed Gaṇeśa's *dhruvas* and *kṣepakas* using Āryapakṣa parameters and informed his students/future users to use his updated parameters for the Sun, Moon, and lunar apogee instead of those Gaṇeśa stated in verses 6–8. The *dhruvas* and *kṣepakas* that

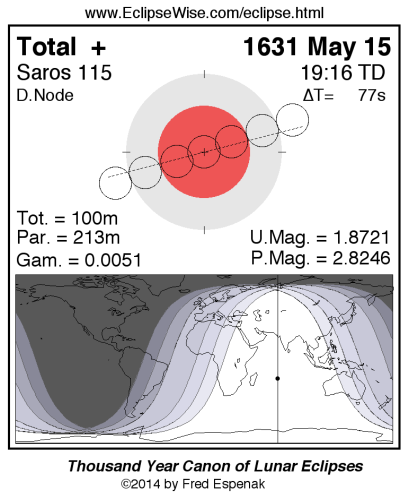

Figure 2.4: The eclipse Viśvanātha is most likely to be talking about in his commentary is the total lunar eclipse on May 15, 1631. Not only is this eclipse 111 years after the *Grahalāghava*'s epoch, maximum visibility of the eclipse occurs closest to modern-day India, unlike the other solar and lunar eclipses that occurred that year.

Viśvanātha states here differ at most by 2 arcseconds from the *dhruvas* and *kṣepakas* for the Sun, Moon, and lunar apogee that we reconstructed using Āryapakṣa period relations as seen in Tables 2.5 and 2.6. This confirms that Viśvanātha did indeed use Āryapakṣa parameters in his recomputed *dhruvas* and *kṣepakas*.[9] The differences in value between Gaṇeśa's stated *dhruvas* and *kṣepakas* and those Viśvanātha just states are the *bīja*-corrections Viśvanātha provides at the end of this commentary passage.

The second half of the third line of Gaṇeśa's verse calls upon an accordance of values with observation, "*ime yānti dṛktulyatāṃ*". This may have been the process Viśvanātha relied on when he recomputed *dhruvas* and *kṣepakas* after discovering that different parameters yielded computations that were in better agreement to his observed data. Commentator Mallāri also claims that Gaṇeśa's parameters and stated corrections arose as products of observation [Dvi25, p.68, lines 8-11]:

> इति तेभ्यः पक्षेभ्यः साधिता इमे ग्रहाः दृशि तुल्यतां दृग्गणितैक्यं यान्ति प्राप्नुवन्तीति । एवं ग्रहणोदयास्तजातकादौ ग्रहाणां साधनं बहुभ्यो ग्रन्थेभ्यः कर्तव्यमिति जडकर्म दृष्ट्वा आचार्यो लाघवार्थममुं ग्रन्थं कृतवान् ।
>
> *iti tebhyaḥ pakṣebhyaḥ sādhitā ime grahāḥ dṛśi tulyatāṃ dṛggaṇitaikyaṃ yānti prāpnuvantīti | evaṃ grahaṇodayāstajātakādau grahāṇāṃ sādhanaṃ bahubhyo granthebhyaḥ kartavyam iti jaḍakarma dṛṣṭvā ācāryo lāghavārtham amuṃ granthaṃ kṛtavān |*
>
> Thus, the [mean longitudes of] these planets found from those [parameters of] *pakṣas* "*yānti*" [meaning] "attain" equality with observation [specifically in] "concordance between the observed and computed values". Thus, reckoning [the longitudes of] the planets at eclipses, rising, setting, nativities and so on is to be done [according to the methods of] many texts; viewing this as stupid task, the teacher made this text for the sake of brevity.

Mallāri cites "*dṛggaṇitaikya*" which is the "concordance between the observed and computed values" as the rationale for Gaṇeśa's selection and modification of *kṣepaka* values. However, since the methodology and data of their observational astronomy is not apparent, and since *kṣepakas* are technically epochal mean longitudes while any observed longitudes of planets would be true longitudes with corrections applied, there

[9]This may seem to be a trivial confirmation, however assuming that a text-author scribe used the method they say they did without verifying whether the calculations affirm the same can lead to discrepancies.

is insufficient evidence to state whether the true longitudes produced using the attributed parameter values and appropriate corrections would align with Gaṇeśa's observed values.[10]

Nonetheless, Gaṇeśa highlights that his efforts in abbreviating previous methodologies and choosing appropriate parameters still arrive at the correct timings of rites such as sacrifices and weddings. The last line of this verse, "*siddhais tair iha parvadharmanayasatkāryādikaṃ tvādiśet*", invokes numerous applications of calculating correct times. Mallāri details these applications in his commentary [Dvi25, p.78, lines 12-15]:

> पर्व ग्रहणं धर्मो यज्ञानुष्ठानैकादशीव्रतादिकम् । नयो नीतिः । राजनीतिः दण्डनीत्यादिकः । सत्कार्यं शुभं कार्यं व्रतबन्धविवाहादि । एभ्यो ग्रन्थेभ्य एतदुत्पन्नतिथ्यादेरेवादिशेत् अयं भावः ।
>
> *parva grahaṇaṃ dharmo yajñānuṣṭhānaikādaśīvratādikam | nayo nītiḥ | rājanītiḥ daṇḍanītyādikaḥ | satkāryaṃ shubhaṃ kāryaṃ vratabandhavivāhādi | ebhyo granthebhya etadutpannatithyāder evādiśet ayaṃ bhāvaḥ |*
>
> *parva* [meaning] "eclipse". *dharmo* [meaning] "rituals, oath on the eleventh [*tithi*], etc." *nayo* [meaning] "conduct", [such as] royal conduct, administration of justice, etc. *satkāryaṃ* [meaning] "an auspicious deed", [such as] thread-initiation [ceremonies], marriages, etc. From the *tithis* produced from the [computations] in these [astronomy] texts, [the appropriate timing of those rites] may be determined; that is the gist.

Cultural practices such as ancestor oblations, fasting done on the eleventh *tithis* of every month, as well as marriages and Vedic education initiation ceremonies require the knowledge of auspicious timings. So, Gaṇeśa composed the *Grahalāghava* as a *karaṇa* to provide more user-friendly methods of computation than *siddhāntas*.

[10] In fact, a contradiction arises between between Mallāri attributing the origin of these parameters to "selecting the correct parameters of each *pakṣa* based on observations" and the *Grahalāghava* being based on Keśava's *Grahakautuka* [Pin81b]. There is insufficient evidence to know the influence of the *Grahakautuka* on the *Grahalāghava*, as Pingree remarks that he had yet to procure a manuscript for Keśava's *Grahakautuka* to note down any influences, and thus far there has not been any study of the text of which we are aware.

Chapter 3

Ravicandraspaṣṭādhikāra

chapter on the true motions of the Sun and Moon

3.1 Introduction

The second chapter of the *Grahalāghava* focuses on determining the true longitudes and motions of the Sun and Moon at a desired time for a user's locale, a process which involves applying corrections to the mean longitudes and motions found in the previous chapter.[1] Verses 2.1–2.4 describe the process of correcting the longitudes and velocities of the Sun and Moon for orbital inequalities in the perspective of an observer at zero latitude and longitude. Verses 5-7 explain how to correct the longitudes for an observer's locale at nonzero longitude and/or latitude. The final two verses discuss how to use these true longitudes of the Sun and Moon to calculate four elements of the *pañcāṅga* ("five parts") calendar/almanac for an observer's locale: the *nakṣatra* (constellation), *tithi* (lunar day), *yoga* (sum of the solar and lunar longitudes), and *karaṇa* (half-*tithi*).

To provide some necessary background for the explanation of Gaṇeśa's verses, we provide a brief summary of the basic features of trigonometric orbital models in Sanskrit astronomy. In the classical geocentric model, the mean planetary motion of all the planets was accepted to be in concentric uniformly circular orbits centered around the Earth, in the order as shown in Figure 3.1. The relationship between mean longitude $\bar{\lambda}$ as a product of mean daily motion $\bar{\lambda}_d$ and elapsed civil days d_c, added to the longitude of the planet at epoch $\bar{\lambda}_0$ is expressed symbolically as:

$$\bar{\lambda} = \bar{\lambda}_0 + \bar{\lambda}_d \cdot d_c$$

However, astronomers observed that each planet's apparent motion seemed to vary in its respective orbit around the zodiac: at times, the planet appeared to move slowly; at other times, quickly. At certain times the planets also appeared closer and larger or further and smaller to the astronomers observing on Earth. Our modern analysis tells us that these observations arise as a result of planetary orbits being elliptical as

[1] The true motion chapter of a standard Sanskrit astronomy text describes the corrections necessary to find true longitudes and motions of all the planets– not just those of the Sun and Moon. This is the case in texts such as the *Āryabhaṭīya* [Shu76, Ch.3, p.85-112], *Laghumānasa* [Shu90, Ch.3, p.120-136], *Siddhāntaśiromaṇi* [Ark80, Ch.2, p.102-222], and the *Karaṇakutūhala* [Mis91, Ch.2, p.18-34]. But Gaṇeśa's decision to split the true motion phenomenon into two chapters is not unique; chapter two of the *Śiṣyadhīvṛddhidatantra* [Cha81a, Ch.2, p.34-47] even has the corrections and concepts relating to the Sun and Moon described in the same order as Gaṇeśa does in the second chapter of the *Grahalāghava*. The first section of the true motion chapter of the *Vaṭeśvarasiddhānta* [Shu86, Ch.2.2, p.159-193] also focuses on the corrections of the Sun and Moon before the subsequent section addresses those of the other planets.

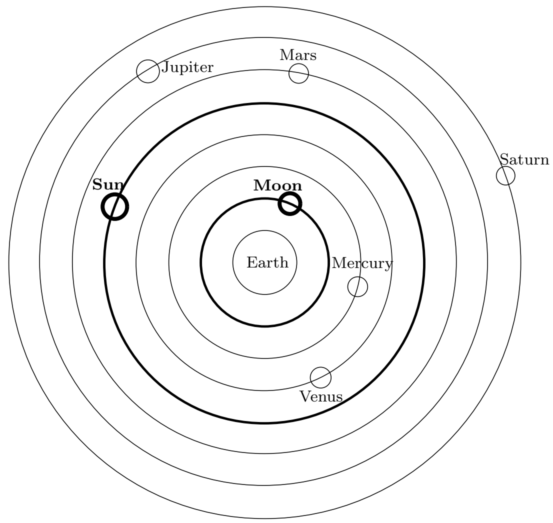

Figure 3.1: The order of the planets, stated in sources such as the *Paitāmahasiddhānta* [TD89, 13.39–41, p.71] (as discussed in [SS85, 4.15.2, p.31]): "Beyond the Moon, are Mercury, Venus, Sun, Mars, Jupiter, and Saturn, and beyond that there are fixed stars. All the planets move in their own individual orbits at constant speed". Thus, the underlying base model depicts uniformly circular planetary motion with concentric circular orbits.

opposed to circular, and planets revolving around focii instead of a center. Instead of rejecting the notion of simple circular motion, astronomers of history incorporated multiple circles in their models to represent this ellipticity. In particular, second millennium Sanskrit astronomers predominantly relied on either of two equivalent geocentric models shown in Figure 3.2: an eccentric model and an epicyclic model.[2]

In both models, the mean planet moves counterclockwise along the concentric circle centered about an observer on Earth. In the eccentric model, the true planet moves uniformly counterclockwise at its rate of mean motion along a circular path called an "eccentric circle", with its center offset from the observer on Earth. This offset is called "eccentricity". In the epicyclic model, the mean planet is the center of a smaller circle called an "epicycle" on which the true planet moves at the rate of its mean motion. The radius of the epicycle is equivalent to the eccentricity. Both models are equivalent in describing the true-motion path of a planet along an eccentric circle. The imaginary line running through the points of an observer on Earth and the center of the eccentric circle is called the "apsidal line" on which the longitudes of the planet's furthest point (apogee) and nearest point (perigee) lie. The longitudes of the apogee and perigee are not fixed, due to what is known in modern astronomy as apsidal precession. Qualitatively, the planetary orbit itself rotates a small amount over time.

The angular difference between the longitudes of the true planet and the mean planet is known as the "*manda*-correction" or "eccentric correction". Also known as the "equation of center", the *manda*-correction is applied to the mean longitude to find the true longitude of the planet. The *manda*-correction μ is found trigonometrically using the *manda*-anomaly κ_M (the angular difference between the longitudes of the mean planet $\bar{\lambda}$ and the *manda*-apogee λ_{A_M}) and the eccentricity, which is equivalent to the epicycle radius r_M.

The trigonometric determination of μ is derived as follows. As seen in Figure 3.3, a perpendicular is dropped from true planet P onto the extended orbital radial $O\bar{P}$ at point Q. Point P' is where the apsidal line intersects the concentric orbit, and a perpendicular is dropped from P' onto $O\bar{P}$ at intersection point

[2]We do not attempt to exhaustively discuss the development of true-motion models in Sanskrit astronomy in this research. We intend to provide just enough general context so that the reader may appreciate Gaṇeśa's verses and salient portions of the commentaries of Mallāri and Viśvanātha.

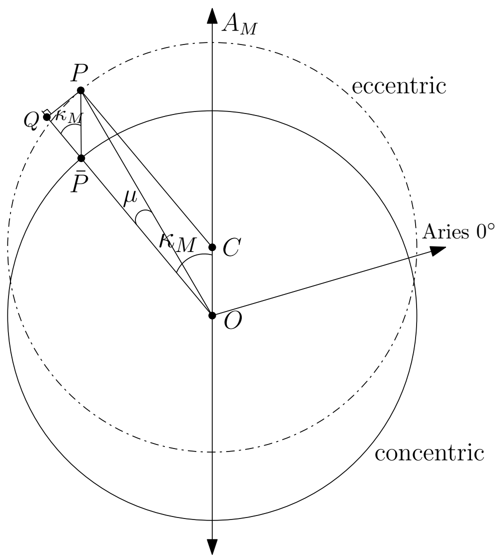

Figure 3.2: The eccentric (left) and epicyclic (right) models are modifications of the basic geocentric simple uniform concentric motion model, which is preserved as the mean planetary path shown as the black circle (labelled "concentric" in the eccentric model). In both models, O is the point of an observer on Earth, the mean longitude of the planet is $\bar{P}$, and the true longitude of the planet is P. PQ is the perpendicular dropped from P onto the orbital radius line $O\bar{P}$. The apsidal line is the line OA_M, which has a longitude measured from the zero point at Aries 0°. The angle $\bar{P}OC$ is the *manda*-anomaly κ_M and $\bar{P}OP$ is the *manda*-correction μ. In the eccentric model, C is the center of the eccentric circle (dot-dashed). In the epicyclic model, the mean planet $\bar{P}$ is the center of a smaller circle called the epicycle (dot-dashed). The true planet P travels uniformly circularly on the epicycle.

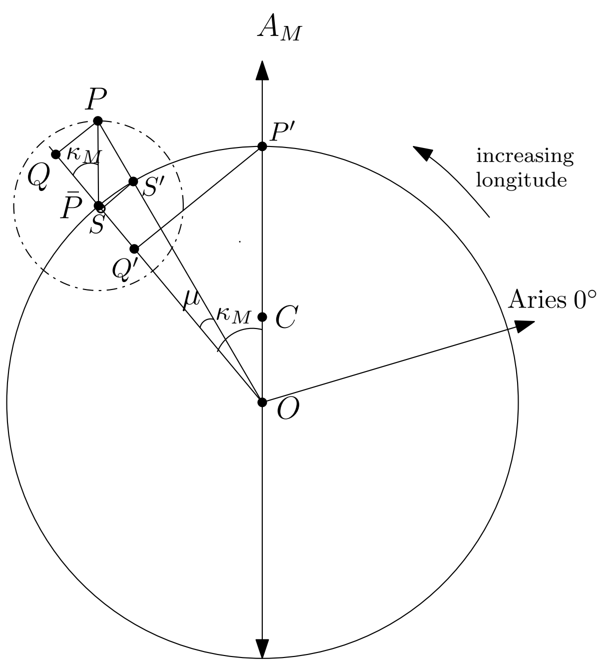

Figure 3.3: The epicyclic model of the *manda*-correction relies on proportionally scaling the orbital radius down to the epicycle radius and using similar triangles to find the *manda*-correction μ.

Q'. Since $\triangle OQ'P'$ is a right triangle, and $OP' = R$ the orbital radius, we see that:

$$\begin{aligned}\sin\kappa_M &= \frac{P'Q'}{OP'}\\ &= \frac{P'Q'}{R}\end{aligned}$$

and so $P'Q' = R\sin\kappa_M$. Since $P\bar{P}$ is parallel to the apsidal line, $P\bar{P}Q = \kappa_M$. Then using a proportion due to triangle similarity, we see that:

$$\begin{aligned}\frac{PQ}{P'Q'} &= \frac{P\bar{P}}{P'O}\\ PQ &= \frac{r_M}{R}R\sin\kappa_M\\ &= r_M\sin\kappa_M\end{aligned}$$

and similarly:

$$\begin{aligned}Q\bar{P} &= \frac{r_M}{R}R\cos\kappa_M\\ &= r_M\cos\kappa_M.\end{aligned}$$

In triangle $\triangle PQO$, the side length QO is the length $Q\bar{P}$ added to the orbital radius R, or $QO = r_M\cos\kappa_M + R$. Note that depending on *manda*-anomaly value, this can be a subtractive expression as well (see the case of mean planet at $\bar{P}_6$ and true planet at P_6 in Figure 3.4): $QO = r_M\cos\kappa_M - R$. So, we express this in its general form: $QO = r_M\cos\kappa_M \pm R$. Then its hypotenuse is the square root of the sum of the squares of its other two sides:

$$OP = \sqrt{(r_M\sin\kappa_M)^2 + (r_M\cos\kappa_M \pm R)^2}$$

Now, consider triangle $\triangle OPQ$ scaled to the concentric as triangle $\triangle OS'S$, with $S'S = R\sin SOS' = R\sin\mu$. By definition, these two triangles are similar, and $OS' = R$. Once again, we use a proportion to see that:

$$\begin{aligned}\frac{S'S}{OS'} &= \frac{PQ}{OP}\\ \frac{R\sin\mu}{R} &= \frac{r_M\sin\kappa_M}{OP},\end{aligned}$$

and we can substitute the expression for OP to get:

$$\sin\mu = \frac{r_M\sin\kappa_M}{\sqrt{(r_M\sin\kappa_M)^2 + (r_M\cos\kappa_M \pm R)^2}}$$

The *manda*-hypotenuse, as OP is called, is conventionally approximated to be commensurate with the orbital radius R. Thus,

$$\sin\mu \approx \frac{r_M\sin\kappa_M}{R}$$

and taking the arcsine of this yields the *manda*-correction μ, which is applied to the mean longitude of the planets to yield the *manda*-corrected longitude λ_M:

$$\lambda_M = \bar{\lambda} \pm \mu.$$

The *manda*-correction is positively applied when $0° \leq \kappa_M \leq 180°$, and negative otherwise.

In the Sanskrit tradition, astronomers consulted tabulated trigonometric function values to use in their

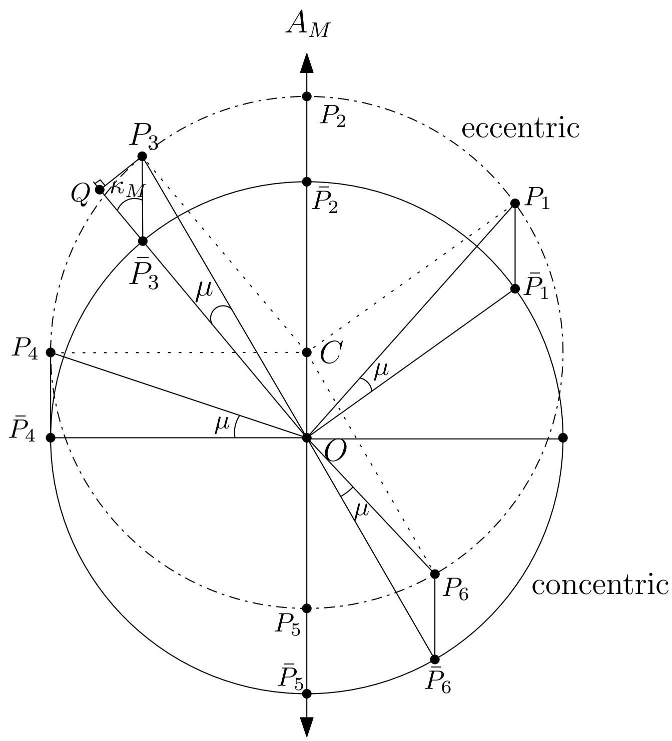

Figure 3.4: The *manda*-correction depicted for a planet using the eccentric model. The mean planet moves eastward from $\bar{P}_1$ to $\bar{P}_6$ along the concentric circle centered at an observer's position on Earth O. The true planet also travels eastward from P_1 to P_6 along the eccentric circle, centered about C, where OC represents the eccentricity of the planet's orbit. The configuration of the *manda*-correction can be seen for every quadrant.

computations. These values were precomputed for degrees of arc[3] between 0 and 90 degrees. So, astronomers had to reduce arcs down to the first quadrant to yield the appropriate arguments for their trigonometric functions. Reduced sine arcs were called *bhuja* and their complementary cosine arcs were called *koṭi*.

Now, we have seen the reliance on the sine function and arcsine computation for the *manda*-correction, which was canonically necessary. Vaṭeśvara for instance provides one section of his true motion chapter detailing the correction of the five planets' longitudes using epicyclic theory and a subsequent separate section detailing corrections using eccentric theory. But interestingly enough, he includes a section explaining how the corrections can be done without using an Rsine table. Specifically, he states a rule equivalent to Bhāskara I's sine approximation formula for an arc restricted to the first quadrant θ:[4]

$$\sin\theta = \frac{4\,(180-\theta)\,\theta}{40500-(180-\theta)\,\theta}$$

Vaṭeśvara states that when the *bhuja* of the *manda*-anomaly is supplied to this formula, and is multiplied by the *manda*-epicycle radius, the Rsine of the *manda*-correction is produced. The *manda*-epicycle radius is also the Rsine of the maximum *manda*-correction (when the *manda*-anomaly is 90 degrees). So, the Rsine

[3] Note that these trigonometric functions are functions of arcs of circles, not of angles [DS83, p.39].

[4] Originally found in his *Mahābhāskarīya* [Sas57, v. 7.17–19, p. 378], this rule is also found in the *Brāhmasphuṭasiddhānta* [Dvi02, v. 14.23–24, p. 281], the *Siddhāntaśekhara* [Mis47, v. 3.17, p. 146]. The rationale for the formula is explained further in [Gup67], as discussed in footnote 5.

of the *manda*-correction μ as a function of the *bhuja* of *manda*-anomaly κ_M is:

$$R\sin\mu = r_M \cdot \frac{4\,(180-\kappa_M)\,\kappa_M}{40500-(180-\kappa_M)\,\kappa_M}$$

Since a planet's velocity is the rate of change of its longitude, a correction applied to its longitude impacts its velocity. We will call this non-constant orbital velocity v, and henceforth will refer to the constant daily velocity or 'mean daily motion' $\bar{\lambda}_d$ of the mean planet by the symbol $\bar{v}$. Thus, a velocity *manda*-correction Δ_v^M is applied to the mean velocity $\bar{v}$ to produce the true velocity of a planet v, just as in the following relation:

$$v = \bar{v} + \Delta_v^M$$

The true velocity is an instantaneous velocity, taken at a specific time since the speed of the planet is not constant. True daily velocity of the planet can be approximated as the difference in true longitude between two successive days.[5] So for two consecutive true longitudes λ_1 and λ_2, and their respective mean longitudes $\bar{\lambda}_1$ and $\bar{\lambda}_2$, we get that the *manda*-velocity correction can be approximated as:

$$\Delta_v^M = \bar{v} - v = (\bar{\lambda}_2 - \bar{\lambda}_1) - (\lambda_2 - \lambda_1) = (\bar{\lambda}_2 - \lambda_2) - (\bar{\lambda}_1 - \lambda_1) = \mu_2 - \mu_1$$

As a result, Sanskrit text authors conventionally present a formula for the *manda*-velocity correction with respect to a change in *manda*-correction values. This change of can be represented either as a difference between successive values of the sine of μ, or as its cosine.[6] Here, we represent the analytic cosine representation as is in the *Siddhāntaśiromaṇi*:

$$\Delta_v^M = (\bar{v} - v_{A_M}) \cdot \cos\kappa_M \cdot (\frac{r_M}{R})$$

where for all planets apart from the moon, the *manda*-apogee velocity v_{A_M} is zero.[7] The *manda*-correction and *manda*-velocity correction are both applicable to all planets, but are the only [orbital] corrections necessary to find the true longitudes (and daily motions) of the Sun and Moon.

The corrections applied thus far account for mean sunrise at a zero point (place at zero degrees latitude and longitude), however observers require the true longitude and daily motions of the Sun and Moon at their local latitude and longitude. Recall that Laṅkā is the conventional zero point with zero terrestrial latitude and longitude. So, local terrestrial latitude and longitude must be found. At true local noon on an equinox day, that is, when the Sun crossing the observer's local meridian intersects the celestial equator, local latitude ϕ is the angular difference between the observer's zenith and the noon equinoctial sun. Local latitude is also found using a fixed upright stick or rod (standard length is 12 *aṅgulas*) called a gnomon. At local noon on the equinox, a right triangle is formed between the upright gnomon, its shadow, and the ray of the noon equinoctial sun. The angle produced from the ray of the noon equinoctial sun and the gnomon is ϕ, as shown in Figure 3.5. The latitude ϕ is trigonometrically related to the gnomon (length $g = 12$) and its noon equinoctial shadow length s_0, shown with a proportion:

$$\frac{s_0}{g} = \frac{R\sin\phi}{R\cos\phi}$$
$$s_0 = 12\tan\phi$$

and thus local terrestrial latitude is found.

[5] We see this observation in sources such as Bhāskara II's *Siddhāntaśiromaṇi* [Ark80, 2.36cd, p.119], and Vaṭeśvara's *Vaṭeśvarasiddhānta* [Shu86, 2.1.96, p.188].

[6] The change in *manda*-correction is represented as some difference of Rsine values scaled by a *manda*-epicycle factor in *Vaṭeśvarasiddhānta* [Shu86, 2.1.97–98, p.188], the *Khaṇḍakhādyaka* [Cha70, 1.1.20, p.51], the *Śiṣyadhīvṛddhidatantra* [Cha81a, 2.15, p.37], the *Karaṇakutūhala* [Mis91, pp. 2.11–12]. The change is represented as Rcosine in the *Siddhāntaśiromaṇi* [Ark80, 2.36cd–2.38, p.155-157], and in the *Laghumānasa* where it is stated as: $\Delta_v^M = \bar{v} \cdot R\cos\kappa_M \cdot (\frac{r_M}{R})$ where the scale factor $\frac{r_M}{R}$ is mentioned as a *manda*-divisor [Shu90, v. 2.4, p. 125].

[7] We discuss the rationale in our commentary of *Grahalāghava* verse 2.1, as this topic is explored in Mallāri's commentary of that verse.

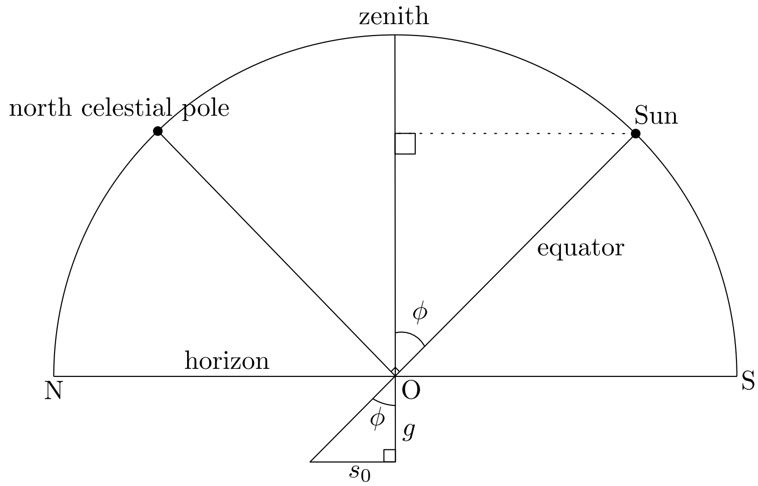

Figure 3.5: On the equinox, when the Sun stationed at the intersection of the ecliptic and the celestial equator, the Sun crosses an observer's local meridian (which is the semicircle shown here) at local true noon. A gnomon of length g (conventionally 12 *aṅgulas*) is fixed upright, and the observer is imagined to be at its tip, at the intersection with the horizon. The imaginary point directly above the observer and gnomon is the zenith. A ray of the noon equinoctial Sun contacting the gnomon casts a horizontal shadow of length s_0. The angle between the sunray and the gnomon is the observer's local latitude ϕ, which is also the angular difference between the zenith and the noon equinoctial Sun.

Then local terrestrial longitude is found as a measure of arc of the equator from the prime meridian or standard meridian, to the meridian of an observer's locale. This can be found by comparing the time astronomers observed phenomena to occur at various places,[8] and establishing a proportion between the time difference Δt and the circumference of the Earth:

$$\frac{\text{terrestrial longitude}}{360^\circ} = \frac{\Delta t}{60} \tag{3.1}$$

Four main corrections are applied to correct for local nonzero latitude and longitude: the *udayāntara*- ("rising difference") correction, *deśāntara*- ("place difference") correction, *bhujāntara*- ("arc difference") correction, and *cara*- ("moving") correction. The *udayāntara*-correction corrects the mean longitude of the Sun along the ecliptic for its right ascension. The *deśāntara*-correction corrects for the difference in terrestrial longitude between the zero-point and the observer. The *bhujāntara*-correction corrects for the right ascension of the Sun, which corrects mean sunrise for true sunrise. And finally, the *cara*-correction (also known as ascensional difference) corrects for the nonzero latitude of the observer on an equatorial arc.

