Holistic Modelling of Car Rental Sub-Problems

A thesis submitted in fulfilment of the requirements for the Degree of Master of Commerce

By

Sean Howson

College of Business and Law

University of Canterbury

2021

Abstract

The car rental fleet management literature from its inception has made bounds in identifying key sub-problems which are faced by car rental companies in operating sustainably in the industry such as pool segmentation, fleet size and mix, fleet deployment, fleet assignment, capacity allocation, and price discovery. Previous research has primarily relied on isolating sub-problems to provide solutions, and thus have been unable to contextualize global interrelatedness, which is a necessary step in the ‘call for realism’ outlined in Oliveira et al. (2017). In my thesis, I thus model up to 7 key sub-problems simultaneously using a Monte Carlo simulation within which the physical and financial dimensions are implemented via Statistical Activity Cost Analysis (SACA). Financial statements are translated from simulated data to display financial outcomes from operational movements which is necessary for informed risk management. The most advanced simulation version considered 133 vehicles, 7 vehicle groups, 3 stations, 27 trip types, rebalancing capability, limited cascading upgrades, seasonal demand variability, and reservation arrivals and price setting based upon real data. These factors demonstrated that the inclusion of a maintenance regime in my simulation extension (V7.1) encompass more realistic assumptions from a fleet and revenue management perspective.

Table of Contents

List of Tables

List of Figures

1 Introduction

My research is motivated by 2 aspects. Firstly, the academic literature is seminal on the problems within the car rental industry and lacks a holistic approach in addressing the key concepts which I address in my thesis. Oliveira et al. (2017) conducted a review article on the car rental literature, which summarised 26 relevant papers for the car rental problem. In these I have identified limitations in that the issues addressed are isolated, which is not representative of the complexity of the car rental industry in a realistic setting. Here, I thus address research directions which jointly deal with a number of car rental sub-problems. The integration of the car rental sub-problems in a holistic setting is therefore necessary to encapsulate the operational complexity of the different levels and time horizons of decision making in the car rental setting. Secondly, New Zealand (NZ) is reliant on tourism. It is of interest for ecological and economic sustainability reasons to optimise operational and financial aspects of the industry.

Due to these aspects, a holistic analysis approach was chosen to be implemented via a simulation study, which allows modelling complex systems. To give the reader a better understanding of the car rental context and its key concepts, Chapter 1.1 describes the car rental industry and the main factors a firm must consider with tactical and strategic decision making to operate sustainably. Chapter 1.2 delves further into my research methodology and how I model and analyse the complexity in and of the car rental industry.

1.1 Contextualising the car rental industry

Car rental firms are composed of a fleet of vehicles within a pool network of rental stations which share capacity. Revenue is only generated when vehicles are on the road and driven by customers, so it is important for a firm to optimise the deployment of vehicles through planning, repositioning, and allocation of different vehicle groups to fulfil reservations. The ultimate goal of a for-profit entity is to generate sustainable profit levels over their planning horizon. For car rental firms, a large proportion of the costs are associated with the operational movements of the fleet, so optimising this process to minimise costs in fulfilling reservations is very important from a sustainability perspective.

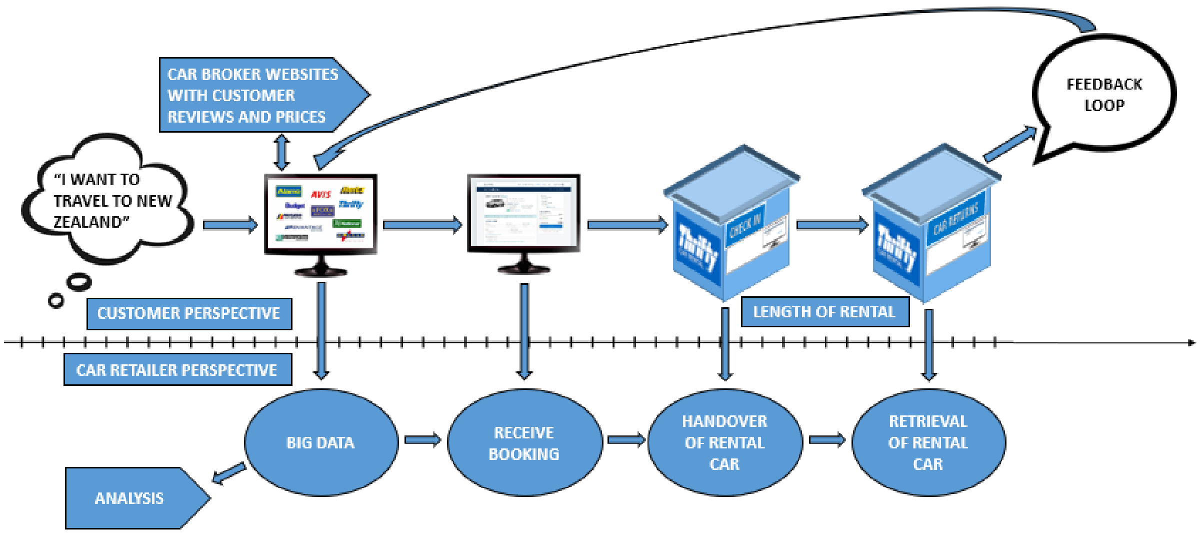


Figure 1 Car rental reservation fulfilment process: Firm and customer perspectives and interactions

Figure 1 displays the steps and actions which occur during the fulfilment of a car reservation. When a customer decides they want to travel to, for example, NZ and rent a car, they select the rental locations at which they wish to arrive and depart, the vehicle group they wish to use, and the duration they want to rent the car for. The characteristics of the reservation along with the duration until the reservation start date partially determine the price level which is displayed on the rental website. Other factors which determine the displayed price, such as intended profit margins, are hidden from customers. The price level displayed is the revenue associated with a reservation from the moment customers realise their booking. Fractional revenues flow to the entity based on reservation cancelling policies. The firm may use big data to analyse, for example, customer behaviour and use the so gained information for price setting and operational strategy. Reservation requests are inherently uncertain, which generates a number of car rental sub-problems that associate with short-term (tactical) and long-term (strategic) decision making horizons. The following list introduces some of these:

- Pool segmentation (strategic) - Two central challenges within this context are i) how to group rental stations into pools which share the same fleet of vehicles, and ii) how to distribute the available fleet between all rental stations that make up the pool structure (Edelstein and Melnyk, 1977). Optimal pool segmentation aims to "share the same fleet of vehicles on a daily basis with low costs and short lead times required to transship [transfer] vehicles between locations within the same pool" (Pachon et al., 2006, p223). A car rental firm will allocate a rental location within each pool as the pool logistics co-ordination and communication centre (Yang et al., 2008).
- Fleet size and mix (strategic) - Linked closely with pool segmentation, fleet size and mix are concerned with determining the appropriate number of vehicles under each vehicle group and their allocation to each pool. Pachon et al. (2006) considers this decision be taken monthly or every trimester to encompass seasonal demand patterns, although this decision can be tactical in nature where rental firms may subcontract capacity to meet large, unforeseen spikes in demand (Carrol and Grimes, 1995). Relationships with car manufacturers are an important part of this problem in determining the acquisition and disposal of vehicles, which requires planning sometimes well in advance of 6 months, furthering why it is part of strategic decision making.
- Fleet deployment (strategic) – Directly attributable to fleet size, this sub-problem is about which rental stations receive the newly acquired vehicles added to the fleet based upon expected reservation arrival frequencies.
- Fleet deployment (tactical) – This is concerned with repositioning vehicles at specific points in time to meet future, and thus, uncertain demand requirements (Oliveira et al., 2014). The planning horizon for this ranges from daily (Li and Tao, 2010; Pachon et al., 2003, 2006; Song and Earl, 2007), the "next few days" given a one-week planning horizon (Fink and Reiner, 2004, p. 286), weekly or every other week (Edelstein and Melnyk, 1977; Fink and Reiners, 2004; Haensel et al., 2011; You and Hsieh, 2014), or monthly planning horizons (Madden and Russell, 2012). Also, firms can trigger empty vehicle rebalancing decisions when demand exceeds the current fleet capacity at a station (Li and Pang, 2017). Empty vehicle rebalancing is defined by transportation time and mode. Transportation times are based on a matrix given the time transfers take between all possible locations within the pool (Fink and Reiners, 2004; Guerriero and Olivito, 2014; Oliveira et al., 2014), an exponential or Poisson distribution (Song and

Earl, 2007), or overnight (Li and Tao, 2010; Pachon et al., 2006, 2003; You and Hsieh, 2014). Transportation mode generally considers a single vehicle transfer via an employee of the car rental firm (Ernst et al., 2010; Haensel et al., 2011; Li and Pang, 2017; Li and Tao, 2010; Madden and Russell, 2012; Oliveira et al., 2014, 2018a, 2018b, 2019; Pachon et al., 2003, 2006; You and Hsieh, 2014) which is the fastest, yet costly, solution. Alternatively, repositioning several vehicles by truck via a transportation company (Fink and Reiners, 2004; Song and Earl, 2007) is cost-efficient but requires more planning time.

- Fleet assignment – This encompasses the availability of each vehicle. Unavailability can come in the form of scheduled maintenance or fulfilling reservations or transfer decisions. Uncertainty in this sub-problem comes in the form of breakdowns, unforeseen maintenance events and late vehicle returns.
- Capacity allocation – Tied in closely with fleet assignment, this is concerned with whether to accept or reject a current reservation request from a customer based on available capacity of each vehicle group. Upgrades and downgrades are considered in this decision making. Booking limits, protection levels and overbooking are part of selecting reservations to serve within a list of reservation requests. If operational intervention via transfer decisions are not feasible and upgrade/ downgrade decisions cannot be made, a reservation is unfulfilled which is likely to result in a reputational cost to the firm.
- Pricing – This sub-problem is concerned with the price level that should be assigned to reservation types. Firms can engage in price setting strategies to incentivise or disincentivise specific reservations as a means of repositioning the fleet through its impact on demand. Macroeconomic factors, competitor pricings, and customer behaviour, especially through the lens of price broker websites which offer a high level of price transparency between competitors (Oliveira et al., 2015), further complicate this problem.

In summary, these elements are pertinent to the car rental problem, and each has unique and interconnected aspects which on the whole create a complex system. Complex systems require holistic modelling, for analyses which only deal with sub-problems in isolation are of limited usefulness.

1.2 Holistic modelling approach

SACA is an analysis system which allows modelling complex systems, and the physical components of the car rental problem (fleet management) jointly with the financial components (revenue management), which is shown in Figure 2.

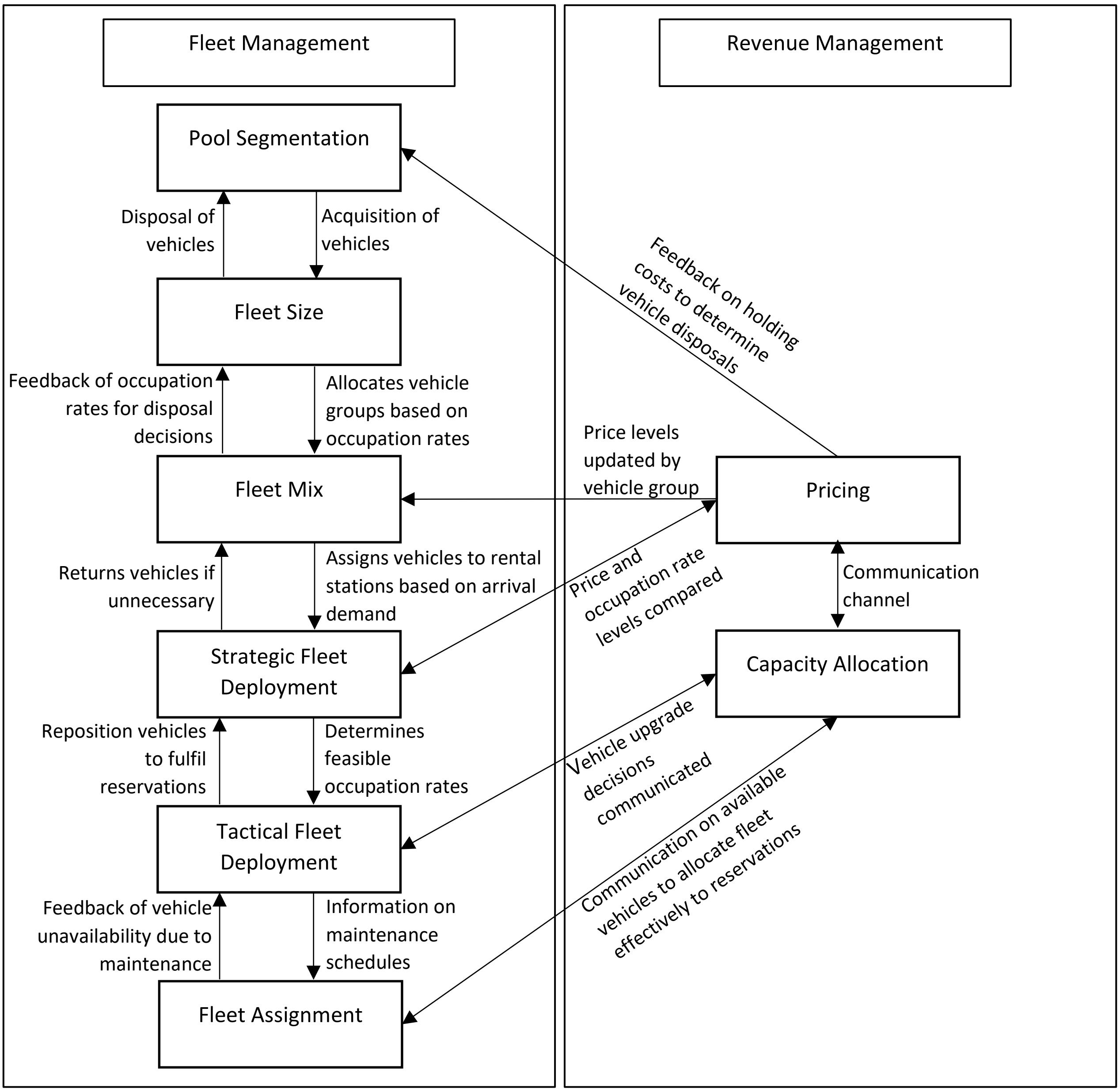


Figure 2 Fleet and revenue management: Elements, connections, and important examples of decision aspects

SACA allows a firm to account for the duality of engineering and accounting aspects in a statistical setting which considers the decisions made during the life cycle of an asset. SACA has been used to model processes in manufacturing (Falta et al., 2006) and naval configuration management (Colin et al., 2012; Colin et al., 2010). The objective of my implementation of SACA is to relate fleet activities to the costs associated with their use, which encapsulates the potential impacts of high-level decisions made within the sub-problems of fleet and revenue management. Financial data consist of sets of information related to the financial health of a business. Financial data are generated from the operation of the fleet, and they are inherently linked to the processes explained in Figure 2, which demonstrates the key concepts and how they communicate and interact with each other within fleet operations.

To usefully analyse the complexity of the car rental industry and the sub-problems outlined in Figures 1 and 2, I have chosen to implement a simulation study. My approach is incremental. Table 1 contains a comparison of the 7 simulation versions, which step-by-step incorporate additional sub-problems and by doing so, bring the simulation context closer to reality. This progression also addresses the limitations outlined in Oliveira et al. (2017) and their demand for increased realism in the academic study of the car rental industry.

Table 1 Simulation versions and their reflection of the car rental context

Simulations	V1	V2	V3	V4	V5	V6	V7	V7.1
Pool Segmentation	2 rental stations	2 rental stations	3 rental stations	3 rental stations	3 rental nodes	3 rental stations	3 rental stations	3 rental stations
Fleet Size	Fixed	Variable	Variable	Variable	Variable	Fixed	Fixed	Fixed
Fleet Mix	1 vehicle group	2 vehicle groups	2 vehicle groups	3 vehicle groups	7 vehicle groups	7 vehicle groups	7 vehicle groups	7 vehicle groups
Strategic Fleet Deployment	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data	Allocated from arrival data
Tactical Fleet Deployment	N/A	N/A	N/A	N/A	Fixed	Matrix of time and costs	Matrix of time and costs	Matrix of time and costs
Fleet Assignment	N/A	N/A	N/A	N/A	N/A	Scheduled Maintenance	Scheduled Maintenance	20–40-point checks at different intensity levels
Capacity Allocation	N/A	N/A	N/A	N/A	N/A	N/A	Limited cascading upgrades	Limited cascading upgrades
Price Setting	Derived from real data	Derived from real data	Derived from real data	Derived from real data	Derived from real data	Derived from real data	Derived from real data	Derived from real data

Simulation V1 is my starting point and models a car rental firm with a single fleet size, 2 rental stations and 1 vehicle group. The fleet is deployed to stations based on expected reservation demand, and reservations arrive to stations based on a uniform arrival process. Customer's reservation characteristics are randomised, although the end location is based upon a variable input which determines whether a trip will be a round trip (station x – station x) or a direct trip (station x – station y). Pricing is based on real data for Hertz rentals. To address both SACA dimensions, financial outcomes are generated and translated into financial statements to display profitability, among other things. V2 adopted a 2nd vehicle group to increase the dimensionality of fleet movements. This is also the case for V3, which added a 3rd rental station, and V4, which added a 3rd vehicle group. V5 then makes an important step towards realism in that the fleet planning process includes a vehicle rebalancing system. V6 updates the vehicle rebalancing system to encapsulate a matrix of durations and distances between stations. Additionally, a scheduled maintenance activity is used to test the impact of increased vehicle unavailability on fleet operations. The final version V7 incorporates revenue management considerations with a limited cascading vehicle upgrade system, which allows a reservation to be adjusted to the next higher vehicle group. This is inherently linked to the rebalancing system in which upgrades are preferred if the cost of the rebalancing decision outweighs the potential revenue generated. V7.1 extends upon V7 with a more complicated maintenance system, which tests different intensities of a basic vehicle defect testing scheme. The costs associated with these events and the potential reputational costs to the firm incurs when a defect is not detected are also considered.

The remainder of my thesis is structured as follows. In Chapter 2, I review the literature pertaining to the car rental context, which allows me to identify the limitations within. This is the bases for Chapter 3 in which I develop my research topic in detail. In Chapter 4, the methodology of my research is discussed, along with the setup of my simulation versions and the limitations in my approach. Chapter 5 discusses the results generated from Chapter 4 and Chapter 6 concludes, summarises my contributed to the research area and provides an outlook with future research directions.

2 Literature review

The following review of the literature aims to present and critique the different interconnected sub-problems that are addressed in this field of research along with the contextualization of the car rental industry and the limitations and further developments to be made within this body of literature. The literature review is conducted using a thematic approach, which assesses the different sub-problems and how they are addressed by the authors. This includes the type of approach used, what sub-problems are considered in their model (if applicable) and the extent to which the authors demonstrate the complexity of realistic problems faced by car rental firms.

The paper by Oliveira et al. (2017) gives the best summary of the complexity of the fleet management decision making literature and its sub-problems, and how the car rental fleet management literature has evolved from its inception since Edelstein and Melnyk (1977). A central piece in the Oliveira paper is a heatmap (reprinted in Figure 3), which displays the number of publications that fall under each of the identified car rental fleet management sub-problems. With the 23 papers included in the heatmap, and few that have been published since 2017, the car rental analysis literature is small.

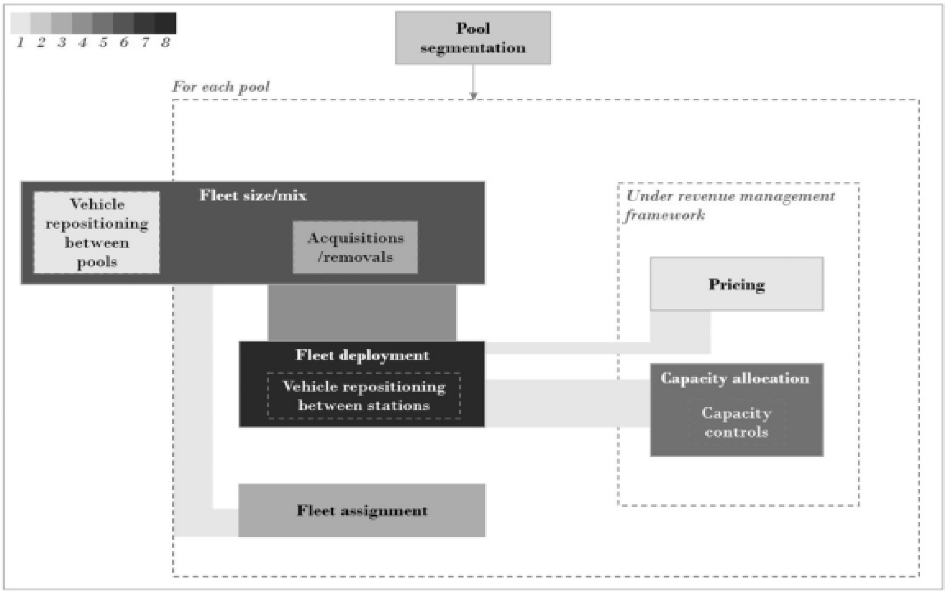


Figure 3 Heatmap of Car Rental Fleet and Revenue Management Literature - Oliveira et al. (2017, p21))

2.1 Pool Segmentation

The first academic work surrounding car rental fleet management is due to Edelstein and Melnyk (1977) who, in a case study, examined the use of the Pool Control System (PCS) that was implemented at Hertz Rent-A-Car. The PCS is used within each pool, in which "The fleet is shared by a group of cities, each city being run by its own management, but fleet administration is centralized" (Edelstein and Melnyk, 1977, p22). The authors state that each pool of Hertz consists of "two to ten cities with fleets of 2000 to 6000 cars" (p22) yet the model does not discuss how the scope of these pools is determined, along with constraints such as mountain ranges, lakes and islands, and state or national borders which inhibit fleet sharing. The PCS is a time-share-based model that aims to assist with tactical decision-making processes to help city managers and the distribution manager evaluate alternatives for answering the following questions for the next 7 days via a rolling time horizon: How many cars will be needed? How many cars will be available or can be moved in from other cities within the same pool? How many reservations can be accepted? And how will the actions taken for any city on any day affect future days and other cities?

The PCS is a daily routine completed via a form by city managers that is heavily reliant on a large number of inputs that includes data from prior, current, and future days. These data include the number of cars available for rental at the start of the day, vehicle acquisitions and disposals, planned maintenance commencing and completing, pre-arranged vehicle transfers and the net effect on reporting cities, the number of "foreign check ins" (vehicles rented outside of the pool and returned inside the pool to reporting city), number of rentals due back into the pool today, one day, two days up to twenty eight days hence, number of rentals completed on the prior day, and a projection of the demand potential for current and future days. Once each form is received, it is checked by the distribution manager (fleet administrator) for consistency and then inputted into the PCS. The model's decision-making process is based off 2 critical risk factors, demand potential and rental capacity for each pool city. A report is produced by the PCS which provides an outlook for the next 7 days. If shortages are apparent, city managers must look at different vehicle transfer and demand control options which are then input back into the model to view the impact of these options on each of the pool cities. Edelstein and Melnyk (1977) use a 3 cities illustrative example in their paper. The PCS information used to make decisions are current reservations, idle vehicles, check-ins, arranged transfers, available fleet, demand forecasts and the net cars remaining for each day over the next 7 days. Different demand control strategies are then proposed in the model, which are then displayed for each of the 3 cities. Managers are then invited to evaluate the trade-offs of each of the proposals, and the final approach is then recorded in the PCS. Demand control procedures are in the form of restricting rentals to reservation customers only, restricting either the number of walk-in customers or putting a limit on the number of reservations that can be accepted. Neither price levels as a form of demand control were considered in this model, nor were any tactical revenue management decisions with an assigned cost. The fleet proposed in their model also does not distinguish vehicle groups, which means that the complexities of fulfilling reservations for different car types are not considered. That also means that this model has not considered upgrade potentials, where vehicle transfers are not apprehensible or certain vehicles are made unavailable due to planned or unplanned maintenance; and late returns which are assumed to be “offset by those who are returning late” (p30). The latter is unrealistic for practice because rentals that are planned to fit into certain time windows and have an earlier return do not guarantee an earlier reservation start time for another customer. Finally, I also note that costs and durations of vehicle transfers are not stated.

The work by Edelstein and Melnyk (1997) served as a great base for the development of the car rental fleet management literature. The general tactical operational decisions faced by Hertz-Rent-A-Car were implemented for a 7-day time horizon. This horizon is acceptable for general tactical operational decisions, and used in later works (e.g., Fink and Reiners, 2004; Haensel et al., 2011; You and Hsieh, 2014). Using reduced input variables, the Edelstein and Melnyk model can be extended to cover a 30-day planning horizon (cf. Madden and Russell, 2012).

The PCS used by Hertz rent-a-car stood the test of time at the pool level, still reported as being utilised for tactical vehicle repositioning decisions in Carrol and Grimes (1995), although renamed to the Daily Planning and Distribution Aid (DPDA). The DPDA model was extended to implement the effect of making interpool transfers, as opposed to intrapool vehicle transfers. The interpool vehicle transfers are made under the authority of intermediate and corporate level management and are only necessitated by major events that fall upon certain pool cities which cannot support the number of reservation requests by their pool specific fleet alone. Carrol and Grimes (1995) contrast the qualitative aspects of the additional cost of interpool vehicle transfers relative to their "improved contribution" (p89). They also discuss how long-term goals of maintaining market share by-way-of customer loyalty and establishing a dependable service must be made at the expense of short-term profitability and maintaining an optimal percentage of the fleet on rent. Having this distinction made gives us a greater understanding of a car rental firm's goals to stay in operation beyond exclusively the operational decisions that come with the movements of the fleet. Unfortunately, the qualitative aspects of operation regarding quality of service and their inherent costs were not implemented as a variable that impacts the DPDA, even though the model aims to optimise fleet occupancy. Just like the PCS, the DPDA fails to recognise the costs that a for-profit car rental company must bear to support the sustainability of their fleet, and how the movement of the fleet contributes so greatly to the costs attributable for a car rental firm. Carrol and Grimes (1995) did neither give an illustration of the DPDA system, nor did the authors show the addition of an interpool transfer input on the adaptation of the original PCS system demonstrated in Edelstein and Melnyk (1977).

Because of this, the fleet planning process portion of Carrol and Grimes work is only descriptive in its nature as a case study on Hertz rent-a-car.

Pachon et al. (2003) adopted a similar definition of the pool to Edelstein and Melnyk (1977): A group of rental locations which share a fleet of vehicles. This allows each location access to a larger fleet, resulting in higher levels of fleet utilisation.[1] Pachon et al. (2003) state that pool structure decision making is based on "distances and demand load correlations among locations" (p907). The fleet planning progress is contextualised to harbour 3 main phases in a sequential hierarchical structure, starting with pool segmentation, followed by strategic fleet planning and then tactical fleet planning. The hierarchical structure of the fleet planning process later became the inspiration for a literature review by Yang et al. (2008) structured around the fleet planning process of car rental and airline management.

Pachon et al. (2006) extended the hierarchical structure of the fleet planning process to be based off recommendations from local city managers and regional management. The ability to intervene local managers by regional management is important to prevent the former to solely focus on the profitability of their own stations rather than the profitability of the entire pool. If local management perceive that being assigned to a certain pool will negatively affect their profitability by harming the utilisation of their fleet, then a station may not join a cluster of stations willingly, disregarding that the whole (pool) may be more profitable even though some stations are less profitable. However, tactical fleet planning is primarily a local management problem. Each station is responsible for optimising the utilisation of their fleet and may conduct their operations in whatever way they see fit to achieve this. If an empty vehicle transfer is deemed to be necessary by local management, then the judgement on

[1] However, clusters and stations are likely to have dissimilar demand loads at different points in time in accordance with weekly and seasonal (trimester) considerations. Only if minimally correlated demand loads are present, better fleet planning is possible due to different load factors being attributable to the number of reservations expected at a station at a given time, which gives a car rental firm sufficient time to arrange intrapool or interpool vehicle transfers (tactical) and vehicle acquisitions via purchase plans with car manufacturers (strategic).

whether it is actuated, and which station the vehicle is taken from within the pool is the responsibility of regional management. This creates conflict between local and regional management and a paradox (the so-called pool segmentation problem) between fleet optimisation at the pool-level and the station-level.

Pachon et al.'s (2006) model to optimise the pool segmentation problem used the (1) the maximum distance between two rental locations in the same pool, (2) the maximum number of rental locations within a pool and (3) the maximum variance of aggregated demand within a pool. They propose a column generation algorithm solution procedure where each column represents a possible pool[2], and display the solution methodology via a case study on Florida, United States of America. While Pachon et al.'s work has advanced the level of analysis complexity, the identified limitation in their work allows for further improvements towards holistic modelling. For example, although they mention that a reduction in empty vehicle repositioning costs is important in pool determination, this is not built into the solution procedure. Similarly, no financial outcomes of specific pool segmentations are addressed. A second example is that Pachon et al. present further models dealing with strategic and tactical fleet planning separately, meaning that the results achieved in each of these models represent local solutions and their validity for practical applications remains unclear. This very point has been strongly formulated by Oliveira et al. (2017), in that sub-problems of the fleet planning process must be integrated and examined with consideration of the time horizons apparent with the different levels of decision making and how they overlap.

[2] Twelve pools are considered, where the majority of the pools only harbour 1 to 2 locations. A few large pools contain a greater number of stations which are interpreted as airport locations surrounded by downtown locations. The authors assume with this configuration that fleet utilisation is improved as different peaks and valleys in demand occur between airport and non-airport locations.

2.2 Fleet size and fleet mix

A large part of understanding fleet size is the distinction between strategic and tactical aspects and how they influence the determination of fleet sizes. Strategic fleet size regulates long-term decision making in regard to the number of vehicles allocated to each pool. These decisions are made at corporate level which may take input from regional management (e.g., Pachon et al., 2006). Tactical fleet size deals with day-by-day adjustments made in which operational decisions are considered (e.g., Carroll & Grimes, 1995). Decision making in this time-horizon falls under regional management for intrapool decisions and is extended to corporate management if an interpool vehicle movement must be made.

