THE MATHEMATICAL SIMULATION OF BATCH-DRYING OF

SOFTWOOD TIMBER

VOLUME I

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by

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SUMMARY

This thesis examines the drying of softwoods with particular reference to the species *Pinus Radiata* as the timber is the most important species commercially grown in New Zealand. The study is relevant to conventional forced convection kiln-drying of wood at temperatures below 100 °C. The work concentrates on clarification of the moisture flow mechanisms responsible for drying of wood with the overall aim of using this knowledge to develop improved drying schedules.

The first stage considers boundary-layer mass transport from the wood surface to the passing airstream. From a comparison of experimental kiln heat and mass transfer coefficients with those predicted by engineering correlations, Sorenson's truncated slab correlation appeared the most relevant. Equations were developed for the evaporative heat and mass transfer driving forces from saturated surfaces. The Chilton-Colburn analogy is assumed to be exact only at the limit of infinitesimally small fluxes and zero moisture vapour concentration. Correction factors, for the often assumed linear humidity and temperature differences, are shown to be essentially functions of wet-bulb temperature only. From these results a novel humidity chart is produced. The driving force calculations are extended to include unsaturated surfaces by assuming the surface layers to be in hygroscopic equilibrium with the humid airstream according to the equilibrium moisture content isotherm. A reduced driving force function is predicted which is a convex-upwards shaped curve with respect to surface moisture content. The convexity increases with increase of the saturated surface driving force.

The batch dryer dynamics are studied using the lumped parameter characteristic drying rate concept. Van Meel's model is extended to include both heat and mass transfer. The two processes are similar provided sensible heat transfer to the solids is small. New solutions, using analytical and finite difference techniques, are presented. By approximating the drying rate curve to a number of linear segments, a convenient solution is derived for problems involving complex shaped rate functions. The existence of a "long-dryer" limit
is shown where the drying zone sweeps through the dryer at constant velocity. Further results are graphed where the simple model is extended to include variable inlet air-conditions, nonconstant transfer coefficients, effects due to solid shrinkage, variable solid equilibrium moisture content, and effects due to maldistribution of air-velocity. The significance of reversals in airflow direction upon dryer performance is also studied. It is shown that one pair of reversals can smooth the moisture-content distribution as effectively as an infinite set of reversals. The batch-drying model can be used to estimate operating costs and it is established that it is desirable to maintain high wet-bulb temperatures to minimise energy consumption.

In chapter 5 the assumption of a unique drying rate curve is examined. Various solutions to the linear and nonlinear moisture diffusion equation are used to predict critical point and drying rate curves as a function of average moisture content and drying conditions. The assumptions of invariant critical point and constant shaped curve will only hold over a narrow range of conditions unless the process is boundary-layer controlled.

Effective diffusion coefficients for the internal moisture flow are predicted as a function of moisture content, density and temperature. Equations for the complex flow mechanisms through the pores in terms of Knudsen flow, vapour diffusion, liquid diffusion, Poiseuille flow and slip flow are combined with pore-size distributions and microscopic structural data. As such predictions proved sensitive to the choice of structural model, an alternative method of fitting experimental moisture content profiles to diffusion equation solutions was used to obtain a reliable diffusion coefficient function.

The nonlinear diffusion equation together with the predicted diffusion coefficient and boundary-layer driving force relationships is solved for typical timber-kiln conditions. The change of wood temperature during drying is shown to accelerate the drying. The effect of density variation in the wood (across growth rings) is also briefly considered. The solutions are coupled with the airstream mass and energy conservation equations to predict the solid moisture content profiles throughout the whole dryer.
These complete dryer simulations provide information for further tests of the accuracy of the simple batch-drying model. If the invariant critical point assumption is relaxed for the batch-drying method, then close agreement is obtained with the detailed simulation.

The final aspect of the thesis relates the internal moisture-content gradients to the development of drying stresses due to restrained shrinkage. An elastic-plastic model is proposed assuming no interaction between stress and moisture movement. Experimentally determined elastic, plastic and shrinkage parameters are used to predict drying stresses across the kiln. The magnitude of the stresses suggest that surface cracking could occur midway during drying. Nevertheless the overall stress history exhibits similar characteristics to that observed experimentally with stress reversal occurring near the end of drying. These results are used to propose an algorithm for the design of a three-stage drying schedule.
1. Background

Drying has been a long practised art. This unit operation was recognised by early man as he used the wind and sun to dry many natural products (e.g., fruit and grain) for preservation, stabilization, or simply to reduce their weight and bulk for storage and transportation. However, drying has been a comparatively neglected field of chemical engineering research perhaps because in many chemical processing plants the drying operation is not crucial compared with a reaction or separation process. Nevertheless, several primary industries are dependent upon the drying operation and it may often be the only treatment of the harvested material in order to produce a saleable product.

One such striking example of this is the kiln-drying of timber in order to enhance its properties as a building material. Although modern tall buildings demand concrete and steel for their construction, wood is popular for domestic buildings because of its insulation properties, workability, flexibility and its natural beauty. Green wood is often used for framing and roof joints, but interior timber demands a low moisture content in order to ensure dimensional stability in terms of the humidity and temperature levels dictated by human comfort. It must also be dry so that it can be machined to develop its decorative qualities.

Until early this century timber was dried in natural-convection kilns heated from under the floor. Forced-circulation kilns soon became an obvious improvement. Because of the large residence times, (e.g., modern kilns require weeks to dry some hardwoods) methods of accelerating the process have been examined by forestry research workers. Many empirical trials have been evaluated using such diverse methods as prefreezing of wood, solar heat, dielectric heating, microwave heating, delta-wing vortex generators, solvent drying, centrifugal kilns, and the pulsing of airflow.
These esoteric methods resulted in the reduction of kiln times. They also established that drying wood in the slower conventional forced-circulation kiln was the most economic method.

Kiln schedules have been developed for forced circulation ventilated kilns by experience and trial and error. As the basic operating variables for a modern kiln are air-temperature and air-velocity most improvement in schedules has been made in these directions. Many existing schedules are complex involving several step changes of the inlet humidity conditions to the stack coupled with air-flow reversals. Due to the different degrade properties of various timbers, there are over ten basic schedules recommended.

The last decade has seen the emergence of two further important factors. The advances in technology of drying with a superheated steam medium has promoted much research of drying at temperatures greater than 100°C, the boiling point of the wood moisture. These methods have greatly accelerated the process but at the expense of a marked deterioration in wood quality. Considerable work is still required to optimise the effect of these two opposing criteria in order to establish "high temperature drying" as a viable commercial process for drying softwoods. The availability of on-line computer control has also introduced the scope for more precise control of kilns and for more complex kiln drying schedules. For example, the drying of oak has been shown to be safely accelerated in an automated kiln as a continuously varying humidity schedule can be used.

The development of drying schedules by trial and error techniques is unsatisfactory. Even when computer controlled, kiln-trials are expensive and time-consuming. Although they may be based on sound empirical principles, the movement of moisture in wood and its relationship to air-velocity, temperature and wood density is not well understood. In order to be able to exploit the capability of the computerised kiln,
it is necessary to develop theoretically the relationships describing wood moisture movement. Consequently this thesis is an attempt to elucidate the movement of water in softwoods with respect to the state variables of temperature, density and air-velocity and to apply this knowledge to the development of optimum drying schedules.

2. Previous work on Simulation and Optimisation of Dryer Performance

Traditionally the drying problem has been split into a number of sub-problems without consideration of the overall process. This approach to studying the dryer kinetics is reflected by the engineering design methods. In some cases batch-drying methods, which depend upon lumped parameter models of the moist material, have been used for dryer design. More often less sophisticated methods have been applied. Even the lumped parameter approach will be inadequate in some circumstances. There is thus a need to improve engineering dryer design techniques by knowing more precisely what happens inside a dryer. Fulford, in discussing Soviet drying research, summarises the current status of dryer simulation:

"The ultimate aim of drying theory must be to set up the separate differential equations for heat, momentum and mass transfer inside the solid material being dried and in the external drying medium, and to solve these simultaneously for the particular conditions prevailing in each case. Considerable advances have been made in studying the internal and external aspects of drying separately but the additional complications arising when attempts are made to link the two processes at the surface of the solid have so far stymied all attempts at finding general theoretical solutions to the overall drying problem."

The external problem, the dynamics of batch drying (see Chapter 4), has been well studied. Nevertheless the examination of real drying schedules and the optimisation of the control
parameters have been largely ignored. To date, there have been only two preliminary studies of these latter problems. In one case Thygeson and Grossmann\textsuperscript{14} studied the optimal operation of a continuous through-circulation dryer by only considering the constant rate period of drying. The optimisation was subject to the inequality constraints of maximum permissible air-horsepower per unit area of the bed and maximum permissible local final moisture content in the bed. The thermodynamic approach to fluid-bed drying optimisation was followed by Sieniutycz\textsuperscript{15} for a silica-gel bed. A dynamic programming algorithm was used to find the optimal temperatures of the inlet gas for the case when the inlet gas humidity is constant. The optimal policy suggests increasing the inlet temperature until an admissible gas temperature is attained. In both studies simple cases of the overall problem are considered.

For the other classical approach to the drying problem, the consideration of the internal moisture movement, there still remains serious computational problems. These arise from the dependence of the transport coefficients on the values of the driving potentials. Except for the simplest cases, it has not been possible to obtain useful analytical solutions to the resulting non-linear equations. Numerical calculations for each specific case will undoubtedly remain important. However, one of the greatest obstacles to completing internal-moisture transfer calculations at present is the lack of transfer parameters for real materials. Methods do not exist for estimating, even roughly, the values of these parameters from the properties of the moist solid. Even a knowledge of the porous structure and moisture bonding within the solid is not sufficient to allow the prediction of these coefficients. Nevertheless Kauh and Peck\textsuperscript{16} evaluated drying schedules for balsa wood by considering the internal moisture transfer. By solving the linear diffusion equation with a complex evaporative boundary condition they obtain good agreement with experiments for the average moisture content of thin balsa slabs as a function of time. However all this work corroborates is the
effectiveness of their rate-determining function for the surface boundary condition. An improved approach to this type of problem is followed by Baughman in studying the concurrent-flow drying of corn. Again the effect of moisture concentration upon the diffusion coefficient is ignored, but an exponential temperature dependence is included. From this model the moisture contents and air-temperature throughout the bed are derived in order to evaluate automatic-control strategies for the dryer.

These previous two models illustrate the tentativeness of dryer simulation. Only one attempt has been presented in the literature to optimise the overall problem. Claxton used a dynamic-programming decision model to find an optimal policy for the kiln-drying of cedar pencil slats. A detailed moisture transport model was proposed by solving the diffusion equation with a concentration-dependent diffusivity according to nonlinear initial and boundary conditions. The nature of these functions is obtained by curve-fitting experimental data, for the drying of cedar wood. Due to an inadequate stress theory, an empirical model based upon kiln-operators' experience was used to estimate the product degradation. Then, by ignoring the changing air conditions across the dryer, a computer algorithm solves for the optimal economic policy taking into account energy requirements and degradation. The shortcomings of this work well illustrates the problems posed earlier by Fulford.

3. Specification of the Problem

The review of previous attempts of the simulation of solids drying emphasises that the problem is complex and its solution difficult. To develop a methodical approach to the problem, we shall consider the aims, scope and constraints of the problem.

The aim is to develop an optimum drying schedule for the removal of moisture from green wood. Overall an optimum
schedule should represent an economic minimization of degradation, kiln time and energy requirements. The energy required for drying will be the summation of the energy required for pumping the air, the energy necessary for evaporation of the wood moisture, and the energy lost through the walls of the dryer. This total energy demand is a function of the temperature level within the dryer, the time of drying, and the velocity of the moisture transport within the wood. Inevitably the process of removing moisture from the wet material has consequences leading to degradation. For hygroscopic materials, such as wood, shrinkage deforms the solid as the moisture is removed. Drying stresses are caused by the non-uniform moisture distribution resulting in severe degradation unless the rate of moisture transfer within the material is high. Further deterioration may result due to the redistribution of foreign substances such as resins and from the reduction of strength due to the prolonged thermal treatment.

The importance of degradation relative to energy costs depends upon the worth of the product relative to the processing cost. One crude criterion for optimising drying schedules would be to maximise wood quality. For wood, this is a reasonable approach as the final value greatly exceeds the production cost and particularly as the product value is a strong function of the final wood quality. Engalichev and Eddy estimated for 1000 board feet of 50 mm thick white pine that the raw material cost was $161, the drying costs to be $24, and the product value to be $218.

So to complete an optimisation, knowledge is required in two major areas; moisture transport and wood quality:
(i) **Moisture Transport** - An elucidation is required of the transport of moisture through the wood to its surface. This transport results from the driving force developed by the convection of hot partially saturated air across the surface. As the humidification of the air increases across the dryer, this will affect the moisture transport within the wood at discrete points within the dryer.
(ii) **Wood Quality** — Information is needed about the stresses developed in the drying wood. From knowledge of the moisture concentration history and elastic-plastic parameters, the wood quality can be evaluated.

Although the wood quality is a consequence of the drying history it imposes a number of constraints upon the final state of the dried wood. For example, the average final moisture content must lie within the equilibrium moisture content range of the environment where the wood is to be used. If the wood is too dry, energy is wasted and, when the material is sold on a weight basis, this bound moisture is saleable. When the wood is dried too quickly, casehardening of the surface may result. This phenomenon indicates the presence of irrecoverable strains due to plastic flow from stresses exceeding the yield limit. As a tough exterior is formed on the wood, the drying treatment in this case violates the constraint that the wood must be able to be machined. A more serious constraint is to ensure that the decorative and structural qualities of wood be preserved. This constraint will be exceeded when the stress exceeds the failure stress causing the material to split.

A large amount of information is required, even for one material, for a full simulation. The material's elasticity, transport, and equilibrium parameters as functions of the state variables of temperature, moisture content and density are necessary. The scope of the problem can be simplified by considering the material and the nature of the dryer relevant to New Zealand conditions. For example, different species of woods can exhibit quite different characteristics. As the predominant species grown in New Zealand for milling is the softwood *Pinus Radiata D. Don*, this species was chosen to be the subject of this study. "High temperature drying" is not used on a commercial scale in New Zealand, so the typical range of operation of a ventilated forced-circulation kiln is taken as the 20 to 100°C temperature constraints. Above 100°C, considerable complications would be introduced to the prediction
Fig. 1  Block diagram of solution method for the timber-drying problem.
of the moisture movement within the material because of the occurrence of internal vaporisation and filtration transfer.

Hence clarification and understanding of the moisture transfer and wood quality interrelationships is required so to calculate optimum drying schedules for the drying of green Pinus Radiata D. Don in a ventilated forced-circulation timber kiln.

4 Solution of Problem

Often in the modelling of engineering plant, simplifications are made to models for convenience of solution. This can lead to considerable deterioration of the physical meaning of the model. The philosophy adopted throughout this work is to consider interactions between variables until they are shown to be insignificant unless the problem is insolvable as a consequence of this added complication. To assist the solution, the problem is broken up into a number of blocks. There is considerable interaction between blocks as illustrated in Fig. 1. At the end of the thesis these various parts of the model are amalgamated for the overall simulation. The rearrangement of the problem into several sub-problems does prove advantageous. This allows some interactions to be studied within a sub-problem which could not otherwise be considered because of the increased complexity within the overall model. The optimisation can also be roughly sub-divided. For example, the external lumped-parameter model largely determines the operating costs of the dryer while the other major cost, that of degradation, is principally determined by consideration of the internal moisture movement.

An overall optimisation is beyond the scope of this thesis. This work concentrates on the study of the moisture transport process and the determination of wood quality. At the end of the thesis the optimization problem is posed but no attempt is made to solve the problem except in the terms of a preliminary exploration of maximising wood quality.
REFERENCES


CHAPTER 2  PRACTICAL ASPECTS OF THE TIMBER-DRYING PROCESS

Before examining the moisture transport process in detail, it is necessary to firstly consider the important aspects of the material to be dried. Of equal importance, is the operational procedure for the timber-dryer. This information provides the basic data for the detailed simulation posed in the following chapters.

PART 1. Nature of the Material

A comprehensive knowledge of the properties of the material under study is essential for an accurate simulation of the transport mechanisms involved. In the case of wood this is complicated by the fact that wood is the product of the metabolism of a living organism, a tree. Hence its properties are subject to wide variations, brought about by the external factors affecting the growth of a tree. Such diversity in wood properties is largely a result of environmental influences such as climate, soil, moisture, growing space, and undoubtedly genetic factors also play an important role. It is important to realise that we are not dealing with one standard, man-made material, manufactured to exact and easily reproducible specifications, but with a variable substance whose basic nature is by and large beyond man's control. Even disregarding the problems of variability wood is a substance of great complexity because of its intricate chemical and anatomical composition. This, in turn, is responsible for the complex nature of its physical properties.

Panshin and De Zeeuw\(^1\) note that wood possesses certain characteristics for the following reasons:

1. All tree stems have a predominantly vertical arrangement;
2. Wood is cellular in structure and its chemical composition, though diverse, is also remarkably similar in that all woods contain cellulose, noncellulosic carbohydrates and lignin;
3. Wood is anisotropic in nature, i.e. it exhibits different physical properties when tested along the three major
directional axes. (This fact arises from the structure and orientation of cellulose in the cell-walls, the elongated shape of the wood cells, and the longitudinal-radial arrangement of the cells in respect to the horizontal and vertical axes of the tree stem); (4) Wood is a hygroscopic substance, i.e. it loses and gains moisture as a result of changes in the atmospheric humidity and temperature. Due to the anisotropic nature moisture variations will produce dimensional changes in wood which are unequal in the three axial directions.

These above characteristics are common to all woods but there is another feature common to fast-growing softwoods, such as Pinus Radiata D. Don. In temperate climates, the vascular cambium is active during the spring and summer months, but passes through a period of partial or total dormancy during the winter months. This cyclic pattern of activity results in a series of concentric rings across the diameter of the tree. The cells formed during the early period of active growth, called earlywood or springwood are thinwalled and large in diameter. The wood formed towards the end of the growing season, the latewood or summerwood, is denser and harder than earlywood, because its cells are generally greater in length, smaller in the radial dimension and possesses thicker walls. This variation in structure and density then means the material is inhomogeneous in the radial direction. A much lesser degree of inhomogeneity is observed in other directions.

These characteristics of wood will be seen later to have important consequences for the drying process.

2. Sawing of Wood

Pinus Radiata has become a popular species in New Zealand forestry as it is particularly suited to the New Zealand climate and grows rapidly. Thirty to fifty years after planting the trees are milled for timber.
Fig. 2  Sawing Diagrams for Softwoods.
(Fig. 122 and 123, Brown and Bethel\textsuperscript{3})
(a) Three methods of quarter-sawing hardwood and southern pine.
(b) Cutting diagram of Douglas fir log,
42 ins. in diameter, showing the position of the various items cut from a log.
Fig. 3  Sawing scheme for New Zealand grown P. Radiata.
The logs are converted into usable timber products through the application of one or more machining processes. These processes reduce the log to smaller pieces having the size, shape and surface condition required of them in use. Efficient and profitable conduct of the machining operation requires that the cutting be done in such a way as to yield the greatest volume of useful and variable material out of the log. Typically for softwoods a 45-55% yield is obtained. These machining operations must also convert the log into products which satisfy accepted specifications and standards.

Consideration of the anisotropic properties of wood show that its greatest strength is in the longitudinal direction, so the log is sawn longitudinally in order to produce planks with the highest possible bending strength.

There are many specialised rules for sawing each species and several schemes of cutting a log may be used to obtain high utilisation as illustrated in Fig. 2. A simplified scheme is used in New Zealand for sawing Pinus Radiata partly because the trees being milled at present are of poorer quality due to the lack of pruning during early growth. The average small end diameter (s.e.d.) of Pinus Radiata logs is 400 to 500 mm and although the actual cuts depend on the diameter of the log the scheme is illustrated in Fig. 3 for a 500 mm diameter log. Firstly, 100 mm is cut from two opposite sides of the log to leave a 300 mm slab (referred to as a "cant" by sawmillers). Depending upon the nature of the pith (this region of the log reflects the lack of pruning) 50 to 100 mm is cut from the centre of the cant in a direction perpendicular to the first cuts. The remainder of the cant is cut into 100 mm by 50 mm dimension lumber, eighteen boards being obtained in this case. The timber is now ready for drying.

3. Description of Kiln

The timber dry-kiln is the most common installation for the artificial drying of timber. The various types of kilns used for the drying of timber consist of one or more chambers.
Fig. 4  Construction of timber kilns.
(a) Cross-shaft overhead fan kiln
(Fig. 5, Stevens and Pratt)
(b) Long-shaft, double track compartment kiln with alternately opposing internal fans
(Fig. 7, Rasmussen).
equipped so that control can be exercised over the temperature, relative humidity, and velocity of the air. On the basis of their construction and operation, kilns designed to dry wood are classified as either compartment or progressive kilns. Both classes include natural circulation and forced-circulation kilns of the ventilated type. However this thesis is only concerned with forced-circulation compartment kilns which operate as batch-dryers.

Modern dry kilns are generally constructed with a concrete floor, brick walls and a wood or reinforced concrete roof so that heat losses are small. Kiln sizes vary considerably but most kilns are large enough to accommodate single or double loading tracks. In general, the height of a kiln with under-load fans is 4 m from floor to ceiling while overload fan kilns are 6 m high. The width of a kiln depends primarily on the number of tracks installed and the method used to pile the lumber. For example, a single-track kiln with end-piled lumber is 4 to 5 m wide but with cross-piled lumber 6 to 7 m width is required.

A number of different types of forced-circulation compartment kilns have been developed. These differ mainly in the arrangement of fans or blowers. Many of these different types are illustrated in references 3, 4 and 5. A double-track, long-shaft, internal fan kiln with reversible circulation is shown in Fig. 4. It has alternate right-hand and left-hand fans mounted on the shaft. This arrangement, together with the fan baffles, forces the air across the kiln and prevents lengthwise drifting. Baffles between the fan floor and the top of the load, together with a combination walkway and baffle on the kiln floor, prevent the air from short circuiting below and above the load. Roof ventilators permit hot moist air to escape from the kiln, and to be replaced by cold dry air. A spray-line supplies steam for the control of relative humidity, and a booster coil reduces the temperature drop between the two tracks of lumber. Heating is usually effected either by steam pipes or by large pipes carrying hot flue-gases from a wood or oil-burning furnace. Only very occasionally
is electricity of gas employed for heating timber-drying kilns.

Kiln conditions are usually automatically controlled. Thermostats are used to automatically control and record dry and wet-bulb temperatures. Some thermostats consist of control bulbs connected to bellows or pressure springs by capillary tubes. Changes in temperature at the control bulbs result in pressure changes in the closed systems that directly or indirectly actuate either electrically or pneumatic driven motor valves. One motor valve controls steam flow into the heating coils, another controls the flow of steam or water used for humidification, and a third controls the opening and closing of the vents. These instruments maintain the temperatures and humidity as constant as possible throughout the kiln.

4. Preparation of the Load for the Kiln

Rasmussen has noted that much of the degrade and waste that occurs during kiln-drying results from poor stacking. Well-stacked lumber dries faster, more uniformly, and with less degrade. So careful preparation is necessary before a load of timber is ready for the kiln.

Sorting the stock simplifies stacking and also aids in collecting together material of like drying characteristics into a kiln charge. The following factors should be considered as far as is practical:

(a) Species - Some species have markedly different drying characteristics than others and so will require different schedules. In some cases, closely similar species can be dried together;

(b) Moisture Content - It is not desirable to mix air-dried and green stock in the same kiln charge. The wetter stock requires milder initial drying conditions and a longer drying time than the drier stock;

(c) Heartwood and Sapwood - For some species these differ appreciably in green moisture content. In addition sapwood generally dries more rapidly and with fewer defects than
Fig. 5 Method of box-piling for random-length lumber (Fig. 79, Rasmussen).
heartwood;
(d) **Grade** - The different grades of timber are preferably separated as the higher grades require milder drying schedules. This is so that they have higher strength, closer control of final moisture content, and better appearance than the lower grades;
(e) **Thickness** - Sorting for thickness is essential. Stock of uniform thickness simplifies stacking and drying. The thicker the material, the longer the drying time and the milder the drying conditions required;
(f) **Length** - For good stacking, one of the best and easiest methods is to pile lumber of a single length on kiln trucks. Overhanging ends of the longer boards in a load of mixed-length stock are likely to warp during drying.

The usual method of piling is to arrange the boards in horizontal layers one above the other, separated by a series of cross-piling sticks (called "stickers"). Stickers are normally made of 25 mm by 25 mm well-seasoned, straight-grained timber that is free from resin that might stain the boards. Within a particular layer, the stickers are spaced at intervals of 300 to 900 mm depending upon the thickness of the wood and its tendency to distort. For *Pinus radiata*, the spacing of 600 mm is recommended. Each vertical line of sticks should be fully supported by a bearer on the kiln floor or kiln trolley. Care should be taken to place the stickers in absolute vertical alignment, so that the weight of the stack is directly transmitted through the lines of stickers to the bearers below.

It is standard practice to build timber piles of rectangular cross-section, mainly for the sake of simplicity and ease of piling. For random length timber particular care is required and the box-piling method is used, as illustrated in Fig. 5. The sides of the stack should be as even as possible with no boards projecting sideways an undue amount beyond the others. Boards which jut out appreciably into the main side air passages will tend to act as air deflectors causing non-uniform air-circulation across the load. In forced-circulation
kilns the boards in each layer are placed edge to edge and the direction of air-circulation through the stack is parallel with the stickers. The last step is to weight the top of the stack with concrete slabs or other suitable material to help minimise the amount of distortion in the otherwise unrestrained top rows. The timber will now be ready to be placed in the kiln chamber for drying.

5. Operation of the Kiln

Once the load has been placed in the kiln there is an operating procedure to be followed so to apply a definite schedule of treatment to the wood. The general procedure is outlined below but a detailed summary is set out in references 4 and 5.

Before starting up a kiln, the operator should make sure that all gaps in and around the pile, that would allow short circuiting of the air, have been sealed. The fans may now be started and the warming of the kiln begun. During the warming, the temperature of the wood will lag appreciably behind that of the air. Consequently if the kiln is warmed quickly, moisture is condensed on the timber surfaces and the moisture content, even of green timber is increased slightly. For this reason a high humidity should not be employed during the warming period. It is suggested as a rough guide, that a constant difference of 5°C between wet- and dry-bulb readings should be maintained until the desired dry-bulb temperature is attained. The wet-bulb temperature should then be set according to the selected schedule.

Once the kiln is up to temperature, it is important to operate the kiln to a particular schedule making changes in temperature and humidity as dictated by the schedule and the state of the load. The dry-bulb temperature is controlled by the steam-heating coils, which have a slow response compared to the steam spray for humidity control. Departures of 1° or 2°C from the schedule temperature will have little effect on the drying provided the operator maintains the correct difference between the wet- and dry-bulb temperatures. A third means is available for controlling the humidity conditions by adjustment.
of the air-vents. Ideally the vents should be brought into use only when it is found with the steam spray off, that the humidity rises above the required value. The vents should then be opened so that a certain amount of damp air is exhausted and cool air drawn in, which when heated, lowers the humidity of the resulting mixture of vapour and air being circulated. Even when the vents are fully closed, moisture may escape due to leakage through badly fitting doors and ports. Apart from this, some moisture may condense on the walls and floor. It is obviously wasteful to open the vents more than necessary. However the best manual control is obtained by opening the vents slightly more than necessary for removing the water evaporated at any given stage of the drying. The deficit is made up by use of the steam spray which is capable of finer control than the air-interchange vents. For a fully automated kiln when the wet-bulb temperature rises above the desired value, the spray is shut off first and if necessary the vents are opened. Conversely when the wet-bulb temperature drops below the set-point, the dampers are closed before the steam spray is brought into operation.

As the timber on the air-entrance side of the stack dries more quickly than the rest it will reach the desired moisture content well before the other side of the load. To improve the uniformity of moisture content and to reduce the length of the conditioning treatment, it is expedient to reverse the direction of airflow from time to time. The advantages of the periodic reversal of airflow are least when the load consists of thick refractory timber slow drying in a well-designed kiln and greatest when seasoning thin free-drying softwood in a kiln with only moderate airspeed through the stack. It is fairly common practice to reverse the fans at fixed intervals of 12 to 24 hours irrespective of drying rates.

At the end of a run a conditioning treatment (4 to 8 hours steaming) may be found necessary. Almost invariably a case-hardening relief treatment (2 to 6 hours at 90 per cent relative humidity) is required prior to the cooling down of the kiln when final moisture content re-estimations have shown the wood to be adequately dried. For example, for interior
the New Zealand Standard requires that not more than 25 per cent of the kiln samples shall vary from a moisture content of 12 per cent by more than plus or minus 4 per cent.

In cooling the load, it is sometimes possible to immediately release the load to the outside atmosphere but if the kiln has been at high temperatures this may lead to a renewal of case-hardening stresses. Theoretically the humidity should be kept high during cooling. However this procedure would be slow and as a compromise a difference of 5°C between wet- and dry-bulb temperatures should be maintained until the temperature has dropped to within 20 to 25°C of that of the outside atmosphere. At this stage the kiln can be safely emptied.

6. Practical Drying Schedules

For the drying of wood a kiln schedule is a carefully worked out set of dry-bulb and wet-bulb temperatures which the operator can use to dry a specific wood product at a satisfactory rate without causing objectionable drying defects. The drying stresses that develop in the wood as it dries are related to the average moisture content of the board. Hence the drying schedules are based upon moisture content and as the moisture content decreases the relative humidity is decreased in a step-wise fashion.

Schedules for the processing of Pinus Radiata timber are presented below from England, Australia, New Zealand and the United States for comparison. As Pinus Radiata is not grown commercially in the United States, the drying schedules for the similar species of Pinus Ponderosa are used for illustration. Higher humidities are used for the thicker timber as can be seen in Table 1.
Table 1
Drying Schedules for Various Thicknesses of Pinus Radiata

(a) Lower Grade 25 mm timber

<table>
<thead>
<tr>
<th>Moisture Content</th>
<th>United States</th>
<th>Australia</th>
<th>New Zealand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_G/\degree C$</td>
<td>$T_W/\degree C$</td>
<td>$T_G/\degree C$</td>
</tr>
<tr>
<td>Green 40%</td>
<td>65.6</td>
<td>54.4</td>
<td>83.2</td>
</tr>
<tr>
<td>35%</td>
<td>65.6</td>
<td>51.6</td>
<td>83.2</td>
</tr>
<tr>
<td>30%</td>
<td>65.6</td>
<td>48.9</td>
<td>83.2</td>
</tr>
<tr>
<td>20%</td>
<td>71.1</td>
<td>51.6</td>
<td>83.2</td>
</tr>
<tr>
<td>15%</td>
<td>76.7</td>
<td>57.2</td>
<td>83.2</td>
</tr>
</tbody>
</table>

N.B. $T_G$ is the dry-bulb temperature and $T_W$ the wet-bulb temperature of the drying medium.

(b) Upper Grade 25 mm timber

<table>
<thead>
<tr>
<th>Moisture Content</th>
<th>United States</th>
<th>England</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_G/\degree C$</td>
<td>$T_W/\degree C$</td>
<td>$T_G/\degree C$</td>
</tr>
<tr>
<td>Green 50%</td>
<td>60.0</td>
<td>51.6</td>
<td>71.1</td>
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<tr>
<td>40%</td>
<td>60.0</td>
<td>48.9</td>
<td>76.7</td>
</tr>
<tr>
<td>35%</td>
<td>60.0</td>
<td>46.1</td>
<td>76.7</td>
</tr>
<tr>
<td>30%</td>
<td>65.6</td>
<td>48.9</td>
<td>82.2</td>
</tr>
<tr>
<td>25%</td>
<td>71.1</td>
<td>51.6</td>
<td>82.2</td>
</tr>
<tr>
<td>20%</td>
<td>71.1</td>
<td>48.9</td>
<td>87.7</td>
</tr>
<tr>
<td>15%</td>
<td>71.1</td>
<td>43.3</td>
<td>87.7</td>
</tr>
</tbody>
</table>
### (c) Upper Grade 38 mm timber

<table>
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<tbody>
<tr>
<td>Content</td>
<td>$T_G/^{\circ}C$</td>
<td>$T_W/^{\circ}C$</td>
<td>$T_G/^{\circ}C$</td>
</tr>
<tr>
<td>Green</td>
<td>54.4</td>
<td>46.1</td>
<td>71.1</td>
</tr>
<tr>
<td>50%</td>
<td>54.4</td>
<td>46.1</td>
<td>76.7</td>
</tr>
<tr>
<td>40%</td>
<td>54.4</td>
<td>43.3</td>
<td>76.7</td>
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<tr>
<td>35%</td>
<td>54.4</td>
<td>40.6</td>
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<tr>
<td>30%</td>
<td>60.0</td>
<td>43.3</td>
<td>82.2</td>
</tr>
<tr>
<td>25%</td>
<td>65.6</td>
<td>46.1</td>
<td>82.2</td>
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<tr>
<td>20%</td>
<td>71.1</td>
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<tr>
<td>15%</td>
<td>71.1</td>
<td>43.3</td>
<td>87.7</td>
</tr>
</tbody>
</table>

### (d) Upper Grade 50 mm timber

<table>
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<th>New Zealand</th>
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<tbody>
<tr>
<td>Content</td>
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<td>$T_W/^{\circ}C$</td>
<td>$T_G/^{\circ}C$</td>
</tr>
<tr>
<td>Green</td>
<td>54.4</td>
<td>48.9</td>
<td>54.4</td>
</tr>
<tr>
<td>50%</td>
<td>54.4</td>
<td>46.6</td>
<td>60.0</td>
</tr>
<tr>
<td>40%</td>
<td>54.4</td>
<td>43.3</td>
<td>60.0</td>
</tr>
<tr>
<td>35%</td>
<td>60.0</td>
<td>46.1</td>
<td>65.6</td>
</tr>
<tr>
<td>30%</td>
<td>65.6</td>
<td>48.9</td>
<td>65.6</td>
</tr>
<tr>
<td>25%</td>
<td>71.1</td>
<td>51.6</td>
<td>71.1</td>
</tr>
<tr>
<td>20%</td>
<td>71.1</td>
<td>43.3</td>
<td>71.1</td>
</tr>
</tbody>
</table>
The schedules given in Table 1 are designed for use in modern forced-air kilns with air velocities of between 1 and 2 ms\(^{-1}\) through the load.

The operational details presented above for typical timber-drying practice provide information for the geometrical arrangement of the solid and the humidity conditions it will encounter within the dryer. In future chapters of this thesis, these data are used to evaluate the experimentally optimised schedules in order to provide insight towards finding theoretically the most economic drying schedule for *Pinus radiata*.
REFERENCES – Chapter 2


CHAPTER 3  THE CALCULATION OF MASS TRANSFER FLUXES

One important aspect of wood-drying is the convective mechanism of moisture transfer from the wood surface to the airstream sweeping the surface. These convective mass transfer fluxes are often expressed as the product of a mass transfer coefficient and a concentration difference driving force. The coefficient is a complex function of the kiln geometry and airstream conditions. Estimations from the engineering literature are compared to experimental observations of timber drying practice in Part 1 of this chapter. In Part 2 an improved method of calculating the evaporative driving force from a saturated surface as a function of the temperature and humidity levels of the airstream is proposed. This theory is extended in Part 3 to predict the drying rates from the unsaturated surface of microporous hygroscopic solids.

Part 1 : The Prediction of Mass Transfer Coefficients for a Timber Kiln

The flow of a fluid along a surface is mathematically described by the equations of motion. It is also well-known from the Reynold's analogy that the convective modes of heat and mass transfer are closely linked to the fluid motion. An exact mathematical analysis requires the simultaneous solution of the equations describing the fluid transport and the transport of mass and energy within the moving fluid. As noted by Kreith¹ this method presupposes that the physical mechanisms are sufficiently well understood to be described in mathematical language. This preliminary requirement limits the scope of exact solutions. Complete mathematical equations describing the fluid flow and the heat and mass transfer mechanisms can be written only for laminar flow in terms of boundary layer approximations to the equations of transport. Even for laminar flow the equations are complicated, but solutions have been obtained for a number of simple geometries such as flow over a flat plate or a circular cylinder.