Using the true local longitudes of the Sun and Moon and their true instantaneous velocities at true sunrise, the last two verses of this chapter of the *Grahalāghava* describe how to find *pañcāṅga* elements, which are luni-solar time divisions based on the longitudinal differences between the Sun and Moon.

[8]While there is some evidence of this sometimes having been done, there is evidence that gnomon noon equinoctial shadow lengths of locations said to lie on the prime meridian yielded different lengths [Pin96].

Degrees of Arc	Measure of *bhuja*	Measure of *koṭi*
$0 \leq \theta \leq 90$	θ	$90 - \theta$
$90 \leq \theta \leq 180$	$180 - \theta$	$90 - (180 - \theta) = \theta - 90$
$180 \leq \theta \leq 270$	$\theta - 180$	$90 - (\theta - 180) = 270 - \theta$
$270 \leq \theta \leq 360$	$360 - \theta$	$90 - (360 - \theta) = \theta - 270$

Table 3.1: For an arc θ that lies in each of the quadrants, the corresponding measures of *bhuja* as per the first two lines of verse 2.1, and the measures of *koṭi* as the complement of *bhuja*.

3.2 Verse 2.1: Finding the *bhuja* of an arc, defining quadrants, and identifying the *mandocca* of the Sun

While Gaṇeśa states that his verses do not describe sine and arc computations, his methodology and formulae still rely on trigonometric relationships. Thus he begins this chapter describing how to reduce an arc to the first quadrant constrained argument called *bhuja*, defining *koṭi* to be its complement, defining the technical term *pada* to be a quadrant, and stating the longitude of the Sun's *manda*-apogee.

दोस्त्रिभोनं त्रिभोर्ध्वं विशोष्यं रसै-
श्चक्रतोऽङ्काधिकं स्याद्भुजोनं त्रिभम् ।
कोटिरेकैककं त्रित्रिभैः स्यात् पदं
सूर्यमन्दोच्चमष्टाद्रयोंऽशा भवेत् ॥ १ ॥ ॥ स्रग्विणी ॥

dos tribhonaṃ tribhordhvaṃ viśeṣyaṃ rasaiś
cakrato'ṅkādhikaṃ syād bhujonaṃ tribham |
koṭir ekaikakaṃ tritribhaiḥ syāt padaṃ
sūryamandoccam aṣṭādrayo'ṃśā bhavet || 1 || || *sragviṇī* ||

[If an arc measure is] less than three signs (*tri-bhā*), [the *bhuja* is] the arc (*dor*) [itself]; [if the arc is] more than three signs [and less than nine signs], [take the absolute] difference [of the measure of the arc] with six (*rasa*) signs [to get the equivalent first quadrant arc]; [if the measure of the arc is] greater than nine (*aṅkā*) [signs], [subtract the measure of the arc] from twelve (*cakra*) [signs] [to yield its first quadrant equivalent]. Three-signs decreased by *bhuja* is *koṭi*. Each and every quadrant (*pāda*) is [taken] with signs three by three. The *manda*-apogee (*mandocca*) of the sun is 78 (*aṣṭa-adri*) degrees.

Commentary: In this verse, Gaṇeśa expresses fundamental trigonometric concepts, as conventionally expressed by Sanskrit astronomers at the beginning of the true motion chapter.[9] In the first two lines, Gaṇeśa describes how to reduce down the measure of an arc between 0 and 360 degrees to its first quadrant analog (between 0 and 90 degrees) termed *bhuja*. He then states the definition of a *pada*, which is the technical term meaning "quadrant", to be commensurate with 90 degrees (or three signs (*tri-bhā*)). Next, he establishes the relationship between the *bhuja* and *koṭi* to be complementary: "three signs reduced by *bhuja* is *koṭi*", or:

$$koṭi\ = 90^\circ - bhuja.$$

See Table 3.1 for the measures of *bhuja* and *koṭi* for arcs occurring in each quadrant. And lastly, he provides a given constant degree value for the *manda*-apogee for the sun, which is necessary to find the Sun's *manda-kendra* (*manda*-anomaly) in the next verse.

[9] See similar verses in, for instance, *Karaṇakutūhala* [Mis91, 2.4, p.20], and *Siddhāntaśiromaṇi* [Ark80, 2.18–19, p.120-139].

The definition of a quadrant or *pāda* is given as "three signs" or 90 degrees. Both commentators Viśvanātha and Mallāri understand "each and every quadrant [taken] three by three" in Gaṇeśa's verse to also indicate the parity of the quadrants. Viśvanātha mentions in his brief commentary on this verse [Dvi25, 20–22, p.71]:

> त्रिभिस्त्रिभी राशिभिरेकैकं[10] पदं स्यात् । तद्यथा । प्रथमं राशित्रयं विषमपदं स्यात् । द्वितीयं समं तृतीयं विषमं चतुर्थं समपदं स्यादित्यर्थः ।
>
> *tribhis tribhī rāśibhir ekaikaṃ padaṃ syāt | tadyathā | prathamaṃ rāśitrayaṃ viṣamapadaṃ syāt | dvitīyaṃ samaṃ tṛtīyaṃ viṣamaṃ caturthaṃ samapadaṃ syad ityarthaḥ |*
>
> Each quadrant is [taken] with signs three by three. How is that? The first trio of signs is an odd quadrant, then the second is even, then the third is odd, the fourth is an even quadrant; thus is the meaning.

Using very similar wording, Mallāri too comments on the parity at the beginning of his commentary. He then launches into an explanation of the quadrants and the *bhuja* and *koṭi* while invoking part of Bhāskara II's *Siddhāntaśiromaṇi* verse [Dvi25, 20–30, p.69]:

> त्रित्रिभैस्त्रिभिस्त्रिभी राशिभिरेकैकं पदं स्यात् । तद्यथा । प्रथमं राशित्रयं विषमं पदं स्यात् ततो द्वितीयं समपदं ततस्तृतीयं विषमं पदं चतुर्थं समपदमित्यर्थः ।
>
> अत्रोपपत्तिः । तत्रादौ दोर्ज्याकोटिज्यास्वरूपमुच्यते । समायां भूमौ इष्टत्रिज्याव्यासार्धेन वृत्तं दिगङ्कितं कृत्वा षष्ट्यधिकशतत्रयमितान् ३६० भागानङ्कयेत् । तत्र तिर्यगूर्ध्वरेखे च । एवं चतुर्भागाः स्युस्तेषां पदसंज्ञा । एवं चक्रे चत्वारि पदानि तत्रैकैकस्मिन् पदे नवतिर्नवतिर्भागाः । प्रथमपदे यद्गतं स एव दोः । द्वितीये एष्यं दोः । एष्यत्वार्थं षड्भशुद्धम् । उक्तं च सिद्धान्तशिरोमणौ ।
>
> अयुग्मे पदे यातमेष्यं तु युग्मे भुजो बाहुहीनं त्रिभं कोटिरुक्तेति ।
>
> अत्र दोर्ज्याकोटिज्ये एकपदमध्ये अतो दोस्त्रिभात् शुद्धः कोटिर्भवतीति युक्तमुक्तम् ।
>
> *tritribhais tribhis tribhī rāśibhir ekaikaṃ padaṃ syāt | tadyathā | prathamaṃ rāśitrayaṃ viṣamaṃ padaṃ syāt tato dvitīyaṃ samapadaṃ tatastṛtīyaṃ viṣamaṃ padaṃ caturthaṃ samapadam ity arthaḥ |*
>
> *atropapattiḥ | tatrādau dorjyākoṭijyāsvarūpam ucyate | samāyāṃ bhūmau iṣṭatrijyāvyāsārdhena vṛttaṃ digaṅkitaṃ kṛtvā ṣaṣṭyadhikaśatatrayamitān 360 bhāgān aṅkayet | tatra tiryagūrdhvarekhā ca | evaṃ caturbhāgāḥ syus teṣāṃ padasaṃjñā | evaṃ cakre catvāri padāni tatraikaikasmin pade navatir navatir bhāgāḥ | prathamapade yadgataṃ sa eva doḥ | dvitīye eṣyaṃ doḥ | eṣyatvārthaṃ ṣaṭbhaśuddham | uktaṃ ca siddhāntaśiromaṇau |*
>
> *ayugme pade yātam eṣyaṃ tu yugme bhujo bāhuhīnaṃ tribhaṃ koṭir ukteti |*
>
> *atra dorjyākoṭijye ekapadamadhye ato dostribhāt śuddhaḥ koṭirbhavatīti yuktam uktam |*
>
> *tritribhais* [meaning] each and every quadrant "[taken] three by three" signs. How is that? The first trio of signs is an odd quadrant, then the second is even, then the third quadrant is odd, the fourth is even; thus is the meaning.
>
> Here [is] the explanation. Then at the beginning, the true nature of the Rsine and Rcosine is stated. Having made a circle marked with cardinal directions on smooth ground, three hundred and sixty 360 degrees are to be marked.
>
> Then, the horizontal and vertical lines [are to be marked]. In this way, four parts [are formed]; "quadrant" is their definition. In this way, in a circle, [there are] four quadrants; there, in each and every quadrant [there are] ninety by ninety degrees. In the first quadrant, whatever [arc] has passed, just that is the *dos* (*bhuja*). In the second, the yet to be is the *dos* (*bhuja*). For the purpose of [knowing what is meant by] "yet to be", [the arc is] subtracted from six signs. It is said in the *Siddhāntaśiromaṇi*:
>
> "The *bhuja* [is] the traversed [arc] in the odd quadrant; but in the even [quadrant], [is] the yet to be traversed [arc]. The *koṭi* is said to be three signs lessened by the *bāhu* (*bhuja*)." Here,

[10] Joshi has राशिभिरेरिकैपदं in [Jos81, 8, p.49].

the Rsine of the *bhuja* and the Rsine of the *koṭi* [are] in one quadrant, so "the *koṭi* is subtracted from the *bhuja*" is added [and] stated.

Mallāri describes how to visualize quadrants and arcs of a circle, which can be seen in Figure 3.6. Bhāskara II uses the parity to identify the *bhuja* corresponding to an arc of anomaly in each quadrant in the *Siddhāntaśiromaṇi* [Ark80, 2.19, p.139].[11] However, Bhāskara II does not explicitly discuss the parity in finding the *bhuja* and *koṭi* in his *Karaṇakutūhala*. In fact, the application or omission of the parity of quadrants in Sanskrit astronomy texts is variable, as observed in [DS83, pg.43].[12] Of course, the parity of quadrants is not even explicitly stated in Gaṇeśa's verse. But its mention in both Mallāri and Viśvanātha's commentaries indicates its nontrivial nature.

In the last line of his verse, Gaṇeśa states the longitude of the solar apogee to be at 78 degrees. Mallāri explains how the solar apogee can be taken to be a fixed quantity rounded up to 78 degrees in his commentary in [Dvi25, 8–12, p.71]:

यद्येकदिनेनैतावती गतिस्तदा कल्पकुदिनैः किमिति एवं प्रसाध्योच्चभगणाः कल्पसौरवर्षैरेते ४८० लभ्यन्ते तदा कल्पगताब्दैः किमिति । अनुपाताद्ग्रन्थादौ रवेर्मन्दोच्चं २ । १७ । ५६ । ४१ सप्तभिर्वर्षै रवेर्मन्दोच्चगतिरेका १ विकला लभ्यते । अत आचार्येण स्थिरं निबद्धम् । बहुकाले ये गणकतिलका उपत्स्यन्ते ते अनेनैवानुपातेन रचयिष्यन्ति ।

yady ekadinenaitāvatī gatis tadā kalpakudinaiḥ kim iti evaṃ prasādhyoccabhagaṇāḥ kalpasauravarṣair ete 480 labhyante tadā kalpagatābdaiḥ kim iti | anupātād granthādau raver mandoccaṃ 2/17/56/41 saptabhir varṣai raver mandoccagatir ekā 1 vikalā labhyate | ata ācāryeṇa sthiraṃ nibaddham | bahukāle ye gaṇakatilakā upatsyante te anenaivānupātena racayiṣyanti

If with one day the velocity is this much, then with the days of a *kalpa*, what is it; having found thus the revolutions of the apogee with the solar years of a *kalpa*, these 480 are obtained; then with the years elapsed [from the beginning] of the *kalpa* what is it? From the proportion, at the beginning of the text, the *manda*-apogee of the Sun $2^s|17°|56'|41''$ with 7 years the *manda*-apogee velocity of the Sun is 1 second. So, [the *manda*-apogee] is fixed as unmoving by the teacher. In a long time, those who rise to be the decorated amongst mathematicians, will make [the *manda*-apogee calculation] with another proportion.

In other words, Mallāri uses period relations to find the mean longitude and daily motion of the *manda*-apogee to justify accepting the longitude as fixed. Since there is no orbital correction applied to the mean longitude of the solar *manda*-apogee, the mean longitude is taken to be its true longitude. The longitude of the solar apogee at the beginning of the *Grahalāghava* at Śaka 1442 is given by Mallāri as $2^s, 17°; 56', 41''$. We can in fact find this using the *Siddhāntaśiromaṇi* revolutions for apogees of planets. The solar *manda*-apogee completes 480 revolutions in 4320000000 years [Ark80, p.18], and the beginning of Śaka 1442 is 1972948621 years from the beginning of the present *kalpa*, as seen in Table 2.4. So, we find the longitude of the solar

[11] "A quadrant [is taken] with three signs; in a circle, [there are] four [of] those [quadrants] of which the definition of odd and even is successively made. The *bhuja* [is] the traversed [arc] in the odd quadrant; but in the even [quadrant], [is] the yet to be traversed [arc]. The *koṭi* is said to be three signs lessened by the *bāhu* (*bhuja*).

[12] Āryabhaṭa for instance states different epicycle dimensions for each of the planets for even and odd quadrants [Shu76, 1.11, p.23]. Meanwhile, Bhāskara I in both [Sas57, 4.1, p.176] and [Shu63, 2.1, p.5] states the basic definition, "three signs form a quadrant." But Lalla and Vaṭeśvara (as well as Bhāskara II of course) note the parity of quadrants and its use in finding the *bhuja* and *koṭi* of arcs. Lalla remarks on the method of reckoning the *bhuja* and *koṭi* as related to the quadrant's parity:"three anomalistic signs form a quadrant. The quadrants are successively distinguished as odd and even [and are respectively] known as *bhuja* or *koṭi*" as in [Cha81a, 2.10, p.32]. Vaṭeśvara in [Shu86, 2.54–55, p.165] mentions: "In the odd (anomalistic) quadrant, the Rsines of the arcs traversed by the planet are defined as *bhuja* and *agra* (*koṭi*), (more correctly, *bhujajyā* and *koṭijyā*), (respectively); in the even (anomalistic) quadrant, the *bhuja* and *agra* are the Rsines of the arcs to be traversed and traversed (respectively)." Both Lalla and Vaṭeśvara discuss "anomalistic" quadrants, which have to do with the arc of anomaly as reckoned from the planet's apogee, which Gaṇeśa explains in the next verse.

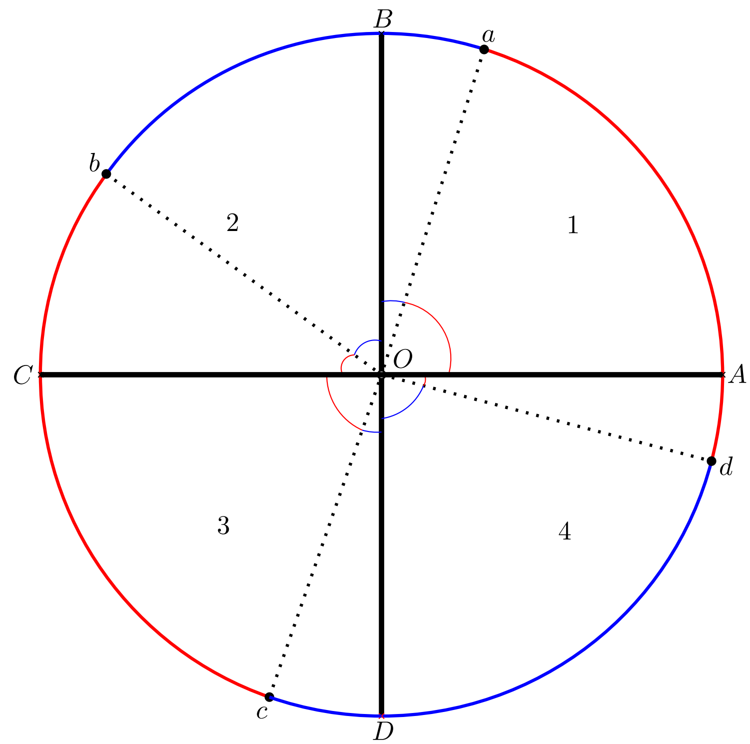

Figure 3.6: A visual representation of the four quadrants and the *bhuja* and *koṭi* as stated by Mallāri in his commentary. Assuming A as the zero point of the quadrants, angles AOa, AOb, AOc, and AOd corresponding to arcs Aa, Ab, Ac, and Ad lie in each of the four quadrants. The arcs in red represent the *bhuja* of an arc occurring in each quadrant, and the arcs in blue represent the *koṭi*. Mallāri quotes Bhāskara II's *Siddhānta-śiromaṇi* observation that in the odd quadrants, the *bhuja* is the arc transversed within the quadrant. The *koṭi* as its complement is the arc yet to be transversed within each quadrant. The opposite holds true for the even quadrants.

manda-apogee λ_{A_M} to be:

$$\begin{aligned}\lambda_{A_M} &= \frac{\text{Revolutions per } \textit{kalpa}}{\text{Years per } \textit{kalpa}} \cdot \text{Years elapsed since beginning of } \textit{kalpa} \\ &= \frac{480}{4320000000} \cdot 1972948621 \\ &= 219\,\text{completed revolutions} + 2^R, 17°; 56', 41''\end{aligned}$$

Then, using this same period relation, the daily motion of the solar *manda*-apogee is:

$$\begin{aligned}v &= \frac{480 \text{ revolutions per } \textit{kalpa}}{4320000000 \text{ years in a } \textit{kalpa}} \cdot 360° \\ &= 0; 0, 0, 8, 38, 24°\end{aligned}$$

So, the *manda*-apogee of the Sun has a velocity of $0; 0, 0, 8, 38, 24°$ per year according to the *Siddhānta-śiromaṇi*. Meanwhile, Mallāri states that average change in *manda*-apogee over 7 years is 1 second, or $0; 0, 0, 8, 34, 17°$ per year, which may possibly be an approximation of the *Siddhāntaśiromaṇi* solar apogee velocity.

Perhaps this difference in solar *manda*-apogee velocity arrives from using the parameters of a different *pakṣa*. When we use *Sūryasiddhānta* parameters (387 revolutions in a *kalpa* 4320000000 years per *kalpa*, and 1955884621 years elapsed until the *Grahalāghava* epoch) [SW81, v. 1.41, p.7], we get that the longitude of the Sun's *manda*-apogee is $2^s, 17°; 16, 44$ and its daily motion is found to be $0; 0, 41, 47, 45, 36°$ per year. These values differ from those Mallāri states significantly. *Āryabhaṭīya* parameters (13 revolutions in 119167416000 years, and 3,244,621 years elapsed until *Grahalāghava* epoch) [Shu76, p.21], we get that the longitude of the Sun's *manda*-apogee is $0; 0, 7, 38$.[13]

In any case, since this daily velocity is so small, it is taken as negligible, and thus the solar apogee's longitude is taken to be fixed. Interestingly, Mallāri's final remark in this passage of commentary addresses that the *manda*-apogee longitudes will have to be updated in the distant future, due to the movement of the apogees however small.

3.3 Verse 2.2: Finding the *manda*-correction of the Sun

After providing the longitude for the solar *manda*-apogee in the previous verse, Gaṇeśa explains how it is used to find the *manda*-anomaly and the solar *manda*-correction in this verse.

मन्दोच्चं ग्रहवर्जितं निगदितं केन्द्रं तदाख्यं बुधैः
केन्द्रे स्यात् स्वमृणं फलं क्रियतुलाद्येऽथो विधेयं रवेः ।
केन्द्रं तद्भुजभागखेचरलवोनघ्ना नखास्ते पृथक्
तद्गोंशोननगेषुभिः परिहृतास्तेंऽशदिकं[14] स्यात् फलम् ॥ ॥ २ ॥ ॥ शार्दूलविक्रीडित ॥

mandoccaṃ grahavarjitaṃ nigaditaṃ kendraṃ tadākhyaṃ budhaiḥ
kendre syāt svam ṛṇaṃ phalaṃ kriyatulādye'tho vidheyaṃ raveḥ |
kendraṃ tadbhujabhāgakhecaralavonaghnā nakhās te pṛthak
tadgoṃśonanageṣubhiḥ parihṛtās te'ṃśādikaṃ syāt phalam || 2 || || *śārdūlavikrīḍita* ||

[13] Shukla notes that the motions of the apogees ascribed to tradition by Bhāskara I and other commentators of the *Āryabhaṭīya* are much less than their actual motions. Āryabhaṭa does not explicitly state the number of revolutions of each planet's apogees in some period; he instead states longitudes for each of the planets but implies minimal movement [Shu76, pp. 20–21].

[14] In [Dvi25, p.71], the *avagraha* "ऽ" is omitted. We defer instead to the verse as presented in [Jos81, p.50].

It is stated by the intelligent ones that the [longitude of] the *manda*-apogee diminished by the [mean longitude of the] planet is called the [*manda*-]anomaly. When the anomaly is in [the six signs] beginning with Aries or Libra, the [*manda*-]correction is positive [or is] negative [respectively]. Now the anomaly of the Sun is to be used. They, the twenty (*nakha*) [degrees] diminished and multiplied by a ninth (*khecara*) part of the *bhuja* of that [anomaly] in degrees, are separately [placed in two places]. [Those degrees in one place are] divided by 57 (*naga-iṣu*) [degrees] diminished by a ninth part of those [degrees placed in the second place]; the [resulting *manda*-]correction is in degrees and so on.

Commentary: In this verse, Gaṇeśa first explains how to find the *manda*-anomaly κ_M used to compute the *manda*-correction μ to correct the Sun's mean longitude. As stated by Gaṇeśa, the *manda*-anomaly angle κ_M is found as a difference between the longitude of the planet's *manda*-apogee λ_{A_M} and its mean longitude $\bar{\lambda}$:

$$\kappa_M = \lambda_{A_M} - \bar{\lambda}.$$

This is the general formula used to find the *manda*-anomaly of all of the planets. Gaṇeśa provided the longitude of the solar apogee as 78 degrees in verse 2.1, so the *manda*-anomaly κ_M of the Sun is:

$$\kappa_M = 78 - \bar{\lambda}$$

and is reckoned counterclockwise from the *manda*-apogee longitude at 78 degrees. Using the quadrant definition from the previous verse, each subsequent 90 degrees from the *manda*-apogee longitude is a quadrant (or sometimes specified as "(anomalistic) quadrant" by editors).So, the "[six signs, or two quadrants] beginning with Aries" is really between 0-180 degrees from the *manda*-apogee longitude, and "[six signs] beginning with Libra" represent between 180-360 degrees. Following the nature of a sine curve, this *manda*-correction is positive and increasing in the first quadrant, positive and decreasing in the second, negative and decreasing in the third and positive and increasing in the fourth.

The reckoning of the *manda*-anomaly angle as the difference between mean longitude of a planet and the longitude of its *manda*-apogee is in fact a well-established notion. Both Mallāri and Viśvanātha begin their commentaries of this verse with Gaṇeśa's definition of the *manda*-anomaly: $\kappa_M = \lambda_{A_M} - \bar{\lambda}$. Mallāri glosses every member of compounds of the first line of this verse in [Dvi25, 2–3, p.72]:

ग्रहेण वर्जितं हीनं यन्मन्दोच्चं तत् तदाख्यं मन्दमेवाख्या नाम यस्येति मन्दकेन्द्रं बुधैरतीन्द्रियदृग्भिराचार्यैर्निगदितं प्रोक्तम् ।

graheṇa varjitaṃ hīnaṃ yanmandoccaṃ tat tadākhyaṃ mandam evākhyā nāma yasyeti mandakendraṃ budhair atīndriyadṛgbhir ācāryair nigaditaṃ proktam |

Whatever [longitude of the] *manda*-apogee "*varjita*" [meaning] "diminished" by [the mean longitude of] the planet is "*tadākhya*" [meaning] "name is just *manda*" [meaning] "that of which the name is *mandakendra*" "*budhai*" [meaning] "by those teachers whose sight surpasses the five senses" "*nigadita*" [meaning] "is stated".

Viśvanātha too explains this in his first sentence of commentary [Dvi25, 21–22, p.74]:

मन्दोच्चं ग्रहेण रहितं कार्यं तदाख्यं बुधैः केन्द्रं निगदितम् ।

mandoccaṃ graheṇa rahitaṃ kāryaṃ tadākhyaṃ budhaiḥ kendraṃ nigaditam |

The [longitude of the] *manda*-apogee is to be diminished by the [mean longitude of the] planet; its name is stated by the intelligent ones [to be] [*manda*-]*kendra*.

In his *Karaṇakutūhala*, Bhāskara II expresses the *manda*-anomaly in this same way, however earlier in his *Siddhāntaśiromaṇi* [Ark80, p. 2.18] he defines the *manda*-anomaly in the opposite order, as in $\kappa_M = \bar{\lambda} - \lambda_{A_M}$.

Viśvanātha does not mention this alternate reckoning of the *manda*-anomaly. He instead adds a brief discussion of another orbital quantity employed in an additional correction for the five star-planets: namely, the so-called *śīghra*-anomaly, which was introduced in Chapter 2 as a proxy for the inferior planets Mercury and Venus, and will be geometrically explained in Chapter 4[15] [Dvi25, 22–23, p.74]:

यदा मन्दोच्चाद्ग्रहः शोध्यते तदा मन्दकेन्द्रं भवति यदा शीघ्रोच्चाद्ग्रहः शोध्यते तदा शीघ्रकेन्द्रं भवति ... ।

yadā mandoccād grahaḥ śodhyate tadā mandakendraṃ bhavati yadā śīghroccād grahaḥ śodhyate tadā śīghrakendraṃ bhavati...

When the [mean longitude of the] planet is subtracted from the [longitude of the] *manda*-apogee, then [it is] the *manda*-anomaly; when the [mean longitude of the] planet is subtracted from the [longitude of the] *śīghra*-apogee, then [it is] the *śīghra*-anomaly...

After finding the *manda*-anomaly angle, its *bhuja* (which we also annotate as κ_M) is supplied as the argument of Gaṇeśa's algebraic formula to find the *manda*-correction μ of the Sun:

$$\mu = \frac{\left(20 - \frac{\kappa_M}{9}\right) \times \frac{\kappa_M}{9}}{57 - \left(\frac{\left(20 - \frac{\kappa_M}{9}\right) \times \frac{\kappa_M}{9}}{9}\right)}$$

This *manda*-correction is applied positively to the mean longitude of the Sun if $0° \leq \kappa_M \leq 180°$ and is applied negatively if $180° \leq \kappa_M \leq 360°$, as seen in Figure 3.4.

We will now attempt a reconstruction of how to proceed from Bhāskara I's algebraic sine rule and Vaṭeśvara's application of it to the *manda*-anomaly sine as discussed in section 3.1 to approximate Gaṇeśa's algebraic formula. We infer from Vaṭeśvara's application that Gaṇeśa was trying a similar approach. So, we use the value of r_M stated in Mallāri's commentary as our best guess at Gaṇeśa's value, since he does not state it [Dvi25, 10–16, p.73]:

अत्र रवेर्मन्दपरिध्यंशाः १३।४३।४२ । अस्मादनुपातः । यदि भांशपरिधे ३६० स्त्रिज्यामितं १२० व्यासार्धं लभ्यते तदा एषां परिधिभागानां किमिति तेषां त्रिज्या १२० गुणो भगणांशाः ३६० भागहारः । अत्र गुणहारौ गुणेनापवर्त्य हरस्थाने त्रयो लब्धास्तस्मात् त्रिभक्ताः परिधयः परिधीनां व्यासार्धानि स्युस्ताः परमफलज्या एवं रवेः परमफलज्या ४।३४।३४ अस्याः धनुः सूर्यस्य परमं मन्दफलम् २।१०।४५ । एवं चन्द्रादीनामपि परममन्दफलानि साध्यानि ।

atra raver mandaparidhyaṃśāḥ 13 | 43 | 42 | asmād anupātaḥ| yadi bhāṃśaparidhes 360 trijyāmitaṃ 120 vyāsārdhaṃ labhyate tadā eṣāṃ paridhibhāgānāṃ kim iti teṣāṃ trijyā 120 guṇo bhagaṇāṃśāḥ 360 bhāgahāraḥ | atra guṇahārau guṇenāpavartya harasthāne trayo labdhās tasmāt tribhaktāḥ paridhayaḥ paridhīnāṃ vyāsārdhāni syus tāḥ paramaphalajyā evaṃ raveḥ paramaphalajyā 4 | 34 | 34 asyāḥ dhanuḥ sūryasya paramaṃ mandaphalam 2 | 10 | 45 | evaṃ candrādīnāṃ api paramamandaphalāni sādhyāni |

Here, the degrees of circumference of the *manda*-[epicycle] circumference of the Sun: 13 | 43 | 42 | From that, a proportion: If 360 of the circumference of the degrees of the orbit produces a half-diameter 120 commensurate with three signs, then those of the degrees of circumference of that [*manda*-epicycle] [produce] what? Of those, the radius 120 is the multiplier and the degrees of revolution 360 are the dividing parts. Here, the multiplier and divider, [simplyfing and] reducing by the multiplier, three is in in the divisor's place; from that, the circumferences divided by 3 [which are] the radii of the circumferences are those Rsines of maximal [*manda*-]correction; in this way, the Rsine of maximal [*manda*-]correction: 4 | 34 | 34 | Its arc [is] the

[15]This discussion of both anomalies in the same verse is quite common in Sanskrit astronomy texts. Of course since Gaṇeśa has chosen to split the planetary corrections into two chapters, the explanation of the *śīghra*-correction phenomenon and finding its anomaly is discussed in the next chapter.

maximal *manda*-correction of the Sun: 2 | 10 | 45 | In this way, the maximal *manda*-corrections of [the planets] the Moon and so on are found.