Pachon et al. (2006) and Patel et al. (2018) dedicate a section and the entirety of their paper, respectively, towards the development of a model to solely optimise the fleet size and fleet mix of a car rental firm. Pachon et al. (2006) embed acquisition and disposal decisions in the strategic fleet planning processes due to commitments with car manufacturers generally needing to be made more than 12 months in advance due to production and importing arrangements. In other words, acquisition and disposal of vehicles is comparable to a leasing contract from the original manufacturer, in which a service deadline is defined for each car (e.g., Hertz et al., 2009). The objective of their network flow model with demand fill rates as a side constraint, is to minimise the sum of lease, transhipment (interpool movements), acquisition and return (disposal) costs.[3] However, the network flow model illustrated by Pachon et al. (2006) underestimates the extent to which a car rental firm can utilise its fleet with appropriate use of intrapool and interpool transfers. The model does not consider the use

[3] Further assumption: 12 pools are allocated fleet sizes under the constraint that 100% forecasted demand would be hypothetically filled each quarterly period; 2 car types are considered for allocation (compact and intermediate) in which upgrades are available to satisfy demand for the lower-priced vehicle group; The fleet size generated for each pool was equal to or greater than the expected demand fill rates for each quarterly period by car type; Having a fleet allocated that is under the 100% fill rate for demand is not allowed, likely because holding a fleet equal to or greater than the reservations expected within the pool for each car type guarantees the reservations being fulfilled at the expense of a greatly under-utilised fleet.

of intrapool vehicle transfers in its allocation of vehicles to each pool, meaning that the daily operations conducted by each station and pool under the oversight of local and regional management is rendered moot. Further, constraints are the demand fill rate and the consideration of only 2 vehicle groups. In summary, I observe that the sub-problem of fleet size, fleet mix and the costs associated with quarterly decision making are in line with a car manufacturer acquisition cost minimisation model.

Patel et al.'s (2018) fleet mix optimisation solution is based on 3 vehicle groups. The authors tested how the different combinations of vehicle types in the fleet affects the (un)availability of the cars to rent for a given capacity allocation process. They tested for the optimal combination of vehicles using a signal to noise ratio (S/N) with the aim to finding the lowest S/N value. This would reduce the shortage of vehicles and the number of necessary upgrade decisions, which in turn would increase the revenue generated. The optimal levels (lowest S/N) were found to be Hatchbacks at 26-30%, Sedans at 21-25% and Small SUV at 21-25%. The sum of these value yields a total of 68-80%, the unallocated proportions to be allocated to prestige cars, people movers or commercial cars, albeit the optimal proportion of the remaining fleet to be allocated is not considered. Interestingly, the authors discuss within the assignment of the fleet compulsory vehicle services to be an important consideration, yet their model is solely based on optimising fleet mix without implementing vehicle unavailabilities due to, for example, physical maintenance constraints or the associated costs.

2.3 Fleet Deployment

Fleet deployment is the most targeted fleet management sub-problem in the literature, with multiple papers synthesizing with the fleet size and mix sub-problems (Fink and Reiners, 2004; Song and Earl, 2007; Li and Tao, 2010; You and Hsieh, 2014).

Fink and Reiners (2004) were the first to synthesize fleet deployment and fleet size. The operational decision-making processes considered acquisitions and disposals from a country wide network in Germany. Multiple periods of rental, up to 15 different car groups, 1800 vehicles and a few hundred stations were investigated using real data with the objective of maximizing profits. The network flow model for profit optimization used a one-week rolling planning horizon in which referencing processes are similar to a DSS for logistics processes (Edelstein and Melnyk, 1977). Simulation iterations are implemented into the network flow model to minimize the costs associated with the arcs (variables) and their effect on network nodes (stations) to optimize profit given different tested fleet sizes, with the constraint that a high service level (>99%) is still maintained. Empty transfers between all possible rental locations were used as a basis to map fleet optimisation, which data were compared to actual transportation times. They find that their initial fleet size (15’500) can be reduced by up to 20% (12’400) before the service level drops below 99%. With holding costs estimated at 10 Euros per day, the authors established that a single reduction “of car holding costs offset lost revenues and additional transportation costs by a factor of more than ten” (Fink and Reiners, 2004, p288). Each iteration of fleet size was tested with differing constraints of upgrade possibilities. Lifting all upgrades constraints proved to outperform normal upgrade (one vehicle group above). This makes sense as the system does not have a capacity allocation constraint built in so rental reservations are accepted on a first come, first served basis. The result of this by default would be an improved service level as any higher vehicle group can satisfy a reservation request which would give the network model greater freedom to reduce the fleet size further, given that holding costs have a greater bearing on operational profit than lost revenues and increased transportation costs. The limitations of Fink and Reiner’s (2004) work lie in the simplification of the fleeting and de-fleeting processes. Leasing contracts were generated with virtual depots for car pickup and return, not representing the actual buy and sell process between car rental firms and vehicle manufacturers, which is exacerbated through large discrepancies in the time horizons between the tactical fleet planning horizon (7 days) and the strategic fleet planning horizon associated with determining fleet additions and disposals via vehicle manufacturers, this generally requiring a planning horizon exceeding 12 months (Carrol and Grimes, 1995; Pachon et al., 2003, 2006).

Song and Earl (2007) produced an event driven model on a two-depot system which aims to optimize fleet size, initial fleet deployment and a vehicle transfer policy based on the minimisation of a cost function associated with leasing, holding, maintenance and empty vehicle repositioning costs. Time and mode of empty vehicle repositioning decision making is investigated using stochastic, deterministic, uniform, and normal distributions, in which the transfer is initiated once the decision is made for the movement of a vehicle, contrary to other papers which initiate the transfer start time once the vehicle starts its movement (Fink and Reiners, 2004; Guerriero and Olivito, 2014; Oliveira et al., 2014). Song and Earl (2007) test 3 different scenarios representing (1) balanced vehicle returns and leasing costs to each depot, (2) a 2-to-1 vehicle return rate to depot 2 with balanced leasing costs and (3) a 2-to-1 vehicle return rate to depot 2 and a 2-to-1 leasing cost at depot 1 relative to depot 2. Optimum fleet size increased by a value of 1 for each scenario up to a total fleet size of 6 vehicles where only a single vehicle group was tested. Initial fleet deployment represented a 50% split for scenario 1 and increased in favour for depot 1 in scenarios 2 and 3 up to 83% due to end locations being in favour of depot 2. Threshold values based on acceptable lower and upper bounds for empty vehicle repositioning decisions were generated in the model based upon single vehicle transfers and transfers via truck which is a unique take on these decisions given other papers covering tactical fleet movements aim to find a single optimal value of transfers to be conducted within a specified time horizon (Fink and Reiners, 2004; Pachon et al., 2003, 2006; Li and Tao, 2010; You and Hsieh, 2014). The dimensionality and generalisability of the model proposed by Song and Earl (2007) has limits even though the concepts proposed for vehicle transfer policy I found interesting. The model lacks dimensionality due to the many undisclosed or omitted attributes that contribute to the decision-making for the strategic and tactical fleet planning processes. Lost sales are not considered in this model as all reservations are assumed to be covered by either owned or leased vehicles.

Li and Tao (2010) conduct a similar analysis to Song and Earl (2007) on the fleet size and fleet deployment sub-problems using a 2-stage dynamic programming model, where fleet size is the decision criterion at stage 1, and transfer policy at stage 2. A 2-station setting, and 1 vehicle group is considered in which the time horizon of the fleet planning process is simplified to an infinite horizon where decision making beyond 1-day is out of the scope of this model. Empty vehicle repositioning is assumed to be completed overnight and available for the next rental day and daily operating costs per vehicle are simplified to 1 arbitrary value which is varied to determine its influence on fleet size. Their modelling maxes out at 29 vehicles for fleet size at optimum values. Reservation demand is based on a uniform distribution where demand values are greater for 1-way trips relative to round-trips resulting in the inter-city reservations demanding a higher rental rate. Time-average profit is negatively correlated with the transfer cost set, where fleet size does not display sensitivity to the different transfer costs tested. Li and Tao (2010) then relax the assumption of lost sales by allowing the ability to subcontract capacity via instant and short-term leasing arrangements. They also allocate lease costs per vehicle depending on local and inter-city depot rates, because of differential combinations of short-term leasing costs and the transportation costs to and from the various stations. However, the way how they added short-term leasing to the model is not fully clear to me because the model assumes that vehicles can be sub-contracted within 24h. Other tactical decision-making aspects are clear. For example, at the end of a reservation day, they assume accurate information for the following reservation day is available and so are estimates for walk-in customers based off historical demand which determine the amount of empty vehicle transfers necessary to fulfil expected demand. If forecasted demand is greater than the amount of vehicles that can realistically be moved via intercity repositioning, then a leasing cost is incurred, and a vehicle is immediately obtained from a near depot. As for transfer decisions, Li and Tao (2010) found that the use of an appropriate transfer policy is important when the transfer cost is close to or larger than the round-trip rental rate charged.

You and Hsieh (2014) produced a case study based on a Taiwanese car rental company and assumed transfer decision assumptions similar to Li and Tao (2010). They used input values based on the number of vehicles transferred overnight and implemented a daily planning process which optimises fleet size and deployment (strategic) and transfer policy (tactical). They analysed a total of 38 rental stations across 12 pools, and single day rentals. Transfer costs are calculated independently of pricing decisions, a constant daily fee is developed for all reservations and a variable travelling fee per km. Unlike earlier papers which consider single day planning horizons (Li and Tao, 2010; Pachon et al., 2003, 2006; Song and Earl, 2007), You and Hsieh (2014) extend to a weekly planning horizon (similarly to Fink and Reiners, 2004; Haensel et al., 2011) for the tactical fleet planning process. The inclusion of 38 stations is a strong improvement on models only considering a 2-station system yet having only a single vehicle group is a limit. You and Hsieh (2014) use a time-dependent demand process[4] to model decisions in vehicle movements.

An interesting case study is presented by Carreia and Santos (2014) using varying fleet sizes of up to 20 electric vehicles (EVs) to an existing fleet of conventional vehicles (CVs). The model optimises against profit from a net of revenue *less* transportation, purchase,

[4] The time-dependency is simulated through an exponential increase in reservations occurring as time tends closer to the reservation start date. Demand requests are modelled as a linear decreasing function of the rental rate charged for a 1-day rental, meaning that a baseline level of price sensitivity of demand is considered although the impact of setting different rental rates is not reflected upon in regard to how it may affect the entire industry within their respective operating regions.

depreciation, and maintenance costs. The model uses 7 rental stations based on a Portuguese car rental firm which maps all possible locations and the transfers conducted between them as a matrix similar to Pachon et al. (2003). The simulation is a discrete time-step implementation in which transfers i) may take a maximum of 8-time steps of approximately 75 mins, and ii) are assumed to fall within 1-day of travel time.

Where no lost sales are assumed, the first state of the model aims to allocate the maximum number of electric vehicles within a given week to acceptable trip types. Due to the lack of an EVs range of about 150-180 kilometres, and their charging time of approximately 8 hours for 90 minutes active use, the number of acceptable reservations that these vehicles can be allocated to is limited. With single day rentals considered, EVs reduce the marginal profit of the car rental firm increasingly by the value of EVs allocated to a fleet of approximately 850 vehicles where maximum demand for EVs must be met. The second state of the MIP model relaxes the constraint of meeting maximum EV demand and CVs were allocated to the majority of reservations, independently of the number of EVs. The charging constraint of the EVs is the most limiting factor and resulted in 43% lower profit. Because the number of EVs simulated is negligible compared to the number of CVs in the fleet, this model essentially covers the tactical fleet planning transportation process of 1 vehicle group. The consideration of 7 rental stations is an improvement on earlier tactical transportation models which only cover 2 rental stations (Song and Earl, 2007; Li and Pang, 2017; Li and Tao, 2010).

Pachon et al. (2003) developed an optimisation model based on stochastic reservation demand to maximise profit for the tactical fleet planning process given a 7-day planning horizon. Six rental stations and 1 vehicle group were considered where the expected profit was determined to be revenue *less* transfer costs. The authors focused on obtaining optimal fleet levels for all 6 rental locations and found that empty vehicle repositioning decisions lead to both improved utilization of the fleet and profit. Using initial fleet values and minimum and maximum expected demand values, initial fleet deployment can be optimised between the 6 rental locations for a set fleet size of 2000 vehicles given a uniform reservation distribution centred on a deterministic mean to minimise the number of empty transfers that are necessary to maintain an acceptable service level (>99%). It appears that Fink and Reiners (2004) took inspiration from Pachon et al. (2003) because they added a service level constraint in their model to optimise financial outcomes within the tactical fleet planning process. Pachon et al. (2006) added to their earlier paper on the tactical fleet deployment process by applying a price-demand function with 2 price states, conditional to whether the following days demand is less than or greater than the inventory at the relevant rental station at the end of the reservation day. This is an improvement on the original model presented.

All papers reviewed in this section do not quite envelope a holistic modelling approach which would incorporate multiple vehicle groups and their respective upgrades, strategic fleet considerations regarding fleet size and the time span of decisions associated along with price sensitive demand, and dynamic booking controls to optimise fleet assignment.

2.4 Fleet Assignment

Hertz et al. (2009) focused on the fleet assignment and fleet size sub-problems using an integer linear programming approach. They used 5 major cost variables that have their own optimisation algorithms, these are (1) assignment costs, (2) holding costs, (3) subcontracting costs, (4) purchase costs and (5) maintenance costs. Fleet size is analysed as a tactical decision, where if reservation requests exceed current stock, the firm must review other options to satisfy customer demand at all costs, meaning lost sales are not considered. The assumption of instantaneous subcontracting of vehicles seems to be commonplace in the literature, being additionally applied in Song and Earl (2007) and Li and Tao (2010). Instances are tested based on 4 fixed inputs, these being (1) reservation requests, (2) vehicle groups, (3) maintenance workshop capacity, and (4) a Boolean value that states whether vehicle purchases are allowed. The authors use complex algorithms[5] and considerations for fleet assignment with maintenance constraints, although the ability to reposition vehicles is not considered. This would mean that vehicle availability is treated in terms of time and not location, which would hurt the efficiency of the maintenance schedule with uncertain demand.

Ernst et al. (2010) present a rental fleet scheduling heuristic (RFSH) to solve the rental fleet scheduling problem (RFSP). A simulation model with a mathematical formulation and a dual Lagrangian multiplier is used to "find a schedule that minimizes the cost of meeting all bookings as well as satisfying maintenance and disposals of vehicles" (p216). The model is used to find the acceptable bounds of the assignment components that consist of bookings, maintenance schedules and planned disposals. A vehicle allocation and scheduling tool (VAST) is employed to build the schedules for fleet assignment. Real data sets were provided from 33 different car rental firm databases, which contained a total of 7768 reservation requests, 23 rental stations, 143 vehicle groups and a fleet size of 2195. The RFSH is designed to rebuild schedules daily while maintaining current accepted reservation requests. The heuristic includes relocations as a duration-dependent process with delays, early pickups and returns updated into the model. The authors gave great insight into the operational complexity of the assignment of vehicles to reservations. The model could be extended to include a longer simulation horizon to incorporate seasonal considerations.

A closed queueing network model is presented by George and Xia (2010) which tests different station capacities[6] with a constant number of reservations that aims to maximise

[5] Allowing vehicle purchases resulted in i) an improvement on other cost functions due to the leeway on the algorithm to better minimize subcontracting assignments, and ii) employees allocated to maintenance schedules. With 12 vehicle groups tested, 200 reservation requests and a workshop capacity of 2 employees for scheduled maintenance, the sum of the cost functions are minimised.

[6] 100 rental stations were the maximum dimensionality tested which resulted in an optimal fleet size of 8655 vehicles.

profit. This is done by optimising fleet size and fleet scheduling with cost variables of scheduled maintenance (hourly) and an unavailability penalty for lost sales. Empty vehicle repositioning is not implemented into the model, so vehicle availability relies on accurate calculations of vehicle return times to create feasible maintenance schedules. The unavailability penalty was found to have a linear relationship with fleet size and a decreasing linear relationship with profit. Maintenance costs displayed an upwards decreasing concave relationship with fleet size, meaning as maintenance costs increase, the rate at which fleet size must decrease is also a decreasing function. The authors place emphasis on the trade-off between revenue obtained and the cost of maintaining the fleet. Fleet size and quality of service are important factors that must be considered by car rental firms in determining a service level to maintain that is acceptable with the costs associated with their active fleet. It would have been interesting if this was implemented into their model as a constraint, similarly, used by Fink and Reiners (2004) and Pachon et al. (2003, 2006). The exclusion of empty vehicle repositioning, and additional vehicle groups limited the power of the model with 100 rental stations considered.

The earlier papers on fleet assignment were superseded by Oliveira et al. (2014) in which a network flow model (also used by Fink and Reiners, 2004; Pachon et al., 2006) is proposed to maximise the profitability of the assignment of vehicles within the special fleet category (luxury vehicles, minivans, off-road vehicles). Due to the small fleet that fit these categories, greater emphasis is placed on the time and cost of empty transfers conducted to fulfil reservation requests. Oliveira et al. (2014) considers the interdependencies between vehicle groups as well as scheduled maintenance and disposal decisions made at different time horizons with respect to different reservation priorities. A relax-and-fix heuristic procedure was conducted[7] using real data from a Portuguese car rental firm. Upgrades and downgrades are implemented using artificial monetary units, where an upgrade serves as an option to fulfil a reservation request for a lower vehicle group. Downgrades, however, are employed as a last resource where an empty transfer or an upgrade is not a feasible option, and a reduction of global profit is incurred. Various simulation settings are tested against the current firms' procedures and improvements are obtained of up to a 51% increase in the objective functions (profit) and a 37% reduction in empty transfer hours. The analysis of the special fleet category is an interesting analysis by Oliveira et al. (2014) as this is not common within the car rental fleet management literature. It would be interesting to see how their model compares to conventional fleet vehicles in the fleet assignment process.

Li and Pang (2017) produce a booking control model using a stochastic reservation formulation, where a transfer cost is only incurred when "the number of bookings accepted is greater than the available capacity and a shuttling movement is triggered" (p853). A decomposition heuristic is proposed[8] that treats multiple-day rentals as multiple single-day rentals with independent demands. The authors link this to the similarity of how a single-leg trip in the airline industry is represented. Real data from a United Kingdom car rental company is used covering over a 90-day rolling planning horizon. Reservation demand is mapped using a Poisson arrival process where greater demand is allocated to the shorter rental lengths. The authors consider empty transfers as an inbound process, meaning the interdependencies between rental stations and their shared capacity is not considered in regard to the transfer costs between each station.

[7] That tested 20 different instances (scenarios) with a maximum of 2696 reservation requests, 39 vehicle fleet size and 66 simultaneous iterations tested based on reservation requests where 5 interconnected vehicle groups are considered.
[8] That utilises 2 rental stations, 2 vehicle groups, 6 days maximum booking horizon and 2, 3, and 4 maximum lengths of rental. This is coupled with varying transfer costs for sensitivity analysis, with the aim of maximising daily revenue.

2.5 Capacity Allocation

Geraghty and Johnson (1997) contained the earliest analysis of a revenue management system in a case study setting. This is implemented and applied by National Car Rental in 1995 and the revenue management system model was used to analyse and manage "capacity, price, and booking requests in a manner that improves revenue per car, revenue per day and utilisation levels" (p110). The system developed looks at empty transfer considerations across stations, vehicle additions as well as upgrade and overbooking to optimise revenue. The authors look at demand variability from a revenue management perspective with a heuristic for setting prices based on supply and demand elasticity. However, the paper focuses on the implementation at National Car Rental, and not on the algorithm itself.

The revenue management literature was furthered by Blair and Anderson (2002) with a case study on the implementation of a performance monitor system (PMS) for Dollar Thrifty Automotive Group (DTAG). The PMS measures the impact of the decisions considered in Geraghty and Johnson (1997) with a performance quality grid and performance metrics. Anderson and Blair (2004) continued their work with the Performance Monitor by looking at dynamic pricing practices. Early rate reductions were identified in a case example for DTAG, which created potential opportunity costs of 13% of addition revenues which could have resulted in a "twofold increase in profits" (p362) if reservation turndowns were avoided.

An interesting theoretical model on capacity allocation was presented by Anderson et al. (2004), where prices are considered to be random variables which follow a mean reverting process, and an increasing linear function as time tends closer to the reservation start time. This was modelled using a stochastic differential equation[9], because demand is volatile, and prices are highly correlated with forecasted demand. A 90-day horizon is considered in a numerical

[9] With a fleet size of 50 cars used to map the impact of a minimum, average and maximum rental price and deviations around these for rental lengths of 1-7 days. A discount rate of 5% is used to represent the rental price as a future price for a reservation today, giving it similarities to a forward price for a commodity.

example where each analysis period is split up into 1-week intervals. The authors looked strictly at rental price elasticity functions and found rental price to be rather inelastic when it is priced between 2 competitors, whereas it was found to be very elastic once a firm's price exceeds a leading competitor. This is likely due to low customer switching costs. The rental car industry is a commodity-based service business, so a firm's decision on pricing may be strongly influenced by their relative pricing decisions in light of their competition.

A great paper for development of revenue management is presented by Haensel et al. (2011) which integrates capacity allocation with fleet deployment decision-making. A network based 2-stage stochastic programming model[10] is utilised where fleet capacity per station is considered a dynamic procedure. With this consideration, deciding the optimal fleet distribution on the network, day-by-day, is possible at given transfer costs. The authors implement a case study for an airport, downtown and suburb rental station that exhibits demand peaks and valleys at different points over the time horizon. A stochastic and deterministic model was applied, in which the stochastic model for demand determination performed better than the deterministic model. This is likely due to the volatile and elastic nature of car rental demand. This study only considered round-trips, meaning the fleet deployment perspective is limited due to the stochastic nature of pickup and drop-off decisions by rental customers.

Steinhardt and Gönsch (2012) look at planned upgrade decision-making within the capacity allocation framework utilising 3 vehicle groups, a 95-vehicle fleet, 6 rental lengths (1-6 days) and 1107 reservation requests. They consider a dynamic programming approach with a heuristic solution method to optimise revenue, where upgrade decisions are based on fairness and scope determinants[11]. Real-world demand and capacity data are provided by a 'major' car

[10] Capacity control via booking limits and vehicle transfers represent the first-stage decisions and demand uncertainty represents the second-stage decision which attempts to approximate this using a finite number of scenarios: 3 rental stations, 2 rental lengths, a 100-vehicle fleet, 1 vehicle group and 467 reservation requests are implemented over a 1 week time horizon.

[11] Fairness in terms of customers who order a higher quality vehicle should hold priority of it, and scope in terms of jumping to the next higher quality product in ascending order.

rental firm, and simulations for restricted and full upgrade availabilities are tested with the inclusion of in-advance and walk-in customers. The authors improved on the fleet size rationalisation of earlier revenue management papers (Anderson et al., 2004; Haensel et al., 2011), yet the exclusion of empty vehicle repositioning limits this model in terms of optimising fleet allocation to better financial outcomes.

Conejero et al. (2014) proposed a time-expanded network model[12] which focuses on the 1-way reservation problem in balancing the fleet across rental stations. EVs (electronic vehicles) were the only vehicle group used in this study which adds additional complexity to allocating reservation assignments due to the constraints on usable parking spaces for charging. Empty vehicle repositioning was not a consideration of this paper as the authors aimed to balance the EV fleet using appropriate capacity control strategies. The scope of feasible transfers for the EVs would be important to examine in future iterations of this model, due the limited range and extensive charging times associated with these vehicles (e.g., Carriera and Santos, 2014).

Guerriero and Olivito (2014) looked at the revenue management problem using a dynamic programming model with linear programming approximations. The authors assume cars must be booked for at least 1 day, and cars used to satisfy rental requests cannot exceed the maximum capacity available. Walk-in requests and upgrade possibilities were also considered under differing circumstances looking at booking limit and bid price policy. The authors examine 2 states of a booking and rental horizon, which a fixed rental price is allocated to each vehicle group in increasing order. The authors do not consider a variance in price levels by vehicle group which incentivises the higher vehicle groups to be rented out over the lower vehicle groups. Additionally, booking limits result in the model not assigning the full range of vehicle groups, because higher vehicle groups generate greater rental revenue.

[12] Which tests across 6 rental stations for the allocation of up to 500 reservations across a 20-day time horizon.

An industry application analysis from a capacity allocation perspective was conducted using a narrative approach by Klein et al. (2019) for various commodity-based service industries. The authors inferred that upgrade decisions are more important for car rental companies relative to the use of capacity control practices used by other industries such as hotel or air cargo. To make this option a distinct possibility, “car rental companies tend to acquire considerably fewer economy cars but more mid-size cars than required” (Klein et al., 2019, p401). The difference in costs between these 2 fleet options is also considerably low, making this decision far more prevalent in car rental than other industries. Car rental companies have additional characteristics that make it more challenging to efficiently allocate capacity, because of 1-way rentals, high levels of upgrade usage, contractual over-the-counter customers, and flexible and uncertain inventory.

2.6 Price Setting

Price setting in this industry is influenced by forecasted demand levels relative to fleet occupation, as well as competitor price levels. Madden and Russell (2012) were the first paper to develop a model to analyze price setting, coupled with fleet deployment characteristics. A mixed integer programming model with linear approximations based on a time-space network[13] of rental locations is proposed where each rental station has its own supply and demand levels for each vehicle group. The model aims to optimise profit through varying price levels which are intended to re-balance the fleet through incentivizing customers (i.e., through impact on demand) rather than vehicle transfers. Transfer decisions are only implemented where a reservation decision was not influenced using price setting. The authors simplify the revenue management process, being that demand can be reasonably forecasted for each vehicle group

[13] The dimensionality of the problem included 53 stations across 13 pools, 5 vehicle groups, 8 different price levels per vehicle group, limited cascading upgrades to the next feasible vehicle group and 1, 2-, 4-, 7-, and 14-day rentals, across a 28-day rolling planning horizon.

and all price levels. The model proposed is certainly an interesting take on the relocation process. However, the use of different price levels per vehicle group does not acknowledge the different prices that can be set to different trip types. Price levels set per rental station only represent an influence to rent from a specific vehicle group, this would not control for trip type which is an important function of the tactical fleet planning process.

Price setting is the focus in Oliveira et al. (2015). A heuristic procedure[14] for price setting is implemented, which corrects prices every 2 hours in response to the changes in market conditions, namely market demand and competitor pricing. They look at price determination from a 'modern' perspective, considering online broker websites that compare competitors pricing for similar vehicle types. Customers can view current offers in the market with full transparency, elevating prices to a more determinant factor in customer preference. This creates the challenge of maximising fleet occupation, at the highest possible price. The model aims to minimise the distance between the actual occupation and the goal occupation levels of the fleet. If real occupation is lower, then a firm would be inclined to set the lowest price in the market, and vice versa. This would be an intriguing model to integrate with the fleet management process, although contextualising how demand is influenced by different price levels set by the firm and its competitors will require some thought.

Yu et al. (2018) takes a unique approach to price setting with differentiation by quality of product.[15] They found this differentiation to have a significant influence on a firm's profitability when considering a customer's willingness to rent a car versus a vehicle's production costs. A narrative approach is taken where the marginal renter is compared to the marginal buyer in making transportation decisions. The authors found that higher-quality

[14] The adaptive heuristic procedure updates prices using a goal occupation curve. This considers actual fleet occupation levels and desired levels over a rolling planning horizon of 30-days until each reservation start date.
[15] The logic behind this statement is that lower production costs should be met with lower per-use rental prices, regardless of the quality of the per-use rental service. Additionally, a comparison is made between smart cars and generic cars and how their differences determine price levels.

vehicles resulted in a higher likelihood an agent would rent rather purchase such a vehicle. Conversely, a consumer's marginal utility was stated to be higher for ownership rather than per-use services.

Alabdulkarim (2018) implemented a model using the ExtendSim for discrete simulations in Microsoft Excel, using various distributions for arrival times, rental durations, reservation start and end times, customer types, vehicle groups selected, and pick-up and drop-off location preferences. This is coupled with fixed price inputs that are used for testing. The model is tested using different levels of car class upgrades: 5 vehicle groups with 15 rental locations are considered, where 5 vehicles are allocated to each vehicle group at each station, making a total fleet size of 375 vehicles. The demand data are retrieved from an "expert in this industry" (p1545), which covers a 1-year period. Customer budgets for each vehicle group are inputted to represent feasible reservation values, and an increasing function per vehicle group represents the upgrade availability threshold for acceptable reservations.

Upgrades up to 2 higher vehicle groups harboured the best results for revenue generation with an opportunity cost function introduced. The function represents the percentage of customers that find the rental price too expensive. With no empty transfers implemented into this model, it limits the operational processes that exist in reality. Therefore, once all upgrades are exhausted, the model must forgo a revenue generating opportunity due to the unavailability. The model is flexible, so the addition of an empty vehicle rebalancing system would be a great extension in future iterations.

Costa (2019) focused on explaining demand by vehicle group through an application of a price elasticity of demand model and 17 different vehicle groups. The context is enhanced by using data from a Portuguese car rental firm.[16] The relationship between firm prices and

[16] The extent of this data includes 50 rental stations, with a peak fleet size of 12'000 vehicles and 10'000 vehicles simultaneously on rent.

their direct competitors were compared to online car broker websites, outlined in Oliveira et al. (2015). Real and goal occupation levels by vehicle group was deemed a significant factor in determining prices to realistically set by a car rental firm relative to its competitors. Broker websites for conventional vehicle groups did not have a significant impact on the variation in occupation rates tested but impacted the absolute occupation values for these vehicle groups. Niche vehicle groups, similar to which was analysed in Oliveira et al. (2014), did not seem to have very elastic tendencies in comparison to more commonly rented vehicle groups due to the smaller fleet held and the lack of substitutability.