However even if the boundary layer equations were solvable for more complex situations much of the solution would be of little value as we are primarily concerned with the
concentration and energy fields at the solid-fluid interface. This circumstance arises because the energy and mass transfer fluxes are defined in terms of the concentration and temperature gradients at the interface:

\[
\text{energy flux} = q_{\text{surface} \rightarrow \text{fluid}} = -\lambda \frac{\partial T}{\partial z}
\]

\[
\text{mass transfer flux} = N_{\text{surface} \rightarrow \text{fluid}} = -D_{AB} \frac{\partial C}{\partial z}
\]

In equations (1) and (2) \(T\) is the fluid temperature, \(\lambda\) is the thermal conductivity of the fluid, \(C\) is the molar concentration of vapour in the fluid, \(D_{AB}\) is the diffusivity of vapour \(A\) through the fluid \(B\), and \(z\) is the direction perpendicular to the fluid motion where \(z\) is equal to zero at the solid surface. For engineering purposes the concept of the convective transfer coefficient has proved much more convenient,

\[
i.e., h_c(T_s - T_\infty) = -\lambda \frac{\partial T}{\partial z}
\]

\[
k_c(C_s - C_\infty) = -D_{AB} \frac{\partial C}{\partial z}
\]

where \(h_c\) and \(k_c\) are the convective heat and mass transfer coefficients respectively and the subscripts \(s\) and \(\infty\) refer to the values attained at the surface and at an infinite distance from the surface. Engineering workers have traditionally described heat and mass transfer in terms of equations (3) and (4). As a consequence the theoretically developed solutions of the transport equations have been expressed in power law correlations of dimensionless geometrical and physical property groups. For complicated situations these correlations have been obtained by experimental measurement.

Before the transfer coefficients relevant to the mass transport of moisture from the wood surface to the passing air-
**Fig. 6**  Flow channels through a timber stack.

**Fig. 7**  Effective hydraulic diameters for timber stack ducts as a function of sticker stickiness and spacing.

**Fig. 8**  Reynolds number for airflow through a timber stack for air-conditions $T_a = 60\,^\circ\text{C}$, $T_w = 50\,^\circ\text{C}$, as a function of air-velocity and hydraulic diameter.
stream can be found, knowledge of the kiln geometry is required. The information summarised in Chapter 2 suggests a typical kiln geometry as illustrated in Fig. 6. The flow of air through a drying stack of timber can be considered as flow through a number of rough-walled rectangular ducts in parallel. This is particularly so, if the boards are stacked edge to edge as is often the case in kiln-drying. An added complication to the flow description is that the stack presents a bluff entrance to the flow. Hence separation of the flow from the wood surface may occur near the stack entrance. Consequently this flow defies analytical description so in this case we are dependent upon empirical observations. This particular geometry represented by the kiln stack has received little attention in the engineering literature. However a number of similar situations have been studied and their applicability to the timber-drying problem will be considered below.

(a) Flow through Rectangular Ducts

This situation closely represents that for the timber stack. Many workers have shown that the description of fully developed flow through rectangular ducts of aspect ratios from 1 to 100 corresponds to that for flow through circular pipes provided the effective hydraulic diameter $D_E$ replaces the pipe diameter in the relevant equations. For a rectangular duct the effective hydraulic diameter is defined as being equal to $2cb/(c+b)$ where $c$ and $b$ are the duct cross-sectional dimensions. The effective diameters for timber stack ducts over the expected range of sticker thickness $c$ (15-40 mm) and the sticker spacing $b$ (300-900 mm) are presented in Fig. 7. Assuming an airstream humidity given by a dry-bulb temperature of 60°C and a wet-bulb temperature of 50°C, the Reynolds numbers $Re_D$ are plotted in Fig. 8 for the range of air-velocities 0.5 to 5 m/s based upon values of the diameter $D_E$ indicated in Fig. 7. From fluid friction studies in rectangular ducts, Hartnett et al. have shown that the critical Reynolds number for the onset of the turbulent flow regime is a weak function of the aspect ratio and the entrance configuration. For the abrupt-
entranced timber stack ducts this critical value is approximately 2500. Hence the results presented in Fig. 8 suggest that much of the operating range for timber drying lies in the turbulent region of flow.

The mass, heat and momentum analogy then suggests that the transfer coefficients may be given by the correlation for turbulent flow in circular ducts:

\[ \tilde{\eta}_M = 0.023 \, \text{Re}_D^{-0.2} \]  

However equation (5) only applies to the case of fully developed boundary layers. The bluff entrance and sudden contraction of flow into the stack will cause large flow disturbances. This means that a certain entrance length \( L_E \) will be required to establish the state of fully-developed flow. Within this entrance region, enhanced transfer rates will be attained. The usually quoted criteria for the length of this region in turbulent flow is 20 hydraulic diameters for the determination of distance-averaged transfer coefficients while for the determination of local transfer coefficients the entrance region is somewhat shorter. The entrance lengths for various hydraulic diameters are tabulated in Table 2. (i.e., \( L_E = 20 \, D_E \)).

<table>
<thead>
<tr>
<th>( D_E/\text{mm} )</th>
<th>( L_E/\text{m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.6</td>
</tr>
<tr>
<td>40</td>
<td>0.8</td>
</tr>
<tr>
<td>50</td>
<td>1.0</td>
</tr>
<tr>
<td>60</td>
<td>1.2</td>
</tr>
<tr>
<td>70</td>
<td>1.4</td>
</tr>
<tr>
<td>80</td>
<td>1.6</td>
</tr>
</tbody>
</table>

As typical stack lengths are 1.5 to 3 m, a significant portion of the stack will sense enhanced transfer rates due to
**Fig. 9**  Air-velocity profiles in flow channel between boards (40 mm apart) at various distances from channel entrance (from Fig. 12, Gilliwald and Tschirnich³).

**Fig. 10** Correction factor for local heat transfer due to entrance flow effects for an abrupt duct entrance with constant wall temperature (from E.S.D.U.⁷⁰, Fig. 4).
entrance effects. An illustration of entrance lengths in timber stacks was observed by Gilliwald and Tschirnich who measured the developing velocity profile between two planks at 40 mm spacing and Re$_D$ approximately equal to 9000. As shown in Fig. 9 the profile is still evolving at a distance 540 mm from the entrance.

The question still remains as to the magnitude and degree of enhancement to transfer rates caused by entrance effects. Romanenko and Krylova experimentally investigated entrance effects for various inlet conditions in turbulent flow and they found that a sudden contraction at the entrance produced the largest influence upon the heat transfer flux. By solving the integral heat transfer and momentum equations Deissler presents an extensive theoretical analysis of turbulent heat transfer and friction in entrance regions. The results indicate that fully-developed local heat-transfer and friction coefficients are, in general, attained in an entrance length less than 10 diameters. Graphs are presented in reference for the ratio of entrance flow coefficients to fully-developed flow coefficients which show this function to be practically independent of Reynolds number at low Prandtl numbers (such as for air). This ratio is also weakly dependent upon the fluid velocity profile at the duct entrance and the duct-wall boundary condition. Substantial agreement was obtained between this analysis and the experiments of Boelter, Young and Iverson for heat transfer to air in the entrance region of smooth tubes. The Engineering Science Data Unit has recently summarised entrance flow studies. Their recommended correction curves for local heat transfer most appropriate to the drying of timber, for the conditions of an abrupt entrance and constant wall temperature, is presented in Fig. 10. For averaged transfer coefficients in short ducts ($2 < L/D_e < 70$) with abrupt entrances, McAdams suggests the expression:

$$\bar{T}_H = \left[ 1 + (D_e/L)^{0.7} \right] 0.023 \text{Re}_D^{-0.2}$$  \hspace{1cm} (6)

Overall, then, this method of simulating the heat and
mass transfer through the timber stack suggests that the local convective transfer coefficients are independent of the distance in the direction of the flow except for a region near the stack entrance.

(b) Flow Over Blunt Plates

It has been observed by Norris and Streid\(^8\) that the Pohlhausen solution\(^9\) for flow over sharp-edged plates also applies to laminar flow through short flat rectangular ducts when \(L/D_E\) is less than \(0.0021 \text{Re}_D\). Although this condition will only apply to a small part of the timber stack, this similarity between the flow through rectangular ducts and flow over plates provides a basis for considering a stack of timber as a stack of blunt plates. This section is primarily concerned with flow over single plates as the study of heat and mass transfer about single boards has been a convenient simplification for experimentation by forestry workers. Another important aspect of this approach is that it has been responsible for considerable advancements in understanding the complex effects of the blunt leading edge.

The heat and mass transfer as a result of laminar flow over a sharp-edged plate is summarised by Pohlhausen's expression

\[
J_M = 0.332 \text{Re}_y^{-0.5}
\]  

(7)

where \(\text{Re}_y\) is the Reynolds number based upon the distance \(y\) from the leading edge in the direction of the flow. The corresponding expression for turbulent flow is

\[
J_M = 0.0288 \text{Re}_y^{-0.2}
\]  

(8)

assuming that the turbulent boundary layer starts at the leading edge. In reality, a laminar boundary layer precedes the turbulent boundary layer up to a \(\text{Re}_y\) of approximately 500,000. Nevertheless Viktorin\(^10\) found that in drying a stack of timber over the velocity range 2 to 6 ms\(^{-1}\) good agreement
FIG. 11 EXPERIMENTAL MEASUREMENTS OF LOCAL VALUES OF HEAT TRANSFER COEFFICIENTS FOR FLOW THROUGH A STACK OF TIMBER (FROM VIKTORIN).
was obtained with eqn (8) over the range of $Re_y$ from 8000 to 900 000, as indicated in Fig. 11.

The effect upon flow by the blunt leading edge was realised by early workers in studies on evaporation from pans or trays. Shepherd, Hadlock and Brewer\textsuperscript{11} and Chu, Finelt, Hoerrner and Lin\textsuperscript{12} observed considerable enhancement in rates with blunt-edged plates compared to sharp-edged plates. The first detailed study of these enhanced rates was by Danckwerts and Anolick\textsuperscript{13} when they examined mass transfer from a grid-packing by naphthalene sublimation techniques. They firstly studied the effect of mass transfer on truncated slabs and discovered that on the leading edge an eddy is formed causing very high mass transfer rates particularly at the point of reattachment. From the point of reattachment a laminar boundary layer is formed. Sørenson\textsuperscript{14} continued these studies with a more detailed investigation of mass transfer on slabs with additional work on the effect of the aspect ratio of the slab (length to depth). The leading edge turbulence also has considerable downstream effects and Sørenson observed that the mass transfer coefficient depended not only on air-velocity and distance from the leading edge but also on the thickness of the slab. The experimental results indicate the existence of a critical $Re_d$ value (based upon slab depth $d$) such that if $Re_d$ is less than 245 the coefficients are independent of $Re_d$ and follow the sharp-edged plate expression, whereas for $Re_d$ values greater than 245 the mass transfer coefficients clearly depend upon $Re_d$. These results also indicate that a laminar boundary layer exists behind the stationary eddy which is formed at the leading edge.

Sørenson correlated the results in terms of laminar flow expressions for plates. For the sharp-edged plate he obtained values 20 per cent higher than Pohlhausen's theoretical analysis.

\[
\begin{align*}
\text{i.e. } & \quad Re_d < 245, \quad j_M = 0.407 \ Re_y^{-0.5} \\
& \quad Re_d > 245, \quad j_M = 0.152 \ Re_d^{0.179} [Re_y^{-49.8} \ Re_d^{0.61}]^{-0.5} 
\end{align*}
\]

(9)
**FIG. 12** COMPARISON OF MASS TRANSFER COEFFICIENTS OBTAINED FROM EXPERIMENTAL DATA OF KOLLMAN AND SCHNEIDER\(^{20}\) FOR DRYING OF SINGLE BOARDS OF TIMBER WITH PREDICTIONS BY SÖRENSON'S CORRELATION.
From a set of averaged j-factors for various ratios of L/d, favourable agreement was obtained with Powell's\textsuperscript{15} and Shepherd's\textsuperscript{11} experiments on trays. These curves indicate a linear dependence on velocity to the power of 0.69. This result compares well with the value of 0.64 observed by Sherwood\textsuperscript{16} for drying pulp slabs (15 mm thick) and the value of 0.62 measured by Powell\textsuperscript{15} for thin truncated slabs.

Several studies have been made by wood researchers on the flow field and transfer rate for drying single planks of saturated timber. Kitahara and Suzuki\textsuperscript{17} studied the wind velocity around wood pieces in a constant velocity field and their results indicate flow separation at the leading edge.

The influence of air-velocity upon the drying rate has received much attention. Kollmann and Schneider\textsuperscript{18} correlated the drying rate by a power law relationship of velocity where the exponent $n$ increases as the velocity increases (Table 3).

<table>
<thead>
<tr>
<th>Velocity/m s\textsuperscript{-1}</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>0.50</td>
</tr>
<tr>
<td>5.7</td>
<td>0.53</td>
</tr>
<tr>
<td>7.9</td>
<td>0.575</td>
</tr>
<tr>
<td>9.13</td>
<td>0.60</td>
</tr>
</tbody>
</table>

In contrast, Ogura and Ohnuma\textsuperscript{19} found the rate proportional to the velocity raised to the power of 0.64 while Isaka\textsuperscript{66} observed a power dependence of 0.69. Both these results agree well with the earlier quoted values. Although there is considerable scatter associated with much of the timber data, additional drying work by Kollmann and Schneider\textsuperscript{20} measured mass transfer coefficients over single boards similar to those predicted by Sörens. These results which cover a wide range of airstream conditions are compared in Fig. 12. In a more detailed study of local evaporation rates along a single board
Fig. 13 Comparison of mass transfer j-factors obtained from data of Terazawa, Tsutsumoto, and Kodama\(^\text{21}\) for the drying of a stack of timber with those predicted by Sørenson's sharpedged plate correlation.
Fig. 14. Velocity profiles for turbulent flow through rectangular ducts of various aspect ratios (from Hartnett et al., Fig. 3)
with a sharp leading edge, Terazawa, Tsutsumoto and Kodama obtain considerably higher $j$-factors than expected from Sørenson's correlation, (see Fig. 13). It is impossible to generalise from the empirical timber data, but Sørenson's equations do appear to suggest reasonable values for the transfer coefficients for the drying of single boards of timber. The rates of transfer in stacked timber are considered in the next section.

(c) Flow Over a Parallel Array of Plates

The geometry of the timber stack can be approximated to a set of long parallel channels or parallel plates as the aspect ratio of the flow ducts is large. This similarity is well illustrated by Fig. 14 where the velocity profiles for duct cross-sections of various aspect ratios are shown from predictions by Hartnett et al. An inspection of these graphs reveals that for large aspect ratios ($b/c \gg 10$) the velocity distribution near the horizontal walls will closely approximate to that expected for flow over a flat plate except for regions near the duct corners. Near the vertical duct walls the flow streamlines correspond more closely to those expected within a circular pipe. As the vertical duct walls in timber stack ducts are due to the dry faces of the stickers, these walls will not participate in the moisture transfer. This last fact establishes the validity of considering the timber stack as a set of parallel plates.

The only work in the engineering literature on heat transfer for flow through a stack of parallel flat plates was undertaken by Sams and Weiland. These experiments were for thin plates (0.45 mm thick) of dimensions 75 mm by 88 mm which were stacked closely together (6 mm gap). For the Reynolds number, $Re_p$, range of 15 000 to 80 000 the averaged transfer coefficients were correlated by a similar expression to equation (6) for flow in short circular tubes.

\[ J_H = 0.034 \left( \frac{L}{D_p} \right)^{0.7} Re_p^{-0.2} \]  

\[ (10) \]
Fig. 15 Examination of effect of sticker thickness $c$ on boundary layer thickness $S$ as a function of air-velocity for flow across boards 300 mm wide x 150 mm long x 25 mm thick. Velocity measured at position 300 mm downstream and temperature of 20°C. (from Fig. 1, 2, 3 and 4, Ogura and Ohnma.)
This work on very thin plates probably only accounts for entrance effects due to the flow contraction and so will underestimate the expected entrance effects for a stack of blunt plates.

If the plates are stacked close enough together it is expected that interference of the boundary layers on the opposite walls of the duct may result. In contrast, the stack should behave as a set of isolated plates provided they are sufficiently spaced. An indication of these limits is given by Ogura and Ohnuma's examination of the effect of sticker thickness on the boundary layer thickness as a function of air-velocity. For typical kiln velocities, Fig. 15 suggests that the boundary layer thickness is independent of the board spacing, provided this spacing is greater than 20 mm. Miller provides further evidence of these effects in preliminary studies on mass transfer within two-dimensional arrays of plates. At an air-velocity of 5 ms\(^{-1}\) and a spacing of 8 mm the mass transfer rates on the slabs closely follow Svenson's analysis for a single plate. This work establishes that the mass transfer rates vary along the plate in the direction of the airflow. This conclusion agrees with Terazawa, Tsutsumoto and Kodama in their study of local evaporation rates along stacked boards. They examined rates across stacks 1 metre wide and at board spacings of 10 mm to 35 mm. The rates are observed to diminish more quickly across the stack than that along one single plate but this phenomenon can be shown to be due to humidification of the airstream as it passes across the stack. The resulting mass transfer coefficients vary with distance across the stack similarly to the variation measured along single plates (Fig. 13).

Some other measurements have been made of drying rates in timber stacks which corroborate previous measurements. Stevens, Johnston and Pratt measured heat transfer coefficients at the entrance of a dummy stack of timber for boards 150 mm wide by 25 mm thick. The mass transfer coefficients calculated from these measurements agree well with averaged coefficients predicted by Svenson for single plates 150 mm wide.
Fig. 16 Comparison of data from Hermann and Rasmussen\textsuperscript{25} for the average mass transfer coefficients in drying a 3m wide stack of timber at various air velocities with turbulent flow in ducts correlation and Sörenson's\textsuperscript{14} truncated slab expression.
Comparison of Stevens et al. correlation and Sørenson's correlation

<table>
<thead>
<tr>
<th>Velocity/ms⁻¹</th>
<th>Stevens et al $\bar{k}_y$/kg m⁻²s⁻¹</th>
<th>Sørenson $\bar{k}_y$/kg m⁻²s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.06 \times 10^{-2}$</td>
<td>$1.72 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>$2.89 \times 10^{-2}$</td>
<td>$2.91 \times 10^{-2}$</td>
</tr>
<tr>
<td>3</td>
<td>$3.72 \times 10^{-2}$</td>
<td>$3.82 \times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>$4.55 \times 10^{-2}$</td>
<td>$4.44 \times 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>$5.38 \times 10^{-2}$</td>
<td>$5.02 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 5 also shows close correspondence between averaged coefficients for single plates 600 mm wide predicted by equation (9) with coefficients calculated from data gathered by Hauka. Hauka examined the effect of air-velocity on average drying rates of 600 mm wide stacks of pine timber.

Comparison of Haufa's measurements and Sørenson's correlations

<table>
<thead>
<tr>
<th>Velocity/ms⁻¹</th>
<th>Haufa $\bar{k}_y$/kg m⁻²s⁻¹</th>
<th>Sørenson $\bar{k}_y$/kg m⁻²s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1.20</td>
<td>0.96</td>
</tr>
<tr>
<td>1.5</td>
<td>1.61</td>
<td>1.21</td>
</tr>
<tr>
<td>2.2</td>
<td>1.82</td>
<td>1.69</td>
</tr>
</tbody>
</table>

Finally data from Hermann and Rasmussen, for the average mass transfer coefficients in drying a 3 m wide stack of timber at various air velocities, are plotted in Fig. 16. As can be seen the experimental data generally lie between averaged coefficients calculated from the turbulent flow in ducts correlation (eqn. 5) and Sørenson's truncated slab expression (eqn. 9).
Fig. 17  Local mass-transfer j-factors for truncated slabs as predicted by Sørenson\textsuperscript{14} as a function of Red and Re$_z$. 

\[ j_m = 0.0288 R_{e_z}^{-0.2} \]

\[ j_m = 0.07 R_{e_z}^{-0.5} \]
DISCUSSION

Three different geometrical analogies to the timber stack have been considered for the prediction of mass transfer coefficients. These results suggest that there is a need for clarification of the transfer of mass, heat and momentum to parallel arrays of blunt-edged ducts of large aspect ratio.

One particular area of concern is the local variation of transfer coefficients across the stack. As will be seen in the next chapter, the nature of this variation has important consequences for the rates of drying within the stack. Nevertheless the experimental evidence presented above indicates that the relationship of local coefficients to distance along the stack is not independent of distance as implied by circular duct equations. Rather the evidence shows agreement with the single plate correlations. Another feature of these experiments is that the turbulent flat-plate expression fits stack data, as does Sørenson's blunt plate correlations, where the flow behaviour has been clearly shown to be laminar behind the turbulent eddy on the leading edge. Although the turbulent plate expression ignores the effect of plate thickness, the correlation lies across a plot of Sørenson's plate equation (eqn. 9), as in Fig. 17. This similarity and the scatter of the data in Fig. 11 suggests that Sørenson's equation would also fit the data equally well.

An additional factor which will affect the rate of transfer, which has been ignored in previous discussion, is the surface roughness. Many workers have studied flow and heat transfer in rough pipes and Dipprey and Sabersky have proposed a similarity rule to correlate, interpret and extend the experimental measurements. These results show that a roughened surface causes higher friction factors than a smooth surface. Hence the related heat and mass transfer j-factors are also larger, but generally to a lesser extent than the increase in fluid friction. An estimation of this effect for the drying of timber can be gained from experiments by Greenhill. These experiments measured friction factors for
the flow of air through wood ducts of various surface roughness. The magnitude of the values are low compared to those from the Fanning friction factor chart but the ratio of friction for planed : average sawn : rough sawn boards was shown to be 1.00 : 1.24 : 1.52. Hence the mass and heat transfer coefficients for the drying of timber will be influenced by the quality of the surface finish after sawing. The enhancement due to the surface roughness could explain the high experimental measurements (presented in Fig. 16) compared to Sørensen's correlation. It has also been observed by Leney for the drying of oak that planing prior to drying results in less degrade. This result could be due to lower rates as a result of a smoother surface.

The results summarised above give a guide as to the choice of transfer coefficients for the kiln-drying of timber. There is still difficulty to estimate accurately the mass transfer rate. Fig. 16 shows that the two correlations agree as the velocity tends to zero but at an air-velocity of 4 m s⁻¹ they differ by 100 per cent. For the remainder of the thesis, Sørensen's correlation is regarded as the most reliable equation for predicting kiln transfer coefficients.
Part 1 has indicated the magnitude of transfer coefficients for the drying of a timber stack. In order to calculate the transfer flux, it is necessary to know the driving forces for heat and mass transfer. This section is concerned with estimating the driving forces for heat and mass transfer for evaporation from a saturated solid surface.

The evaporation of water from the surface of saturated solids has been shown to correspond to the rate of evaporation of a free water surface for the same geometry and airstream conditions\(^2\). These two phenomena have generally been regarded to be equivalent but the reason for this is not obvious. The reason for this result was clarified by Suzuki et al\(^7\) in their study on mass transfer from discontinuous sources where the flow over the porous solid is described by solutions to simplified models for the boundary layer. The mass transfer flux is shown to be a function of the ratio \(L_{DS}/\delta\), the length of the discontinuous source to the thickness of the boundary layer, and the fractional area occupied by the sources. When the ratio \(\delta/L_{DS}\) is greater than 30, the flux is practically independent of the discontinuous nature of the sources and the magnitude of the fractional area. This condition will pertain to the drying of most finely-pored materials as shown below for the example of drying wood.

\[ L_{DS} = 40 \text{ } \mu\text{m} \text{ and } \delta = \frac{\rho_{G} \beta}{k_{y}} \]

and for drying at 50°C with an air-velocity = 2 m s\(^{-1}\)

\[ \rho_{G} = 1.1 \text{ kg m}^{-3}, \beta_{AB} = 2.9 \times 10^{-5} \text{ m}^{2} \text{ s}^{-1}, k_{y} = 0.01 \text{ kg m}^{-2} \text{ s}^{-1} \]

\[ \therefore \delta/L_{DS} = 75 \]

It is also often assumed\(^2\),\(^7\) that, under conditions
that prevail in drying equipment, moisture evaporation rates are directly proportional to the difference in humidity between that at the exposed surface of the material being dried and that in the bulk air. However this proportionality is expected to hold only for the limit of small humidity differences at low temperatures (<40°C). At higher temperatures, this proportionality does not hold as the data from Stevens and Johnston\(^75\) for evaporation of water demonstrates in Table 5. Further data on the drying of saturated slabs of aerated cement measured by Schneider\(^28\) also emphasises this fact (Table 6).

Table 5

<table>
<thead>
<tr>
<th>(T_{\text{air}} / ^\circ \text{C})</th>
<th>(T_{\text{water}} / ^\circ \text{C})</th>
<th>(W_A / \text{kg m}^{-2} \text{s}^{-1})</th>
<th>(\pi)</th>
<th>(k_y / \text{kg m}^{-2} \text{s}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>59.95</td>
<td>50.5</td>
<td>(1.73 \times 10^{-4})</td>
<td>0.00413</td>
<td>(4.14 \times 10^{-2})</td>
</tr>
<tr>
<td>69.9</td>
<td>60.7</td>
<td>(1.73 \times 10^{-4})</td>
<td>0.00453</td>
<td>(3.82 \times 10^{-2})</td>
</tr>
<tr>
<td>79.85</td>
<td>71.0</td>
<td>(1.70 \times 10^{-4})</td>
<td>0.00513</td>
<td>(3.31 \times 10^{-2})</td>
</tr>
<tr>
<td>89.8</td>
<td>80.1</td>
<td>(1.97 \times 10^{-4})</td>
<td>0.00708</td>
<td>(2.78 \times 10^{-2})</td>
</tr>
<tr>
<td>99.95</td>
<td>90.05</td>
<td>(2.15 \times 10^{-4})</td>
<td>0.01168</td>
<td>(1.84 \times 10^{-2})</td>
</tr>
<tr>
<td>89.7</td>
<td>60.2</td>
<td>(5.35 \times 10^{-4})</td>
<td>0.01430</td>
<td>(3.74 \times 10^{-2})</td>
</tr>
</tbody>
</table>

(Air velocity = 1.2 ms\(^{-1}\))

N.B.: The coefficient of proportionality \(k_y\) is equal to \(W_A/\pi\).
These two tables of data show that for high temperatures the relationship between the evaporative flux and the humidity potential is more complex than a simple linear proportionality. Hence the driving force for mass transfer under these conditions is not given by the simple humidity difference.

These humidities are commonly estimated from psychrometric readings whenever the air-steam system is encountered. The calculational procedure assumes that the wet-bulb and adiabatic saturation temperatures are identical. Although these approximations are freely acknowledged, there has been little assessment of the errors so introduced. Moreover, these charts tend to be inaccurate when read to determine humidity differences, and often do not cover conditions of commercial interest in drying.

New expressions are developed below which allow more exact calculation of the driving forces for heat and mass transfer. This method does not assume the equivalence of the wet-bulb and adiabatic-saturation temperatures. The stagnant film approximation is assumed for the boundary layer equations. Although such a model is a gross simplification, the stagnant film has been shown to provide an excellent basis for correlation. These equations are then solved over the complete range of

### Table 6

Examination of Proportionality Between Drying Rates and Humidity Differences From Data of Schneider

<table>
<thead>
<tr>
<th>$T_{air}/^\circ C$</th>
<th>$T_{surface}/^\circ C$</th>
<th>$w_A/kg \ m^{-2} \ s^{-1}$</th>
<th>$\kappa$</th>
<th>$k_y/kg \ m^{-2} \ s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>70</td>
<td>$9.70 \times 10^{-4}$</td>
<td>0.0435</td>
<td>$2.23 \times 10^{-2}$</td>
</tr>
<tr>
<td>130</td>
<td>70</td>
<td>$7.35 \times 10^{-4}$</td>
<td>0.0331</td>
<td>$2.22 \times 10^{-2}$</td>
</tr>
<tr>
<td>110</td>
<td>70</td>
<td>$4.90 \times 10^{-4}$</td>
<td>0.0223</td>
<td>$2.20 \times 10^{-2}$</td>
</tr>
<tr>
<td>150</td>
<td>90</td>
<td>$8.10 \times 10^{-4}$</td>
<td>0.0690</td>
<td>$1.18 \times 10^{-2}$</td>
</tr>
<tr>
<td>130</td>
<td>90</td>
<td>$4.90 \times 10^{-4}$</td>
<td>0.0464</td>
<td>$1.06 \times 10^{-2}$</td>
</tr>
<tr>
<td>110</td>
<td>90</td>
<td>$2.54 \times 10^{-4}$</td>
<td>0.0233</td>
<td>$1.09 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

(Air velocity = $3 \ \text{ms}^{-1}$)
Fig. 18 Schematic diagram of moisture evaporation from a saturated surface. (Note the mass transfer film is thicker than the heat transfer film for air/water-vapour).
conditions encountered in drying practice.

**Humidity Potential Driving Force**

The rate of diffusion of a vapour through an essentially still gas is obtained by integrating Fick's first law. The flux of vapour $N_A$ is given by:

$$N_A = c \frac{D_{AB}}{\delta M} \ln \left[ \frac{1-y_G}{1-y_S} \right]$$  \hspace{1cm} (11)

where $c$ is the total molal density, $D_{AB}$ is the vapour diffusivity of $A$ through $B$, $\delta M$ is the "thickness" of the film across which the vapour transport is occurring, $y_G$ is the mole fraction of vapour ($A$) in the bulk gas $B$ and $y_S$ is the mole fraction of $A$ at the surface (Fig. 18). A mass transfer coefficient $k_F$ may be defined as

$$k_F = \frac{c_B D_{AB}}{\delta M}$$  \hspace{1cm} (12)

hence

$$N_A = k_F c_B \frac{D_{AB}}{\delta M} \ln \left[ \frac{1-y_G}{1-y_S} \right]$$  \hspace{1cm} (13)

The coefficient $k_F$ is the Colburn-Drew coefficient which is independent of the driving force, but depends upon the surface geometry and the gas-flow conditions (temperature, velocity etc.)

The mole-fractions of moisture may be related to the humidity (mass of moisture/mass of moisture-free gas) thus:

$$y = \frac{Y}{M_A} \left[ \frac{1}{M_B} + \frac{1}{M_A} \right] = \frac{Y}{[D+Y]}$$  \hspace{1cm} (14)
where $D$ is the vapour/gas molecular weight ratio ($M_A/M_B$) and $Y$ is the humidity. Equation (13) then becomes after some simplification:

$$\ln \frac{D+Y_S}{D+Y_G}$$

For convenience, we shall define:

a humidity potential $\pi = Y_S - Y_G$ (16)

and a humidity-level correction $\beta = \frac{1}{\text{factor}} (D+Y_S)$ (17)

so that equation (15) can be rewritten as

$$N_A = -k_F c \ln \frac{1-\beta \pi}{c_B}$$

The flux in mass units (e.g. kg/m²s) is given by

$$W_A = -k_F c \frac{M_A}{c_B} \ln \frac{1-\beta \pi}{c_B}$$

where $M_A$ is the molar mass of the vapour. However, it is often assumed that

$$W_A \propto \pi$$

and the coefficient of proportionality is defined as the mass transfer coefficient $k_y$. It follows from equation (19) that:

$$k_y = k_F \left( \frac{c}{M_A} \right) \beta \left[ \frac{\ln (1-\beta \pi)}{-\beta \pi} \right]$$

The significance of equation (21) is more clearly seen if we consider the product $(-\beta \pi)$ as a single parameter, say $B$. 
Then the term within the square brackets is given by

\[ E = \frac{\ln(1+\beta)}{B} \]  \hspace{2cm} (22)

The parameter \( E \) is thus similar to the driving force corrections proposed by Ackermann\(^3\) and others\(^3\) for high transfer fluxes, and the parameter \( B \) to Spalding's\(^3\) generalised driving force.

When \( \beta \) approaches \( 1/D \) and \( B \) is very small, equation (21) becomes:

\[ k_y \rightarrow k \frac{M_A}{M_B} = k \frac{c - c_B}{D} \]  \hspace{2cm} (23)

which is the conversion listed by Treybal (op. cit.) in Table 3.1. Let the limiting mass flux in this situation be \( W_A^* \), then

\[ W_A^* = k_y^* \pi \]  \hspace{2cm} (24)

say, where

\[ k_y^* = k \frac{M_B}{M_A} \]  \hspace{2cm} (25)

from equation (16). In general:

\[ W_A = k_y \left[ k_y \phi_M \right] \pi = k_y^* \phi_M \pi \]  \hspace{2cm} (26)

where \( \phi_M \) may be regarded as a correction factor to the linear driving force, \( \pi \). Comparison of equation (21) with (25) and (26) yields

\[ \phi_M = \frac{c_D}{c_B} \left[ \frac{\ln(1-\beta \pi)}{-\beta \pi} \right] \]  \hspace{2cm} (27)
Under many convective-drying circumstances, $-B = \beta \cdot \varphi < 0.1$ and the term within the square brackets can be approximated by $(1-B/2)$.

**Temperature Potential Driving Force**

The foregoing analysis leading to $\varphi_M$ implicitly assumes isothermal conditions. However, the evaporation is coupled to the heat received by the evaporating interface. In many cases the thermal transport is by convection from the surrounding gas at a higher temperature.

The influence of the vapour flux on the heat transfer has been derived by a number of workers$^{30, 31, 33}$. The convective heat transfer coefficient $h_c$ may be written as

$$h_c = \frac{\lambda}{\delta_H} \left[ \ln(1+B_T) \right]$$

where $B_T = C_{PA}(T_G-T_S)/\Delta H_{VS}$. In equation (28), $\lambda$ is the thermal conductivity, $\delta_H$ is the "film thickness" for heat transfer, $C_{PA}$ is the isobaric heat capacity of the vapour, $T_G$ is the bulk-gas temperature, $T_S$ is the surface temperature and $\Delta H_{VS}$ is the enthalpy change on vaporisation at $T_S$.

The vapour flux may be expressed in terms of the temperature potential ($\gamma = T_G - T_S$):

$$W_A = h \frac{\gamma}{\Delta H_{VS}}$$

where $h$ is the sum of the convective and radiative heat-transfer coefficients. It is convenient to define

$$h_r = \frac{\gamma}{\delta_H} h_c$$

so that equation (22) becomes

$$W_A = (1+\frac{h_r}{\delta_H}) h_c \frac{T_G - T_S}{\Delta H_{VS}}$$
Fig. 19 Radiative correction factor $\Theta_H$ as a function of wet-bulb temperature and wet-bulb depression for typical timber-drying conditions. (beam length = 25 mm, $h_0 = 30 \text{ W m}^{-2}\text{K}^{-1}$).
In the limiting case when the vapour flux becomes very small,

\[ W_A^* = h_c \sqrt{\frac{j}{\Delta H_{VS}}} \]  

(32)

where

\[ h_c^* = \frac{\lambda_B}{\delta_H} \]  

(33)

in which \( \lambda_B \) is the thermal conductivity of the moisture-free gas. In general,

\[ W_A = h_c^* \left( \frac{h_c}{\Delta H_{VS} h_c} \right) (1 + \theta_H) \]

\[ = h_c^* \phi_H (1 + \theta_H) \]  

(34)

where \( \phi_H \) may be regarded as a correction factor to the driving force \( j \). Comparison of equation (34) with (28) and (33) yields

\[ \phi_H = \frac{\ln(1 + B_H)}{\ln(B_T)} \]  

(35)

Again, often the term within square brackets can be replaced by a simpler expression \([1 - B_H/2]\).

The radiative correction factor \( \theta_H \) is often small. In the absence of radiating surfaces, the thermal radiation is principally derived from the moisture vapour and is thus strongly dependent upon humidity (Fig. 19). Under usual timber-kiln conditions, for example, \( \theta_H \approx 0.05 \). On the other hand, reflective surfaces may direct thermal radiation to the drying surface from emitters such as steam coils, and the radiative contribution to the total heat transferred can no longer be ignored.