Here, Mallāri expresses a proportion between the orbital radius and its circumference and the *manda*-epicycle radius and its circumference. He states the Sun's *manda*-epicycle circumference to be $13°43'42''$, the circumference of the orbit to be $360°$ and the orbital radius to be 120. This value of $R = 120$ comes from the *Karaṇakutūhala*, [Mis91, 2.6–7, p.21]. Then the proportion to find the *manda*-epicycle radius r_M is:

$$\begin{aligned}\frac{r_M}{c_M} &= \frac{R}{C}\\ \frac{r_M}{13°43'42''} &= \frac{120}{360}\\ r_M &= \frac{13°43'42''}{3}\\ &= 4°34'34''\end{aligned}$$

and its arcsine is the maximal correction $\mu = 2°10'45'' = \frac{7845}{3600}^{\circ}$ in fractional form. We now supply this into our *manda*-correction formula. We will then rewrite and simplify the equation to attempt to match Gaṇeśa's formula in verse 2.2:

$$\begin{aligned}R\sin\mu &= \frac{7845}{3600}\cdot\frac{4\,(180-\kappa_M)\,\kappa_M}{40500-(180-\kappa_M)\,\kappa_M}\\ &= \frac{523}{60}\cdot\frac{(180-\kappa_M)\,\kappa_M}{40500-(180-\kappa_M)\,\kappa_M}\end{aligned}$$

and since Gaṇeśa's formula has divisors of 9 we express this equivalently as:

$$R\sin\mu = \frac{523}{60}\frac{81}{81}\cdot\frac{\left(20-\dfrac{\kappa_M}{9}\right)\dfrac{\kappa_M}{9}}{\dfrac{40500}{81}-\left(20-\dfrac{\kappa_M}{9}\right)\dfrac{\kappa_M}{9}}$$

and the denominator is further divided by 9 to yield:

$$\begin{aligned}R\sin\mu &= \frac{523}{60}\frac{81}{81}\frac{1}{9}\cdot\frac{\left(20-\frac{\kappa_M}{9}\right)\times\frac{\kappa_M}{9}}{\dfrac{500}{9}-\left(\dfrac{\left(20-\frac{\kappa_M}{9}\right)\times\frac{\kappa_M}{9}}{9}\right)}\\ &= \frac{523}{540}\cdot\frac{\left(20-\frac{\kappa_M}{9}\right)\times\frac{\kappa_M}{9}}{\dfrac{500}{9}-\left(\dfrac{\left(20-\frac{\kappa_M}{9}\right)\times\frac{\kappa_M}{9}}{9}\right)}.\end{aligned}$$

Two small adjustments then need to be made: approximating $\frac{523}{540}\approx 1$ and $\frac{500}{9}\approx 57$. So if we make these two approximations, the right hand side of the *manda*-correction equation now resembles Gaṇeśa's formula.

Now, we refer to Bhāskara II's *Karaṇakutūhala*'s *manda*-correction formula for the Sun, where the *manda*-correction μ varies linearly with $R\sin\kappa_M$ multiplied by a scale factor [MMP20, p. 203]:

$$\mu \approx \frac{10R\sin\kappa_M}{550}.$$

Here, the scale factor $\frac{10}{550}$ represents $\frac{r_M}{R}$, as in the general Rsine *manda*-correction trigonometric formula:

$$R\sin\mu = \frac{r_M}{R}R\sin\kappa_M.$$

Bhāskara II states values for Rsine (for R = 120) as well as the differences between the sine values and the method of interpolation so users can find $R\sin\kappa_M$ using a smaller table of sines. Note that the left hand side of the equation contains μ itself. Since the maximal *manda*-correction for the Sun is such a small angle ($2°10'54''32'''$ in the *Karaṇakutūhala*, $2°10'45''$ in the *Grahalāghava*), Bhāskara II uses a small angle approximation to approximate $R\sin\mu \approx \mu$.

Thus, assuming Gaṇeśa too employs such a small angle approximation, we get:

$$\mu \approx R\sin\mu = \frac{\left(20 - \frac{\kappa_M}{9}\right) \times \frac{\kappa_M}{9}}{57 - \left(\frac{\left(20 - \frac{\kappa_M}{9}\right) \times \frac{\kappa_M}{9}}{9}\right)}$$

as is stated in Gaṇeśa's verse.

Figure 3.7 compares three different methods of calculating *manda*-correction values for the Sun for 0 to 90 degrees of *manda*-anomaly: an analytical method derived from the trigonometric formula stated by Bhāskara II in his *Siddhāntaśiromaṇi*: $\mu = \arcsin\left(\frac{41}{1080}\frac{120\sin\kappa_M}{120}\right)$, the formula presented in the *Karaṇakutūhala*: $\mu = \frac{10 * 120\sin\kappa_M}{550}$, and the algebraic formula presented in this verse by Gaṇeśa. Table 3.2 presents the maximum *manda*-correction values for each of these three methods.

Text	max. μ
Siddhāntaśiromaṇi	$2°;10'31''0'''$
Karaṇakutūhala	$2°;10'54''32'''$
Grahalāghava	$2°;10'45''0'''$

Table 3.2: Maximal *manda*-corrections to the Sun's mean longitude, calculated from formulae in the *Siddhāntaśiromaṇi*, the *Karaṇakutūhala*, and the *Grahalāghava*.

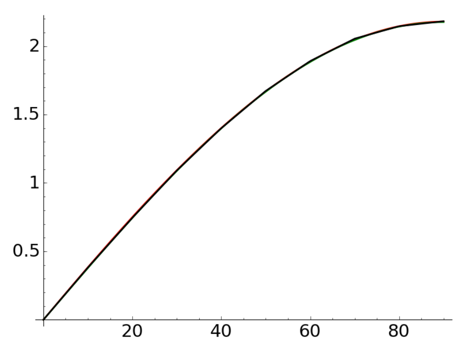

Figure 3.7: The *manda*-correction values μ in degrees for the Sun as a function of the *manda*-anomaly κ_M in degrees of arc as calculated by three different methods: derived from the *Siddhāntaśiromaṇi* (green), the *Karaṇakutūhala* (black), and the *Grahalāghava* (red).

3.4 Verse 2.3: Finding the *manda*-equation of the Moon

After stating the formula for the equation of center of the Sun in the previous verse, Gaṇeśa now presents the formula for the equation of center of the Moon.

विधोः केन्द्रदोर्भागषष्ठोननिघ्नाः
खरामाः पृथक् तन्नखांशोनितैश्च ।
रसाक्षैर्हृतास्ते लवाद्यं फलं स्या-
द्रवीन्दू स्फुटौ संस्कृतौ स्तश्च ताभ्याम् ॥ ॥ ३ ॥ ॥ भुजङ्गप्रयात ॥

vidhoḥ kendradorbhāgaṣaṣṭhonanighnāḥ
kharāmāḥ pṛthak tannakhāṃśonitaiś ca |
rasākṣair hṛtāste lavādyaṃ phalaṃ syād
ravīndū sphuṭau saṃskṛtau staś ca tābhyām || 3 || || *bhujaṅgaprayāta* ||

Thirty (*kha-rāma*) [degrees] diminished by a sixth (*ṣaṣṭha*) part of the *bhuja* of the anomaly of the moon and multiplied [by that same sixth part] [are placed] separately [in two places]. [The degrees in one place] are divided by fifty-six (*rasa-akṣa*) diminished by a twentieth part (*nakha-aṃśa*) of those [degrees in the second place], and the result is in degrees and so on. The [mean longitudes of the] moon and sun corrected by these two [*manda*-corrections] are the true [longitudes].

Commentary: In this verse, Gaṇeśa states the formula for the *manda*-correction μ for the moon as a function of the *bhuja* of the *manda*-anomaly κ_M:

$$\mu = \frac{\left(30 - \frac{\kappa_M}{6}\right) \times \frac{\kappa_M}{6}}{56 - \left(\frac{\left(30 - \frac{\kappa_M}{6}\right) \times \frac{\kappa_M}{6}}{20}\right)}.$$

Since the lunar apogee's mean daily motion is significant, its longitude is not taken to be fixed as was that of the solar apogee. The longitude is instead found in the same manner as the mean longitudes of the other planets, as seen in verse 2.5.2. The lunar apogee, like the other planetary apogees has no orbital anomaly so its mean longitude requires no correction.

Again, Gaṇeśa's algebraic formula is an approximation of the standard trigonometric formula for the *manda*-correction μ as a function of the *manda*-anomaly κ_M:

$$\sin\mu = \frac{\frac{r_M}{R} \times R\sin\kappa_M}{R}.$$

Using Vaṭeśvara's rule with Bhāskara I's sine approximation formula, we see that the Rsine of the *manda*-correction is:

$$R\sin\mu = r_M \cdot \frac{4\,(180 - \kappa_M)\,\kappa_M}{40500 - (180 - \kappa_M)\,\kappa_M}$$

We try to arrive at Gaṇeśa's formula now by deducing Gaṇeśa's value of r_M. Mallāri states that the Rsine of maximal *manda*-correction is $5°1'40'' = \frac{181}{36}°$ [Dvi25, 8, p.76]. We now use this in our *manda*-correction formula, and rewrite and simplify the equation to attempt to match Gaṇeśa's formula in verse 2.3:

$$\begin{aligned} R\sin\mu &= \frac{181}{36} \cdot \frac{4\,(180 - \kappa_M)\,\kappa_M}{40500 - (180 - \kappa_M)\,\kappa_M} \\ &= \frac{181}{9} \cdot \frac{(180 - \kappa_M)\,\kappa_M}{40500 - (180 - \kappa_M)\,\kappa_M} \end{aligned}$$

and since Gaṇeśa's formula has divisors of 6 we express this equivalently as:

$$R\sin\mu = \frac{181}{9}\frac{36}{36}\cdot\frac{\left(30-\frac{\kappa_M}{6}\right)\frac{\kappa_M}{6}}{\frac{40500}{36}-\left(30-\frac{\kappa_M}{6}\right)\frac{\kappa_M}{6}}$$

and the denominator is further divided by 20 to yield:

$$\begin{aligned} R\sin\mu &= \frac{181}{9}\frac{36}{36}\frac{1}{20}\cdot\frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{\frac{1125}{20}-\left(\frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{20}\right)} \\ &= \frac{181}{180}\cdot\frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{\frac{1125}{20}-\left(\frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{20}\right)}. \end{aligned}$$

Two small adjustments then appear to have been made: approximating $\frac{181}{180}\approx 1$ and $\frac{1125}{20}\approx 56$. Thus the right hand side of the *manda*-correction equation now resembles Gaṇeśa's formula. Then assuming a small angle approximation as we did in the previous verse, we have:

$$\mu \approx R\sin\mu = \frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{56-\left(\frac{\left(30-\frac{\kappa_M}{6}\right)\times\frac{\kappa_M}{6}}{20}\right)}$$

as is stated in Gaṇeśa's verse.

In his commentary of this verse, Mallāri attempts to explain Gaṇeśa's choice of divisor 20 and constant 56 in [Dvi25, lines 8-17, p.76]:

> परमं चन्द्रफलं भागाद्यम् ५ । १ । ४० अत्र चन्द्रमन्दफलानयने त्रिज्या पञ्चविंशत्यधिकशतद्वयमिता धृता यावद्यावदधिका तावत्तावत् फलस्य सूक्ष्मत्वमतः सूक्ष्मत्वार्थमेतावती त्रिज्या २२५ । परमभागा नवतिः ९० । अत्रैषां भुजभागानां षडंशेन १५ ऊनास्त्रिंशत् १५ ततस्तेनैव हता परमदोर्ज्या भवति २२५ । एवमिष्टभागेभ्योऽपीष्टजीवा भवन्ति । अत उक्तं केन्द्रदोर्भागषष्ठोननिघ्नाः खरामा इति । सा त्रिज्या केन भक्ता परमं मन्दफलं स्यादिति ज्ञानार्थं परमफलेनैव भक्ता जातो हरः सावयवः ४४।४५।० असौ सावयवोऽतो लाघवार्थं रसाक्षा गृहीताः । अनयोरन्तरं ११।१५।० एतद्विंशत्या २० सवर्णितं त्रिज्या भवति २२५ । अत एवोक्तं तन्नखांशोनितै रसाक्षैस्ते हृता इति स्वस्वमन्दफलसंस्कृतावेव सूर्येन्दू स्फुटौ भवतस्तयोः शीघ्रफलाभावात् ।
>
> *paramaṃ candraphalaṃ bhāgādyam 5 | 1 | 40 atra candramandaphalānayane trijyā pañcaviṃśatyadhikaśatadvayamitā dhṛtā yāvadyāvadadhikā tāvattāvat phalasya sūkṣmatvamataḥ sūkṣmatvārthametāvatī trijyā 225 | paramabhāgā navatiḥ 90 | atraiṣāṃ bhujabhāgānāṃ ṣaḍaṃśena 15 ūnāstriṃśat 15 tatas tenaiva hatā paramadorjyā bhavati 225 | evamiṣṭabhāgebhyo'pīṣṭajīvā bhavanti | ata uktaṃ kendradorbhāgaṣaṣṭhonanighnāḥ kharāmā iti | sā trijyā kena bhaktā paramaṃ mandaphalaṃ syād iti jñānārthaṃ paramaphalenaiva bhaktā jāto haraḥ sāvayavaḥ 44 | 45 | 0 asau sāvayavo 'to lāghavārthaṃ rasākṣā gṛhītāḥ | anayor antaraṃ 11 | 15 | 0 etadviṃśatyā 20 savarṇitaṃ trijyā bhavati 225 | ata evoktaṃ tannakhāṃśonitai rasākṣaiste hṛtā iti svasvamandaphalasaṃskṛtāv eva sūryendū sphuṭau bhavatas tayoḥ śīghraphalābhāvāt*
>
> The maximum [*manda*-]correction of the moon in degrees and so on: 5 | 1 | 40. Here in the finding of the *manda*-correction of the Moon, however large the radius taken commensurate with 225, that quantity is due to the minuteness of the [*manda*-]correction, so for the sake of accuracy the radius is just this large: 225. The maximum degrees: 90. Here with with a sixth

part of these degrees of *bhuja* 15 diminished from 30: 15, then the maximal sine multiplied by this is 225. Thus from the degrees of the desired arc also the desired sines arise. So it is stated "*kendradorbhāgaṣaṣṭhonanighnāḥ kharāmāḥ*..." That radius divided by what is the maximal *manda*-correction? Thus, for the purpose of knowing [it] is divided by just the maximal correction; the divisor is with fractional parts: 44|45|0. This is with fractional parts so for the sake of brevity 56 [degrees] are taken. The difference of those two: 11|15|0; this is put in the same category (without remainders) by 20, so the radius is 225. So, it is just stated "*tannakhāṃśonitai rasākṣai hṛtā*." The [longitudes of the] Sun and Moon corrected by each of their *manda*-corrections are [their] true [longitudes], due to the nonexistence of their *śīghra*-correction.

Mallāri explains the presence of the 56 and the divisor 20 in the denominator in his commentary by using his knowledge of the maximum *manda*-correction, $5°1'40''$ which occurs when the *manda*-anomaly is 90 degrees. Since at this point of the explanation 56 and 20 are unknown constants, we denote them as x and y respectively, and we represent Mallāri's calculation in the following form:

$$5°1'40'' = \frac{\left(30 - \frac{90}{6}\right) \times \frac{90}{6}}{x - \left(\frac{\left(30 - \frac{90}{6}\right) \times \frac{90}{6}}{y}\right)} = \frac{225}{x - \frac{225}{y}}$$

Now, he divides 225 by $5°1'40''$ to find out what the denominator should equal, as in the following:

$$x - \frac{225}{y} = \frac{225}{5°1'40''} = 44°45'0''$$

We are unsure why he chose 56 for the constant value we represent by x. He then derives the remaining constant by a calculation equivalent to the following solution for y:

$$\frac{225}{y} = 56° - 44°45'0'' = 11°15'0''$$
$$y = \frac{225}{11°15'0''} = 20$$

which is Mallāri's justification of the subtraction from 56 and division by 20. Clearly, Mallāri's numerical rationale is not so much a derivation as an explanation working backward; it cannot be definitively said that this process of reverse engineering is simply Mallāri's explanatory style, and not evidence of Gaṇeśa's "trial and error" process at arriving at memorable and round constants and divisors. At the end of his commentary he mentions that the *manda*-corrected longitudes of the Sun and Moon are the true longitudes, which is his explanation of the last line of Gaṇeśa's verse.

Figure 3.8 compares three different methods of calculating *manda*-correction values for the Moon for 0 to 90 degrees of *manda*-anomaly: an analytical method derived from the trigonometric formula stated by Bhāskara II in his *Siddhāntaśiromaṇi*: $\mu = \arcsin\left(\frac{79}{900}\frac{120 \sin \kappa_M}{120}\right)$, the formula presented in the *Karaṇakutūhala*: $\mu = \frac{10 * 120 \sin \kappa_M}{238}$, and the algebraic formula presented in this verse by Gaṇeśa. Table 3.3

presents the maximum *manda*-correction values for each of these three methods.

Text	max. μ
Siddhāntaśiromaṇi	$5°;2'26''33'''$
Karaṇakutūhala	$5°;2'31''15'''$
Grahalāghava	$5°;1'40''0'''$

Table 3.3: Maximal *manda*-corrections to the Moon's mean longitude, calculated from formulae in the *Siddhāntaśiromaṇi*, the *Karaṇakutūhala*, and the *Grahalāghava*.

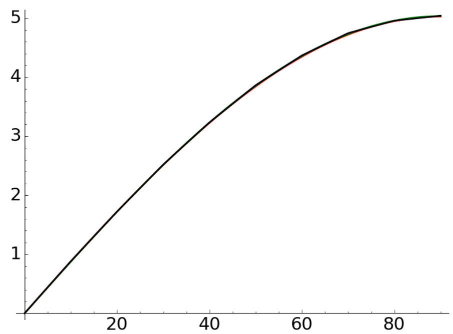

Figure 3.8: The *manda*-correction values μ in degrees for the Moon as a function of the *manda*-anomaly κ_M in degrees of arc as calculated by three different methods: derived from the *Siddhāntaśiromaṇi* (green), the *Karaṇakutūhala* (black), and the *Grahalāghava* (red).

3.5 Verse 2.4: True motions of the Sun and Moon

The *manda*-corrections described in the previous two verses have produced the corrected true longitudes of the Sun and Moon. Since a planet's velocity is the rate of change of its longitude, this angular correction to the longitude necessitates a corresponding change in otherwise constant mean velocity. Now, Gaṇeśa states the velocity *manda*-correction formulae for the Sun and Moon.

केन्द्रस्य कोटिलवखाश्विलवोननिघ्ना
रुद्रा रवेस्त्रिकुहृताः शशिनो द्विनिघ्नाः ।
स्वाङ्गांशकेन सहिताश्च गतौ धनर्णं
केन्द्रे कुलीरमृगषड्कगते स्फुटा सा ॥ ॥ ४ ॥ ॥ वसन्ततिलका ॥

kendrasya koṭilavakhāśvilavonanighnā
rudrā raves trikuhṛtāḥ śaśino dvinighnāḥ |
svāṅgāṃśakena sahitāś ca gatau dhanarṇaṃ
kendre kulīramṛgaṣaṭkagate sphuṭā sā || 4 || || *vasantatilakā* ||

Eleven (*rudra*) [degrees] diminished by a twentieth part (*kha-aśvi*) of the *koṭi* of anomaly in degrees and multiplied by [that same twentieth part]; [the velocity *manda*-correction in arcminutes] of the Sun: [those degrees] divided by thirteen (*tri-ku*); [the velocity *manda*-correction in arcminutes] of the moon: multiplied by two (*dvi*) joined by a sixth (*aṅga*) part of [that quantity] itself; and [the velocity *manda*-correction is applied] to the [mean] motion positively or negatively when the [*manda*-]anomaly is in the six [signs] from Cancer or Capricorn [respectively]; that is the true [velocity].

Commentary: The argument of the *manda*-velocity correction is the *koṭi* of the *manda*-anomaly, and is the complement of the *bhuja* of the *manda*-anomaly. Thus for every mention of the *koṭi* in Gaṇeśa's formula we supply: $(90 - \kappa_M)$.

The *manda*-velocity correction Δ_v^M given for the Sun is:

$$\begin{aligned}\Delta_v^M &= \frac{\left(11 - \dfrac{koṭi}{20}\right) \times \left(\dfrac{koṭi}{20}\right)}{13} \\ &= \frac{\left(11 - \dfrac{(90-\kappa_M)}{20}\right) \times \left(\dfrac{(90-\kappa_M)}{20}\right)}{13}\end{aligned}$$

The *manda*- velocity correction given for the Moon is:

$$\Delta_v^M = \left(11 - \frac{(90-\kappa_M)}{20}\right) \times \left(\frac{(90-\kappa_M)}{20}\right) \times (2 + \frac{2}{6})$$

Gaṇeśa's verbal economy is particularly evident here in this verse, as he explains the common expression between the two velocity correction formulae for the Sun and Moon before neatly addressing their particular differences in divisors or multipliers.

The *manda*-velocity correction Δ_v^M of the Sun or the Moon is applied to the mean daily motion $\bar{v}$ to yield its *manda*-corrected velocity v_M as follows:

$$v_M = \bar{v} \pm \Delta_v^M$$

If the *manda*-anomaly is in the six zodiacal signs beginning with Cancer, meaning it lies in the second and third quadrant, or $90° \leq \kappa_M \leq 270°$, then the *manda*-velocity correction is added to the planet's mean velocity. If the anomaly lies in the six zodiacal signs beginning with Capricorn, meaning it lies in the fourth and first quadrants $(270° \leq \kappa_M \leq 90 + 360°)$ then the *manda*-velocity correction is subtracted from the planet's mean velocity. Qualitatively, the speed of a planet is slowest at apogee, where $\kappa_M = 0$ (in the subtractive half of the domain) and fastest at perigee where $\kappa_M = 180$ in the additive half.

Mallāri opens his commentary on this verse by discussing the *bhuja* and *koṭi* of the *manda*-anomaly angle and the values where their sines are zero or maximum. He is clearly thinking of a trigonometric interpretation of the velocity-correction, such as the formula stated by Bhāskara II that we discussed in 3.1:

$$\Delta v_M = \frac{\bar{v} - v_{A_M} \cdot R \cos \kappa_M \cdot \left(\frac{r_M}{R}\right)}{R}$$

where $\bar{v}$ is the mean velocity of the planet, and v_{A_M} is the velocity of its apogee. In the case of the Sun, since its apogee velocity is negligible, we have:

$$\Delta v_M = \frac{\bar{v} \cdot R \cos \kappa_M \cdot \left(\frac{r_M}{R}\right)}{R}$$

Mallāri's discussion begins as follows [Dvi25, lines 17–21, p.77]:

> प्रथमपदादौ भुजज्या शून्यं तत्र ग्रहफलमपि शून्यं तत्र कोटिज्या परमा तत्र गतिफलमति परमं यथायथा ग्रहफलस्य वृद्धिस्तथातथा गतिफलस्यापचयो दृश्यते । एवं कोटिज्यायाः परमत्वे गतिफलस्य परमत्वं कोटिज्याभावे गतिफलाभावः । अतः केन्द्रकोटिज्यातो गतिफलसाधनं कर्तुं युज्यते ।
>
> *prathamapadādau bhujajyā śūnyaṃ tatra grahaphalam api śūnyaṃ tatra koṭijyā paramā tatra gatiphalam api paramaṃ yathāyathā grahaphalasya vṛddhis tathātathā gatiphalasyāpacayo dṛśy-ate | evaṃ koṭijyāyāḥ paramatve gatiphalasya paramatvaṃ koṭijyābhāve gatiphalābhāvaḥ | ataḥ*

kendrakoṭijyāto gatiphalasādhanaṃ kartuṃ yujyate |

> At the beginning of the first quadrant, the sine of the *bhuja* is zero; there the correction of the planet is also zero; there the sine of the *koṭi* is maximal, there the the velocity correction goes to maximum; just as the increase of the correction of the planet, the decrease of the velocity correction is seen. Thus in the maximality of the sine of the *koṭi* is the maximality of the velocity correction; in the nonexistence of the sign of the *koṭi* is the nonexistence of the velocity correction. So, from the sine of the *koṭi* of the anomaly, finding the velocity correction is connected.

Mallāri explains the dependence of the velocity-correction on the sine of the *koṭi* (the cosine) of the *manda*-anomaly.

Then, Mallāri provides his explanation for Gaṇeśa's formulae while once again working back from the maximal *manda*-velocity correction values to explain constants (see lines 20-27 and lines 1-2 of [Jos81, pp. 58–59]):

> अत्रोभयत्रापि त्रिज्या सपादैकोनत्रिंशन्मिता २९।१५ धृता । तत्साधनं यथा । कोटिभागानां परिमाणं ९० नखांशेन ४।३० ऊना रुद्रास्ततो हता जाता त्रिज्या २९।१५ एवमिष्टांशेभ्य इष्टा स्यादेव । अत एवोक्तं कोटिलवखाश्विलवोननिघ्ना इति । ततो दोर्ज्यातः फलसाधनं खे परमं गतिफलं २।१५ त्रिज्या २९।१५ केन भक्ता सतीदं स्यादतस्तेनैव त्रिज्या भक्ता जातो हरस्त्रयोदश १३ । अतो खेस्त्रिकुहृता इति । एवं चन्द्रस्य परमं गतिफलम् ६८।१५ । अत्र दोर्ज्या केन गुणिता सतीदं फलं स्यादतस्त्रिज्याभक्तं फलं जातं गुणस्थाने २।२० अत्र द्वावेव गृहीतावत उक्तं शशिनो द्विनिघ्ना इति । एवं द्विगुणत्रिज्यायां जातं ५८।३० अस्य परमगतिफलस्य चान्तरमिदं ९।४५ षड्भिः सवर्णितं जातं तत्तुल्यमेव । अतः स्वाङ्गांशकेन सहिता इति ।
>
> *atrobhayatrāpi trijyā sapādaikonatriṃśanmitā 29/15 dhṛtā | tatsādhanaṃ yathā | koṭibhāgānāṃ parimāṇaṃ 90 nakhāṃśena 4/30 ūnā rudrās tato hatā jātā trijyā 29/15 evamiṣṭāṃśebhya iṣṭ=a syād eva | ata evoktaṃ koṭilavakhāśvilavonanighnā iti | tato dorjyātaḥ phalasādhanaṃ khe paramaṃ gatiphalaṃ 2/15 trijyā 29/15 kena bhaktyā satīdaṃ syād atas tenaiva trijyā bhaktā jāto haras trayodaśa 13 | ato khestrikuhṛtā iti | evaṃ candrasya paramaṃ gatiphalam 68/15 | atra dorjyā kena guṇitā satīdaṃ phalaṃ syād atastrijyābhaktaṃ phalaṃ jātaṃ guṇasthāne 2/20 atra dvāveva gṛhītāv ata uktaṃ śaśino dvinighnā iti | evaṃ dviguṇatrijyāyāṃ jātaṃ 58/30 asya paramagatiphalasya cāntaram idaṃ 9/45 ṣaḍbhiḥ savarṇitaṃ jātaṃ tattulyam eva | ataḥ svāṅgāṃśakena sahitā iti |*
>
> Here and in both places, the radius is taken commensurate with one less than thirty and a quarter degrees: 29|15. Its reckoning is just so. The measurement of the degrees of *koṭi*: 90; eleven diminished by a twentieth part, 4|30, and multiplied [by that twentieth part] is the radius 29|15; just in this way, the desired [velocity correction] is [produced] from the desired degrees. So, it is just stated "*koṭilavakhāśvilavonanighnā*". Then from the sine at 0, the reckoning of the [*manda*-velocity] correction, the maximum velocity correction 2|15; the radius, 29|15, divided by what [yields] this? So, the radius is divided by just that divisor thirteen 13. So "*trikuhṛtā*". In this way the maximum velocity correction of the Moon: 68|15. Here the sine multiplied by what [yields] this correction? So, the correction is divided by the radius; in the multiplier place 2|20; here two [units] are [instead] taken so "*śaśino dvinighnāḥ*". Thus 58|30 is [the result] of the two multiplied radius; and the difference of its maximal velocity correction is 9|45; put in the same category [as improper fraction] by 6 equivalent to just that. So "*svāṅgāṃśakena sahitāś*."

Once again, Mallāri uses his knowledge that the maximal velocity corrections for the Sun $2'15''$ and the Moon $1°8'15''$ occur when the *manda*-anomaly is 0 degrees.

Table 3.4 presents the maximum *manda*-velocity correction values for the Sun and Moon as derived from the formulae of the *Siddhāntaśiromaṇi*, the *Karaṇakutūhala*, and Mallāri's commentary for this verse of the *Grahalāghava*.

Text	Sun max. Δv_M	Moon max. Δv_M
Siddhāntaśiromaṇi	$2';14''41'''$	$1°;8'48''32'''$
Karaṇakutūhala	$2';20''0'''$	$1°;8'15''0'''$
Grahalāghava	$2';15''0'''$	$1°;8'15''0'''$

Table 3.4: Maximal *manda*-velocity corrections to the mean daily velocities of the Sun and Moon, calculated from formulae in the *Siddhāntaśiromaṇi*, the *Karaṇakutūhala*, and the *Grahalāghava*.