The most developed literature to date surrounding revenue management looks at the integration of capacity allocation and the pricing sub-problems. A discrete programming approach using a mixed integer non-linear program and a constraint programming model[17] was applied by Oliveira et al. (2018a) with the aim to maximise revenue *less* transfer and holding costs for a given number of rental requests. The constraint programming model outperformed other models proposed in the paper in terms of the profit obtained. Price levels set by the authors are based on an index, similar to Madden and Russell (2012). Higher price levels are allocated where higher demand occupation levels are apparent. The authors found that computational problems quickly become an issue when the dimensionality of the problems addressed are increased, and any additional value added, or differing magnitude of demand values tested, will impact on the optimal value obtained.

Oliveira et al. (2018b) extended their workings with the addition of a genetic algorithm to test the frequency of rental reservations by start time and rental length with real data from a Portuguese car rental firm. Most reservations arrived within 400 hours of the start time, with the majority of rental requests being 3-to-4-week periods. This is an interesting finding when

[17] A space-time network of up to 10 rental stations and 5 time periods is applied to map vehicle occupation and location across the time horizon. Each reservation represents a node which triggers a movement of cars with a reservation started or a transfer movement incurred, with a maximum of 2369 reservations tested.

considering the past literature which tends to analyse 1-and-2-days rental lengths. The values tested were a total of 40 rental stations across 4 pools and a 10’000-vehicle fleet across 5 vehicle groups, in which the model allows for upgrades. Additionally, acquisition, leasing, and upgrade penalty costs are added into the profit maximisation objective function. The model proposed is very representative of the revenue management processes in reality, however, the price-demand relationship still lacks in justification, especially because competitor pricing is not included.

Oliveira et al. (2019) address the above capacity-pricing problem with the addition of a competitor-based demand function. This harbours 2 states of being above or below the minimum price in the market. Different scenarios are created which are able to generate a set of possible solutions.[18] The authors give a good description on uncertainty parameters within the model, stating that an exact probability associated to a scenario does not necessarily have to be defined, only acceptable bounds for which the solutions can be representative of the problems addressed. This paper has a strong appeal to holistic modelling, particularly within revenue management practices. The level of the simulation complexity required higher computational capacity (3.46 gigahertz CPU and 48GB RAM). Extending this model to test variable fleet capacity and the relationship between capacity and fleet operational decisions would be an interesting perspective, especially with time horizons that exceeds 1-week which was used in the model.

The success of the implementation of revenue management systems at Europcar is outlined in Guillen et al. (2019). The systems have a broad base which accurately communicates capacity available, expected vehicle demand, and competitor pricing information into simulating own optimal prices. The revenue management systems are

[18] A scenario is represented by a set number of reservations, vehicle groups and a small or large market size factor which reflects the set price levels relative to the number of competitor’s in the market. “The minimum competitor price in the market is an uncertain parameter, within a limited range” (Oliveira et al., 2019, p641).

estimated to have improved daily revenue and fleet utilisation by 2% and 3%, respectively, across European chains.

3 Methodology

My analysis in Chapters 1 and 2 demonstrates that the contributions in the extant car rental literature, which covers contributions up to the year 2019[19], focus on i) 1 to 2 sub-problems alone, and ii) the optimisation of physical outcomes such as the movement and allocation of vehicles in the fleet planning process. The heatmap presented by Oliveira et al. (2017) and reprinted in Figure 3 concurs with my analysis. As a consequence, Oliveira et al. request the car rental literature produce contributions which increase realism to specific problems achievable via operational and overall integrations of sub-problems which I have discussed and displayed in Figure 2.

In terms of the individual sub-problems, price setting is the least pursued topic in the literature which, on one hand, may be understandable given the dynamic and unpredictable nature of consumer elasticity to different price levels set and different points in time; On the other hand, considering its importance to practice, the relevance of the literature is impeded because car rental companies' primary focus is financial sustainability (profitability).

My research aim therefore is to attempt to incorporate simultaneously a maximum number of realistic elements and their interaction within the car rental context. I achieve this through simulations which integrate the different sub-problems with their overlapping time horizons and the physical and financial aspects of the assets that are associated with the fleet planning process. This level of holistic modeling may be achieved using Statistical Activity Cost Analysis (SACA). SACA is designed to mutually consider how the costs and physical considerations for the vehicles and related decisions interrelated. All these elements make up the net of the input-output activities which represent aspects of car rental business processes. Additionally, the treatment of costs in the extant car rental literature is not accounting-based,

[19] I have not found many papers that are relevant to my research in the past 2 years, which is likely to Covid-19 and its occurrence at the beginning of 2020.

which allows me to further elevate the realism of my work by demonstrating how decisions within the physical dimensions affect the financial reporting in financial statements through to business analytics. The methodology was tested using a machine with specifications given in Table 2.

Table 2 CPU specifications

CPU specifications	10^{th} Gen Intel Core i7-10710U 4.70GHz, 1100MHz
Cores	6
Threads	12
RAM	4x8GB 2666MHz DDR4 CL16 DIMM Memory
Operating system	Windows 10 64bit

3.1 Research Design

I adopt a bottom-up simulation approach using Microsoft Excel 365, augmented with real data where available. Microsoft Excel is sufficient in terms of its flexibility to the addition of inputs and functions to generate scenarios that evolve alongside the outputs that can be fine-tuned to be representative and diverse concerning the derived outputs.

A total of 7 simulations were conducted. The focus in each simulation was to increase the complexity of the simulation from one implementation to the next. The increase in complexity is displayed from the additional functions, inputs and sub-problems added into the model. The simulations are run over a 6-month time horizon, in which data are generated from a range of scenarios that are derived from a multifarious combination of input variables.

The simulations translate their results into financial statements to display financial data in the car rental context. For example, gross profit is obtained from deducting the sum of revenue *less* transfer costs attributable to revenue earning vehicles over the simulation horizon, thus,

$$Gross\ Profit = \sum_{t=0}^{T} (Revenue - Transfer\ Costs). \tag{1}$$

To be more specific, the financial statements that are created from simulated data consist of the income statement and the balance sheet. The income statement is utilized to display the revenues and costs generated from the holding and movement of mobile assets; the balance sheet is utilized to display the extent of operational dimensionality in regard to vehicle groups, rental stations, and the flow on effects from the results obtained in the income statement.

The following sections explain the choices made in the implementation of the simulations. Section 3.2 illustrates the employment of Statistical Activity Cost Analysis (SACA) in the car rental context and the underlying limitations and assumptions used. In the same section I also explain the use of the Monte Carlo methodology which complements the implementation of SACA. Section 3.3 defines and explains the setup and execution of the incremental simulations, and following sections detail the methodology in primary versions.

3.2 Implementation of SACA

SACA is a useful tool for examining engineering assets (vehicles) due to their mutually dependent physical and financial dimensions, which is reflected in their time-dependent economic value (Colin et al., 2010). Optimising the management of these value calculations comes down to the risk and return of utilizing a certain number of vehicles in a fixed asset-like configuration system (Colin et al., 2006). SACA can be applied with tracking the life cycle of vehicles within rental companies from their acquisition to their disposal.

3.2.1 Aspects of SACA

The physical aspect of SACA in the car rental context can include the distance travelled via vehicle reservations and empty vehicle rebalancing actions incurred, maintenance schedules tracking vehicle downtimes as well as reductions of vehicle reliability with their use over time which can be used to trigger a maintenance event.

The financial aspect of SACA is utilized to measure the impacts of high-level decisions made within the operation of fleet management, which will determine how the fleet is allocated and utilised over time (Colin et al., 2012). Logically, as a vehicle is utilised over time, a reduction in its reliability follows. Understanding the impact of this in cost form encapsulates the essence of the implementation of SACA and how it can improve our understanding of the nature of the car rental industry being comprised of for-profit entities.

3.2.2 Limitations

For this simulation, tracking the movement of each individual vehicle and its reliability over time was outside of the bounds of this study. Fleet size is reflected by each individual vehicle being recorded as either "on-rent" or "idle", although the extent of physical tracking only goes as far as displaying the number of vehicles that are available for use at any station, for any vehicle group, and at any point in time. For unavailability considerations, a vehicle can be clearly distinguished from either fulfilling a reservation, being in the workshop for scheduled maintenance or being transferred from one station to another to fulfill an empty rebalancing decision.

The financial side of the vehicles is captured in my simulations more detailed through SACA, taking advantage of the translated financial statements from the simulations which assist in apprehending the financial aspect of the operation of the fleet between the rental stations over the planning horizon. For example, my simulations allow deriving cost distributions rather than point estimates. Varying the inputs directly influences the movement of vehicles in the system, and having independent costs associated with each reservation, vehicle held, empty rebalancing decision commenced, upgrade decisions made, and mandatory maintenance incurred, positively benefits the justification of testing different operational movements on the system given their financial risk and return consequences. Car rentals are full of uncertainty in terms of how many reservations will arrive at any point in time along with what vehicle and trip type is chosen. The ability to test the sensitivity of the inputs to the car rental pool system reinforces the quantification of the risks of certain scenarios coming to fruition in a realistic setting, which facilitates better decision making.

3.2.3 Monte Carlo Methods

To get a better understanding of how SACA is applied to a fixed asset-like system configuration, Monte Carlo simulation methodology was chosen to distribute and randomize input parameters and attach these to scenario functions. Simulation settings can be modified over time to be more diverse and representative of the physical and financial outputs and how they reflect realistic firm outcomes.

3.3 Simulation setup elements

Table 3 gives a list of indices which are included in the 7 Monte Carlo simulations. The following sub-sections then explore thematic aspects.

Table 3 Parameter and input indices for simulation versions

Parameter and input indices	Parameter and input definition
N	Fleet Size
$\mathbf{N_{CHC}}$	Fleet Size allocated to station CHC
$\mathbf{N_{ZQN}}$	Fleet Size allocated to station ZQN
$\mathbf{N_{DUD}}$	Fleet Size allocated to station DUD
M	Number of rental locations in the pool network
G	Vehicle group
i	Origin destination, $i = 1, 2, \ldots M$
j	Destination location, $j = 1, 2, \ldots M$
l	Alternate destination location (used for vehicle rebalancing critical value constraint)
d	Demand load per month to origin destination i
P	Price level
T	Vehicle rebalancing decision
V	Revenue obtained for vehicle group G from i to j
R_i	Round trip from station i
D_ij	Direct trip from station i to j

3.3.1 Arrival data and location

The geographical region of my pool network is the South Island of NZ (cf. Figure 4), which inspired a number of assumptions in my simulations, such as number of rental stations, number of vehicles, demand, trip possibilities etc. Based on the realistic assumptions, I now can attempt to optimise financial outcomes in the car rental context with respect to the operational movement of the fleet.

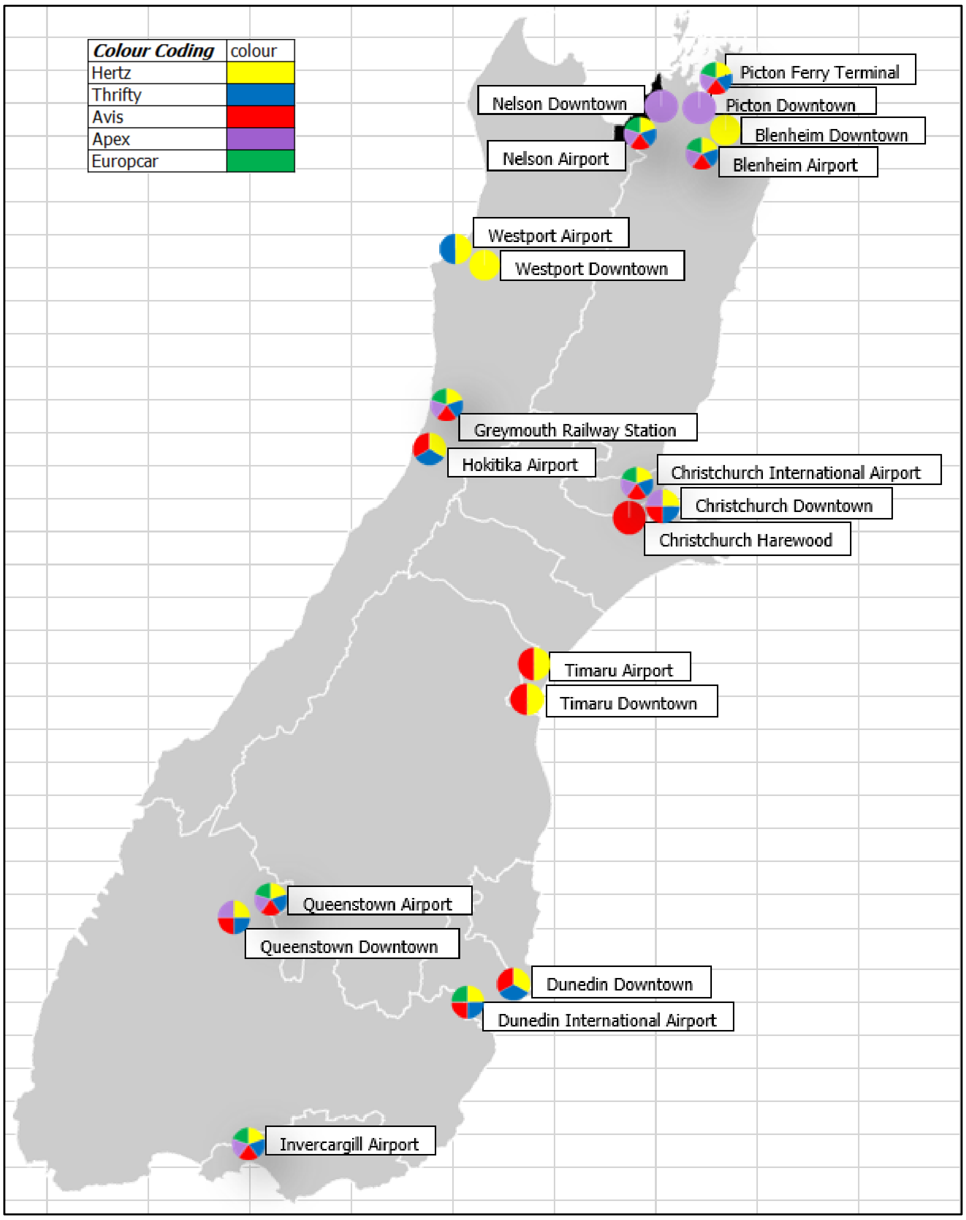


Figure 4 Geographical map of the South Island pool network by rental firms

Due to the dynamic and unpredictable nature of the reservation arrival times, booking requests were generated according to a uniform distribution. The arrival (and departure) frequency at various locations was chosen to mimic monthly domestic and international arrival data obtained from StatsNZ (https://www.stats.govt.nz/tools/nz-dot-stat). Figures 5 and 6 display 2 and 3 station simulation contexts and the associated trip types. The figures demonstrate that increasing the number of stations from 2 to 3 may seem trivial, however, the modelling becomes complex quickly: For rebalancing decisions, for example, it is clear from

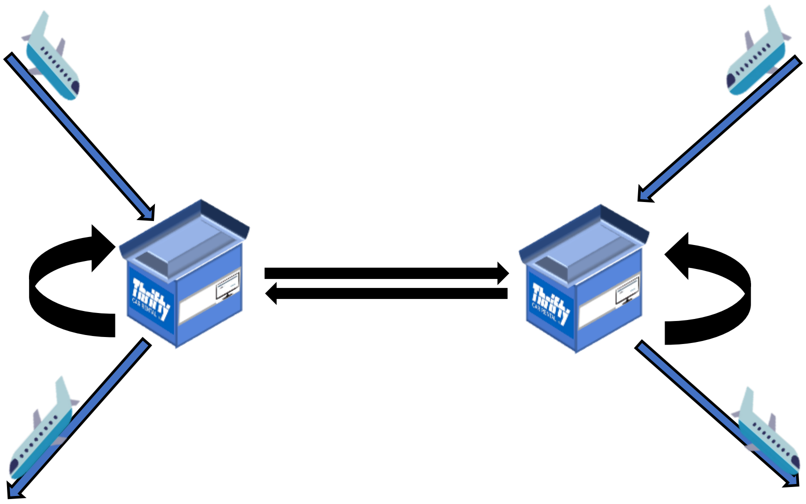


Figure 5 Two rental station simulation: trip types (black) and customer movements (blue)

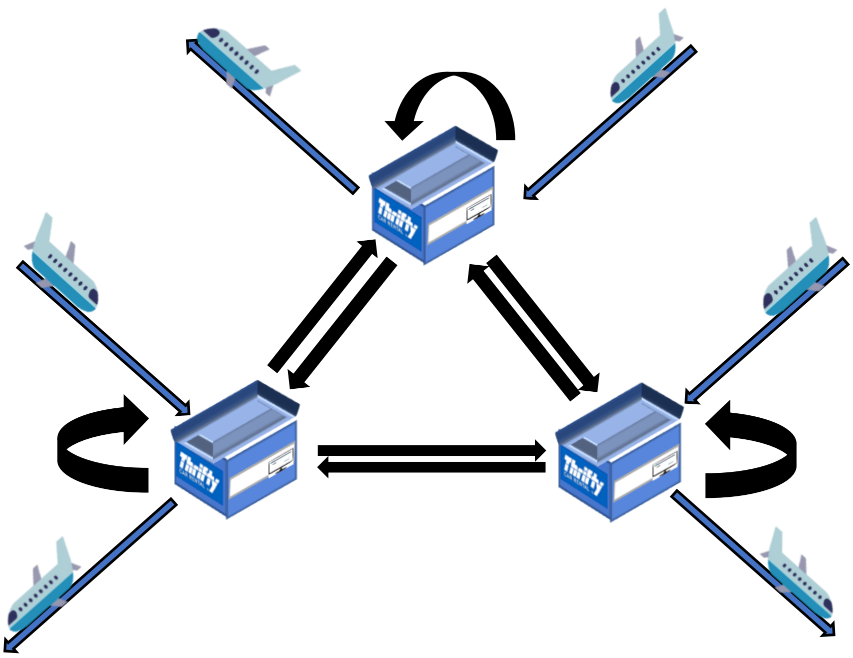


Figure 6 Three rental station simulation: trip types (black) and customer movements (blue)

where the empty vehicle transfers are made; however, in the 3 stations case, real options modelling needs to be considered.

3.3.2 Conceptualizing demand

Given the access to monthly arrival data to New Zealand airports, we can rationally assume that these arrival values are proportional to the number of customers that are expected to rent-a-car each month. Therefore, seasonality and demand load on the firm per month can be tested when using these arrival values. Even though these data give us access to the number of arrivals and departures to and from each port, they do not allow tracking of individual journeys. Therefore, the conceptualization of demand only goes as far as to cover the rationalization of the start location for each month. The end location, determined by the trip type chosen by the customer at each reservation, was then allocated as a varied input variable that exists to test the sensitivity of the system to different demand in terms of trip types.

3.3.3 Price levels

Price levels are derived from real data from Hertz' rental website which display real time prices to be charged to consumers based upon start location, rental length, end location and vehicle group. In February 2021, I have extracted 14 days' worth of price levels for each trip type and vehicle group and calculated average daily values to simplify the price allocation process.

3.3.4 Trip types

The trip type input is one of the primary variables in the simulation model which is utilized for sensitivity analysis in the aim to optimise financial outcomes for a car rental company. Trip type is differentiated by two different states: round trips and direct trips. A round trip R_I is defined by the start location I of the reservation being the same as the end location of the reservation. A direct trip D_{IJ} is defined by the start location I of the reservation differing from the end location J. To define an arbitrary trip type by reservation arrival, trip types are inputted and varied by rental station, which is representative of the start location, and cut-off values for these inputs are generated based on a co-efficient of variation independent of the start location to determine the end location for each reservation. This method was chosen due to the stochastic nature of the reservation arrival process.

3.3.5 Vehicle groups

Vehicle groups are selected based upon a co-efficient of variation independent of the start location and end location for each reservation. The rationalization of this is similar to the trip type process, in which the vehicle group chosen by the customer is of stochastic nature, although cut-off values can be set that are more representative of preferred car types and their upgrade hierarchy. This will be explained in greater detail in Section 3.3 where specific simulation versions are broken down in detail in order to display the dimensionality and complexity that is contained in each version.

3.3.6 Empty vehicle rebalancing

Empty vehicle rebalancing R is based upon vehicle groups at specific stations and controlled by critical values over the time horizon. For a vehicle rebalancing decision to be triggered, a vehicle group at a given station must reach a cumulative value of *less than* or *equal to* zero. This count is based on actual fleet values by vehicle group at each station *and* received reservations that inform us whether a reservation is already allocated for a specific vehicle to be taken from a station within the next reservation day. If a critical value is reached, a vehicle rebalancing commences and a vehicle rebalancing cost is incurred. Earlier simulation versions have a fixed cost set for all rebalancing decisions (similarly to Guerriero and Olivito, 2014; Li and Pang, 2017; Li and Tao, 2010; Oliveira et al., 2014, 2018a, 2018b, 2019; and You and

Hsieh, 2014). This is extended in later simulation versions to display transfer costs in terms of a matrix of the rental stations (similarly to Fink and Reiners, 2004; and Pachon et al., 2003, 2006) in which the transportation time is a linear function of the transportation cost (Song and Earl, 2007). In terms of the unavailability period during the vehicle rebalancing decision being conducted, it was decided to simplify that all rebalancing decisions could be conducted on an overnight basis (similarly to Li and Tao, 2010; Pachon et al., 2003, 2006; and You and Hsieh, 2014). Due to the size of the pool and the geographic features of the NZ South Island, it is not unreasonable to assume that any rebalancing decision could be completed overnight before the beginning of the next reservation day. This could be extended in the future to consider more transportation modes, for example, trucking or railway.

3.3.7 Scheduled maintenance

The scheduled maintenance constraint input serves 2 functions. Firstly, a cost function, which is represented by a fixed cost value of $200. A fixed value was used to simplify the maintenance state to be conducted on particular vehicle groups. My models can be easily extended to consider quality information on vehicle group specifications. Secondly, an unavailability function which reduces a rental station's fleet by 1 vehicle, subject to a particular vehicle group, for a length of 1 day. The vehicle is in unavailability status during the maintenance period and automatically returned to the fleet by the start of the next reservation day. The scheduled maintenance constraint is tested at 3 different states: 10%, 15%, and 20% unavailability status yet is increased to 1% granularity in the simulation V7.1. This technique was chosen to simulate random maintenance events that could come to fruition at any point in time over the simulation horizon when a reservation request arrives.

3.3.8 Vehicle upgrades

Vehicle group upgrades U are subject to a limited cascading function, being that a vehicle upgrade can only be to the next higher rental group in hierarchical order in terms of vehicle size (similarly to Ernst et al., 2010); Guerriero and Olivito, 2014); Madden and Russell, 2012; and Oliveira et al., 2014, 2018b, 2019). To determine whether an upgrade decision is to be considered, an empty vehicle rebalancing decision must first be triggered; An upgrade decision is only made where the transfer cost is greater than the revenue that can be achieved from the reservation. Logically, a firm would not want to fulfill a reservation that would cost more to incur than would be received in rental revenue.

3.4 Simulation V1

Simulation V1 aims to demonstrate the most basic composition of the car rental setting. Figure 7 displays the interrelated decisions that encapsulate the reservation fulfilment process for simulation V1 from the firm (grey outline) and customer perspective (black outline).

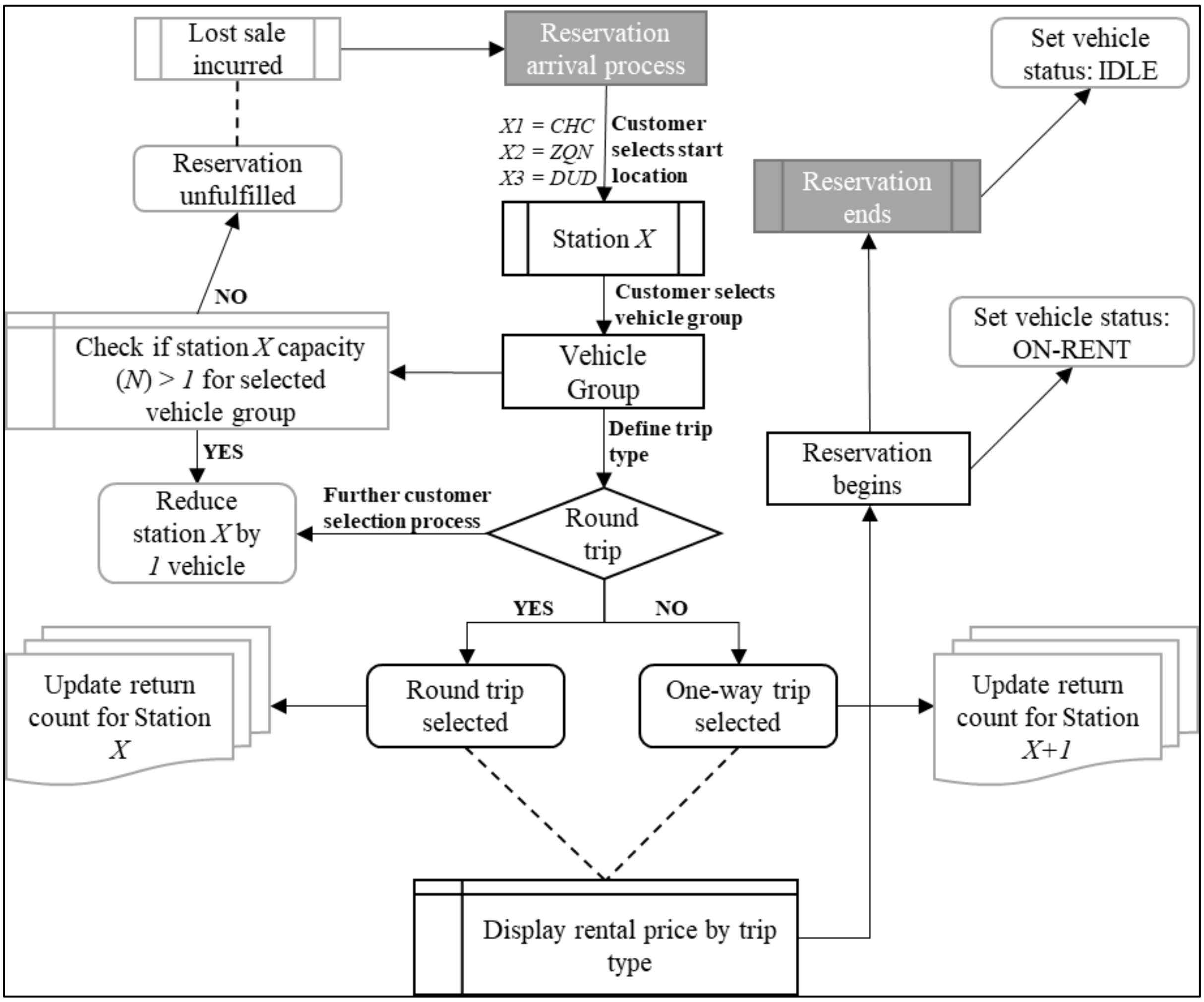


Figure 7 Flow chart for the simulation V1 car reservation fulfillment and renting processes

The pool network is composed of a total of 2 rental stations, Christchurch International Airport (CHC) and Queenstown International Airport (ZQN), and cars can be rented out for 1 day only. The fleet size N_i, put at both rental stations at the beginning of the simulation is determined as follows. It is 50% of the proportion of Real Arrival numbers RA_i, i={CHC, ZQN}, from August 2016 divided by the total real arrival numbers at both airports, thus,

$$N_{CHC} = \frac{AR_{CHC}}{(AR_{CHC}+AR_{ZQN})} \times 50\% \qquad \text{and} \qquad N_{ZQN} = \frac{AR_{ZQN}}{(AR_{CHC}+AR_{ZQN})} \times 50\%.$$

Both the 50% and August 2016 assumptions are simulation inputs which can be varied. Actual values are displayed in Table 3 below.

Reservation demand which is derived from arrival data to rental stations is static to test the basic capabilities of the model V1 at different combinations of input variables. From the starting arrival data collected, 429 monthly reservation values are randomly allocated across the starting month; the random allocation process is replicated based on a monthly sum of random values generated on the interval [0, 1], making a total of 2494 reservations over the 6-month time horizon. To generate a random value on this interval, the RAND() function is used and replicated over 2494 rows. The start time of the reservation can then be calculated by taking a cumulative of the inter-arrival times of the start process. The process which simulates car returns adds 1 day to the starting time.

For the system to determine the location from which a rental starts, I use the proportions between arrivals at each rental station and total arrivals to the 2-station pool network, thus,

$$Arrivals_{CHC} = \frac{AR_{CHC}}{(AR_{CHC}+AR_{ZQN})}, \text{ and } \quad Arrivals_{ZQN} = \frac{AR_{ZQN}}{(AR_{CHC}+AR_{ZQN})},$$

where $Arrivals_{CHC} + Arrivals_{ZQN} = 1$. I then use RAND() to determine in which rental station the individual trip begins. In summary, both processes which determine start time and trip start location generate random event arrivals using uniform distributions.

```
Start location
IF start process ≤ arrival prop CHC
        THEN
                PRINT "CHC"
        ELSE
                PRINT "ZQN"
ENDIF
```

```
End location
IF trip_type = "R_C" OR "D_QC"
        THEN
                PRINT "CHC"
        ELSE
                PRINT "ZQN"
ENDIF
```

```
Reservation day end
IF start_time ≤ daily arrival SUM
        THEN
                RETURN 1
        ELSE
                RETURN 2…,n
ENDIF
```

```
Trip type
IF start location = "CHC"
        THEN
                IF end_process ≤ R_C input
        THEN
                PRINT "R_C"
        ELSE
                PRINT "D_CQ"
FALSE;
IF start location ≠ "CHC"
        THEN
                IF end_process ≤ R_Q input
        THEN
                PRINT "R_Q"
        ELSE
                PRINT "D_QC"
ENDIF
```

Figure 8 Pseudo codes for V1 simulation components

Figure 8 represent pseudo codes for the main inputs in V1. The trip type is randomly determined by a set cut-off value between 0 and 1, and RAND(): If RAND() generates a number larger than the cut-off value, that particular trip-type is deemed a round-trip, otherwise it is a direct trip. Over 121 different simulation instances of V1, I have tested the influence of systematically changing these cut-off values.