**Psychrometric Ratio**

A psychrometer still has wide acceptance as a hygrometer, partly because of the availability of humidity charts that serve for calibration, and partly because of the simplicity of
the components. The wet-bulb depression \( \psi \), when the wetted surface has no dry spots and the areas for heat and mass transfer are identical, is given by equating (26) and (34):

\[
\frac{h_c^* (1-x_H) c_H \psi}{\Delta H_{VS}} = k_y^* \phi_m^\kappa
\]  

(36)

The ratio of coefficients \( h_c^* / k_y^* \) may be equated to the ratio of Stanton numbers:

\[
\frac{St_H}{St_m} = \frac{k_y^* C_P B}{h_c^*} = \Lambda
\]

(37)

The parameter \( \Lambda \) is also known as the psychrometric ratio. Two theoretical boundary layer models have been used to predict the magnitude of \( \Lambda \). Wasan and Wilke\(^{33} \) propose a correlation for the psychrometric ratio based on their analysis of the transfer of heat and mass in pipe flow\(^{34} \). The assumption was made that the velocity distribution outside a single cylinder can be represented by the distribution for the flow in the vicinity of a cylindrical pipe. This assumption is reasonable for a cylinder with moderate curvature. They proposed the following relationship for a Re number of 10\(^5\).

\[
\Lambda = 1 + 0.7 \left( \frac{Pr^{0.77} - 1}{1 + 0.7 \left( \frac{Sc^{0.77} - 1}{10} \right)} \right)
\]

(38)

In further work, Kauh, Peck and Wasan\(^{39} \), used Spalding functions to predict \( \Lambda \) for flow over flat-plates and cylinders. These results suggest that \( \Lambda \) is a function of Reynolds number. The psychrometric ratio is also closely related to the ratio of the \( j \)-factors for heat and mass transfer. Numerous investigators\(^{35-38} \) have measured this ratio and over a wide range of Reynolds numbers \( \frac{J_H}{J_m} \) has been shown to lie between 0.86 to 1.26. At first these results suggest that the equality between the \( j \)-factors is approximate. However this ratio is particularly difficult to measure experimentally and furthermore Gupta\(^{38} \)
suggests that the ratio may have been calculated incorrectly in some cases. These errors arose because of incorrect definitions of the driving forces and because radiation effects were not isolated. Nevertheless, examination of the Reynolds analogy and the Chilton and Colburn analogy suggests that these relationships can only be expected to hold strictly if the transport properties are constant across the boundary layer. This condition will be attained exactly for the fictitious state of infinitesimally small fluxes as the concentration of the vapour tends to zero. Hence it is postulated that, under these limiting conditions, the Chilton-Colburn analogy will hold, thus:

$$\frac{j_H*}{j_M*} = \frac{St_H*}{St_M*} \left( \frac{Pr*}{Sc*} \right)^m = 1$$  \hspace{1cm} (39)

where the value of the exponent $m$ depends upon the shape of the temperature and moisture concentration profiles. Levich shows that $m \rightarrow 2/3$ for thin boundary layers (the so-called infinite Schmidt number limit which holds to $Sc \approx 0.5$). This value of $2/3$ was confirmed in recent psychrometric studies by Henry and Epstein for flat-plates but for spheres and cylinders $m$ took the value of $0.5$. In further experiments these relationships were also shown to hold for both low and high levels of turbulence.

The limiting Prandtl and Schmidt numbers are determined from the properties of the moisture-free gas and the vapour diffusivity $D_{AB}$. From equation (39), it follows that

$$A = \frac{k^*}{h^*_c} C_{PB} = \left( \frac{Pr^*}{Sc^*} \right)^{2/3}$$  \hspace{1cm} (40)

The psychrometric ratio, so defined, thus excludes the distortion of profiles caused by high transfer fluxes. These effects are contained solely in the $\phi$-factors. Because of the small variation of the limiting Pr and Sc numbers with temperature for air, the transfer coefficients $k^*_y$ and $h^*_c$ can
be shown to be practically independent of the temperature level. A comparison of the "film thicknesses" for heat and mass transfer can be obtained from equations (12), (25), (33) and (40):

$$\frac{\delta_H}{\delta_M} = \frac{\lambda_B}{C_{PB} \beta_{AB} H_B h c} = \left(\frac{S_{c^*}}{Pr^*}\right)^{1/3} \tag{41}$$

For the air-water vapour system, this result suggests that the mass transfer film is approximately 10% thicker than that for heat transfer.

The wet-bulb thermometer equation (eqn. 36) relating the heat and mass transfer fluxes can now be rearranged to give:

$$C_{PB} (1+\theta_H) \rho_H \frac{\varphi}{\Delta H_{VS}} = \varphi_M \tag{42}$$

On expanding the $\varphi$ terms, we find:

$$\frac{C_{PB}}{C_{PA}} (1+\theta_H) \frac{\lambda_B}{\lambda_{c^*}} \frac{c_B}{c_{c^*}} \frac{\ln (1+\alpha_{PA} \frac{\varphi}{\Delta H_{VS}})}{\Delta D} = -\ln (1-\beta \pi) \tag{43}$$

which simplifies to:

$$1-\beta \pi - \exp \left[ \frac{\lambda}{c} S (1+\theta_H) \right] = 0 \tag{44}$$

where

$$S = -\frac{C_{PB}}{C_{PA}} \frac{c_B}{\lambda_B} \ln [1+C_{PA} \frac{\varphi}{\Delta H_{VS}}] \tag{45}$$

Equation (35) may be solved by an algorithm based on Newton's method. Consider the case when $\theta_H$ is known. Let

$$f(\pi) = 1-\beta \pi - \exp \left[ \frac{\lambda S (1+\theta_H)}{c} \right] \tag{46}$$
Since $\lambda S/c$ is only weakly dependent upon $\pi$, we may assume

$$f'(\pi) = -\beta \quad (47)$$

so that the algorithm becomes

$$\pi(n+1) = \pi(n) - \frac{f(\pi(n))}{f'(\pi(n))}$$

$$= \pi(n) + \frac{1}{\beta} \left[ 1 - \exp\left( \frac{\lambda(n) S(1+\delta)}{\exp(\lambda(n)) c(n)} \right) \right] \quad (48)$$

where $(n)$ is the $n$th estimate of the parameter so super­scripted. The algorithm gives rapid convergence to

$$\left| \frac{\pi(n+1) - \pi(n)}{\pi(n)} \right| \leq 10^{-6}.$$

The equation for adiabatic saturation is derived by many workers, e.g. Treybal (op. cit. p. 190) to be

$$(T_d - T_{AS}) C_S / \Delta H_{VAS} = (Y_{AS} - Y_d) \quad (49)$$

where $T_{AS}$ is the adiabatic-saturation temperature, $C_S$ is the mean humid heat, evaluated at humidity $Y_G$ and temperature averaged over the range $T_{AS}$ to $T_G$, $\Delta H_{VAS}$ is the enthalpy of vaporization at the adiabatic-saturation temperature and $Y_{AS}$ is the adiabatic saturation humidity. For a particular set of dry- and wet-bulb conditions, there is no a priori reason to suppose that the wet-bulb and the adiabatic saturation temperatures are the same.

For this reason, a humidity chart that is independant of this presumption is desirable. By solving equation (48) for known wet- and dry-bulb conditions, the variation of $\pi$ with wet-bulb temperature $T_S$ for contours of wet-bulb depression $\delta$ can be determined. Since $Y_S(T_S)$ is known from the vapour-liquid equilibrium, the bulk-gas humidity $Y_G$ follows from the difference $(Y_S - \pi)$. 
The foregoing expressions are general and can be applied to any gas-vapour system. These expressions are illustrated by reference to the frequently encountered system of air-water vapour, which is also applicable to the drying of timber.

Before the flux equations can be solved, physical property data as a function of temperature and concentration are required for the vapour-gas mixture. As experimental data for the properties of air-water vapour mixtures are sparse, rigorous kinetic gas theory expressions were used to correlate the mixture properties in terms of experimentally measured data for the single components. These expressions also include an interaction term due to the interference of the unlike molecules and this term is evaluated by fitting available experimental mixture data to the rigorous expression.

For convenience, the physical properties of the component gases taken from reliable data sources were curve-fitted by a Gaussian least-squares method using the computer program "CURVE2" (see Appendix 2A). These property equations are presented in Table 7 as polynomials in temperature for the temperature range of interest 0°C - 200°C.

### Table 7

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_B$</td>
<td>Nsm$^{-2}$</td>
<td>1.71687E-5</td>
<td>4.96315E-8</td>
<td>-3.85243E-11</td>
<td>--</td>
</tr>
<tr>
<td>$C_{PB}$</td>
<td>J kg$^{-1}$K$^{-1}$</td>
<td>1.00745E3</td>
<td>1.05751E-1</td>
<td>2.19757E-4</td>
<td>-1.35059E-7</td>
</tr>
<tr>
<td>$\lambda_B$</td>
<td>Wm$^{-1}$K$^{-1}$</td>
<td>2.41240E-2</td>
<td>7.17753E-5</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Physical Properties of Vapour-Gas Mixture

The foregoing expressions are general and can be applied to any gas-vapour system. These expressions are illustrated by reference to the frequently encountered system of air-water vapour, which is also applicable to the drying of timber.
The specific volumes of the gas and vapour are correlated by the truncated virial equation of state,

\[ V = \left(1 + \sqrt{1 + \frac{4B_{IJ}P}{RT}}\right) \frac{RT}{RT + 2PM} \]  

where \( R \) is the Universal gas constant, \( P \) is the pressure and \( B_{IJ} \) is the second virial coefficient. The second virial coefficient for water vapour is given as a function of temperature by Keenan, Keyes, Hill and Moore's curve-fitted equation:

\[ B_{BB} = 2.0624E-3 - 2.61204 \times 10^{-3} \left(\frac{T}{T + 3.49E4}\right)^2 \]  

where \( T \) is the temperature.
For air the second virial coefficient is calculated from kinetic theory assuming an intermolecular potential function of the form of the Lennard-Jones 6:12 potential with the force constants \(\sigma_{AA} = 0.359\ \text{nm}\) and \(\epsilon_{AA}/k = 100\ \text{K}\). These parameters represent an average of values observed by several workers from measurements on the viscosity and second virial coefficient of dry air.

The following methods were used to calculate the mixture fluid properties.

1. **Specific Volume** - This property is calculated from equation (50) where the second virial coefficient of the mixture, \(B_M\), is given by the expression

\[
B_M = B_{AA} y_A^2 + 2y_A y_B B_{AB} + y_B^2 B_{BB}
\]

where \(B_{AB}\) represents the second virial coefficient due to the interaction of the unlike molecules. \(B_{AB}\) is predicted from kinetic theory assuming the mixture obeys the Lennard-Jones 6:12 intermolecular potential function with force constants \(\sigma_{AB} = 0.309\ \text{nm}\) and \(\epsilon_{AB}/k = 220\ \text{K}\). These parameters are obtained from the single component force constants using the empirical combining laws suggested by Hirschfelder, Bird and Curtiss for a polar-nonpolar gas mixture.

\[
\sigma_{AB} = \frac{1}{2} (\sigma_{AA} + \sigma_{BB}) \eta^{-1/6}_M \tag{53a}
\]

\[
\epsilon_{AB} = (\epsilon_{AA} \epsilon_{BB})^{1/4} \eta^{2}_M \tag{53b}
\]

\[
\eta = 1 + 0.892E24 \left[ \frac{\infty}{(B_o)_{AA}} \right]^{1/2} \left[ \frac{\epsilon_{BB}^{t*}}{(\epsilon_{AA})} \right]^{1/2} \tag{53c}
\]

In equations (53) \(\alpha_{AA}\) is the polarizability (Chaddock suggests the value 1.726E-30 m³ for air) and \((B_o)_{AA}\) is the rigid-sphere second virial coefficient of the nonpolar molecule. Rowlinson has shown that steam obeys the Stockmayer inter-
molecular potential where the intermolecular parameters take the values $\sigma_{BB} = 0.265 \text{ nm}, \epsilon_{BB} = 380 \text{ K}$ and $t^*_{BB} = 1.2$.

2. **Specific Heat Capacity** - As rigorous kinetic gas theory expressions are not available for the specific heat capacity of a mixture of polyatomic gases, the simple mole-fraction weighted ratio of the pure component values was assumed in order to calculate the mixture specific heat capacity.

\[ C_{PM} = C_{PA} y_A + C_{PB} y_B \]  \hspace{1cm} (54)

3. **Dynamic Viscosity** - Chapman and Cowling\textsuperscript{57} present an expression for the first approximation to the dynamic viscosity of a polyatomic gas mixture derived from the Chapman-Enskog hard-sphere gas model:

\[ \mu_M = \frac{y_A^2 R_A + y_B^2 R_B + y_A y_B R_{AB}}{y_A^2 R_A / \mu_A + y_B^2 R_B / \mu_B + y_A y_B R_{AB}} \]  \hspace{1cm} (55)

where

\[ R_A = 1 + \frac{M_A \cdot A_{AB}}{M_B} \]  \hspace{1cm} (56a)

\[ R_B = 1 + \frac{M_B \cdot A_{AB}}{M_A} \]  \hspace{1cm} (56b)

\[ R_{AB}' = E_{AB} \left( \frac{1}{\mu_A} + \frac{1}{\mu_B} \right) + 2(1-A_{AB}) \]  \hspace{1cm} (56c)

\[ R_{AB} = \frac{E_{AB} \cdot A_{AB} \cdot (M_A + M_B)^2}{\mu_A \mu_B \cdot E_{AB} \cdot M_A M_B} \]  \hspace{1cm} (56d)

and

\[ A_{AB} = \frac{3}{10} \frac{Q_{AB}(2,2)}{Q_{AB}(1,1)} \]  \hspace{1cm} (56e)
The functions $\Omega_{AB}^{(1,1)}$ and $\Omega_{AB}^{(2,2)}$ are probability integrals representing the intermolecular collisions and these are tabulated in reference 54 for the Lennard-Jones 6:12 potential. The term $\mu_{AB}$ represents the interaction viscosity of the unlike molecules which can be calculated theoretically from the expression

$$\mu_{AB} = \frac{5}{16} \left( \frac{2kT M_1 M_2}{\pi M_1 + M_2} \right)^{\frac{3}{2}} \sigma_{AB}^{2 \Omega_{AB}^{(2,2)}}$$

However, it is more accurate to estimate the force constants of interaction by fitting Kestin and Whitelaw's experimental mixture viscosities to equations (55) and (56) rather than using the interaction force constants suggested for estimating the specific volume. This approach leads to viscosity intermolecular parameters of $\sigma_{AB}^{\mu} = 0.308\ \text{nm}$ and $\varepsilon_{AB}^{\mu}/k = 220\ \text{K}$.  

4. **Thermal Conductivity** - Because of the complexity of polyatomic gas mixtures, rigorous kinetic gas theory expressions are not available for the prediction of mixture thermal conductivities. Several empirical schemes have been suggested for calculating polyatomic mixture thermal conductivities by applying correction factors to values predicted for monatomic gas mixtures. However, Hirschfelder, Bird, and Curtiss note that the equation for the thermal conductivity of monatomic gas mixtures should reproduce well the concentration dependence of the thermal conductivity of more complex mixtures. Hence the Chapman-Enskog hard-sphere gas model first approximation to the thermal conductivity of a monatomic gas mixture is used to correlate the thermal conductivities of air-water vapour mixtures.
\[ \lambda_M = \frac{y_A^2 Q_A + y_B^2 Q_B}{y_A^2 Q_A / \lambda_A + y_B^2 Q_B / \lambda_B + y_A y_B Q_{AB}} \]

where

\[ Q_A = 1 + C_{AB} \cdot D_{AB} \cdot M_A + \frac{(M_A - M_B)^2}{M_B} \]

\[ Q_B = 1 + C_{AB} \cdot D_{AB} \cdot M_B + \frac{(M_B - M_A)^2}{M_A} \]

\[ Q_{AB}' = 2 + \left( \frac{M_A + M_B}{M_A M_B} \right) \frac{F_{AB}}{2} \left( \frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) - 2C_{AB} - D_{AB} \]

\[ Q_{AB} = \frac{2C_{AB}(M_A + M_B)(1 - D_{AB}) + (M_A + M_B)}{F_{AB}} \left[ \frac{F_{AB}}{\lambda_A \lambda_B} - (2D_{AB} - 1)(M_A - M_B)^2 \right] \]

\[ C_{AB} = \frac{4}{15} \frac{Q_{AB}(2,2)}{Q_{AB}(1,1)} \]

\[ D_{AB} = \frac{Q_{AB}(1,2) - 4/5 Q_{AB}(1,3) + 1}{Q_{AB}(1,1)} \]

and

\[ F_{AB} = \frac{2}{15} \frac{Q_{AB}(2,2)}{Q_{AB}(1,1)} \]

The term \( \lambda_{AB} \) represents the interaction thermal conductivity which is given by the following expression

\[ \lambda_{AB} = 8.32796E18 \sqrt{\frac{2(M_A + M_B)}{2M_A M_B}} \frac{Q_{AB}(2,2)}{Q_{AB}^2} \]

where the intermolecular parameters have to be evaluated from
Fig. 20 to 23  Physical and transport properties of air/water-vapour mixtures as a function of temperature (0 to 200°C) and mole fraction of water vapour.

Fig. 22 and 23 compare these predictions with Mason and Monchick's calculations.
experimental data. The air-water vapour mixture thermal conductivities measured by Grüss and Schmick when fitted to equations (58) and (59) suggest values for the thermal conductivity interaction force constants of $\sigma_{AB}^0 = 0.270 \text{ nm}$ and $\sigma_{AB}^0/k = 220 \text{ K}$.

Equations (52)-(60) are illustrated in Figs. 20-23 where the mixture properties are plotted as functions of temperature and concentration. The viscosity and thermal conductivity predictions of Mason and Monchick are presented in Figs. 22 and 23 for comparison and although they indicate a similar concentration dependence there is some discrepancy between the pure component values. These graphs also indicate the significant positive deviations from ideal gas mixture behaviour because of the unlike molecule interactions.

**DISCUSSION**

The flux equations and physical property equations can now be coupled by consideration of the air-water system. As temperature and concentration vary across the "film" it is necessary to take into account this variation in determining the mixture physical properties. In this case the mixture physical property parameters appearing in equation (48) are determined from temperature averaged pure component values

\[
\overline{C}_{PA} = \frac{1}{T_G - T_0} \int_{T_0}^{T_G} C_{PA}(T) \, dT \quad (61)
\]

for a mixture composition based on the average concentration over the film

\[
y_M = (y_S + y_G)/2 \quad (62)
\]

The parameter $D$, which is needed to estimate $\beta$, is defined as a molecular-weight ratio, but some non-idealities (excluding volume of mixing effects) may be taken into account by evaluating $D$ as the mass density ratio. The radiative factor is taken to be zero. The computer program "HUM" (see Appendix 2B) evaluates the physical property equations (50)-(62) and
Fig. 24. Humidity-potential driving force for air/water-vapour system as a function of wet-bulb temperature and wet-bulb depression.
solves the convergence algorithmic equation (48) for various combinations of wet- and dry-bulb temperatures to predict the driving force and humidity.

1. **Humidity-potential driving force**

The humidity-potential driving force \( \phi_H \), equations (26), (27), and (44), is plotted in Fig. 24 as a function of wet-bulb temperature \( T_w \) (0-98°C) and wet-bulb depression (2-80°C). At low depressions \( \Psi < 10^0 \) the driving force is virtually independent of temperature level, and directly proportional to the temperature difference. This result illustrates the conclusions drawn by Wylie \(^6\) where it is shown for small fluxes that the effect of the variability of fluid properties on laminar convective heat and mass transfer processes is negligible. At higher depressions, nonlinearities result due to the effect of the positive interaction of the air and water vapour molecules upon the mixture transport properties. Furthermore, the potential-correction factor \( \beta \) falls significantly with temperature and the driving force diminishes as a consequence.

Not all wet-bulb temperature/depression combinations are possible. There is a forbidden area that corresponds to a humidity difference that is greater than the saturation humidity at the wet-bulb temperature. The boundary of this region is defined by equation (44) for the limits \( \kappa = Y_w \) and \( \theta_H = 0 \).

\[
1 - \beta Y_w = \exp \left( \frac{\lambda}{c} \right) = 0 \tag{63}
\]

Equation (63) is not readily soluble since all the parameters are nonlinear functions of \( T_w \). However, a Wegstein iteration may be used as an algorithmic scheme, i.e., we put

\[
T_w^{(n+1)} = 1 - Y_w^{(n)} \beta^{(n)} \cdot \exp \left[ \frac{\lambda^{(n)}}{c^{(n)}} \right] + T_w^{(n)} \tag{64}
\]

The iteration converges rapidly to

\[
\left| \frac{T_w^{(n+1)} - T_w^{(n)}}{T_w^{(n)}} \right| < 10^{-6}.
\]
Fig. 25 and 26. Heat and mass transfer driving force correction factors as a function of wet-bulb depression and wet-bulb temperature.
The resultant envelope is plotted on Fig. 24.

2. Driving-force correction factors, $\phi_M$ and $\phi_H$

The correction factors $\phi_M$ and $\phi_H$ are presented in Figs. 25 and 26, respectively. Both $\phi$-factors are strongly dependent upon the wet-bulb temperature, but are virtually independent of wet-bulb depression. Since

$$W_A = k_y \phi_M \pi$$

(26)

it follows that the evaporation flux is directly proportional to the humidity potential at a given wet-bulb temperature, as the coefficient $k_y \pi$ has been observed to be practically independent of temperature. Many industrial dryers are operated under constant wet-bulb temperature conditions, and expressions that assume a direct proportionality between the flux and the humidity potential are thus valid for these circumstances. The validity of these theoretical predictions is verified by the experimental data presented in Tables 5 and 6. These data are reworked for Tables 8 and 9 to examine the proportionality between the flux $W_A$ and the driving force $\phi_H \pi$.

**Table 8**

Examination of Proportionality Between Drying Rates and Humidity Driving Forces from Data of Stevens et al.

<table>
<thead>
<tr>
<th>$T_{\text{air}}$ $^\circ$C</th>
<th>$T_{\text{water}}$ $^\circ$C</th>
<th>$W_A$/kg m$^{-2}$s$^{-1}$</th>
<th>$\phi_M \pi$</th>
<th>$k_y \pi$/kg m$^{-2}$s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>59.95</td>
<td>50.5</td>
<td>$1.73 \times 10^{-4}$</td>
<td>0.00367</td>
<td>$4.71 \times 10^{-2}$</td>
</tr>
<tr>
<td>69.9</td>
<td>60.7</td>
<td>$1.73 \times 10^{-4}$</td>
<td>0.00363</td>
<td>$4.77 \times 10^{-2}$</td>
</tr>
<tr>
<td>79.85</td>
<td>71.0</td>
<td>$1.70 \times 10^{-4}$</td>
<td>0.00350</td>
<td>$4.85 \times 10^{-2}$</td>
</tr>
<tr>
<td>89.8</td>
<td>80.1</td>
<td>$1.97 \times 10^{-4}$</td>
<td>0.00375</td>
<td>$5.25 \times 10^{-2}$</td>
</tr>
<tr>
<td>99.95</td>
<td>90.05</td>
<td>$2.15 \times 10^{-4}$</td>
<td>0.00361</td>
<td>$5.96 \times 10^{-2}$</td>
</tr>
<tr>
<td>89.7</td>
<td>60.2</td>
<td>$5.35 \times 10^{-4}$</td>
<td>0.01153</td>
<td>$4.63 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

(Air velocity = 1.2 m/s$^{-1}$)
Fig. 27 and 28  Errors generated by using adiabatic saturation lines for wet-bulb depression lines in hygrometry. Temperature error \((T_{as} - T_s)\) and humidity error \((\frac{Y_{as} - Y_s}{Y_s})\) as a function of wet-bulb depression and wet-bulb temperature.
Examination of Proportionality Between Drying Rates and Humidity Driving Forces from Data of Schneider

<table>
<thead>
<tr>
<th>$T_{air}/^\circ C$</th>
<th>$T_{surface}/^\circ C$</th>
<th>$w_A/kg \cdot m^{-2} \cdot s^{-1}$</th>
<th>$\varphi_M$</th>
<th>$k_y^* /kg \cdot m^{-2} \cdot s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>70</td>
<td>$9.70 \times 10^{-4}$</td>
<td>0.0309</td>
<td>$3.14 \times 10^{-2}$</td>
</tr>
<tr>
<td>130</td>
<td>70</td>
<td>$7.35 \times 10^{-4}$</td>
<td>0.0233</td>
<td>$3.15 \times 10^{-2}$</td>
</tr>
<tr>
<td>110</td>
<td>70</td>
<td>$4.90 \times 10^{-4}$</td>
<td>0.0157</td>
<td>$3.12 \times 10^{-2}$</td>
</tr>
<tr>
<td>150</td>
<td>90</td>
<td>$8.10 \times 10^{-4}$</td>
<td>0.0217</td>
<td>$3.73 \times 10^{-2}$</td>
</tr>
<tr>
<td>130</td>
<td>90</td>
<td>$4.90 \times 10^{-4}$</td>
<td>0.0145</td>
<td>$3.38 \times 10^{-2}$</td>
</tr>
<tr>
<td>110</td>
<td>90</td>
<td>$2.54 \times 10^{-4}$</td>
<td>0.0073</td>
<td>$3.48 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

(Air velocity = 3 ms$^{-1}$)

Close examination of Tables 5 and 6 also shows the direct relationship between the flux and the humidity potential at a constant wet-bulb temperature.

However, if the mass transfer coefficient $k_y$ is extrapolated to a new set of wet-bulb temperature conditions, then allowance must be made for the difference in the values of $\varphi_M$. The correction factors $\varphi_M$ vary less with changes in wet-bulb temperature and thus a description of the drying process in terms of temperature differences is sometimes more convenient. Especially as temperatures are more easily measured than humidities.

3. Adiabatic Saturation

For a particular set of dry- and wet-bulb conditions, the bulk air humidity $Y_g$ was calculated from equation (44) for $\pi$. This value of $Y_g$ was substituted in equation (49) and $T_{AS}$ was found by an iterative interpolation. The difference between $T_{AS}$ and $T_{W}$ is plotted in Fig. 27 as a function of wet-bulb temperature and depression. When the latter is small, the absolute magnitude of this difference is also small. However, even for very small depressions of 2$^\circ C$, the relative magnitude
Fig. 29 Humidity chart for air-water-vapour system for humidity potentials less than 0.03 kg kg\(^{-1}\) as a function of wet-bulb temperature and wet-bulb depression.
can be significant at low surface temperatures (it is 6 per cent at a wet-bulb temperature of 0°C): the ratio \( \frac{T_{AS} - T_W}{\sqrt{q}} \), rather than the absolute difference, indicates the error generated in using adiabatic-saturation lines for wet-bulb depression lines in hygrometry. The plotted curves show clearly that the temperature difference \( T_{AS} - T_W \) becomes less as the wet-bulb temperature rises, and at the boiling point, the adiabatic-saturation and wet-bulb temperatures are identical. At wet-bulb temperatures, above 60°C and depressions in excess of 20°C, the ratio \( \frac{T_{AS} - T_W}{\sqrt{q}} \) is never more than 1 per cent. Most convective driers are operated under such conditions. However this temperature error belies the magnitude of the humidity errors as presented in Fig. 28. As can be seen these errors tend to asymptotes at the boiling points at values proportional to the magnitude of the temperature depression. The coincidence of \( T_{AS} \) and \( T_W \) at the boiling point is general to any system, since \( \frac{dY_S}{dT_s} \) becomes infinitely large as the boiling point is approached.

4. Humidity Chart

A humidity chart, based on equation (48), for the air-water-vapour system is presented in Fig. 29. The chart covers wet-bulb temperatures from 0 to 98°C and humidity potentials below 0.03 kg/kg, so including wet-bulb depressions up to 80°C. This chart covers a wider range of hygrothermal conditions than normally presented in Grosvenor charts and is corrected for the deviations from linear driving forces. For operations at a constant wet-bulb temperature, the profile of humidity potentials is given by a vertical line. The advantages of the new chart are twofold. Firstly, plotting humidity difference \( \pi \) rather than absolute humidity as ordinate will often yield a reading accuracy of \( Y_G \) which is an order greater than that provided by conventional charts. Secondly the construction of the chart does not hinge on the psychrometric ratio \( A \) being one, and thus similar charts can be drawn for systems involving non-aqueous vapours.
Fig. 30 Relative humidity of water-vapour as a function of wet-bulb depression and wet-bulb temperature.
The former advantage is illustrated by using the charts to evaluate the bulk air humidity at a dry-temperature of 110°C and at a wet-bulb temperature of 70°C, i.e. $V_0 = 40\,^\circ C$. The humidity potential is read as 0.0221 kg/kg from Fig. 29. From the table of saturation humidities in Appendix 1, the bulk-air humidity is hence 0.2602 kg/kg. In comparison, we read a value of 0.252 kg/kg from Krischer's high-temperature Mollier chart. A value of 0.254 kg/kg is read from the Grosvener chart of Zimmerman and Lavine.

In drying practice, schedules are often specified in terms of relative humidity. Fig. 30 presents the relative humidity as a function of wet- and dry-bulb temperatures.

PART 3 Driving Forces for Moisture Evaporation from Unsaturated Surfaces

The drying of wet materials has been categorised into two drying periods, exclusive of the initial warmup interval. In the first period, the drying rate per unit area of drying surface is constant and the magnitude of this rate is given by the equations presented in Part 2. This period is succeeded by a falling-rate period in which the moisture removal rate is gradually decreased. The critical point of change between the two periods has been observed for wood to occur when the surface moisture content reaches the maximum hygroscopic moisture content. Luikov has also suggested that this critical point will be close to the maximum hygroscopic moisture content for many materials.

The subsequent reduction in rate may be due to a combination of the following causes:

1) the recession of the evaporative plane from the surface to within the solid;
2) the effective surface area for heat and mass transfer decreases as the solid dries out;
3) below the hygroscopic moisture content the partial pressure of the water vapour present will be less than the saturation vapour pressure;
Fig. 31. Significance of reeding evaporative plane model for falling-rate period of drying. Pore radius difference needed to overcome head $\Delta h$ (solid thickness) as a function of pore radius.
4) Extra energy is required to vaporise the moisture because of adsorptive bonding to the solid skeleton.

The significance of these various mechanisms is considered below.

The Receding Evaporative Plane Model

The mechanism of a receding evaporative plane falling rate has been examined by several workers for non-hygroscopic solids. Evaporation of the moisture will take place from the surface until the capillary tension forces can no longer overcome the gravitational forces. It is expected that the receding plane mechanism should only apply for the drying of coarse-pored materials. For such materials, the gravitational forces play an important role for the capillary motion of the moisture. A criterion for this surface evaporation limit can be derived from Newitt's analysis of capillary flow.

For two pores of radii \( r_1 \) and \( r_2 \) connected together by a liquid thread, the capillary tension forces can move the moisture against a head \( \Delta h \) according to equation (65).

\[
\frac{2\sigma}{r_1} \left( \frac{1}{r_1} - 1 \right) = \rho g \Delta h, \quad r_2 > r_1
\]

By defining \( \Delta r = r_2 - r_1 \), one can derive the following equation

\[
\Delta r = r_1 \frac{\rho g \Delta h}{\frac{2\sigma}{r_2} - \frac{\rho g \Delta h}{2\sigma} - \frac{1}{1 - \frac{\rho g \Delta h}{2\sigma}}}
\]

which predicts the difference in pore radius necessary to overcome the head \( \Delta h \). If \( \Delta h \) is defined as the thickness of the solid, then the value of \( \Delta r \) for a given pore radius \( r_1 \) can be calculated for when the evaporation will always take place from the surface. Values of \( \Delta r \) are plotted in Fig. 31 against \( \Delta h \) for lines of constant radii over the range \( r_1 = 1 \) to 100 \( \mu m \).
Any porous solid will have a distribution of pore radii, so provided a reasonable percentage of the pores cover the range \( r_1 \) to \( r_1 + \Delta r \) then recession of the evaporative interface will not occur. The ratio \( \Delta r/r \) is presented tabularly on Fig. 31 for \( \Delta h = 0.05 \) as a function of \( r \). It can be seen that when \( r < 20 \mu\text{m} \), a very narrow pore distribution is sufficient to satisfy the surface evaporation criterion. For drying beds of glass spheres, hexagonally or cubic-packed, two distinct pore sizes are observed. For example for cubic packing pores of radius \( 0.41R \) and \( 0.73R \) are observed where \( R \) is the sphere radius. The dotted lines on Fig. 31 indicate the surface evaporative criteria for drying beds of spheres. So provided the diameter of the beads is less than \( 50 \mu\text{m} \) no recession of the evaporative interface should be observed for drying solid beds less than \( 100 \text{mm} \) thick.

The Falling Rate period for Drying Hygroscopic Solids

The above results have established the range of application for the evaporative interface model. The analysis is perfectly general for the drying of all solids provided the two moisture content ranges for the drying of a hygroscopic solid are recognised. When the moisture content is greater than \( X_{\text{hyg}} \), the maximum hygroscopic moisture content, the moisture is termed "free moisture". For moisture contents below \( X_{\text{hyg}} \), the term "bound moisture" is used. Also associated with these two regimes are particular pore-size distributions. The free moisture is considered to be stored in capillaries of radius \( r > 0.1 \mu\text{m} \) while the bound moisture is contained in pores of radius \( r < 0.1 \mu\text{m} \).

For the two types of solid, hygroscopic and non-hygroscopic, the onset of the falling-rate period will result from the same surface condition, i.e., \( X_{\text{surf}} = X_{\text{hyg}} \) (for non-hygroscopic solids \( X_{\text{hyg}} = 0 \)). In general the decreasing drying rate will then result from a combination of the receding plane model and effects due to the solid hygroscopicity. Because the hygroscopic moisture is stored in very small pores no receding plane will be observed for that regime. For most hygroscopic
Experimental measurements by Isaka (Tables 5 to 15) indicating the linearity between the drying-rate of Kunugi and Hinoki wood (-thickness 0.5 to 2.0 cm) and the temperature difference between the wood surface and the air in both the longitudinal and tangential directions. (T₀ = 40°C, T₀ = 35.15°C, r.h. = 75%).
solids the maximum pore diameters for free water storage are less than 20 μm (for wood $r_{\text{max}} = 15 \mu m$). Thus it is believed that the receding plane mechanism will contribute negligibly to the falling-rate period drying of hygroscopic solids. There are two possible exceptions to this rule. The first refers to the drying of solids when the free water is contained in coarse capillaries greater than 20 μm radius. The second case applies to drying at temperatures greater than the moisture boiling point. Under the latter conditions an evaporative interface recedes into the solid because of internal boiling which is heat transfer controlled. Such a phenomenon was observed by Chen for the drying of sand and porous metals under conditions of a high heat flux.

Studies of the falling-rate period of drying for wood have revealed that the drying rate is proportional to the temperature difference between the wood surface and the surrounding air. The experimental observations of Isaka, summarised in Figs. 32 and 33, show some scatter because the temperature difference was only resolved to 0.1°C. More importantly these data indicate a unique relationship of surface temperature to drying rate which is independent of the species, direction of internal moisture movement and the sample thickness. These results also show that the heat and mass transfer coefficients remain constant throughout both drying periods. This last finding can also be inferred from Suzuki et al.'s work on mass transfer from discontinuous sources. If the pore diameter to boundary-layer thickness is small enough then Suzuki's analysis shows that the mass transfer flux is independent of the effective wetted surface area. Even if this surface area does change as the solid dries, then the rate will be unaffected. The range of validity of this assumption will be closely related to the assumption for negligible significance of the receding plane mechanism. If the boundary layer thickness is 3 mm then the effect of the changing surface area can be ignored provided $r_{\text{max}}$ is less than 100 μm (i.e., $S/L_{\text{DS}} > 30$).

Although Keey and Suzuki's derivation of their receding-plane model assumes a direct proportionality between the
Fig. 34. Function \( P_R \) measured by Ogura\(^{62}\) with the variation of surface moisture content for the drying of wood at 50\(^\circ\)C.
temperature difference \((T_G - T_{SURF})\) and the drying rate, Isaka's results cannot be explained by a combined receding plane and hygroscopicity model. The receding evaporative plane model will not predict a rate to surface temperature relationship that is independent of the direction of internal moisture movement or the material thickness.