He chooses the Sun to first examine to justify the divisor 13 (equivalent to applying a scale factor to a general formula) which we denote x for the purpose of its identification, as below:

$$\begin{aligned}
\Delta v_M &= \frac{\left(11 - \dfrac{(90-\kappa_M)}{20}\right)\cdot\left(\dfrac{(90-\kappa_M)}{20}\right)}{x} \\
2'15'' &= \frac{\left(11 - \dfrac{(90)}{20}\right)\cdot\left(\dfrac{(90)}{20}\right)}{x} \\
&= \frac{(11 - 4|30)\cdot 4|30}{x} \\
x &= \frac{29|15}{2|15} \\
&= 13
\end{aligned}$$

Next, he explains how to find the multiplier $\left(2+\frac{2}{6}\right)$ by using the maximal velocity correction $1°8'15''$. In this procedure, we represent this scale factor multiplier as $y_1 + y_2$:

$$\begin{aligned}
\Delta v_M &= \left(11 - \frac{(90-\kappa_M)}{20}\right)\cdot\left(\frac{(90-\kappa_M)}{20}\right)\cdot(y_1+y_2) \\
1°8'15'' &= 29|15\cdot(y_1+y_2) \\
y_1 + y_2 &= 2|20
\end{aligned}$$

Then, y_1 is taken as 2, and we have yet to find y_2:

$$\begin{aligned}
1°8'15'' &= 29|15\cdot(2+y_2) \\
1°8'15'' &= 58|30 + (29|15\cdot y_2) \\
9|45 &= 29|15\cdot y_2 \\
y_2 &= \frac{9|45}{29|15} \\
&= \frac{1}{3}
\end{aligned}$$

which we express as having the same multiplier 2, so we get that $y_2 = \frac{2}{6}$ and indeed the multiplier is Gaṇeśa's $\left(2+\frac{2}{6}\right)$.

3.6 Verse 2.5: Finding *carakhaṇḍas*

After explaining the *manda*-corrections to mean longitude and velocity of the Sun and Moon at mean sunrise at Laṅkā, Gaṇeśa moves on to introduce the *cara*-correction. The *cara*-correction (correction due to ascensional difference) accounts for the difference in a planet's rising time as seen by an observer at Laṅkā (where the planet has a right ascension), as opposed to an observer with a nonzero terrestrial latitude (where the planet has an oblique ascension). While the ascensional differences and related corrections are expounded upon in more depth in the subsequent *pañcatāraspaṣṭīkara* and *tripraśna* chapters of the *Grahalāghava*, Gaṇeśa introduces the necessary elements in applying the *cara*-correction to the Sun and Moon's longitudes to find elements of an observer's *pañcāṅga*.

मेषादिगे सायनभागसूर्ये
दिनार्धजा भा पलभा भवेत् सा ।
त्रिष्ठा हता स्युर्दशभिर्भुजङ्गैर्
दिग्भिश्चरार्धानि गुणोद्धृताऽन्त्या ॥ ॥ ५ ॥ ॥ वाणी उपजाति ॥

meṣādige sāyanabhāgasūrye
dinārdhajā bhā[16] *palabhā bhavet sā* |
triṣṭhā hatā syur daśabhir bhujaṅgair
digbhiś carārdhāni guṇoddhṛtā'ntyā || 5 || || *vāṇī upajāti* ||

When the sun with degrees of tropical [longitude] is at the beginning of Aries, that shadow which is produced at half-day is the noon equinoctial shadow (*palabhā*). That [shadow length] multiplied [separately] in three places by ten (*daśa*), eight(*bhujaṅga*), and ten (*dig*)– the last divided by three (*guṇa*)– are the ascensional differences (*carārdha*).

Commentary: In this verse, Gaṇeśa describes the procedure for finding two important quantities for the *cara*-correction: the noon equinoctial shadow (*palabhā*) and acsensional differences (*carārdha*). Since the *cara*-correction accounts for a latitudinal difference between Laṅkā and an observer's locale, first Gaṇeśa explains how to find local terrestrial latitude.

By using the word "*palabhā*", Gaṇeśa references the process of using a gnomon to find terrestrial latitude ϕ. The angle formed between the ray of equinoctial sun and the gnomon is equivalent to the angular difference between the observer's zenith and noon equinoctial sun, and is the local latitude ϕ. Terrestrial latitude ϕ, the gnomon (of length g), and its noon-equinoctial shadow s_0 are trigonometrically related as:

$$\frac{s_0}{g} = \frac{R\sin\phi}{R\cos\phi}$$
$$s_0 = g\tan\phi$$

Gaṇeśa also states the three *carakhaṇḍas* or ascensional differences ω_1, ω_2, and ω_3, where:

$$\omega_1 = 10 \cdot s_0, \qquad \omega_2 = 8 \cdot s_0, \qquad \omega_3 = \frac{10}{3} \cdot s_0,$$

Gaṇeśa's stated scale factors multiply the length of the noon equinoctial shadow s_0. Mallāri clearly defines the relevant technical vocabulary in his commentary, and also quotes the *Golādhyāya* of the *Siddhāntaśiromaṇi* to describe a gnomon [Dvi25, 13–21, p.79]: [Dvi25, p. 79]):

अयनस्य भागा अयनांशा अग्रे वक्ष्यमाणाः । तैः सह वर्तमानो युक्तो यः सूर्यस्तस्मिन् सूर्ये मेषादिगे राशिभागकलादिना शून्यमिते सति तस्मिन् दिने दिनार्धे मध्याह्ने द्वादशाङ्गुलशङ्कुर्निवेश्यः ।

शङ्कुलक्षणमुक्तं भास्करेण ।
समतलमस्तकपरिधिर्भ्रमसिद्धो दन्तिदन्तजः शङ्कुरिति ।

[16] Amended based on Penn Collection 390 Item 699 3r and Item 1785 3r. Joshi had निनार्द्धभा या [Jos81, 20, p.60], and Dvivedi retains दिनार्द्धजा भा या while noting that this verse should be read as we have amended, in his footnote in [Dvi25, p.79].

> एवं तस्य शङ्कोर्मध्याह्ने भा चाया या भवति सा पलभा भवेदित्यर्थः सा पलभा त्रिष्टा त्रिषु स्थानेषु तिष्ठतीति त्रिष्ठा । दशभि १० भुंजङ्गैरष्टभि ८ र्दिग्भि १० र्हता गुणिता ततोऽन्तिमा गुणैस्त्रिभि३ रुद्धृता सती त्रीणि चरखण्डकानि भवन्ति ।
>
> *ayanasya bhāgā ayanāṃśā agre vakṣyamāṇāḥ | taiḥ saha vartamāno yukto yaḥ sūryas tasmin sūrye meṣādige rāśibhāgakalādinā śūnyamite sati tasmin dine dinārdhe madhyāhne dvādaśāṅgulaśaṅkurniveśyaḥ |*
>
> *śaṅkulakṣaṇamuktaṃ bhāskareṇa | samatalamastakaparidhirbhramasiddho dantidantajaḥ śaṅkuriti |*
>
> *evaṃ tasya śaṅkor madhyāhne bhā cāyā yā bhavati sā palabhā bhaved ityarthaḥ sā palabhā triṣṭā triṣu sthāneṣu tiṣṭhatīti triṣṭhā | daśabhir 10 bhujaṅgair aṣṭabhir 8 digbhir 10 hatā guṇitā tato'ntimā guṇais tribhir 3 uddhṛtā satī trīṇi carakhaṇḍakāni bhavanti |*
>
> The degrees of precession [called] "*ayanāṃśā*" is subsequently described. Whatever present [longitude of the] Sun is joined along with those [degrees of precession], when that Sun is "*meṣādige*" [meaning] when "[its longitude] with *rāśis*, degrees, minutes, and so on [is] commensurate with 0", on that day "*dinārdhe*" [meaning] "noon" a twelve-*aṅgula* gnomon is displayed.
>
> The nature of the gnomon is stated by Bhāskara [II]: "The gnomon, made of a tooth of an elephant, is free from imperfections [of circumference, everywhere between] the top and bottom [which have] the same circumference..."[17]
>
> Thus at noon, whatever is the "*bhā*" [meaning] "shadow" of that gnomon is the "*palabhā*" thus is the meaning. That noon equinoctial shadow "*triṣṭhā*" [meaning] "situated in three places." "*daśabhi*" [meaning] "[by] 10" "*bhujaṅgai*" [meaning] "[by] 8" "*digbhi*" [meaning] "[by] 10" "*hatā*" [meaning] "multiplied" then the final one "*guṇai*" [meaning] "by three 3" is divided; [those] are the three *carakhaṇḍas*.

This quote is yet another example of Mallāri providing both context in both astronomy and with the body of literature preceding the *Grahalāghava* that may have influenced Gaṇeśa. Since Gaṇeśa only references the practice of using a gnomon, Mallāri clarifies what a gnomon is and that it is a perfect cylinder made of ivory, by citing the *Siddhāntaśiromaṇi*. He then proceeds to gloss Gaṇeśa's verse further, which states the three *carakhaṇḍas* or ascensional differences ω_1, ω_2, and ω_3 which multiply the length of the noon equinoctial shadow s_0. These are the ascensional differences for the ends of the first three signs in the first quadrant of the ecliptic: Aries, Taurus, and Gemini. This rule is a modification of the trigonometric relation, which uses terrestrial latitude ϕ and solar declination (how much the Sun is north or south of the celestial equator) δ to find the ascensional difference ω:

$$\sin\omega = \tan\delta\tan\phi$$

and ω_1, ω_2, and ω_3 are the ascensional differences calculated for declinations found for tropical longitudes at 30°, 60°, and 90°. So first, the solar declination is found, which can be done using similar triangles, as shown in Figure 3.9. The Sun at position D has tropical longitude λ equivalent to the arc AD, the Rsine of which is DH. The arc corresponding to the dropped perpendicular DG is the solar declination. Arc AC represents the first quadrant of the ecliptic, from the beginning of Aries at A at the equinox to the beginning of Cancer at C. $OC = R$ is the distance from an observer at O to C where the local meridian intersects the ecliptic at C which is also the point of maximum obliquity of the ecliptic, meaning the maximal angular difference between the celestial equator and the ecliptic. So, the arc of CF is taken to be 24° by numerous Sanskrit astronomers; its Rsine is the perpendicular CF. Then we have the two similar right triangles: DGH and

[17] The full quote is: समतलमस्तकपरिधिर्भ्रमसिद्धो दन्तिदन्तजः शङ्कुः । तच्छायातः प्रोक्तं ज्ञानं दिग्देशकालानाम् ।।
samatalamastakaparidhir bhramasiddho dantidantajaḥ śakuḥ | tacchāyṭaḥ proktaṃ jñānaṃ digdeśakālānām | |
"The gnomon, made of a tooth of an elephant, is free from imperfections [of circumference, everywhere between] the top and bottom [which have] the same circumference. The knowledge of direction, place, and time are stated from its shadow." [Jos88, 12.9, p.423]

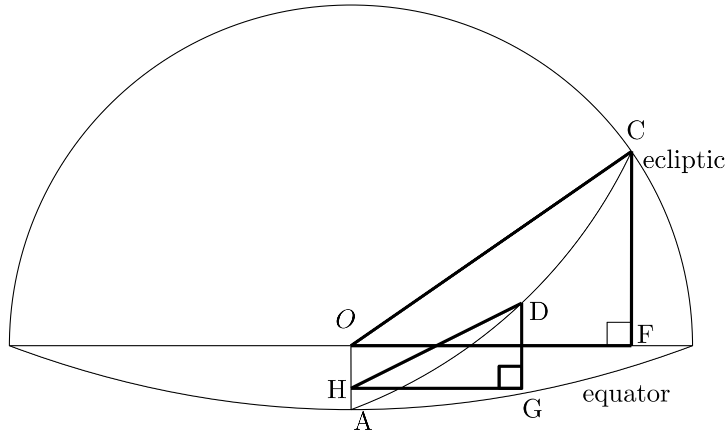

Figure 3.9: The solar declination for the Sun at position D on the ecliptic can be found using similar triangles. An observer is at O. Arc AD measured from the vernal equinox at the start of Aries A is the tropical longitude of the Sun, arc AC is the first quadrant of the ecliptic from the beginning of Aries to the beginning of Cancer. At point C, the distance between the ecliptic and the equator (the obliquity of the ecliptic) is at its greatest, which is taken to be 24°. The declination δ is the arc corresponding to DG.

CFO and can use a proportion to find the length DG to find its arc of solar declination:

$$\begin{aligned}
\frac{DG}{HD} &= \frac{CF}{OC} \\
DG &= \frac{CF}{OC} \cdot HD \\
R \sin \delta &= \frac{R \sin 24^\circ}{R} \cdot R \sin \lambda.
\end{aligned}$$

Then, this solar declination relates to the previously found terrestrial latitude, which can be seen with similar triangles once again. The solar declination corresponds to the arc DF in Figure 3.10, which is how much the Sun is north of the celestial equator (which runs through CEF). Its Rsine is OG and its Rcosine is DG, which is the radius of the day-circle (the daily solar revolution for the Sun at local latitude) which runs through RBD. CE is the corresponding equatorial arc to RB which is the amount of the day-circle between local sunrise and 6'oclock (one quarter of the Sun's daily revolution) that has risen. This is the ascensional difference ω. The terrestrial latitude triangle is ZOF. If a perpendicular is dropped from the zenith at Z onto the equator at J, then the corresponding right triangle ZOJ which is similar to right triangle OHG, results in the relation:

$$GH = \frac{ZJ}{OJ} \cdot OG = \frac{R \sin \phi}{R \cos \phi} \cdot OG = \tan \phi \cdot R \sin \delta = \frac{s_0}{12} \cdot R \sin \delta$$

This is currently scaled for the day-circle with radius $R \cos \delta$ so a proportion is used to yield Rsine of the equatorial arc of ascensional difference CE:

$$\begin{aligned}
\frac{R \sin EC}{R} &= \frac{GH}{R \cos \delta} \\
R \sin EC &= \frac{GH}{R \cos \delta} \cdot R \\
R \sin \omega &= \frac{\frac{s_0}{12} \cdot R \sin \delta}{R \cos \delta} \cdot R \\
&= s_0 \cdot R \tan \delta
\end{aligned}$$

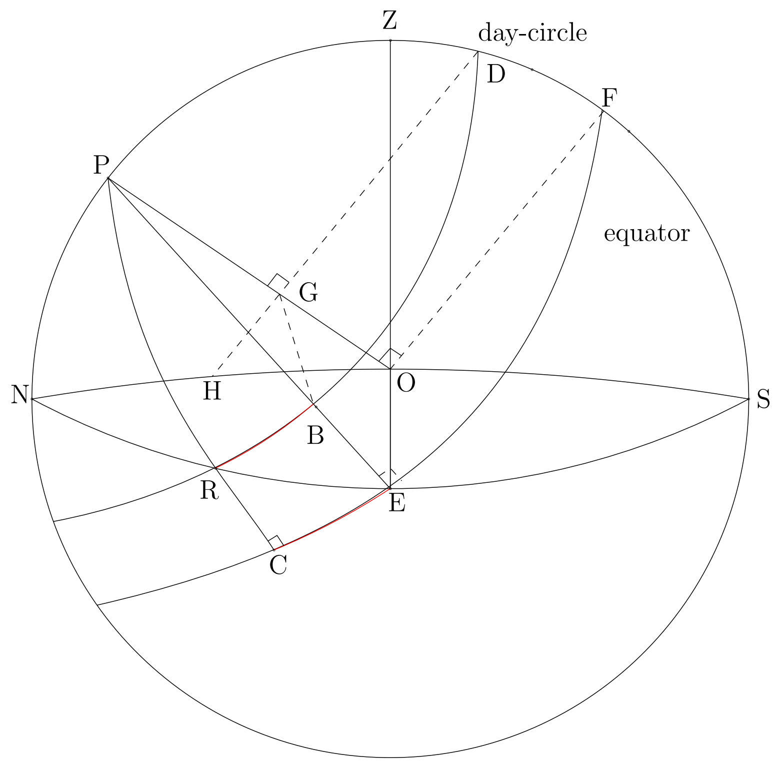

Figure 3.10: The horizon at an observer's northern latitude locale. The zenith is at Z, and the celestial equator runs through CEF with the northern celestial pole at P. The Sun is at R, and the day-circle runs through RBD. EF represents one quarter of the equator, which is equivalent to one quarter of the Sun's daily revolution or 6 hours of time when the corresponding arc BD of the day-circle too will rise. Then the arc BR is the arc from local sunrise to six-o-clock is measured by its corresponding equatorial arc CE which is the ascensional difference ω in red. This can be found by using terrestrial latitude arc ZF ϕ and solar declination δ, the Rsine of which is OG and the Rcosine is DG which is the radius of the day-circle.

Then, Gaṇeśa's scale factors, which appear in the *Karaṇakutūhala* [Mis91, 2.19, p.30] and in the *Siddhānta-śiromaṇi* [Ark80, 2.49–51, p.180], seem to be approximations for $R \tan \delta$ for declinations taken at 30°, 60°, and 90° for the three ascensional differences ω_1, ω_2, and ω_3 corresponding to the end of Aries, Taurus, and Gemini respectively.

3.7 Verse 2.6: Cara correction of the Sun

Gaṇeśa now states a verse to use the *carakhaṇḍas* found in the previous verse to find the *cara*-correction of the Sun.

स्यात् सायनोष्णांशुभुजर्क्षसङ्ख्य-
चरार्धयोगो लवभोग्यघातात्।
खाग्न्याप्तियुक्तस्तु चरं धनर्णं
तुलाजषट्के तपनेऽन्यथाऽस्ते ॥ ॥ ६ ॥ ॥ भद्रा उपजाति ॥

syāt sāyanoṣṇāṃśubhujarkṣasaṅkhya-
carārdhayogo lavabhogyaghātāt |

khāgnyāptiyuktas tu caraṃ dhanarṇaṃ
tulājaṣaṭke tapane'nyathā'ste || 6 || || *bhadrā upajāti* ||

The addition of [the number of] *carārdhas* [corresponding to] the number of *rāśis* (*rkṣa*) of the *bhuja* of the precession increased [longitude of the] Sun [expressed as *rāśis*, degrees, and so on], and the joined quotient obtained from [dividing] the multiplication of the [degrees and so on of the] next [*carakhaṇḍa*] (*bhogya*) and the degrees [and so on of the precession increased longitude of the Sun], by 30 (*kha-agni*) yields the *cara* [in *palas*], which is positive or negative when the Sun is in the six [zodiacal signs] from Libra or Aries [respectively]; at sunset, it is reversed.

Commentary: Gaṇeśa describes how to use the *carakhaṇḍas* given in the last verse to compute the *cara*-correction. The *cara*-correction is a half-equation of daylight, which means the correction made to account for the difference in sunrise time between sunrise at the equator and sunrise at a given locality.

As per Gaṇeśa's verse, the formula to calculate *cara*-correction for the Sun ω_S takes into account the longitude of the tropical Sun (the sidereal longitude of the Sun increased by degrees of precession) expressed in *rāśis*, degrees, minutes, and seconds. This value is expressed as the equivalent first quadrant argument *bhuja* in $R_i^R, D°; M'S''$ *rāśis*, degrees, minutes, and seconds. The i in R_i corresponds to how many *carakhaṇḍas* have elapsed, for $0 \leq i \leq 2$. So the $(i+1)$th *carakhaṇḍa* denoted c_{i+1} is the next, or "*bhogya*". So, the *cara* correction ω_S in arcseconds given the elapsed *carakhaṇḍa* c_i, the next *carakhaṇḍa* c_{i+1}, the *bhuja* sans *rāśis* $D°; M'S''$ is as follows:

$$\omega_S = c_i + \frac{c_{i+1} \times (D°; M'S'')}{30}$$

This equation is the same as what is presented in the *Karaṇakutūhala* [Mis91, v. 2.20, p. 30].

Mallāri explains how the Sun's *cara*-correction is derived from a proportion: [Dvi25, 27–30, 1–8, p.80-81]:

अत्रोपपत्तिः । चरं नाम लङ्कार्कोदयरेखार्कोदययोरन्तरमतस्तद्दक्षिणोत्तरम् । तत्साधनायोपायः । अत्र प्रतिराशिखण्डानि सन्त्यतो भुजराशिमितखण्डयोगः कर्तव्यः । शेषात् त्रैराशिकम् । यदि त्रिंशद्भि ३० भागैरेष्यखण्डतुल्यं चरं लभ्यते तदा शेषभागैः किमिति सुगतम् ।

अथ धनर्णोपपत्तिः । जाता ग्रहा लङ्कार्कोदयकालीना रेखार्कोदयकालीनाः कार्याः । तत्र लङ्कायां यत् क्षितिजं तस्योन्मण्डलसंज्ञा । अन्यदेशीयस्य क्षितिजस्य क्षितिजसंज्ञैव । उत्तरगोले उन्मण्डलार्कोदयात् पूर्वं क्षितिजार्कोदयः । उन्मण्डलास्तात् पश्चात् क्षितिजास्तमयो यतः क्षितिजादुपर्युन्मण्डलम् । अत उत्तरगोले उदये चरमृणमस्ते च धनम् । दक्षिणगोलेऽस्माद्विपरीतम् । तद्यथा । उन्मण्डलार्कोदयान्तरं क्षितिजार्कोदयः । उन्मण्डलास्तमयात् पूर्वं क्षितिजास्तमयो यतः क्षितिजादध उन्मन्डलमतो दक्षिणगोले उदये चरं धनमस्ते ऋणमित्युपपन्नम् ।

atropapattiḥ | caraṃ nāma laṅkārkodayarekhārkodayayor antaram atas taddakṣiṇottaram | tatsādhanāyopāyaḥ | atra pratirāśikhaṇḍāni santy ato bhujarāśimitakhaṇḍayogaḥ kartavyaḥ | śeṣāt trairāśikam | yadi triṃśadbhir 30 bhāgair eṣyakhaṇḍatulyaṃ caraṃ labhyate tadā śeṣabhāgaiḥ kim iti sugatam |

atha dhanarṇopapattiḥ | jātā grahā laṅkārkodayakālīnā rekhārkodayakālīnāḥ kāryāḥ | tatra laṅkāyāṃ yat kṣitijaṃ tasyonmaṇḍalasaṃjñā | anyadeśīyasya kṣitijasya kṣitijasaṃjñyaiva | uttaragole unmaṇḍalārkodayāt pūrvaṃ kṣitijārkodayaḥ | unmaṇḍalāstāt paścāt kṣitijāstamayo yataḥ kṣitijād upary unmaṇḍalam | ata uttaragole udaye caram ṛṇam aste ca dhanam | dakṣiṇagole'smād viparītam | tadyathā | unmaṇḍalārkodayāntaraṃ kṣitijārkodayaḥ | unmaṇḍalāstamayāt pūrvaṃ kṣitijāstamayo yataḥ kṣitijād adha unmanḍalam ato dakṣiṇagole udaye caraṃ dhanam aste ṛṇam ityupapannam |

Here is the explanation. The *cara* is the difference between [the time of] sunrise at Laṅkā and [the time of] sunrise [at a different latitude] on the [meridian] line, so [the locale] is north or south of that [zero latitude at Laṅkā]. A method for its reckoning. Here the *carakhaṇḍas* corresponding to every *rāśi* are here because the addition of the *khaṇḍa* corresponding to the *rāśis* of the *bhuja* [of tropical longitude] is to be done. From the remainder is a proportion. If with thirty 30 degrees a *cara* is obtained equivalent to the desired *khaṇḍa*, then with the remaining degrees what is it? Arrived [at] easily.

Here is the explanation of positivity and negativity [of the *cara*-correction]. The planets are [reckoned] at the time of sunrise at Laṅkā [but] are to be [reckoned] at sunrise at the line. There at Laṅkā whatever is the "*kṣitija*" (horizon) its name is the "*unmaṇḍala*" (six o'clock circle). The name of the horizon of another place is just "the horizon". In the northern hemisphere, before the rising of the sun on the six o'clock circle, the rising of the sun on the horizon [occurs]. After the setting on the six o'clock circle the setting on the horizon because the six o'clock circle is above the horizon. So in the northern hemisphere at sunrise the *cara* is negative, and at sunset it is positive. In the southern hemisphere, due to that [very fact], it is the reverse. That is just so. The difference of sunrise on the six o'clock circle [and] the sunrise on the horizon. Before the setting on the six o'clock circle, the setting on the horizon because the six o'clock circle is below the horizon; in the southern hemisphere at rising the *cara* is positive [and] at setting it is negative is explained.

Mallāri once again includes a proportion in his commentary. The *bhuja* of the tropical longitude of the Sun, is $R_i^R, D°; M'S''$ *rāśis*, degrees, minutes, and seconds. Each elapsed *rāśi* is taken corresponding to a *carakhaṇḍa* since one *carakhaṇḍa* corresponds to the ascensional difference of one zodiacal sign which is 30°. So the remaining degrees [minutes, and seconds] $D°; M'S''$ are between 0 and 30 degrees corresponding to some portion of the next *carakhaṇḍa*, c_{i+1}. Hence the proportion to find $(\omega_S - c_i)$:

$$\frac{(\omega_S - c_i)}{(D°; M'S'')} = \frac{c_{i+1}}{30°}$$
$$(\omega_S - c_i) = \frac{c_{i+1}}{30°} \cdot (D°; M'S'')$$

which is what Gaṇeśa has in his stated formula.

3.8 Verse 2.7: Time and place corrections

Gaṇeśa states rules for corrections to the longitude of the Moon based on the user's locality, as well as a formula for precession to convert sidereal longitudes to tropical ones.

देयं तच्चरमरुणे विलिप्तिकासु
मध्येन्दौ द्विगुणनवोद्धृतं कलासु ।
भाप्तं च द्युमणिफलं लवेऽथ वेदा-
ब्ध्यब्ध्यूनः खरसहृतः शकोऽयनांशाः ॥ ॥ ७ ॥ ॥ प्रहर्षिणी ॥

deyaṃ taccaram aruṇe viliptikāsu
madhyendau dviguṇanavoddhṛtaṃ kalāsu |
bhāptaṃ ca dyumaṇiphalaṃ lave'tha vedā-
bdhyabdhyūnaḥ kharasahṛtaḥ śako'yanāṃśāḥ || 7 || || *praharṣiṇī* ||

That *cara*-correction [which is] to be applied to the Sun in arcseconds, is multiplied by two and divided by 9, [and is to be applied] in minutes to the mean [longitude of the] Moon, as well as the [*manda*-]correction of the Sun divided by 27 (*bha*) in degrees. Now, the Śaka [year] diminished by 444 (*veda-abdhi-abdhi* [and] divided by 60 (*kha-rasa*) are the degrees of precession.

Commentary: In this verse, Gaṇeśa describes the three corrections to be made to the Moon before the *manda*-correction is applied and yields the true longitude of the Moon. In the previous verse, Gaṇeśa provided the formula for the *cara*-correction for the Sun that was applied in arcseconds. That *cara*-correction with some modifications is the *cara*-correction ω_M to be applied to the Moon in arcminutes, as shown below:

$$\omega_M = \frac{2 \cdot \omega_S}{9}$$

Then, the *bhuja*-correction is provided as an argument of the Sun's *manda*-correction μ_{Sun}:

$$\lambda_b = \frac{\mu_{\text{Sun}}}{27}$$

And lastly, the formula for the degrees of precession for a given Śaka year Y_D is stated:

$$p = \frac{Y_D - 444}{60}$$

which is added to the otherwise sidereal longitude of the planet to get its tropical longitude.

3.9 Verses 2.8–2.9: Finding *tithi*, *karaṇa*, *nakṣatra*, and *yoga*

In these last two verses of the chapter, Gaṇeśa explains how the true longitudes and velocities of the Sun and Moon at true sunrise at an observer's locale are used to find four out of the five elements of the *pañcāṅga*: *tithi*, *karaṇa*, *nakṣatra*, and *yoga*.

भक्ता व्यर्कविधोर्लवा यमकुभिर्याता तिथिः स्यात् फलं
शेषं यातमिदं हरात् प्रपतितं भोग्यं विलिप्तास्तयोः ।
भुक्त्योरन्तरभाजिताश्च घटिका यातैष्यकाः स्युः क्रमात्
पूर्वार्धे करणं बवाद्गततिथिर्द्विघ्न्याद्रितष्टा भवेत् ॥ ॥ ८ ॥ ॥ शार्दूलविक्रीडित ॥

तत् सैकं त्वपरे दलेऽथ शकुनेः स्युः कृष्णभूतोत्तरा-
दर्धाच्चाथविधोश्च सार्कसितगोर्लिप्ताः खखाष्टोद्धृताः ।
याते स्तो भयुती क्रमाद्गगनषण्णिघ्ने गतैष्ये तयो-
रिन्दोर्भुक्तिहृते जवैक्यविहृते यातैष्यनाड्यः क्रमात् ॥ ॥ ९ ॥ ॥ शार्दूलविक्रीडित ॥

bhaktā vyarkavidhor lavā yamakubhir yātā tithiḥ syāt phalaṃ
śeṣaṃ yātam idaṃ harāt prapatitaṃ bhogyaṃ viliptās tayoḥ |
bhuktyor antarabhājitāś ca ghaṭikā yātaiṣyakāḥ syuḥ kramāt
pūrvārdhe karaṇaṃ bavād gatatithir dvighnyādritaṣṭā bhavet || 8 || || *śārdūlavikrīḍita* ||

tat saikaṃ tvapare dale'tha śakuneḥ syuḥ kṛṣṇabhūtottarād
ardhāc cāthavidhoś ca sārkasitagor liptāḥ khakhāṣṭoddhṛtāḥ |
yāte sto bhayutī kramād gaganaṣaṇṇighne gataiṣye tayor
indor bhuktihṛte javaikyavihṛte yātaiṣyanāḍyaḥ kramāt || 9 || || *śārdūlavikrīḍita* ||

The degrees of [the longitude of] the moon diminished by [the longitude of] the sun [are] divided by 12 (*yama-ku*); the quotient is the [number of elapsed *tithis*]. This elapsed *tithi* remainder subtracted from the [previous] divisor [is] the yet to elapse [part of the current *tithis*]. [The elapsed part of the current *tithi* and the yet to elapse part] in arcseconds divided by the difference of the velocities of those two [planets, the Moon and Sun]. The elapsed and yet to elapse [parts of the current *tithi* expressed in] *ghaṭikas* are respectively [the elapsed and yet to elapse parts of the current *tithi* in] arcseconds divided by the difference of velocities of those two [planets– the Sun and the Moon]. In the first half [of the *tithi*], the elapsed [number of] *tithis* multiplied by two and divided (*taṣṭa*) by seven is [the current] *karaṇa* from Bava.