I assume that the reservation for every simulated trip arrives 1 day in advance, so the incoming reservation at the start of the simulation horizon is expressed as day zero. From this, the start of a reservation which is defined as the vehicle being collected by the customer and leaving the rental station is expressed as day 1. Before vehicles can be recorded as departing from each rental station, a cumulative count must first be created to reflect the vehicles returning to each station on the following reservation day given their destination station. This cumulative count follows the reservation process that is calculated on each row and cumulatively counts all vehicles currently present and arriving within 1 day at each station. With reference to Figure 7, this count is represented as ‘update return count for station *X*’ and ‘update return count for station *X+1*’.

The fleet capacity at each station CHC and ZQN is determined by the trip start location and trip-type processes. In my simulation the cumulative count for these capacities will thus change over time the initial numbers N_CHC and N_ZQN (cf. Figure 9). Without rebalancing, it is expected that certain simulation parameter settings yield a zero capacity at 1 of the 2 rental stations.

Station CHC

```
Input N_CHC
        PLUS
IF start_location = "CHC"
        THEN
                RETURN -1
        ELSE
                RETURN 0
ENDIF
        PLUS
IF reservation day end X ≠ reservation day end X-1
        THEN
                RETURN cumulative count CHC
        ELSE
                RETURN 0
ENDIF
```

Station ZQN

```
Input N_ZQN
        PLUS
IF start_location = "ZQN"
        THEN
                RETURN -1
        ELSE
                RETURN 0
ENDIF
        PLUS
IF reservation day end X ≠ reservation day end X-1
        THEN
                RETURN cumulative count ZQN
        ELSE
                RETURN 0
ENDIF
```

Figure 9 Pseudo code for dynamic control of vehicle numbers at rental stations CHC and ZQN

.

Finally, a revenue process must be created for each reservation fulfilled. A basic VLOOKUP function can be used to display this information with an additional IF statement that checks whether the rental station has the sufficient capacity to fulfil the reservation. If the station is unable to fulfil the reservation, a lost sale is incurred. Otherwise, if a trip has been realised, revenues shown in Table 4 flows to the company. The amounts, as mentioned earlier, are based on actual 2021 rental costs for Hertz compact auto rentals.

Table 4 Trip costs

Trip type	Revenue [$]
CHC-CHC	169.07
ZQN-ZQN	110.45
CHC-ZQN	216.60
ZQN-CHC	234.49

The financial outcomes of a V1 simulation instance are summarized in financial statements. In Figures 10 and 11, I show a default income statement and balance sheet, respectively, which will be used in all simulation versions 1 to 7.

Tables 5 and 6 explain the individual levels contained in the income statement and balance sheet, respectively. Some of the values for assets, liabilities, equity, expenses and revenues are taken from Hertz’ 2019 financial statements (https://ir.hertz.com/financials).

Income Statement		
Sales revenue		$ -
Cost of sales		
Wages attributable to vehicle transfers	$ -	
Fuel costs attributable to vehicle transfers	$ -	
Empty vehicle transfers, net		$ -
Gross Profit		$ -
Expenses		
Depreciation of revenue earning vehicles	$ -	
Insurance	$ -	
Maintenance expense	$ -	
Selling, general and administrative expense	$ -	
Wages and Salaries *not* attributable to vehicle transfers	$ -	
Airport parking expense	$ -	
Operating profit		$ -
Lease Interest expense		
Vehicle	$ -	
Non-vehicle	$ -	
Interest expense, net		$ -
Net profit before tax		$ -
Income tax (28%)	$ -	
Net profit after tax		**$ -**
add back depreciation	$ -	
Net profit after depreciation addition		**$ -**

Figure 10 Simulation V1 income statement layout

Assets		
Cash and cash equivalents	$ -	
Accounts Receivable	$ -	
Revenue earning vehicles		
Compact Auto	$ -	
Total revenue earning vehicles		$ -
Leased airport buildings		
CHC rental building	$ -	
ZQN rental building	$ -	
Total leased airport buildings		$ -
Total assets		**$ -**
Liabilities		
Credit card authorisation bond	$ -	
Lease Interest Payable	$ -	
Lease Principle Payable	$ -	
Net vehicle loans	$ -	
Leased airport buildings liability	$ -	
Total Liabilities		**$ -**
Equity		
Contributed Capital	$ -	
Retained Earnings	$ -	
Total Equity		**$ -**
Total liabilities and equity		**$ -**

Figure 11 Simulation V1 balance sheet layout

Table 5 Simulation V1 income statement inputs

Income statement input	Explanation
Depreciation	Based on a 5% yearly value relative to the total value attributable to revenue earning vehicles in the balance sheet.
Insurance	Based on a proportion of a total firm insurance cost of $10M per year, segmented and reduced by number of stations considered and multiplied by 43% which is the proportion of Hertz' operations that are strictly based upon the leisure segment for car rental revenue generating activities.
Maintenance expense	Not implemented in this iteration
Selling, general and administrative expense	Rationalised from Hertz financial statements, 11% of sales revenue.
Wages and salaries not attributable to vehicle transfers	Assumed at 2 employees working at each station at one time at $20 an hour, 8 hours a day, 7 days a week, for 26 weeks (half year). A total of 4 employees are included in this version.
Airport parking expense	Data from the CHC and ZQN airport website was taken for yearly parking costs, and a 50% discount per park for the rental car firm was assumed. 15% of the starting fleet size for both CHC and ZQN are assumed to have allocated parking and are shuttled in and out of these parks when necessary to fulfill a reservation that is about to begin.
Interest expense; net	Based upon 7.68% of sales revenue, this value was achieved from averaging the interest expense for years 2017, 2018, and 2019 of accumulated interest.
Interest expense; non-vehicle	Based upon a 1% half annual interest payment on a cumulative of the total cost of the leased buildings at CHC and ZQN airport nodes
Interest expense; vehicle	Interest expense; net *less* Interest expense; non-vehicle
Net profit before tax	deducting expenses from gross profit
Net profit after tax	based on a 28% tax rate

Table 6 Simulation V1 balance sheet inputs

Balance sheet input	Explanation
Cash and cash equivalents	Based on 6.27% of sales revenue, as attributable to 2019 Hertz financial statements.
Accounts receivable	Based upon the reservations that were placed on the last day of the simulation horizon, yet to be fulfilled.
Revenue earning vehicles	Vehicle group is based on the market value of each vehicle type used by Hertz, multiplied by the number of vehicles held in each vehicle group.
Leased airport buildings; CHC	Assumed to be leased at a value of $1.1M.
Leased airport buildings; ZQN	Assumed to be leased at a value of $0.9M.
Credit card authorization bond	Based upon the accounts receivable value, representing a contingency held for possible damage, cleaning fees or additional fuel needed for the vehicle once it is returned to the relevant rental station and prepared for the next booking.
Lease interest payable	Based upon a 1% half annual value for total leased airport buildings, relative to lease interest expense; non-vehicle
Lease principle payable	Based upon a 5% half annual value for total leased airport buildings
Net vehicle loans	All vehicles assumed to be purchased outright, N/A
Leased airport buildings liability	Liability value for total leased airport buildings
Total equity	Assets - Liabilities
Retained earnings	Net profit after depreciation addition value, added to balance sheet
Contributed capital	Total equity – Retained earnings

The setup for 1 instance of simulation V1 was contained within 27 columns and approximately 2’500 rows in Excel, and more space was used to analyse results. In total, I ran 121 instances of simulation V1 to test 121 different parameter settings: I have changed in 10% steps the proportion of roundtrips R_{CHC} and R_{ZQN}. In order to store and display the simulation outputs in Excel, a series of 121 rows must be created containing all possible combinations. For the system to communicate between the scenario functions and the input variables which mutually influence each other, inputs R_{CHC} and R_{ZQN} are converted into INDEX and MATCH functions to return a particular scenario. The INDEX function acts as the return function down each column, whereas the MATCH function is a lookup function that links to an absolute referenced cell which harbours the scenario input. The scenario array is based upon the series that was created to display all possible combinations. Table 7 gives a summary of the car rental sub-problems tested in V1.

Table 7 Simulation V1 dimensionality and complexity tested

Car rental sub-problem test values	**Value**
Input scenarios	121
Number of rental pools	1
Number of rental stations	2
Fleet size	214 (N=50%)
N_{CHC}	135
N_{ZQN}	79
Number of vehicle groups	1
Empty vehicle rebalancing	N/A
Maintenance constraints	N/A
Customer types	N/A
Vehicle breakdowns	N/A
Vehicle acquisition and disposal	N/A
Vehicle upgrades	N/A
Price level	Based upon trip type
Price level strategy	N/A
Reservation cancellations	N/A
Number of competitors	N/A
Financial statements	Yes

3.5 Simulation V4

Simulation V4 differentiates from V1 with the extension of the pool network to a 3 rental station network and the implementation of 2 more vehicle groups. The rental length remains set at 1 day only. Figure 12 displays the reservation fulfilment process.

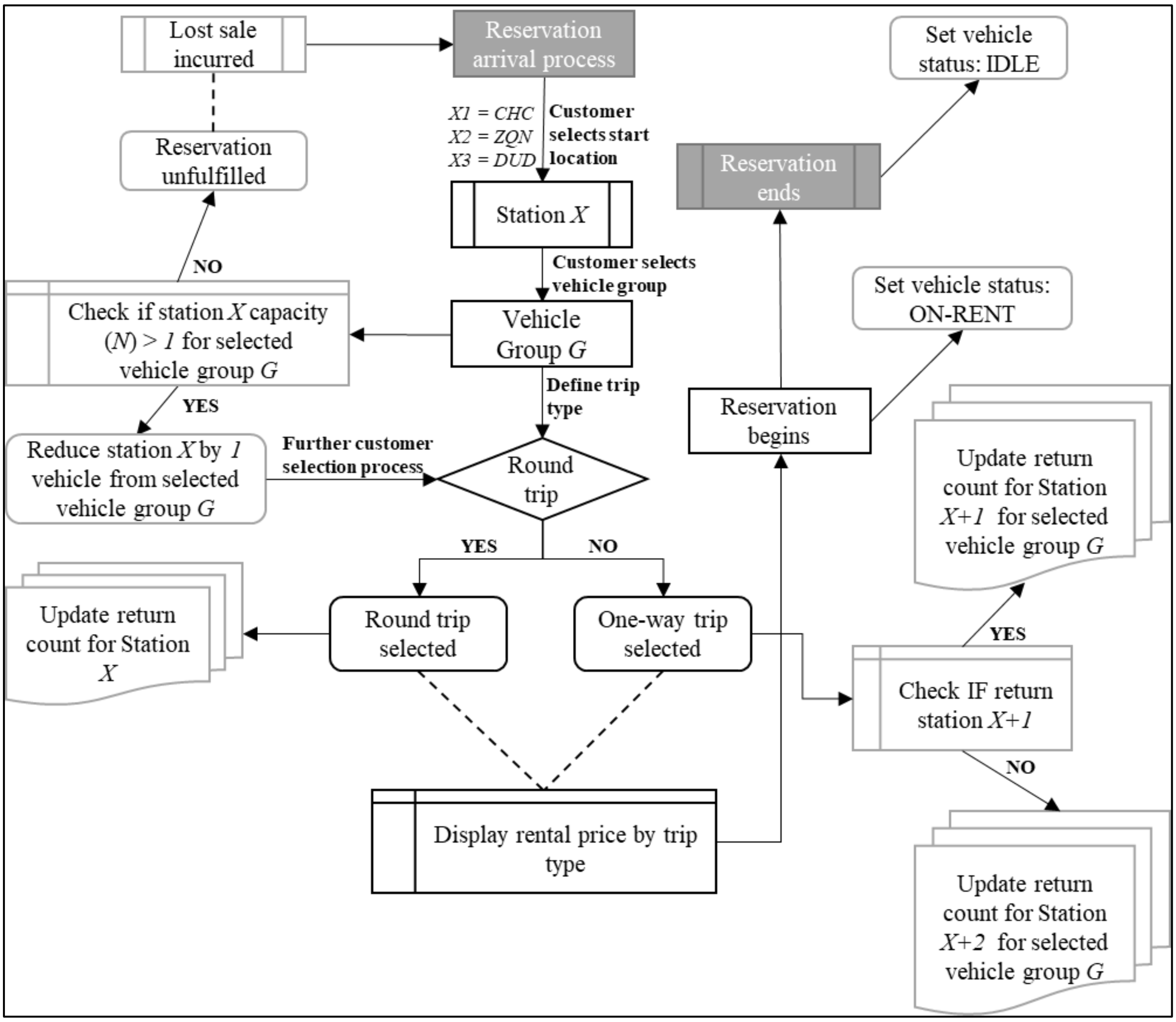


Figure 12 Simulation V4 pseudo code for reservation fulfilment process

The 3 rental stations considered are CHC, ZQN, and Dunedin International Airport, DUD. To interpret the physical and financial outcomes of fleet capacity, multiple fleet sizes N_i, i={CHC, ZQN, DUD}, are tested. To vary the fleet size, a variable factor f is additionally used. The strategic fleet deployment is more complex in V4 over V1 due to an additional rental station in the pool network. Thus, $N_{CHC} = \frac{CHC}{(CHC+ZQN+DUD)} \times N$, $N_{ZQN} = \frac{ZQN}{(CHC+ZQN+DUD)} \times N$, and $N_{DUD} = \frac{DUD}{(CHC+ZQN+DUD)} \times N$.

The event arrival processes for V4 are the same as for V1. Due to randomness, I have obtained 427 monthly reservation values allocated across the starting month, creating a total of 2’550 reservations over the 6-month time horizon. To determine the start locations, an additional Excel column must be added with an additional arrival calculation and a more complex IF statement. The new arrival location calculation is therefore, $Arrivals_{CHC} = \frac{AR_{CHC}}{(AR_{CHC}+AR_{ZQN}+AR_{DUD})}$, $Arrivals_{ZQN} = \frac{AR_{ZQN}}{(AR_{CHC}+AR_{ZQN}+AR_{DUD})}$, and $Arrivals_{DUD} = \frac{AR_{DUD}}{(AR_{CHC}+AR_{ZQN}+AR_{DUD})}$. The start locations are also similarly generated as in V1, using 2 fixed values between 0 and 1 which will vary the start location over the time horizon. Arrival probability $Arrivals_{ZQN}$ input is thus not used directly in the calculation, due to the ELSEIF statement encompassing the cut-off values for this proportion. The full combination goes in order of reservation arrival size as follows: 0 – CHC – ZQN – DUD – 1.

The trip type function must be adjusted with the addition of DUD and results in 9 possible trip types (3 rental stations and 2 trip types).

The 3 vehicle groups used in simulation V4 are Economy Car (EC), Compact Auto (CA), and Compact SUV (CSUV). These are deployed to rental stations at 35%, 40% and 25% proportions, respectively. CA is set at the highest proportion, as it is perceived as the most rented vehicle, followed by EC (Klein et al., 2019; Patel et al., 2018). The cumulative count for V4 is not only calculated on a per rental basis, but now includes a cumulative count per vehicle group to record idle and on-rent vehicles for each reservation day.

The main station process for recording vehicles leaving the station on each reservation day now has an additional column to represent the DUD rental station, and additional columns record each vehicle group G leaving and returning each reservation day. The additional functions for this process are shown in Figure 13 (station DUD is not displayed).

Station CHC	***Station ZQN***
Input G_CHC	Input G_ZQN
PLUS	PLUS
IF start_location = "CHC"	IF start_location = "ZQN"
THEN	THEN
RETURN -1	RETURN -1
ELSE	ELSE
RETURN 0	RETURN 0
ENDIF	ENDIF
PLUS	PLUS
IF reservation day end $X \neq$ reservation day end $X\text{-}1$	IF reservation day end $X \neq$ reservation day end $X\text{-}1$
THEN	THEN
RETURN cumulative count G_CHC	RETURN cumulative count G_ZQN
ELSE	ELSE
RETURN 0	RETURN 0
ENDIF	ENDIF

Figure 13 Pseudo code for vehicle control at rental stations CHC and ZQN

Three vehicle groups yield 9 unique trip types and thus require a total of 27 different price levels. As in V1, the rental prices are actual prices from Hertz and retrieved in the simulation from a matrix using INDEX and MATCH functions. An IF statement is linked with the revenue return function to check whether the station has the sufficient capacity for a particular vehicle group requested to fulfil a reservation. If the station is unable to fulfil the reservation, a lost sale is incurred, as was in V1.

Simulation V4 has 2 additional inputs that are implemented into the simulated scenario, being the R_D (round trip DUD) and a fleet size parameter. The extent to which the inputs are tested is displayed in Table 8.

Table 8 Simulation V4 primary input intervals simulated

Input	Intervals tested
Fleet size	2 - 20% and 30%, increments of 10%
R_{CHC}	21 – 55% to 75%, increments of 1%
R_{ZQN}	41 – 10% to 50%, increments of 1%
R_{DUD}	19 – 10% to 100%, increments of 5%

The total number of scenario iterations tested in simulation V4 is a multiplicative of the 4 input values, thus, $number\ of\ iterations = Fleet\ size \times R_C \times R_Q \times R_D$, which means a total of 32’718 scenarios are simulated. In running this simulation, a better understanding of the physical movements of each vehicle group from each station over time is necessary. Across all 32’718 V4 instances simulated, I obtain 327’180 output values relative to each unique combination of the scenario inputs. Table 9 summarises which of the car rental sub-problems that are tested in V4.

Table 9 Simulation V4 dimensionality and complexity tested

Car rental sub-problem test values	Value
Input scenarios	32’718
Number of rental pools	1
Number of rental stations	3
Fleet size	85 (N=20%); 128 (N=30%)*
N_{CHC}	60; 90
N_{ZQN}	23; 35
N_{DUD}	2; 3
Number of vehicle Groups	3
Empty vehicle rebalancing	N/A
Maintenance constraints	N/A
Customer types	N/A
Vehicle breakdowns	N/A
Vehicle acquisition and disposal	N/A
Vehicle Upgrades	N/A
Price level	Based upon trip type and vehicle group
Price level strategy	N/A
Reservation cancellations	Yes
Number of competitors	N/A
Financial statements	Yes
***:** The chosen values for N yield most stable and realistic simulation environments and are informed by analysis in simulations V2 and V3.	

Financial statement additions in V4 include additional revenue earning vehicles being included in the balance sheet (based upon market value), as well as an increased rental station lease value, with DUD valued at $0.7M, derived proportionately from the estimated values for CHC and ZQN. CHC is estimated from the lease values apparent on Hertz' balance sheet, divided by the number of rental locations in NZ, and a factor of 1.2 is added to reflect size.

3.6 Simulation V5

Simulation V5 adds to V4 in terms of tactical fleet deployment considerations and an increase to 7 vehicle groups. Figure 14 displays the interrelated decisions that encapsulate the reservation fulfilment process for simulation V5. The main additions to the process include a vehicle relocation system, which allows the car rental firm to reposition vehicles between stations, therefore increasing operational intervention and greatly reducing the number of unfulfilled reservations.

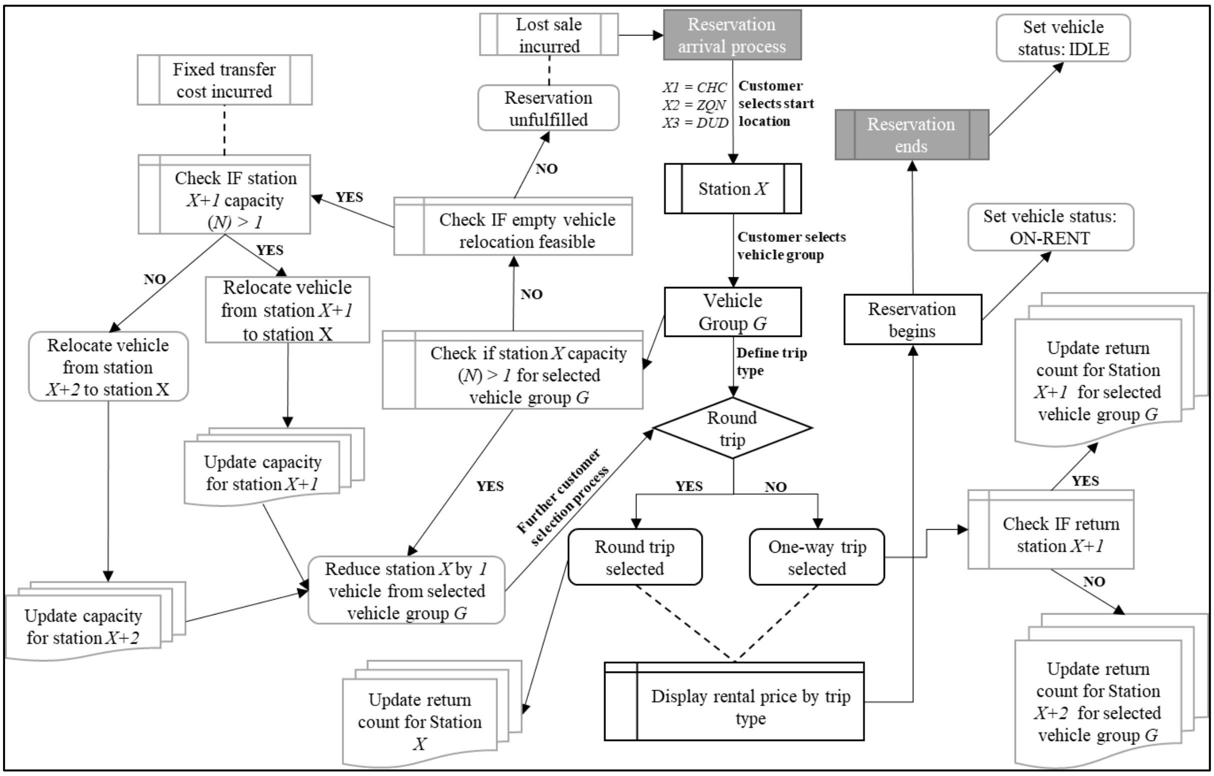


Figure 14 Simulation V5 reservation fulfilment process

Based on the same procedures described for V1 and V4, 441 monthly reservations are created from arrival data, which resulted in 2’590 reservations by simulation end. The vehicle groups added to the fleet are: Intermediate Sedan (ISED), intermediate SUV (ISUV), full-size SUV (FSUV) and the OTHER category (4x4 Utes, 12-seater commuter vehicles). The OTHER vehicle group includes multiple categories due to the niche attributes of the vehicles, and their rental frequency. I assumed a descending order of reservation preference for the vehicle groups as follows: $CA > EC > FSED > CSUV > ISUV > FSUV > OTHER$. The assumed rental frequencies are 26%, 18%, 16%, 14%, 10%, 8% and 8%, respectively. The proportions chosen for each vehicle group do not explicitly follow the literature in terms of their allocation, although inspiration was taken from their work and applied to the analysis of the fleet in the car rental context (e.g., from, Costa, 2019; and Patel et al., 2018).

The number of cumulative count functions is increased to 21, which reflects the 7 vehicle groups across 3 rental stations, and the main station columns are similarly increased. Additionally, the revenue return function array must be increased to encompass 9 trip types across 3 rental stations and 7 vehicle groups, creating a possible of 63 different price levels that exist in the simulation.

The largest addition to this simulation comes in the form of executing a working empty vehicle rebalancing function to coincide with the main stations. The empty vehicle rebalancing problem must operate in regard to the incoming reservations to each station and for each vehicle group. Therefore, an additional 21 columns must be added with their own unique function to control the 2’590 reservations.

To make the empty vehicle rebalancing system work, firstly, an IF statement with corresponding AND/ OR functions for the logical test are used to lookup the rental station by vehicle group *G* at its last incoming reservation point (*t – 1)*; secondly, a binary number system is used to display a value of 1 if an empty vehicle rebalancing decision is necessary based upon a critical value of 0 vehicles; Third, additional functions are included into the primary station functions to lookup which station is rebalancing its fleet to satisfy the decision of the pool network (cf. Figure 15)

EC_CHC example for rebalancing array

```
IF EC_CHC ≤ 0 AND EC_ZQN > 0 OR
        EC_DUD > 0
        THEN
                RETURN 1
        ELSE
                RETURN 0
```

EC_CHC main station function addition example

```
IF EC_CHC > 1 AND EC_ZQN_transfer_array = 1
        THEN
                RETURN -1
        ELSE
                RETURN 0
ENDIF
        PLUS
IF EC_ZQN ≤ 1 AND EC_DUD_transfer_array = 1
        THEN
                RETURN -1
        ELSE
                RETURN 0
ENDIF
```

Figure 15 Pseudocode for rebalancing functions V5

At first, the function ran into some problems with CHC tested as the primary station to satisfy all rebalancing decisions if possible. Unfortunately, this created circular references[20] which resulted in incorrect calculations where the station functions attempted to fulfil the same task in tandem. After some thought, a solution was conjured through the designation of each rental station as primary and secondary rebalancing node to a particular station. Given the geographical distribution of the pool network, CHC was set as the primary node for ZQN, ZQN the primary node for DUD, and DUD the primary node for CHC. The secondary rebalancing

[20] A circular reference is where a formula refers to another cell (or its own) more than once in a chain of simultaneous calculations which creates a loop and confuses the system.

nodes are as follows: CHC for DUD, ZQN for CHC, and DUD for ZQN. Since each rebalancing decision can only be in one state at a particular reservation, the simulation runs smoothly with the addition of the empty vehicle rebalancing sub-problem. An example is given in Figure 14 in the form of a pseudocode. Each time an empty vehicle repositioning decision takes place, a fixed transfer cost of $280 is incurred to represent a rounded average of assumed wage and fuel costs, based upon the distance between the matrix of rental stations in the pool network.

The rebalancing array function is simple: it looks up whether the station has reached 0 vehicles, and if the primary or secondary station can fulfil the rebalancing request. The functions added to main station cells required more thought, however. Firstly, an INDEX and MATCH function is used to lookup the rebalancing array to return the vehicle to necessary station by the next reservation day. Secondly, an IF statement looks up whether the primary rebalancing node has the capacity to rebalance its vehicles, based on whether the node has greater than 1 vehicle for that particular vehicle type. This is followed by the secondary IF statement which actuates if the primary IF statement does not return a true value. For example, if EC_CHC needed an empty vehicle repositioning for a particular vehicle group, this could be fulfilled by either ZQN or DUD nodes depending on how the vehicles are distributed at each stage of the simulation. The employment of an empty vehicle rebalancing process greatly improves the robustness of the model, as it gives insight into the extent to which empty vehicle rebalancing decisions can be used, as well as the costs that result from exhausting the rebalancing system.

The extent to which the inputs are tested in simulation V5 is displayed in Table 10.

Table 10 Simulation V5 primary input intervals simulated

Input	Intervals tested
Fleet size	2 - 20% and 30%, increments of 10%
$\mathbf{R_{CHC}}$	21 – 55% to 75%, increments of 1%
$\mathbf{R_{ZQN}}$	41 – 10% to 50%, increments of 1%
$\mathbf{R_{DUD}}$	19 – 10% to 100%, increments of 5%

Using a total of 23 output variables that are generated for 32'718 scenarios, the simulation will output a total of 752'514 output values relative to each unique combination of the scenario inputs. To conclude, the implementation of simulation V5, a summary of the car rental sub-problems tested, and their inherent complexity is displayed in Table 10.

Table 11 Simulation V5 dimensionality and complexity

Car rental sub-problem test values	**Value**
Input scenarios	32'718
Number of rental pools	1
Number of rental stations	3
Fleet size	88; 131
N_{CHC}	54; 80
N_{ZQN}	32; 47
N_{DUD}	2; 4
Number of vehicle Groups	7
Empty vehicle rebalancing	Based upon primary and secondary rebalancing nodes
Maintenance constraints	N/A
Customer types	N/A
Vehicle breakdowns	N/A
Vehicle acquisition and disposal	N/A
Vehicle Upgrades	N/A
Price level	Based upon trip type and vehicle group
Price level strategy	N/A
Reservation cancellations	Yes
Number of competitors	N/A
Financial statements	Yes

Financial statement additions include additional revenue earning vehicles displayed in the balance sheet, and values for wages (70%) and fuel costs (30%) are translated into the income statement.

3.7 Simulation V7

Simulation V7 adds to V5 a more robust fleet assignment and capacity allocation algorithms. These come in the form of i) a vehicle rebalancing matrix which returns costs incurred to the pool network based on transfer locations, and ii) a limited cascading upgrade system which can allocate vehicles of the next higher vehicle group to a customer if a rebalancing decision is not feasible. Figure 16 displays the reservation fulfilment process for simulation V7.