From further drying studies, Ogura describes the falling-rate period for the drying of wood by defining a function \(p_R\) which corrects the constant-rate period flux.

\[
W_A = k_p (p_S - p_G - p_R)
\]  

(67)

where \(p_S\) is the saturation vapour pressure of the moisture at the wet-bulb temperature of the bulk air stream, \(p_G\) is the partial pressure of the water vapour in the bulk gas stream and \(k_p\) is the mass transfer coefficient based upon the partial pressure driving force. The nature of the function \(p_R\) (Fig. 34) was found to be uniquely related to the surface moisture content at a dry-bulb temperature of 50°C for various relative humidities. The shape of the curve for \(p_R\) is similar to the hygroscopic solid moisture equilibrium isotherm. If the receding plane mechanism contributed significantly to the falling-rate period then Isaka and Ogura's results would not predict a unique surface moisture content to rate relationship.

The Driving Force Model for Unsaturated Surfaces

It has been shown above that the falling rate drying behaviour of hygroscopic solids cannot be explained by the mechanism of a receding evaporative plane or due to a decreasing surface area function. This then means that the decreasing rate must be due to the third and fourth mechanisms that were originally proposed. A model is derived below that predicts the driving force for the drying of an unsaturated surface from the vapour-pressure depression relationships. The effect of the energy of bonding is included with the energy of vaporisation.
Fig. 35, 36 and 37 Diagrams of solid/vapour equilibria for wood and water-vapour as measured by Stamm and Loughborough for the temperature range 20 to 100°C. Equilibrium moisture content as a function of temperature and relative humidity (Fig. 35) and as a function of wet-bulb depression and wet-bulb temperature (Fig. 37). Normalised equilibrium moisture content as a function of temperature and relative humidity (Fig. 36).
It is postulated that for the drying of hygroscopic microporous solids such as wood that the evaporation of moisture takes place exclusively at the surface under normal convective conditions. The rate of drying is assumed to be small enough so that the surface layers remain in hygroscopic and psychrometric equilibrium with the surrounding airstream. This means that the partial pressure of the surface moisture is given by the equilibrium moisture content-relative humidity isotherms, such that any hysteresis effects are ignored. The equilibrium moisture content isotherms for pine wood are presented in Fig. 35 and 36 for moisture content versus relative humidity over the temperature range 20°C to 100°C. These isotherms are similar for all woods because of the physical and chemical similarity of the cell-wall structure between species. As drying schedules are normally specified in terms of temperatures it is convenient to know the equilibrium moisture content as a function of the wet- and dry-bulb temperatures of the surrounding air (see Fig. 37).

As vaporisation of moisture is assumed to occur at the surface, an expression of the form of equation (36) can be proposed to predict the driving force for evaporation. During the falling rate period the surface temperature rises to meet the dry-bulb temperature of the surrounding air. This period is said to start when the surface moisture content is equal to the hygroscopic moisture content, \( \chi_{\text{hyg}} \). The hygroscopic moisture content is defined as the moisture content corresponding to a 0.5 per cent vapour pressure depression. For wood this limit corresponds to the fibre-saturation point (FSP). At this limit the temperature of the surface will be given by the wet-bulb temperature of the surrounding air.

Provided the heat flow to the solid is small in comparison with the energy required for evaporation (see Chapter 6) we can write:

\[
\frac{k_y \cdot \frac{\partial \chi}{\partial \chi}}{(\chi_{\text{SURF}} - \chi_{\text{G}})} = \frac{h_c \cdot \frac{\partial \chi}{\partial (T_{\text{G}} - T_{\text{SURF}})}}{\left( \Delta H_{\text{VS}} + \Delta H_{\text{B}} \right)} 
\]
Fig. 38 Enthalpy of bonding of water to wood and its ratio to the latent heat of vaporisation as a function of normalised equilibrium moisture content (based on calculations by Stamm and Loughborough and Weichert).
The humidity \( Y_{\text{SURF}} \) is the equilibrium humidity at the drying surface given by the equilibrium moisture content isotherm for the moisture content \( X_{\text{SURF}} \) and temperature \( T_{\text{SURF}} \), the temperature of the drying surface. The term \( \Delta H_B \) represents the energy of bonding of the moisture to the solid skeleton. This can be calculated from the desorption isotherm where

\[
\Delta H_B = \frac{RT}{M} \left( \int \frac{d\varphi}{\sigma dT} \right) \ln \left( \frac{1}{\varphi} \right),
\]

(69)

\( \varphi \) is the relative humidity of the vapour and \( \sigma \) is the surface tension of the liquid. Hence \( \Delta H_B \) will be a function of moisture content and temperature as suggested by the equilibrium moisture isotherms presented in Fig. 35. Stamm and Loughborough\(^79\) and Weichert\(^67\) have calculated \( \Delta H_B \) for wood using equation (69) and the results are plotted in Fig. 38. It can be seen that \( \Delta H_B \) is small compared to \( \Delta H_{VS} \) until \( X/X_{h_{\text{hyg}}} \) is equal to 0.5. Under most drying conditions for wood the endpoint of drying corresponds to \( X/X_{h_{\text{hyg}}} \geq 0.3 \).

Since the equilibrium data are given as a function of the relative humidity and the surface temperature, it is convenient to express \( Y_{\text{SURF}} \) in terms of these quantities. The surface humidity in terms of the partial vapour pressure of the surface moisture, \( P_{\text{SURF}} \), is:

\[
Y_{\text{SURF}} = \frac{D P_{\text{SURF}}}{P - P_{\text{SURF}}},
\]

(70)
where $P$ is the total pressure. Substituting the expressions

$\Phi = \frac{p_{\text{SURF}}}{p_S}$ and $p_S = \frac{p_Y}{p_Y + \phi_Y S}$

into equation (70) leads to the expression:

$$Y_{\text{SURF}} = \frac{D\phi Y_S}{D + Y_S - \phi Y_S} \tag{71}$$

where $Y_S$ is the saturation humidity at temperature $T_{\text{SURF}}$.

Combining equations (68) and (71) gives the final nonlinear equation.

$$k_y^* \varphi_0 \left( \frac{D\phi Y_S}{D + Y_S - \phi Y_S} \right) = \frac{h_c^* \varphi_0 (T_G - T_{\text{SURF}})}{(\Delta H_{\text{WS}} + \Delta H_B)} \tag{72}$$

For a given surface moisture content and bulk gas humidity, this equation is a complex function of the parameter $T_{\text{SURF}}$.

The Wegstein iterative root-finding scheme suggests the following algorithm:

$$T_{\text{SURF}}^{(n+1)} = k_y^* \varphi_0 (n) \left[ \frac{D(n) \cdot Y_S(n)}{D(n) + Y_S(n) - \varphi(n) Y_S(n)} - \varphi_T \right]$$

$$= \frac{h_c^* \varphi_0 (n) (T_G - T_{\text{SURF}}(n)) + T_{\text{SURF}}^{(n)}}{(\Delta H_{\text{WS}}(n) + \Delta H_B(n))} \tag{73}$$

which gives rapid convergence to

$$\left| \frac{T_{\text{SURF}}^{(n+1)} - T_{\text{SURF}}^{(n)}}{T_{\text{SURF}}^{(n)}} \right| \leq 10^{-6}.$$ 

The computer program "DRIVE" (see Appendix 2C) solves equation (73) by using the physical property equations (50) to (62) and the equilibrium moisture content data (e.g., Fig. 35).

The program predicts the transfer flux from the unsaturated surface at various surface moisture contents and surrounding air conditions.

The transfer flux for the unsaturated surface as a function
Fig. 39, 40 and 41
Reduced surface driving-force function \( R \) for three simple temperature independent equilibrium moisture content curves at wet-bulb temperature \( T_w = 40°C \) and various wet-bulb depressions.

Fig. 39 linear case
Fig. 40 Two linear segment curve - break point at \( (\Psi = 0.4, \frac{x_E}{x_{hyg}} = 0.8) \)
Fig. 41 Two linear segment curve - break point at \( (0.8, 0.4) \).
Fig. 4.2 to 4.7 Reduced surface driving-force function for pinewood as a function of surface moisture content at various wet-bulb temperatures (20–70°C) and wet-bulb depressions (based on equilibrium moisture content isotherms in Fig. 3.7).
of moisture content can be summarised by defining the reduced surface driving-force function $R(X_{SURF})$. This is given by the ratio of the flux from an unsaturated surface to that for a saturated surface.

\[
R = \frac{X_{SURF} - X_E}{X_{HYG} - X_E}
\]

To demonstrate the method, the driving force curves for the case of a linear equilibrium moisture content curve are presented in Fig. 39 for various temperature differences $\varphi$ from 2 to 50°C at a wet-bulb temperature of 40°C. The curves, which are plotted as a function of the dimensionless surface moisture content $w_s ((X_{SURF} - X_E)/(X_{HYG} - X_E))$, show that the function $R$ becomes more convex downwards in shape as the flux increases. Two further examples are presented in Figs. 40 and 41 for nonlinear solid-vapour equilibrium curves. In these cases the shape of the curves exhibit a complex trend as the temperature difference increases. This complexity arises because as the temperature difference increases the equilibrium moisture content changes. To generate the driving force curve, the equilibrium curve is required for the range $X_E$ to $X_{HYG}$. If this curve is nonlinear then the effective shape of the equilibrium curve will change with the temperature difference. These first three examples have assumed a temperature independent equilibrium curve. But for wood the equilibrium curves are dependent upon temperature. This factor produces further complex changes to the nature of the driving force curves. Figs. 42-47 present the surface driving forces curves for the drying of wood from wet-bulb temperatures of 20°C to 70°C at depressions up to 50°C. In all cases the shape of the curves increase in downwards convexity. However for a constant temperature difference $\varphi$ the shape of the curves is practically independent of the wet-bulb temperature. This result is illustrated in
Fig. 48 to 50
Significance of wet-bulb temperature upon shape of reduced surface driving force function for pine wood at wet-bulb depressions of 2, 10 and 20°C.
Comparison of reduced surface driving-force function for pinewood at $T_w=40^\circ C$, $T_0=50^\circ C$, defined in terms of humidity differences or temperature differences.
Fig. 52. Comparison of predicted reduced surface driving-force function for pinewood (Fig. 4.2 to 4.7) with curves suggested by Ogura's experimental function $P_R$ (Fig. 34) at a dry-bulb temperature of 50°C.
Figs. 48 to 50 for $\varphi = 2, 10$ and $20^\circ C$.

All of the driving force curves presented above have been defined in terms of the mass transfer flux. It can also be shown that a reduced driving force defined in terms of the temperature differences $\left[\frac{T_G - T_{\text{SURF}}}{\varphi}\right]$ yields practically identical curves to those defined on a more rigorous basis. This result is illustrated in Fig. 51 for the drying of wood at $T_w = 40^\circ C$ for $\varphi = 10^\circ C$ and $50^\circ C$.

The driving force model proposed above has been adopted by several other workers $^{36,87,88,89,91}$ to describe the drying of solids. In these cases the model is assumed without any supporting experimental evidence. The experimental measurements of Ogura (Fig. 34) can be recalculated in terms of the dimensionless falling rate function to show good agreement with theoretical curves. These two sets of curves are compared in Fig. 52 for the drying of wood for the air-conditions of $T_G = 50^\circ C$ and $\varphi = 10, 20$ and $30^\circ C$. The differences between the theoretical and experimental curves possibly suggest that effects due to sorption hysteresis or the heat transfer flux are significant. Poor agreement is observed between theory and experiment from Sergovskii's $^{68}$ measurements on the drying of pine at $T_G = 75^\circ C$ and $T_w = 56^\circ C$. In this case the driving force function was slightly convex downwards in shape. Nevertheless Sergovskii observed a unique driving force curve that was independent of the drying rate, species, thickness and the initial moisture content of the wood.

There is little evidence available, apart from that quoted above, to test the driving force model for evaporation from unsaturated surfaces. However there are other results that offer indirect evidence for such a model. Kotok, Lowery and Jensen $^{69}$ note that the average moisture content of drying timber can be monitored by following the surface temperature of the wood. They present calibration curves for the average wood moisture content as a function of time and surface temperature for the drying of Ponderosa Pine at various rates. During these tests they also frequently noted a plateauing of
surface temperature which appeared to be associated with the constant rate period. Further evidence is offered by Harbert\textsuperscript{90}. He proposes a new automatic control strategy for the drying of solids by monitoring the difference in temperature between the solid surface and the wet-bulb temperature. Nomograms are presented which also relate the average moisture content to this surface temperature variation for a wide range of materials.

The foregoing evidence shows that the surface temperature and surface moisture content variation plays an important role in the determination of the drying rate of unsaturated solids. It is believed that the proposed model should accurately describe the drying behaviour of microporous hygroscopic solids such as wood.
REFERENCES - Chapter 3


67. L. Weichert - "Investigations on sorption and swelling of spruce, beech and compressed beech wood between 20°C and 100°C", Holz als Roh und Werkstoff, 21, 8, p. 290-300.


In order to be able to follow and evaluate the worth of batch-drying schedules, a method is required to predict the moisture content and temperature fields as functions of time at various locations throughout the dryer. The batch drying behaviour can be predicted by solving the expressions that result from coupling the mass and energy conservation equations with rate equations. Such an approach assumes a lumped-parameter model for the solid kinetics.

The design and operation of batch-dryers has long been a trial-and-error technique with allowances for large factors of safety. The main preoccupation of the designer has been to provide a machine capable of drying the solid as fast as possible, often ignoring the consequence. This implied policy of time minimisation possibly provides a reasonable basis for dryer design. But it completely ignores the consideration of the economic aspects of dryer operation, such as whether that policy would optimise the energy required per unit mass of dry material produced. The solutions for the batch-drying equations potentially offer much useful information towards comparing dryer designs and their operating characteristics. To date, as discussed in Chapter 1, this avenue of exploration has been ignored except for the optimisation study of Thygeson and Grossmann.

Initially in this chapter, the batch-drying simulation literature is criticised in order to pose an elegant method for studying batch-dryer behaviour in Part 2. The equations are developed for drying a nonshrinking solid where the changes take place in one dimension only, that of the direction of the airflow. The dryer is operated such that the inlet temperature and humidity potentials remain constant. The solid is assumed to be arranged so that the transfer coefficients are constant throughout the dryer. This method of describing batch-dryer dynamics closely parallels existing work, but care is taken in
defining variables so that some assumptions made by other
workers are not necessary. The rigorous equations for the
heat and mass transfer interchange between the solid and gas
are shown to be justifiably simplified, provided certain
restrictions hold. These restrictions are generally satisfied
by practical gas-solid systems.

In Part 3, new solutions for the batch-drying model are
presented. These are developed using analytical and computer
techniques. Some of these techniques allow complex drying
problems to be solved with ease and accuracy. The model is
rigorously examined in Part 4 and it is extended to include
such phenomena as varying mass transfer coefficients, changing
inlet potentials, and shrinking solids. These extensions then
allow real dryers to be simulated more accurately. Another
operating procedure used often in practice, flow reversing of
the airstream, is considered in Part 5. The consequences and
benefits of such a policy are shown and some simple optimal
procedures are studied. From the results of the previous parts
of the Chapter, Part 6 examines some particular features of the
drying process with the view of process improvement. For
example, knowledge of the drying rate maxima allows the
calculation of how much the rate can be accelerated so that the
initial inlet rate is maintained as long as possible at positions
within the dryer. To further illustrate the usefulness of the
batch-drying model, the energy requirements for the moisture
evaporation are calculated by monitoring the outlet potentials
from the dryer.

Part 1 Review of Literature

During the drying of moist solids in stationary beds,
temperatures and moisture contents of both the material and the
drying medium change continuously with time and location in the
dryer. Moreover, the rates at which these changes occur depend
on the composition, structure and packing arrangement of the
solid, and on the composition and flowrate of the drying
medium. This problem has received considerable attention in
the literature by considering the batch-dryer as a packed bed. All these studies have examined the dryer behaviour in terms of one-dimensional changes: that is, the state of the moist solids and the humid gases is assumed constant in the direction at right angles to the gas flow. In order to fully describe the system, the conservation equations of mass and energy are solved together with rate and equilibrium expressions. For an adiabatic dryer packed uniformly with a non-shrinking solid, the interchange of mass and energy between the stationary solid bed and the fluid moving in plug flow with constant transport properties, is given by equations (1) and (2),

\[
\text{mass} \quad \frac{(1-e) \rho_s}{G_B} \frac{\partial \bar{x}}{\partial \theta} = \frac{\partial y_g}{\partial \theta} + \frac{\partial (\rho_g \frac{\partial y_g}{\partial \theta})}{\partial \theta}
\]

\[
\text{energy} \quad \frac{(1-e) \rho_s}{G_B} \frac{\partial h}{\partial \theta} = \frac{\partial y_g}{\partial \theta} + \frac{\partial (\rho_g \frac{\partial h}{\partial \theta})}{\partial \theta}
\]

In equations (1) and (2), \(e\) is the volumetric fraction of voids, \((1-e)\) is the volumetric fraction of solid, \(\rho_s\) is the solid density (mass of dry solid per unit volume of moist solid), \(\rho_g\) is the humid gas density (reciprocal of humid volume), \(G_B\) is the specific mass flowrate of dry air through an empty dryer, \(\bar{x}\) is the average solids moisture content (mass of moisture per unit mass of dry solid), \(y_g\) is the gas humidity (mass of moisture per unit mass of dry air), \(\theta\) is the time, \(y\) is the distance along the dryer in the direction of the airflow, \(h\) is the humid enthalpy of the moist solid and \(H\) is the humid enthalpy of the moist gas. The various batch-drying investigators have chosen conservation relationships based upon equations (1) and/or (2). However the diverse choice of rate expressions determines the subsequent method of solution to the set of equations. The worth of the many approaches is considered below.

Terazawa, Tsubumoto and Kodama\(^2\) present a straightforward
Fig. 53. Drying rate curves determined by Terazawa et al. for Ezymatsu wood (thickness 23 mm, specific gravity = 0.38) at dry-bulb temperature of 60°C and air-velocity 0.7 ms⁻¹ as a function of wet-bulb depression and average moisture content.
but time-consuming graphical method of predicting moisture content profiles throughout a timber dryer. An empirically derived form of the energy balance equation (2) gives the temperature drop $\Delta T_G$ across a single board of width $\Delta z$ and depth $d$ within a stack of timber by the expression,

$$\frac{\Delta T_G}{\Delta y} = \frac{\Delta H_{VS} \Delta \rho_s \Delta X}{\Delta y \beta v \rho_g C_{PG}}$$

This formula is coupled with a set of drying-rate curves, such as presented in Fig. 53. The drying rate can then be calculated for a given wet- and dry-bulb temperature difference at various moisture contents. The first step in the method is to predict the drying rate for the first board on the air entering side of the stack. This is obtained from the drying-characteristic curves according to the temperature and depression fixed by the drying schedule and the initial moisture content. By substituting this rate into equation (3) the decrease in temperature across this board can be found. Secondly, the drying rate for the second board is obtained from Fig. 53 for the reduced temperature depression resulting from the air-passage between the first boards. In this manner the average evaporation rate and the moisture content for each board along the airflow during the first time period (1 hour) is calculated. By repeating this explicit finite difference procedure, the moisture contents of each board can be calculated after further time periods.

In contrast to Terazawa and co-authors' rigorous drying rate expression, Van Meel proposes one unique characteristic rate curve $f$ for the solid.

$$-\frac{\delta F}{\delta \theta} = k_y \gamma (Y_s - Y_g) f\left(\frac{F - F_s}{F - F_g}\right)$$

In equation (4) $F$ is the moisture content defined as the mass of moisture per unit surface area of the solid and $\gamma$ is a correction factor to the linear humidity driving force. The
function $f$ takes the value of unity for $F > F_U$, the critical moisture content which is assumed to be a constant. The shape of the curve for values of $F < F_U$ is assumed to be independent of the range of drying rates encountered within the dryer. Although this concept is clearly an approximation to real behaviour, this assumption provides the basis for an elegant mathematical analysis of the batch-drying problem.

Van Meel\textsuperscript{3} notes that the variation of the gas humidity with respect to time can be neglected. After coupling equations (1) and (4), they are further simplified by defining dimensionless parameters. These equations are fully solved for the case of a linear characteristic curve. The general case is partially solved. These solutions are applied to the problem where the inlet gas humidity continuously varies by recirculating a constant fraction of the humid exhaust gases. Experimental results gathered by Jaeschke\textsuperscript{4,5} for drying bars of concrete were found to agree well with predictions by Van Meel's model for the case of a constant inlet humidity potential.

Toei, Hayashi et al\textsuperscript{6} improved Van Meel's model by expressing the partial differential equations in terms of more convenient parameters. For his model, Van Meel\textsuperscript{3} assumes a particular solid arrangement within the dryer while Toei's analysis is applicable to any possible solid arrangement. In this case the flux equation is given by:

$$-\mathcal{Q}_S (1-\varepsilon) \frac{\partial \bar{X}}{\partial \theta} = k_y a (Y_S - Y_G) \frac{f \left( \frac{\bar{X} - \bar{X}_E}{\bar{X}_{CR} - \bar{X}_E} \right)}{\bar{X}_{CR} - \bar{X}_E} \quad (5)$$

where $a$ is the solid surface area per unit volume of the dryer. The mass balance is given by equation (1). The equations are solved for the case of a linear characteristic curve when the inlet humidity potential is held constant. In particular, expressions are derived for the average moisture content and drying rate over the whole dryer. Good agreement with these averaged expressions was obtained for drying beds of silica gel.
The same set of equations as proposed by Toei et al. was studied by Keye for the constant recirculation ratio problem. Van Meel's solutions are extended to predict limiting values of the recirculation ratio such that the drying rate at the outlet will not exceed the initial drying rate at the inlet. However, these values may be in error as the predictions are based upon an incorrect assumption. It is assumed that the maximum drying rate at the outlet occurs when the moisture content is equal to the critical moisture at that point. This assumption will only hold for a limited range of conditions as will be shown later in Part 6.

The value of the characteristic curve concept has been recognised by many of the above workers. Viktorin also uses this approach as part of a method for evaluating the moisture content and humidity distributions across a timber pile. The rate expression is given by a function of the form of equation (3) except that the mass transfer coefficient is taken to be a function of distance. A characteristic curve of the form \( f = (\bar{X} - \bar{X}_E)^n / \{A_F + B_F(\bar{X} - \bar{X}_E)^n\} \), is assumed where \( A_F, B_F \) and \( n \) are constants. This function was shown by Filonenko and Lebedev to approximate the drying behaviour of several hygroscopic materials. The combination of this \( f \)-function and the varying mass transfer coefficient produces a set of equations that cannot be solved by analytical methods. Another analysis which assumes a unique characteristic curve is Nykelstad's approximate method. By simplifying the conservation equations a method is posed for studying the simultaneous heat and mass transfer in batch dryers where the drying-rate \( f \)-curve has a complex shape. The batch-drying dynamics are described by the equations

\[
\begin{align*}
\frac{d\bar{X}}{d\theta} &= -K \left( Y_s - Y_d \right) = -C_B \frac{dy}{y} \\
\frac{d\bar{X}}{d\theta} &= \frac{C_{PL} \bar{X} dT_S}{\Delta H_{VS} d\theta} - \frac{C_{PS} dT_p}{\Delta H_{VS} d\theta} - \frac{U}{\Delta H_{VS} \rho_S} (T_G - T_S) \quad (7)
\end{align*}
\]
where $K$ is the volumetric mass transfer coefficient, $U$ is the volumetric heat transfer coefficient, $C_{PL}$ is the specific heat capacity of the moisture, $\bar{T}_S$ is the average solids temperature, and $C_{PS}$ is the specific heat capacity of the dry solid. These equations are solved in conjunction with linearised drying rate characteristics based upon Krischer and Jaeschke's experimental observations. This analysis implies a unique characteristic curve which is linearised into two segments. Further unnecessary simplifications and incorrect assumptions invalidate this analysis. For example, Krischer and Jaeschke's curves for bed positions far from the inlet are approximated to four linear segments: the constant-rate region, the increasing falling-rate region, the first falling-rate region and the second falling-rate region. The slope of the increasing-rate region is assumed to be constant throughout the bed and the second falling-rate period is regarded as identical throughout the dryer. The increasing-rate period is taken to end when the critical point reaches that bed position. This assumption has already been noted to be in error.

A more serious error is the description of the driving forces over the increasing and falling-rate periods. Mykelstad suggests that, from the beginning of the increasing-rate period, the surface temperature increases as the bulk air temperature $T_G$ and the humidity $Y_G$ improves. Throughout the remainder of the drying process, the surface humidity is assumed to be given by the saturation humidity at the corresponding surface temperature $T_S$. This assumption suggests that the temperature driving force decreases while the humidity driving force increases. Both these driving forces must decrease as the solid dries out. This will be the case if $Y_S$ is corrected for the vapour-pressure depression due to the solid's hygroscopicity.

The increasing-rate region has been shown by other workers to be due to improved potentials at that point in the dryer, These result because of reduced fluxes upstream. This rate will increase while the surface of the solid is still saturated.
and the surface temperature remains constant. Once the critical point has been reached the rate may still increase to a maxima while the surface temperature rises in magnitude. Notwithstanding these errors, Mykelstad's solutions are complex. A full batch-drying experiment would be necessary in order to evaluate the necessary constants for the description of the drying of a new material. Such a requirement negates the value of the predictive method.

A superior method is suggested by Muraev'ev¹¹ which also depends upon an approximate energy conservation relationship similar to that proposed by Mykelstad (eqn. 7). In this case the term involving the drying rate is eliminated by noting that the ratio of the energy required to evaporate the moisture compared to the energy required to heat the solid (Kossovich number) is practically constant throughout drying. After further approximations and elimination of one of the variables, an analytical solution is obtained for the surface temperature as a function of time and distance. The solution is presented in terms of integral Laplace-Carson transforms. His analysis of the experimental results obtained for drying flax fibres showed that a unique characteristic drying curve was obtained to within the experimental error.

The heat and mass transfer in the drying of packed beds has also been studied by Sieniutycz¹², Banks¹³,¹⁴ and Thygeson¹. Equations (1) and (2) were coupled by Sieniutycz to the following rate expressions:

\[-\rho_s (1-\varepsilon) \frac{\partial \bar{x}}{\partial \theta} = K_y a (Y_E - Y_G) \quad (8)\]

\[-\rho_s (1-\varepsilon) \frac{\partial \bar{h}}{\partial \theta} = h_C a (H_E - H) \quad (9)\]

where \(Y_E\) is the equilibrium humidity and \(H_E\) is the equilibrium enthalpy of the vapour at the surface of the drying solid. These equations are solved graphically on two-phase enthalpy
diagrams where the enthalpies are unique functions of the average moisture content. The construction of these diagrams implies a unique characteristic rate curve. A simpler model is proposed by Banks in studying the drying of deep beds of grain. The conservation equations are solved by assuming that the transfer coefficients are infinite. This means the air and solids are in thermal and moisture sorption equilibrium at every location. The solution indicates that cooling and drying fronts move through the bed at constant velocity. This analysis is hardly applicable to the constant rate drying regime. But it probably offers a simple description of falling-rate period drying for changes of air-conditions which result in small changes of the equilibrium moisture content of the solid.

In contrast, Thygeson's optimisation study only considers the constant drying rate period. The batch-drying behaviour is described by an energy conservation equation. This equation is similar to equation (2) except a dispersion term has been added. All the previous studies have ignored dispersion effects by assuming a plug-flow model. The significance of this dispersion term is not evaluated, although the equations are solved for the constant rate period.

The convenience of using a humidity difference driving force, and, the use of molal units for the fluid properties is illustrated by the equations proposed by Peck et al and Doe for batch-drying. Peck combines the approximate conservation relationships with his empirically derived rate expression.

\[ \rho_s \frac{\partial \bar{x}}{\partial \theta} = G_B \frac{\partial y}{\partial y} \frac{\partial p}{\partial y} + \frac{\partial p}{\partial y} \frac{\partial \bar{y}}{\partial y} \frac{\Delta H_{\text{VS}}}{\partial y} \]

\[ = -k_{\text{GA}} (p_s - p_g) \quad \bar{x} > \bar{x}_{\text{CR}} \]

\[ = -k_{\text{GA}} \left( \frac{\bar{x}}{\bar{x}_{\text{CR}}} \right)^2 (p_s - p_g) \quad \bar{x} < \bar{x}_{\text{CR}} \quad (10) \]
In equation (10), $p_s$ and $p_G$ are the partial vapour pressures of the moisture at the surface and in the bulk-gas stream respectively. No attempt is made to solve these equations, but they are clearly more complex with the moisture vapour concentration expressed in terms of partial pressures rather than humidities. A similar problem arises with the set of batch-drying equations proposed by Doe,

$$
\frac{G(p-p_G)}{(p-0.378p_G)} \left[ C_{PA} + 0.622 \frac{C_{PV}}{p_G} \right] \frac{\partial T_G}{\partial y} - \Delta H_{VS}(T_S) \frac{\partial \eta}{\partial \theta} + C_{PS} \frac{\partial \bar{T}_S}{\partial \theta} = 0 \quad (11)
$$

$$
\frac{\partial \bar{T}_S}{\partial \theta} = -0.622 \frac{Gp_G}{(p-0.378p_G)^2} \frac{\partial p_G}{\partial y} \quad (12)
$$

$$
\approx -k_w (p_s - p_G) \quad (13)
$$

where $G$ is the mass flow rate of moist gas per unit area and $\bar{\eta}$ is the solid packing density (mass of moist solid per unit length of dryer). Equation (13) is assumed to apply for the constant rate region which is said to have been completed when the surface moisture content of the drying solid reaches the equilibrium moisture content. This critical point is defined by the instant the surface moisture content is equal to the equilibrium moisture content according to the diffusion equation solution for the case of a constant diffusion coefficient and a constant drying-flux boundary condition. The falling rate period is then described by the exponential relationship

$$
\frac{\bar{\eta} - \bar{\eta}_E}{\bar{\eta} - \bar{\eta}_E} = \exp[-(\bar{\eta} - \bar{\eta}_E)/\bar{\eta}] \quad (14)
$$

which implies a linear drying-rate curve. However, this curve is not unique, as $\bar{\eta}_E$ is a function of the drying rate, according to the diffusion equation solution. But for a given inlet
drying flux, this curve will be unique with regard to the whole dryer. These equations are solved by a finite difference technique, but if the parameters were defined analogously to equations (1) and (2), then analytical solutions could be obtained.

The most recent batch-drying study was by Schicketanz. Rigorous rate expressions are coupled to the conservation equations where the characteristic curve is not assumed to be a unique function of moisture content $\bar{X}$ but also a function of the air-temperature $T_g$ and distance $y$. In this case, the equations have to be solved by finite-difference techniques, but no solution is attempted. The drying of 60 mm deep beds of porous particles is examined to see if the average drying-rate of the whole bed can be predicted in terms of a characteristic curve measured for a thin layer of particles. As is expected from Toei's analysis, considerable differences are observed, the discrepancy in this case being approximately 16 per cent. However he shows that if the bed depth is less than nine layers of particles thick, then the drying rate of the shallow bed can be extrapolated from the known drying behaviour of a single layer of particles.

This review indicates that batch-drying theory to date has been based upon simplified conservation relationships, which assume gas plug flow. Although these equations appear to be confirmed experimentally, no analysis has been presented to indicate the expected range of validity of these models. This gap in our knowledge is considered in the next section. By starting with the conservation relationships, where dispersion effects are included, the validity of ignoring certain terms is shown by studying typical magnitudes of the parameters.

Certain aspects of real behaviour observed in batch-drying have been ignored in the literature, because of the added complexity imposed on the models. These aspects include such phenomena as variable transfer coefficients, tranverse air velocity distributions and shrinking solids. In a further part
Fig. 54. Schematic diagram of moisture transfer process between solids and the passing airstream in a well-insulated batch-dryer.
of the chapter it is shown that some of these effects can be included in the simple model, without imposing any serious constraints. In some cases the equations can still be solved analytically without recourse to finite difference techniques.

The characteristic curve concept has been used widely to model batch-drying behaviour because of its simplicity. Diffusion theory for moisture transport in solids suggests that this concept is only approximate, but this model is assumed for the rest of this chapter. The validity of the concept is scrutinised in the next chapter by considering the diffusion equation solutions.

Part 2 Constitutive Equations

We shall consider a well-insulated batch dryer (so that moisture and heat losses due to condensation on the walls are negligible) which is circulated uniformly with air and evenly filled with non-shrinking solids. (Fig. 54). Hence the dryer behaviour can be considered in terms of one dimensional changes in the direction perpendicular to the airflow direction provided the solid and gas properties are constant across the dryer.

The conservation of moisture, over an infinitesimally short zone in the dryer, between a moving Newtonian fluid with constant transport properties and a stationary solid bed in the absence of chemical reaction is given by:

$$
\frac{(1-\varepsilon) \rho_s \partial Y}{\partial y} = \frac{\varepsilon \rho_g \partial Y}{\partial y} + \frac{\varepsilon D \rho_g \partial^2 Y}{\partial y^2}
$$

A similar equation can be derived by considering the conservation of energy over an infinitesimally small zone:

$$
\frac{\rho_s (1-\varepsilon) \partial H}{\partial y} = \frac{\varepsilon \rho_g \partial H}{\partial y} + \frac{\varepsilon D \rho_g \partial^2 H}{\partial y^2}
$$
In equations (15) and (16), $\bar{D}_M$ and $\bar{D}_R$ are total mass and thermal diffusivities respectively. This equation ignores the effects due to heats of mixing and heat transfer due to the emission and absorption of radiant energy in the gas. The viscous dissipation terms have also been neglected as the velocity gradients in the direction of flow are small.

Simplification of the Energy Balance Equation

It is convenient to express equation (16) in terms of temperature changes of the airstream and solid. The specific humid enthalpies of the moist solids and the moist gas can be written as:

$$ h = (C_{PS} + C_{PL} \bar{x}) (\bar{T}_S - T_0) - \Delta H_B $$

$$ H = C_{PG} (\bar{T}_G - T_0) + Y_G [\Delta H_{VW} + C_{PV} (T_G - T_W) + C_{PL} (T_W - T_0)] $$

In equations (17) and (18), all the specific heat capacities are defined as temperature-averaged values over the range of interest, and $\Delta H_B$ is the integral enthalpy of bonding of moisture to the solid, $T_0$ is the reference basis for the enthalpies, and $\Delta H_{VW}$ is the latent-heat of vaporisation of the moisture evaluated at the wet-bulb temperature of the incoming air. By substituting eqns (17) and (18) into (16) and neglecting products of derivatives, we get:

$$ -\frac{G}{G_B} [C_{PL} (T_S - T_0) \frac{\partial \bar{x}}{\partial \theta} + (C_{PS} + C_{PL} \bar{x}) \frac{\partial \bar{T}_S}{\partial \theta} - \Delta H_B \frac{\partial \bar{x}}{\partial \theta}] $$

$$ = \frac{C_{PH} \frac{\partial \bar{T}_G}{\partial y}}{G_B} + \frac{C_{PH} C_{PH} \frac{\partial \bar{T}_G}{\partial y}}{G_B} - \frac{C_{PH} C_{PH} \frac{\partial^2 \bar{T}_G}{\partial y^2}}{G_B} + \frac{\Delta H_{VW} + C_{PV} (T_G - T_W)}{G_B} $$

$$ + \frac{C_{PL} (T_W - T_0)}{G_B} \left\{ \frac{\partial \bar{x}}{\partial y} + \frac{C_{PH} \frac{\partial \bar{x}}{\partial \theta}}{G_B} - \frac{C_{PH} C_{PH} \frac{\partial^2 \bar{T}_G}{\partial y^2}}{G_B} \right\} $$

(19)
In equation (19) \( C_{PH} \) is the humid specific heat capacity, defined as

\[
C_{PH} = C_{FG} + C_{PV} Y_G
\]  

(20)

Equation (19) can be simplified further as \( \Theta_H \) and \( \Theta_M \) are generally regarded\(^\text{39} \) as numerically equal. Together equations (15) and (19) give:

\[
\frac{\rho_S (1-c) \partial \bar{x}}{C_B \Delta H_W} \left\{ 1 + E_T \right\}
\]

\[
= C_{FG} \frac{\partial T_G}{\partial y} + \frac{C_{PH} \epsilon Q_G}{C_B} \frac{\partial T_G}{\partial \Theta} - \frac{\epsilon Q_H}{C_B} \frac{\partial^2 T_G}{\partial y^2}
\]

(21)

where

\[
E_T = \left\{ \Delta H_B + C_{PV} (T_G - T_W) - C_{PL} (\bar{T}_S - T_W) - (C_{PS} + C_{PL}) \frac{dT_S}{d\bar{x}} \right\} / \Delta H_W
\]

(22)

as \( \bar{T}_S \) will be a function of \( \bar{x} \) only. If the term \( E_T \) can be shown to be small then equation (21) will be of the same form as equation (15).