When it is in the other half [of the *tithi*], [the number of elapsed *karaṇas*] is increased by one. And from the latter half of the fourteenth *tithi* of the *kṛṣṇapakṣa*, [are the fixed-*karaṇas* starting with] Śakuni. [The longitudes] in arcminutes of the moon and of the moon together with the sun, divided by 800 (*khakhāṣṭa*) [arcminutes] are respectively the [number of] elapsed *nakṣatras* and *yogas*. When the [number of] elapsed and yet to elapse [parts of the current *nakṣatra* and *yoga*, as found with the remainder of the prior division and the remainder subtracted from 800 minutes] are multiplied by 60 (*gagana-ṣaṭ*) [and] when they are divided by the velocity of the moon and the combined velocity of the two [planets– the Sun and the Moon] [the results are] respectively the elapsed and yet to elapse [parts of the current *nakṣatra* and *yoga*] [expressed in] *nāḍis*.

Commentary: These last two verses of this chapter describe how to find elapsed and coming *tithis*, *karaṇas*, *nakṣatras*, and *yogas*, which along with the *vāra* (day) are the five components that make up the *pañcāṅga*. First, Gaṇeśa describes how to find the *tithi*. A lunar month, also called a synodic month, is reckoned from syzygy to syzygy of either new moon to new moon or full moon to full moon. So, the lunar month is considered the length of time it takes for the Sun and Moon to complete a full 360° of separation. A *tithi*, also called a lunar day, is defined to be the length of time for the angular distance between the Sun and Moon to be 12 degrees. So there are 30 *tithis* in one lunar month– a result used in *ahargaṇa* calculations such as those found in verses 1.4–5. Hence, Gaṇeśa states how to find the number of elapsed *tithis* τ of the current synodic month, and the amounts of elongation elapsed T_e and yet to elapse T_y in degrees within the current *tithi*. He finds these three quantities using the longitudes of the Sun and the Moon λ_{Sun} and λ_{Moon}:

$$\tau = (\lambda_{Moon} - \lambda_{Sun}) \quad \text{div } 12, \qquad T_e = (\lambda_{Moon} - \lambda_{Sun}) \quad \text{mod } 12, \qquad T_y = 12 - T_e.$$

If the difference is negative, that is, when the longitude of the Moon is less than that of the Sun, then add 360° to the longitude of the Moon to yield a positive difference. If the number of elapsed *tithis* is $1 \leq \tau \leq 15$, then the current *tithi* is in the *śuklapakṣa*. If it is $\tau \geq 16$, the current *tithi* is in the *kṛṣṇapakṣa*, and its enumeration in the *kṛṣṇapakṣa* from the first *tithi* onwards is given by: $\tau \mod 15$.[18]

Then, these amounts of elongations T_e and the part yet to elapse T_y are expressed as time transpired in the current *tithi* in *ghaṭikās*. Since a *tithi* measures 12 degrees of separation between the longitudes of the Sun and Moon, we need to find the rate of separation between the Sun and Moon to scale it to the two parts of the current *tithi*. Now, we can use the difference of their instantaneous true velocities at local true sunrise (which are some number of degrees and arcminutes traversed over 1 civil day). At that instant of true sunrise, this difference of true velocities is also the difference in their longitudes over 1 civil day. So, the amount of luni-solar elongation in degrees, divided by the true rate of elongation in degrees per day produces a time interval of partial days expressed in *ghaṭikās*. Then, dividing the elapsed elongation T_e and yet to elapse elongation T_y by the difference of true daily motions of the Sun and Moon v_{Sun} and v_{Moon} yields the two corresponding time intervals γ_e and γ_y:

$$\gamma_e = \frac{T_e}{v_{Moon} - v_{Sun}}, \qquad \gamma_y = \frac{T_y}{v_{Moon} - v_{Sun}}.$$

And thus we have the duration of both the elapsed and yet to elapse parts of the current *tithi* in *ghaṭikās*.

Then, Gaṇeśa states how to find the current *karaṇa*. A *karaṇa* is a half-*tithi*, so in a lunar month there are 60 total *karaṇas*. In the *kṛṣṇapakṣa* (waning half of the lunar month), starting from the latter

[18] See [SD96, p. 13] for a table of the conventional names of *tithis* (for instance *dvitīyā* "second") along with alternative names.

half of the fourteenth *tithi* are the four consecutive *sthira* ("fixed") *karaṇas*: Śakuni, Catuṣpāda, Nāga, and Kimstughna. These are fixed in order to be the *karaṇas* from that latter half of the fourteenth *tithi* until the first half of the first *tithi* of the *śuklapakṣa* (waxing half of the lunar month). Then, from the second half of the first *tithi* of the *śuklapakṣa*, begin the *cala* ("moving") *karaṇas* starting with Bava. They are: Bava, Bālava, Kaulava, Taitila, Gara, Vaṇija, and Viṣṭi. There are seven moving *karaṇas* that cycle through, in order, eight times through the course of a lunar month until the start of Śakuni and the four fixed *karaṇas*. Since the four fixed *karaṇas* occur at known times of the lunar month, Gaṇeśa provides his formula to find out what the current moving *karaṇa* k_i is, where $0 \leq i \leq 6$ is the index corresponding to the moving *karaṇa* with $i = 0$ as the seventh *karaṇa*, Viṣṭi, and $i = 1$ as Bava. Using the number of elapsed *tithis* τ and depending on the magnitude of the elapsed elongation in the current *tithi* T_e:

$$\text{If } T_e \leq 6^\circ : i = (2 \cdot \tau) \mod 7, \qquad \text{If } T_e > 6^\circ : i = ((2 \cdot \tau) \mod 7) + 1$$

He does not mention how to find duration of the *karaṇa*; however since *karaṇas* are just half a *tithi*, the same process applies to calculating *karaṇa* timings.

Then, Gaṇeśa explains how to find the current *nakṣatra* and its timings. The ecliptic is divided into 27 equal parts called *nakṣatras*, so each *nakṣatra* measures $360^\circ/27 = 13^\circ;20'$ or $800'$ of the ecliptic. In one sidereal month, the moon completes one revolution about the ecliptic, so the "current *nakṣatra*" as per *pañcāṅga* specifications refers to the current $800'$ part of the zodiac in which the Moon is situated. So, for the true longitude of the moon λ_{Moon} at an instant expressed in arcminutes, the number N of elapsed *nakṣatras* is:

$$N = \lambda_{Moon} \quad \text{div } 800.$$

Since these are the number of elapsed *nakṣatras*, the current one is the $(N + 1)$th *nakṣatra* counted from the first *nakṣatra* Aśvinī to the twenty-seventh *nakṣatra* Revatī.[19] Then the amount of the current *nakṣatra* elapsed N_e and yet to elapse N_y are:

$$N_e = \lambda_{Moon} \mod 800, \qquad N_y = 800 - N_e.$$

The duration of each of these two parts of the current *nakṣatra* in *nāḍīs* (synonymous to *ghaṭikās*) is:

$$\eta_e = \frac{N_e}{v_{Moon}} \cdot 60, \qquad \eta_y = \frac{N_y}{v_{Moon}} \cdot 60.$$

Lastly, Gaṇeśa explains how to find the current *yoga* and its timings. While the *tithi* and *karaṇa* were both measures of luni-solar separation, the *yoga* is the time in which the Sun and Moon together travel $800'$ or $13^\circ;20'$ of the zodiac. Of these too there are 27. So, the number of elapsed *yogas* ψ is found by dividing the sum of the longitudes of the Sun λ_{Sun} and the Moon λ_{Moon} by $800'$:

$$\psi = (\lambda_{Sun} + \lambda_{Moon}) \quad \text{div } 800.$$

Since these are the number of elapsed *yogas*, the current one is the $(\psi + 1)$th *yoga* counted from the first *yoga* Viṣkambha to the twenty-seventh *yoga* Vaidhṛti. Then the amount of the current *yoga* elapsed ψ_e and yet to elapse ψ_y are:

$$\psi_e = \lambda_{Moon} \mod 800, \qquad \psi_y = 800 - \psi_e.$$

The duration of each of these two parts of the current *yoga* in *ghaṭikās* is:

$$\zeta_e = \frac{\psi_e}{v_{Moon}} \cdot 60, \qquad \zeta_y = \frac{\psi_y}{v_{Moon}} \cdot 60.$$

[19] See [SD96, p. 14] for the entire list of *nakṣatras*, *yogas*, and *karaṇas* from a tabulated example.

Chapter 4

Pañcatāraspaṣṭīkaraṇādhikāra chapter on the true motions of the five star-planets

4.1 Introduction

The previous chapter of the *Grahalāghava* focused on finding the true longitudes of the Sun and the Moon by applying the orbital *manda*-correction. This third chapter, the *pañca-tāra-spaṣṭa* 'correction of five star[-planets]' explains how to apply the appropriate corrections to the mean longitudes found in Chapter 1 to find the true longitudes of the five star-planets Mars, Mercury, Jupiter, Venus, and Saturn. As explained in the preceding chapter, the Sun and Moon only require the *manda*-correction. The other star-planets also require a *śīghra*-correction, or "synodic correction" to account for their apparent reversal of motion and change in velocity as observed from the Earth [MP18, pp. 38–39].

This chapter's content begins with discussions of the *śīghra*- and *manda*-corrections, but comprises other material relating to the five star-planets:

- A versified table of so-called *śīghra*-numbers (verses 1–5) and the rule for applying them to produce each planet's *śīghra*-correction (verse 6)
- A similar table of *manda*-numbers (verses 7–8) and their corresponding rule for the planetary *manda*-corrections (verse 9)
- A description of the iterative method of applying the *śīghra*- and *manda*-corrections to each of the planet's mean longitudes to produce the orbitally corrected true longitudes (verse 10)
- A corresponding *śīghra*- and *manda*-correction to the mean motions of each of the planets to yield true motions of the planets (verses 11–12)
- An additional correction to the *śīghra*-corrections of Mars and Venus (verse 13) along with an additional velocity *śīghra*-correction for Mercury, Mars, and Venus (verse 14)
- All relevant information pertaining to the so-called synodic phenomena, or celestial events depending on the relative positions of the planet, Earth, and the Sun, such as apparent retrograde motion (verses 15–20).

For each of the above topical verse groupings we provide Sanskrit text and transliteration with translation, and where relevant a tabular representation of the data stated in the verses. For most of these topics

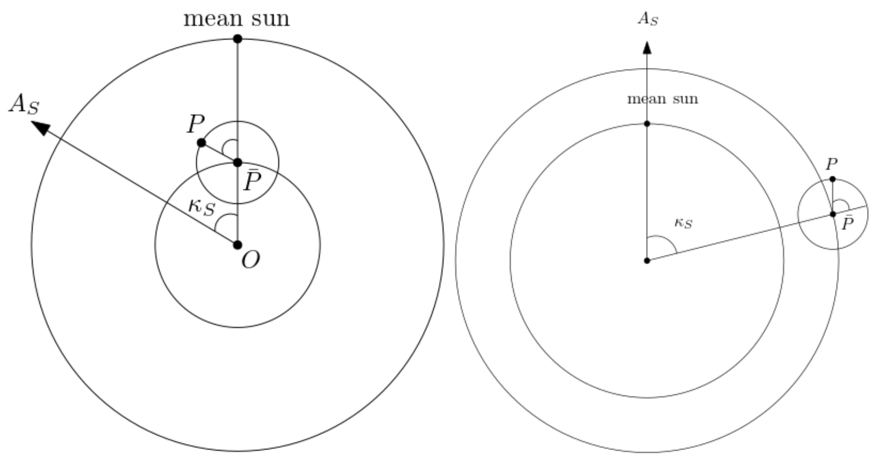

Figure 4.1: The *śīghra*-configuration of an inferior planet (left) and a superior planet (right). The *śīghra*-apogee is taken to be the planet itself in the case of the inferior planets, since their mean longitudes are the same as the mean longitude of the Sun. The *śīghra*-apogee of the superior planets is the mean longitude of the Sun.

we also include a detailed mathematical commentary. However, for the topics discussed in verses 3.11 and 3.14 and 16–20 our study is still very incomplete, so we defer the presentation of commentary and analysis until a later publication.

In the heliocentric model, the relative motion of the Earth and other planets around the Sun produces apparent irregularities that are explained in geocentric models as anomalies depending on relative positions of the planets and the Sun, and hence termed "synodic". In Indian astronomy this synodic dependence is accounted for via a second orbital anomaly known as *śīghra* applied to the five star-planets.

4.1.1 Astronomical background: the *śīghra*-corrections to longitude and velocity

As was the case in determining the *manda*-correction, the first step in calculating the *śīghra*-correction involves reckoning the *śīghra*-anomaly. The *śīghra* represents a synodic relation between a slower and a faster celestial body, one of which is the Sun, and the role of the *śīghra*-apogee is played by the faster body. In the case of the inferior planets, the true planet itself is the *śīghra*-apogee. In the case of the superior planets, the mean Sun is the *śīghra*-apogee. Figure 4.1 shows the difference in *śīghra*-configuration between an inferior planet and a superior planet. In the following explanation of computing the *śīghra*-correction trigonometrically, we use the configuration of a superior planet, shown in more detail in figure 4.3

The definition of the *śīghra*-anomaly can be indicative of the order in which the *manda* and *śīghra* corrections are applied to the planets, as we will see in Section 4.6. In many texts we find the *śīghra*-anomaly to be defined as "the apogee diminished by the *manda*-corrected mean longitude of the planet" cf. *Siddhāntaśiromaṇi* [Ark80, v. 2.18, p.190], *Śiṣyadhīvṛddhidatantra* [Cha81a, v. 3.18–19, p.56], but interestingly the *Khaṇḍakhādyaka* too defines the *śīghra*-anomaly as Gaṇeśa does [Cha70, v. 2.18, p.56].

We first explain the trigonometric method of computing the *śīghra*-correction. Figure 4.2 shows the *śīghra*-configuration of a superior planet. O represents an observer on Earth, $\bar{P}$ is the mean longitude of the planet, and P is the true longitude of the planet. $O\bar{P}$ is the distance between the mean planet and Earth, which is the orbital radius R. $P\bar{P}$ is the *śīghra*-epicycle radius r_S. The *śīghra*-anomaly κ_S is the angle from the apogee P' to the mean planet, and the *śīghra*-correction σ is the angular difference between the true longitude of the planet and the mean longitude of the planet. Q is the perpendicular dropped down from

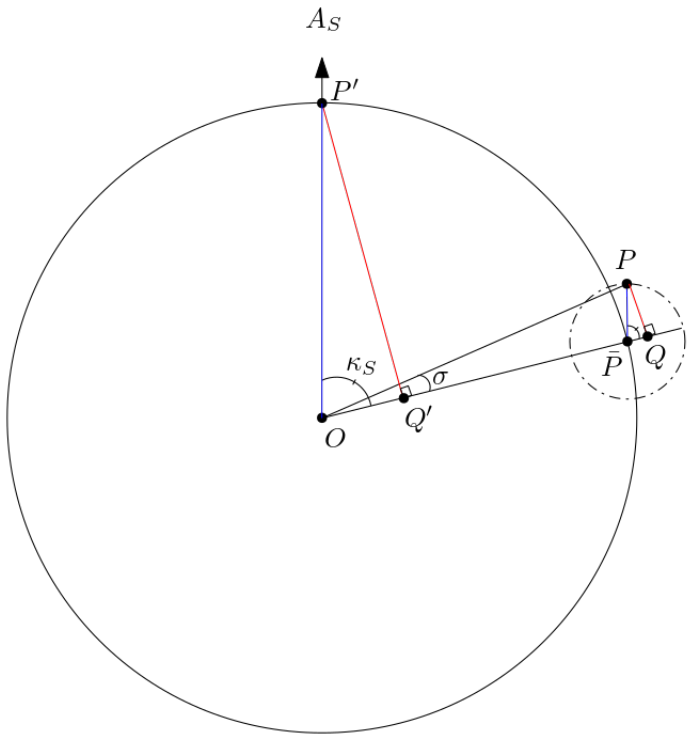

Figure 4.2: The triangle $\Delta P'OQ'$ formed between the apogee and a perpendicular Q' dropped onto radial line $O\bar{P}$ and the triangle $DeltaP\bar{P}Q$ are similar.

P onto the radial line $O\bar{P}$, and Q' is the perpendicular dropped down from P' onto the same line. Two main proportions come into play. The first uses the similarity of the two triangles $\Delta P'OQ'$ and $\Delta P\bar{P}Q$, as $\Delta P'OQ'$ is scaled to the mean circular orbit of the planet with radius R, and $\Delta P\bar{P}Q$ is scaled to the epicycle with radius r_S. When the sine of the *śīghra*-anomaly κ_S is maximal, at 90 degrees ("*trijyā*"), it is equivalent to R. This is also the hypotenuse of triangle $\Delta P'OQ'$. Then, we also have that the sine of the maximum *śīghra*-correction to be the radius of the epicycle r_S, which is the hypotenuse of the triangle $\Delta P\bar{P}Q$. So, the proportionality he describes are of the side lengths:

$$\frac{P'O}{\bar{P}P} = \frac{P'Q'}{PQ}$$

which is:

$$\frac{R}{r_S} = \frac{R\sin\kappa_S}{PQ}$$
$$PQ = \frac{r_S}{R}\cdot R\sin\kappa_S$$

Then, the second proportion relates the right triangle ΔPOQ and the radially scaled triangle $\Delta SOS'$, which is formed with Point S on the mean orbital path and Point S' as the perpendicular dropped onto line OQ, as shown in Figure 4.3. In triangle ΔPOQ, OP is the hypotenuse and PQ is the side length produced from the previous proportion that corresponds to this hypotenuse. Then in triangle $\Delta SOS'$, OS is the hypotenuse which is equivalent to the orbital radius R, and its corresponding side length SS'. So, this

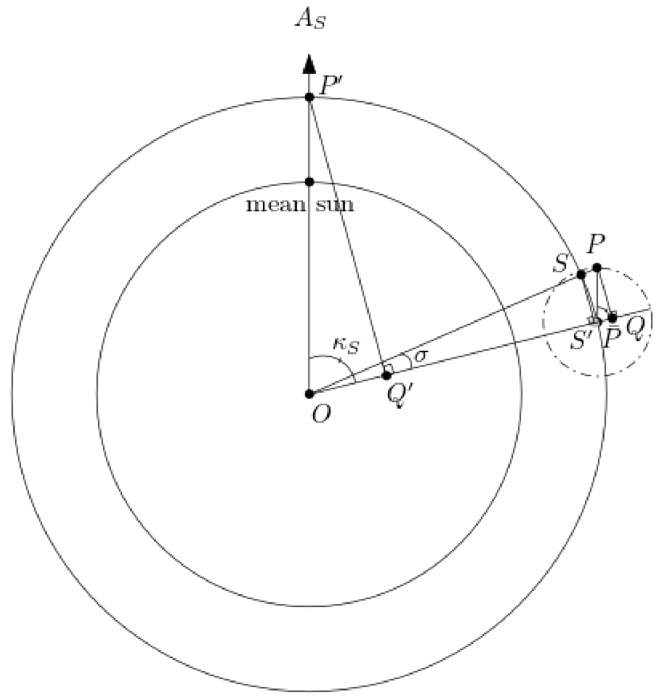

Figure 4.3: The *śīghra*-correction σ for a superior planet as dependent on the *śīghra*-anomaly κ_S and epicycle radius r_S, which is equal to $P\bar{P}$.

second proportion is symbolically:

$$\begin{aligned}
\frac{PQ}{OP} &= \frac{SS'}{OS} \\
\frac{PQ}{H_S} &= \frac{R\sin\sigma}{R} \\
R\sin\sigma &= PQ \cdot \frac{R}{H_S} \\
&= \left[\frac{r_S}{R} \cdot R\sin\kappa_S\right] \cdot \frac{R}{H_S} \\
&= \frac{r_S}{H_S} \cdot R\sin\kappa_S
\end{aligned}$$

Combining our understanding from the two proportions, we refer to Figure 4.3 to find the *śīghra*-correction for a given *śīghra*-anomaly κ_S. The two right triangles of importance are ΔOPQ and $\Delta\bar{P}PQ$. Again, angle $P\bar{P}Q$ is commensurate with the *śīghra*-anomaly κ_S. Triangle $\bar{P}PQ$ has as its hypotenuse (*karṇa*) the length of the *śīghra*-epicycle radius, $\bar{P}P$, or r_S. Its *bhuja* is defined to be the length PQ. The *koṭi* of triangle $\Delta\bar{P}PQ$ is the length $\bar{P}Q$. As we found with the first proportion:

$$PQ = \frac{r_S}{R} \cdot R\sin\kappa_S$$

so side length $\bar{P}Q$ is given by:

$$\bar{P}Q = \frac{r_S}{R} \cdot R\cos\kappa_S$$

Now we look at triangle OPQ. Angle POQ is the *śīghra*-correction σ. The *bhuja* of this triangle is also PQ.

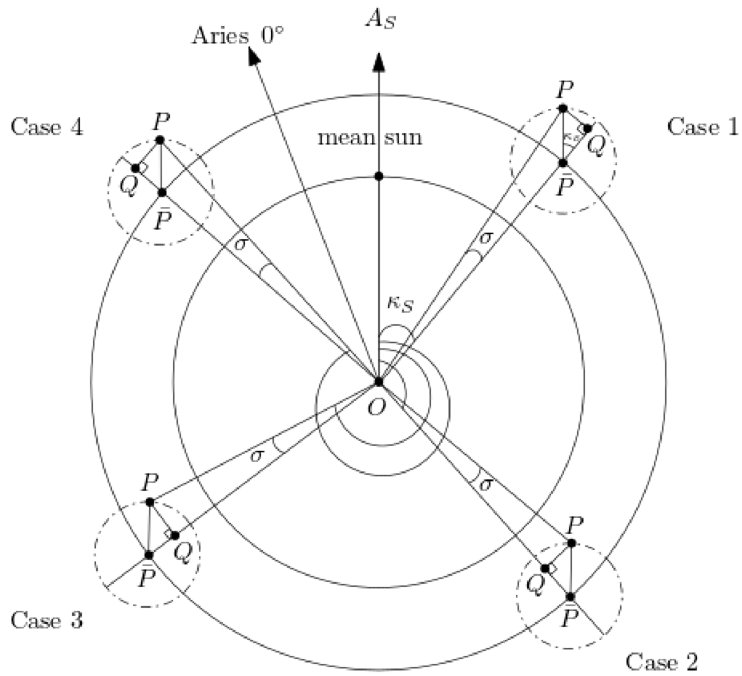

Figure 4.4: The configurations for *śīghra*-correction of a superior planet, given various *śīghra*-anomaly arguments. While the values for *śīghra*-anomaly may range from 0 to 360 degrees, the argument of the *śīghra*-correction *bhuja*, which is the angle $P\bar{P}Q$, is constrained between 0 and 90 degrees. In Cases 1 and 4, length $\bar{P}Q$ occurs outside the mean orbital path circle, so the length is an additive quantity when finding the length OQ. In Cases 2 and 3, when the angle of *śīghra*-anomaly lies between $90 \leq \kappa_S \leq 270$, the length $\bar{P}Q$ is subtractive since it lies within the circle of the mean orbit of the planet and is thus less than R.

The *koṭi* is the length OQ:

$$\begin{aligned} OQ &= O\bar{P} + \bar{P}Q \\ &= R + \frac{r_S}{R} \cdot R\cos\kappa_S \end{aligned}$$

But $\bar{P}Q$ is added or subtracted to R depending on the *śīghra*-anomaly. If the *śīghra*-anomaly is $90 \leq \kappa_S \leq 270$ then the *koṭi* is $R - R\cos\kappa_S \cdot \frac{r_S}{R}$, but if the *śīghra*-anomaly is $0 \leq \kappa_S \leq 90$ or $270 \leq \kappa_S \leq 360$, then the *koṭi* is $R + R\cos\kappa_S \cdot \frac{r_S}{R}$, which are the cases Mallāri describes for the "*antara*" ("difference") or "*yoga*" ("addition"). We see this in Figure 4.4.

Then the hypotenuse of this triangle is indeed the square root of the sum of the *koṭi* squared and the *bhuja* squared, as in:

$$\begin{aligned} H_S &= \sqrt{(bhuja)^2 + (koṭi)^2} \\ &= \sqrt{\left(R\sin\kappa_S \cdot \frac{r_S}{R}\right)^2 + \left(R \pm R\cos\kappa_S \cdot \frac{r_S}{R}\right)^2} \end{aligned}$$

The application of the *manda*- and *śīghra*-corrections are concisely described in *karaṇa* texts to be an iterative process, alternating between taking the *manda*-corrected and *śīghra*-corrected planet as each iteration's new mean planet. However, the prescribed order differs from text author to text author. Bhāskara II in his *Karaṇakutūhala* (see Verses 2.13-2.14) describes the application of the *śīghra*-correction after the *manda*-correction, as the *manda*-corrected planet is subtracted from its *śīghra*-apogee to yield the *śī*-

ghra-anomaly which is then used to find the *śīghra*-correction. He also states that the procedure is done repeatedly. He also states that the correction of Mars' longitude involves applying a half-*manda* and half-*śīghra* correction, then the full corrections to find its true longitude. Since in actuality, the effects of the *manda*- and *śīghra*-corrections on the longitudes of the planets are simultaneous, the iterative procedure with half-corrections described by Gaṇeśa and Bhāskara II and others are presumably done to account for this.

Analogous to how the *manda*-correction to velocity accompanied *manda*-corrections to planetary longitudes, as discussed in section 3.1, *śīghra*-corrections to velocity Δ_v^S correspond to *śīghra*-corrections to longitude. The true velocity of a planet v is:

$$\begin{aligned} v &= \bar{v}_M + \Delta_v^M \\ &\approx \bar{v}_M + \Delta\sigma \cdot v_{\kappa_S}. \end{aligned}$$

We can see this from finding instantaneous velocity as the derivative of true longitude λ, where $\lambda = \bar{\lambda}_M + \sigma$ [MP18, p.43]:

$$\begin{aligned} v &= \frac{d}{dt}(\lambda) \\ &= \frac{d}{dt}\left(\bar{\lambda}_M + \sigma\right) \\ &= \bar{v}_M + \frac{d}{d_{\kappa_S}}(\sigma) \cdot \frac{d}{dt}(\kappa_S) \\ &= \bar{v}_M + \frac{d}{d_{\kappa_S}}(\sigma) \cdot v_{\kappa_S} \end{aligned}$$

for the *śīghra*-anomaly's rate of change $v_{\kappa_S} = v_{A_S} - \bar{v}_M$. Text authors seem to approximate the derivative of σ to be the difference of successive *śīghra*-correction values.

4.1.2 Astronomical background: synodic phenomena

The celestial phenomena known as 'synodic' (because of their dependence on the relative positions of the planet and Sun) follow periodic patterns. A planet will become temporarily invisible when its longitude gets close to that of the Sun, then reappear at twilight as they separate, then seem to stop and reverse the direction of its motion among the stars, and so forth. The sequence of such phenomena will be different for superior and inferior planets, because the inferior planets never elongate very far from the Sun. The sequence of synodic phenomena of the superior planets are as follows:

- **Heliacal Rising:** when the planet is first visible before sunrise in the East.
- **First Station:** when the planet is changing direction of apparent motion from direct to retrograde, so velocity is 0 and the planet appears to be stationary.
- **Retrograde Motion:** when the planet appears to be moving backwards against a background of the fixed stars.
- **Second Station:** when the planet changes direction of apparent motion from retrograde to prograde, so the planet once again appears to be stationary with velocity 0.
- **Heliacal Setting:** the last phase of visibility, when the planet appears just after sunset in the West.

In the case of the superior planets, retrograde motion occurs when the planet is in opposition, and periods of visibility are marked from conjunction to conjunction.

The sequence of synodic phenomena of the inferior planets includes two periods of invisibility according to observers on Earth:

- **Western Rising:** when the planet moving quickly eastward becomes visible on the western horizon after sunset, when the planet is somewhat east of the Sun.

- **First Station:** when the planet nears its greatest elongation, it appears to stop and changes direction of apparent motion from direct to retrograde. In a heliocentric model, this is effectively the planet entering the nearer semicircle of its heliocentric orbit so it appears to have reversed course.

- **Retrograde Motion:** when the planet appears to be moving westward, or backwards, against a background of the fixed stars.

- **Western Setting:** the planet, while still East of the Sun, is moving westward until it disappears in the Sun's light and ceases to be visible even after the Sun has set below the western horizon. At some point of this period of invisibility, the planet will be in inferior conjunction.

- **Eastern Rising:** when the planet is so far West of the Sun that while the Sun has yet to rise in the eastern horizon, the planet is visible once again, just before sunrise.

- **Second Station:** when the planet once again reaches its greatest elongation in its westward direction, and appears to stop and change direction of apparent motion from retrograde to prograde, moving eastward once again.