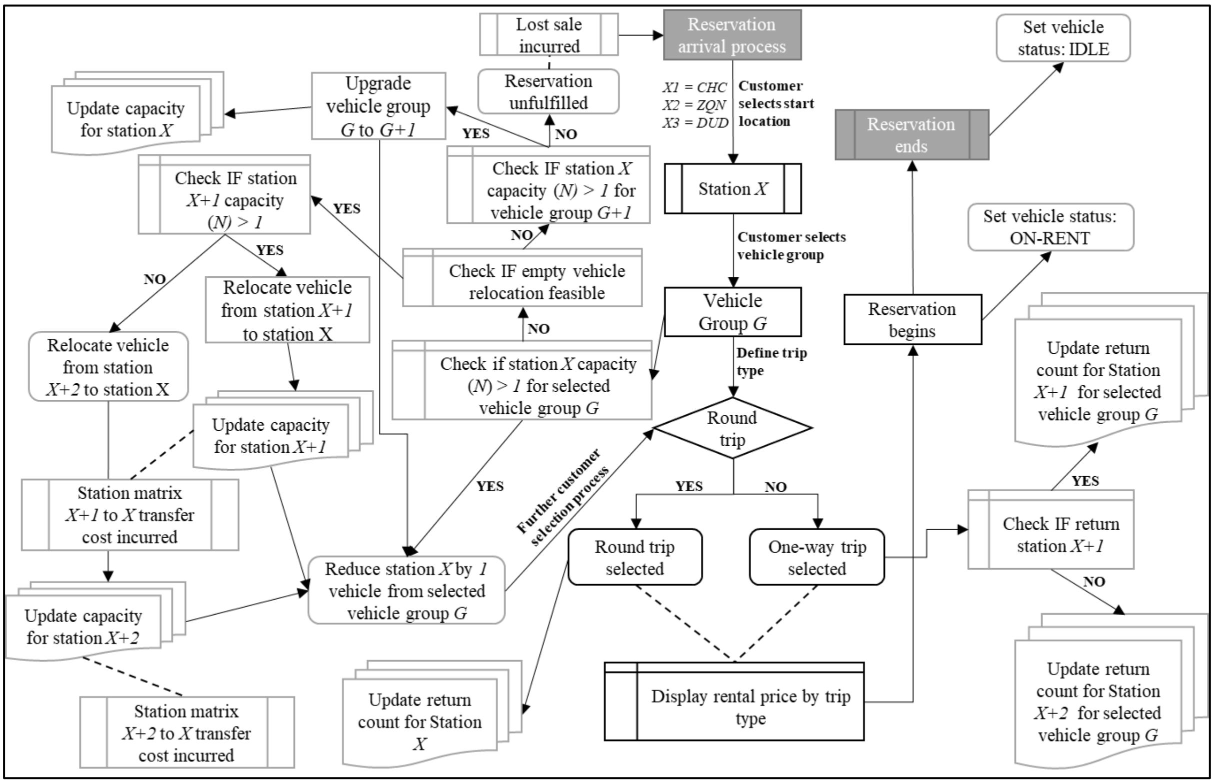


Figure 16 Simulation V7 reservation fulfilment process

The outputs for the fuel and wage matrix (cf. Figure 17) are calculated using the input variables, and the distance and time matrices. The distance matrix determines how many kilometres are travelled which determines how many litres of fuel are consumed between each rental station. The time matrix determines how many hours an employee must be allocated to complete a rebalancing decision and return back to the station where they originated from, plus a vehicle turnover window at the rebalancing station of 15 minutes. The wage and fuel matrix values are then combined to create the rental station cost matrix which is used with an INDEX and MATCH function to display the total cost for each empty vehicle rebalancing decision incurred across the simulation horizon. The lookup function has the capability to return both the primary and secondary station node costs depending on the node used to rebalance vehicles to the station in need of additional capacity. The implementation of this matrix strongly improves the simulations' ability to further reflect reality in the car rental context, as it is directly reflective of the operational movements from each individual station and the inherent costs associated with these operational decisions.

distance matrix (km)			
	CHC	*ZQN*	*DUD*
CHC	0	473	384
ZQN	473	0	251
DUD	384	251	0

time matrix (minutes)			
	CHC	*ZQN*	*DUD*
CHC	0	349	299
ZQN	349	0	251
DUD	299	251	0

wage matrix ($)			
	CHC	*ZQN*	*DUD*
CHC	0	242.67	209.33
ZQN	242.67	0	134.00
DUD	209.33	134.00	0

fuel matrix ($)			
	CHC	*ZQN*	*DUD*
CHC	0	104.06	84.48
ZQN	104.06	0	55.22
DUD	84.48	55.22	0

Input	Value
Changeover interval	0.25 hours
Employee hourly rate	$20
Fuel usage in kilometres per litre	10
Fuel cost	$2.20 per litre
Number of employee trips	2
Minute to hour adjustment	60

cost matrix ($)			
	CHC	*ZQN*	*DUD*
CHC	0	346.73	293.81
ZQN	346.73	0	189.22
DUD	293.81	189.22	0

Figure 17 Matrices used for vehicle rebalancing decisions

To better demonstrate the interplay between the fleet assignment and capacity allocation sub-problems, a scheduled maintenance allocation and an upgrade system was added into V7 also. The scheduled maintenance implementation was created using a maintenance downtime variable input, which was tested at 10%, 15% and 20% values. These values are added into the scenario array and tested with the other round-trip inputs. Three random value arrays must once again be created for rental station, vehicle group and scheduled maintenance respectively on the interval [0, 1]. A maintenance index array is then created which lookups whether the scheduled maintenance input falls within the cut-off ranges, and if yes, then the station at which the particular vehicle is stationed is removed from the fleet for the current day, a fixed maintenance cost of $200 is incurred, and the car returned the following day.

Reservation demand was upgraded to be of a dynamic and seasonal nature based upon each monthly arrival values given by StatsNZ to each rental station across the simulation horizon. Reservations are set between August and January to get an appreciation of the buildup of reservation demand from the mid-year to the greater demand loads attributable to the end-year summer months of December and January. Monthly reservation values are 441, 427, 465, 453, 584 and 575, respectively, making a total of 2945 reservations over 184 days, based upon a simulation start day of August 1, 2016.

As stated earlier, the upgrade system that has been allocated into this model is a limited cascading upgrade allocation function which is incurred if the empty vehicle rebalancing cost would be greater than the revenue obtained from the reservation. The order of the upgrade hierarchy is

$$EC < CA < FSED < CSUV < ISUV < FSUV < OTHER,$$

which is similar to the order of reservation preference by vehicle group. Note that EC itself cannot be used as an upgradable vehicle, although the vehicle can be upgraded to the next vehicle group class, being the CA. Secondly, the OTHER category cannot feasibly be used as an upgradable vehicle due to the niche properties associated with it in comparison to the FSUV. Therefore, it is not used as part of the upgrade hierarchy, and consequently, FSUV can only be used as a vehicle group to fulfill reservations for ISUV but cannot be upgraded itself. To create this function in Microsoft Excel 365, an upgrade array must be made for each vehicle group and station which utilizes 2 IF statements to lookup whether the vehicle group in the cell can feasibly be upgraded, which returns a value of 1, and a lookup function to determine whether a vehicle group that is 1 class lower on the hierarchy needs an upgrade decision, which returns a value of -1. This array is used with another INDEX and MATCH function that looks up which vehicles are “removed” and “added” to the station based upon which upgrade was used, and an auxiliary constraint to prevent an upgrade and empty rebalancing decision are made within the same row. The reason an addition lookup function was used was to not adjust the original functions but to remove vehicles from the fleet based upon their original start location. By adding the same vehicle back to the fleet at the same point and removing the upgraded vehicle simultaneously, it essentially cancels out the original statement as if the original vehicle had never left the station. This made the simulation layout easier to comprehend and kept the original functions that were used in the simulation’s inception in V1.

Simulation V7 uses the same round-trip input intervals implemented in V5. Another important addition in V7 comes in the form of the fleet size input being fixed at 30% and a variable maintenance input being added between 10-20%. The reason the fleet size input was fixed to this particular value is made based on its performance during earlier simulation versions. The round-trip proportions R_i, i={CHC, ZQN, DUD}, tested are displayed in Table 12.

Table 12 Simulation V7 primary input intervals simulated

Input	**Intervals tested**
Maintenance	3 – 10% to 20%, increments of 5%
$\mathbf{R_{CHC}}$	21 – 55% to 75%, increments of 1%
$\mathbf{R_{ZQN}}$	41 – 10% to 50%, increments of 1%
$\mathbf{R_{DUD}}$	19 – 10% to 100%, increments of 5%

The number of physical output variables being tested stays the same as for V5, although an additional financial variable is added which comes in the form of a SUM of maintenance costs over the simulation horizon. This makes a total of 21 physical output variables and 3 financial output variables. With 49’077 scenarios and 24 output variables tested, the simulation will output a total of 1’177’848 output cells relative to each unique combination of the scenario inputs.

To conclude the implementation of simulation version 7, a summary of which car rental sub-problems are tested and what their complexity is, is contained in Table 13.

Table 13 Simulation V7 dimensionality and complexity

Car rental sub-problem test values	**Value**
Input scenarios	49’077
Number of rental pools	1
Number of rental stations	3
Fleet size	133
N_{CHC}	81
N_{ZQN}	48
N_{DUD}	4
Number of vehicle Groups	7
Empty vehicle rebalancing	Based upon primary and secondary rebalancing nodes
Maintenance constraints	Variable maintenance downtime, fixed cost
Customer types	N/A
Vehicle breakdowns	Yes
Vehicle acquisition and disposal	N/A
Vehicle Upgrades	Limited cascading upgrades considered
Price level	Based upon trip type and vehicle group
Price level strategy	N/A
Reservation cancellations	Yes
Number of competitors	N/A
Financial statements	Yes

The financial statements section is supplemented with the addition of a value for the maintenance expense, 3 financial statements are created based upon the testing of 10%, 15% and 20% maintenance constraints. The vehicle upgrade process does neither have a direct revenue nor cost associated with it. The extent to which the number of upgrades used assists the financial outcomes in each scenario iteration, can be investigated using sensitivity analysis. This will be evaluated in Chapter 4.

With the addition of a more representative empty vehicle rebalancing cost system along with fleet assignment and capacity allocation consideration, it is justifiable that the integration of a number of car rental sub-problems, as outlined in Oliveira et al. (2017), have been successfully implemented to represent a plausible reflection of reality in the car rental context.

4 Results and discussion

The goal of this section is to describe and discuss the results from implementing the proposed simulations from Chapter 3. This includes descriptive statistics and regression analyses with which I highlight potential relationships (if any) between input parameters and how they translate into scenario values in terms of their mutual influence on the financial outcome variables. Secondly, financial statements are displayed to give an overview of the extent to which simulation versions translate into financial data to represent how useful the simulations are at generating financial statements, and thus would include useful information for potential investors, lenders, and other stakeholders. Thirdly, vehicle occupation rates and availability at each station for each vehicle group over time is mapped and tested to get an idea of the operational complexity of the pool network and the fleet management concepts that are expanded throughout the simulation versions.

4.1 Simulation V1

To analyse the results, I have ordered the 121 V1 simulation instances from largest to smallest revenue values generated. Additionally, if at the end of the simulation horizon a rental station has run out of vehicles, it is marked as 'failure node', and not further considered. The rational for excluding non-viable simulation instances is that we are interested in optimising financial outcomes within a feasible operational context. Therefore, all rental stations in the pool network must have sufficient vehicles at the end of the simulation horizon to carry on their operations into the foreseeable future, i.e., for at least 6 months. Out of the 121 scenarios tested, 106 scenarios received a 'failure node'. The characteristics of the 15 'passing node' instances for proportions of round-trips R_{CHC} and R_{ZQN} are given in Table 14.

Table 14 V1 input variable characteristics which yield passing node instances, and revenue generated. Starting vehicle numbers are 135 for CHC, and 79 for ZQN.

Revenue		R_C	R_Q	CHC	ZQN
$	**791'494**	40%	0%	97	117
$	**778'384**	40%	10%	21	193
$	**716'672**	50%	10%	199	15
$	**699'724**	50%	20%	99	115
$	**678'725**	50%	30%	4	210
$	**638'142**	60%	30%	135	79
$	**623'307**	60%	40%	49	165
$	**569'142**	70%	40%	206	8
$	**554'479**	70%	50%	121	93
$	**537'919**	70%	60%	25	189
$	**484'789**	80%	60%	179	35
$	**468'401**	80%	70%	84	130
$	**395'607**	90%	80%	166	48
$	**378'012**	90%	90%	64	150
$	**309'529**	100%	100%	135	79

Thus, 'passing node' instances heavily depend on an appropriate combination of input values for R_{CHC} and R_{ZQN}. These combinations are displayed in Figure 18 and could serve for a 2nd iteration of simulation instances which test a finer granularity of input value combinations for R_{CHC} and R_{ZQN} in the vicinity of those which yield 'passing nodes'. This could, for example, be used to assess maximum revenue generated holding either of the 2 proportions constant.

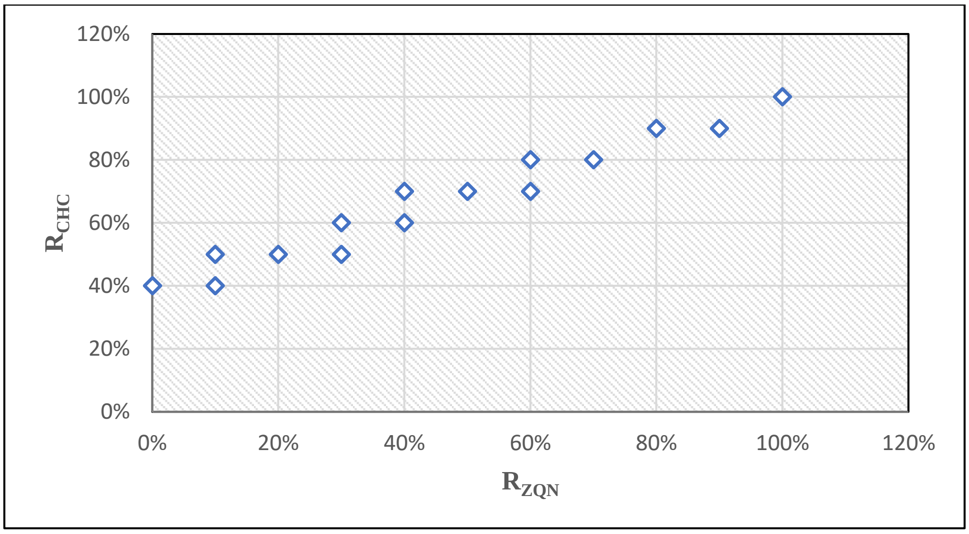


Figure 18 Simulation V1 optimised combinations of R_{CHC} and R_{ZQN} scenarios

The final vehicle numbers at either station shown in Table 14 are obtained from 1 simulation instance for the various input value combinations. Future work may explore, through a Monte Carlo setup, the dependence of the vehicle holdings at the 2 rental stations and the simulation length.

Tables 15 and 16 display the income statement and balance sheet, respectively, for V1. While there is no preference for which of the 15 simulation instances I display financial information, I have randomly chosen the R_{CHC}=60% and R_{ZQN}=30%. Note that the final vehicle numbers at each station equals the starting number of vehicles. Apart from this result being obtained by chance (the uniform distributions which determine start times and trip types will generate different results if the seed values for RAND() are altered), it also does not mean that no direct trips have occurred because it may not be the same 135 vehicles which end up in CHC.

A direct consequence from the trip prices assumed originally, Table 14 reveals that total revenue is negatively correlated with the fraction of round-trips in both ZQN and CHC. For practical purposes, and based on the financial statements shown below, a manager now has available simulation results which translate into what-if scenarios, which can be used to guide strategy. If, for example, in the past few months he or she has observed that the fraction of round-trips is high (or higher) than direct trips, the following two options present themselves: firstly, round-trip prices, relatively direct-trip prices, can be increased; secondly, the manager may entice customers to choose direct trips more frequently through combination offers with accommodation in or around the other rental station and corresponding flight arrangements.

Table 15 Simulation V1 income statement. Entries based on V1 instance No 40, in which RCHC=60% and RZQN=30%.

Car Rental V1 Ltd Income Statement For 6-month ended March 31st, 2017		
All values in NZ dollars ($)		
Sales revenue		638 142
Cost of sales		
Wages attributable to vehicle transfers	0	
Fuel costs attributable to vehicle transfers	0	
Empty vehicle transfers, net		0
Gross Profit		638 142
Expenses		
Depreciation of revenue earning vehicles	95 096	
Insurance	100 000	
Maintenance expense	0	
Selling, general and administrative expense	70 196	
Wages and Salaries *not* attributable to vehicle transfers	116 480	
Airport parking expense	54 690	
Operating profit		201 680
Lease Interest expense		
Vehicle	29 013	
Non-vehicle	20 000	
Interest expense, net		49 013
Net profit before tax		152 667
Income tax (28%)	42 747	
Net profit after tax		**109 920**
add back depreciation	95 096	
Net profit after depreciation addition		**205 016**

Table 16 Simulation V1 balance sheet. Entries according to assumptions discussed in Section 3.4

Car Rental V1 Ltd **Balance Sheet** **For 6-month ended March 31st, 2017**		
All values in NZ dollars ($)		
	Assets	
Cash and cash equivalents	238 620	
Accounts Receivable	3 239	
Revenue earning vehicles		
Compact Auto	3 803 850	
Total revenue earning vehicles		3 803 850
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
Total leased airport buildings		2 000 000
Total assets		**6 045 709**
	Liabilities	
Credit card authorisation bond	3 239	
Lease Interest Payable	20 000	
Lease Principle Payable	100 000	
Net vehicle loans	0	
Leased airport buildings liability	2 000 000	
Total Liabilities		**2 123 239**
	Equity	
Contributed Capital	3 717 453	
Retained Earnings	205 016	
Total Equity		**3 922 470**
Total liabilities and equity		**6 045 709**

The profitability ratios and the Z-score which are the result of our financial statement example are displayed below in Table 17 with reference to Hertz' ratios (https://www.gurufocus.com/term/zscore/OTCPK:HTZGQ/Altman-Z-Score/Hertz-Global) as of 2021.

Table 17 Car Rental V1 Ltd financial ratios and z-scores compared with Hertz (2021)

	Hertz 2021	**Car Rental V1 Ltd**
ROE	(132) %	16%
ROA	(0.78) %	11%
Profit margin	(15) %	32%
Z-score	0	0.31

Similar to Hertz, our car rental firm exhibits a low Z-score (<1), which indicates that the current trip structure and fleet size, although profitable and exhibiting good liquidity, indicates a probability of financial distress across a range of financial data existent in Car Rental V1 Ltd.'s financial statements. These findings give a strong motivation for local and regional management to influence the trip-type proportions through price setting.

Figure 19 displays how the operational movements occur across the simulation time horizon for outcome scenario 40. The fluctuation of vehicles at each rental station over time is fairly stable for this scenario. ZQN reached a low point of 48, and CHC a maximum of 156 capacity at which point there must be 10 cars on the road. Figure 20 contains the number of reservations that arrive each day across the simulation horizon. The pattern shown also suggests a constant moving average which is a direct consequence from how the reservation arrival process has been modelled.

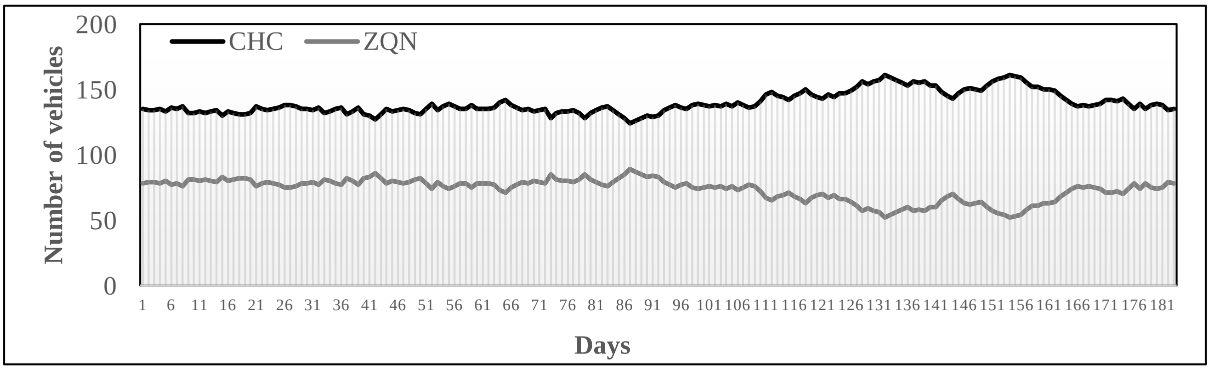


Figure 19 Simulation V1 fleet movements: scenario 40

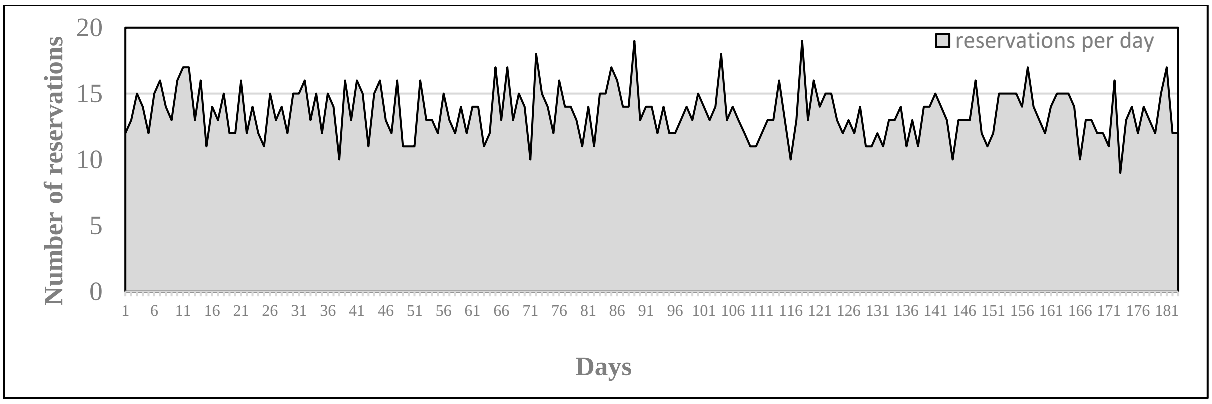


Figure 20 Simulation V1 reservations per day

4.2 Simulation V4

Simulation V4 improves upon V3 with the addition of a 3rd vehicle group, being of the compact SUV category. Since this simulation does not exhibit as large of an increase in terms of car rental fleet management sub-problems. The top 15 (out of 1000) instances which are characterized as a 'passing node' for proportions of round-trips R_{CHC}, R_{ZQN} and R_{DUD} are displayed in Table 18.

Table 18 V4 input variable characteristics which yield passing node instances, and revenue generated. Starting vehicle numbers are 95 for CHC, 35 for ZQN, and 3 for DUD

	Round trips			CHC			ZQN			DUD		
Revenue	R_{CHC}	R_{ZQN}	R_{DUD}	EC	CA	CSUV	EC	CA	CSUV	EC	CA	CSUV
$ 604'266	64%	23%	10%	12	15	10	21	30	21	12	6	2
$ 602'041	61%	10%	20%	23	23	12	6	13	17	16	15	4
$ 602'002	61%	10%	15%	24	24	12	6	13	17	15	14	4
$ 601'982	61%	10%	10%	22	26	13	8	13	17	15	12	3
$ 601'880	64%	23%	15%	14	13	9	19	30	21	12	8	3
$ 601'753	61%	10%	25%	22	21	12	6	13	17	17	17	4
$ 601'638	61%	10%	30%	22	21	13	6	12	16	17	18	4
$ 601'318	64%	23%	20%	13	12	9	19	30	21	13	9	3
$ 600'309	64%	23%	25%	12	10	9	19	30	21	14	11	3
$ 600'194	64%	23%	30%	12	10	10	19	29	20	14	12	3
$ 599'228	64%	22%	10%	13	19	11	20	26	19	12	6	3
$ 598'920	64%	18%	10%	22	27	16	11	16	14	12	8	3
$ 598'555	64%	19%	10%	21	28	16	12	16	14	12	7	3
$ 598'359	61%	11%	10%	18	25	11	12	15	19	15	11	3
$ 597'871	61%	11%	20%	19	22	10	10	15	19	16	14	4

Figure 21 displays the fluctuations for the 'passing node' instances do not harbour much fluctuation for R_{CHC}, yet values for R_{ZQN} and R_{DUD} fluctuate between 10%-34% and 10%-

100%, respectively. The R_{CHC} input exhibits a strong influence over the generation of maximum financial outcomes, which allows inputs R_{ZQN} and R_{DUD} to fluctuate and still yield a 'passing node' which exhibits a high revenue value. Unlike V1, there is not a clear negative relationship between round-trips and revenue, although, R_{ZQN} and R_{DUD} are able to operate at low proportion if R_{CHC} has an appropriate amount of round-trips to supplement the capacity of rental stations ZQN and DUD. The relationship of parameters R_{CHC}, R_{ZQN} and R_{DUD} over the top 1000 'passing nodes' are better displayed with a 3D mesh, which gives a far better interpretation of these values.

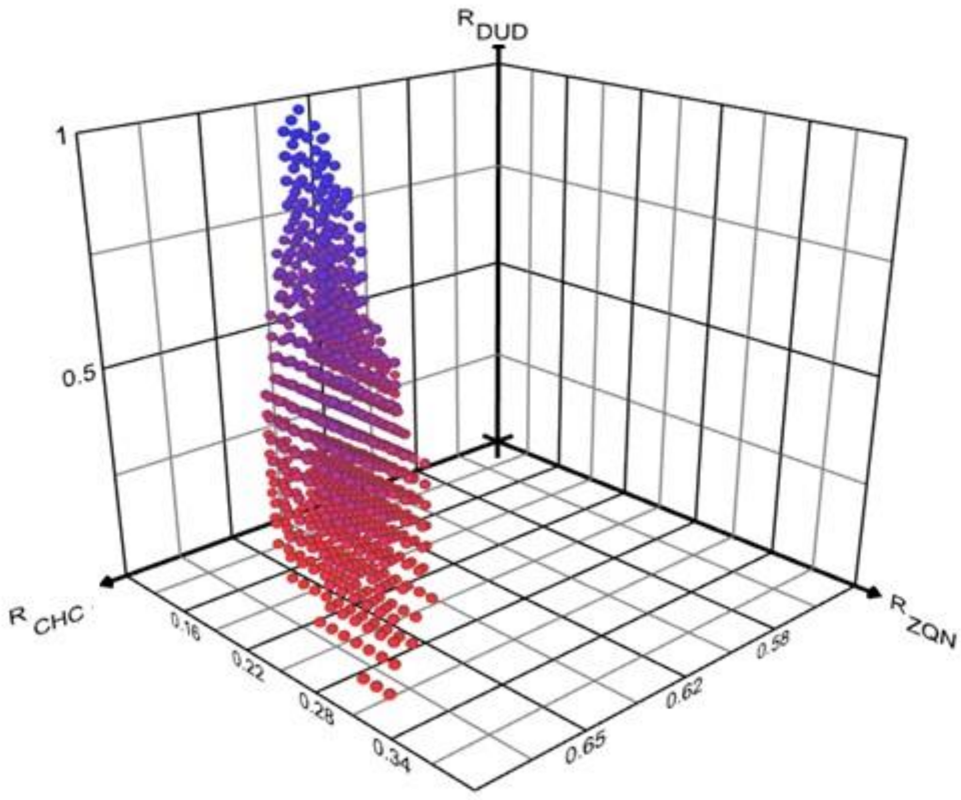


Figure 21 Simulation V5 optimised combinations of R_{CHC}, R_{ZQN} and R_{DUD} 3D model

We can observe in Figure 21 that CHC operates around 65% to 68% round-trips which has no visible relationship with other parameters. DUD, on the other hand, has more freedom to trend closer to 100% where ZQN has a lower level of round-trips, and therefore, supplementing station DUD with greater capacity due to a higher level of direct trips.

Tables 19 and 20 display the income statement and balance sheet, respectively, for V4 at a fleet size of *f*=20%. I have chosen the R_{CHC}=64%, R_{ZQN}=17% and R_{DUD}=10% as the instance to be displayed in the financial statements below. The range of revenues is between $604’266 and $559’263.

Table 19 Simulation V4 income statement; N = 85 (f=20%)

Car Rental V4 Ltd Income Statement For 6-month ended January 31st, 2017		
All values in NZ dollars ($)		
Sales revenue		561 209
Cost of sales		
Wages attributable to vehicle transfers	0	
Fuel costs attributable to vehicle transfers	0	
Empty vehicle transfers, net		0
Gross Profit		561 209
Expenses		
Depreciation of revenue earning vehicles	38 419	
Insurance	150 000	
Maintenance expense	0	
Selling, general and administrative expense	61 733	
Wages and Salaries *not* attributable to vehicle transfers	174 720	
Airport parking expense	21 625	
Operating profit		114 713
Lease Interest expense		
Vehicle	16 104	
Non-vehicle	27 000	
Interest expense, net		43 104
Net profit before tax		**71 609**
Income tax (28%)	20 050	
Net profit after tax		**51 558**
add back depreciation	38 419	
Net profit after depreciation addition		**89 977**

Table 20 Simulation V4 balance sheet; N = 85 (f=20%)

Car Rental V4 Ltd Balance Sheet For 6-month ended January 31st, 2017		
All values in NZ dollars ($)		
	Assets	
Cash and cash equivalents	96 402	
Accounts Receivable	2 960	
Revenue earning vehicles		
Compact Auto	604 350	
Economy Car	481 545	
Compact SUV	450 846	
Total revenue earning vehicles		1 536 741
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
DUD rental building	700 000	
Total leased airport buildings		2 700 000
Total assets		**4 336 103**
	Liabilities	
Credit card authorisation bond	2 960	
Lease Interest Payable	27 000	
Lease Principle Payable	135 000	
Net vehicle loans	0	
Leased airport buildings liability	2 700 000	
Total Liabilities		**2 864 960**
	Equity	
Contributed Capital	1 381 166	
Retained Earnings	89 977	
Total Equity		**1 471 143**
Total liabilities and equity		**4 336 103**

Tables 21 and 22 display the income statement and balance sheet, respectively, for V4 at a fleet size of 30%. I have chosen the R_{CHC}=64%, R_{ZQN}=18% and R_{DUD}=10% as the instance to be displayed in the financial statements below.