Magnitude of the parameter \( E_T \)

We now wish to consider the magnitude of the term \( E_T \), which effectively represents the inverse of the Kossovich number (Ko). It was stated in Part 3 of Chapter 3 that if the Lewis number is large (for wood \( Le \approx 100 \)) then the surface temperature will be practically identical to the average temperature of the solid. In this case we might expect that the solids temperature \( \bar{T}_S \) will vary with \( \bar{x} \) according to the characteristic drying rate curve \( f(\phi) \).
Fig. 5.5  Parameter $E_T$ (ratio of sensible heat transfer to evaporative heat transfer) for the drying of wood as a function of the characteristic moisture content.
Substituting (23) into (22):

\[
\Delta H_{VW}E_T = \Delta H_B + (T_G - T_W) \left\{ C_{PV}C_{PL}(1-f) + \left[ (C_{PS} + C_{PL}x_E) (x_{CR} - x_E) \right] \frac{df}{d\phi} \right\}
\]

The parameter \( E_T \) can now be evaluated for the example of drying wood in air.

\( i.e. \) \( C_{PV} = 1800 \text{ Jkg}^{-1}\text{K}^{-1}, \)
\( C_{PL} = 4200 \text{ Jkg}^{-1}\text{K}^{-1}, \)
\( C_{PS} = 2000 \text{ Jkg}^{-1}\text{K}^{-1} \)
\( \Delta H_{VW} = 2400 \text{ kJkg}^{-1}, \)
\( x_{CR} - x_E = 1\text{ kgkg}^{-1}, x_E = 0.1 \text{ kg kg}^{-1} \)
\( f(\phi) \approx \phi, \) and so we have

\( E_T = 0.0018 (T_G - T_W) \) for \( x > x_{CR} \) \hspace{1cm} (25a)
\( E_T = \frac{\Delta H_B}{\Delta H_{VW}} + 0.0035(T_G - T_W) \phi \) for \( x < x_{CR} \) \hspace{1cm} (25b)

This function is plotted in Fig. 55 for the most severe drying schedule presented in Table 1 (U.S. lower grade 25 mm schedule). As can be seen, \( E_T \) is small for \( \phi > 0.1 \), but for lower moisture contents this term becomes significant because of the influence of the energy of bonding. However it will still be a reasonable assumption to ignore the contribution of \( E_T \) to the energy balance. Although if the initial temperature driving force exceeds 20°C then this assumption will be questionable. In general the same result will apply to other solids than wood. This analysis demonstrates that Mykelstad's model over-
Fig. 56  Characteristic drying-rate curve concept.
emphasises the magnitude of the surface-temperature variation, particularly as he ignored the contribution of the energy of sorption.

For the case where \( E_t \) can be ignored the conservation relationships are given by:

\[
\frac{(1-e) \rho_s}{a} \frac{\partial \bar{X}}{\partial \theta} = \frac{\partial Y_G}{\partial y} + \frac{e \theta G}{a} \frac{\partial Y_G}{\partial \theta} - \frac{e \theta M G}{a} \frac{\partial^2 Y_G}{\partial y^2} \quad (26)
\]

and

\[
\frac{(1-e) \rho_s}{\Delta H_{vw} a} \frac{\partial \bar{X}}{\partial \theta} = \frac{c_{PH} \partial G}{a} \frac{\partial G}{\partial y} + \frac{c_{PH} \theta G}{a} \frac{\partial G}{\partial \theta} - \frac{\theta M G}{a} \frac{\partial^2 G}{\partial y^2} \quad (27)
\]

The Characteristic Drying Rate Curve Concept

For drying from a saturated surface, the driving forces have been shown (see Chapter 3) to take the form

\[
W_A = k_y \phi_H (Y_W - Y_G) = h_y \phi_H (T_G - T_W) = -\frac{\rho_g (1-e) \partial \bar{X}}{\Delta H_{vw}} \quad (28)
\]

Many drying operations are performed either under constant wet-bulb conditions or with stepwise changes in the wet-bulb temperature, as in timber seasoning. Under these conditions, \( (k_y \phi_H) \) can be regarded as a constant parameter over the dryer and the flux to be directly proportional to the humidity potential. When the solids are no longer saturated with moisture, we invoke the characteristic curve concept

\[
\log \left[ \frac{\bar{X} - \bar{X}_{CR}}{\bar{X}_{CR} - \bar{X}_E} \right] \quad (29)
\]

(see Fig. 56) to factor the maximum drying flux to estimate the diminished flux. This concept is closely related to the surface driving-force relationship examined in Part 3 of Chapter 3. The important property assumed of the characteristic curve is that the curve will hold for any set of drying
conditions irrespective of the magnitude of the driving force. If the following criteria are satisfied, then this property will hold:

(a) the critical moisture content $\bar{X}_{CR}$ is independent of external conditions (e.g., humidity and gas-rate) and is unaffected by the initial moisture content of the sample;

(b) all drying curves for a specific substance are geometrically similar, that is, the shape of the characteristic curve is independent of the humidity driving force;

(c) the equilibrium-moisture content is independent of external conditions.

None of these criteria can ever hold strictly, but for interpolation purposes the variation of these quantities with position in the dryer (particularly when the characteristic curve has been carefully determined in terms of the changes that occur at the dryer inlet) are assumed to be second order effects.

It has further been shown in Chapter 3 that the surface drying rate curves defined in terms of both the humidity and temperature driving forces are practically coincidental. This equivalence is also assumed to hold for the characteristic drying-rate curve.

By coupling the rate and conservation expressions, assuming constant heat and mass transfer coefficients throughout the dryer, the behaviour of the batch-dryer is fully described by the following equations:

\[
\begin{align*}
\frac{\partial Y}{\partial \theta} - \frac{\bar{G} Y}{\bar{G} B} &= \frac{\partial Y}{\partial \theta} - \frac{\bar{G} Y}{\bar{G} B} + \frac{\partial Y}{\partial \theta} - \frac{\bar{G} Y}{\bar{G} B} \frac{\partial^2 Y}{\partial Y^2} \\
&= \frac{\partial Y}{\partial \theta} \left( \frac{Y - Y_G}{Y - Y_E} \right) \cdot f\left( \frac{\bar{X} - \bar{X}_E}{\bar{X}_{CR} - \bar{X}_E} \right) 
\end{align*}
\]

(29)
The Dimensionless Batch-Drying Equation

It is convenient to simplify the above equations by defining the following dimensionless parameters:

\[ \pi_M = \frac{Y_W - Y_G}{Y_W - Y_G^\circ} \]
\[ \pi_H = \frac{T_G - T_W}{T_G^\circ - T_W} \]
\[ \phi = \frac{X - X_E}{X_CR - X_E} \]
\[ z = \frac{y}{L} \]
\[ \tau_M = \frac{k_y a \phi_M (Y_W - Y_G^\circ) \Theta}{(X_CR - X_E) \phi_S (1 - \varepsilon)} \]
\[ \tau_H = \frac{h_c a \phi_H (T_G^\circ - T_W) \Theta}{(X_CR - X_E) \Delta H_{VW} \phi_S (1 - \varepsilon)} \]
\[ \lambda_M = \frac{k_y \beta_M a L}{G B} \]
\[ \lambda_H = \frac{h_c a \beta_H L}{C_{PH} G B} \]
\[ P_e_M = \frac{G_B L}{\varepsilon \beta_M \rho_G} \]
\[ P_e_H = \frac{G_B L}{\varepsilon \beta_H \rho_G} \]

where \( L \) is the length of the dryer, and \( Y_G^\circ \) and \( T_G^\circ \) are the gas humidity and gas temperature respectively evaluated at the dryer inlet. These parameters are defined assuming that the inlet air conditions remain constant. The \( \pi \) functions can be
regarded as transfer potentials while the \( \lambda \) functions represent the corresponding NTU. There are also the corresponding time constants \( \tau \), which represent the extent of drying for unit loading of the dryer and the Peclet number determines the proportion of heat and mass transfer due to the air bulk flow compared to diffusive transfer (i.e. dispersion).

By substituting the dimensionless parameters into equations (29) and (30), we find both equations reduce to the form:

\[
\frac{\partial \Phi}{\partial \tau} = \frac{1}{\lambda} \frac{\partial \pi}{\partial \tau} + k \frac{\partial \pi}{\partial \tau} - \frac{1}{\psi} = \pi f(\Phi)
\]

where the coefficient \( k \) is the air to bound moisture capacity ratio for unit mass flows:

\[
k_M = \frac{\varepsilon (y_w - y_G^o)}{\rho_s (1-\varepsilon) (\bar{x}_{CR} - \bar{x}_E)}\]
\[
k_H = \frac{\varepsilon G_c (T_G - T_w)}{\rho_s (1-\varepsilon) (\bar{x}_{CR} - \bar{x}_E) \Delta H_{vw}}
\]

Equation (37) is more general than that derived by Van Meel\(^3\) as this equation is completely dimensionless. It is also based upon more rigorous driving force expressions, which account for the coupled transfer of heat and mass. This equation will be unique when the psychrometric ratio is numerically equal to 1 as the heat and mass transfer dimensionless parameters will all be numerically equivalent. These equalities arise from the wet-bulb thermometer equation and the adiabatic saturation equation in the limit of the wet-bulb temperature being identically equal to the adiabatic saturation temperature. Under these conditions the heat and mass transfer potentials will vary in the same manner. For the air-water system the difference between the wet-bulb and adiabatic saturation temperatures is small as seen in Chapter 3, so this unique relationship may be assumed. For other systems, although the mass transfer and heat transfer phenomena appear to be
associated but uncoupled, it will be necessary to consider the implicit coupling due to the adiabatic equilibrium. In this case the temperature of the drying solids would no longer be equal to the wet-bulb temperature everywhere in the dryer for constant rate drying. Instead the solids temperature will vary across the dryer in relation to the change in potential.

The adoption of equation (37) to describe the batch-drying dynamics also implies constant physical properties for the fluid and solid. The humid specific heat capacity $C_{ph}$ and the gas density $\rho_g$ are regarded as uniform over the dryer. As the temperature falls along the dryer and the humidity rises, these physical properties will vary to some extent. The cumulative error in assuming these parameters to be constant depends upon the extensiveness of the dryer (i.e. NTU), and the stage of the drying. For example, the variation of these physical properties will be greatest during the constant rate period, as this is when the largest changes in the $\kappa$ potentials are observed across the dryer. As the specific heat capacities vary little with temperature and the humidity difference across the dryer will rarely exceed 0.03 (see Fig. 29) the change in this property can be ignored. For timber drying the maximum $\Delta \kappa$ observed will be less than 0.01. The variation of the density, however, is not so small. Although the temperature and humidity changes are in opposite senses, the temperature effect dominates. At high temperatures, this variation can be ignored provided the temperature change is not greater than 10°C. In general, it will be most accurate to estimate these parameters at the inlet conditions. Since for a greater part of the drying the variation of air-conditions over the bed is small, particularly if the material is to be dried to a moisture content close to the equilibrium moisture content.

Magnitude of the Capacity Coefficient $k$

Equation (37) can be simplified further by considering the magnitude of the capacity coefficient $k$.

* e.g. for drying 50 mm thick Pinus radiata boards, spaced 25 mm apart at $T_g = 60°C$, $T_w = 50°C$ then:
The significance of the dispersion term in equation (39) is best considered by studying the solution of this equation for the constant rate period of drying \((f(\phi) = 1)\). This is the period of drying where these effects will be most dominant. Thygeson and Grossmann present a solution to these equations, which for the boundary conditions of constant initial moisture content \(\Phi_0\) and constant inlet humidity potential (i.e. \(\pi(t, \infty) = 1\)) results in the expressions,

\[
\pi = \exp\left[1 - (1 + \frac{\Phi_0}{\pi})^2\right] \frac{PeZ}{2} \tag{40}
\]

and

\[
\Phi = \Phi_0 - \pi \tau
\]

Equation (40) shows that the potential is a simple exponential function of distance. The parameters \(\lambda\) and \(Pe\) determine the shape of the curve. Examples of the potential profile are
Fig. 57. Humidity potential profiles across a dryer showing the significance of dispersion for dryers of various length.
presented in Fig. 57. The dispersion has the effect of reducing the extensiveness of the dryer, compared to the plug-flow solution.

If the Pe number is large, then the term \( \frac{4\lambda}{Pe} \) will be small, so that

\[
\left( 1 + \frac{4\lambda}{Pe} \right)^2 \approx 1 + \frac{2\lambda}{Pe}
\]

and equation (40) will simplify to:

\[
\kappa = \exp(-\lambda Z)
\]

which is the solution for the plug-flow case, where the dispersion effects are negligible. The equality in equation (42) will hold provided that \( \frac{4\lambda}{Pe} \ll 0.25 \).

The magnitude of the term \( \frac{4\lambda}{Pe} \) within dryers depends largely on the arrangement of the solid in the dryer. For the case where the material is stacked, such as for the drying of bricks and timber, or for tray driers, Sherwood and Woertz's correlation for the dispersion coefficient in turbulent flow through ducts should closely simulate the situation.

\[
i.e. \quad 10^{4}\mu_{g}D_{g} = 2.7 \times 10^{-4} \quad Re_{D} + 0.8
\]

At \( Re_{D} = 100,000 \), \( \mu_{g} \approx 4 \times 10^{-3} \) m²/s for air-water vapour flow. For the worst possible case of an air-velocity of 1 m/s and a dryer of length 1 m, this result suggests \( Pe = 500 \). So in this case, the inequality will be satisfied.

The Peclet number will be smaller for the case of drying a granular bed of solids. De Maria and White summarised measurements on dispersion in gas flows through beds of particles and found that the Peclet number based on the particle diameter \( D_{P} \) was numerically equal to 2 and independent of Reynolds number.

\[
i.e. \quad Pe = \frac{2D}{D_{P}}
\]
So, provided the bed is longer than 100 particle diameters, the dispersion inequality will hold over the range of NTU expected in batch dryers. In summary, dispersion effects can be ignored in all industrial dryers using air as the drying medium.

By defining

\[ \zeta_M = \lambda_M z = k \frac{\delta_M a y}{G_B}, \quad \zeta_H = \lambda_H z = h \frac{\delta_H a y}{C_B} \tag{46} \]

then equation (39) simplifies to the final equations:

\[ \frac{\partial \Phi}{\partial t} + \frac{\partial \pi}{\partial \zeta} = -\pi f(\Phi) \tag{47} \]

Hence it has been shown that the equations of heat and mass transfer for batch-drying reduce to the same expression for mass transfer in batch-drying as proposed by Van Meel\(^3\), when the adiabatic saturation and wet-bulb temperatures are equal. Van Meel presents a neat solution to equation (47) and the elements of this method will be outlined here because of the powerful relationships that are developed from them. The elimination of \( \pi \) from equation (47) leads to an equation in \( \Phi \):

\[ \frac{\partial^2 \Phi}{\partial \zeta \partial t} - \frac{1}{f(\Phi)} \cdot \frac{df(\Phi)}{d\Phi} \frac{\partial \Phi}{\partial \zeta} + f(\Phi) \frac{\partial \Phi}{\partial t} = 0 \tag{48} \]

This second-order equation may be integrated to yield

\[ \Phi = \frac{1}{f(\Phi) + P(\Phi)} \zeta \tag{49} \]

where \( P(\zeta) \) is an arbitrary function, dependent upon the initial conditions.

Van Meel\(^3\) developed the solution taking into account possible recirculation of the humid air from the exhaust.
Fig. 58 Exhaust recycle loop for humid air in a batch dryer.
Such recirculation is common in cross-circulated dryers for reasons of thermal economy. However, to maintain a suitable drying environment with fresh-air makeup, often very high recirculation ratios (> 0.9) are necessary. Poor control of the dryer may result unless the inlet humidity potential of the fresh-air admitted is decreased by spraying with water. We shall assume that the inlet air, the result of mixing the recycled air with some fresh less humid air, has a constant humidity potential \((Y_w - Y_g^0)\) throughout the drying period. The assumed arrangement is shown in Fig. 58. The analysis will hold for either cross-circulation or through circulation; in both cases one considers the variation in process conditions in the direction of the airflow through the material charged to the dryer.

**Calculation of the Drying Process**

1st period \(\bar{X} > \bar{X}_{UR}\)

In general the initial moisture distribution will be uniform, but after the period of heating up the material to the temperature at which drying can begin, a non-uniform distribution may occur. This non-uniformity may arise because of condensation of water-vapour on the wet material at the moment drying really starts, or because of uneven heating of the material to be dried. A more general moisture-content distribution may be assumed such as

\[
\Phi(\zeta, 0) = \Phi_o + \Phi_D e^{-\zeta}
\]

(50)

in which \(\Phi_D\) can be positive, negative or zero as long as \(\Phi(\zeta, 0) > 1\) for \(0 < \zeta < \lambda\), where \(\lambda\) is the total NTU in the dryer. If the warm-up period is negligible then \(\Phi_D\) will be zero. However, for significant warming-up times, where this phase is carefully controlled so that condensation does not occur, \(\Phi_D\) will be negative as drying will occur. The drying rate during this period will increase with time to the desired rate as the surface reaches the dynamic steady-state temperature.
(i.e. the wet-bulb temperature of the air). However if the air stream is heated too rapidly in relation to the solid, then the condition can be obtained where the gas humidity exceeds the saturation humidity at the solid surface and condensation occurs. If condensation predominates evaporation during this period then \( \Phi_D \) will be positive. Essentially the first drying period is said to have begun, when the solids have reached the dynamic wet-bulb temperature imposed by the air-stream conditions.

During the first period \( f(\Phi) = 1 \), and hence equations (47) and (49) have the form:

\[
\frac{\partial \Phi}{\partial \zeta} + \Phi = \Phi_0
\]

(51)

and

\[
\frac{\partial \Phi}{\partial \zeta} = -\pi
\]

(52)

The initial boundary conditions are:

1. \( \pi = \text{cst} = 1 \) at \( \zeta = 0 \)
2. \( \Phi = \Phi_0 + \Phi_D e^{-\zeta} \) at \( \tau = 0 \)

Integration of equations (51) and (52) then yields:

\[
\pi(\zeta, \tau) = e^{-\zeta}
\]

(53)

and

\[
\Phi(\zeta, \tau) = \Phi_0 + \Phi_D e^{-\zeta - e^{-\zeta} \tau}
\]

(54)

Since in this period \( \pi \) is independent of \( \tau \), \( \Phi \) decreases linearly with \( \tau \) for a given value of \( \zeta \). The first period is fully described by equations (53) and (54).

The second period starts at \( \tau = \tau_1 \), when \( \Phi(0, \tau_1) = 1 \).
and, from eqn. (54)

\[ r_1 = \Phi_o + \Phi_D - 1 \]  \hspace{1cm} (55)

The distribution of moisture content \( \Phi \) at that moment is:

\[ \Phi(\zeta, r_1) = \Phi_o - (\Phi_o - 1)e^{-\zeta} \]  \hspace{1cm} (56)

which is independent of \( \Phi_D \).

**Second Period** - (when moisture contents both higher and lower than \( \bar{X}_{GR} \) are present within the dryer).

Substitution of the initial distribution for this period (eqn. (56)) into the general expression (eqn. (49)) yields

\[ \frac{1}{f(\Phi)} \frac{\partial \Phi}{\partial \zeta} + \Phi = \Phi_o \]  \hspace{1cm} (57)

Integration of (57) gives

\[ \int_{\Phi}^{1} \frac{ds}{(\Phi_o - s)f(s)} = -\zeta + N(r) \]  \hspace{1cm} (58)

If the critical moisture content \( \Phi = 1 \) is situated at \( \zeta_{GR} \), then from eqn. (58)

\[ \zeta_{GR} = N(r) \]

and hence

\[ \int_{\Phi}^{1} \frac{ds}{(\Phi_o - s)f(s)} = \zeta_{GR} - \zeta \]  \hspace{1cm} (59)

Thus \( \Phi \) depends on \( (\zeta_{GR} - \zeta) \).
The distribution for $\zeta < \zeta_{CR}$ is given by:

$$\int_0^\zeta \frac{d\Phi}{d\tau} = \zeta - \zeta_{CR}$$

(60)

and hence

$$\Phi = \Phi_o - (\Phi_o - 1) e^{\zeta_{CR} - \zeta}$$

(61)

(which agrees with eqn. (56) when $\Phi = 1$ at $\zeta = 0$ for $\zeta_{CR} = 0$).

From equation (47) and on differentiating eqn. (59), we find

$$\pi = \frac{\Phi_o - \Phi}{f(\Phi) \partial \tau}$$

(62)

where $d\zeta_{CR} / d\tau$ is the dimensionless velocity of the critical moisture content through the dryer. This velocity can be calculated from the inlet potential $\Phi(0, \tau)$ from eqn. (62):

$$\frac{d\zeta_{CR}}{d\tau} = \frac{1}{\Phi_o - \Phi(0, \tau)}$$

(63)

where $\Phi(0, \tau)$ can be found as a function of $\zeta_{CR}$ from eqn. (59).

Equations (62) and (63) can be combined to give:

$$\pi = \frac{\Phi_o - \Phi(\zeta, \tau)}{\Phi_o - \Phi(0, \tau)}$$

(64)

The position of the critical point $\zeta_{CR}$ as a function of $\tau$, can be obtained by integration of equation (63).
Third Period ($\bar{Y} < \bar{Y}_{CR}$)

If for $\tau = \tau_2$, $\bar{Y}_{CR}$ has reached the value $\lambda$ (the total NTU in the dryer), the third period begins. Calculations for this period are similar to those for the second period, if $\bar{Y}_{CR}$ is permitted to take values greater than $\lambda$. No longer is there any physical meaning attached to the magnitude of $\bar{Y}_{CR}$.

Part 3 Methods of Solution of the Batch-Drying Equations

The equations developed in Part 2 can be solved when the nature of the function $f(\phi)$ is known. The required method of solution will be dependent upon the shape of this curve. Several new methods of solving these equations are presented below by use of the techniques of analytical integration, finite differences, quasilinearisation and analogue computation. These methods offer the means of being able to solve the whole spectrum of batch-drying problems. In addition, it is shown that in the limiting case of extensive dryers simplified but accurate solutions can be obtained.

(a) Graphical Solution

Van Meel\(^3\) presents a graphical method of solution to this set of equations. By graphical integration of equation (59), the moisture content $\bar{\phi}$ can be obtained as a function of ($\bar{Y}_{CR} - \bar{Y}$). These results together with equation (63) allow calculation of $d\tau/d\bar{Y}_{CR}$, which when integrated with respect to $\bar{Y}_{CR}$ yield the time $\tau$. At low moisture contents the term $1/f(\bar{\phi} - \bar{\phi}_0)$ becomes very large so for the integration of equation (59) small errors can aggregate. Further to the two graphical integrations, Van Meel's method also involves tedious and extensive graphical manipulations, so there is an incentive to find alternative procedures. Some of the later solutions proposed are also more accurate.
Direct Analytical Solutions

To obtain an analytical solution to this problem, one necessary condition is the ability to integrate equation (59). As the characteristic curve is generally an experimentally determined function, there is the other requirement that the experimental measurements can be fitted to a simple analytical function. However, for the range of simple functions of \( f(\Phi) \) for which equation (59) is integrable, given by \( \Phi^{1/n} \) or \( \Phi^n \) where \( n \) is a positive integer, the solution is an implicit function of \( \Phi \) involving trigonometrical expressions, except, for two special cases. The first of these is the linear case \( f(\Phi) = \Phi \) which has been solved by Van Meel. This solution for the second and third drying periods is presented below together with the solution for the other special case \( f(\Phi) = \Phi^{1/2} \). The drying of some hygroscopic materials has been observed to approximate to these curves.

1. For \( f(\Phi) = \Phi \)

The instantaneous local moisture content is evaluated as

\[
\Phi(\zeta, \tau) = \frac{\Phi_0}{(\Phi_0 - 1) \Phi_0 (\zeta_{CR} - \zeta)} + 1 \quad 1 < \Phi < 0
\]  

(65)

and

\[
\Phi(\zeta, \tau) = \Phi_0 - (\Phi_0 - 1) \zeta_{CR} - \zeta \quad \text{for } \Phi \geq 1
\]  

(66)

The velocity of the critical point is given by

\[
\frac{d\zeta_{CR}}{d\tau} = \frac{(\Phi_0 - 1) + e^{-\Phi_0 \zeta_{CR}}}{(\Phi_0 - 1) \Phi_0}
\]  

(67)

The potential profile becomes:

\[
\pi = \Phi_0 - 1 + e^{-\Phi_0 \zeta_{CR}} \quad 1 < \Phi < 0
\]  

(68)
Fig. 59 to 62. Batch-drying curves for solids with initial moisture content $\Phi_0 = 2$ and linear characteristic drying-rate curve $f(\Phi) = \Phi$ at various positions along a dryer of length $\lambda = 10$. 
and 
\[ Y = \frac{e^{\frac{\phi}{\phi_0}}}{\phi_0} \left[ (\phi - 1)e^{\phi_0 CR} + 1 \right]^{1/\phi} \geq 1 \] \tag{69}

The extent of drying for the second period is

\[ \tau - \tau_1 = \ln \left\{ \frac{(\phi - 1)e^{\phi_0 CR} + 1}{\phi_0} \right\} \tag{70} \]

where \( \tau_1 = \) length of the constant rate period (eqn. 55), and the extent of drying for the third period is:

\[ \tau - \tau_2 = \ln \left\{ \frac{(\phi - 1)e^{\phi_0 CR} + 1}{\phi_0} \right\} \tag{71} \]

where \( \tau_2 = \) time elapsed to the end of the second drying period:

\[ \tau_2 = \ln \left\{ \frac{(\phi - 1)e^{\phi_0 \lambda} + 1}{\phi_0} \right\} + \tau_1 \tag{72} \]

The total drying time \( T_D \) required to obtain a value \( \phi_{END} \leq 1 \) at the outlet \( \phi = \lambda \) for drying under constant inlet conditions is:

\[ T_D = \frac{\phi_0}{\phi_{END}} + \phi_0 - 1 + \ln \left\{ \frac{(\frac{\phi}{\phi_0} - 1)e^{\phi_0 \lambda} + 1}{\phi} \right\} \tag{73} \]

These equations are illustrated in Figs. 59-62 for the case of \( \phi_0 = 2 \) for dryers of various lengths up to \( \lambda = 10 \). The interesting feature of the graphs is the asymptotic nature of the curves in the limit when the dryer is long.
In this case the solution is more complex, particularly as the moisture content diminishes to zero within a finite time instead of an infinite time as in the linear case. The reason for this unusual behaviour is not known. The instantaneous local moisture content is given by:

\[
\Phi(z, \tau) = \Phi_0 - \left(\frac{\Phi_0}{z_{CR}}\right) \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\] (74)

and

\[
\Phi(z, \tau) = \Phi_0 \left[1 - \frac{\frac{\Phi_0}{z_{CR}} (z_{CR} - z)}{\frac{\Phi_0}{z_{CR}} (z_{CR} - z) + 1 + e^{-\Phi_0 z_{CR}}}\right]^2, \quad 1 < \Phi \leq 0
\]

\[
z > \frac{z_{CR} - \ln(1)}{\Phi_0}
\] (75)

where

\[
l = \frac{\frac{\Phi_0}{z_{CR}} + 1}{\frac{\Phi_0}{z_{CR}} - 1}
\] (75a)

and the other case

\[
\Phi(z, \tau) = 0 \quad \zeta < z_{CR} - \frac{\ln(1)}{\Phi_0}
\] (76)

The velocity of the critical point is

\[
\frac{dz_{CR}}{d\tau} = \frac{(1 + e^{-\Phi_0 z_{CR}})^2}{4l \Phi_0 z_{CR}}, \quad z_{CR} < 1 \frac{\ln(1)}{\Phi_0}
\] (77)

and

\[
\frac{dz_{CR}}{d\tau} = \frac{1}{\Phi_0}, \quad z_{CR} \geq 1 \frac{\ln(1)}{\Phi_0}
\] (78)

Thence the humidity potential profiles are:

\[
\kappa = \left(\frac{\Phi_0}{z_{CR}} - \zeta\right) \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\]

\[
\kappa = \frac{\Phi_0}{z_{CR}} \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\]

\[
\kappa = \frac{\Phi_0}{z_{CR}} \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\]

\[
\kappa = \frac{\Phi_0}{z_{CR}} \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\]

\[
\kappa = \frac{\Phi_0}{z_{CR}} \left(1 + e^{-\Phi_0 z_{CR}}\right), \quad \Phi > 1
\]
and 
\[ \kappa = \frac{\sqrt{\Phi_o} \left( 1 + e^{\Phi o \zeta_{CR}} \right)^2}{\left[ 1 + e^{\Phi o \zeta_{CR}} \right]^2}, \quad 1 < \Phi < 0 \] and 
\[ \zeta_{CR} < 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(80)

and finally
\[ \kappa = \frac{41 o \left( \zeta_{CR} - \zeta \right)}{\sqrt{\Phi_o} \left( \zeta_{CR} - \zeta \right)} \], \quad 1 < \Phi < 0 \] and 
\[ \zeta_{CR} \gg 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(81)

The extent of drying for the second period is
\[ \tau - \tau_1 = 2 \frac{(\sqrt{\Phi_o} + 1) (e^{\Phi o \zeta_{CR}} - 1)}{e^{\Phi o \zeta_{CR} + 1}} \], \quad \zeta_{CR} < 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(82)

\[ \tau - \tau_1 = 2 + \frac{\Phi_o (\zeta_{CR} - \frac{\ln(1)}{\sqrt{\Phi_o}})}{\sqrt{\Phi_o}} \], \quad \zeta_{CR} \gg 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(83)

and for the third period:
\[ \tau - \tau_2 = \frac{4\sqrt{\Phi_o} \zeta_{CR} \sqrt{\Phi} \lambda}{\sqrt{\Phi_o} \zeta_{CR}(1 + e^{\Phi o \zeta_{CR}})}, \quad \zeta_{CR} < 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(84)

\[ \tau - \tau_2 = 2 + \frac{\Phi o \zeta_{CR} - \sqrt{\Phi_o} \ln(1) - 2(\sqrt{\Phi_o} + 1)(e^{\Phi o \zeta_{CR} + 1})}{e^{\Phi o \zeta_{CR} + 1}} \], \quad \zeta_{CR} \gg 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(85)

\[ \lambda < 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(86)

\[ \zeta_{CR} \gg 1 \frac{\ln(1)}{\sqrt{\Phi_o}} \]  
(87)
Fig. 63 to 66  Batch-drying curves for solids with initial moisture content $\bar{\theta}_0 = 2$ and characteristic drying-rate curve $f(\bar{\theta}) = \sqrt{\bar{\theta}}$ at various positions along a dryer of length $\lambda = 10$. 
and
\[ \nu = \nu_2 = \phi_o (\gamma_{CR} - \lambda), \quad \lambda > \frac{1}{N\phi_o} \ln(1) \] (86)

The total drying time required to reach the value \( \phi_{END} \) at the outlet is:

\[ T_D = \phi_o + \phi_D - 1 + 2(\sqrt{\phi_o} - 1) \left[ \frac{1}{N\phi_o} \ln \left( \frac{\sqrt{\phi_o} \lambda}{\sqrt{\phi_{END}} - \sqrt{\phi_o}} \right) \right] \]

\[ \lambda < \frac{1}{N\phi_o} \ln \left( \frac{\sqrt{\phi_o} + \sqrt{\phi_{END}}}{\sqrt{\phi_o} - \sqrt{\phi_{END}}} \right) \quad (87) \]

and

\[ T_D = \phi_o (1 + \lambda) + \phi_D + 1 + \sqrt{\phi_o} \ln \left( \frac{\sqrt{\phi_o} - \sqrt{\phi_{END}}}{\sqrt{\phi_o} + \sqrt{\phi_{END}}} \right) \]

\[ \lambda \geq \frac{1}{N\phi_o} \ln \left( \frac{\sqrt{\phi_o} + \sqrt{\phi_{END}}}{\sqrt{\phi_o} - \sqrt{\phi_{END}}} \right) \quad (88) \]

The solutions illustrated by the above equations are displayed graphically in Figs. 63-66 for the case of \( \phi_o = 2 \) and for dryers of various lengths up to \( \lambda = 10 \).

(c) Quasilinearisation

This technique is useful for handling characteristic curves of complex shape. Such curves can often be broken up into straight-line segments as shown in Fig. 67. In many cases, three straight lines will be sufficient to fit the whole falling-rate region, but the technique can be applied to any number of segments.

The curve \( f(\phi) \) can be expressed in terms of the \( N \)
Fig. 67 Quasi-linearised characteristic drying-rate curve.
breakpoints \((f_M, \Phi_M)\):

For \(K = 1\)
\[
f = \frac{(1-f_2) \Phi + f_2 \Phi_2}{(1-\Phi_2)}
\]

for \(K = M\) \((1 < M < N)\)
\[
f = \frac{(f_{N-M+1} - f_{M+1}) \Phi + \Phi_{M+1} f_{M+1} f_M}{(\Phi_{M+1})}
\]

for \(k = N\)
\[
f = \frac{f_N}{\Phi} \frac{\Phi_{M+1}}{N}
\]

Equations (57-64) can be expressed in terms of the relative drying rate \(f(\Phi)\) rather than the moisture content \(\Phi\). By putting

\[
f = a_M + b_M \Phi
\]

we can transform eqn. (57) to

\[
\frac{1}{f} \frac{\partial f}{\partial \zeta} + f = f_0
\]

where

\[
f_0 = a_M + b_M \Phi_0
\]

The potential \(\pi\) is given by

\[
\pi = \frac{-1}{b_M f} \frac{\partial f}{\partial \tau} = \frac{(f_0 - f)}{b_M} \frac{\partial \zeta_{CR,M}}{d \tau}
\]

where \(\zeta_{CR,M}\) is the position of the \(M\)th breakpoint in the dryer, and the velocity of this breakpoint will be:

\[
\frac{d \zeta_{CR,M}}{d \tau} = \frac{b_M}{f_0 - f(\phi, \tau)}
\]
The potential profile then becomes

\[ \pi = \frac{f - f_0}{f_0 - f(o, \tau)} \quad (96) \]

The integrated equations will take a similar form of solution to the linear case when \( f(\Phi) = \Phi \) except here \( \Phi \) is replaced by \( f \) everywhere.

Equation (93) integrates to give:

\[ \int_{f}^{f_M} \frac{ds}{(f_0 - s) s} = \frac{\xi - \xi}{CR_M} \quad (97) \]

where \( f_M > f \geq f_{M+1} \), and hence

\[ f = \frac{f_0}{(f_0/f_M - 1) \exp f_0 (\xi_{CR_M} - \xi) + 1} \quad (98) \]

For the first segment \( f_M = 1 \), and thus

\[ f = \frac{f_0}{(f_0 - 1) \exp f_0 (\xi_{CR_1} - \xi) + 1} \quad (99) \]

which is identical to equation (65), except that the moisture content \( \Phi \) is replaced everywhere by \( f \).

The distribution for \( f \) for \( \xi > \xi_{CR_1} \) is given by an equation similar to (66):

\[ f = f_0 - (f_0 - 1) \exp \left( \xi_{CR_1} - \xi \right) \quad (100) \]
If the moisture content $\Phi$ at the entrance of the dryer is in the same segment of the $f$ curve (say the $M$th segment) as that at the position $\zeta$ under consideration, then the velocity of the break-point and the humidity potential profile can be described in terms of the position of the $M$th break-point $\zeta_{CR_M}$.