- **Eastern Setting:** when the planet is still West of the Sun so it can only be seen before sunrise when the Sun is below the eastern horizon, but is moving eastward until it disappears in the Sun's light. In the middle of the prograde arc, the planet will be in superior conjunction, then it will be far enough East of the Sun to appear on the western horizon after sunset again to start a new synodic cycle.

4.2 Verses 3.1–3.5: The *śīghra*-numbers of Mars, Mercury, Jupiter, Venus, and Saturn

Gaṇeśa begins the third chapter with five verses listing what he calls the "*śīghra*-numbers" of Mercury, Venus, Mars, Jupiter, and Saturn. Provided for each 15 degrees of *śīghra*-anomaly, these *śīghra*-numbers are scale factors of the *śīghra*-corrections.

खमष्टमरुतोऽद्रिभूभुव उदध्यगोर्व्योऽष्टदृग्-
दृशो नवनगाश्विनोऽक्षदशनाः शराङ्गाग्नयः ।
गुणाङ्कदहनाः खखाब्धय इभाङ्गरामाः क्रमान्-
नवाम्बुधिदृशो नभः क्षितिभुवश्चलाङ्का इमे ॥ १ ॥ ॥ पृथ्वी ॥

kham aṣṭamaruto'dribhūbhuva udadhyagorvyo'ṣṭadṛg-
dṛśo navanagāśvino'kṣadaśanāḥ śarāṅgāgnayaḥ |
guṇāṅkadahanāḥ khakhābdhaya ibhāṅgarāmāḥ kramān-
navāmbudhidṛśo nabhaḥ kṣitibhuvaś calāṅkā ime || 1 || || *pṛthvī* ||

0 (*kha*) 58 (*aṣṭa-marut*) 117 (*adri-bhū-bhuva*) 174 (*udadhi-aga-urvī*) 228 (*aṣṭa-dṛś-dṛś*) 279 (*nava-naga-aśvina*) 325 (*akṣa-daśana*) 365 (*śara-aṅga-agni*) 393 (*guṇa-aṃka-dahana*) 400 (*kha-kha-abdhi*) 368 (*ibha-aṅga-rāma*) 249 (*nava-ambudhi-dṛś*) 0 (*nabhas*). These are the *śīghra*-numbers of Mars (*kṣitibhu*) in order.

खं भूकृताः कुवसवोऽद्रिभवाः खतिथ्यो-
ऽष्टाद्रीन्दवो नवनवक्षितयोऽर्कपक्षाः ।
अर्काश्विनः शरखगक्षितयोऽक्षतिथ्यो
गोऽष्टौ खमाशुफलजाः स्युरिमे विदोऽङ्काः ॥ २ ॥ ॥ वसन्ततिलका ॥

khaṃ bhūkṛtāḥ kuvasavo'dribhavāḥ khatithyo-
'ṣṭādrīndavo navanavakṣitayo'rkapakṣāḥ |
arkāśvinaḥ śarakhagakṣitayo'kṣatithyo
go'ṣṭau kham āśuphalajāḥ syur ime vido'ṅkāḥ || 2 || || *vasantatilakā* ||

0 (*kha*) 41 (*bhū-kṛta*) 81 (*ku-vasu*) 117 (*adri-bhava*) 150 (*kha-tithi*) 178 (*aṣṭa-adri-indu*) 199 (*nava-nava-kṣiti*) 212 (*arka-pakṣa*) 212 (*arka-aśvina*) 195 (*śara-khaga-kṣiti*) 155 (*akṣa-tithi*) 89 (*go-aṣṭa*) 0 (*kha*). These are the [*śīghra*-]numbers of Mercury produced from the *śīghra*-correction.

खं तत्त्वानि नगाब्धयोऽष्टषट्काः
पञ्चेभा गजखेचरा रसाशाः ।
नागाशा द्विदिशो नवाहयः षट्-
षष्टिः षट्कगुणा नभो गुरोः स्युः ॥ ॥ ३ ॥ ॥ एकरूप ॥

khaṃ tattvāni nagābdhayo'ṣṭaṣaṭkāḥ
pañcebhā gajakhecarā rasāśāḥ |
nāgāśā dvidiśo navāhayaḥ ṣaṭ-
ṣaṣṭiḥ ṣaṭkaguṇā nabho guroḥ syuḥ || 3 || || *ekarūpa* ||

0 (*kha*) 25 (*tattva*) 47 (*naga-abdhi*) 68 (*aṣṭa-ṣaṭka*) 85 (*pañca-ibha*) 98 (*gaja-khecara*) 106 (*rasa-āśā*) 108 (*nāga-āśā*) 102 (*dvi-diś*) 89 (*nava-ahi*) 66 (*ṣaṭ ṣaṣṭi*) 36 (*ṣaṭka-guṇa*) 0 (*nabhas*) are [the *śīghra*-numbers] of Jupiter.

खमग्न्यङ्गैस्तुल्या रसयमभुवः षट्कधृतयो-
ऽरिसिद्धाः पक्षाभ्राग्नय उदधिनाराचदहनाः ।
द्विशून्योदन्वन्तः खजलधिकृता भूरसकृता-
स्त्रिवेदोदन्वन्तो रसयमगुणाः खं भृगुजनेः ॥ ॥ ४ ॥ ॥ शिखरिणी ॥

kham agnyaṅgaistulyā rasayamabhuvaḥ ṣaṭkadhṛtayo
'risiddhāḥ pakṣābhrāgnaya udadhinārācadahanāḥ |
dviśūnyodanvantaḥ khajaladhikṛtā bhūrasakṛtās
trivedodanvanto rasayamaguṇāḥ khaṃ bhṛgujaneḥ || 4 || || *śikhariṇī* ||

0 (*kha*) 63 (*agni-aṅga*) 126 (*rasa-yama-bhuva*) 186 (*ṣaṭka-dhṛti*) 246 (*ari-siddha*) 302 (*pakṣa-abhra-agni*) 354 (*udadhi-nārāca-dahana*) 402 (*dvi-śūnya-udanvat*) 440 (*kha-jaladhi-kṛta*) 461 (*bhū-rasa-kṛta*) 443 (*stri-veda-udanvat*) 326 (*rasa-yama-guṇa*) 0 (*kha*) are [the *śīghra*-numbers] of Venus.

खमिषुक्षितयो गजाश्विनो[1] गो-
दहना नागकृताः पयोधिबाणाः ।

[1]Dvivedin says गजास्विनो while Joshi and Ms. Coll. 390 Item 1785 attest गजाश्विनो. Given the *bhūtasaṃkhyā* being expressed, we retain the latter.

द्विरगेषुमिता हुताशबाणाः
शारवेदास्त्रिगुणाः धृतिः खमार्केः ॥ ॥ ५ ॥ ॥ मालाभारिणी ॥

kham iṣukṣitayo gajāśvino go-
dahanā nāgakṛtāḥ payodhibāṇāḥ |
dvir ageṣumitā hutāśabāṇāḥ
śaravedās triguṇāḥ dhṛtiḥ kham ārkeḥ || 5 || || *mālābhāriṇī* ||

0 (*kha*) 15 (*iṣu-kṣiti*) 28 (*gaja-aśvina*) 39 (*go-dahana*) 48 (*nāga-kṛta*) 54 (*payodhi-bāṇa*) [and] two [values] measured by 57 (*aga-iṣu*) 53 (*hutāśa-bāṇa*) 45 (*śara-veda*) 33 (*tri-guṇa*) 18 (*dhṛti*) 0 (*kha*) [are the *śīghra*-numbers] of Saturn.

Commentary: The values of these *śīghra*-numbers are tabulated in Table 4.1.

Name of Planet	0	15	30	45	60	75	90	105	120	135	150	165	180
Mars	0	58	117	174	228	279	325	365	393	400	368	249	0
Mercury	0	41	81	117	150	178	199	212	212	195	155	89	0
Jupiter	0	25	47	68	85	98	106	108	102	89	66	36	0
Venus	0	63	126	186	246	302	354	402	440	461	443	326	0
Saturn	0	15	28	39	48	54	57	57	53	45	33	18	0

Table 4.1: Table of *śīghra*-numbers of each planet, for *śīghra*-anomaly measures from 0-180 degrees, in 15-degree increments.

4.3 Verse 3.6: Finding the *śīghra*-correction

After stating the *śīghra*-numbers in the first five verses, Gaṇeśa starts this sixth verse by describing the *śīghra*-anomaly for the superior planets, then how to find its corresponding *śīghra*-number from the previously stated versified tables. This *śīghra*-number divided by the scale factor 10 yields the *śīghra*-correction.

भौमार्कीज्यविहीनमध्यमरविः स्यात् स्वाशुकेन्द्रं तु विद्-
भृग्वोरुक्तमिदं रसोर्ध्वमिनभाच्छुद्धं तदंशा दिनैः
भक्ता खादिफलक्रमादिह गताङ्कोऽसौ क्षयर्द्ध्याहता-
च्छेषाद्बाणकुलब्धिहीनयुगयं दिग्हृल्लवाद्यं फलम् ॥ ६ ॥ ॥ शार्दूलविक्रीडित ॥

bhaumārkījyavihīnamadhyamaraviḥ syāt svāśukendraṃ tu vid-
bhṛgvor uktam idaṃ rasordhvam inabhācchuddhaṃ tadaṃśā dinaiḥ |
bhaktā khādiphalakramād iha gatāṅko'sau kṣayarddhyā hatāc
cheṣād bāṇakulabdhihīnayug ayaṃ dighṛl lavādyaṃ phalam || 6 || || *śārdūlavikrīḍita* ||

The mean [longitude of the] Sun diminished by [the mean longitude of] Mars, Saturn, [or] Jupiter is [the planet's] own *śīghra*-anomaly, but the [*śīghra*-anomaly] of Mercury or Venus is [as] stated [in the first chapter]. [When] this [anomaly measure] is greater [than] six [signs], then [it is] subtracted from twelve (*inabha*) [signs]. Those degrees [of the anomaly], divided by 15 (*dina*) correspond here to the elapsed integer [*śīghra*-]correction [numbers] beginning [with] zero. This [elapsed integer] is increased or decreased by the quotient from the remainder multiplied by the increasing or decreasing [difference of *śīghra*-correction values], and fifteen (*bāṇa-ku*). [This] divided by ten (*dik*) is the [*śīghra*-]correction in degrees and so on.

Commentary: First, Gaṇeśa describes how to find the *śīghra*-apogee for each planet. The *śīghra*-anomaly κ_S for the superior planets is stated to be the difference between the longitude of the *śīghra*-apogee λ_{A_S} and the mean longitude of the planet $\bar{\lambda}$:

$$\kappa_S = \lambda_{A_S} - \bar{\lambda}$$

where the *śīghra*-apogee is the mean longitude of the Sun. Gaṇeśa expedited the process of finding the *śīghra*-anomalies of Mercury and Venus by providing their parameters in Chapter 1 itself, instead of providing parameters for their *śīghra*-apogees as other texts do.

Verse 6 then provides insight into how to use the versified table presented in Verses 1-5, and how to interpolate the values of the *śīghra*-anomaly from these 15-degree increments.Since Gaṇeśa has stated the *śīghra*-numbers for degrees of arc between 0 and 180, for a *śīghra*-anomaly value between 0 and 180, the argument is taken as is. However, degrees of arc between 180 and 360 are subtracted from 360 to yield the argument. This arc measure should be divided by 15 to correspond to the tabulated values presented previously. This will yield a quotient q and a remainder r:

$$\frac{\kappa_S}{15} = q + \frac{r}{15}$$

If the quotient corresponds exactly to a 15-degree increment with no remainder, then the value should be directly taken from the table. Otherwise, a linear interpolation must be used to account for the remainder. First, find the closest *śīghra*-number to the quotient. This is considered the elapsed *śīghra*-number s_n. Find the difference between this elapsed number and the *śīghra*-number for the next 15-degree increment s_{n+1}. This difference multiplied by the remainder and divided by 15, should then correspond to the *śīghra*-correction multiplied by 10:

$$10 \cdot \sigma = s_n + r \cdot \frac{s_{n+1} - s_n}{15}$$

Gaṇeśa distinguishes between his stated *śīghra*-numbers and *śīghra*-correction values with clearly different nomenclature ("number" *aṅka* as opposed to "correction" *phala*), since these *śīghra*-numbers are to be divided by 10 to yield the *śīghra*-correction values.

In his commentary for verses 1–5, Mallāri explains that since this trigonometric calculation is so difficult and tedious, Gaṇeśa presents pre-calculated *śīghra*-corrections values for every 15 degrees of *śīghra*-anomaly between 0 and 180 degrees, and thus simply states the *śīghra*-number values for the user to use [Dvi25, p.90, lines 6–8]:

> अत्रेदं जडकर्म दृष्ट्वाचार्येण शीघ्रकेन्द्रं पञ्चदशभागवृद्ध्या प्रकल्प्य शीघ्रफलानि प्रसाध्य तानि सावयवान्यतो दशगुणानि । राशिषट्कमध्ये द्वादशः सर्वेषां ग्रहाणां पृथक् पृथगुत्पादितानि ।
> *atredaṃ jaḍakarma dṛṣṭvācāryeṇa śīghrakendraṃ pañcadaśabhāgavṛddhyā prakalpya śīghraphalāni prasādhya tāni sāvayavānyato daśaguṇāni | rāśiṣaṭkamadhye dvādaśaḥ sarveṣāṃ grahāṇāṃ pṛthak pṛthag utpāditāni |*
>
> Here, [teacher [Gaṇeśa]] having seen this tedious process, having determined the *śīghra*-anomaly with 15 degree-increments, having calculated the [corresponding] *śīghra*-corrections, those [*śīghra*-corrections] that are with [fractional] portions are multiplied by ten by the teacher [Gaṇeśa]. In [the first] six signs, twelve [*śīghra*-corrections] of all the planets are produced severally.

Mallāri states that true to his intent, Gaṇeśa has performed all the trigonometric calculations himself in order to present values that users can algebraically manipulate in a linear interpolation scheme in order to produce true planetary longitudes without as much effort. He also clarifies the relationship between the *śīghra*-corrections and the *śīghra*-numbers Gaṇeśa has stated. Since the *śīghra*-correction values have fractional parts, Gaṇeśa has multiplied them by 10, presumably for both ease of computation and to retain more significant figures while preserving the economy of verbiage. Due to the symmetricality over two quadrants, Gaṇeśa has only provided the *śīghra*-numbers for degrees 0 to 180; users will use the negative values for degrees 181 to 360.

Interestingly, in his commentary of verse 6, Mallāri observes when the differences of consecutive *śīghra*-numbers in the interpolation are increasing and decreasing, and he presents them as two separate cases [Dvi25, lines 28–30, 1–2, pp.91–92].

> अत्रोपपत्तिः । यदि पञ्चदशभागैरेकः शीघ्राङ्कस्तदेष्टैः केन्द्रभागैः किम् । एवं यल्लब्धं तन्मितो गतः स्यात् । ततः शेषादनुपातः । यदि पञ्चदशभागैर्गतैष्यान्तरतुल्या ह्रासर्वृद्धिर्लभ्यते तदा शेषाङ्कैः किमिति । फलेन क्षये हीनो वृद्धौ युक्तो गताङ्कः कार्य एव । ततो दशगुणाङ्काः सन्त्यतो दशभिर्भक्तो भागाद्यं शीघ्रफलं भवतीत्युपपन्नम् ।
>
> *atropapattiḥ | yadi pañcadaśabhogair ekaḥ śīghrāṅkas tadeṣṭaiḥ kendrabhāgaiḥ kim | evaṃ yallabdhaṃ tanmito gataḥ syāt | tataḥ śeṣād anupātaḥ | yadi pañcadaśabhāgair gataiṣyāntaratulyā hrāsar vṛddhir labhyate tadā śeṣāṅkaiḥ kim iti | phalena kṣaye hīno vṛddhau yukto gatā.nkaḥ kārya eva | tato daśaguṇāṅkāḥ santy ato daśabhir bhakto bhāgādyaṃ śīghraphalaṃ bhavatīty upapannam |*
>
> Here is the explanation. If one *śīghra*-number [corresponds to] fifteen degrees, then what [corresponds to] the desired degrees of anomaly? Thus whatever is obtained [as quotient], its amount is the elapsed [*śīghra*-number]. Then from the remainder a proportion. If an increment or decrement equivalent to the difference of the elapsed or next [*śīghra*-number values] is obtained with fifteen degrees, then what [*śīghra*-number quantity] corresponds to the remainder [of dividing the degrees of *śīghra*-anomaly by 15]? When decreasing, the elapsed value is to be decreased by the decrement; when increasing, increased [by the increment]. Then the [*śīghra*-]numbers have been multiplied by ten, so divided by ten yields the *śīghra*-correction in degrees and so on; thus it is demonstrated.

Mallāri uses a proportion to explain the interpolation of degrees of anomaly that are not fifteen degree multiples. He states that after dividing:

$$\frac{\kappa_S}{15} = q + \frac{r}{15}$$

then these remaining degrees r are interpolated using the proportion:

$$\frac{s_{n+1} - s_n}{15^\circ} = \frac{\mathrm{x}}{r}$$

$$x = r \cdot \frac{s_{n+1} - s_n}{15^\circ}$$

which is indeed what Gaṇeśa states in his verse. Mallāri also explicitly describes the increasing and decreasing cases of interpolation. When the next *śīghra*-number is greater than the previous, then the difference (which then is multiplied by the remaining degrees of *śīghra*-anomaly and divided by 15) is additive and added to the elapsed *śīghra*-number. When the next *śīghra*-number is less than the elapsed, then the difference is negative and is subtracted from the elapsed *śīghra*-number. Then, Mallāri specifies that when this correction is to be applied to the longitudes of the planets, these interpolated *śīghra*-numbers should be divided by 10 to yield the actual *śīghra*-corrections.

4.4 Verses 3.7–3.8: The *manda*-numbers of Mars, Mercury, Jupiter, Venus, and Saturn

As discussed in Section 3.1, all the planets have the *manda*-correction applied to their mean longitudes to account for the eccentricity of orbits. We have seen in Sections 3.3 and 3.4 that Gaṇeśa provided algebraic sine-approximation formulas to find the *manda*-corrections of the Sun and Moon. Here in verses 7–8, Gaṇeśa instead states the *manda*-numbers of the five star-planets in the same versified table manner as he did for their *śīghra*-numbers in verses 1–5.

खं गोऽश्विनोऽद्रिमरुतोऽक्षगजा नवाशाः
सिद्धेन्दवः खदहनक्षितयोऽसृजोऽङ्काः
मान्दा बुधस्य खमिनाः कुदृशोऽष्टपक्षा
देवाः शरानलमिता रसवह्नयः स्युः ॥ ७ ॥ ॥ वसन्ततिलका ॥

khaṃ go' śvino'drimaruto'kṣagajā navāśāḥ
siddhendavaḥ khadahanakṣitayo'sṛjo'ṅkāḥ |
māndā budhasya kham ināḥ kudṛśo'ṣṭapakṣā
devāḥ śarānalamitā rasavahnayaḥ syuḥ || 7 || || *vasantatilaka* ||

0 (*kha*) 29 (*go-aśvin*) 57 (*adri-marut*) 85 (*akṣa-gaja*) 109 (*nava-āśā*) 124 (*siddha-indu*) 130 (*kha-dahana-kṣiti*) are [*manda*-]numbers.

The [*manda*-numbers] related to the *manda*-correction of Mercury are 0 (*kha*) 12 (*ina*) 21 (*ku-dṛś*) 28 (*aṣṭa-pakṣa*) 33 (*deva*) 35 (*śara-anala*) 36 (*rasa-vahni*).

खेन्द्रर्क्षाणि नवाग्नयोऽह्युदधयोऽक्षाक्षा नगाक्षा गुरोः
शुक्रस्याभ्ररसेशविश्वमनवो द्विर्बाणचन्द्राः क्रमात्
खं गोऽब्जाः खकृताः खषड्नगनगा गोऽष्टौ त्रिनन्दाः शनेः
शुद्धोऽब्ध्यद्रिषडग्निनागगृहतः स्यान्मन्दकेन्द्रं कुजात् ॥ ८ ॥ ॥ शार्दूलविक्रीडित ॥

khendrarkṣāṇi navāgnayo'hyudadhayo'kṣākṣā nagākṣā guroḥ
śukrasyābhrarasesaviśvamanavo dvir bāṇacandrāḥ kramāt |
khaṃ go'bjāḥ khakṛtāḥ khaṣaṭ-naganagā go'ṣṭau trinandāḥ śaneḥ
śuddho'bdhyadriṣaḍagnināgagṛhataḥ syān mandakendraṃ kujāt || 8 || || *śārdūlavikrīḍita* ||

Of Jupiter [the numbers are sequentially] 0 (*kha*) 14 (*indra*) 27 (*ṛkṣa*) 39 (*nava-agni*) 48 (*ahi-udadhi*) 55 (*akṣa-akṣa*) 57 (*naga-akṣa*.

Of Venus [the numbers are] sequentially 0 (*abhra*) 6 (*rasa*) 11 (*īśa*) 13 (*viśva*) 14 (*manu*) twice 15 (*bāṇa-candra*).

Of Saturn [the numbers are] 0 (*kha*) 19 (*go-abja*) 40 (*kha-kṛta*) 60 (*kha-ṣaṭ*) 77 (*naga-naga*) 89 (*go-aṣṭa*) 93 (*tri-nanda*).

From 4 (*abdhi*), 7 (*adri*), 6 (*ṣaḍ*), 3 (*agni*), [and] 8 (*nāga*) signs [respectively, the mean longitude of the planet] is subtracted, and is the *manda*-anomaly [of each planet starting] from Mars.

Commentary: Gaṇeśa now states the *manda*-numbers for every 15 degrees of arc from 0 to 90 for the five planets as a versified table. Just as *śīghra*-numbers were distinct values divided by the scale-factor 10 to produce *śīghra*-correction values, the *manda*-numbers are also divided by ten to yield *manda*-correction values. The *manda*-numbers as stated in these two verses, for every 15 degrees of *manda*-anomaly from 0 to 90 is provided in tabular form in Table 4.2. The *manda*-corrections have sine quadrant symmetry, and thus only values for 0-90 degrees of anomaly are necessary.

The *manda*-anomaly angle is reckoned as the difference between the longitude of the apogee of a planet and its mean longitude, as was discussed in Chapter 2 when finding the *manda*-correction for the Sun and Moon:

$$\kappa_M = \lambda_{A_M} - \bar{\lambda}$$

Like the solar apogee, the apogees of the other planets move so slowly they are taken to be fixed longitudes. Thus, Gaṇeśa states these longitudes in Verse 8cd. Bhāskara II also states the longitudes of the apogees of the planets in his *Karaṇakutūhala* Verse 2.1. As discussed in Chapter 2, the *Siddhāntaśiromaṇi* and other

Name of Planet	0	15	30	45	60	75	90
Mars	0	29	57	85	109	124	130
Mercury	0	12	21	28	33	35	36
Jupiter	0	14	27	39	48	55	57
Venus	0	6	11	13	14	15	15
Saturn	0	19	40	60	77	89	93

Table 4.2: Table of *manda*-values of the planets, for *manda*-anomaly angle measures from 0-90 degrees in 15-degree increments.

texts presented the number of integer revolutions of the apogees in a *kalpa* and the longitudes of the apogees can be calculated using that period relation. The longitudes for the planets as computed in all three texts (with the *Siddhāntaśiromaṇi* longitudes rounded to the nearest minute) are in Table 4.3.

Name of Planet	*Grahalāghava* λ_{A_M}	*Karaṇakutūhala* λ_{A_M}	*Brāhmasphuṭasiddhānta* λ_{A_M}
Mars	120°	128°30′	128°25′
Mercury	210°	225°	224°55′
Jupiter	180°	172°30′	172°35′
Venus	90°	81°	81°17′
Saturn	240°	261°	260°54′

Table 4.3: The longitudes of the apogees of the planets in degrees and minutes as stated in Verses 3.7-8 of the *Grahalāghava*, Verse 2.1 of the *Karaṇakutūhala*, and calculated from the number of apogee revolutions (292, 332, 855, 653, 41 for Mars, Mercury, Jupiter, Venus, and Saturn respectively) for the 1972948621 years elapsed from the start of the *kalpa* to the *Grahalāghava*'s epoch given in Verses 1.2.1–6 in the *Siddhāntaśiromaṇi*.

4.5 Verse 3.9: Finding the *manda*-correction

As was done previously for the *śīghra*-correction, Gaṇeśa now describes the method of finding the *manda*-correction by interpolating the aforementioned *manda*-numbers and dividing them by scale factor 10 for a computed *manda*-anomaly.

मृदुकेन्द्रभुजांशका दिनाप्ताः
फलमङ्कः प्रगतस्तदूनितैष्यः
परिशेषहतो दिनाप्तियुक्तो
दशभक्तः फलमंशकादि मान्दम् ॥ ९ ॥ ॥ मालाभारिणी ॥

mṛdukendrabhujāṃśakā dināptāḥ
phalam aṅkaḥ pragatas tadūnitaiṣyaḥ
pariśeṣahato dināptiyukto
daśabhaktaḥ phalam aṃśakādi māndam || 9 || || *mālābhāriṇī* ||

The degrees of the arc of the *manda*-anomaly are divided by 15 (*dina*). The result [corresponds to] the elapsed [*manda*-]number. The next [*manda*-number] diminished by that [elapsed *manda*-number corresponding to the quotient] multiplied by the remainder [of the division by 15], is joined to the quotient of [the division by] 15 [of the difference and the remainder], [and] divided by 10 is the *manda*-correction in degrees and so on.

Commentary: This verse describing the *manda*-interpolation closely resembles Gaṇeśa's verse 6. The desired arc of *manda*-anomaly, is divided by 15 as per the provided incremental arguments. This division results in a whole number quotient, and a remainder:

$$\frac{\kappa_M}{15} = q + \frac{r}{15}$$

Again, the quotient corresponds to an elapsed *manda*-number. The difference between the elapsed and the next 15-degree increment value is divided by the previous division's remainder. This quantity is then added to the previous quotient (the elapsed):

$$10 \cdot \mu = m_n + r \cdot \frac{m_{n+1} - m_n}{15}$$

Again, Gaṇeśa states a division by 10 to arrive at the actual correctional value, because he has multiplied by 10 to get whole numbers.

Recall that in Section 3.1, we discuss approximating the *manda*-hypotenuse to be equivalent to the orbital radius R. We do not do so in the computation of the *śīghra*-correction, as seen in Section 4.1.1. Interestingly, in his commentary of the previous two verses, Mallāri engages in a brief discussion of why the hypotenuse is approximated in the case of the *manda*-correction and not the *śīghra*-correction [Dvi25, lines 16–22, p.93]:

> अतो मन्दफलानयने मन्दकर्णोऽपि ग्राह्यः । स सर्वैरपि नाङ्गीकृतः । तत्र ग्रहकर्णाग्रहणे एकं कारणं वक्तव्यम् । शीघ्रफलान्मन्दफलस्योनत्वात् स्वल्पान्तरत्वान्मन्दकर्मणि कर्णो न गृहीतः । एवं चेत् तर्हि स्वल्पेऽपि शीघ्रफले कर्णो गृह्यते । तदधिके मन्दफले न गृह्यते । एवं कथमिति चेन्नो । यतोऽत्र युक्त्या हेतुज्ञानं नैव भवति । फलवासना विचित्रास्ति । एतादृशेनैव कर्मणा आकाशे ग्रहस्पष्टत्वं दृश्यते । अतः प्रत्यक्षप्रमाणोपलब्ध्या एतत् कृतमिति वक्तव्यम् इति सर्वं निरवद्यम् ।
>
> *ato mandaphalānayane mandakarṇo'pi grāhyaḥ | sa sarvairapi nāṅgīkṛtaḥ | tatra grahakarṇāgrahaṇe ekaṃ kāraṇaṃ vaktavyam | śīghraphalānmandaphalasyonatvāt svalpāntaratvānmandakarmaṇi karṇo na gṛhītaḥ | evaṃ cet tarhi svalpe'pi śīghraphale karṇo gṛhyate | tadadhike mandaphale na gṛhyate | evaṃ kathamiti cenno | yato'tra yuktyā hetujñānaṃ naiva bhavati | phalavāsanā vicitrāsti | etādṛśenaiva karmaṇā ākāśe grahaspaṣṭatvaṃ dṛśyate | ataḥ pratyakṣapramāṇopalabdhyā etat kṛtamiti vaktavyam iti sarvaṃ niravadyam |*
>
> So, in calculating the *manda*-correction, the *manda*-hypotenuse is also to be taken. That is not agreed upon by everyone. There, in [support of the case of] the [*manda*-]hypotenuse of the planet not being taken, one reason is to be stated. Due to the *manda*-correction being less than the *śīghra*-correction [with] little difference, the hypotenuse is not taken in the *manda*-process. Indeed if [it is] thus, then even when the *śīghra*-correction is small, the [*śīghra*-]hypotenuse is taken. When the *manda*-correction is greater than that [small *śīghra*-correction], the [hypotenuse] is not taken. If [it is] not thus, [then] how is it? Because here even with the rationale there is no knowledge of the cause at all. The demonstration of the correction is strange. The trueness of the planets can be seen in the sky only with this kind of [strange] process. So, it must be said that this was done with realization [attained] through direct perception; thus all [of this] is unobjectionable.

The convention is approximating the *manda*-hypotenuse to be equivalent to the orbital radius R. Mallāri follows one line of reasoning that states that since the *manda*-corrections of the planets are generally smaller than their *śīghra*-corrections, so the *manda*-hypotenuse could be approximated as R. The difference between the *manda*-hypotenuse and R is quite small, so the approximation is quite accurate. He identifies a contradiction because when the *manda*-corrections of some planets are larger than the *śīghra*-corrections of some planets, astronomers still insist on restricting the hypotenuse approximation to the *manda*-correction. The *śīghra*-hypotenuse is always computed and never approximated with R even when the difference between

R and the *śīghra*-hypotenuse is negligible. However, Mallāri points out that this is not a very rigorous explanation, and indicates that the true longitudes of the planets can only be verified through observation.