Table 21 Simulation V4 income statement; 30% fleet size

Car Rental V4 Ltd **Income Statement** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Sales revenue		598 920
Cost of sales		
Wages attributable to vehicle transfers	0	
Fuel costs attributable to vehicle transfers	0	
Empty vehicle transfers, net		0
Gross Profit		598 920
Expenses		
Depreciation of revenue earning vehicles	57 835	
Insurance	150 000	
Maintenance expense	0	
Selling, general and administrative expense	65 881	
Wages and Salaries *not* attributable to vehicle transfers	174 720	
Airport parking expense	33 470	
Operating profit		117 014
Lease Interest expense		
Vehicle	19 000	
Non-vehicle	27 000	
Interest expense, net		46 000
Net profit before tax		**71 013**
Income tax (28%)	19 884	
Net profit after tax		**51 129**
add back depreciation	57 835	
Net profit after depreciation addition		**108 965**

Table 22 Simulation V4 balance sheet; 30% fleet size

Car Rental V4 Ltd **Balance Sheet** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Assets		
Cash and cash equivalents	145 123	
Accounts Receivable	2 960	
Revenue earning vehicles		
Compact Auto	906 525	
Economy Car	730 620	
Compact SUV	676 269	
Total revenue earning vehicles		2 313 414
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
DUD rental building	700 000	
Total leased airport buildings		2 700 000
Total assets		**5 161 497**
Liabilities		
Credit card authorisation bond	2 960	
Lease Interest Payable	27 000	
Lease Principle Payable	135 000	
Net vehicle loans	0	
Leased airport buildings liability	2 700 000	
Total Liabilities		**2 864 960**
Equity		
Contributed Capital	2 187 572	
Retained Earnings	108 965	
Total Equity		**2 296 537**
Total liabilities and equity		**5 161 497**

The number of unfulfilled reservations for the N = 85 fleet is 317, whereas it is 111 for the N=128 case. This results in a service level of 88% and 96%, respectively. Operating with a service level <95% over the long term may be detrimental for the reputation of a car rental firm, and obviously it will also generate lower profits. The V4 simulation thus allows the manager to optimise the fleet size against the service level. Another option may be in that a firm attempts to fulfill as many reservation requests as reasonably possible (e.g., reservation upgrades, reposition vehicles, acquire more vehicles), even if the cost of fulfilling this request may be greater than the revenue generated in the short-term. This is important as we move to simulation V5, where empty vehicle rebalancing is considered in the operational decision making of the pool network. Vehicle rebalancing would be a further option to increase the service level, of course.

Table 23 Simulation V4 maximum profit values by fleet size input

	20% Fleet	30% Fleet
Net profit before tax	$ 71 609	$ 71 013
Net profit after tax	$ 51 558	$ 51 129
Net profit after depreciation addition	$ 89 977	$ 108 965

The differences between the financial outputs generated between the two fleet size inputs are very negligible, the only telling difference is the $18’988 increase in “net profit after depreciation” attributable to the 30% fleet size input. The 30% fleet size generated $37’711 more revenue than the 20% fleet size, this is offset however from a $11’845 increase in airport parking expenses, a $4’148 extra sales and admin cost, and increased depreciation expense for the larger fleet held.

Figure 22 displays the distributions of the number of reservations per day over simulation V4. The moving average of the reservations is constant during the simulated 6-month period. However, the inter-daily fluctuation from a minimum of 8 reservations and a maximum of 20 reservations puts creates a car demand uncertainty, which makes it challenging for operational decision-making agents to achieve a target service level.

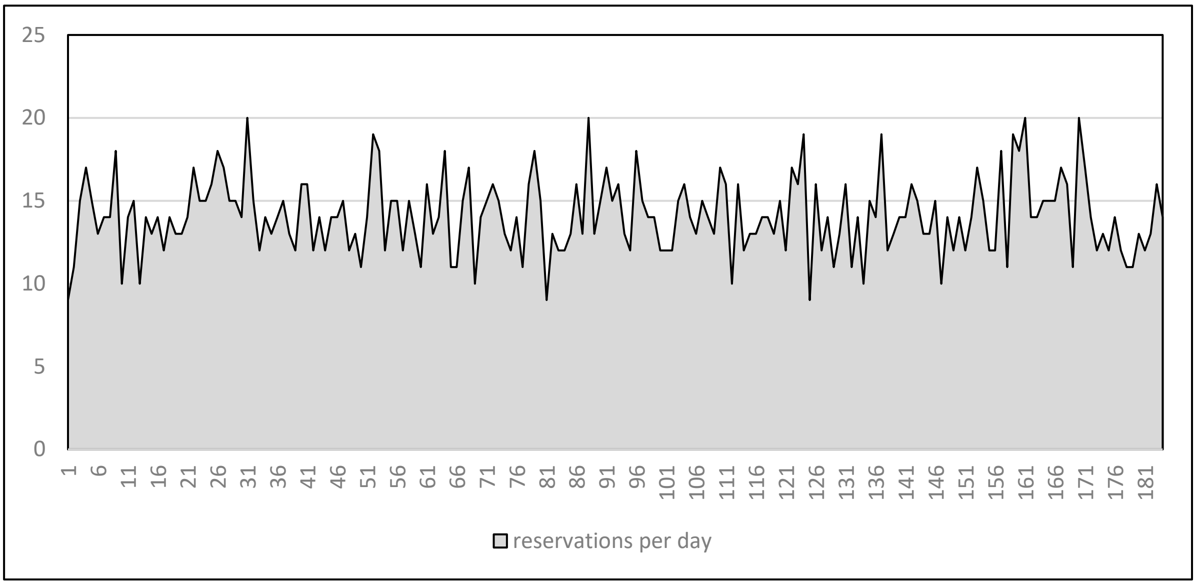


Figure 22 Simulation V4 reservations per day

The profitability ratios and Z-score which are the result of our financial statement examples are displayed below in Table 24 with reference to Hertz' ratios as of 2021. Interestingly, the 20% fleet size outperformed the 30% fleet size in terms of profitability ratios and the marker for firm health (Altman z-score), this likely due to the greater operating profit generated relative to the total assets held and sales being a higher proportion of the total assets.

Table 24 Car Rental V4 Ltd financial ratios and z-scores compared with Hertz (2021)

	Hertz 2021	**Car Rental V4 Ltd (20% fleet size)**	**Car Rental V4 Ltd (30% fleet size)**
ROE	(132)%	38.15%	26.08%
ROA	(0.78)%	12.94%	11.60%
Profit margin	(15)%	20.44%	19.54%
Z-score	0.0	0.26	0.25

4.3 Simulation V5

Simulation V5 improves on V4 with the introduction of an empty vehicle repositioning system for tactical fleet deployment operations. Additionally, 4 vehicle groups have been added to the fleet at a total of 7 vehicle groups in operation, and rebalancing decisions incurring a fixed cost of $280. From the 32’718 instances of V5, I have chosen to analyse the top 1000 by revenue outcomes of which 910 produce a ‘passing node’. For reference the top 15 ‘passing node’ instances for gross profit and their characteristics for the proportion of round-trips R_{CHC}, R_{ZQN} and R_{DUD} are given in Figure 23.

	Round trips			CHC							ZQN							DUD						
Revenue	RCHC	RZQN	RDUD	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER
$ 753,647	55%	25%	20%	15	5	10	11	6	6	4	9	28	8	1	2	14	2	1	1	1	1	2	1	3
$ 753,206	55%	35%	20%	14	5	10	10	6	6	4	9	28	8	1	2	14	2	2	1	1	2	2	1	3
$ 754,180	55%	20%	20%	15	5	10	11	6	6	4	9	28	8	1	2	14	2	1	1	1	1	2	1	3
$ 753,331	55%	30%	20%	14	5	10	11	6	6	4	9	28	8	1	2	14	2	2	1	1	1	2	1	3
$ 752,381	55%	25%	21%	15	5	10	11	6	6	4	9	28	8	1	2	14	2	1	1	1	1	2	1	3
$ 751,939	55%	35%	21%	14	5	10	10	6	6	4	9	28	8	1	2	14	2	2	1	1	2	2	1	3
$ 754,703	55%	25%	19%	15	5	10	11	6	6	4	9	28	8	1	2	14	2	1	1	1	1	2	1	3
$ 751,524	55%	25%	22%	14	5	10	11	6	5	4	10	28	8	1	2	15	2	1	1	1	1	2	1	3
$ 752,644	55%	40%	20%	14	5	10	10	6	6	4	9	28	8	1	2	14	2	2	1	1	2	2	1	3
$ 752,913	55%	20%	21%	15	5	10	11	6	6	4	9	28	8	1	2	14	2	1	1	1	1	2	1	3
$ 761,307	55%	25%	12%	17	5	12	10	6	6	5	7	28	6	1	2	14	1	1	1	1	2	2	1	3
$ 752,064	55%	30%	21%	14	5	10	11	6	6	4	9	28	8	1	2	14	2	2	1	1	1	2	1	3
$ 752,057	55%	20%	22%	14	5	10	11	6	5	4	10	28	8	1	2	15	2	1	1	1	1	2	1	3
$ 754,855	55%	15%	20%	16	5	11	11	6	6	4	8	28	7	1	2	14	2	1	1	1	1	2	1	3
$ 755,078	55%	10%	20%	16	5	11	11	6	6	4	8	28	7	1	2	14	2	1	1	1	1	2	1	3

Figure 23 V5 input characteristics for largest revenue generating instances

The reason that static R_{CHC} outcomes are part of the feasible round-trip combinations is due to the addition of an empty fleet rebalancing system. This allows rental stations to operate more effectively on their own accord as the system allows individual stations to increase rental capacity when necessary from other rental stations. To give a better appreciation of the distribution of round-trips R_{CHC}, R_{ZQN} and R_{DUD}, a 3-dimensional state space is displayed in Figure 24 which contains the top 1000 combinations of R_{CHC}, R_{ZQN} and R_{DUD}.

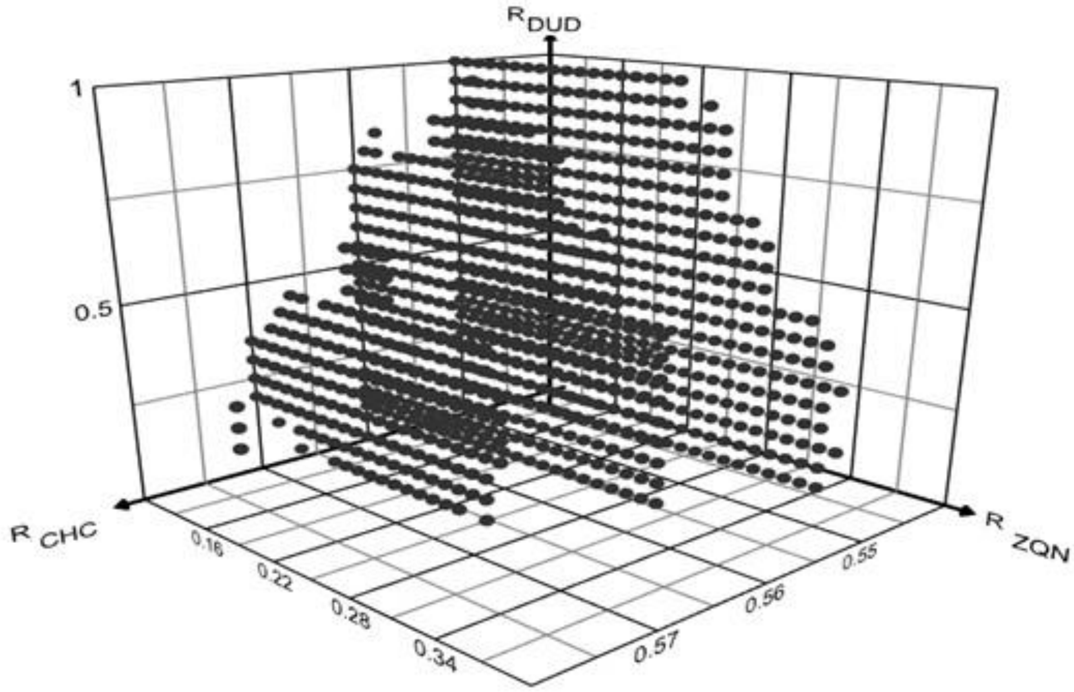


Figure 24 Simulation V5 3D model of input combinations for R_{CHC}, R_{ZQN} and R_{DUD}

For R_{CHC} = 55%, round-trips for DUD are able to range between 10% and 100% for a given R_{ZQN}, although values for R_{DUD} only reach 100% round-trips where R_{ZQN} is equal to or below 21%. As round-trips for CHC increase to 56% and 57%, the possible round-trips for R_{ZQN} and R_{DUD} decrease, reflecting the lesser capacity of vehicles that is obtained from the CHC rental station over the simulation instances.

Scenarios that were chosen as an example to translate into the financial statements at each level of fleet size are displayed below in Table 25.

Table 25 Simulation V5 input combinations chosen to translate into the financial statements

Scenario	Gross profit	R_{CHC}	R_{ZQN}	R_{DUD}	Fleet input
4348	$ 663 705	55%	12%	0.35	0.2
19153	$ 699 327	55%	20%	0.25	0.3

Tables 26 and 27 display the income statement and balance sheet, respectively, for V5 at a fleet size of 20%.

Table 26 Simulation V5 income statement; 20% fleet size

Car Rental V5 Ltd Income Statement For 6-month ended January 31st, 2017		
All values in NZ dollars ($)		
Sales revenue		760 865
Cost of sales		
Wages attributable to vehicle transfers	68 012	
Fuel costs attributable to vehicle transfers	29 148	
Empty vehicle transfers, net		97 160
Gross Profit		663 705
Expenses		
Depreciation of revenue earning vehicles	46 406	
Insurance	150 000	
Maintenance expense	0	
Selling, general and administrative expense	83 695	
Wages and Salaries *not* attributable to vehicle transfers	174 720	
Airport parking expense	23 642	
Operating profit		185 243
Lease Interest expense		
Vehicle	31 439	
Non-vehicle	27 000	
Interest expense, net		58 439
Net profit before tax		**126 804**
Income tax (28%)	35 505	
Net profit after tax		**91 299**
add back depreciation	46 406	
Net profit after depreciation addition		**137 705**

Table 27 Simulation V5 balance sheet; 20% fleet size

Car Rental V5 Ltd **Balance Sheet** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Assets		
Cash and cash equivalents	116 445	
Accounts Receivable	3 372	
Revenue earning vehicles		
Compact Auto	426 600	
Economy Car	265 680	
Compact SUV	245 916	
Intermediate SUV	259 128	
Full-size SUV	187 683	
Full-size Sedan	238 140	
OTHER (12-seater vans/ 4x4 wild track)	233 100	
Total revenue earning vehicles		1 856 247
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
DUD rental building	700 000	
Total leased airport buildings		2 700 000
Total assets		**4 676 064**
Liabilities		
Credit card authorisation bond	3 372	
Lease Interest Payable	27 000	
Lease Principle Payable	135 000	
Net vehicle loans	0	
Leased airport buildings liability	2 700 000	
Total Liabilities		**2 865 372**
Equity		
Contributed Capital	1 672 987	
Retained Earnings	137 705	
Total Equity		**1 810 692**
Total liabilities and equity		**4 676 064**

Tables 28 and 29 display the income statement and balance sheet, respectively, for V5 at a fleet size of 30%.

Table 28 Simulation V5 income statement; 30% fleet size

Car Rental V5 Ltd Income Statement For 6-month ended January 31st, 2017		
All values in NZ dollars ($)		
Sales revenue		753 647
Cost of sales		
Wages attributable to vehicle transfers	38 024	
Fuel costs attributable to vehicle transfers	16 296	
Empty vehicle transfers, net		54 320
Gross Profit		699 327
Expenses		
Depreciation of revenue earning vehicles	69 056	
Insurance	150 000	
Maintenance expense	0	
Selling, general and administrative expense	82 901	
Wages and Salaries *not* attributable to vehicle transfers	174 720	
Airport parking expense	33 849	
Operating profit		188 802
Lease Interest expense		
Vehicle	30 884	
Non-vehicle	27 000	
Interest expense, net		57 884
Net profit before tax		**130 917**
Income tax (28%)	36 657	
Net profit after tax		**94 260**
add back depreciation	69 056	
Net profit after depreciation addition		**163 316**

Table 29 Simulation V5 balance sheet; 30% fleet size

Car Rental V5 Ltd **Balance Sheet** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Assets		
Cash and cash equivalents	173 279	
Accounts Receivable	3 372	
Revenue earning vehicles		
Compact Auto	604 350	
Economy Car	415 125	
Compact SUV	389 367	
Intermediate SUV	421 083	
Full-size SUV	268 119	
Full-size Sedan	357 210	
OTHER (12-seater vans/ 4x4 wild track)	306 990	
Total revenue earning vehicles		2 762 244
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
DUD rental building	700 000	
Total leased airport buildings		2 700 000
Total assets		**5 638 895**
Liabilities		
Credit card authorisation bond	3 372	
Lease Interest Payable	27 000	
Lease Principle Payable	135 000	
Net vehicle loans	0	
Leased airport buildings liability	2 700 000	
Total Liabilities		**2 865 372**
Equity		
Contributed Capital	2 610 206	
Retained Earnings	163 316	
Total Equity		**2 773 523**
Total liabilities and equity		**5 638 895**

The larger fleet size is associated with a larger holding cost, which reduce the discrepancies in the revenue generated, the profit values for the 20% and 30% fleet size are displayed below in Table 30.

Table 30 Simulation V5 maximum profit values by fleet size

	20% Fleet	**30% Fleet**
Net profit before tax	$ 126 804	$ 130 917
Net profit after tax	$ 91 299	$ 94 260
Net profit after depreciation addition	$ 137 705	$ 163 316

The profitability ratios and the Z-score which are the result of our financial statement examples are displayed below in Table 31 with reference to Hertz' ratios as of 2021.

Table 31 Car rental V5 Ltd financial ratios and z-scores compared with Hertz

	Hertz 2021	**Car Rental V4 Ltd (20% fleet size)**	**Car Rental V5 Ltd (30% fleet size)**
ROE	(132)%	42.02%	27.17%
ROA	(0.78)%	16.27%	13.37%
Profit margin	(15)%	24.35%	25.05%
Z-score	0	0.35	0.31

The 30% fleet size generates a slightly greater profit margin than the 20%. However, this is at the expense of lower liquidity (ROA) and profitability (ROE) ratios. The values relative to Hertz exhibit similar profitability ratios to simulation V1 and V4, however better z-scores are generated than V4 with the empty vehicle rebalancing system.

Figure 25 displays the distribution of the reservations per day for simulation V5. Distribution appears uniform across the simulation, the random element of the start time, however, results in a large spike at the 3-month mark over the time horizon.

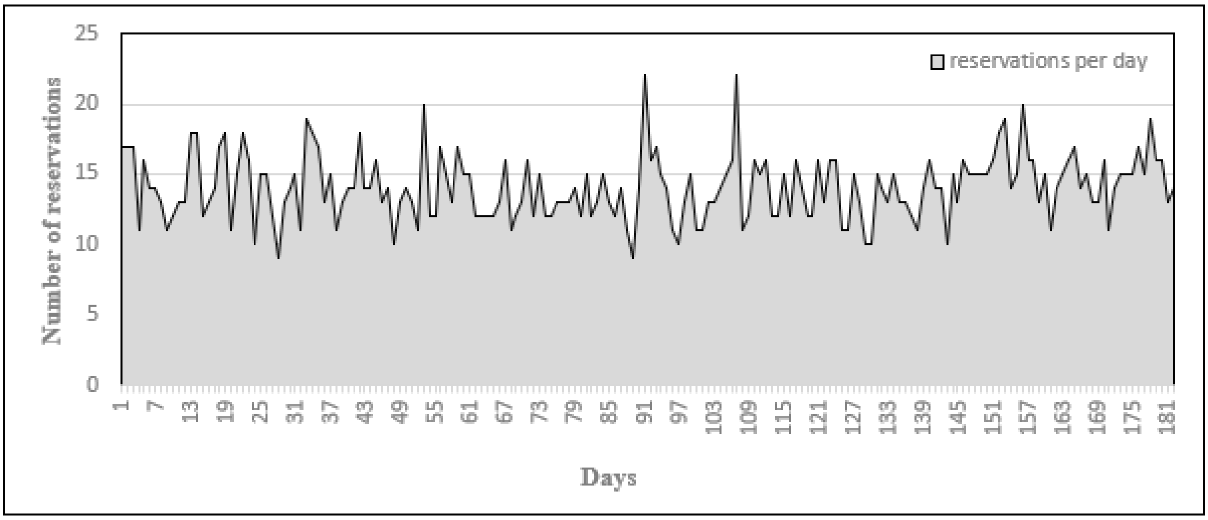


Figure 25 Simulation V5 number of reservations per day

For physical fleet movements, Figures 26 displays movements for CHC, ZQN and DUD over the simulation horizon for optimized financial values given a 20% fleet size input. When a vehicle group at a particular station is zero. The critical value engages the rebalancing decision if a reservation arrives at that station for the particular vehicle group, and the auxiliary constraint prevents a rebalancing decision from starting and ending at the same station with the allocation of our primary and secondary rebalancing nodes.

It appears that our CA vehicle group was the most popular vehicle group, which is apparent from the intensity of fleet movements for CA_CHC and CA_ZQN. EC_DUD fleet capacity became quite large, which is due to unfavorable direct trips to DUD in the middle of the simulation horizon, this balanced out however as the simulation neared its end in this instance. The revenue value used as an example in the income statement (relative to the scenario instance in Table 30) for the 20% fleet input is \$760’865 with rebalancing costs of \$97’160, this generates a gross profit of \$663’705. The rebalancing costs are attributable to a total of 347 empty vehicle rebalancing decisions incurred, meaning that an average of 1.89 empty rebalancing decisions are being incurred every day over the simulation horizon.

Figure 27 displays the fleet movements for CHC, ZQN and DUD over the simulation horizon for optimized financial values given a 30% fleet size input. Less empty vehicle rebalancing decisions being incurred (194) the fleet movements are more stable.[21]Equivalently, this yields an average of 1.05 rebalancing decisions per day. The revenue value used as an example in the income statement (relative to the scenario instance in Table 30) for the 30% fleet size is $753’647 with rebalancing costs of $54’320, giving us a gross profit of $699’327. Even though our 20% fleet size generated $7’218 greater revenue than the 30% input, the operational movements resulted in a greater transfer cost burden of $42’840 which generated a gross profit $35’621 lower than our 30% fleet size.

The 30% fleet size outperforms the 20% fleet size in both physical and financial aspects. Our larger fleet size results in less volatile operational movements which gives the system more flexibility to deal with consecutive unfavorable trip types. The unfavorable trip types place particular stations in a position of reaching critical values for vehicle group capacity. Having a financial value bound to each operational fleet rebalancing decision gives us a good perspective into long-term consequences of making financial decisions and how they translate into the firm financial statements. Additionally, it shows that revenue is not the best financial outcome to optimise, as it fails to consider the operational decisions of the pool network and the costs associated.

[21] 153 fewer rebalancing decisions with f=30%, and an average of 0.85 less per day.

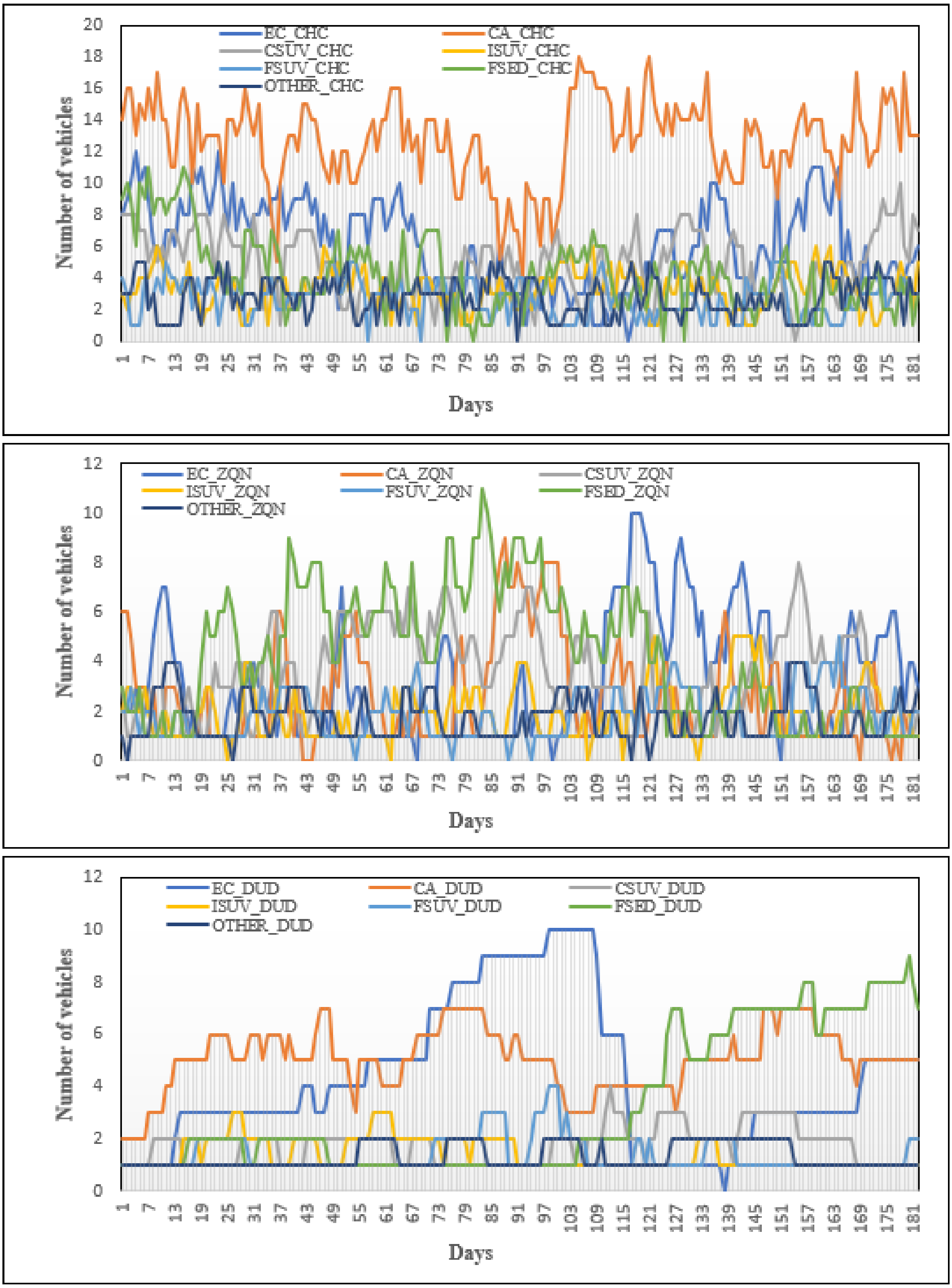


Figure 26 Simulation V5 fleet movements by station; 20% fleet size

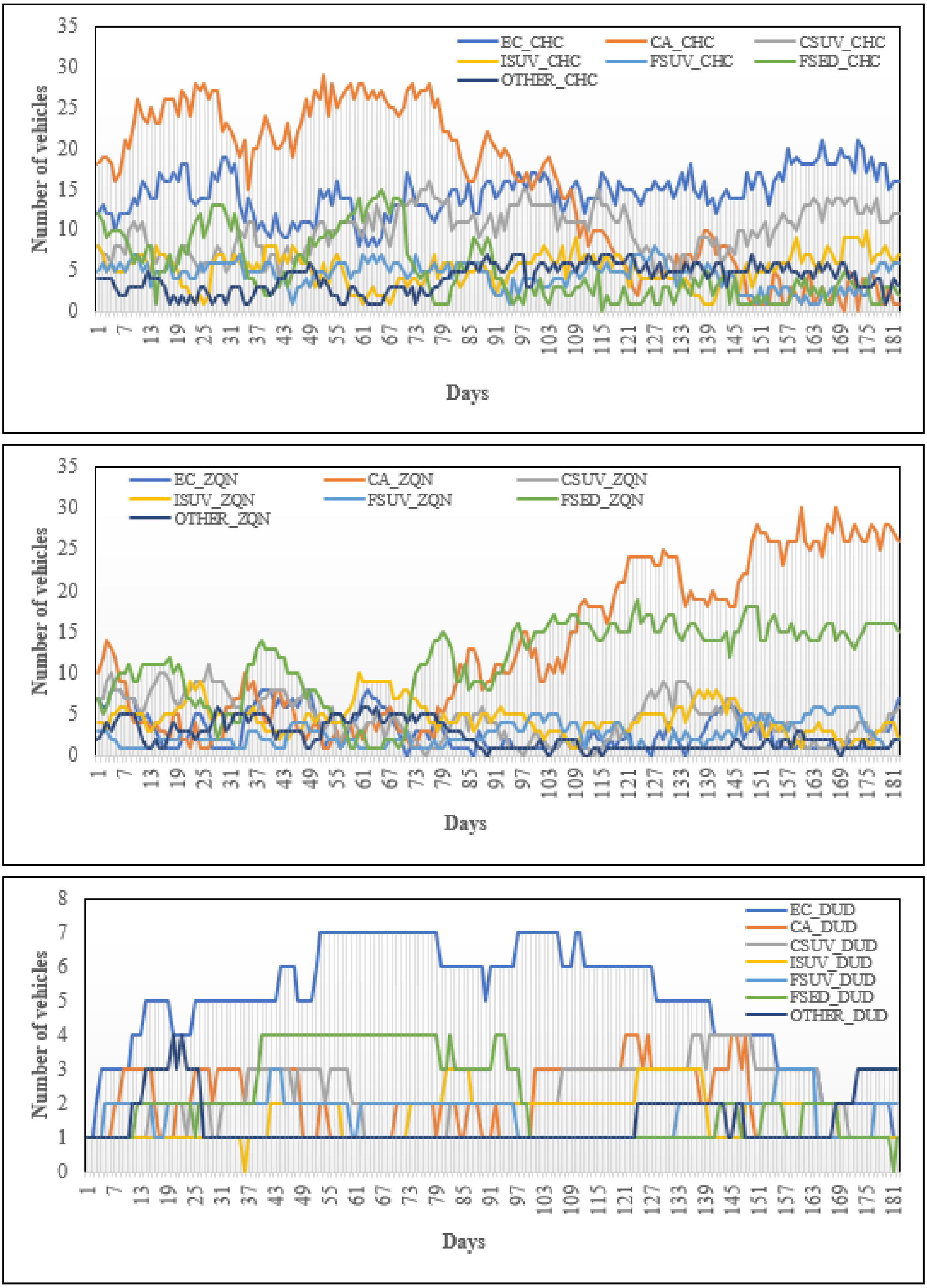


Figure 27 Simulation V5 fleet movements by station; 30% fleet size

To contextualize the relationship between revenue and transfer costs, Figures 28 and 29, respectively, show the 1000 ‘passing node’ instances and how revenue and transfer costs change. The scenario samples (x-axis) have been ordered based on gross profit maximums.