From equation (95) the velocity of the break-point becomes

$$
\frac{d\zeta_{CR_M}}{d\zeta} = \frac{b_M}{f_M} \left\{ \frac{f_0}{f_M} - 1 + e^{-f_0 \zeta_{CR_M}} \right\}
$$

(101)

and the potential profile is given by eqn. (96):

$$
\pi = \frac{f_0/f_M - 1 + e^{-f_0 \zeta_{CR_M}}}{f_0/f_M - 1 + e^{-f_0 (\zeta_{CR_M} - \zeta)}}
$$

(102)

When the moisture content $\Phi$ at the point under consideration is in the $M$th segment of the characteristic curve, and the moisture content at the entrance of the dryer is a different segment of the curve (say the $K$th segment), then the potential profile and the break-point velocity must be described in terms of both the positions of the $M$th and $K$th break-points. Hence in this case, the break-point velocity is:

$$
\frac{d\zeta_{CR_M}}{d\zeta} = \frac{b_K}{f_K} \left\{ \frac{f_0}{f_K} - 1 + e^{-f_0 \zeta_{CR_K}} \right\}
$$

(103)

where $K > M$, and the corresponding potential profile is given by

$$
\pi = \frac{b_K \left( f_0/f_K - 1 + e^{-f_0 \zeta_{CR_K}} \right)}{b_M \left( f_0/f_M - 1 + e^{-f_0 (\zeta_{CR_M} - \zeta)} \right)}
$$

(104)
Thus it follows by comparing the velocity of the $M^{th}$ and $K^{th}$ break-points that

$$\frac{d\zeta_{CR_M}}{d\tau} = \frac{d\zeta_{CR_K}}{d\tau} = \cdots = \frac{d\zeta_{CR_1}}{d\tau}$$

(105)

and the break-points move through the dryer with uniform velocity.

From equation (105) the relationship between $\zeta_{CR_M}$ and $\zeta_{CR_K}$ can be obtained so eqns. (103) and (104) can be evaluated.

On integrating eqn. (105) we find

$$\zeta_{CR_M} = \zeta_{CR_K} + C_{M,K}$$

(106)

where $C_{M,K}$ is a constant. In general

$$C_{M,K} = \zeta_{CR_M} - \zeta_{CR_K} = \Sigma_{t=M}^{K-1} C_t$$

(107)

and so $C_{M,K}$ takes the value of the position of the $M^{th}$ break-point when the $K^{th}$ break-point reaches the inlet of the dryer, or the end of the ($K-1)^{th}$ segment falls at the inlet.

The individual values of $C_t$ can be obtained by putting $f = f_{M+1}$ at the inlet $(\zeta = 0)$ in eqn. (98), and solving to find the position of the $M^{th}$ break-point at that instant. Thus,

$$C_M = \zeta_{CR_M} \quad \text{(for } f = f_{M+1} \text{ at } \zeta = 0)$$

$$= \frac{1}{f} \ln \left[ \frac{f_{M+1}}{f_M} - 1 \right]$$

(108)

Drying times can be estimated by summing the times required to reach each of these $C$ values. That is, the
Fig. 68  Two and three segment quasi-linear approximations to characteristic drying-rate curve $f(\phi) = \sqrt{\phi}$. 
sum of the time periods for which the moisture content \( \bar{\phi} \) is in each segment of the \( f \) curve at the inlet of the dryer.

Hence we define the period \( \tau_M \) by:

\[
\tau_M = \tau \text{ (when } f = f_{M+1} \text{ at } \zeta = 0 \text{)} - \tau \text{ (when } f = f_M \text{ at } \zeta = 0 \text{)}
\]

Let \( \tau \) time elapsed since \( f = f_M \) at the inlet of the dryer, then from equation (101)

\[
\tau = \frac{1}{b_M} \ln \left[ \frac{f_0/f_M - 1}{b_M f_0/f_M^{CR} + 1} \right] \quad (109)
\]

Since \( \tau = \tau_M \) when \( \zeta_{CR_M} = C_M \), we get from equation (90):

\[
\tau_M = \frac{1}{b_M} \ln \left[ \frac{f_M}{f_{M+1}} \right] \quad (110)
\]

Once \( \zeta_{CR_M} \) reaches \( \lambda \), the end of the dryer, we are no longer interested in the \( M \)th segment of the curve, unless \( M = N \). In this latter case, the drying problem can be treated identically to the case of the third period in the analysis of the general problem.

The total drying time \( T_D \) to reach a value \( \bar{\phi}_{END} \) (\( \bar{\phi}_{END} < \bar{\phi}_N \)) at the outlet of the dryer (\( \zeta = \lambda \)) for constant external conditions, is:

\[
T_D = \bar{\phi}_o + \bar{\phi}_D - 1 + \sum_{M=1}^{N-1} \frac{1}{b_M} \ln \left[ \frac{f_M}{f_{M+1}} \right] + 1 \ln \left[ \frac{f_0/f_{END} - 1}{f_0/f_{END} + 1} \right] \quad (111)
\]

A computer program "QUASI" (see Appendix 2D) was
Fig. 69 to 72. Comparison of analytical solution for batch-drying curves for solids with initial moisture content $\theta_0 = 2$ and characteristic drying-rate curve $f(\theta) = \sqrt{\theta}$ (Fig. 63 to 66), with two-segment quasi-linear approximation method (computer program "QUASI").
Fig. 73 to 76  Comparison of analytical solution for batch-drying curves for solids with initial moisture content \( \theta_0 \approx 2 \) and characteristic drying-rate curve \( f(\theta) = \sqrt{\theta} \) (Fig. 63 to 66) with three segment quasilinear approximation method (computer program "QUASI").
written to solve the batch-drying equations using this quasilinearisation technique. The effectiveness of the method is illustrated in Figs. (69-76) where the exact analytical solution is compared to solutions generated using two and three segment straight-line approximations for the characteristic curve \( f(\Phi) = \Phi^{\frac{1}{2}} \), (see Fig. 68). The dotted lines indicate the approximate solution. The drying rate curves provide the toughest test of the method, and in the case of the three-segment approximation the two solutions match very closely, for the parameter values of \( \Phi_0 = 2 \) and \( \lambda = 2 \).

These results then suggest that the quasilinear method provides a neat way of solving the problem when \( f(\Phi) = \Phi^{\frac{1}{2}} \) compared to the complex set of equations (eqns. 74-88) which comprise the exact solution.

(d) Analogue Computer Methods

If a set of partial differential equations is expressible in terms of total differentials, then an analogue computation can provide solutions of satisfactory accuracy.

Equation (47) may be solved numerically using the method of characteristics as shown by Tetzlaff with the equations defining the total differentials \( d\Phi \) and \( d\Phi \):

\[
d\Phi = \frac{\partial \Phi}{\partial \tau} d\tau + \frac{\partial \Phi}{\partial \zeta} d\zeta \tag{112}
\]
\[
d\tau = \frac{\partial \tau}{\partial \tau} d\tau + \frac{\partial \tau}{\partial \zeta} d\zeta \tag{113}
\]

A matrix set is formed with equation (47), (112) and (113), with the partial derivatives as the independent
Fig. 77  Analogue computer circuit for solution of batch-drying equations.

Fig. 78  Complete scaled analogue computer circuit for solution of batch-drying equations using three distance increments with a half-step initialisation.
variables, which is then solved to give the following four equations defining a characteristic strip:

\[ \frac{d\xi}{d\tau} = 0, \text{ i.e. } \xi = \text{cst}; \tag{114} \]

\[ \frac{d\eta}{d\xi} = 0, \text{ i.e. } \eta = \text{cst}; \tag{115} \]

\[ \frac{d\eta}{d\zeta} = \frac{-\pi f(\phi)}{d\zeta} = \frac{\partial \phi}{d\tau} \tag{116} \]

Equation (116) only holds if it is solved along either of the characteristic directions, defined by equations (114) and (115). Equation (116) is solved successively along lines of constant \( \zeta \) by integrating with respect to \( \tau \), so it is necessary to form the space derivative by finite differences.

As illustrated in Fig. 77, the analogue circuit comprised an iterative scheme (using a three-step difference formula) to calculate the space derivative \( \frac{d\eta_1}{d\zeta} \) from \( \xi_1, \eta_{i-1}, \eta_{i-2} \):

\[ \frac{d\eta_1}{d\zeta} = \frac{1}{2\Delta\zeta} \left( -3\pi_1 + 4\pi_{i-1} - \pi_{i-2} \right) \tag{117} \]

where \( \Delta\zeta \) is the space interval. This derivative is equated to \( \frac{\partial \phi_1}{d\tau} \), which is factored to give the real-time derivative \( \frac{\partial \phi}{d\theta} \), and then integrated to give \( \phi \). The relative drying-rate \( f(\phi) \) is obtained from \( \phi \) through a diode function generator (DFG). The loop is completed through a quarter-square multiplier (QSM) by multiplying \( f(\phi) \) by \( \pi_1 \) to obtain the derivative \( \frac{d\eta_1}{d\zeta} \) according to equation (116).

A dryer decomposed into three distance increments, with a half-step initialisation, was found to be sufficient for the simulation of a dryer of 1 NTU. A ten-segment function generator was used to set up a complex shaped characteristic curve \( f(\phi) \) from \( \phi \). The final scaled circuit consisted of four
Fig. 79 to 81  Batch-drying curves generated by analogue computer technique for *Pinus Radiata* wood with initial moisture content $\phi_0 = 1.22$ and characteristic drying-rate function given in reference 22 for dryer of length $\lambda = 1.05$. 
Fig. 22. Finite-difference grid for explicit numerical solution to batch-drying equations.
blocks, like Fig. 77, connected in series as shown in Fig. 78. By interpolation the inlet profile may also be generated.

An example of this method of solving the batch-drying equations is furnished by Figs. 79-81. This case assumes the characteristic drying rate curve given in reference 22 for the drying of Pinus Radiata.

The dryer parameters are $\Phi_0 = 1.22$ and $\lambda = 1.05$. This simulation closely approximates the behaviour of a timber kiln. In this case the solutions are presented as a function of real time instead of dimensionless time.

(c) Digital Computer Solutions

Tetzlaff\textsuperscript{21} has used a Heun predictor-corrector method to integrate the characteristic-strip equations (114)-(116). However he found that 500 time intervals were necessary to attain stability and accuracy for the simulation of a dryer of length 1 NTU divided into 7 distance intervals.

In this study, a simple explicit finite-difference scheme was used to calculate the moisture content profiles throughout the dryer. Two successive time levels of the finite difference grid are illustrated in Fig. 82. The first step of this procedure is to obtain the moisture content at the inlet at the $(n+1)^{th}$ time level in terms of the moisture content at the $n^{th}$ time level from the finite difference analogue of equation (47).

By centering this equation about the point $(\zeta_1, \tau_{n+\frac{1}{2}})$ this leads to the expression

$$\Phi_{1,n+1} = \Phi_{1,n} - \Delta\tau \cdot f(\Phi_{1,n+\frac{1}{2}}) = \Phi_{1,n} - \Delta\tau \cdot f\left(\frac{\Phi_{1,n+1} + \Phi_{1,n}}{2}\right)$$

(118)

This equation is nonlinear in the variable $\Phi_{1,n+1}$ so the Wegstein iterative root-finding method was used to find the solution. From the value at the inlet for the new time level, $\Phi_{1,n+1}$, the remainder of the profile across the dryer is obtained from the marching formula given by the finite difference
Fig. 83 to 86. Accuracy analysis of finite difference solutions to batch-drying equations by examining errors in outlet moisture content.

**Fig. 83** - Constant timestep scheme, $\Phi_0=2$, $\lambda=2$, $f(\Phi)=0$.

**Fig. 84** - Constant timestep scheme, $\Phi_0=2$, $\lambda=2$, $f(\Phi)=(1,0.5,0.8,0.2,0.05)$.

**Fig. 85** - Compares constant timestep scheme ($\Delta t = \Phi_0 \Delta Y$) with variable timestep scheme ($\Delta t = (\Phi_0 - \Phi_0(t)) \Delta Y$).

- $\Phi_0=2$, $\lambda=2$, $f(\Phi)=0$, $\Delta Y=0.2$.

**Fig. 86** - Variable timestep scheme.

- (a) $f(\Phi)=0$, $\Phi_0=4$, $\lambda=2$.
- (b) $f(\Phi)=(1,0.5,0.8,0.1,0.4)$, $\Phi_0=2$, $\lambda=2$.
- (c) $f(\Phi)=(1,0.6,0.4,0.1)$, $\Phi_0=2$, $\lambda=2$.
- (d) $f(\Phi)=0$, $\Phi_0=2$, $\lambda=4$.
- (e) $f(\Phi)=0$, $\Phi_0=2$, $\lambda=2$. 
analogue of equation (57) centred on the point $(\bar{y}_{i-1},
abla_{n+1})$.

\[ \Phi_{i,n+1} = \Phi_{i-1,n+1} + \Delta \zeta \cdot \left\{ \frac{\Phi_{i-1,n+1} + \Phi_{i,n+1}}{2} \right\} \left\{ \frac{\Phi_{i,n+1} - \Phi_{i-1,n+1}}{2} \right\} \]  

(119)

Once again, the equation is nonlinear in the unknown variable, $\Phi_{i,n+1}$, and the Wegstein method is used to accelerate the solution of equation (119).

By dividing the dryer length, $L$, into $I$ distance intervals ($L = I \Delta \zeta$) and the drying time, $\tau$, into $N$ intervals ($\tau = N \Delta \tau$), the complete moisture content history of the dryer can be evaluated by alternate use of equations (118) and (119), starting from the initial condition that $\Phi = \Phi_0$ everywhere throughout the dryer. The potential profile $\bar{\pi}$ can be obtained from the moisture content profile by the finite difference analogue of equation (64).

\[ \bar{\pi}_{i,n} = \frac{\Phi_0 - \Phi_{i,n+1}}{\Phi_0 - \Phi_{i-1,n+1}} \]  

(120)

The accuracy and stability of the solution depends upon the step sizes chosen. The combination of equations (47), (57) and (64) leads to an expression which clearly indicates the optimum step size ratio $(\Delta \tau/\Delta \zeta)$ for the finite difference solution. This expression is given by:

\[ -(\Phi_0 - \Phi(0, \tau)) \frac{\partial \Phi}{\partial \tau} = \frac{\partial \Phi}{\partial \zeta} \]  

(121)

which is the simple convective transport equation where the convective velocity is $1/[\Phi_0 - \Phi(0, \tau)]$. Von Rosenberg\(^{32}\) shows
Fig. 87 to 90  Comparison of batch-drying curves generated by finite difference solution using variable timestep scheme with analytical solution given by quasi-linearisation method ("QUASI").

\[ F(\Phi) = (1, 1, 0.6, 0.4, 0.4, 0.2), \quad \Phi_0 = 2, \quad \lambda = 2, \quad \Delta \Phi = 0.2. \]
that this equation is stable for all step-sizes but in order to minimise truncation error the optimum step-size ratio is:

\[ \Delta \tau = \left\{ \Phi_0 - \Phi(0, \tau) \right\} \Delta \zeta \] (122)

As equation (122) suggests \( \Delta \tau / \Delta \zeta \) is very small when \( \Phi(0, \tau) \) is approximately equal to \( \Phi_0 \), the ratio was taken to be given by \( \Delta \tau = (\Phi_0 - 1)\Delta \zeta \) until the end of the constant-rate period was reached. The computer program "RATBAG" (see Appendix 2E(i)) was written to simulate the batch-drying dynamics using the above equations.

Clearly with such an explicit finite difference scheme the largest errors will accrue at the outlet of the dryer. The maximum errors between the finite difference solution using a constant time step and the exact analytical solution are shown in Figs. 83-84 for two types of \( f(\phi) \) curve as a function of the distance interval. These results show that the particular choice of step-size ratio can greatly affect the order of the accuracy. The superiority of the variable time-step scheme is demonstrated in Fig. 85, where the accrued error developed during the finite difference solution is compared. Further results are plotted in Fig. 86 which indicate some of the relationships between the accuracy and the shape of the characteristic curve and the dryer parameters. In the case of a fast characteristic curve (i.e. concave-down type curve) the advantage of the variable time-step scheme is small because the inlet moisture content tends to zero more quickly. Nevertheless for the simulation of more complex problems involving flow switching a step-size ratio of \( \Phi_0 \Delta \zeta / \Delta \tau \) should provide accurate solutions.

The applicability of the finite difference solution for the simulation of problems involving complex shaped characteristic curves is demonstrated in Figs. 87-94. Here the solutions for a dryer of length \( \lambda = 2 \) and initial moisture content \( \Phi_0 = 2 \) are presented for the quasilinear \( f(\phi) \) curves
Fig. 91 to 94. Comparison of batch-drying curves generated by finite difference solution using variable timestep scheme with analytical solution given by quasilinearisation method ("QUASI").

\[ f(\Theta) = \left(1, 0.5, 0.8, 1, 0.4\right) \quad \Theta_0 = 2, \quad \lambda = 2, \quad \Delta \gamma = 0.4. \]
with breakpoints (1, 1; 0.5, 0.8; 0.1, 0.4) and (1, 1; 0.6, 0.4; 0.4, 0.2). In these cases the exact analytical solutions can be generated using "QUASI" and these solutions are shown by the dotted lines. Even by using a coarse finite difference grid the accuracy is acceptable.

The results presented above show that accurate solutions can be obtained using approximately 400 grid points compared to 3500 points required for Tetzlaff's solution to the same problem. In this case solutions required 5 secs on a Burroughs 6700 high-speed computer.

(f) Simplified Solution for Long Dryers

It can be seen from the drying curves presented in Fig. 59 that in the limit of a long dryer the curves tend to an asymptotic shape. These curves also appear to be regularly spaced suggesting that the drying wave moves through the dryer with constant velocity. This latter observation is confirmed by considering equation (63), in the limit as $\Phi(0, \tau)$ tends to zero.

i.e. \[ \lim_{\tau \to 0} \frac{d\phi_{cr}}{d\tau} = 1 \] (123)

Thus at this limit the critical point will move through the dryer with a constant velocity of $1/\Phi_c$. An examination of equation (63) shows that the critical point velocity decreases from the value of $1/(\Phi_c - 1)$ when the critical point is at the inlet to the limiting value of $1/\Phi_c$ when the critical point is deep within the dryer. Generally the limit of $\Phi(0, \tau)$ being equal to zero is only reached after an infinite time of drying has elapsed (the solution to the case $f(\phi) = \phi^2$ provides an exception). But for engineering accuracy, this limit can be said to have been reached when $\Phi(0, \tau) = 0.01 \Phi_c$. 


Fig. 95 and 96. "Long-dryer" distance limit for spectrum of characteristic drying-rate curves $f(\Phi) = (1, 1, \Phi_2, f_2)$ for an initial moisture content $\Phi_0 = 2$. Correction factor $C(\Phi_0)$ for initial moisture contents different from 2.
The position of the critical point at this instant also determines the dimensionless length of the dryer from where this approximation will hold. This length $\zeta_L$ is a function of the initial moisture content and the shape of the characteristic curve. By approximating the characteristic curve to two straight lines which intersect at the point $(\phi_2, f_2)$ such that:

$$
\begin{align*}
  f &= 1 & \phi &> 1 \\
  f &= a_1 + b_1 \phi & 0 < \phi < 1 \\
  f &= b_2 \phi & \phi > 0
\end{align*}
$$

Then equations (106)-(109) give the following expression for $\zeta_L$, 

$$
\zeta_L = \frac{1}{b_2 \phi_0} \ln \left[ \frac{\phi_0/\phi(0, \tau_L) - 1}{\phi_0/\phi_2 - 1} \right] + \frac{1}{a_1 + b_1 \phi_0}
$$

$$
\ln \left[ \frac{a_1 + b_1 \phi_0 - 1}{a_1 + b_1 \phi_2 - 1} \right]
$$

where $\phi_0/\phi(0, \tau_L) = 100$. This function is plotted in Fig. 95 for $\phi_0 = 2$ against various values of $\phi_2$ and $f_2$ which cover the complete spectrum of characteristic curves. For initial moisture contents different from 2, Fig. 96 provides approximate correction factors $C$ for $\zeta_L$, 

$$
i.e. \quad C(\phi_0) = \frac{\zeta_L(f, \phi_0)}{\zeta_L(f, 2)}
$$

From these graphs it is apparent that $\zeta_L$ decreases as $\phi_0$ increases and as the characteristic curve becomes increasingly
more concave downwards in shape. To fully describe this problem we also require the time taken, \( \tau_L \), for the critical point to reach the limiting value \( \xi_L \). From equation (111) the elapsed time is:

\[
\tau_L = \hat{\Phi}_0 - 1 + \frac{1}{b_4} \ln \left( \frac{1}{a_1 + b_1 \hat{\Phi}_2} \right) + \frac{1}{b_2} \ln \left( \frac{\Phi_2}{\Phi(0, \xi_L)} \right) \quad (127)
\]

Within this region of the dryer, the moisture content versus time and moisture content versus rate curves will all have the same shape. Integration of equation (123) gives the position of the wave at time \( \tau \):

\[
i.e. \quad \xi_{CR} = \frac{\tau - \tau_L}{\Phi_0} + \xi_L \quad (128)
\]

The moisture content profiles are given by equations (59), (61) and (127). For \( \hat{\Phi} \geq 1 \), \( \hat{\Phi} = \hat{\Phi}_0 - (\hat{\Phi}_0 - 1) e^{\frac{\Phi}{\hat{\Phi}_0}} \quad (129) \)

for \( \hat{\Phi} \leq \hat{\Phi} \leq 1 \),

\[
\frac{\tau - \tau_L}{\Phi_0} = \int_{\Phi_0}^{\bar{\Phi}} \frac{ds}{(\Phi_0 - s)f(s)} = \frac{\tau - \tau_L}{\Phi_0} - \xi_L + \xi_L \quad (130)
\]

These curves will have the same shape at different bed positions as the two independent variables which describe the moisture content are linearly connected. The movement of this constant velocity moisture content wave leads to analogous behaviour to that predicted by Banks 13, 14 for drying with infinite transfer coefficients. However in this case the drying front will be somewhat wider. The drying-rate curve is given by equation (62) and (123) so that:

\[
\frac{\partial \Phi}{\partial \tau} \left( \hat{\Phi}_0 \right) = g(\hat{\Phi}) \quad (131)
\]
Fig. 97 Three examples of the characteristic drying-rate curves at inlet $f(\phi)$ and positions within a long dryer $g(\phi)$, when $\xi_0 = 2$. 
Hence the rate is a unique function of the moisture content, so that the rate versus moisture content curves will be identical for all positions in a long dryer. This conclusion contrasts with Mykelstad's assumption for deep bed drying that the rate curves were only identical for the second falling-rate region. In fact, his analysis will only apply to short dryers. So at this limit a second unique rate curve \( g(\phi) \) is obtained which is a more complex function than the original characteristic curve \( f(\phi) \). Three examples of these two curves are given in Fig. 97. The potential curves will also have the same shape and these are also a unique function of the moisture content as suggested by equation (64).

\[
\pi = \frac{\phi - \phi}{\phi_0}
\]

Hence equations (125-132) completely describe the drying process within an extensive dryer.

Part 4 Extensions of Batch-Drying Equations to Include Complex Phenomena

The batch-drying model which has been solved in Part 3 incorporates several simplifications which may not be observed in practice. This model is hereafter referred to as the simple model. Such phenomena as a shrinking solid, varying inlet potentials, non-constant velocity across the dryer in a direction at right angles to the airflow, transfer coefficients that change with position and the equilibrium moisture content which varies with the gas relative humidity, will be encountered in a real dryer. These effects are examined below, and methods of calculation are suggested so that these variations can be included in the simple model.

(a) Velocity Distribution

One of the assumptions of the simple model is that the gas velocity is constant in the direction perpendicular to the gas flow. However it has been observed in several studies
Fig. 98. Distribution of air-velocity for various layers of boards (1 to 6) within stacks of timber of height 2.25 m in a kiln as measured by Malmquist and Meichsner.\(^\text{28}\).

(a) Wood thickness \(b = 31\text{ mm}\), sticker thickness \(c = 25\text{ mm}\);
(b) \(b = 25\text{ mm}, c = 20\text{ mm}\);
(c) \(b = 37\text{ mm}, c = 25\text{ mm}\);
(d) \(b = 50\text{ mm}, c = 25\text{ mm}\).
on timber dryers that this is not always the case. The distribution of air-velocity depends upon the board-spacing and their thickness as Nalquust and Neichauer's data show in Fig. 98. It is also probable that a non-uniform temperature distribution may be associated with this velocity distribution, but this additional factor is ignored in the analysis below.

If there is no mixing of the gas stream between flow channels, as is the case in a pile of timber, then the following analysis may be used to predict the drying history. The problem is decomposed by splitting the dryer into a number of small sections. The air-velocity being constant for each section. Strictly, this rearrangement of the problem also implies that the air-velocity distribution is locally smooth with respect to a single board so that each surface of the board dries at the same rate. The simple batch-drying equations (eqn. 47) can now be applied to each section. Each portion of the dryer will have different parameters $\tau_h$ and $\zeta_h$ as given by equations (34) and (46), because the air-velocity $v(h)$ is a function of the height $h$ in the direction perpendicular to the gas-flow. By solving equation (47) at the various heights in the dryer, the moisture content $\Phi$ is generated as a function of $\tau_h$, $\zeta_h$ and $h$. In order to obtain a more meaningful comparison of the drying at various levels in the dryer it is necessary to calculate $\Phi$ as a function of the real distance and time, i.e. $\Phi(y, h, \Theta)$. In Chapter 3, it was shown that $k_{v^*}$ is approximately proportional to $v^{0.6}$ and $G_B$ is directly related to $v$. By defining the velocity distribution $v^*$ as

$$v^* = \frac{v(h)}{v}$$

then the distance $y$ and the time $\Theta$ will be given by the
Fig. 99 to 101

Moisture content $\bar{\Phi}$ variation with time at inlet, middle and outlet of dryer when air-velocity distribution is non-uniform.

Example 4(e), Fig. 93.

$f(\bar{\Phi}) = \bar{\Phi}$, $\bar{\Phi}_0 = 2$, $\bar{\lambda} = 1.0$
expressions below.

\[ Y = \frac{\bar{G}_b \; V^{0.4} \; \zeta}{k_y \; \alpha \; \phi_M} \]  
(134)

\[ \Theta = \frac{(1-E) \; \rho_S (\bar{x} - \bar{x}_E) V^{0.6} \zeta}{k_y \; \alpha \; \phi_M \; (\bar{y}_w - \bar{y}_E)} \]  
(135)

The solution to the simple model for the linear \( f(\Theta) \) case is used together with equations (134) and (135) to solve this problem (see computer program "ELDIS", Appendix 2S). From the velocity distributions presented in Fig. 98, example (C4) was chosen because it was measured for a geometry identical to that for drying Pinus Radiata wood. This example is used to show how the moisture content distributions are affected by a non-uniform velocity profile. The parameters correspond approximately to those for a timber kiln where \( \bar{x} = 1.0 \) and \( \zeta = 0.05 \Theta \) ( \( \Theta \) has the units of hours). These results are presented in Fig. 99-101 for the inlet, middle and outlet of the dryer in the direction of the airflow. The disparity of the moisture content distributions illustrates the ill effects resulting from this phenomenon.

It can be shown a priori that one necessary condition towards optimising dryer performance is to ensure that the air-velocity is uniform across the dryer. These aerodynamic considerations are often neglected in dryer design and operation but it is apparent from the results above that this aspect should receive careful attention. Werner and Sturany have demonstrated that a uniform air-velocity distribution can be attained under practical conditions. Werner proposes a slot distributor to ensure good air-distribution and a mixing duct to maintain heat distribution. From model studies, Sturany shows the effectiveness of smoothing the air-distribution by rounding the bend in the inlet air duct to the dryer and inserting flow splitters. These photographic results are presented in Fig. 102. Re
Fig. 102  Flow visualisation studies through models of a timber kiln using aluminium powder in a water-tunnel (from Sturany).  
(a) Rounded corner after heating coil.  
(b) Flow splitters added to inclined wall at stack entrance.  
(c) Rectangular bend after heating coil and staggered flow splitters.  
(d) Rounded corner with staggered flow splitters.  
Note flow in (a) and (b) predominantly through top part of stack.  Option (d) offers little advantage over (c).
further notes that sophisticated air inlet ducts may not be necessary in wood kilns provided there is a sufficiently large cross-section between the front wall of the dryer and the stack. This, in conjunction with the resistance of a suitably wide pile, ensures a uniform flow over the whole cross-section because a region of slight excess static pressure is created in the front of the stack. These latter results then conclude that the performance of any batch-dryer, including timber kilns, can be optimised with respect to the inlet air-velocity distribution.

(b) Varying Inlet Potential

Most timber-drying schedules involve step-changes in humidity potential and the temperature potential as the wood dries out. This can be seen in Table 1. Many other materials are dried by similar types of schedule where the inlet potential may even be continuously varied. To account for this complication, the simple model can include this effect by generalising the definition for the time constant parameters. By putting \( Y_W - Y_G^o = \pi_o^M \) and \( T_G^o - T_W^o = \pi_o^H \), then the time constants \( \tau \) will be defined by the more general relationships below.

\[
\tau_M = \frac{k_o^s a_G^o \int_0^\Theta \pi_o^M d\Theta}{(X_{GR} - X_E) \xi_S (1 - c)} \quad \tau_H = \frac{h_o^s a_H^o \int_0^\Theta \pi_o^H d\Theta}{(X_{GR} - X_E) M W^o \pi_s (1 - c)}
\] (136)

Equation (136) leads to the identical expressions as presented in equation (34) when the inlet potential \( \pi_o^s \) remains constant.

Substitution of equations (31), (32), (46) and (136) into equations (29) and (30) and neglecting the small dispersion term gives the following dimensionless equation

\[
\frac{\partial \xi}{\partial \tau} + k \frac{\partial \xi}{\partial \xi} + k \pi \frac{\partial \pi}{\partial \tau} = -\kappa f(\chi)
\] (137)

The term \( k \pi \partial \pi / \partial \tau \) has earlier been shown to be negligible and the term \( k \pi / \pi_o \partial \pi / \partial \tau \) can also be shown to be small even when \( \pi_o \)
increases rapidly with \( \tau \). For example, if \( \pi_0 = \pi_0^T \),

\[
(1 + 100\tau) \quad \text{where} \quad \pi_0^T \quad \text{is the initial inlet potential then:}
\]

\[
\frac{k\pi}{\pi_0} \frac{d\pi}{d\tau} = \frac{100kRe}{\pi_0 (1 + 100\tau)} \sim 10^{-3} \quad \text{at} \quad \tau = 0
\]  

(138)

as \( k \) is of the order of \( 10^{-5} \) and \( \pi \) cannot exceed unity. So, as the other terms in equation (137) are of the order of unity, the time derivative terms can be neglected. Hence equation (47) is a valid approximation to this problem provided the time constants are defined according to equation (136).

Before the varying inlet potential problem can be fully described, a relationship between dimensionless time and real time is required. As drying schedules are normally specified in terms of moisture content changes, equation (136) has to be inverted to obtain the real time.

\[
\Theta = \int_0^\tau \frac{d\tau}{C \pi_0} = \int_0^\phi \frac{d\phi}{\phi(0,\tau) \pi_0 C^T f}
\]  

(139)

where

\[
C^T = \frac{k_Y aM}{(X_{GR} - X_E) \phi_S (1 - \epsilon)}
\]  

(140)

The above equations presuppose that all the other dryer parameters are independent of the changing inlet potential. In fact the constants \( \rho_M, \phi_H, \phi_M, k_Y^*, h_C^*, \text{and} C_{PH} \) are likely to change with the inlet potential. As the time derivative \( \partial\pi/\partial\tau \) is neglected, the variation in gas density is unimportant. If the wet-bulb temperature of the airstream is held constant then the remaining parameters will all stay essentially constant. The equilibrium moisture content will also change with the inlet conditions as well as along the dryer. The magnitude of this effect is considered in Part 4(c). But in
Fig. 103 to 106  Batch-drying curves for constant inlet potential at various positions within the dryer. $\Theta_0 = 2$, $n = 2$, $f(\Theta) = \Theta$. 
the case where the wet-bulb temperature does not remain constant, then the number of transfer units for the dryer will vary with the changing inlet conditions. Strictly this case will require finite difference methods for solution, but some of the effects due to the changing physical properties can be included in the characteristic rate function by careful definition of the characteristic curve concept. The drying rate curve is generally determined from bench-scale tests for drying experiments at constant external conditions. In this case the curve is defined as

\[ f(\Theta) = \frac{N_A}{N_J} \frac{Y_{\text{surf}} - Y_G^O}{Y_W - Y_G^O} \]  

(141)

But if the curve is defined as

\[ f(\Theta) = \frac{k_Y \phi_M (Y_{\text{surf}} - Y_G^O)}{k_Y^* \phi_M (Y_W - Y_G^O)} \]  

(142)

where \( \phi_M^* \) and \( k_Y^* \) are the values of these constants determined at the initial inlet conditions, then the solutions to the simple batch-drying model can be applied to this problem using this modified characteristic curve.

Hence the solutions to equation (47) presented in Part 3 can be applied to the solution of the varying inlet potential problem except the relationship between real time and dimensionless time is more complex. The drying curves generated by the simple model do not immediately give a true appreciation of the manner in which the drying rates are enhanced by the increasing inlet potential. By factoring the rate \( \partial \Theta / \partial \tau \) obtained from the simple model solution by the ratio \( \pi_o / k_o^F \), where \( \pi_o^F \) is the final inlet potential, then the improved rates become apparent (see Appendix 2E(ii)). Figs. 103-121 show
Fig. 107 to 111. Batch-drying curves at various positions within the dryer for variable inlet humidity potential schedule used for drying 50mm thick *Pinus Radiata* in New Zealand. (see Table 1).

\[ f(\Phi) = \Phi, \quad \Phi_0 = 2, \quad \lambda = 2. \]
Fig. 112 to 116  Batch-drying curves at various positions within the dryer for variable inlet humidity potential schedule used for drying 50mm thick Pinus Radiata in Australia (see Table 1).

\[ f(\Phi) = \Phi, \quad \Phi_0 = 2, \quad \lambda = 2. \]
Fig. 117 to 121  Batch-drying curves at various positions within the dryer for variable inlet humidity potential schedule used for drying 50mm thick Pinus Radiata in U.S.A. (see Table 1).  \( f(\phi) = \phi, \phi_0 = 2, \lambda = 2 \).
typical patterns obtained for complex drying schedules involving step-changes in humidity potential presented in Table 1 for drying 50 mm thick Pinus Radiata wood. The first four diagrams show the dimensionless solution for the case of a constant inlet potential for the initial condition $\Phi_0 = 2$ and a dryer of length $\lambda = 2$, and a linear characteristic curve. The subsequent graphs for the New Zealand three-step schedule, the Australian five-step schedule and the American seven-step schedule are compared by plotting the enhanced rates as a function of the dimensionless time $C \pi_o \Theta$. By examining the moisture content and humidity changes as a function of time $C \pi_o \Theta$, the subsequent reduction in drying times associated with the enhanced rates becomes clear.

It is difficult to compare the worth of the three schedules. The American schedule is clearly the fastest schedule but at the expense of rates within the kiln that greatly exceed the initial inlet rate. In contrast the New Zealand schedule enhances the rate such that the maximum rates within the dryer are nearly equal to the initial inlet rate. However this comparison of the three schedules for the same characteristic curve is not a true test. It is shown in Chapter 5 that the shape of the curve becomes increasingly more convex downwards as the rate increases. Under these conditions the observed rate maxima for the faster schedules may be considerably smaller than those predicted.

Another example of a continuously varying inlet potential drying schedule is the constant recirculation ratio problem studied by Van Meel$^3$. i.e.

$$\pi_o = (1 - r)\pi^0 + r\pi(\lambda, \nu) \quad (143)$$

where $\pi^0$ is the fresh-air potential and $r$ is the recirculation ratio. The analytical solution to this problem, in the case of a linear $f(\Phi)$ curve, can be obtained by use of equations (53),
This method of calculating the drying behaviour for a varying inlet humidity potential policy is potentially very powerful. Once the solution for the simple model has been generated, many varying potential policies can be studied. It is straightforward to consider the worth of a range of such policies. Once an optimal criteria for the dryer operation has been chosen, this method could rapidly isolate an optimal policy.