Then Mallāri quotes part of the *Golādhyāya* of the *Siddhāntaśiromaṇi* [Cha81b, v. 21.36a, 37b] [Dvi25, lines 24–25, p.93]:

> स्वल्पान्तरत्वान्मृदुकर्मणीह कर्णः कृतो नेति च केचिदूचः ।
> नाशङ्कनीयं न चले किमित्थं यतो विचित्रा फलवासनात्र ।
>
> *svalpāntaratvānmṛdukarmaṇīha karṇaḥ kṛto neti ca kecidūcaḥ |*
> *nāśaṅkanīyaṃ na cale kim itthaṃ yato vicitrā phalavāsanātra | |*
>
> Many others stated that the hypotenuse is not made here in the *manda*-process because of its having only little difference [between orbital radius and hypotenuse]. It is not to be doubted [and] not in question, why? Since the demonstration of the correction here is peculiar.

Mallāri justifies his previous statement that the explanation behind choosing the *manda*-hypotenuse to be approximated with R is "strange" (*vicitra*) by citing the *Siddhāntaśiromaṇi*, which similarly states this is an issue that has a complicated rationale.

Mallāri's commentary of verses 3.7–8 also features an example computation of the tabulated first and second *manda*-numbers for Mars [Dvi25, lines 26–33, p.93]:

> अत्र त्रिज्यातुल्यया मन्दकेन्द्रदोर्ज्यया यदि परमं मन्दफलं तदेष्टदोर्ज्यया किमिति । एवं पञ्चदशभागवृद्ध्या मन्दकेन्द्रं प्रकल्प्य अनया युक्त्या मन्दफलानि प्रसाध्यानि । तानि सावयवान्यतो दशगुणानि कृत्वा राशित्रयमध्ये ग्रहाणां पृथक् पृथक् षडङ्का मान्दा भवन्तीत्युपपन्नम् । अत्र धूलीकर्म । प्रथमाङ्को भुजाभावाच्चुन्यम् । ततः पञ्चदश १५ भागास्तेषां ज्या ३१ । भौमपरममन्दफलेन गुणिता जाता ३४७।१२ । इयं खार्क१२० भक्ता जातं फलम् २।५४ । इदं साव्यवत्वाद्दशगुणं २९ जातो भौमस्य द्वितीयो मान्दाङ्कः । एवं सर्वेषां सर्वेऽङ्का उत्पादनीया ।
>
> *atra trijyātulyayā mandakendradorjyayā yadi paramaṃ mandaphalaṃ tadeṣṭadorjyayā kimiti | evaṃ pañcadaśabhāgavṛddhyā mandakendraṃ prakalpya anayā yuktyā mandaphalāni prasādhyāni | tāni sāvayavānyato daśaguṇāni kṛtvā rāśitrayamadhye grahāṇāṃ pṛthak pṛthak ṣaḍaṅkā māndā bhavantītyupapannam | atra dhūlīkarma | prathamāṅko bhujābhāvāccunyam | tataḥ pañcadaśa 15 bhāgāsteṣāṃ jyā 31 | bhaumaparamamandaphalena guṇitā jātā 347 | 12 | iyaṃ khārka 120 bhaktā jātaṃ phalam 2 | 54 | idaṃ sāvyavatvāddaśaguṇaṃ 29 jāto bhaumasya dvitīyo māndāṅkaḥ | evaṃ sarveṣāṃ sarve'ṅkā utpādanīyā |*
>
> Here, if the *manda*-correction is maximum [is obtained] with the sine of the *manda*-anomaly equal to R, then what [*manda*-correction is obtained] with the sine of the desired [*manda*-anomaly]? In this way, having considered the *manda*-anomaly with [a constant] 15-degree increment, the *manda*-corrections are to be computed with that rationale. Those [*manda*-corrections] are with fractional parts, so having multiplied [them] by 10, one by one are the six *manda*-numbers of the planets in one quadrant; thus it is explained. Here is the board work. The first number from the non-existence of an arc is 0. Then 15 degrees; their sine 31. Multiplied by the maximum *manda*-correction of Mars is 347|12. This divided by 120 is the result 2|54. Because this [has] fractional-parts, it is multiplied by ten, 29, [and] Mars' second *manda*-number is produced. In this way, all of the numbers of all [of the planets] are to be obtained.

Here, Mallāri relies on the approximate trigonometric formula which we saw in sections 3.3 and 3.4:

$$\mu \approx \frac{R \sin \kappa_M \cdot \frac{r_M}{R}}{R}$$

For $\kappa_M = 0$, $\sin\mu = 0$ which explains Gaṇeśa's first *manda*-number for Mars. Then, Mallāri explains Gaṇeśa's second *manda*-number, for $\kappa_M = 15$. Since Gaṇeśa does not state trigonometric methods, his values for r_M for each of the planets can only be recomputed and approximated. In a previous portion of Mallāri's commentary, we see that he uses $R\sin 15 = 31$ [Dvi25, line 11, p. 90]. So in Mallāri's example:

$$\begin{aligned} R\mu &\approx \frac{R\sin\kappa_M \cdot r_M}{R} \\ &= \frac{120\sin 15 \cdot r_M}{120} \\ &= \frac{347|12}{120} \\ &\approx 2|54 \end{aligned}$$

which multiplied by 10 yields 29 as Gaṇeśa states. To find out what value of r_M Mallāri is using, we can divide:

$$\begin{aligned} 347|12 &= 31 \cdot r_M \\ \frac{56}{5} &= \cdot r_M \\ 11|12 &= r_M \end{aligned}$$

If Mallāri's commentary is indeed representative of Gaṇeśa's methodology, then Gaṇeśa is most likely consulting trigonometry tables such as that of the *Karaṇakutūhala* [2] to compute values he otherwise presents in his tables. It still remains to be seen as to why Gaṇeśa did not choose to provide such clever formulae for the *manda*-corrections for each of the five planets as he did for the Sun and Moon. We compare Gaṇeśa's *manda*-correction values to those calculated using the *Siddhāntaśiromaṇi* and the *Karaṇakutūhala*, as shown in Figure 4.5.

4.6 Verse 3.10: Applying the *śīghra*-and *manda*-corrections to planets

Gaṇeśa now describes the iterative process of applying the *śīghra* and *manda* corrections to find the true longitudes of the planets in this verse.

प्राङ्मध्यमे चलफलस्य दलं विदध्यात्
तस्माच्च मान्दमखिलं विदधीत मध्ये
द्राक्केन्द्रकेऽपि च विलोममतश्च शीघ्रं
सर्वं च तत्र विदधीत भवेत् स्फुटोऽसौ ॥ १० ॥ ॥ वसन्ततिलका॥

prāṅmadhyame calaphalasya dalaṃ vidadhyāt
tasmāc ca māndam akhilaṃ vidadhīta madhye
drākkendrake'pi ca vilomamataś ca śīghraṃ
sarvaṃ ca tatra vidadhīta bhavet sphuṭo 'sau || 10 || || *vasantatilakā* ||

First, one should apply half of the *śīghra*-correction to the mean [longitude of the planet]. From that, the complete *manda*-[correction] may be applied to the mean [longitude of the planet]. And also in [finding] the *śīghra*-anomaly, the *śīghra*-[correction] is [applied to the previously found

[2] Mallāri's value R $sin 15 = 31$ is the value found from interpolating the sine table in the *Karaṇakutūhala* presented for every 10 degrees of arc and $R = 120$ [Mis91, v.2.6–7, p. 21].

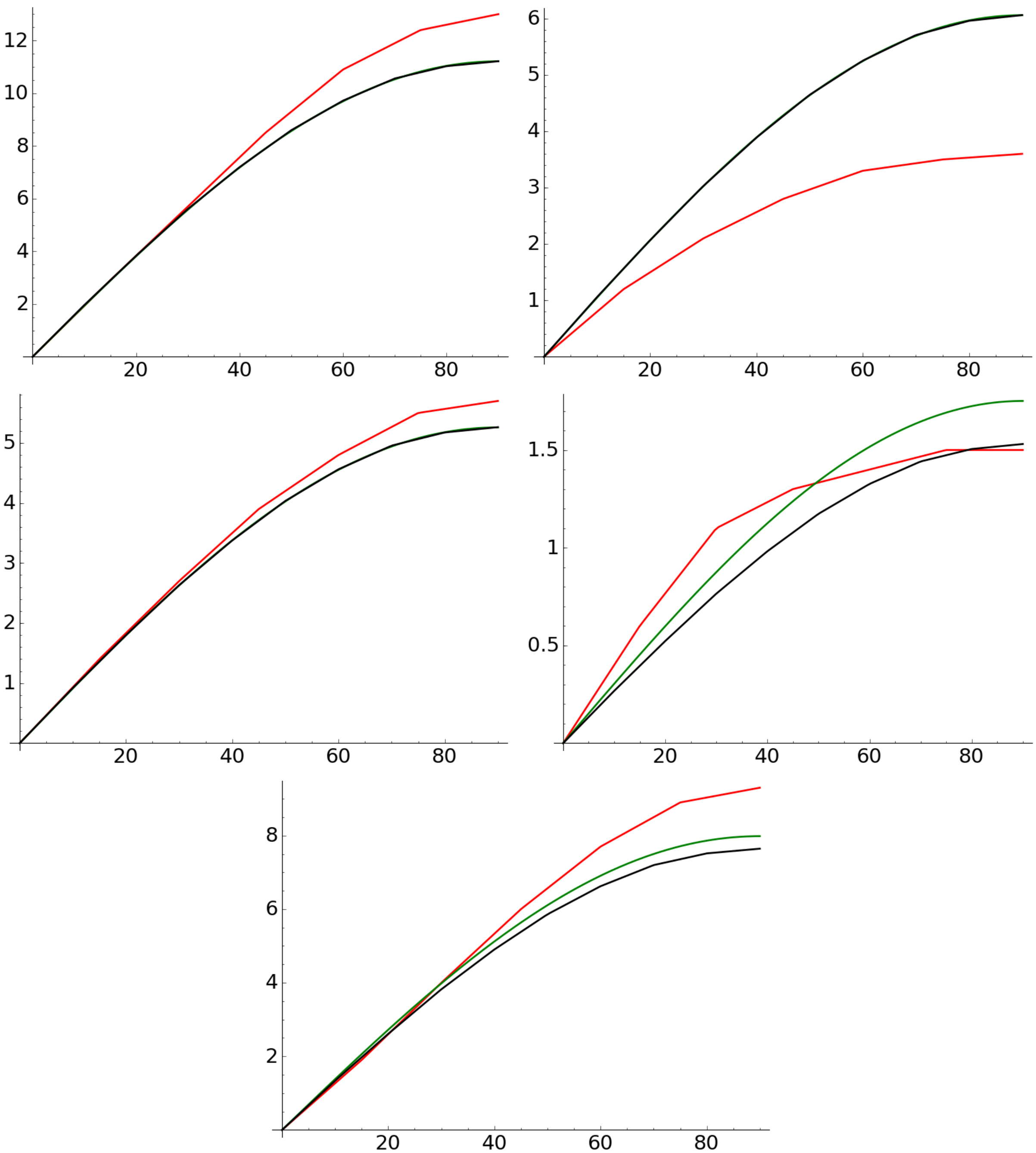

Figure 4.5: The *manda*-correction values in degrees for *manda*-anomaly values 0-90 degrees as calculated from the *Grahalāghava* (red), the *Siddhāntaśiromaṇi* (green), and the *Karaṇakutūhala* (black) for each star-planet, clockwise from top left: Mars, Mercury, Venus, Saturn, and Jupiter.

śīghra-anomaly in] reverse. So one should apply the full *śīghra*[-correction]; that is the true [longitude of the planet].

Commentary: In this verse, Gaṇeśa states that first the *śīghra*-correction is calculated, again with the *śīghra*-anomaly κ_S found using the mean longitude of the planet $\bar{\lambda}$ and the *śīghra*-apogee λ_{A_S}:

$$\kappa_S = \lambda_{A_S} - \bar{\lambda}$$

and half of the *śīghra*-correction σ is applied to the mean longitude of the planet $\bar{\lambda}$ to produce a half-*śīghra*-corrected longitude of the planet:

$$\lambda_S = \bar{\lambda} \pm \frac{\sigma}{2}$$

This half-*śīghra* corrected longitude is now taken to identify the *manda*-anomaly:

$$\kappa_M = \lambda_{A_M} - \lambda_S$$

Then, the *manda*-corrected longitude is taken to calculate the *śīghra*-anomaly for the full *śīghra*-correction:

$$\kappa_S = \lambda_{A_S} - \lambda_M$$

and Gaṇeśa states that this is now the true longitude of the planet.

While other text authors state that this process should be repeated until convergence is reached, Gaṇeśa abbreviates this process.

4.7 Verse 3.11: The *manda*-correction to the mean motions of planets

After explaining the *manda*-correction to each planet's mean longitudes, Gaṇeśa now states the corresponding *manda*-corrections to mean motions of each planet in this verse. In Section 3.5, we saw that the *manda*-correction μ to the mean motions of the Sun and Moon $\bar{\lambda}$ also appeared to be algebraic approximations of the cosine function. In this verse, Gaṇeśa presents these mean motion *manda*-corrections as scale factors of consecutive *manda*-numbers.

मान्दाङ्कान्तरमार्क्यसृग्गुरूणां
भक्तं बाणनगैः शरैः खरामैः
विद्भृग्वोर्द्विहताशुगोद्धृतं तद्-
दध्यात् प्राग्वदितौ मृदुस्फुटा सा ॥ ११ ॥ ॥ एकरूप ॥

māndāṅkāntaram ārkyasṛggurūṇāṃ |
bhaktaṃ bāṇanagaiḥ śaraiḥ kharāmaiḥ |
vidbhṛgvor dvihatāśugoddhṛtaṃ tad |
dadhyāt prāgvaditau mṛdusphuṭā sā || 11 || || *ekarūpa* ||

The difference of the *manda*-numbers of Saturn, Mars, and Jupiter, divided by 75 (*bāṇa-naga*), 5 (*śara*), [and] 30 (*kha-rāma*) [respectively] [and the difference of the *manda*-numbers] of Mercury and Venus multiplied by 2 (*dvi*) and divided by 5 (*āśuga*), that applied to the previously stated is the *manda*-velocity.

Commentary: The *manda-gatiphala*s of the planets Δ_v^M as presented in this verse, rely on the *manda*-numbers listed in verses 7 and 8, which were listed for every 15 degrees of arc. For two consecutive *manda*-numbers m_i and m_{i+1}, the *gatiphala*s Δ_v^M for each planet are as following:

Saturn:

$$\Delta_v^M = \frac{m_{i+1} - m_i}{75}$$

Mars:

$$\Delta_v^M = \frac{m_{i+1} - m_i}{5}$$

Jupiter:

$$\Delta_v^M = \frac{m_{i+1} - m_i}{30}$$

Mercury:

$$\Delta_v^M = 2 \cdot \frac{m_{i+1} - m_i}{5}$$

Venus:

$$\Delta_v^M = 2 \cdot \frac{m_{i+1} - m_i}{5}$$

4.8 Verse 3.12: The *śīghra*-correction of motions of planets

In this verse, Gaṇeśa states the *śīghra*-correction to be applied to the *manda*-correction motions to find true motions of the planets.

भौमाच्चलाङ्कविवरं शरहृत् स्वबाणां-
शाढ्यं त्रिहृत् कृतहृतं द्विगुणाक्षभक्तम् ।
तद्धीनयुक् क्षयचये तु मृदुस्फुटा स्यात्
स्पष्टाथ चेद्बहु ऋणात् पतिता तु वक्रा ॥ १२ ॥ ॥ वसन्ततिलका ॥

bhaumāccalāṅkavivaraṃ śarahṛt svabāṇāṃ-
śāḍhyaṃ trihṛt kṛtahṛtaṃ dviguṇākṣabhaktam |
taddhīnayuk kṣayacaye tu mṛdusphuṭā syāt
spaṣṭātha ced bahu ṛṇāt patitā tu vakrā || 12 || || *vasantatilakā* ||

The difference of *śīghra*-numbers [of the planets starting] from Mars [are respectively] divided by 5 (*śara*), increased by its own fifth part (*bāṇa-aṃśa*), divided by 3 (*tri*), divided by 4 (*kṛta*), divided by 5 (*akṣa*)and multiplied by 2 (*dvi*). That [scaled correction] is added or subtracted when positive or negative [to the *manda*-corrected velocity]. The *manda*-corrected velocity is then the true [velocity]. Now, if, from the largeness of the subtracted [correction] [the *manda*-corrected velocity] falls, then [that is] the retrograde [motion].

Commentary: Just as he did in the previous verse, Gaṇeśa presents the formulae for computing the *śīghra-gatiphala* of each planet Δ_v^S as the difference of successive *śīghra*-numbers previously presented, $s_{i+1} - s_i$, multiplied by a scale factor.
Mars:

$$\Delta_v^S = \frac{s_{i+1} - s_i}{5} \tag{4.1}$$

Mercury:

$$\Delta_v^S = (s_{i+1} - s_i) + \frac{s_{i+1} - s_i}{5} \tag{4.2}$$

Jupiter:

$$\Delta_v^S = \frac{s_{i+1} - s_i}{3} \tag{4.3}$$

Venus:

$$\Delta_v^S = \frac{s_{i+1} - s_i}{4} \tag{4.4}$$

Saturn:

$$\Delta_v^S = 2 \cdot \frac{s_{i+1} - s_i}{5} \tag{4.5}$$

Mallāri explains these scale factors in his commentary on this verse:

अत्रोपपत्तिर्गतिमन्दफलवत् । अत्र शीघ्रफलान्तरं गतेः शीघ्रफलं तत्रानुपातः । यदि पञ्चदशभागकलाप्रमाणेन ९०० इदं शीघ्राङ्कान्तरं तदा शीघ्रकेन्द्रगतिकलाप्रमाणेन किमिति । ततः शीघ्राङ्कानां दशगुणितत्वात् तद्दशभिर्भज्यं कलार्थं च षष्ट्या गुण्यम् । एवं शीघ्राङ्कान्तरस्य हरघातो हरः ९००० । षष्टि । ६० र्गुणः । गुणहरौ गुणेनापवर्त्य जातो हरः १५० । अस्य केन्द्रगतिगुणोऽस्ति । अत्र भौमगुरुशुक्राणां केन्द्रगतिभिराभिः २८ । ५४ । ३७ सार्धशते १५० हरे भक्ते जाता हराः । ५ । ३ । ४ बुधकेन्द्रगतिर्गुणः १८६ अत्र गुणहरौ त्रिंशतापवर्तीतौ जातो गुणः ६ । हरः ५ । यो राशिः षड्भि ६ र्गुण्यते पञ्चभि ५ र्भज्यते स स्वबाणांशाढ्य एव भवति । तथा शनेः केन्द्रगतिः ५७ । अत्र गुणहरौ गुणार्धेनापवर्त्त्य जातो गुणः २ । हरः ५ अतो द्विहताक्षभक्तं शीघ्राङ्कान्तरं शनेर्गतिफलं स्यादित्युपपन्नम् । एवमेतद्गतेः शीघ्रफलं मन्दस्पष्टगतौ देयं स्पष्टा स्यादेव । तत्र धनर्णोपपत्तिः । अङ्कान्तरेऽग्रे चेत् क्षयस्तदा ग्रहे स्वल्पफलत्वाद्गतिरपि न्यूना । अग्रे चेद्वृद्धिस्तदा ग्रहे फलाधिकत्वात् स्पष्टगतिरधिका । अतः क्षयर्द्धी ऋणधनसंज्ञोक्ता । चेत् फलं मन्दस्पष्टगर्ते न शुध्यति तदा विपरीतशोधनेन विपरीतगतिर्वक्रा गतिर्भवतीत्युपपन्नम् वक्रत्ववासनामग्रे सविस्तरां वक्ष्यामः ।

Here is the explanation similar to the *manda*-[velocity]-correction [explanation]. Here, the difference of the *śīghra*-correction is the *śīghra*-correction of velocity; (since?) there is the proportion. If this *śīghra*-number difference [corresponds] to the amount in arcminutes of 15 degrees, 900, then what corresponds to the measure in arcminutes of the *śīghra*-anomaly velocity? Therefore, from the decaplicity of the *śīghra*-numbers that [unknown quality of the proportion] is divided by 10 and multiplied by 60 for the sake of [converting to] arcminutes. In this way, the product of the *śīghra*-number difference [with] the divisor (10) is 9000. 60 is the multiplier. Reduce/simplify the multiplier and the divisor by the multiplier is 150 the divisor [of the *śīghra*-number difference]. The anomaly velocity is the multiplier of it [the *śīghra*-number difference]. Here, when the divisor 150 is divided by the anomaly velocities of Mars, Jupiter, and Venus [respectively] 28, 54, 37, the divisors 5, 3, 4 are [respectively] produced. The anomaly velocity of Mercury is the multiplier 186, here the multiplier and divisor are reduce/simplified by [the factor] 30, and the multiplier is 6. Divisor 5. Whatever quantity is multiplied by 6 and divided by 5 [can also be stated to be] increased by its own fifth part. Then, the anomaly velocity of Saturn is 57. Here, reduce/simplifying the multiplier and divisor, by half the multiplier, the multiplier 2 is produced. The divisor is 5. So, the difference of *śīghra*-numbers multiplied by 2 and divided by 5 is the velocity correction of Saturn. In this way, *śīghra*-correction of the velocity to be applied to the *manda*-corrected velocity, is the true [velocity]. Then the explanation of positive and negative. When the difference of numbers , then [the *śīghra*-correction] is subtracted because of the nature of being a small-correction the velocity is also small. Ahead, increases, then the from the largeness of the correction the true velocity is increased. So, the subtractive or additive is defined to be the positive or negative. If the correction is not subtracted from the *manda*-corrected velocity, then by the reversed subtraction the reversed motion [meaning] retrograde motion as described. The knowledge of the retrogradation we will extensively explain later.

The difference of the *śīghra*-correction is the *śīghra*-correction of velocity.

Ultimately, this explanation relies on the idea that velocity can be expressed as the change in position over an interval time. Mallāri notes that the change in consecutive *śīghra*-corrections over the difference in arc of two consecutive *śīghra*-corrections (which is 15 degrees, or 900 arcminutes) is proportional to the velocity *śīghra*-correction over the velocity of the *śīghra*-anomaly. The proportion as stated by Mallāri of the *śīghra*-number difference $s_{n+1} - s_n$, the *śīghra*-anomaly velocity v_{κ_S}, and the *śīghra*-velocity correction Δ_v^S is as follows:

$$\frac{60\ (s_{n+1} - s_n)\,/10}{900'} = \frac{\Delta_v^{S\,(\prime)}}{v_{\kappa_S}{}^{(\prime)}}$$

Since Gaṇeśa has stated that the *śīghra*-corrections have been multiplied by ten to be the *śīghra*-numbers he presented in the first five verses of this chapter, we see that the *śīghra*-number difference should be divided by 10 to yield the *śīghra*-corrections.

Mallāri then rearranges this equation so all multiplied coefficients are on one side of the equation, as follows:

$$\frac{60 \cdot (s_{n+1} - s_n)}{9000'} = \frac{\Delta_v^S}{v_{\kappa_S}}$$

The next step as described is to reduce the fraction to produce:

$$\frac{(s_{n+1} - s_n)}{150'} = \frac{\Delta_v^S}{v_{\kappa_S}}.$$

The last step involves rewriting this equation to isolate the unknown *śīghra*-velocity correction:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150'} \cdot v_{\kappa_S}$$

which is just an interpolation between first differences of successive *śīghra*-numbers, scaled to the current *śīghra*-anomaly velocity.

Recall that the *śīghra*-anomaly is taken to be the difference of the *manda*-corrected mean longitude of the planet and the longitude of the *śīghra*-apogee. For the inner planets Mercury and Venus, the mean longitude of the sun coincides with the *manda*-corrected mean longitude of the planet. For the outer planets, the *śīghra*-apogee is taken to be the mean sun.

Gaṇeśa presents the *śīghra*-anomaly velocities of Mercury (186') and Venus (37') when listing mean daily motions in the first chapter. The *śīghra*-anomaly velocities of each of the outer planets is the following:

$$v_{\kappa_S} = \bar{v}_{A_S} - \bar{v}$$

$$v_{\kappa_S} \text{ of Mars} = 59'8'' - 31'26'' = 27'42''$$

which Mallāri takes to be 28'. Similarly, we see:

$$v_{\kappa_S} \text{ of Jupiter} = 59'8'' - 5'0'' = 54'8''$$

which Mallāri takes to be 54', and

$$v_{\kappa_S} \text{ of Saturn} = 59'8'' - 2'0'' = 57'8''$$

which Mallāri takes to be 57'. Note that we have supplied mean daily motions in their rounded state as was presented by Gaṇeśa in Chapter 1, so it is likely that Mallāri is using the mean daily motions of the planets in their original unrounded state.

Now, if we substitute these values for *śīghra*-anomaly velocities in our *śīghra*-velocity correction equation for Mars, we get:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150} \cdot 28$$

and if we round 28 to 30, this fraction simplifies to:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{5}$$

which is the same as equation 4.1.

For Jupiter, we have:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150} \cdot 54$$

and if we round 54 to 50, this fraction simplifies to:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{3}$$

which is the same as equation 4.3.

For Venus, we have:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150} \cdot 37$$

and with some rounding we get:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{4}$$

which is the same as equation 4.4.

For Mercury, we have:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150} \cdot 186$$

and with some rounding we get:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{5} \cdot 6$$

which is the same as the difference of successive *śīghra*-numbers being increased by its own fifth part, as in equation 4.2.

For Saturn, we have:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{150} \cdot 57$$

and if we round 57 to 60, then reduce the fraction by a factor of 30:

$$\Delta_v^S = \frac{(s_{n+1} - s_n)}{5} \cdot 2$$

which is the same as equation 4.5.

4.9 Verse 3.13: Additional correction to the *śīghra*-corrections of Mars and Venus

In this verse, Gaṇeśa states an additional correction to the *śīghra*-corrections of Mars and Venus to be applied in the interval from 165-180 degrees of *śīghra*-anomaly (and consequently in the interval between 180-195 degrees due to symmetry about 180 degrees).

> शुक्रारयोश्चलभवोऽन्त्यगतो यदाङ्कः
> शेषांशकाश्च पतिताः पृथगक्षभूम्यः ।
> येऽल्पा भृगोस्त्रिविहृता असृजोऽक्षभक्ता
> देयाः स्वशीघ्रफलवत् स्फुटयोः स्फुटौ तौ ॥ १३ ॥ ॥ वसन्ततिलका ॥
>
> *śukrārayoś calabhavo'ntyagato yadāṅkaḥ*
> *śeṣāṃśakāś ca patitāḥ pṛthagakṣabhūmyaḥ |*
> *ye'lpā bhṛgos trivihṛtā asṛjo'kṣabhaktā*
> *deyāḥ svaśīghraphalavat sphuṭayoḥ sphuṭau tau || 13 ||* || *vasantatilakā* ||
>
> When the *śīghra*-number which was the last elapsed *śīghra*-entity of Mars and Venus and the remaining degrees fallen/subtracted from the separate [divisions] by 15 (*akṣa-bhūmi*) [respectively], whatever are small of Venus divided by 3 (*tri*) [and] of Mars divided by 5 (*akṣa*), are to be applied in a manner similar to its own *śīghra*-correction [calculation] of those two corrected [planets]; [now] those two [are] true.

Commentary: Gaṇeśa uses this one verse to address an additional correction to be made to the previous *śīghra*-corrections. However, this correction is only applied to the *śīghra*-corrections of Venus and Mars. Since the *śīghra*-correction differences of Venus and Mars are so large in this last interval (as seen in table 4.1), this additional correction has to be applied. The *Karaṇakutūhala* also states an additional orbital correction, but is only stated for Mars. In verse 2.14, the *manda*-apogee of Mars is adjusted by *śīghra*-anomaly, however there is no mention of Venus getting any extra correction.

Gaṇeśa presents the procedures for computing an additional *śīghra*-correction to Mars and Venus s^*, based on the *śīghra*-anomaly κ.

First, divide the *śīghra*-anomaly by 15. The result of this division yields an integer quotient q, along with a remainder r:

$$\frac{\kappa}{15} = \mathrm{q} + \mathrm{r}$$

Then, we check which quantity is smaller: the remainder itself, or the remainder subtracted from 15. Choosing the smaller entity we get the equations for this additional correction:
Mars:

$$s^* = \frac{\text{smaller}(r, 15 - r)}{5}$$

and for Venus:

$$s^* = \frac{\text{smaller}(r, 15 - r)}{3}$$

and this correction is to be either added or subtracted to the longitude of the *śīghra*-corrected planet. If the *śīghra*-correction was added to the *manda*-corrected planet, then this additional correction should also be added and vice versa.

4.10 Verse 3.14: The new *śīghra*-velocity correction of Mars, Mercury, and Venus

After describing the additional *śīghra*-correction to the longitudes of Mars and Venus in the last verse, Gaṇeśa states in this verse an additional correction to be made to the *śīghra*-corrected motions of Mars, Mercury, and Venus when the *śīghra*-anomaly is in that same final interval between 165-180 degrees.