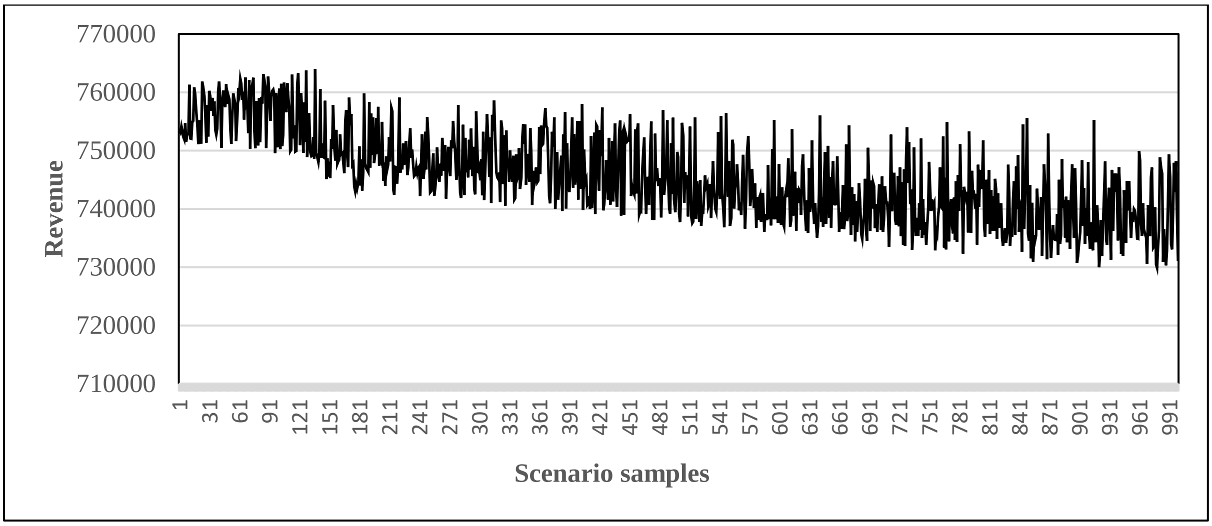


Figure 28 Simulation V5 revenue by optimal scenario values

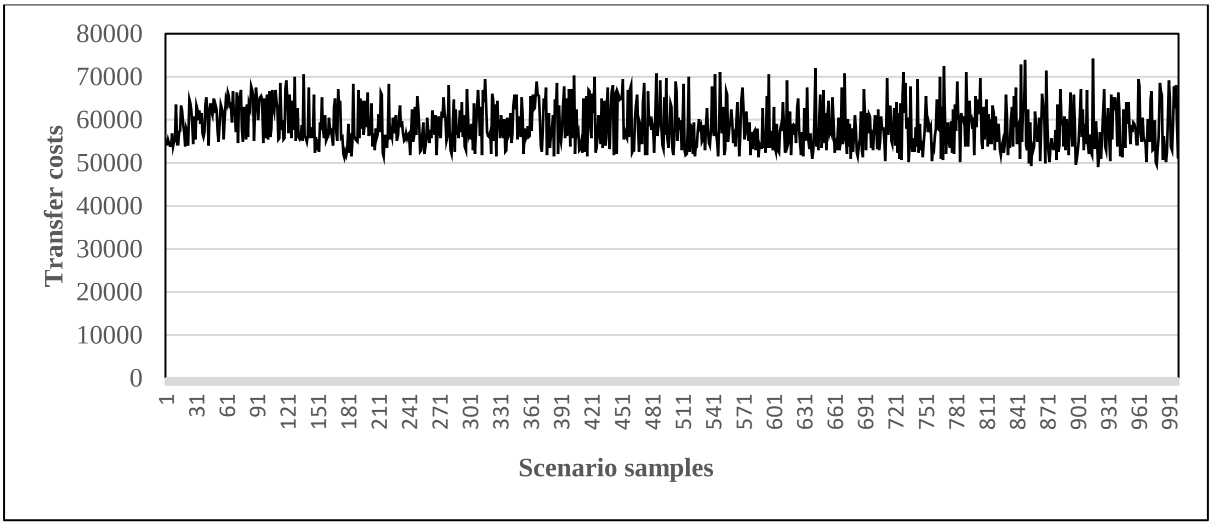


Figure 29 Simulation V5 transfer costs by optimal scenario values

Revenue has a clear downward trend although transfer costs vary between \$50’000 and \$70’000 over these values, getting more volatile as scenarios flow to the 1000th ‘passing node’ instance.

The SACA activities in simulation V5 have gained in complexity by incorporating the empty vehicle rebalancing system. The addition of this sub-problem further reduces the disconnect in the modelling of fleet management and revenue management decision-making. Revenue management is concerned with incentivizing customers to book trips and vehicle groups which generate the greatest amount of revenue, which in most cases are direct trips. This creates a disconnect between the individual rental station operational decision making and pool network revenue management, as direct trips result in rental stations reaching critical values at a faster rate. Hence, it is important for a model to incorporate rebalancing activities.

Physical fleet movements can vary greatly over the simulation horizon by each instance due to the initial simulation settings determining how reservations are distributed over the simulation horizon. I believe this is a very important condition in representing realism within our simulation settings. For example, station DUD starts with a maximum fleet size of 2 for the 20% fleet input and 4 for the 30% fleet input. As we know, the pool network contains 7 vehicle groups, this means DUD starts with 5 and 3 vehicle groups, respectively, with zero capacity to fulfill reservations. The system is predisposed to an empty vehicle rebalancing decision if a direct trip does neither arrive in DUD from CHC nor ZQN nodes for the specific vehicle groups, before a reservation would arrive for DUD and request a vehicle from that particular vehicle group.

We can observe in Figures 26 and 27 that even with a smaller fleet size, the evolution of the simulation caused a large number of direct trips to rental station DUD for EC, CA, and FSED vehicle groups, creating a large capacity held (up to 10 vehicles) over the simulation horizon due to the randomized uniform arrival process. This was not the case for the other vehicle groups. For the larger fleet size, CA experienced an increased capacity up to 7 vehicles in the simulation horizon (for DUD), although the other vehicle groups did not experience this variability. This emphasizes the point that a simulation with identical R_{CHC}, R_{ZQN} and R_{DUD}

inputs will not generate the same operational outcomes. Even with information of monthly arrival demand to each rental station, a car rental firm has minimal control for which reservations arrive at which station at particular points with regard to their demand model's time window. For practical applications, simulation round-trip ranges may be informed on past observations.

Because the conceptualization of fleet management is important in reflecting upon financial outcomes inherent with revenue management decision making, it was deemed necessary to update the empty vehicle repositioning cost functions to be representative of the duration, distance and costs associated with repositioning decisions between specific rental stations. A rebalancing matrix is utilized for simulations V6 and V7 to conceptualize this in the most appropriate form. Additionally, a maintenance scheduled is created for the following simulations to represent uncertainty in the form of vehicle unavailability. A vehicle could require maintenance at any point in time and there is no guarantee on which station the vehicle resides at when the maintenance must be commenced. This displays the physical and financial consequences in a realistic setting from the fleet management planning process. Additionally, a limited cascading upgrade system is added for simulation V7 to give the car rental firm more control over deleterious operational outcomes that are the result of unfavorable reservation types.

4.3 Simulation V7

Simulation V7 improves upon earlier simulations with multiple revenue management factors. Firstly, a limited cascading upgrade function is added to coincide with the empty vehicle rebalancing tool. Secondly, a dynamic seasonal arrival process is utilized to test how the simulation operates with different levels of reservation intensity. This corresponds with the maintenance schedules and the round-trip scenarios to test a full array of overlapping time horizons and operational processes that exist in a realistic setting for a car rental firm. The characteristics of the top 15 (out of 1000) 'passing node' instances for proportions of round-trips R_{CHC}, R_{ZQN} and R_{DUD} are given in Figure 30.

	Round trips			CHC							ZQN							DUD						
Revenue	RCHC	RZQN	RDUD	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER	EC	CA	CSUV	ISUV	FSUV	FSED	OTHER
$ **857,644**	55%	10%	15%	14	21	10	5	6	16	6	2	4	4	3	1	1	2	9	11	5	5	3	4	1
$ **858,121**	55%	10%	10%	14	22	11	5	6	16	6	2	4	4	3	1	1	2	9	10	4	5	3	4	1
$ **857,185**	55%	10%	20%	14	21	10	5	6	16	6	2	4	4	3	1	1	2	9	11	5	5	3	4	1
$ **856,473**	55%	11%	15%	14	21	10	5	7	16	6	2	4	4	3	1	1	2	9	11	5	5	2	4	1
$ **856,951**	55%	11%	10%	14	22	11	5	7	16	6	2	4	4	3	1	1	2	9	10	4	5	2	4	1
$ **856,726**	55%	10%	25%	14	21	10	5	5	16	7	2	4	4	3	1	1	1	9	11	5	5	4	4	1
$ **856,014**	55%	11%	20%	14	21	10	5	7	16	6	2	4	4	3	1	1	2	9	11	5	5	2	4	1
$ **855,000**	55%	13%	15%	14	21	10	5	7	16	6	2	4	4	3	1	1	2	9	11	5	5	2	4	1
$ **857,427**	55%	10%	15%	14	21	10	5	6	16	6	2	4	4	3	1	1	2	9	11	5	5	3	4	1
$ **855,839**	55%	12%	15%	14	21	10	5	7	16	6	2	4	4	3	1	1	2	9	11	5	5	2	4	1
$ **855,478**	55%	13%	10%	14	22	11	5	7	16	6	2	4	4	3	1	1	2	9	10	4	5	2	4	1
$ **857,905**	55%	10%	10%	14	22	11	5	6	16	6	2	4	4	3	1	1	2	9	10	4	5	3	4	1
$ **856,317**	55%	12%	10%	14	22	11	5	7	16	6	2	4	4	3	1	1	2	9	10	4	5	2	4	1
$ **856,332**	55%	10%	30%	14	20	10	5	5	16	7	2	4	4	3	1	1	1	9	12	5	5	4	4	1
$ **855,555**	55%	11%	25%	14	21	10	5	6	16	7	2	4	4	3	1	1	1	9	11	5	5	3	4	1

Figure 30 V7 input characteristics for largest revenue generating instances

The instances of R_{CHC} displayed in Figure 30 show the impact of the simulation setup with an upgrade and rebalancing system. A clear preference exists for engaging in a larger number of direct trips, as they generate the greatest amount of revenue. This leads to rental stations reaching critical capacity values at a faster rate, although the upgrade system offsets this with a "zero sum" cost for upgrading vehicles to the next feasible vehicle group.

Figure 31 represents all combinations of 'passing node' instances of R_{CHC}, R_{ZQN} and R_{DUD}. R_{CHC} rarely deviates from its 55% round-trips, whereas R_{ZQN} has minimal leeway in the maximum values between 15% and 19%, and R_{DUD} ranges between 10% and 80%. The station size (which reflects the number of reservations arriving at the start location) has a strong impact on the capacity of other stations in fulfilling their capacity. With 55% of round-trips occurring in CHC, DUD is able to maintain its fleet through a proportion of direct trips from CHC ending in DUD. The same is apparent for ZQN, in which low round-trips still yield successful operational outcomes where the options of vehicle upgrades and repositioning exists.

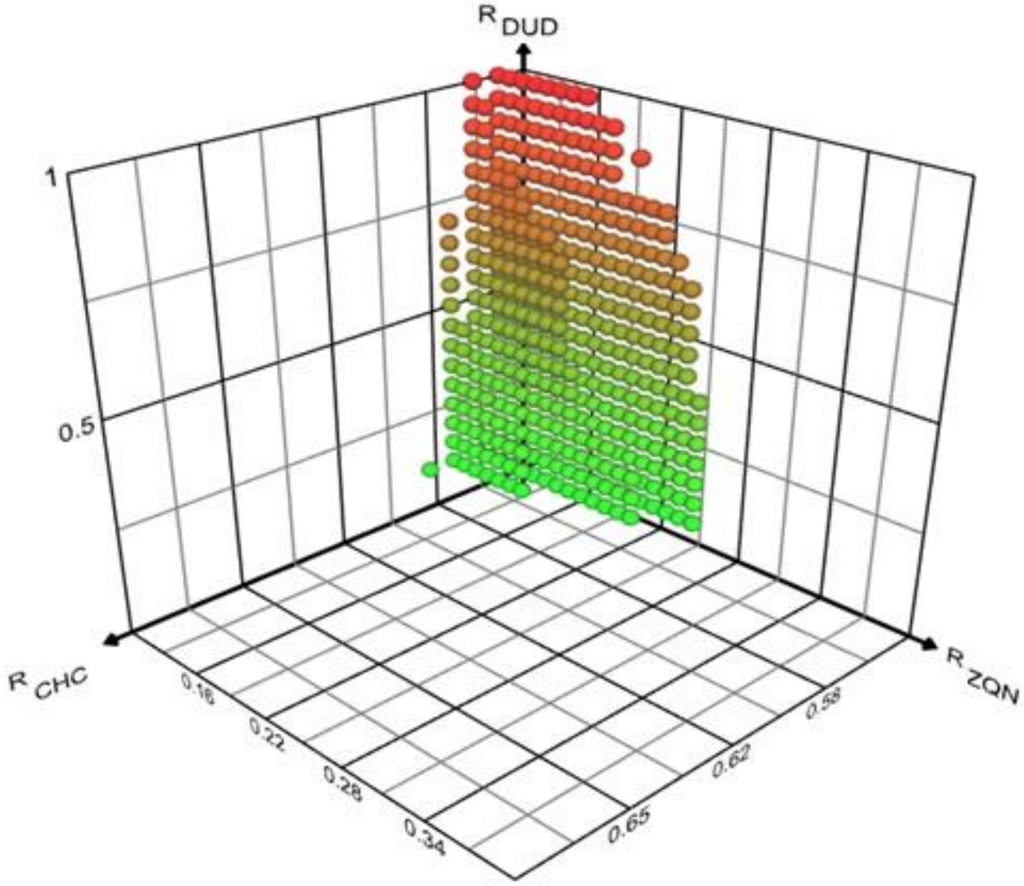


Figure 31 Simulation V7 3D model of combinations of R_{CHC}, R_{ZQN} and R_{DUD}

Tables 32 and 33 display the income statement and the balance sheet, respectively, for a particular 'passing node' instance. The starting fleet size for CHC, ZQN and DUD is 81, 48 and 4, respectively.

Table 32 Simulation V7 income statement

Car Rental V7 Ltd **Income Statement** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Sales revenue		857 644
Cost of sales		
Wages attributable to vehicle transfers	21 029	
Fuel costs attributable to vehicle transfers	9 012	
Empty vehicle transfers, net		30 041
Gross Profit		827 603
Expenses		
Depreciation of revenue earning vehicles	69 763	
Insurance	150 000	
Maintenance expense	53 000	
Selling, general and administrative expense	94 341	
Wages and Salaries *not* attributable to vehicle transfers	174 720	
Airport parking expense	33 849	
Operating profit		251 931
Lease Interest expense		
Vehicle	38 872	
Non-vehicle	27 000	
Interest expense, net		65 872
Net profit before tax		**186 059**
Income tax (28%)	52 097	
Net profit after tax		**133 963**
add back depreciation	69 763	
Net profit after depreciation addition		**203 725**

Table 33 Simulation V7 balance sheet

Car Rental Ltd **Balance Sheet** **For 6-month ended January 31st, 2017**		
All values in NZ dollars ($)		
Assets		
Cash and cash equivalents	175 052	
Accounts Receivable	5 644	
Revenue earning vehicles		
Compact Auto	639 900	
Economy Car	415 125	
Compact SUV	389 367	
Intermediate SUV	421 083	
Full-size SUV	268 119	
Full-size Sedan	357 210	
OTHER (12-seater vans/ 4x4 wild track)	299 700	
Total revenue earning vehicles		2 790 504
Leased airport buildings		
CHC rental building	1 100 000	
ZQN rental building	900 000	
DUD rental building	700 000	
Total leased airport buildings		2 700 000
Total assets		**5 671 200**
Liabilities		
Credit card authorisation bond	5 644	
Lease Interest Payable	27 000	
Lease Principle Payable	135 000	
Net vehicle loans		
Leased airport buildings liability	2 700 000	
Total Liabilities		**2 867 644**
Equity		
Contributed Capital	2 599 830	
Retained Earnings	203 725	
Total Equity		**2 803 556**
Total liabilities and equity		**5 671 200**

The profitability ratios and the Z-score which are the result of our financial statement examples are displayed below in Table 34 with reference to Hertz' ratios as of 2021. V7 exhibited the greatest profit margin of the simulations, as well as the largest z-score. However, the z-score still indicates that the company may be in financial stress in the future. The generation of this value is likely due to the assumption of purchasing the fleet outright with cash held before the simulation begins. This would lead the formula to assume that there is poor utilisation of financials relative to the fixed assets held, which is not necessarily the case.

Table 34 Car rental V7 Ltd financial ratios and z-scores compared with Hertz

	Hertz 2021	**Car Rental V7 Ltd**
ROE	(132)%	30.59%
ROA	(0.78)%	15.12%
Profit margin	(15)%	29.37%
Z-score	0	0.38

Fleet movements for the seasonal arrival processes is displayed in Figure 33. The fleet capacity available for CHC and ZQN rental stations was exhausted later in the simulation horizon due to peak demand. Conversely, DUD fared well from the peak demand stages, with many reservations ending in DUD creating a buildup of capacity triggered a multitude of empty vehicle rebalancing decisions back to CHC. This was especially prevalent for the EC vehicle group.

Figure 32 displays the reservation arrival frequency over the simulation horizon with the addition of seasonal arrival data. The peak rental months occur at the end of the simulation horizon. First 4-months had a moving average of 13 reservations per day, this jumped to 18 reservations per day in peak months

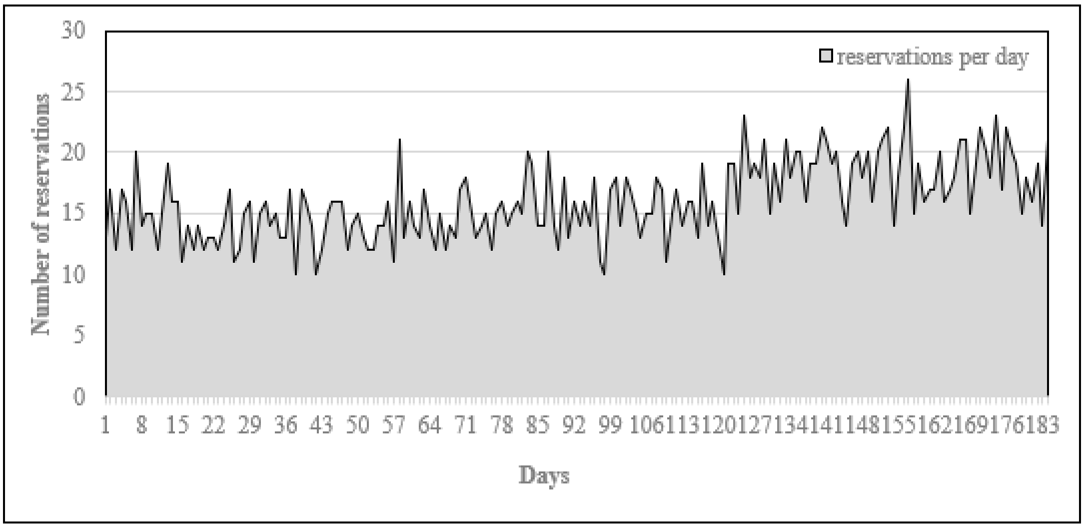


Figure 32 Simulation V7 reservations per day

With the addition of the upgrade system, Figure 34 displays that empty vehicle repositioning costs are more stable over the simulation horizon for the top 1000 ‘passing node’ instances: here the range is approximately $9’000 while for transfer costs in V5 the range is approximately $24’00 . Greater stability in operational decision making, results in better predictability of financial outcomes. The upgrade system specifically had a positive outcome on the generation of financial values, due to its assistance in stabilising operational functions to fulfil reservations with net benefits.

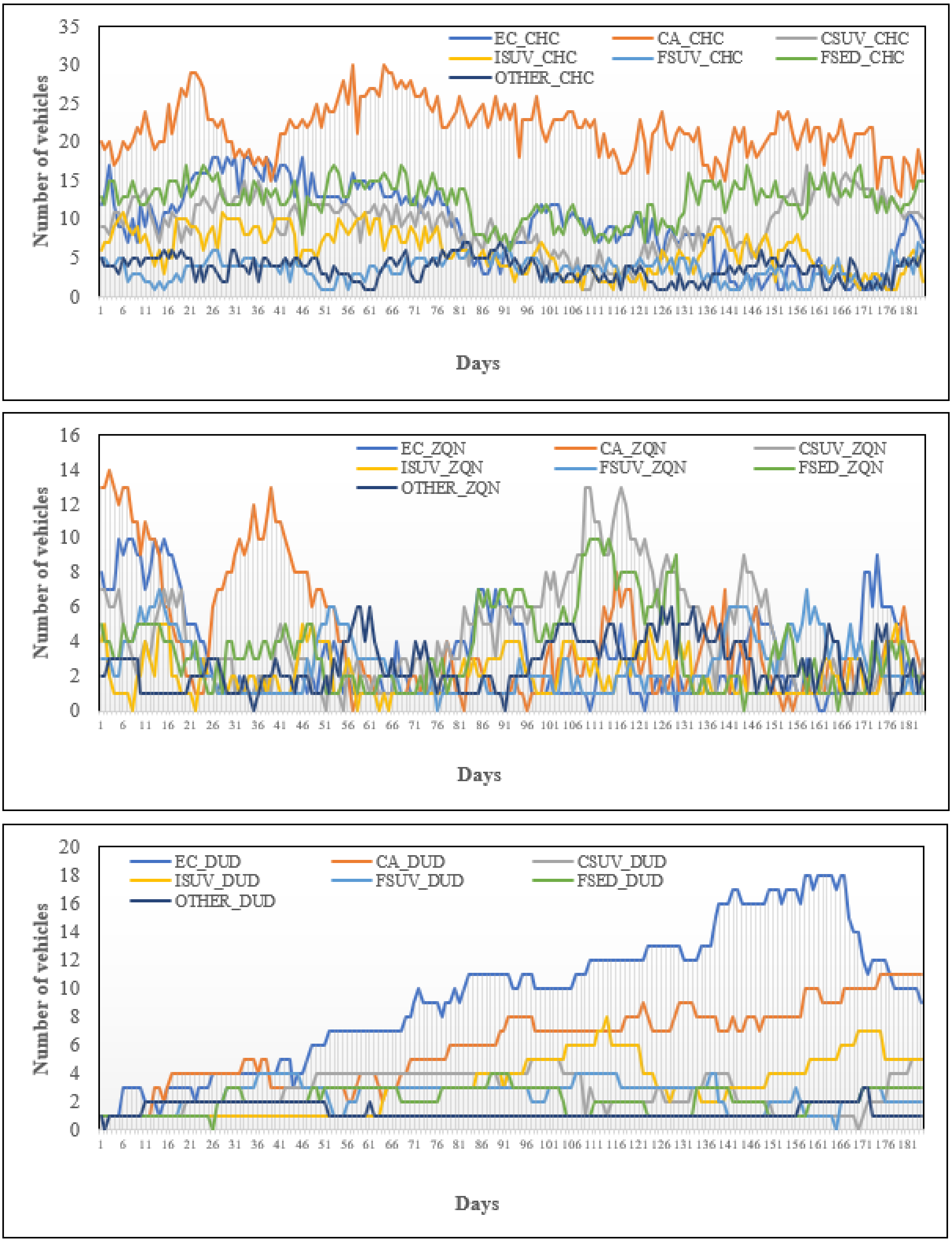


Figure 33 Simulation V7 fleet capacity at the end of each day over the simulation horizon

Figure 35 shows that the corresponding in the top 1000 'passing node' instances which are ordered by descending gross profit. The variation of revenue is less volatile as we move from the 1st to 1000th revenue value which is similar to the pattern observed in simulation V5; however, the uncertainty in the range has more than doubled between the two simulation versions. In V7 (V5) we start with a revenue range of approximately \$5'000 (>\$10'000) at the more profitable simulation instances, and a revenue range of approximately \$8'000 (>\$20'000) at the less profitable simulation instances. The overall smaller revenue range is due to the dimensionality of the car rental sub-problems considered in this version, with the updated rebalancing matrix and limited cascading vehicle upgrades.

Figure 34 Simulation V7.0 transfer costs over top 1000 scenario combinations

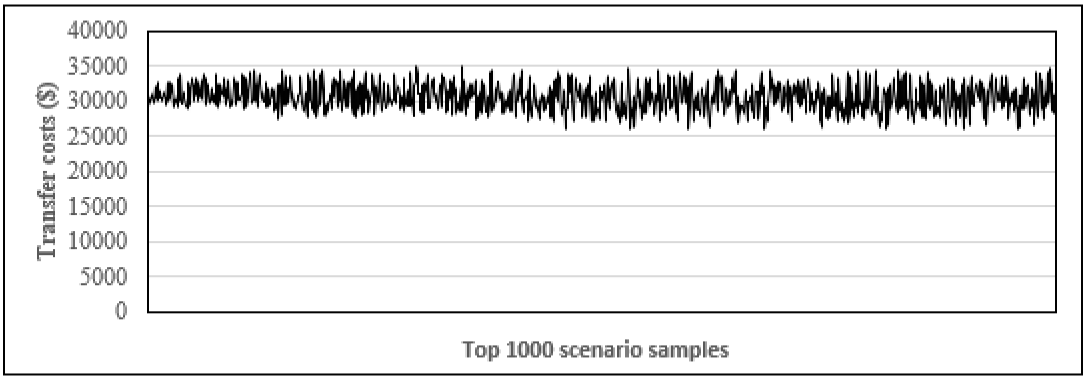
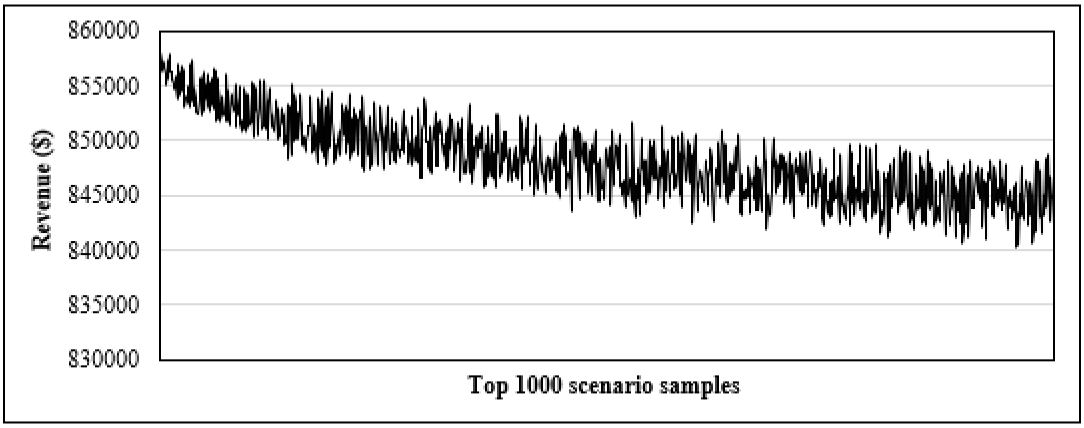


Figure 35 Simulation V7 revenue values over top scenario combinations



5.4 Simulation V7.1: maintenance extension

Maintaining the fleet is a necessary a costly part of operations for a car rental firm. A firm's maintenance strategy and maintenance tasks may vary depending on the type of trips particular vehicles make over time and the state in which they are returned to rental stations. The majority of firms engage in cosmetic 20–40-point checks after a vehicle is returned as a preparation for the next reservation. In these checks, maintenance is only engaged in if a problem has been detected such as low tire treads or cosmetic damage which may require panel beating. More engaging maintenance tasks such as warrant updates, COFs and oil changes occur upon certain mileage cut-offs.

The Simulation V7 was extended to test how scheduled maintenance affects the representation of physical and financial operations in the real world. The maintenance type that is the focus in this extension represents the 20-40-point checks; and every such event incurs a $200 cost (or any suitable or realistic cost distribution). The particular maintenance we are analyzing is thus an event which can be triggered anytime over the simulation horizon: we assume it starts one day before the reservation date for a particular vehicle. The frequency with which we trigger maintenance task across the whole fleet is called maintenance intensity. A 100% maintenance intensity means that the 20-40-point check is invoked every time. A <100% intensity means that only the designated percentage of cars, selected randomly, are 20-40-point checked the day before they are rented out.

Extension 7.1 is tested against a reputationally damaging event (RDE) which is generated at a cost of $500 based off the following: Let y = the maintenance intensity, N = number of vehicles, x = percentage of defects, and z = probability of defect detected (fixed at 20%). Thus, an RDE is trigger with a probability equal to N*(1-y)*z and randomly assigned to a reservation.

Figure 36 displays common income statement levels (revenue, expenses, and profit) as a function of maintenance intensity, including test statistics for linear trends in the simulated data.