(c) Varying Equilibrium Moisture Content

As the airstream passes through the dryer, the air accumulates moisture causing the dry-bulb temperature to drop. The equilibrium moisture content (e.m.c.) of the solids is a strong function of the relative humidity. This means the e.m.c. will vary across the dryer in association with the changing air conditions. The variation in e.m.c. would be unimportant except for the fact that the drying rate has been defined by equations (29) and (30) as being dependent upon the value of the e.m.c. During the constant-rate period, \( f(\Theta) \) is equal to one so that the rate is independent of the e.m.c. This can be seen more clearly by expressing equation (54) in terms of the actual moisture content \( \bar{X} \), distance \( y \) and time \( \theta \).

\[
\bar{X} = \bar{X}_0 - e^{-\frac{k_Y a_y}{G_B} \frac{y}{y_0} (Y_w - Y_o) \Theta \frac{Q_B}{G_B (1 - \Theta)}}
\]

So for the constant-rate period the change in e.m.c. across the dryer is independent of the drying process.

The importance of the equilibrium moisture content variation becomes apparent during the falling-rate period. As \( f(\Theta) \) decreases, a particular value of \( f \) will correspond to a different value of the moisture content \( \bar{X} \) as \( X_B \) varies across the dryer.

The potential is observed to change greatly across a dryer of moderate length during the constant-rate period, so one may
expect large errors during the latter stages of drying because of the e.m.c. variation. However, as the critical point sweeps through the bed, the potential rises rapidly. By the time the critical point reaches the end of the dryer for the case of a linear \( f(\Phi) \) curve, the potential takes the value:

\[
\pi(\lambda, \tau_2) = \frac{\Phi_0 - 1 + e^{-\Phi_0 \lambda}}{\Phi_0}
\]

compared to \( \pi = e^{-\lambda} \) during the constant-rate period. This second period potential smoothing is illustrated in Table 10.

<table>
<thead>
<tr>
<th>( \Phi_0 )</th>
<th>Potential ( \pi(\lambda, \tau_2) ) (eqn. 145)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda = 0.5 )</td>
</tr>
<tr>
<td>1.5</td>
<td>0.65</td>
</tr>
<tr>
<td>2.0</td>
<td>0.68</td>
</tr>
<tr>
<td>3.0</td>
<td>0.74</td>
</tr>
</tbody>
</table>

So the change in potential across the dryer is considerably reduced but the magnitude of the error of assuming a constant e.m.c. will depend upon the equilibrium relationship and the magnitude of \( \bar{\chi}_{CR} \) relative to \( \bar{\chi}_{E} \). A guide to this latter criterion is the ratio of the maximum moisture content to the maximum hygroscopic moisture content. Luikov notes that for brick, glass and concrete that this ratio varies from 1:12 to 1:30. For the drying of wood this ratio varies from 1:2 for heartwood to 1:6 for sapwood. If this ratio is large and the solid mass transfer controlling then errors due to the varying e.m.c. will be small.
Fig. 122 to 125. Batch-drying curves at various positions within the dryer where variation of equilibrium moisture content of the solids is included (based on Fig. 35). Drying of wood for inlet air-conditions of $T_a = 60^\circ \text{C}$, $T_w = 50^\circ \text{C}$.

$$f(\bar{\phi}) = \bar{\phi}, \lambda = 2, \bar{\omega}_n = 2, \bar{X}_{cr} = 1.0.$$
Fig. 126 to 129  Batch-drying curves at various positions within the dryer where variation of equilibrium moisture content of the solids is included (based on Fig. 35). Drying of wood for inlet air-conditions $T_0 = 60^\circ \text{C}$, $T_w = 50^\circ \text{C}$.

$f(\bar{\phi}) = \bar{\phi}$, $\lambda = 2$, $\bar{\phi}_0 = 2$, $\bar{X}_{cr} = 0.3$. 
In order to examine the solutions of the batch-drying equations for the magnitude of this error it is first necessary to define the dryer parameters so that they are independent of the variable $\bar{X}_E$. It is convenient to define $\phi$ and $\tau$ in terms of the value of $\bar{X}_E$ at the inlet, $\bar{X}_E^I$, which is known to be constant, for constant inlet-air conditions.

\[ \phi = \frac{\bar{X} - \bar{X}_E^I}{\bar{X}_E - \bar{X}_E^I} \]  

\[ \tau_h = \frac{b \cdot e^\phi}{(\bar{X}_E - \bar{X}_E^I)(\Delta H_W \cdot \rho_s)(1 - e)} \]

\[ \tau_m = \frac{k \cdot e^\phi}{(\bar{X}_E - \bar{X}_E^I)(\rho_s)(1 - e)} \]  

So that the batch-drying equations, eqn. (47), are correctly modified to give:

\[ \frac{\partial \phi}{\partial \tau} = \frac{\partial \phi}{\partial X} = -\pi f(\phi') \]  

where

\[ \phi' = \frac{\bar{X} - \bar{X}_E}{\bar{X}_E - \bar{X}_E^I} + \frac{\bar{X}_E^I - e(\pi)}{\bar{X}_E - e(\pi)} \]  

as the equilibrium moisture content $\bar{X}_E$ is a function of the potential $e(\pi)$. This equation can only be solved by finite difference methods together with equilibrium moisture content data (see Appendix 2E(iii)). Two examples are graphed in Figs. 122-129 showing the errors generated by assuming the equilibrium moisture content to be constant (shown by dotted lines) for the drying of wood from an initial moisture content of 2 in a dryer of length 2 NTU when $\bar{X}_E$ is 12 and 3. The inlet-air conditions are given by $T_G = 60^\circ C$ and $T_W = 50^\circ C$, and the equilibrium moisture content function $e(\pi)$ is based on data presented in Fig. 35. The magnitude of the ratio $\bar{X}_E^I$ is
approximately equal to 12 for the drying of softwoods. In this case, the errors that arise from ignoring the variable 12. When $\frac{X_{CR}}{X_E}$ is equal to 3, which approximately corresponds to the drying of hardwoods, this variation can be seen to be significant.

If the drying schedule involves a continuously varying inlet potential policy then the variation in e.m.c. could be more important. In this case, the dimensionless moisture content $\Phi$ should be defined in terms of the equilibrium moisture content based on the final inlet potential,

\[(d)\text{ Varying mass transfer coefficients}\]

It was noted in Part 1 of Chapter 3 that for the drying of wood the local transfer coefficients decrease with distance across the load. Such a variation may also be expected for drying other solids if they are stacked in regular arrays similar to wood or if the solids are arranged on trays. In the case of drying granular beds of solids there is evidence that the transfer coefficients remain substantially constant throughout the dryer.

Hence for some particular arrangements of the solid within the dryer, the assumption in the simple model that the transfer coefficients are constant will no longer hold. For this situation it is necessary to define the dryer parameters according to the expressions,

\[
\tau_M = \frac{k^*_Y (L) a \phi_M (Y_w - Y_G^0)}{(X_{CR} - X_E) \rho_B (1 - \varepsilon)}
\]

\[
\tau_H = \frac{h^*_C (L) a \phi_H (T_G^0 - T_w)}{\Delta h_{ww} (X_{CR} - X_E) \rho_s (1 - \varepsilon)}
\]

\[
\xi_M = \frac{k^*_Y (L) a \phi_M y }{\phi_B}
\]

\[
\xi_H = \frac{h^*_C (L) a \phi_H y }{C_{P_H} G_B}
\]

\[
\xi = \frac{k^*_Y (\xi)}{k^*_Y (L)}
\]
Fig. 130 Variation of humidity potential with distance across dryer during constant rate period of drying when mass transfer coefficient is a function of distance.
then the batch-drying equations become

\[ \frac{\partial \Phi}{\partial \tau} = \frac{\partial \xi}{\partial \tau} = -\pi \xi (\Phi) \]  \hspace{1cm} (153)

where \( \xi = \xi (\zeta) \). The variable \( \pi \) can be eliminated from equation (153) to give the following equations in \( \Phi \):

\[ \frac{\partial}{\partial \tau} \left( \frac{1}{\xi} \frac{\partial \Phi}{\partial \tau} \right) = -\frac{1}{\xi} \frac{\partial \Phi}{\partial \tau} \frac{d(1/\xi)}{d\tau} \]  \hspace{1cm} (154)

and

\[ \frac{\partial}{\partial \tau} \left( \Phi + \frac{1}{\xi} \frac{\partial \Phi}{\partial \tau} \right) = -\frac{1}{\xi} \frac{\partial \Phi}{\partial \tau} \frac{d(1/\xi)}{d\tau} \]  \hspace{1cm} (155)

Equation (153) cannot be solved analytically except for the constant-rate period when \( f(\Phi) = 1 \). For the boundary conditions \( \pi = 1 \) at \( \zeta = 0 \) and \( \Phi = \Phi_0 \) at \( \tau = 0 \), integration of equation (153) yields:

\[ \pi(\zeta, \tau) = \exp(-\bar{\xi} \zeta) \]  \hspace{1cm} (156)

\[ \Phi(\zeta, \tau) = \Phi_0 - \xi, \exp(-\bar{\xi} \zeta), \tau \]  \hspace{1cm} (157)

where

\[ \bar{\xi} = \frac{1}{\zeta_0} \int_{\zeta_0}^{\zeta} \xi d\zeta = \frac{\bar{k}_Y^*(\zeta)}{\bar{k}_Y^*(L)} \]  \hspace{1cm} (158)

The potential profile for a dryer of two NTU for a constant transfer coefficient is compared in Fig. 130 with the case of a coefficient predicted from the turbulent flat-plate correlation and also for the transfer coefficient based upon Sørensen's correlation. It can be seen that when the transfer coefficient decreases with distance that the drying zone within the dryer is widened as the drying-rate falls off more slowly across the dryer.

In order to solve the overall problem, a finite difference solution is necessary similar to the method proposed earlier for the simple model. By expressing equation (154) about the
point \((\xi_{i-1}^+, \tau_{n+1}^-)\), we obtain the finite difference analogue:

\[
\Phi_{i,n+1} = \Phi_{i,n} + \left( \frac{\Phi_{i-1,n+1} - \Phi_{i,n}}{\xi_{i,n+1}^- \cdot \xi_{i,n}^+} - \frac{\Delta \xi}{2} \right) \left\{ \frac{1}{\xi_{i,n+1}^- \cdot \xi_{i,n}^+} + \frac{\Delta \xi}{2} \right\}
\]

Equation (159) is clearly nonlinear in the variable \(\Phi_{i,n+1}\) and a root-finding method is required to find the solution. Equation (159) is analogous to the marching formula for the finite difference solution of the simple model, equation (119). Before the marching formula can be used to generate the profiles across the dryer, it is necessary to predict the inlet profile. By centering equation (153) about the point \((\xi_1, \tau_{n+2})\) the following equation results:

\[
\Phi_{1,n+1} = \Phi_{1,n} - \Delta \tau \cdot \xi_1 \cdot f(\Phi_{1,n+2}^-)
\]

Equations (159) and (160) are solved alternatively in the same manner as the finite-difference solution for the simple model outlined in Part 3(e). Except in this case, the potential profile \(\pi\) is not given simply by the dimensionless moisture content difference ratio, equation (64). No simple relation can be derived to predict \(\pi\) from the moisture content \(\Phi\). It is necessary to solve equation (153) by finite differences about the point \((\xi_{i-1}^+, \tau_{n+1}^-)\) to obtain the potential profile:

\[
\pi_{i,n+1} = \pi_{i-1,n+1} \left\{ \frac{1 - \Delta \xi(f_{\xi_{i,n}^-} + f_{\xi_{i-1,n}^-}) f(\Phi_{i-1,n+1}^-)}{h + \Delta \xi(f_{\xi_{i,n}^-} + f_{\xi_{i-1,n}^-}) f(\Phi_{i-1,n+1}^-)} \right\}
\]

To illustrate the significance of a non-constant transfer coefficient, the drying equations were solved for a dryer of 2 NTU filled with a solid of an initial moisture content \(\Phi_0\) of 2 which dries according to the linear rate curve \(f(\Phi)\) (see Appendix 2E(iv)). The drying profiles for the case of a
Fig. 131 to 134. Batch-drying curves at various positions across dryer showing significance of variable mass transfer coefficient (based on Sörenson's correlation).

\[ f(\theta) = \theta, \lambda = 2, \theta_0 = 2. \]
Fig. 135 to 138 Batch-drying curves at various positions across dryer showing significance of variable mass transfer coefficient (based on turbulent flat-plate correlation)

\[ f(\theta) = \theta, \quad \lambda = 2, \quad \theta_0 = 2. \]
transfer coefficient which varies with distance according to the turbulent flat-plate correlation and for the transfer coefficients predicted by Sørenson's correlation are both compared with the results for the case of a constant transfer coefficient (dashed lines) in Figs. 131-138.

It can be seen in these graphs that a transfer coefficient that decreases with distance across the dryer has the detrimental consequence of increasing the lag between the outlet and the inlet moisture content. This result suggests that another a priori requirement for optimal performance of a batch-dryer is to keep the transfer coefficients constant or, if feasible, to increase the transfer coefficients with distance until the rates are constant across the dryer. Such a feasible policy may be obtained by using turbulence promotion devices.

(e) Drying of Shrinking Solids

It has been seen in Part (d) that the arrangement of the solids within the dryer can have an important effect upon the heat and mass transfer coefficients for the evaporation of the moisture from the solid surface. This factor also is important when discussing the drying of shrinking solids.

In general three particular methods of arranging the solid are observed in practice. These three ways A, B and C are outlined below.

A: the material is arranged on trays within the dryer such that their position is fixed externally.

B: the material is stacked in regular arrays and it is spaced by an inert material which is not fixed in position.

C: the material is packed in heaps so that subsequent layers are in contact with one another and only constrained by the dryer walls.

The behaviour and analysis of the drying of a shrinking solid within a batch dryer depends upon the type of solid arrangement and each case demands separate consideration.

For solids arranged according to class A subsequent
shrinkage of the solid will modify the void ratio within the dryer and the specific surface area to volume ratio, of the particles. The first effect may have the consequence of changing the air velocity and flowrate according to the response of the fan to a reduced pressure drop across the load. This change will be approximately related to the average moisture content of the solids within the dryer. The shrinkage effects can probably be adequately included within the characteristic curve concept. The shrinkage and relative drying rate are closely related, as both are functions of the solid average moisture content. As the critical point is defined to be reached when the hygroscopic limit is reached on the drying surface, then shrinkage will commence at a similar instant. Under these conditions the characteristic curve must be determined from dryer tests as bench-scale experiments will not include the shrinkage side-effects.

For the second case it is necessary to consider in addition the directional aspects of the shrinkage. If the solid shrinks in the directions perpendicular to the airflow then the free cross-sectional area will remain unchanged while the volume of the solids decreases. In this circumstance the air velocity will remain constant as the layers of material will always remain a constant distance apart, although the void ratio and the specific surface area/unit volume of the solid will change. These latter effects can be accounted for within the characteristic curve. Some small effects may arise due to shrinkage in the direction of airflow as if the material was originally stacked edge to edge, then gaps will appear. This spacing of the solid in the longitudinal direction will have negligible influence upon the transfer coefficients as shown by Miller's study for mass transfer to spaced slabs. But when the shrinkage is so great that the gap is of the order of the slab width this effect will not be negligible. However the associated changes of air-velocity and void-ratio with the longitudinal shrinkage will necessitate a more complex analysis using finite difference methods.
Fig. 139  Volumetric shrinkage curve for wood-chips and wheat as a function of characteristic moisture content $\phi$. 
The third type of arrangement of solids produces interesting effects as the solid moves relative to the direction of airflow as it shrinks. Although the whole bed of moist solids shrinks as the individual particles shrink, the void ratio will remain practically constant, particularly if the particles are spherical. Once again pressure drop changes may influence the air-flow but more importantly the actual position of the drying particles will change as the solid shrinks except for the solids at the front of the dryer. This is provided the moist solids are free-flowing and no arches of solid are formed. Because of this complexity a finite difference analysis is required which accounts for the changing co-ordinates of the particles. By defining a moving distance co-ordinate \( \zeta' \)

\[
\zeta' = \int_0^{\zeta} s \, d\zeta
\]

then the drying of the shrinking solids will be described approximately by equation (57) where the variable \( \zeta' \) replaces \( \zeta \) everywhere.

\[
\frac{1}{f(\Phi)} \frac{d\Phi}{d\zeta'} = \Phi_0
\]

The function \( s \) appearing in equation (162) is the volumetric shrinkage function such that:

\[
\begin{align*}
    s(\Phi) &= 1 \quad \Phi > 1 \\
    s &= s(\Phi) \quad 0 \leq \Phi \leq 1 \\
    s &= C_s \quad \Phi = 0
\end{align*}
\]

where \( C_s \) is a constant. The shrinkage functions \( s = 0.75 + 0.25 \Phi \) and \( s = 0.9 + 0.1 \Phi \) which correspond approximately to the volumetric shrinkage of wheat and wood-chips respectively are presented in Fig. 139.

If equation (163) is solved along a line of constant \( \zeta' \)
Fig. 14.0 to 14.3  Batch-drying curves at various positions across dryer showing the significance of effects due to shrinkage \( s = 0.9 + 0.1 \phi \) when the particles are free to move.

\[ f(\phi) = \phi, \quad \lambda = 2, \quad \phi_0 = 2. \]
Fig. 14.4 to 14.7  Batch-drying curves at various positions across the dryer showing significance of effects due to shrinkage (s = 0.75 + 0.25 $\phi$) when the particles are free to move. 

$f(\phi) = \phi$, $\lambda = 2$, $\phi_0 = 2$. 
then the solution will effectively follow a given particle as it moves with the shrinking bed. By combining equations (162) and (163) then the finite difference algorithm, analogous to equation (119) for the solution of the simple model, will be:

\[
\Phi_{i,n+1} = \Phi_{i-1,n+1} + \Delta z \cdot f(\Phi_{i-1,n+1} + \Phi_{i,n+1}) \cdot \frac{1}{2} \\
+ \bar{s}(\Phi_{i-1,n+1} + \Phi_{i,n+1}) \cdot \left\{ \Phi_0 - \left( \frac{\Phi_{i,n+1} + \Phi_{i-1,n+1}}{2} \right) \right\}^{165}
\]

This equation is solved together with equation (118) in the same manner as outlined in Part 3(e), for a dryer of 2 NTU and an initial moisture content \( \Phi_0 \) equal to 2 assuming a linear \( f(\Phi) \) rate curve. (see Appendix 2E(v)). This analysis presumes that the local moisture gradients be similar everywhere in the dryer. Although this condition is unlikely, differential shrinkage over the dryer will be small as its reduction is always sought by the process operator to preserve the structural qualities of the dried product. The solution for a nonshrinking solid (indicated by the dashed lines) is compared with that for the drying of a shrinking solid (shown by the solid lines) in Figs. 140-147 for both shrinkage curves illustrated in Fig. 139. It can be seen that the shrinking solid has the advantage of reducing the drying lag over the dryer. A more serious implication of these results is that the shrinkage phenomena cannot be ignored when the overall shrinkage is greater than 10 per cent.

The shrinkage problem discussed above is not directly applicable to the drying of wood but if offers a check on the likely errors caused by the wood shrinkage. The volumetric shrinkage of wood on drying is approximately 10 per cent. Five per cent of this shrinkage occurs in the direction of airflow and the remaining shrinkage is in the direction at right angles to the airflow. It was discussed earlier that only the shrink-
age in the direction of airflow is significant. The drying of a class C solid is expected to cause the largest deviations from the solution of the simple model. Hence for the drying of wood the shrinkage effects should be negligible.

It is interesting to compare the deviations from the simple model that result due to the various phenomena discussed in Part 4. Both the equilibrium moisture content and the shrinkage cause corrections that are in the same sense. For these phenomena the deviation is nil at the inlet and the corrections increase negatively with distance across the dryer. The corrections for a variable mass transfer coefficient are more complex. In this case negative corrections are observed for the inlet drying while positive corrections are necessary for the outlet moisture content profile. For a real dryer a combination of these phenomena may be observed. It is possible in some cases for these effects to partially cancel out so that the simple model may adequately simulate a real dryer.

Discussion

The previous parts of this Chapter have dwelled at length upon many features of the batch-drying problem. This analysis is general and it is expected to hold for all batch-dryers. Some experimental evidence has been quoted in Part 1 to support such a batch-drying model. The applicability of such a model to drying wood in timber kilns has only received cursory attention.

The batch-drying characteristics of a timber kiln has been little studied by timber physicists. It is difficult to compare the experimental results with the theoretical model for most of the measurements that exist. Generally insufficient data are given so that the parameters cannot be estimated for the model. Furthermore, often inadequate care has been taken to match wood samples so that all the boards initially have the same moisture content and grain type. However the change
in drying rates across a timber stack has been measured by several workers. The results of the studies of Viktorin and Terazawa et al. are summarised in Chapter 3.4 where the drying rates across the timber stack have suggested a mass transfer coefficient which is a function of distance. Greenhill has measured the change of drying rates across a 1.5 m wide pile of blackwood for drying from 100 to 80 per cent moisture content. In contrast with Viktorin's and Terazawa's study, these measurements are shown in Fig. 148 to compare well with constant-rate period potential profiles when the transfer coefficient is constant.

Another way to examine the batch-drying characteristics of a drying stack of timber is to examine the moisture content variation as a function of time at various positions in the stack. Stevens and Pratt performed such experiments for the drying of 25 mm boards of Corsican pine at the inlet, middle and outlet of a 3 m wide pile. Fig. 149 shows these results together with further experiments which illustrate the significance of air-flow reversals. The same observations were made by Bateson (see Fig. 150) for the drying of 50 mm beech. Neither of these studies can be simulated theoretically because of the non-uniform initial moisture contents. The theoretical model can handle such moisture content distributions but knowledge is required of the initial moisture content distribution at all positions across the stack.

The most thorough study has been undertaken by Miller. In this case, the drying of a 2.5 m wide stack of 25 mm thick Pinus radiata was studied. The wet- and dry-bulb temperatures of the air, the surface temperature of the wood and the average moisture content of the wood were monitored at three positions across the stack. These results are presented in Fig. 151-4 for drying at an air-velocity equal to 3.56 m s⁻¹. The trends of these curves are similar to the batch-drying solutions presented earlier in Parts 3 and 4 but some aspects of the curves are unusual. The drying-rate curves do not show a constant-rate period. Also the change in dry-bulb temperature with time is linear in shape rather than sigmoid. Best agreement for these results is found with the
Fig. 14-8 Average drying-rates across stack of timber 1.5m wide at various air-velocities during drying from 100 percent to 80 percent moisture content (from Greenhill\textsuperscript{42}). These are compared with theoretical rate profiles for constant-rate period drying (dotted lines) where the mass transfer coefficient is assumed constant.
Fig. 149

Drying of 10ft wide pile of 1in thick Corsican pine at airspeed of 7 fps⁻¹.
(from Stevens and Pratt⁴⁻³).

Fig. 150

Drying of stack of 50mm thick beech wood
(from Bateson⁴⁻₆).
Fig. 151 to 154. Batch-drying curves for a 2.5 m wide stack of 25 mm thick Pinus Radiata wood at an air-velocity of 3.56 ms⁻¹ and inlet temperatures, $T_0 = 72°C$ and $T_w = 53°C$. Temperatures, drying-rate, and moisture-content variation with time and position (from Miller 35, Fig. 4, 5, 8 and 12).
varying mass-transfer coefficient solution presented in Figs. 131 to 134. For these solutions the constant-rate period is short and the humidity potential curves are more linear in shape than the case of a constant transfer coefficient.

Miller's study also considered the significance of air-velocity upon the drying time of a stack of timber. For the range of velocities 2 to 6.6 m s⁻¹ he concludes that above 3 m s⁻¹ increasing the air-velocity does not reduce the drying time significantly. A similar conclusion can be gained from results presented by Torgeson. He derived experimentally, the following table of values which show the drying time as a function of air velocity at definite intervals across a 1.2 m wide stack of 25 mm thick Sugar Maple sapwood.

Table 11

Drying time of 25 mm Sugar Maple sapwood from 70 to 40 per cent moisture content at definite intervals across a 1.2 m wide pile for inlet air-conditions of \( T_g = 61.2\,^\circ C \) and \( T_w = 55\,^\circ C \).

(from Torgeson)

<table>
<thead>
<tr>
<th>Air velocity /m s⁻¹</th>
<th>Drying Time / hour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( z=0.0,m )</td>
</tr>
<tr>
<td>1.30</td>
<td>15</td>
</tr>
<tr>
<td>2.77</td>
<td>12</td>
</tr>
<tr>
<td>4.33</td>
<td>10</td>
</tr>
</tbody>
</table>

The significance of air-velocity and the timber arrangement within the kiln upon the drying time can be considered theoretically. Firstly, we require knowledge of the parameters \( T, \lambda \) and \( \Phi \). For timber stacked edge to edge in layers the parameters \( a \) and \( \varepsilon \) are simply given by the following expressions.

\[
i.e. \quad \varepsilon = \frac{b}{b+d} \quad (166)
\]
where $b$ is the spacing between layers of boards and $d$ is the board thickness. The difficulty of estimating the mass transfer coefficient was discussed in Chapter 3. A power-law dependence is assumed where $k_y \propto v^{0.6}$. Kinmonth's measurements on the drying of 100 mm x 50 mm Pinus Radiata suggest $k_y \propto 2 \times 10^{-2} \text{kgm}^{-2} \text{s}^{-1}$ for $v = 1.5 \text{ m s}^{-1}$. Hence $k_y^* \approx 0.0157 \nu^{0.6}$.

For drying at $T_G = 60^\circ \text{C}$ and $T_W = 50^\circ \text{C}$, $\phi_H = 0.88$ and $\kappa = 0.005$. The density of the air $\rho_G = 1.1 \text{ kgm}^{-3}$ and the density of the dry wood $\rho_S = 400 \text{ kgm}^{-3}$. Under these drying conditions it will be seen in Chapter 6 that $\bar{x}_o = 1.7 \text{ kgkg}^{-1}$, $\bar{x}_{CR} = 0.90 \text{ kgkg}^{-1}$, and $\bar{x}_B = 0.06 \text{ kgkg}^{-1}$ such that the characteristic drying rate curve is approximately linear in shape. The mass flowrate of the air through the empty drier $G_B$ is given by:

$$G_B = \nu \rho_G \varepsilon$$

By using the results expressed above for drying a stack of timber 3 m wide, the parameters are then given by:

$$\lambda = \frac{0.075 \nu^{-0.4}}{b} , \quad \tau = \frac{0.0015 \nu^{0.6}}{d} , \quad \phi = 2$$

where $\theta$ is in hours. Generally for drying softwoods the spacing between layers of the boards $b$ is 25 mm. Hence $\zeta$ will essentially be a function of velocity while $\tau$ is a function of
velocity and the board thickness. The values of $\xi$ and $\nu$ are presented in Table 12.

Table 12

Batch-Drying Parameters for Timber Kiln

<table>
<thead>
<tr>
<th>Air velocity /m s$^{-1}$</th>
<th>$\tau /\theta \times$ hour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d=25$ mm</td>
</tr>
<tr>
<td>1</td>
<td>0.060</td>
</tr>
<tr>
<td>2</td>
<td>0.091</td>
</tr>
<tr>
<td>3</td>
<td>0.116</td>
</tr>
<tr>
<td>4</td>
<td>0.137</td>
</tr>
<tr>
<td>5</td>
<td>0.158</td>
</tr>
</tbody>
</table>

For most modern kilns the air velocity is 2.5 m s$^{-1}$ so that the NTU for the dryer is approximately 2. So most of the solutions presented in Parts 3 and 4 are directly applicable to the simulation of a timber kiln. The final moisture content desired for kiln-drying is $\bar{X} = 0.012$ kg kg$^{-1}$ which implies $\Phi_{\text{END}} = 0.07$. The drying times applicable to the parameters presented in Table 12 can be generated from eqn. (73) where the end of drying is given by $\Phi = 0.07$ at $\xi = \lambda$.

Table 13

Estimated Times for the Drying of Wood in a Timber Kiln

<table>
<thead>
<tr>
<th>Air velocity /m s$^{-1}$</th>
<th>$\theta$/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d=25$ mm</td>
</tr>
<tr>
<td>1</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>89</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>43</td>
</tr>
</tbody>
</table>
Fig. 155 to 158  
Moisture content lag $\Delta \theta = \tilde{\phi}(\lambda, \tau) - \tilde{\phi}(0, \tau)$ between dryer inlet and outlet as a function of time and outlet moisture content for dryers of various lengths (0.1 to 10) for characteristic drying-rate curves $f(\phi) = \phi$ and $f(\phi) = \sqrt{\phi}$.
These drying times are similar to those obtained in practice as 25 mm thick timber normally requires 3-4 days while 50 mm thick timber takes 5-7 days to dry. The times for the thicker wood are underestimated because the identical characteristic curve has been assumed for all thicknesses. In reality the drying time is roughly proportional to the square of thickness rather than directly proportional to the thickness as suggested by these results (see eqn. 171). These results probably also exaggerate the significance of the air-velocity. In order to simulate the wood drying times more accurately it is necessary to consider the internal moisture movement.

**Part 5**

**Batch-Dryer Operation with Air-Flow reversals**

One of the unsatisfactory features of batch drying is the inherent lag of the solids moisture content between the dryer outlet and inlet. This phenomenon leads to uneven moisture content distributions which can impair the final product quality. Nevertheless, as the drying proceeds the moisture content difference between the inlet and outlet of the dryer, $\Delta\phi$, increases from zero until it reaches a maximum value and then decreases to zero again as the outlet moisture content tends to the equilibrium moisture content. This natural smoothing is illustrated in Figs. 155-6 for an initial moisture content $\phi_0 = 2$ and a linear $f(\phi)$ curve, for dryers of various lengths. For the limit of a long dryer the difference $\Delta\phi$ is directly proportional to the outlet moisture content for all $\tau$. The effect of the shape of the characteristic curve is illustrated by Figs. 157-8. It can be seen that for the case of $f(\phi) = \sqrt{\phi}$ that $\Delta\phi$ is greater for short dryers than the linear case, while for long dryers the difference $\Delta\phi$ appears to be independent of the shape of the curve. Although adequate natural smoothing is obtained if the outlet solids are dried close to the equilibrium moisture content this may require a long drying time. Figs. 155 and 157 show that this problem is magnified within short dryers, while for long dryers the extra time required is probably insignificant compared to the
total drying time. Consequently to reduce drying times for short dryers (NTU < 1) flow reversals are used to accelerate the smoothing of the moisture content distribution. It will be seen later that drying using a flow reversal policy will decrease the magnitude of $\Delta Q$ as well as saving time.

Similar problems may be encountered in the operation of other types of batch equipment but the benefit of flow-reversals has received little attention in the engineering literature, except for work by Brenner$^{36}$ and Horn$^{37}$. Brenner studied the mass transfer from an assembly of particles in a flow field while Horn considered the effects of flow reversal in a system of interconnected tanks containing sources and sinks for a reacting solute. Neither studies are directly applicable to this problem, but some of the results will also apply to batch-drying with flow reversals. Further empirical observations were made by Stevens and Johnston$^{44}$ in an examination of flow-reversals during timber drying. They suggest that if the direction of airflow is switched frequently the inlet potential can be intensified to readjust the inlet rate back to that attained for one-way circulation.

The rest of this chapter is devoted to a study of flow-reversals and batch-drying. The problem is posed and after an examination of the constant-rate period of drying, a special case of falling-rate period drying is solved to illustrate the advantages of flow-reversing. By further considerations of a policy for an infinite number of flow reversals, optimal finite flow-reversal sequences are suggested.

Batch-Drying Model with Flow Reversals

Brenner$^{36}$ observed that under certain conditions, reversal of flow, i.e., replacing the fluid velocity vector $v$ by $-v$ at every position, does not change the overall rate of mass transfer. In general, the local transfer rate changes. For batch-drying, the fluid velocity is constant and independent of position in the direction of airflow but the same result will apply.
For drying in the forward direction (hereafter referred to as the positive direction), equations (47) and (49) will describe the moisture transfer for the positive direction

\[ \frac{\partial \Phi}{\partial t} = \frac{\partial \pi}{\partial \xi} = -\pi f(\Phi) \]  
\[ \frac{\partial \pi}{\partial \xi} \]  

and

\[ \frac{1}{f} \frac{\partial \Phi}{\partial \xi} + \Phi = \Gamma(\xi) \]  

When drying with the airflow in the opposite direction, the equations (47) and (49) will still describe the moisture transfer provided a new distance co-ordinate is defined which increases in magnitude with the direction of flow. However it is more convenient to calculate the mass transfer in terms of the original distance co-ordinate \( \xi \). So, the sense of the distance co-ordinate will be reversed in order to describe drying with flow in the negative direction. Hence the adjoint equations of equations (47) and (49) will describe the moisture transfer during this period for the negative direction.

\[ \frac{\partial \Phi}{\partial t} = \frac{\partial \pi}{\partial \xi} = -\pi f(\Phi) \]  
\[ \frac{\partial \pi}{\partial \xi} \]  

and

\[ \frac{1}{f} \frac{\partial \Phi}{\partial \xi} + \Phi = \Pi'(\xi) \]  

In order to describe the drying process fully, the functions \( \Pi(\xi) \) and \( \Pi'(\xi) \) must be determined at the end of each flow reversal period. The solution of equations (47), (49), (172) and (173) will now be studied for both the constant-rate and falling-rate periods of drying.

**Flow reversals during the Constant-Rate Period**

If the first flow period is in the positive direction, and the moisture content distribution is initially uniformly equal to \( \Phi_o \), then the solution to equations (47) and (49) will
be the same as that developed earlier for one-way circulation.

\[ \Phi = \Phi_0 - e^{-\zeta \tau} \]
\[ \pi = e^{-\zeta} \]  

(174) \hspace{1cm} (175)

Then if the first reversal period ends after time \( \tau = R_1 \), integration of equations (172) and (173) yields the following equations (where \( \pi(\lambda, \tau) = 1 \) now):

\[ \Phi = \Phi_0 - e^{-\zeta} R_1 - e^\zeta - \lambda \tau \]
\[ \pi = e^{\zeta - \lambda} \]  

(176) \hspace{1cm} (177)

where \( \tau \) is the elapsed time since the change of flow direction. By repeating the above procedure, it can be shown, in general, that for flow in the positive direction, the drying during the \((2N)\)th flow reversal period is described by equa.

(175) and (178).

\[ \pi = e^{-\zeta} \]  

(175)

\[ \Phi = \Phi_0 - e^{-\zeta} \lambda (R_2 + R_4 + \ldots + R_{2N-2}) - e^\zeta (R_1 + R_2 + \ldots + R_{N-1} + \ddots) \]  

(178)

Similarly the drying during the \((2N+1)\)th flow reversal period, in the negative direction, is described by equations (177) and (179).

i.e., \( \pi = e^{\zeta - \lambda} \)  

(177)

\[ \Phi = \Phi_0 - e^{-\zeta} (R_2 + R_4 + \ldots + R_{2N}) - e^\zeta - \lambda (R_2 + R_4 + \ldots + R_{2N-2} + \ddots) \]  

(179)

The end of the constant-rate period will be reached when the solids moisture content at either end of the dryer has been reduced to the critical moisture content. In fact, the best
use is made of the constant-rate period when the solids at both ends of the dryer attain the critical point simultaneously. By substituting the condition $\theta = 0$ at $\chi = 0$ and $\chi = \chi$ into equations (178) and (179) after an even number of flow-reversal periods, $2N$, have elapsed, we obtain the following equality.

\[ R_1 + R_2 + \ldots + R_N = R_2 + R_4 + \ldots + R_{2N} = \tau_R \]  

(180)

Hence the total time spent drying with the airflow moving in the positive direction is identically equal to the total time expended while drying with the airflow circulating in the opposite direction. From equation (178), the time spent drying in one direction is given by

\[ \tau_R = \frac{\Phi_0 - 1}{1 + e^{-\lambda}} \]  

(181)

So the total length of the constant-rate period will now be $2\tau_R$.

\[ \tau_1 = 2\left(\frac{\Phi_0 - 1}{1 + e^{-\lambda}}\right) \]  

(182)

A comparison of equation (182) with equation (55) when $\Phi_D = 0$ indicates why a time advantage is gained from flow-reversing. This is because the length of the constant-rate period is increased by flow reversing which means the highest possible average drying rate for the whole stack of solids is maintained for a longer period of time. The significance of this time saving will depend upon the relative lengths of both the constant and falling-rate periods.