कुजबुधभृगुजानां चेच्चलाङ्कोऽन्तिमः स्या-
द्दशहतपरिशेषांशा नगाद्र्यग्निभक्ताः ।
फलमिषुदहनैर्युक् सप्तगोभिस्त्रिबाणै
र्भवति गतिफलं तत् स्यात् तदा नैव पूर्वम् ॥ १४ ॥ ॥ मालिनी ॥

kujabudhabhṛgujānāṃ ceccalāṅko'ntimaḥ syād
daśahatapariśeṣāṃśā nagādryagnibhaktāḥ |
phalam iṣudahanair yuk saptagobhis tribāṇair
bhavati gatiphalaṃ tat syāt tadā naiva pūrvam || 14 || || *mālinī* ||

If the *śīghra*-number of Mars, Mercury, or Venus is final [in the final interval], the degrees of remainder [from division of the anomaly angle by 15], multiplied by 10, are divided by 7 (*naga*), 7 (*adri*), and 3 (*agni*) [respectively]. The result increased by 35 (*iṣu-dahana*), 97 (*sapta-go*), [and] 53 (*tri-bāṇa*) [respectively] is the [*śīghra*-]velocity correction; not the previous [*śīghra*-velocity correction].

Commentary: Gaṇeśa now states a corresponding additional *śīghra*-velocity correction when the *śīghra*-anomaly is in the last interval.

The formulas are provided below for the *śīghra*-correction to the velocity, Δ_v^{S*}, and r the remainders of the *śīghra*-anomaly divided by 15:

Mars:

$$\Delta_v^{S*} = 35 + \frac{10 \cdot r}{7}$$

Mercury:

$$\Delta_v^{S*} = 97 + \frac{10 \cdot r}{7}$$

Venus:

$$\Delta_v^{S*} = 53 + \frac{10 \cdot r}{3}$$

We postulate that Mercury is included because of its difference in *śīghra*-correction in that last interval, while not as drastic as Venus and Mars, may have been considered by Gaṇeśa to not be trivial. We defer concrete conclusions and the derivation of these formulas to future publications.

Planet	**Gaṇeśa**	Āryabhaṭa I	Brahmagupta, Lalla, Śrīpati, and Bhāskara II	*Sūryasiddhānta*	Āryabhaṭa II	Vaṭeśvara
Mars	163	164	163	164	163	163
Mercury	145	146	145	144	145	145
Jupiter	125	130	125	130	125	126
Venus	167	165	165	163	166	165
Saturn	113	116	113	115	113	113

Table 4.4: The *śīghra*-anomaly degrees of arc corresponding to the first station of each planet when the planets turn retrograde, as stated by different text authors in their works *Grahalāghava* [Dvi25, v. 3.15, pp.108-109], *Khaṇḍakhādyaka* [Cha70, v. 1.2.8–17, pp.49–75], *Brāhmasphuṭasiddhānta* [Dvi02, v. 2.48–49, pp.41–42], *Śiṣyadhīvṛddhidatantra* [Cha81a, p. v. 3.20], *Siddhāntaśekhara* [Mis47, v. 3.58, p.191], *Siddhānta-śiromaṇi* [Cha81b, p. v. 1.2.41], *Sūryasiddhānta* [SW81, v. 2.53–54, p.22], *Mahāsiddhānta* [Dvi10, v. 3.31, pp.63], *Karaṇaprakāśa* [Dvi99, v.3.8, p.26], and *Vaṭeśvarasiddhānta* [Shu86, v. 2.4.9, p.219] (cf. Table 15 of [Shu86, p.220].)

4.11 Verse 3.15: The stationary points of each planet

In this verse, Gaṇeśa states the *śīghra*-anomaly values corresponding to the starts of retrograde and direct motion (first and second station respectively) for each of the planets.

> त्रिनृपैः शरजिष्णुभिः शरार्कैर्
> नगभूपैस्त्रिभवैः क्रमात् कुजाद्याः ।
> चलकेन्द्रलवैः प्रयान्ति वक्रं
> भगणात् तैः पतितैर्व्रजन्ति मार्गम् ॥ १५ ॥ ॥ मालभारिणी ॥

> *trinṛpaiḥ śarajiṣṇubhiḥ śarārkair*
> *nagabhūpais tribhavaiḥ kramāt kujādyāḥ* |
> *calakendralavaiḥ prayānti vakraṃ*
> *bhagaṇāt taiḥ patitair vrajanti mārgam* || 15 || || *mālabhāriṇī* ||

> [At points commensurate] with 163 (*tri-nṛpa*), 145 (*śara-jiṣṇu*), 125 (*śara-arka*), 167 (*naga-bhūpa*), 113 (*tri-bhava*) degrees of *śīghra*-anomaly respectively the [planets] starting with Mars go backwards. [The planets] go forward [at points commensurate] with those [*śīghra*-anomalies] subtracted from twelve [signs].

Commentary: When planets seem to be undergoing a change in motion from retrograde to direct or vice versa, these time periods are called stations since the planet appears to be stationary to an observer on Earth. In this verse, Gaṇeśa states the stationary points for each planet in the order Mars, Mercury, Jupiter, Venus, and Saturn as: 163, 145, 125, 167, and 113 for first station, and their explements 197, 215, 235, 193, and 247 for second station. These stationary points values are also provided in earlier works, as shown in Table 4.4 for comparison.

Reconstructing Gaṇeśa's *śīghra*-anomaly values for stationary points can be done by first considering the true planetary longitude equation λ in terms of the *manda*-corrected longitude λ_M and the *śīghra*-correction σ:

$$\lambda = \lambda_M + \sigma.$$

Since we are looking for stationary points, this is where true velocity is zero. So,we differentiate to get the equation for the true velocity of a planet v given the *manda*-corrected velocity v_M in terms of the anomaly velocity v_{κ_S} and the change in *śīghra*-correction with respect to change in anomaly $\frac{d(\sigma)}{d\kappa_S}$:

$$v = v_M + v_{\kappa_S} \cdot \frac{d(\sigma)}{d\kappa_S}.$$

When we substitute that the *manda*-corrected velocity is $v_M = v_{A_S} - v_{\kappa_S}$, and that $\frac{d(\sigma)}{d\kappa_S} = 1 - \frac{R\cos\sigma}{H_S}$ we get:

$$v = v_{A_S} - v_{\kappa_S} \cdot \frac{R\cos\sigma}{H_S}$$

Mallāri's commentary of this verse explains the conditions for stationary points as they relate to velocities:

> अत्रोपपत्तिः । ग्रहस्य वक्रारम्भे मार्गारम्भे च गतिः शून्यम् ० । तच्च यदोच्चगतिसमा केन्द्रगतिस्तदैव । अत्र ग्रहाणां शीघ्रोच्चगतिर्ज्ञातैवास्ति तया स्पष्टकेन्द्रगतितुल्यया भवितव्यम् ।
>
> *atropapattiḥ | grahasya vakrārambhe mārgārambhe ca gatiḥ śūnyam 0 | tacca yadoccagatisamā kendragatis tadaiva | atra grahāṇāṃ śīghroccagatir jñātaivāsti tayā spaṣṭakendragatitulyayā bhavitavyam |*
>
> Here is the explanation. The [true] velocity of the planet at the beginning of retrogradation and at the beginning of direct motion is 0. And then that [is the case] when the anomaly velocity [modified by the *śīghra*-velocity correction] is equivalent to the apogee velocity. Here [at this point], the *śīghra*-apogee velocity of the planets is given; [the stationary point] is to occur because of it [the *śīghra*-apogee velocity] being equivalent to the corrected/modified anomaly velocity.

The stationary points occur when the true instantaneous velocity of the planet is 0, which occurs whenever the apogee velocity is equivalent to the planet's anomaly velocity.

Thus we have:

$$v_{A_S} = v_{\kappa_S} \cdot \frac{R\cos\sigma}{H_S}$$

and we rearrange this equation and expand both $\cos\sigma$ and H_S as:

$$\frac{v_{A_S}}{v_{\kappa_S}} = \frac{R^2 + r_S R\cos\kappa_S}{R^2 + r_S^2 + 2r_S R\cos\kappa_S}$$

and with some algebraic manipulation:

$$\frac{v_{A_S}}{v_{\kappa_S}} \cdot (R^2 + r_S^2) + \frac{v_{A_S}}{v_{\kappa_S}} \cdot (2r_S R\cos\kappa_S) - R^2 = r_S R\cos\kappa_S$$

which again can be rewritten:

$$\frac{v_{A_S}}{v_{\kappa_S}} \cdot (R^2 + r_S^2) - R^2 = \cos\kappa_S \cdot (Rr_S - 2Rr_S \cdot \frac{v_{A_S}}{v_{\kappa_S}})$$

with cosine isolated:

$$\cos\kappa_S = \frac{\frac{v_{A_S}}{v_{\kappa_S}} \cdot (R^2 + r_S^2) - R^2}{Rr_S - 2Rr_S \cdot \frac{v_{A_S}}{v_{\kappa_S}}}$$

and we finally take the arccosine of both sides of the equation to find the angle of anomaly in radians (which we can then convert to degrees) at which we find the stationary point at the beginning of retrogradation:

$$\kappa_S = \arccos\left(\frac{\frac{v_{A_S}}{v_{\kappa_S}} \cdot (R^2 + r_S^2) - R^2}{Rr_S - 2Rr_S \cdot \frac{v_{A_S}}{v_{\kappa_S}}}\right)$$

To find the stationary points, again, we would set the true velocity of the planet equal to zero in order to arrive at the above relation, and substitute in the values for the *śīghra*-apogee velocity, *śīghra*-anomaly velocity, and *śīghra*-epicycle radius. We state this process generally here, leaving the computations for future work.

4.12 Verse 3.16: Rising and setting *śīghra*-anomalies for Mars, Jupiter, and Saturn

In this verse, Gaṇeśa states the degrees of *śīghra*-anomaly corresponding to the eastern heliacal risings and western heliacal settings of Mars, Jupiter, and Saturn.

क्षितिजोऽष्टयमैरुदेति पूर्वे
गुरुरिन्द्रै रविजस्तु सप्तचन्द्रैः ।
स्वस्वोदयभागसंविहीनैर्
भगणांशैर् ३६० अपरत्र यान्ति चास्तम् ॥ १६ ॥ ॥ मालभारिणी[3] ॥

kṣitijo'ṣṭayamair udeti pūrve
gurur indrai ravijas tu saptacandraiḥ |
svasvodayabhāgasaṃvihīnair
bhagaṇāṃśair 360 aparatra yānti cāstam || 16 || || *mālabhāriṇī* ||

In the east, Mars rises with [*śīghra*-anomaly 28 [degrees of *śīghra*-anomaly], Jupiter [rises] with 14 [degrees] and/but Saturn with 17 [degrees]. And in the west, [the planets] set with 360 degrees diminished by their own rising degrees [of *śīghra*-anomaly].

Commentary: In this verse, Gaṇeśa states the degrees of *śīghra*-anomaly corresponding to the visibility of the superior planets Mars, Jupiter, and Saturn as 28, 14, 17 degrees respectively. Their heliacal settings in the west then occur at explementary degrees of *śīghra*-anomaly: 332, 346, 343 respectively. The start of visibility begins with heliacal rising in the East just before sunrise, and the planet remains visible until heliacal setting in the West just after sunset. The superior planets are far enough from the Sun that they undergo just one period of visibility and invisibility unlike the inferior planets. As visibility of planets was an important feature of observational astronomy, several previous text authors have listed out measures of *śīghra*-anomaly corresponding to the heliacal eastern risings and western settings of the superior planets, as shown in Table 4.5.

[3] For the *mālabhāriṇī* meter, the first two syllables are supposed to short followed by a long syllable. Here, we only have two long syllables. The rest of the line follows the scheme of *mālabhāriṇī* well. Checking the text to see if the verse was intended to be स्वस्वो- we see that both editions retain this reading as well as the glosses in Mallāri and Viśvanātha's commentaries.

Planet	**Gaṇeśa**	Āryabhaṭa I, Lalla, and Brahmadeva	Brahmagupta and Bhāskara II	Āryabhaṭa II	Śrīpati	Vaṭeśvara
Mars	28	28	28	28	28	28
Jupiter	14	14	14	14	14	13
Saturn	17	20	17	17;30	17	17

Table 4.5: The *śīghra*-anomaly degrees of arc corresponding to the eastern heliacal rising of each planet, as stated by different text authors in their works *Grahalāghava* [Dvi25, v. 3.15, pp.108-109], *Khaṇḍakhādyaka* [Cha70, v. 1.2.8–17, pp.49-75], *Brāhmasphuṭasiddhānta* [Dvi02, v. 2.48–49, pp.41–42], *Śiṣyadhīvṛddhidatantra* [Cha81a, p. v. 3.20], *Siddhāntaśekhara* [Mis47, v. 3.58, p.191], *Siddhāntaśiromaṇi* [Cha81b, p. v. 1.2.41], *Sūryasiddhānta* [SW81, v. 2.53–54, p.22], *Mahāsiddhānta* [Dvi10, v. 3.31, pp.63], *Karaṇaprakāśa* [Dvi99, v. 3.10, p. 27], and *Vaṭeśvarasiddhānta* [Shu86, v. 2.4.9, p.219] (cf. Table 15 of [Shu86, p.220].)

4.13 Verse 3.17: Rising and setting *śīghra*-anomalies for Mercury and Venus

As he did for the superior planets in the previous verse, Gaṇeśa states in this verse the degrees of *śīghra*-anomaly to denote the beginnings of periods of visibility and invisibility for the inferior planets.

खशरैश्च जिनैः परे ज्ञभृग्वो
रुदयोऽस्तोऽक्षदिनैर्नगाद्रिभूभिः ।
उदयोऽक्षनखैस्त्र्यहीन्दुभि प्रा-
गस्तो दिग्दहनैश्च षड्सुरैः स्यात् ॥ १७ ॥ ॥ मालभारिणी[4] ॥

khaśaraiś ca jinaiḥ pare jñabhṛgvor
udayo'sto'kṣadinair nagādribhūbhiḥ |
udayo'kṣanakhais tryahīndubhi prāg
asto digdahanaiś ca ṣaṭsuraiḥ syāt || 17 || || *mālabhāriṇī* ||

In the west, the rising of Mercury and Venus is with [a synodic elongation of] fifty (*kha-śara*) and twenty-four (*jina*) [degrees of *śīghra*-anomaly], [and] the setting is with one hundred and fifty-five [and] one hundred and seventy-seven [degrees of *śīghra*-anomaly respectively]. In the east, the rising is with two hundred and five [and] one hundred and eighty-three [degrees of *śīghra*-anomaly] and the setting is with three hundred and ten and three hundred and thirty six [degrees of *śīghra*-anomaly respectively].

Commentary: The degrees of elongation corresponding to the western rising, to western setting, and eastern rising to eastern setting are all presented in Table 4.6. Due to the proximity of Mercury and Venus to the Sun, the two undergo two periods of visibility/invisibility unlike the superior planets.

4.14 Verse 3.18: Number of days elapsed/coming for synodic phenomena

वक्रोदयादिगदितांशकतोऽधिकाल्पाः
केन्द्रांशकाः क्षितिसुताद् द्विगुणास्त्रिभक्ताः ।

[4]In the last *pāda*, the first two syllables should be short instead of the singular long syllable to fit the scheme of *mālabhāriṇī*.

Planet	Western Rising	Western Setting	Eastern Rising	Eastern Setting
Mercury	50	155	205	310
Venus	24	177	183	336

Table 4.6: Gaṇeśa states degrees of elongation (or degrees of *śīghra*-anomaly) when the eastern rising and setting and western rising and setting occur for Mercury and Venus.

Planet	**Gaṇeśa**	and Āryabhaṭa I	Brahmagupta, Śrīpati, and Bhāskara II	Lalla and Brahmadeva	Āryabhaṭa II	Vaṭeśvara
Mercury	50	51	50	51	49	49
Venus	24	24	24	23	20	24

Table 4.7: The *śīghra*-anomaly degrees of arc corresponding to the western heliacal rising of Mercury and Venus, as stated by different text authors in their works *Grahalāghava* [Dvi25, v. 3.15, pp.108-109], *Khaṇḍakhādyaka* [Cha70, v. 1.2.8–17, pp.55–56], *Brāhmasphuṭasiddhānta* [Dvi02, v. 2.48–49, pp.41–42], *Śiṣyadhīvṛddhidatantra* [Cha81a, pp. v. 3.20], *Siddhāntaśekhara* [Mis47, v. 3.58, p.191], *Siddhāntaśiromaṇi* [Cha81b, pp. v. 1.2.41], *Sūryasiddhānta* [SW81, v. 2.53–54, p.22], *Mahāsiddhānta* [Dvi10, v. 3.31, p.63], *Karaṇaprakāśa* [Dvi99, v. 3.11, p.27], and *Vaṭeśvarasiddhānta* [Shu86, v. 2.4.9, p.219] (cf. Table 15 of [Shu86, p.220].)

Planet	**Gaṇeśa**	Āryabhaṭa I, Lalla, and Sumati	Brahmagupta and Bhāskara II	Āryabhaṭa II	Śrīpati	Vaṭeśvara
Mercury	205	205	205	205	205	203
Venus	183	183	183	182;30	183	183

Table 4.8: The *śīghra*-anomaly degrees of arc corresponding to the eastern heliacal rising of Mercury and Venus, as stated by different text authors in their works *Grahalāghava* [Dvi25, v. 3.15, pp.108-109], *Khaṇḍakhādyaka* [Cha70, v. 1.2.8–17, pp.49-75], *Brāhmasphuṭasiddhānta* [Dvi02, v. 2.48–49, pp.41–42], *Śiṣyadhīvṛddhidatantra* [Cha81a, p. v. 3.20], *Siddhāntaśekhara* [Mis47, v. 3.58, p.191], *Siddhāntaśiromaṇi* [Cha81b, p. v. 1.2.41], *Sūryasiddhānta* [SW81, v. 2.53–54, p.22], *Mahāsiddhānta* [Dvi10, v. 3.31, pp.63], *Karaṇaprakāśa* [Dvi99, v. 3.10, p.27], and *Vaṭeśvarasiddhānta* [Shu86, v. 2.4.9, p.219] (cf. Table 15 of [Shu86, p.220].)

साङ्कांशका दशहताङ्गहृताः कुभक्ता
वक्राद्यमाप्तदिवसैः क्रमशो गतैष्यम् ॥ १८ ॥ ॥ वसन्ततिलका ॥

vakrodayādigaditāṃśakato'dhikālpāḥ
kendrāṃśakāḥ kṣitisutād dviguṇās tribhaktāḥ |
sāṅkāṃśakā daśahatāṅgahṛtāḥ kubhaktā
vakrādyam āptadivasaiḥ kramaśo gataiṣyam || 18 || || *vasantatilakā* ||

The degrees of anomaly greater or less than the stated degrees of [the synodic phenomena of] retrogradation, rising, and so on [of the planets starting] from Mars are multiplied by two (*dvi*), divided by 3 (*tri*), joined with a ninth part [of itself] (*sa-aṅka-aṃśa*), multiplied by 10 (*daśa*) and divided by six (*aṅga*), [and] divided by one (*ku*), respectively, is the elapsed or coming [time interval], with the result in days [of synodic phenomena] beginning with retrogradation.

Commentary: This verse provides a formula for calculating the number of days until or since a particular synodic phenomenon. When the degrees of *śīghra*-anomaly of a planet align with the values stated in the previous verse, then the synodic phenomenon occurs. But with each passing day, the angle of *śīghra*-anomaly changes and we can now use the *śīghra*-anomaly velocity to find out how many days removed any given day is from a particular phenomenon.

We know that the formula for displacement d in terms of a given velocity v and time t is expressed as:

$$v = \frac{d}{t}$$

which can then be rearranged to find time, as:

$$t = \frac{d}{v}$$

On any arbitrary day, we have a difference in *śīghra*-anomaly angle between the current planet's *śīghra*-anomaly angle and the degrees of anomaly stated in previous verses for synodic phenomena. We can use this difference as our displacement in the above formula, along with the given *śīghra*-anomaly velocity, to find the time in days until or since the phenomenon.

Since the velocity is given in arcminutes, but the displacement is in degrees, we need to multiply our displacement by 60 to convert degrees to arcminutes, like so:

$$t = \frac{d \cdot 60}{v}$$

Name of Planet	Scale Factor	*śīghra*-anomaly velocity
Mars	2	27
Mercury	1/3	186
Jupiter	10/9	54
Venus	10/6	37
Saturn	1	57

Table 4.9: Table of scale factors multiplying degrees of difference between stated degrees of anomaly for synodic phenomena and the current degrees of anomaly for synodic elongation of the planets.

We can see that Gaṇeśa has slightly approximated the *śīghra*-anomaly velocity values in order to get nicer scale factors.

Planet	Western Rising	Retrograde Motion	Western Setting	Eastern Rising	Direct Motion	Eastern Setting
Mercury (days)	32	32	3	16	3	32
Venus (months)	2	8	0.75	0.25	0.75	8

Table 4.10: Gaṇeśa states the beginning of each synodic phenomenon (heliacal rising, retrograde motion, direction motion, and heliacal setting) after the end of the prior phenomenon, for the two inferior planets. For instance, the western rising of Mercury begins 32 days after its eastern setting.

4.15 Verse 3.19: The time intervals of synodic phenomena of Mercury and Venus

In this verse, Gaṇeśa states the beginnings of each synodic phenomenon of Mercury and Venus, which are the intervals for each phenomenon.

पूर्वास्तादुदयः परेऽनृजुगतिस्तोयास्तमैन्द्र्युद्गमो
मार्गोऽस्तोऽत्र च दन्तदन्तदहनाष्ट्याज्याशदन्तैर्दिनैः ।
चान्द्रेस्तत्परतत्परं त्वथ भृगोस्तद्वद्द्विमास्यात्ततोऽ
ष्टाभिर्व्यङ्घ्रिभुवाङ्घ्रिणा विचरणैकेनाष्टमासैः क्रमात् ॥ १९ ॥ ॥ शार्दूलविक्रीडित ॥

pūrvāstād udayaḥ pare'nṛjugatis toyāstamaindryudgamo
mārgo'sto'tra ca dantadantadahanāṣṭyājyāśadantair dinaiḥ |
cāndres tatparatatparaṃ tvatha bhṛgos tadvaddvimāsyāt tato'
ṣṭābhir vyaṅghribhuvāṅghriṇā vicaraṇaikenāṣṭamāsaiḥ kramāt || 19 || || *śārdūlavikrīḍita* ||

From the eastern setting, the rising in the west, the retrograde motion (first station), the western setting, eastern rising, direct motion (second station) and here the setting of Mercury [occur in a time interval commensurate with] 32 (*danta*), 32 (*danta*), three (*dahana*), 16 (*aṣṭi*), three (*ājyāśa*), 32 (*danta*) days one after another; but of Venus, similarly, after two months, then [the time interval commensurate] with eight (*aṣṭa*) [months], with one less quarter (*vi-aṅghri-bhū*), with one-quarter (*ṅghri*) [month], with one less quarter (*vi-caraṇaika*) [month], [and] with eight (*aṣṭa*) months respectively.

Commentary: These intervals of each synodic phenomenon for Mercury and Venus are presented in Table 4.10.

Mallāri comments about the method of calculating these intervals [Dvi25, lines 22–24, p.114]:

अत्रोपपत्तिः । पूर्वास्तशीघ्रकेन्द्रांशाः पश्चिमोदयशीघ्रकेन्द्रांशकेभ्यो यावदन्तरितास्तावदंशानां कलाः केन्द्रगतिभक्ता दिनानि स्युः । एवं वक्रमार्गादीनामपि तत्तत्केन्द्रान्तराद्दिनानि स्युरित्युपपन्नम् ।

atropapattiḥ | *pūrvāstaśīghrakendrāṃśāḥ paścimodayaśīghrakendrāṃśakebhyo yāvad antaritās tāvadaṃśānāṃ kalāḥ kendragatibhaktā dināni syuḥ* | *evaṃ vakramārgādīnām api tattatkendrāntarād dināni syur ityupapannam* |

Here is the explanation. Whatever the difference [of] the degrees of *śīghra*-anomaly of setting in the east from the degrees of *śīghra*-anomaly of rising in the west [and], the minutes of those

Planet	Rising	Retrograde Motion	Direct Motion	Setting
Mars	4	10	2	10
Jupiter	1	4.25	4	4.25
Saturn	1.25	3.5	4	3

Table 4.11: Gaṇeśa states the beginning of each synodic phenomenon (heliacal rising, retrograde motion, direction motion, and heliacal setting) after the end of the prior phenomenon, in months. For instance, the heliacal rising of Mars begins 4 months after its setting.

degrees [produced from the difference], multiplied by the anomaly velocity are the days. Thus, the days of retrogradation, direct motion, and so on are produced from each of the respective anomaly differences; thus is the demonstration.

4.16 Verse 3.20: The time intervals of synodic phenomena of Mars, Jupiter, and Saturn

Just as Gaṇeśa stated the time intervals for synodic phenomena of Mercury and Venus in the last verse, he does so in this last verse for the superior planets.

भौमस्यास्तादुदयकुटिलर्जुत्वमौढ्यं क्रमात् स्या-
न्मासैर्वेदैरथ दशमितैर्लोचनाभ्यां च दिग्भिः ।
जीवस्योर्व्या सचरणयुगैः सागरैः साङ्घ्रिवेदैः
साङ्घ्र्येकेन त्रियुगदहनैरर्धयुक्तैस्तथार्कः ॥ २० ॥ ॥ मन्दाक्रान्ता ॥

bhaumasyāstād udayakuṭilarjutvamauḍhyaṃ kramāt syān
māsair vedair atha daśamitair locanābhyāṃ ca digbhiḥ |
jīvasyorvyā sacaraṇayugaiḥ sāgaraiḥ sāṅghrivedaiḥ
sāṅghryekena triyugadahanair ardhayuktais tathārkeḥ || 20 || || *mandākrāntā* ||

From the setting of Mars, the rising, retrograde motion, direct motion, and setting occur respectively [in time intervals commensurate] with months: four (*veda*), now ten (*daśa*), two (*locana*), and ten (*dik*). Of Jupiter, with one (*urvī*), with four and a quarter (*sa-caraṇa-yuga*), with four (*sāgara*), [and] with four and a quarter (*sa-aṅghri-veda*) [months]. In that way, of Saturn, with one and a quarter (*sa-aṅghri-eka*), with one half plus three (*tri*), four (*yuga*), and three (*dahana*) [months].

Commentary: We present these time intervals of heliacal rising, retrograde motion, direct motion, and heliacal setting of the superior planets Mars, Jupiter, and Saturn in Table 4.11.

Chapter 5

Thesis Discussion

At the end of his *Grahalāghava*, after ten more chapters of content we must defer for future detailed study, Gaṇeśa reaffirms his pride in constructing a *karaṇa* work that eliminates all explicit use of trigonometric tables [Dvi25, lines 19-22, pg. 367]:

> पूर्वे प्रौढतराः क्वचित्किमपि यच्चक्रुर्धनुर्ज्ये विना
> ते तेनैव महातिगर्वकुभृदुच्छृङ्गेऽधिरोहन्ति हि ।
> सिद्धान्तोक्तमिहाखिलं लघुकृतं हित्वा धनुर्ज्ये मया
> तद्गर्वो मयि मास्तु किं न यदहं तच्छास्त्रता वृद्धधीः ॥
>
> *pūrve prauḍhatarāḥ kvacit kimapi yac cakrur dhanurjye vinā*
> *te tenaiva mahātigarvakubhṛducchṛṅge 'dhirohanti hi |*
> *siddhāntoktam ihākhilaṃ laghukṛtaṃ hitvā dhanurjye mayā*
> *tadgarvo mayi māstu kiṃ na yad ahaṃ tacchāstratā vṛddhadhīḥ ||*
>
> In the past, the most excellent scholars made some [procedures and texts] without sine and arc in some places; but they ascend the topmost peak of immense ego. Here the exposition of a *siddhānta* is [completely] abbreviated by me, omitting the arc and sine; [to] whatever [extent] I [am] intelligent, that [is] from the *śāstras*".

How far was Gaṇeśa's achievement in the *Grahalāghava* indebted to the great *jyotiṣa* tradition which certainly formed the core of his own learning? We can see in his work clear evidence of ideas and techniques derived from earlier authorities, but also many features where he appears to carve out a new path of his own. For instance, Gaṇeśa's mean longitudinal increment formulas resemble those of the *Karaṇakutūhala*, modified with his attempt at simplifying and reducing divisors wherever possible, and adhering to his own parameters. In the mean longitude computation process, Gaṇeśa divides the time elapsed since his epoch into 11-year *cakras* and redefines a *dhruva* to instead refer to an explement of mean longitudinal displacement of a planet per *cakra*. Why did he have *dhruvas* represent a subtractive longitudinal displacement? Were his motivations possibly *koṣṭhaka* driven with easier computations for smaller time divisions?

More generally:

1. What are the motivations underlying the development of approximation methods?
2. What was the role of theoretical exactitude in forming and evaluating approximation formulas? What was the role of empirical observation in adopting and modifying mathematical models that provide the theory?

3. How did the different *pakṣas* in their various textual forms influence the *Grahalāghava* and the formation of a new *pakṣa*?

Mallāri's commentary occasionally mentioned direct observation as a means of justifying corrections or approximations, as we saw in his discussion of approximating the *manda*-hypotenuse with the orbital radius R. Systematically analyzing the two commentaries of Mallāri and Viśvanātha, while used occasionally in this thesis as rationales and possible derivations for Gaṇeśa's formulas, could give us more insight into how demonstrations and examples and proofs were structured and the roles they served in exposition. What pedagogical methods do they use, and can their commentaries give us clues about Gaṇeśa's and possibly Keśava's methods as well? The two commentaries have two different exposition styles: Mallāri's commentary was primarily comprised of theoretical explanations and Viśvanātha's commentary has an ongoing worked-through example. Since these two commentaries were commonly transcribed and edited together with the *Grahalāghava*, what could be the rationale behind this intentional choice?

Our time spent investigating the first three chapters of the *Grahalāghava* has helped uncover intriguing lines of future inquiry to expand our knowledge of not only Gaṇeśa and his *Grahalāghava*, but also about *pakṣas* and scholarship, and the *karaṇa* tradition as a whole.

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