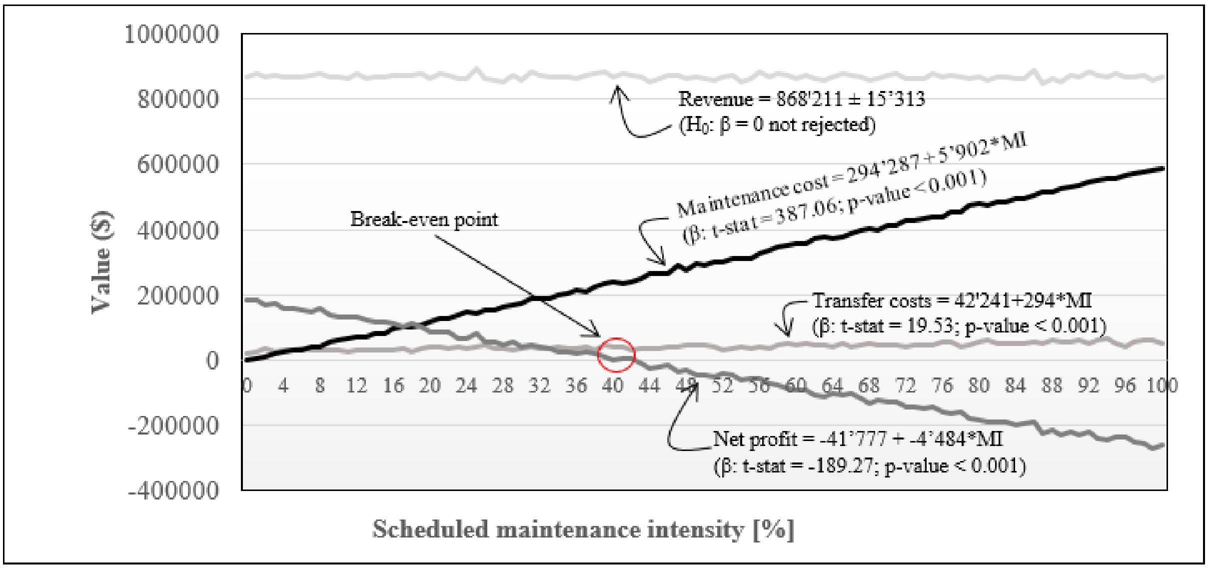


Figure 36 Financial outcomes per scheduled maintenance intensity

Because revenues are independent from maintenance intensity, they remain constant while maintenance expenses increase linearly. This yields negative relationship between net profit after tax and maintenance intensity, as expected. A rental car firm is now in a position to determine at which maintenance intensity level it remains profitable. With the selected simulation settings, the break-even point lies around 40%. The risk management benefits demonstrated in my simulation above allow analyzing operational breakeven levels and other financial strategies, such as sustainable growth targets.

Notably, I also show in Figure 36 that transfer costs have a statistically significant positive linear relationship with maintenance intensity (and because of transitivity, transfer costs are negatively correlated with profits). The reason for this is that the greater the number of cars which are unavailable for reservations at a rental station due to maintenance, the more frequently car transfers need to be done to that station in order to fulfill the reservation requests.

Figure 37 displays the number of transfer (black) and upgrade (grey) decisions against maintenance intensity.

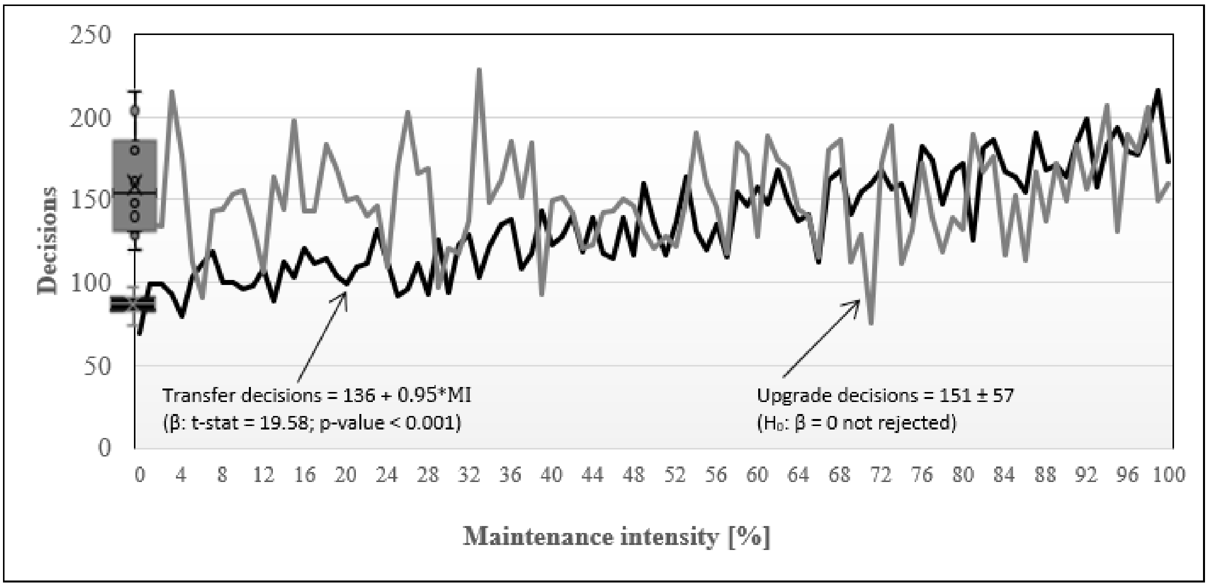


Figure 37 Number of transfer and upgrade decisions per maintenance intensity; overlayed with box plots for decision frequency at 0% maintenance intensity

Figure 37 also includes 2 box-plots for the number of transfer and upgrade decisions at MI = 0%. MI = 0% actually means that maintenance does not exist. This figure is thus an exemplar of how useful it is to holistically model the car rental because the inclusion of maintenance (MI>0%) obtains differential function relationships between fleet (transfers) and revenue (upgrade) management decisions which otherwise would be held constant at say, average represented by the respective box-plots.

Generally, maintenance intensities create delays such that vehicles are less likely to fulfill reservations. This creates more uncertainty from an operational perspective. A second option to vehicle transfers are vehicle upgrades. Figure 37 shows that the number of upgrade decisions is constant across maintenance intensities for the following reason: When a rental station cannot fulfill an incoming reservation request for a particular vehicle, the simulation is implemented such that it first looks at the cost of rebalancing a vehicle from the primary repositioning node (if possible) and the revenue associated with the reservation. Only if i) the transfer cost exceeds the potential revenue, and ii) the proposed upgradable vehicle stock is above its critical value (<1), the system suggests the reservation be upgraded to the next higher vehicle type. Apparently in my simulation settings, the rebalancing option caters for all levels of maintenance intensities as the upgrade decision proves to be a more feasible option from a cost perspective (until possible upgrade decisions are exhausted).

Upgrading is preferred 3:1 over transfer (at MI=0%). However, at MI=100%, the ratio is approximately 1:1 between upgrade and transfer decisions. This is because it is generally more profitable to the firm to upgrade rather than transfer. However, at high MI, where more reservation requests need altering, the upgrading does not increase relative to transfers because the capacity of the next higher vehicle group is exhausted. Additionally, because the arrival and trip selection process are randomized, an incoming reservation that proposes a rebalancing decision may be for vehicle that cannot perform an upgradable reservation event (FSUV, OTHER). These are all factors which would reduce the feasible amount of upgrade decisions made, and because scheduled maintenance is linked with the operation and movement of the fleet, it influences the tactical deployment of vehicles, which is why empty transfers incur the greatest operational burden where fleet unavailability is increased.

Table 35 Simulation V7.1: Change in fleet availability for each station and vehicle group versus Maintenance Intensity (MI) after 6 months of fleet movement simulation. [-17;-12) = ■; [-12;-7) = ■; [-7;-2) = ■; (-2;2) = ■; (2;7] = ■; (7;12] = ■; (12;17] = ■ *

Notes

*: There are 6 occasions where the difference is larger than 17 subsumed in this category.

	VEHICLE GROUP - LOCATION																				
	EC-CHC	CA-CHC	CSUV-CHC	ISUV-CHC	FSUV-CHC	FSED-CHC	OTHER-CHC	EC-ZQN	CA-ZQN	CSUV-ZQN	ISUV-ZQN	FSUV-ZQN	FSED-ZQN	OTHER-ZQN	EC-DUD	CA-DUD	CSUV-DUD	ISUV-DUD	FSUV-DUD	FSED-DUD	OTHER-DUD
START SIM	15	22	11	8	6	13	6	9	13	7	4	3	7	2	1	1	1	1	1	1	1
MI(%)																					
0																					
1																					
2																					
3																					
4																					
5																					
6																					
7																					
8																					
9																					
10																					
11																					
12																					
13																					
14																					
15																					
16																					
17																					
18																					
19																					
20																					
21																					
22																					
23																					
24																					
25																					
26																					
27																					
28																					
29																					
30																					
31																					
32																					
33																					
34																					
35																					
36																					
37																					
38																					
39																					
40																					
41																					
42																					
43																					
44																					
45																					
46																					
47																					
48																					
49																					
50																					
51																					
52																					
53																					
54																					
55																					
56																					
57																					
58																					
59																					
60																					
61																					
62																					
63																					
64																					
65																					
66																					
67																					
68																					
69																					
70																					
71																					
72																					
73																					
74																					
75																					
76																					
77																					
78																					
79																					
80																					
81																					
82																					
83																					
84																					
85																					
86																					
87																					
88																					
89																					
90																					
91																					
92																					
93																					
94																					
95																					
96																					
97																					
98																					
99																					
100																					

Table 35 displays a heatmap of the ending capacity at each station by vehicle group for the different maintenance intensities. It is clear that the schedule influences the end positions of the fleet, although, the fleet management and revenue management systems put in place ensure that the firm can still run with the greater operational burden. Having the ability to encapsulate the many different fleet management sub-problems improves the physical and financial outcomes of the pool network when it placed under a tremendous load of operational stress.

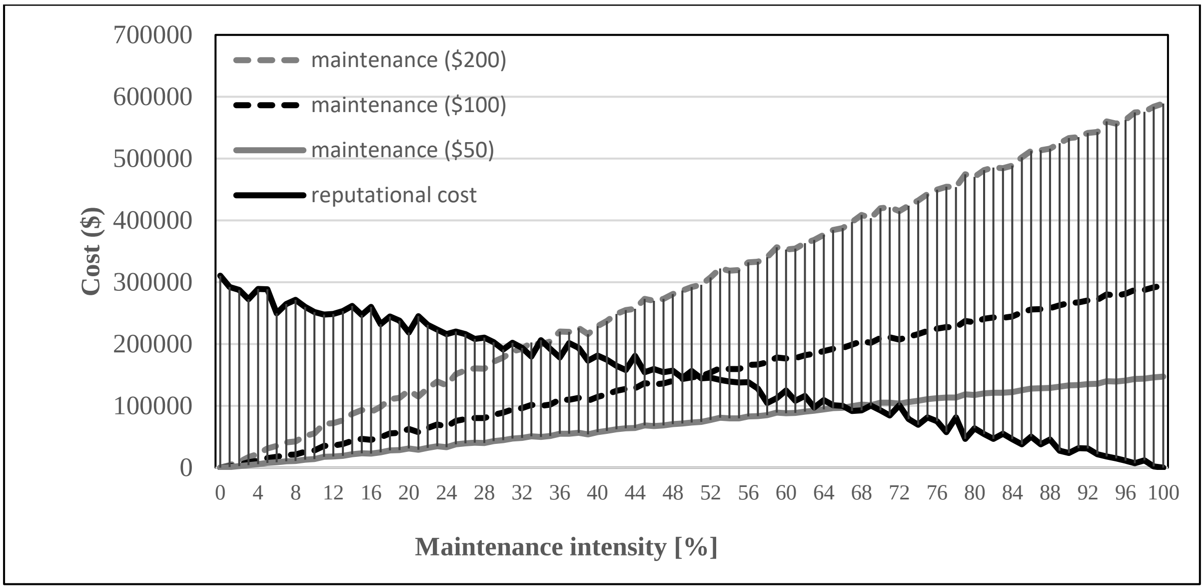


Figure 38 maintenance and reputational costs per maintenance intensity

The scheduled maintenance model is extended in Figure 38 with the consideration of a reputationally damaging event to the firm where a maintenance event is not engaged upon, and a vehicle defect is detected. If a customer discovers a defect in the rented vehicle, a reputational cost to the firm of $500 is recognised. This value is modelled against difference maintenance costs per scheduled maintenance events in which the trade-off between reputational cost and maintenance events can be observed. At a reputational cost of $500 and a 20% proportional chance of an RDE, which represents a customer recognizing a defect in an unchecked vehicle, break-even points per maintenance intensity are generated (cf. Figure 37). With a maintenance cost of $200, the simulation settings break-even at an intensity around 35% and as maintenance cost decreases the intensity is able to increase up to 68% with a maintenance cost of $50. This is important analysis for a car rental firm to conduct in determining optimal maintenance intensities at relevant maintenance costs.

5 Conclusion

The majority of papers which analysis the car rental fleet management process does so through the lens of an isolated sub-problem. There is a clear disconnect between the fleet management and revenue management focus (Oliveira et al., 2017), in which fleet management generally aims to optimise physical fleet outcomes through cost minimization and revenue management aims to optimise financial outcomes through revenue or profit. Modelling and optimising an isolated sub-problem of the fleet management process fails to characterize the interconnectedness of the fleet management and revenue management sub-problems, along with the overlapping time horizons and the physical and financial elements that envelop the operation of a firm that utilizes mobile fixed assets as the core of their business operations. There are a number of important sub-problems: pool segmentation, fleet size, acquisition and disposals, fleet mix, strategic fleet deployment, tactical fleet assignment, capacity allocation, price setting, price level strategy, reservation cancellations, late vehicle returns, dynamic and uncertain demand, competitors in the industry, customer types, vehicle breakdowns. Each sub-problem has mutual influence over the physical and financial outcomes for the financial sustaining operations of a car rental firm.

To address the limitations identified in the literature, I have opted for a Monte Carlo simulation model developed in Excel 365 combined with an adaptation of Statistical Activity Cost Analysis (SACA). I have chosen an incremental approach to the simulation methodology in which I have gradually increased the complexity of selected fleet management sub-problems and demonstrated how these increase the dimensionality of the simulations (which equals the generalizability of the model). Notably, with respect to financial modelling, my work demonstrates how individual financial simulation variables translate into financial statements. This allows for appropriate financial statement analyses.

The most important results are best represented in the final simulation version (V7), which is robust in adapting to deleterious outcomes that are the result of the dynamic and uncertain nature of the car rental industry. How the simulation inputs have changed from V4 to V7 is displayed in Figure 39, which shows the evolution of the inputs when I implemented additional sub-problems.

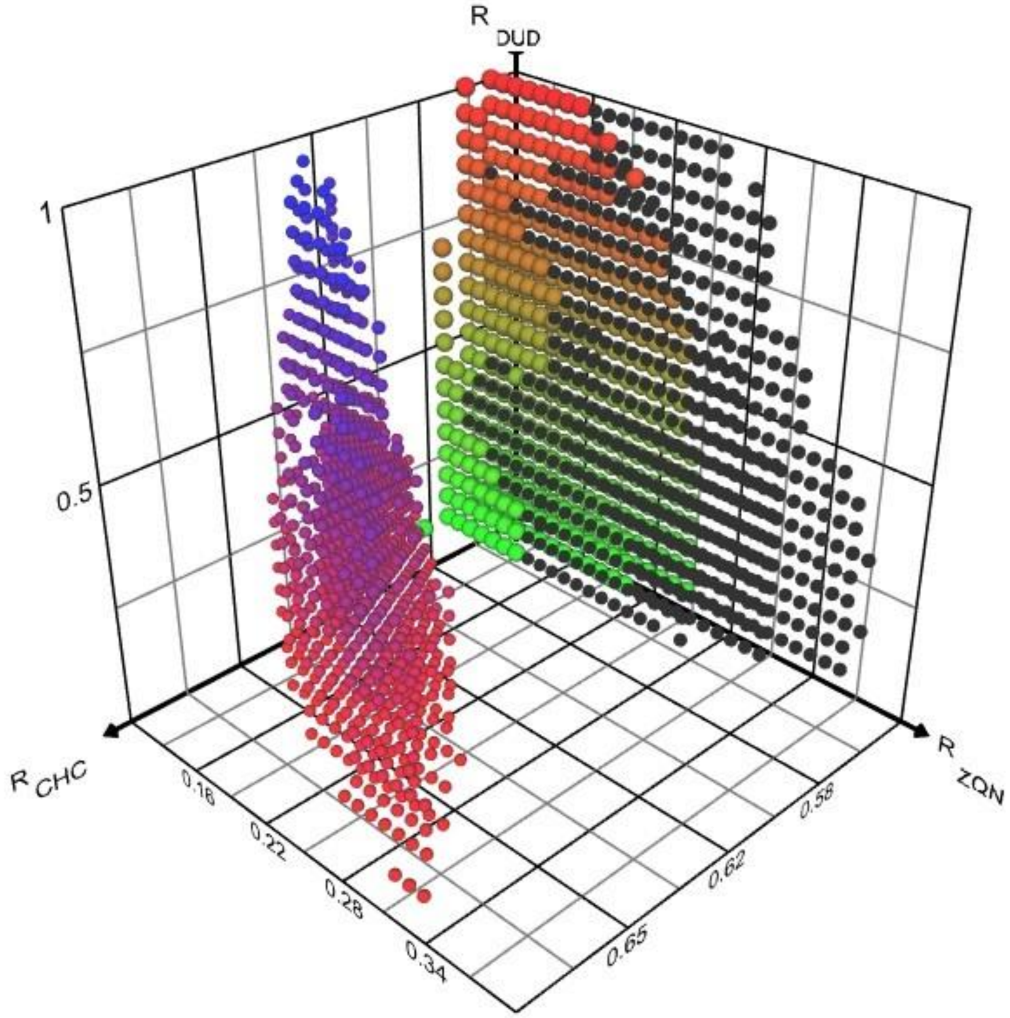


Figure 39 3D of round-trip inputs for simulations V4 (blue/red), V5 (black), and V7 (green/red)

V7 considers the following car rental sub-problems: pool segmentation, fleet size, fleet mix, strategic fleet deployment, tactical fleet assignment, capacity allocation, price setting, dynamic and uncertain demand, variable maintenance intensity. With a realistic adaptation of an empty vehicle rebalancing and limited cascading upgrade process, vehicle availability is modelled in terms of location and not time. This gives the system options in terms of how to allocate and rebalance capacity, which is especially prevalent in fleet assignment, where maintenance schedules and their intensity require a higher level of operational intervention to conduct their business sustainably. Maintenance is an unavoidable issue in the car rental context, where much of the literature generalizes this as an inherent cost (e.g., Carriera and Santos, 2014; Oliveira et al., 2014; Song and Earl, 2007). The literature has been extended to encapsulate maintenance schedules (e.g., Ernst et al., 2010; George and Xia, 2010; Hertz et al., 2009), although these are not tested vigorously at different intensities to encapsulate the physical and financial consequences. Ernst et al. (2010) integrated empty vehicle rebalancing decisions into their model, which improved the generalizability their work. However, just like the majority of fleet management papers, their model is structured in the form of cost minimization, which is not representative of the profitability of the car rental firm to continue operations into the foreseeable future.

In summary, V7 models various physical processes and how these are translated into financial outcomes via financial statements which capture real problems faced by car rental business. Simulation studies can reliably incorporate uncertainties in any variable and thus allow for scenario testing. Any simulation study can be extended. I believe that the following would be the most interesting extensions for future simulations V8+.

Firstly, modelling of walk-in customers and a price sensitive demand model, which adapts based on consumer price elasticity as time trends closer to the reservation start time, would be interesting to see how the prices influence the occupation rates and the price levels set. This would necessitate modelling the interplay between rental firms and how they compete with setting prices to optimise fleet occupation while operating sustainably. Secondly, physical SACA activities would be further developed to better represent fixed asset behaviour through, for example, by tracking individual vehicles and their movements over time with monitoring the reduction in reliability that follows. Seeing the interplay of these features over a longer simulation horizon (1+ years), and the stability of the simulation would be a third avenue worth looking into. A final note relates to the scalability of my simulation environment. The adaptability of the Excel 365 software is limited, unlike programming language which is adaptable through lines of code. Excel must encompass a large array of cells over many columns and rows to represent a single process at a particular moment in time. Because of this, excel modelling reaches a point where the simulation of a large array of parameters over a large number of operational functions is too much for any system to handle.

References

Alabdulkarim, A. A. (2018). Simulating different levels of car class upgrades in a car rental company's operation. *2018 Winter Simulation Conference (WSC)*. 1539-1550. https://doi.org/10.1109/WSC.2018.8632511

Anderson, C. K., & Blair, M. (2004). Performance Monitor*:* The opportunity costs of Revenue Management. *Journal of Revenue and Pricing Management*, *2*(4). 353-367. https://doi.org/10.1057/palgrave.rpm.5170083

Anderson, C. K., Davison, M., & Rasmussen, H. (2004). Revenue Management: A Real Options Approach. *Naval Research Logistics*, *51*(5). 686-703. https://doi.org/10.1002/nav.20026

Blair, M., & Anderson, C. K. (2002). Performance Monitor. *Journal of Revenue and Pricing Management*, *1*. 57-66. https://doi.org/10.1057/palgrave.rpm.5170006

Carroll, W. J., & Grimes, R. C. (1995). Evolutionary Change in Product Management: Experiences in the Car Rental Industry. *Interfaces*, *25*(5). 84-104. https://doi.org/10.1287/inte.25.5.84

Conejero, J. A., Jordán, C., & Sanabria-Codesal, E. (2014). An Iterative Algorithm for the management of an electric car-rental service. *Journal of Applied Mathematics*, *27*. 1-11. https://doi.org/10.1155/2014/483734

Carreia, G. H. A., & Santos, R. F. G. (2014). Optimizing the Use of Electric Vehicles in a Regional Car Rental Fleet. *Transportation Research Record: Journal of the Transportation Research Board*, *2454*(1). 76-83. https://doi.org/10.3141%2F2454-10

Colin, A., Lambrineas, P., Weir, T., & Willet, R. J. (2012). A Statistical Activity Cost Analysis of Fleet Schedule Data. *Engineering Asset Management and Infrastructure Sustainability*. 121-131. http://dx.doi.org/10.1007/8663_2015_4

Colin, A., Lambrineas, P., & Willet, R. J. (2010). Optimizing budget allocations in naval configuration management. *Engineering Asset Management Review, Amadi Echendu JE*. 137-149. http://dx.doi.org/10.1007/978-1-84996-178-3_5

Colin, A., Falta, M., Su, S., Turner, L., Willet, R. J., & Wolff, R. (2006). A statistical activity cost analysis of the relationship between physical and financial aspects of fixed assets. *Engineering Asset Management : Proceedings of the First World Congress on Engineering Asset Management (WCEAM).* 1-8. http://dx.doi.org/10.1007/978-1-84628-814-2_43

Costa, S. (2019). Understanding Demand and Pricing Behavior in the Car Rental Business. *University of Portugal*. https://hdl.handle.net/10216/121292

Edelstein, M., & Melnyk, M. (1977). The Pool Control System. *Interfaces*, *8*(1). 21-36. https://doi.org/10.1287/inte.8.1pt2.21

Ernst, A. T., Gavriliouk, E. O., & Marquez, L. (2010). An efficient Lagrangean Heuristic for Rental Vehicle Scheduling. *Computers & Operations Research*, *38*(1). 216-226. https://doi.org/10.1016/j.cor.2010.04.014

Falta, M., Gallagher, L., & Willet, R. (2006). Modelling ‘Hard-to-Measure’ Costs in Environmental Management Accounting. *Asia-Pacific Management Accounting Journal*, *1*(1). 127–144. http://arionline.uitm.edu.my/ojs/index.php/APMAJ/article/view/360

Fink, A., & Reiners, T. (2004). Modelling and solving the short-term Car Rental Logistics Problem. *Transportation Research Part E: Logistics and Transportation Review*, *42*(4). 272-292. https://doi.org/10.1016/j.tre.2004.10.003

Geraghty, M. K., & Johnson, E. (1997). Revenue Management saves National Car Rental. *INFORMS Journal on Applied Analytics*, *27*(1). 107-127. https://doi.org/10.1287/inte.27.1.107

Guerriero, F., & Olivito, F. (2014). Revenue Models and Policies for the Car Rental Industry. *Journal of Mathematical Modelling and Algorithms in Operations Research, 13*. 247–282. https://doi.org/10.1007/s10852-013-9234-8

Guillen, J., Ruiz, P., Dellepiane, U., Maccarrone, L., Maccioni, R., Pinzuti, A., & Procacci, E. (2019). Europcar Integrates Forecasting, Simulation, and Optimization Techniques in a Capacity and Revenue Management System. *INFORMS Journal on Applied Analytics*. *49*(1). 1-91. https://doi.org/10.1287/inte.2018.0970

Haensel, A., Mederer, M., & Schmidt, H. (2011). Revenue Management in the Car Rental Industry: A Stochastic Programming Approach. *Journal of Revenue and Pricing Management, 11*. 99–108. https://doi.org/10.1057/rpm.2010.52

Hertz, A., Schindl, D., & Zuffery, N. (2009). A Solution Method for a Car Fleet Management Problem with Maintenance Constraints. *Journal of Heuristics, 15*(5). 425-450. https://doi.org/10.1007/s10732-008-9072-4

Klein, R., Koch, S., Steinhardt, C., & Strauss, A. K. (2019). A review of Revenue Management: Recent generalizations and advances in industry applications. *European Journal of Operational Research*, *284*(2). 397-412. https://doi-org.ezproxy.canterbury.ac.nz/10.1016/j.ejor.2019.06.034

Li, D., & Pang, Z. (2017). Dynamic Booking Control for Car Rental Revenue Management: A Decomposition Approach. *European Journal of Operational Research, 256*(3). 850-867. https://doi.org/10.1016/j.ejor.2016.06.044

Li, Z., & Tao, F. (2010). On determining Optimal Fleet Size and Vehicle Transfer Policy for a Car Rental Company. *Computers & Operations Research*, *37*(2). 341-350. https://doi.org/10.1016/j.cor.2009.05.010

Lohmann, G., & Zahra, A. (2010). The influence of international tourists' travel patterns on rental car fleet management in New Zealand. *Drive tourism* (1st ed, pp. 159 – 171). Routledge. https://doi.org/10.4324/9780203880395

Oliveira, B. B., Carravilla, M. A., Oliveira, J. F., & Toledo, F. M. B. (2014). A Relax-and-fix-based Algorithm for the Vehicle-Reservation Assignment Problem in a Car Rental Company. *European Journal of Operational Research*, *237*(2). 729-737. https://doi.org/10.1016/j.ejor.2014.02.018

Oliveira, B. B., Carravilla, M. A., Oliveira, J. F., Raicar, P., Acácio, D., Ferreira, J., & Araújo, P. (2015). Pricing for Internet Sales Channels in Car Rentals. *Operations Research and Big Data*, *15*. 139-147. https://doi.org/10.1007/978-3-319-24154-8_17

Oliveira, B. B., Carravilla, M. A., & Oliveira, J. F. (2017). Fleet and Revenue Management in Car Rental Companies: A Literature Review and an Integrated Conceptual Framework. *Omega*, *71*. 11-26. https://doi.org/10.1016/j.omega.2016.08.011

Oliveira, B. B., Carravilla, M. A., & Oliveira, J. F. (2018). A Dynamic Programming Approach for Integrating Dynamic Pricing and Capacity Decisions in a Rental Context. *Operational Research*, *223*. 297-311. https://doi.org/10.1007/978-3-319-71583-4_20

Oliveira, B. B., Carravilla, M. A., & Oliveira, J. F. (2018). Integrating Pricing and Capacity Decisions in Car Rental: A Matheuristic approach. *Operations Research Perspectives*, *5*. 334-356. https://doi.org/10.1016/j.orp.2018.10.002

Oliveira, B. B., Carravilla, M. A., Oliveira, J. F. & Costa, A. M. (2019). A co-evolutionary matheuristic for the car rental capacity-pricing stochastic problem. *European Journal of Operational Research*, *276*(2). 637-655. https://doi.org/10.1016/j.ejor.2019.01.015

Pachon, J. E., Iakovou, E., Ip, C., & Aboudi, R. (2003). A synthesis of Tactical Fleet Planning Models for the Car Rental Industry. *IIE Transactions*, *35*(9). 907-916. https://doi.org/10.1080/07408170304409

Pachon, J. E., Iakovou, E., & Ip, C. (2006). Vehicle Fleet Planning in the Car Rental Industry. *Journal of Revenue and Pricing Management*, *5*(3). 221-236. https://doi.org/10.1057/palgrave.rpm.5160041

Patel, D., Patel, A., & Patel, D. (2018). Optimization of a Car Rental Fleet to fulfill the Demand. *International Journal of Trend in Research and Development*, *5*(2). 508-512. https://www.researchgate.net/publication/324830841_Optimization_of_A_Car_Rental_Fleet_To_Fulfill_The_Demand

Russell, R. A., & Madden, T. (2012). Modelling the Rental Car Pricing and Relocation Problem. *International Journal of Revenue Management*, *6*(3/4). 246–257. https://doi.org/10.1504/IJRM.2012.050400

Song, D. P., & Earl, C. F. (2007). Optimal Empty Vehicle Repositioning and Fleet-Sizing for Two-Depot Service Systems. *European Journal of Operational Research*, *185*(2). 760–777. https://doi.org/10.1016/j.ejor.2006.12.034

Steinhardt, C., & Gönsch, J. (2012). Integrated Revenue Management approaches for Capacity Control with Planned Upgrades. *European Journal of Operational Research*, *223*(2). 380-391. https://doi.org/10.1016/j.ejor.2012.05.047

Yang, Y., Jin, W., & Hao, X. (2008). Car Rental Logistics Problem: A review of literature. *IEEE International Conference on Service Operations and Logistics, and Informatics*. 2815-2819. https://doi.org/10.1109/SOLI.2008.4683014

Yang, Y., Jin, W., & Hao, X. (2008). Dynamic Pool Segmentation Model and Algorithm in the Car Rental Industry. *Journal of Computers*, *4*(12). 1202–1208. https://doi.org/10.4304/jcp.4.12.1202-1208

You, P. S., & Hsieh, Y. C. (2014). A study on the Vehicle Size and Transfer Policy for Car Rental Problems. *Transportation Research Part E: Logistics and Transportation Review*, *64*. 110-121. https://doi.org/10.1016/j.tre.2014.01.007

Yu, Y., Dong, Y., & Guo, X. (2018). Pricing for sales and per-use Rental Services with Vertical Differentiation. *European Journal of Operational Research*, *270*(2). 586-598. https://doi-org.ezproxy.canterbury.ac.nz/10.1016/j.ejor.2018.03.035