The result expressed in equation (180) also suggests that strictly only two flow-reversal periods of length $\tau_R$ are
necessary during the constant-rate period to attain the
smoothed distribution given in equation (183) at the time the
critical point is reached.

\[ \Phi(\zeta, \tau^R_f) = \Phi_o - (\Phi_o - 1) \left( e^{-\zeta} + e^{\zeta-\lambda} \right) \frac{1}{1 + e^{-\lambda}} \]  

(183)

In practice, more reversals may be necessary particularly if
drying stresses developed within the solid are related to
moisture content gradients in the direction of the airflow.

An example of the improved smoothing of the moisture
content distribution obtained by flow switching is shown in
Table 14. For the parameters of \( \Phi_o = 2 \) and \( \lambda = 1 \), the moisture
content profiles at the end of the constant-rate period are
compared for the cases of with (eqn. (183)) and without flow
reversals (eqn. (56)).

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>( \Phi(\zeta, \tau^R_f) - \text{eqn. (183)} )</th>
<th>( \Phi(\zeta, \tau^R_f) - \text{eqn. (56)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.2</td>
<td>1.072</td>
<td>1.181</td>
</tr>
<tr>
<td>0.4</td>
<td>1.108</td>
<td>1.330</td>
</tr>
<tr>
<td>0.5</td>
<td>1.110</td>
<td>1.394</td>
</tr>
<tr>
<td>0.6</td>
<td>1.108</td>
<td>1.451</td>
</tr>
<tr>
<td>0.8</td>
<td>1.072</td>
<td>1.551</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000</td>
<td>1.632</td>
</tr>
</tbody>
</table>

Thus \( \Delta \Phi \) is reduced from 0.632 to 0.110. For the flow-
reversal case, the distribution is symmetrical about the
Fig. 159  Humidity potential profiles for constant-rate period drying with and without flow reversals. Time saving that arises due to enhanced inlet potential (i.e., time averaged potential over pair of flow-reversal periods at inlet equal to that for one-way circulation) and the rate enhancement factor as a function of dryer length.
middle of the dryer so under these conditions the lag between the inlet (or outlet) of the dryer and the middle provides the criterion of uniformity of the solids moisture content. This smoothed moisture content distribution is a direct result of the improved potentials attained across the dryer. The time averaged potential \( \bar{\pi} = (e^{-\xi} + e^{\xi - \lambda})/2 \) over a pair of flow-reversal periods is plotted in Fig. 159 and compared to the potential profile for one-way circulation for a dryer of 1 NTU. The dashed line shows the enhanced potential profile if Steven's and Johnston's proposal is followed and the averaged inlet potential increased to equal the inlet potential for one-way circulation. As the actual drying time is directly proportional to the inlet potential for constant inlet conditions, then this enhanced rate could reduce the drying time by one-way circulation for a dryer of 1 NTU by 32 per cent. This result will hold providing that the shape of the drying-rate curve is independent of the magnitude of the potential. The time-saving that arises from this cause is illustrated in Fig. 159 for dryers of various lengths. It can be seen that the time saving in the limit of long dryers can be as high as 50 per cent. The rate enhancement factor \( C_R(\sqrt[\pi]{R_o}) \) is also shown in Fig. 159. There will also be further reductions in time because of the flow-reversal smoothing.

Flow Reversals During the Falling-Rate Period

The same type of analysis using equations (47) and (49) and equations (172) and (173) can be followed in order to describe the drying with flow reversals during the second and third periods. It has already been noted that the functions \( P(\xi) \) and \( P'(\xi) \) are not given by the constant \( \phi_o \) but instead are exponential functions of \( \xi \) and \( \phi_o \). One necessary condition for obtaining an analytical solution to equations (47), (49), (172) and (173) is the ability to integrate the functions \( P(\xi) \) and \( P'(\xi) \) with respect to \( \xi \). However it can be shown that
even for the linear \( f(\Phi) \) case that for a period of falling-rate drying preceded by a constant-rate period that the nature of the functions \( f(\zeta) \) and \( f'(\zeta) \) is such that analytical integration is impossible. So in order to solve the general flow-reversal problem, analogue or digital computer methods are necessary.

There is one special case that can be solved analytically. This problem is for a linear falling-rate curve, with an initial moisture content everywhere uniformly equal to the critical moisture content. Although this special case will be rarely met in practice, the solution illustrates some interesting features of flow-reversal policies that will be common to the general problem. Furthermore, if the dryer is short, the moisture content distribution at the end of the constant-rate period after flow-reversaling, may approximate to the initial moisture content distribution for this case.

**Solution for falling-rate period drying with flow reversals when \( f(\Phi) = \Phi \) and \( \Phi_0 = 1 \)**

The direction of flow during the first flow reversal period is taken to be positive so that the moisture content will be given by the solution for the one-way circulation problem presented by Van Meel when the initial moisture content is less than or equal to the critical moisture content and \( f(\Phi) = \Phi \).

\[
\phi(\zeta, \tau) = \frac{\phi(0, \tau)e^\zeta}{1 - \Phi(0, \tau) + \Phi(0, \tau)e^\zeta} \tag{134}
\]

Say at same value of \( \phi(0, R_1) = p_1 \), we desire to reverse the direction of flow then the moisture content distribution at this instant will be given by:

\[
\phi(\zeta, R_1) = \frac{p_1e^\zeta}{1 - p_1 + p_1e^\zeta} \tag{135}
\]
As the second flow-reversal period is described by equations (172) and (173) with \( f(\theta) = \theta \), the distribution in equation (185) must satisfy equation (173). Hence the function \( P'(\zeta) \) is given by

\[
P'(\zeta) = \frac{1 - 2(1 - p_1)}{p_1 e^\zeta + 1 - p_1}
\]

so that the moisture content changes during the second flow reversal period according to the solution of equation (187).

\[
\frac{-1}{\theta} \frac{\partial \psi}{\partial \zeta} = 1 - 2(1 - p_1) \quad \frac{p_1 e^\zeta + 1 - p_1}{(1 - p_1)}
\]

This partial differential equation is of the form of a Riccati differential equation and can be linearised by the substitution:

\[
\eta = A(\tau) e^\int \psi d\zeta
\]

\[
(188)
\]

as

\[
\frac{1}{\eta} \frac{\partial \eta}{\partial \zeta} = -\psi
\]

\[
(189)
\]

and

\[
\frac{\partial \psi}{\partial \zeta} = \frac{1}{\eta} \frac{\partial^2 \eta}{\partial \zeta^2} - \frac{1}{\eta^2} \left( \frac{\partial \eta}{\partial \zeta} \right)^2
\]

\[
(190)
\]
By substituting equations (189) and (190) into equation (187), the simpler partial differential equation (191) results.

\[
\frac{\partial^2 \eta}{\partial \zeta^2} = \frac{\partial}{\partial \zeta} \left[ \frac{2(1 - p_1)}{p_1 e^{\zeta} + 1 - p_1} \right] \tag{191}
\]

This equation can be integrated once to yield

\[
\frac{\partial \eta}{\partial \zeta} = \frac{B(\tau) e^\zeta}{(1 - p_1 + p_1 e^\zeta)^2} \tag{192}
\]

and integrating once again the function \( \eta \) is obtained

\[
\eta = \frac{B(\tau) (e^\zeta - 1)}{1 - p_1 + p_1 e^\zeta} + C(\tau) \tag{193}
\]

By substituting back into equation (189) the moisture content is given by the expression

\[
\phi(\zeta, \tau) = \frac{e^\zeta}{(1 - p_1 + p_1 e^\zeta)[E(\tau)(1 - p_1 + p_1 e^\zeta) + 1 - e^\zeta]} \tag{194}
\]

where the function \( E(\tau) \) (\( E = C/B \)) remains to be determined.

It is convenient to relate the moisture content to the inlet moisture content (at \( \zeta = 0 \)) so this implies

\[
E(\tau) = 1/\Phi(0, \tau) \tag{195}
\]

and so the moisture content during the second flow reversal period is given by the expression below.

\[
\Phi(\zeta, \tau) = \frac{\Phi(0, \tau)e^\zeta}{(1 - p_1 + p_1 e^\zeta)[\Phi(0, \tau)(1 - e^\zeta) + 1 - p_1 + p_1 e^\zeta]} \tag{196}
\]

The end of the second flow reversal period will be reached when \( \Phi(0, \tau) = \Phi_2 \), so that equation (196) with \( \Phi(0, \tau) = \Phi_2 \)
will become the initial moisture content distribution for the third reversal period. This distribution must satisfy equation (49).

For the third flow reversal period the following differential equation is required to be solved,

$$\frac{1}{\phi} \frac{\partial \phi}{\partial \zeta} + \phi = 1 - 2(p_1 - p_2)e^{\zeta}$$

$$\phi \frac{\partial \zeta}{\partial t} = \frac{1 - (p_1 - p_2) + (p_1 - p_2) e^\zeta}{1 - (p_1 - p_2) + (p_1 - p_2) e^\zeta}$$

By defining \( \eta = A(\tau) \int_0^\tau \phi \, d\zeta \), this equation can be solved by the same procedure as before, to give the solution:

$$\Phi(\zeta, \tau) = \frac{\Phi(0, \tau)e^\zeta}{1 - (p_1 - p_2) + (p_1 - p_2) e^\zeta}$$

It can further be shown that, in general, the moisture content distribution for the \( n \)th flow reversal period is given by

$$\Phi(\zeta, \tau) = \frac{\Phi(0, \tau)e^\zeta}{1 - \sum_{m=1}^n (-1)^m p_{m-1}(e^{\zeta} - 1)[\Phi(0, \tau)(1 - e^\zeta)(-1)^{n+1}] + \sum_{m=1}^n (-1)^m p_{m-1}(e^{\zeta} - 1)]}$$

where \( p_0 = 0 \).

By putting \( \sum_{m=1}^n (-1)^m p_{m-1} = p \), equations (199), (47) and (172) can be combined to generate the potential profiles.
Fig. 160 to 163 Characteristic moisture content distributions for a batch-dryer operated with airflow reversals at constant time-intervals $\Delta t_s$.

- $f(\phi) = \phi$, $\phi_0 = 1$, $\Pi_0 = 0.000.79$, $\lambda = 2$
- $\Delta t_s = 20\Pi_0$ and $100\Pi_0$. 
Figs. 164 to 167  Characteristic moisture content distributions for batch-dryer operated with airflow reversals when the maximum moisture content difference \[ \Delta \phi (\lambda T) - \phi(0 T) \] reaches a given value \( \Delta \phi_{s} \).

\[ f(\bar{\phi}) = \bar{\phi} , \quad \bar{\phi}_{0} = 1 , \quad \Pi_{0} = 0.00679 , \quad \lambda = 2 \]

\( \Delta \phi_{s} = 0.1 \) and 0.4.
i.e. flow in positive direction

\[ n(\xi, \tau) = \frac{(1 - p + pe^\xi)}{[\phi(0, \tau)(e^\xi - 1) + 1 - p + pe^\xi]} \]  

and flow in negative direction

\[ n(\xi, \tau) = \frac{[\phi(0, \tau)(1 - e^\lambda) + 1 - p + pe^\lambda]}{[\phi(0, \tau)(1 - e^\lambda) + 1 - p + pe^\lambda]} \]  

The time required for each flow reversal period, \( \tau_{RP} \), can be similarly calculated,

i.e. flow in positive direction

\[ \tau_{RP}^n = \ln(p_{n-1}/p_n) \]  

and flow in negative direction

\[ \tau_{RP}^n = \ln \left( \frac{\left[ p_n(1 - e^\lambda) + 1 - p_n + p_{n-1}e^\lambda \right] p_{n-1}}{\left[ p_{n-1}(1 - e^\lambda) + 1 - p_{n-2} + p_{n-1}e^\lambda \right] p_n} \right) \]  

where \( p_o = 1 \) for these two equations.

By using equations (199)-(203) a batch-drying flow-reversal policy can be generated by a sequence of \( \phi(0, \tau) \) values which signify the various switch points. Two types of flow reversal schemes are illustrated in Figs. 160-167 where the moisture content profiles for each case are plotted against time and against distance. The first scheme involves flow-reversing after constant intervals of time \( \Delta \tau_s \) while the second method flow-switches when the maximum moisture-content difference across the dryer reaches a given value \( \Delta \theta_s \). These results were generated by the computer program "SWITCH" (see Appendix 2F) which evaluates the above equations.

Both flow-switching schemes are effective in smoothing the moisture content distribution. The second scheme appears potentially more effective at smoothing the distributions when
Fig. 168 and 169  Limiting moisture content difference $\Delta \Phi \left[ \Phi(\gamma_2, \tau) - \Phi(0, \tau) \right]$ across dryer for infinite flow reversals for dryers of various length ($\lambda = 0.2$ to $1.5$) as a function of $p$ and inlet moisture content $\Phi(0, \tau)$. 

$f(\Phi) = \Phi, \quad \Phi_e = 1.$
the total number of flow switches is small. When large numbers of flow switches are made, both schemes have similar effectiveness. An interesting feature of the moisture content difference switching scheme is that there are limiting values of $\Delta \Phi_S$ for a given length dryer which cannot be achieved at certain stages of the drying. The magnitude of these values is considered in the next section by studying infinite flow reversal policies.

**Infinite flow reversal policies**

It is interesting to consider the ultimate smoothing of the moisture-content distribution by using an infinite number of flow reversals. At this limit, the moisture content of the solids at the inlet and the outlet of the dryer will be identical and the maximum moisture content will occur at the centre of the dryer. Substituting these conditions into equation (199), gives the following expression for $\Delta \Phi$.

\[
\Delta \Phi = \frac{\left[ e^{\lambda} \right]^{\lambda/2}}{(1-p+pe^{\lambda})(1-e^{\lambda})} \left[ \left( e^{\lambda} \right)^{\lambda/2} + (1-e^{\lambda}) \right]
\]

This function is plotted in Fig. 168 as a function of $p$ and $\lambda$. Fig. 169 shows $\Delta \Phi$ as a function of $\Phi(0, \tau)$ and $\lambda$. ($\Phi(0, \tau) = [e^{\lambda} \left(1-p+pe^{\lambda}\right)^{\lambda/2}]/\left(1-e^{\lambda}\right)/(1-p+pe^{\lambda})$). The values of $p$ presented in Fig. 168 will give the smoothest possible moisture content distribution when the end of drying is desired at a particular value of $\Phi(0, \tau)$.

The effectiveness of flow reversals for smoothing the moisture content distribution is indicated in Table 15. Here the maximum moisture content difference $\Delta \Phi_{\text{max}}$ obtained in drying by one-way circulation for $\Phi = 1$ and a linear $f(\Phi)$.
curve is compared with the difference that results from an infinite flow reversal policy, for dryers of various lengths. A further example, that of drying from $\Phi_0 = 1$ according to the $f(\Phi) = \Phi^2$ curve, is also presented for the case of one-way circulation.

Table 15

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(\Phi_{\text{max}}) one-way circulation</th>
<th>(f(\Phi) = \Phi^{1/4}) one-way circulation</th>
<th>(f(\Phi) = \Phi) Infinite flow reversals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.025</td>
<td>0.039</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.2</td>
<td>0.050</td>
<td>0.077</td>
<td>0.001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.124</td>
<td>0.191</td>
<td>0.006</td>
</tr>
<tr>
<td>1.0</td>
<td>0.245</td>
<td>0.370</td>
<td>0.024</td>
</tr>
<tr>
<td>1.5</td>
<td>0.358</td>
<td>0.528</td>
<td>0.052</td>
</tr>
<tr>
<td>2.0</td>
<td>0.462</td>
<td>0.660</td>
<td>0.089</td>
</tr>
<tr>
<td>3.0</td>
<td>0.635</td>
<td>0.841</td>
<td>0.183</td>
</tr>
<tr>
<td>5.0</td>
<td>0.848</td>
<td>0.974</td>
<td>0.398</td>
</tr>
</tbody>
</table>

These results corroborate an earlier conclusion that flow-reversal, for smoothing the moisture content distribution, is most advantageous for short dryers. The effect of the shape of the characteristic curve suggests that for "fast" characteristic curves flow reversal will be more beneficial than for drying solids according to "slow" rate curves. For drying from initial moisture contents greater than 1, a similar order of improvement for the moisture content difference is expected.

Optimal Flow-reversal Policies

The previous section has indicated the advantages of flow
reversing for smoothing the moisture content distribution and for reducing the drying time. Hence there is incentive to use the flow reversals to obtain the optimal advantage. An infinite flow reversal policy by definition is the global optimum in all respects. However a large number of flow reversals may be expensive because of the demands placed upon the dryer operator. This section now considers the use of a finite number of flow reversals to obtain the maximum improvement possible in the batch dryer. Such an optimal policy will depend upon the criterion of optimality. The following criteria are considered:

(i) to obtain the smoothest possible moisture content distribution at the end of drying, i.e. minimise the integral

\[ \int_0^\infty \left| \phi - \phi_{\text{min}} \right| d\xi \]

(ii) to minimise the drying time required to ensure that all the solids within the dryer have been dried to a moisture content less than a limiting value \( \phi_{\text{END}} \).

Each case will now be examined in turn to examine the worth of each criterion. The solution for the case of a linear falling rate drying curve, where the initial moisture content is uniformly equal to the critical moisture content, is used to consider the optimal flow-switching times when only two flow-reversal periods are used.

Case (i)

For this case we wish to find the minimum of the integral

\[ \int_0^\infty \left( \phi - \phi_{\text{min}} \right) d\xi \]

From an examination of the moisture content distribution during the second flow reversal period one can distinguish two phases. During the first stage the minimum moisture content will be at the inlet. As the outlet moisture content decreases faster than the inlet moisture content during this flow period, the condition will be reached when the inlet and outlet moisture contents are identical. After this point is reached the minimum moisture content will
be given by the outlet moisture content. From equation (199), we then wish to minimise $E_\phi$ where $E_\phi$ is given by:

\[
E_\phi = \int_0^\lambda \frac{\Phi(0, \tau)e^\zeta}{(1-p+pe^\zeta)(\Phi(0, \tau)(1-e^\zeta)+1-p+pe^\zeta)} - \Phi(0, \tau) \frac{d\zeta}{(1-p+pe^\zeta)(\Phi(0, \tau)(1-e^\zeta)+1-p+pe^\zeta)}
\]

\[
\Phi(0, \tau) > e^{\lambda} - (1-p+pe^\lambda)^2
\]

\[
(1-p+pe^\lambda)(1-e^\lambda)
\]

(205a)

and

\[
E_\phi = \int_0^\lambda \frac{\Phi(0, \tau)e^\zeta}{(1-p+pe^\zeta)(\Phi(0, \tau)(1-e^\zeta)+1-p+pe^\zeta)} - \frac{\Phi(0, \tau)e^\lambda}{(1-p+pe^\lambda)(\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda)} \frac{d\zeta}{(1-p+pe^\lambda)(\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda)}
\]

\[
\Phi(0, \tau) > e^{\lambda} - (1-p+pe^\lambda)^2
\]

\[
(1-p+pe^\lambda)(1-e^\lambda)
\]

(205b)

By evaluating the integrals, the simpler expressions for $E_\phi$ are obtained.

\[
E_\phi = \ln \left\{ \frac{1-p+pe^\lambda}{\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda} \right\} - \frac{\Phi(0, \tau)e^\lambda}{(1-p+pe^\lambda)(\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda) + 1-p+pe^\lambda}
\]

\[
\Phi(0, \tau) > e^{\lambda} - (1-p+pe^\lambda)^2
\]

\[
(1-p+pe^\lambda)(1-e^\lambda)
\]

(206a)

and

\[
E_\phi = \ln \left\{ \frac{1-p+pe^\lambda}{\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda} \right\} - \frac{\Phi(0, \tau)e^\lambda}{(1-p+pe^\lambda)(\Phi(0, \tau)(1-e^\lambda)+1-p+pe^\lambda)}
\]

\[
\Phi(0, \tau) < e^{\lambda} - (1-p+pe^\lambda)^2
\]

\[
(1-p+pe^\lambda)(1-e^\lambda)
\]

(206b)
Fig. 170 and 171. Moisture content difference integral which represents smoothness of moisture content profile across dryer due to flow reversals. Fig. 170 shows that the smoothest distribution results when $\xi = 0.5$, or the maximum moisture content occurs at dryer midpoint. Fig. 171 presents parameters $\rho$ and $\Phi_{\text{max}}$ for smoothest moisture content distributions at end of drying.

$f(\xi) = \Phi, \; \Phi_0 = 1, \; \lambda = 1$. 
In order to find the minimum of $E_D$, it is necessary to examine the derivative with respect to $\phi(0, \tau)$. However in this case the derivative is discontinuous so no simple relationship can be derived from equations (206). It can be shown by iterative calculation that the smoothest distribution results when the two end moisture contents are identical. The computer program "FLOS" (see Appendix 21) solves equations (206) for the moisture content to be less than or equal to $\phi_{END}$ throughout the dryer. Figs. 170 and 171 show how the minimum value of $E_D$ is obtained when $\phi_{END}$ is equal to the moisture content at the middle of the dryer. At this instant the inlet and outlet moisture contents are identical.

**Case (ii)**

For this criterion we wish to know the minimum time required to reduce the moisture content of the solids throughout the whole dryer below a certain moisture content $\phi_{END}$. Hence we need to know the maximum moisture content, $\phi_{max}$, within the dryer at any time. The position of this maximum, $\zeta_{max}$, can be obtained by operating on equation (199) and putting $\phi/\phi_{END} = 0$.

\[
\phi_{max} = \sqrt{\frac{(1-p)[1-p+\phi(0, \tau)]}{p[\phi_{END} - \phi(0, \tau)]}}
\]

The final inlet moisture content $\phi(0, \tau)$ required in order to obtain $\phi_{max}$ equal to $\phi_{END}$ is given by

\[
\phi(0, \tau) = \frac{\phi_{max}^2 \phi_{END}}{\phi_{max} - \phi_{END}(1-p+pe)}
\]

The time taken to reach this position is given by equations (202) and (203)

\[
T_D = \ln \left[1-e^\lambda + (1-p+pe^\lambda)/\phi(0, \tau)\right]
\]
Fig. 172 to 175. Flow switching for optimal flow-reversal policy to minimise the drying time. Fig. 172 and 173 present parameters \( p \) (switch inlet moisture content), final inlet moisture content, and final maximum moisture content as functions of dryer length \( \lambda \). Fig. 174 and 175 compare the drying time with and without flow reversals and show the time savings due to an optimal flow-reversal policy as a function of the final maximum moisture content.

\[
f(\Phi) = \Phi, \quad \Phi_0 = 1.
\]
These three equations (207)-(209) were solved iteratively to find the minimum value of $T_D$ for a given value of $\Phi_{\text{END}}$ using the computer program "FLOREV" (see Appendix 2K). In this case it was also found that a flow-reversal switch to obtain the final moisture content identically equal at both ends of the dryer provides the optimal time policy. These results are plotted in Figs. 172 to 174 to indicate the switch-points and drying times for drying with flow reversals in dryers where $\lambda = 0.5, 1, 2$ and 4.

That both criteria of the smoothest moisture content distribution and minimum time are satisfied by the same end condition makes a flow reversal policy very attractive. The time savings by flow-reversal policy are indicated in Fig. 175. For long dryers the reduction in time can be considerable. It is interesting to note that for short dryers the flow-reversal policy greatly smooths the moisture-content distribution with small savings in time while for a long dryer the time reduction is large for small improvements in the moisture-content distribution.

The above results can be extended to the case of an initial moisture content less than 1. As the moisture content can be redefined in terms of the initial moisture content to make $\Phi_0 = 1$, the results expressed earlier will be applicable to this problem where the switch-points are linearly related to the ratio of the initial moisture contents. The solution for the optimal policy for two flow-reversal periods can also be extended to include policies for n flow reversal periods, where $n$ is an even number. For the criterion of the smoothest possible moisture content distribution, examination of equation (199) shows the result expressed earlier for the two-period case equally applies to this case. This is because the parameter $p$ that determines the final optimal switch-point is independent of all except the $(n-1)^{\text{th}}$ flow switch parameter. The drying time $T_D$ that has elapsed after n flow reversal periods is obtained by summing the n values of $\tau_k$ as given by equations (202) and (203).
On simplification we find

\[ T_D = \ln \left( \frac{1 - e^{-\lambda}}{(1 - P_{n-1} + P_{n-1} e^\lambda) / P_n} \right) \]  

which is identical in form to equation (209). So if this equation is solved to find the minimum drying time for the \( n \) flow-reversal periods, exactly the same results will be obtained as for the case of two flow-reversal periods. This result is analogous to that discussed earlier for the constant rate period.

These conclusions for the use of finite numbers of flow reversals show that it is not necessary to follow the locus of the infinite flow reversal policy. But that the optimal final moisture content distribution can be obtained with only two flow reversal periods just as during the constant rate period. This result will hold provided that the prior history of the moisture content distribution does not affect the material properties by degradation. Furthermore the information required for predicting the optimal flow-reversal policy can be obtained from the infinite flow reversal solution. The only advantage in using more than two flow reversal periods is that for the case of two periods the timing of the switch-point is critical. When many flow-reversal periods are used the switching need not be particularly accurate in order to produce a reasonably optimal result.
The General Problem of Batch-Drying with Flow Reversals

The special case of drying solids in a batch dryer when the characteristic drying rate curve is linear and the initial moisture content is equal to the critical moisture content has been rigorously studied. Infinite and finite flow-reversal policies have been examined for this case. It now remains to consider the generalised problem with airflow reversals where the initial moisture content of the solid may exceed the critical moisture content. The solid may also dry according to a characteristic drying rate curve of complex shape.

This problem cannot be solved analytically but the finite difference and analogue computer methods outlined in Part 2 can be used to obtain a solution. The method presented earlier for the finite-difference solution requires modification. After the first flow reversal the function \( P(\chi) \) in equation (49) will not be given by the constant \( \phi_0 \) for the case of batch-drying without flow reversals. So it is necessary to solve the batch-drying equations by using equations (47) and (48) rather than equations (47) and (57).

The finite difference analogue of equation (48) about the point \( (\chi_{i-\frac{1}{2}}, \tau_{n+\frac{1}{2}}) \) gives the following expression for generating \( \phi_{i,n+1} \):

\[
\Phi_{i,n+1} = \Phi_{i,n} + \left( \Phi_{i-1,n+1} - \Phi_{i-1,n} \right) \left\{ \frac{1}{f_{i-1,n+1}} - \frac{\Delta \chi}{2} \right\} \\
\left\{ \frac{1}{f_{i,n+\frac{1}{2}}} + \frac{\Delta \chi}{2} \right\}
\]

The equations are solved exactly as outlined in Part 3(e). The inlet moisture content is generated from equation (118) and the moisture content profile across the dryer at a given time level is generated by equation (212). The potential profile must also be calculated iteratively. The finite
Fig. 176 to 179

Moisture content distributions for batch-dryer as function of time and distance when airflow direction reversed at constant intervals of time $\Delta t_0$ (0.1, 0.2, 0.5, 1.0). Flow reversal solution compared to case of one-way circulation (dotted lines).

$f(\Theta) = \Theta$, $\Theta_0 = 2$, $\lambda = 2$. 
difference analogue of equation (47) about the point \((x_{i-\frac{1}{2}},\tau_{n+1})\) gives the following equation for predicting \(\tau\) from the known moisture content profile.

\[
P_{i,n} = P_{i-1,n+1} + \left( \frac{2-\Delta \tau}{2+\Delta \tau} \right) \left( f_{i-\frac{1}{2},n+1} - f_{i-\frac{1}{2},n+1} \right)
\]

(213)

These equations are solved by a modified version of the computer-program "RAMBAG" (see Appendix 2E(vi)).

It is straightforward to simulate batch-drying of solids when the airflow direction is reversed at discrete intervals of time. This is provided the moisture content values are stored in an array as a function of distance across the dryer. Instead of simulating the airflow reversal it is mathematically convenient to reverse the positions of the drying solid then at each switch-point it is only necessary to invert the array of moisture content values. From that point on the solution can be generated in the same manner as before.

The simulation technique is illustrated by the case of a linear drying rate curve, an initial moisture content \(\phi_0 = 2\) for a dryer of length \(\lambda = 2\). The moisture content \(\phi\) is plotted as a function of \(\tau\) in Figs. 176-179 when the flow has been reversed at constant intervals of time \(\Delta \tau = 0.1, 0.2, 0.5, 1, 0\). The improvement in the moisture content distribution is clearly shown. The same qualitative effects were also observed by Bateson and Stevens and Pratt from timber-kiln studies with frequent flow reversing (Fig. 150 and 151).

The analogue computer technique is also well-suited to modelling such a complex problem. A similar procedure is followed to that proposed for the finite difference method. At each switch-point the solution is stopped and the values of the moisture contents are noted at various positions within the dryer (i.e., from the integrator outputs shown on Fig. 78). The solution is then restarted after the initial conditions for the integrators have been reset in the opposite order using the
Fig. 180 to 182. Analogue computer solution for batch-dryer showing interaction of variable inlet potential schedule (25 mm low-grade Australian schedule) and flow-reversal switch.

f(Φ) given by reference 22, Φ₀ = 1.22, λ = 1.05.
Fig. 183 to 186  Batch-drying curves comparing one-way circulation with an infinite flow reversal policy at various positions across the dryer.

\[ f(\bar{\Phi}) = \bar{\Phi}, \quad \lambda = 2, \quad \bar{\Phi}_0 = 2. \]
**Fig. 187 to 190**  Batch-drying curves comparing one-way circulation with an infinite flow-reversal policy at various positions across the dryer.

\[ f(\Phi) = (1, 1; 0.5, 0.8; 0.1, 0.4), \lambda = 2, \Phi = 2. \]
Fig. 191 to 194. Batch-drying curves comparing one-way circulation with an infinite flow reversal policy at various positions across the dryer.

\[ f(\mathcal{D}) = (1, 1; 0.6, 0.4; 0.4, 0.2), \lambda = 2, \mathcal{D}_0 = 2 \]
measured values. By using comparators and function relays a step-changing inlet potential policy was superimposed. This solution is illustrated in Figs. 180-2 for the 25 mm low-grade Australian drying schedule presented in Table 1 with one switch in airflow direction. These results indicate that if both the airflow direction and potential changes are in phase then dangerously high drying rates can be obtained for a short time.

It has been shown for one special case that the infinite flow reversal problem also provides the information for an optimal flow reversal sequence when the number of reversals is finite. Hence the infinite flow reversal policy is considered for the general problem. Such a policy can be approximated to the solution of the batch-drying equations for a large number of flow reversals \( N > 100 \) provided each second flow reversal period ends when the moisture content is identical at both ends of the dryer. The drying curves for a dryer of parameters \( \Phi_0 = 2 \) and \( \lambda = 2 \) are compared for one-way circulation and infinite flow reversals for three types of characteristic curve. These graphs, presented in Figs. 183-194, show the improvement of uniformity of the drying-rates and moisture content gained from the infinite flow reversal policy.

The parameters for the infinite flow reversal case are presented in Figs. 195-203. The drying time, inlet moisture content and the moisture content at the centre of the dryer are presented as functions of \( p \) for drying from an initial moisture content \( \Phi_0 = 2 \) according to the three types of characteristic curve. The function \( p \) is defined as earlier such that it is given by \( \frac{\lambda^2}{\lambda(p^2+1)} (-1)^{\lambda} \), the summation of the inlet moisture contents at the \( (n-1) \) switchpoints in the airflow direction.

As the special cases for flow-reversal studied earlier enabled the optimal flow reversal policy to be generated from the infinite flow reversal solution, it is of interest to see if such a result applies to the general case. That is, for a given value of \( p \) can the same end condition (i.e. the instant when the inlet and outlet moisture contents are equal) be obtained using only one flow reversal switch as for the case of infinite flow reversals. This idea is considered below in Table 16 for two situations when drying from an initial moisture content \( \Phi_0 = 2 \) for a dryer of length \( \lambda = 2 \).
Fig. 195 to 203  Infinite flow reversal parameters for a batch dryer. Drying time, inlet moisture content $\Phi(0;\tilde{p})$, and maximum moisture content $\Phi(\lambda_2;\tilde{p})$ as functions of switch moisture contents $\tilde{p}$ for dryers of various lengths ($\lambda=0.5$ to 4) and various characteristic drying-rate curves.

Fig. 195, 196, 197 - $f(\Phi) = \Phi$, $\lambda=2$, $\Phi_0=2$.
Fig. 198, 199, 200 - $f(\Phi) = (1,1; 0.5,0.8; 0.4,0.4)$, $\lambda=2$, $\Phi_0=2$.
Fig. 201, 202, 203 - $f(\Phi) = (1,1; 0.6,0.4; 0.4,0.2)$, $\lambda=2$, $\Phi_0=2$. 
Comparison of Moisture Content Distribution when Inlet and Outlet Moisture Contents Identical for One Flow Reversal Switch and Infinite Flow Reversal Switches

<table>
<thead>
<tr>
<th>$f(\Phi)$</th>
<th>One Flow Reversal Switch</th>
<th>Infinite Flow Reversal Switches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>(1.1; 0.5, 0.8, 0.1, 0.4)</td>
<td>0.354</td>
<td>3.589</td>
</tr>
<tr>
<td>Linear</td>
<td>0.405</td>
<td>4.122</td>
</tr>
</tbody>
</table>

It can be seen that the infinite flow reversal solution predicts a flow reversal switch that is too early for the case of one switch. Nevertheless, the time taken and the centre moisture contents are similar in each case. These differences could be due to errors developed in the numerical solution. But even if the numerical solutions are accurate, the infinite flow reversal solutions provide much useful information as to the required drying times to achieve particular end moisture contents at the middle of the dryer. Equally important it appears the result can be generalised that one or a few flow-reversal switches will produce as smooth as possible moisture content distribution as with infinite switching and that the minimum drying time is also independent of the number of flow switches provided that at the end of drying the inlet and outlet moisture contents are identical. The switchpoints to attain this latter condition will have to be determined iteratively from the finite difference solution to the required problem.
Part 5 Further features of the Batch-Drying Process with regards to Schedule Design

The previous sections of this chapter have examined the dynamics of a batch-dryer under various conditions including complex operating policies such as flow-reversing and variable inlet potentials. In order to apply these involved procedures a detailed knowledge of the drying rates is often helpful.

Some researchers have considered for drying that there are limiting drying rates above which degradation will take place. Strictly such an assumption is incorrect (see Chapter 8) but it still provides a reasonable criterion for schedule design. It has also been noted previously (see Part 1) that several workers have falsely assumed the maximum rate to occur within the dryer when the critical moisture content reaches that point in the dryer. The first section of Part 6 calculates the maximum drying rate within the dryer and it is shown that this rate maximum decays with time as the maximum sweeps through the dryer. A knowledge of this decaying rate maximum provides a basis for enhancing the inlet potential.

The second section calculates the theoretical energy efficiency of a batch-dryer. In order to provide controlled inlet conditions it is necessary to sacrifice energy. This arises because of two reasons. Firstly, the dryer is operated at temperatures greater than the surroundings to promote heat transfer and the moisture movement through the solid. Most batch-dryers rely on recirculation of the humid exhaust air to maintain thermal economy. However, in order to reduce the humidity of the recirculation stream to keep the inlet conditions constant, it is necessary to expel some of the humid air and to replace it with cool fresh air. So some energy is lost when the humid air is expelled and further energy is required to heat the cool dry air to the desired inlet temperature. The magnitude of these energy requirements compared with the energy required for the vaporisation of the moisture is examined to predict the theoretical energy efficiency of a batch-dryer.
A The Drying Rate Maximum within the Dryer

The rate maximum will be derived below for the case of one-way circulation for any type of characteristic drying-rate curve. As the f-curve is discontinuous at \( \Phi = 1 \) it is necessary to consider each period of drying separately.

During the constant-rate period the maximum drying rate is attained at the inlet and is numerically equal to 1. For subsequent drying periods the rate maxima shifts from the inlet and sweeps through the dryer.

From equations (47) and (64) the drying rate for the second period as \( f = 1 \) is given by:

\[
\frac{\partial \Phi}{\partial t} = -\left( \frac{\Phi}{\Phi_0} - \Phi \right)
\]

For a maximum of \( \partial \Phi / \partial t \) we require

\[
\frac{\partial^2 \Phi}{\partial t^2} = 0 = -\left( \frac{\Phi}{\Phi_0} - \Phi \right)(1 - f(0, \tau))
\]

which suggests that a maximum is obtained for the trivial cases when \( \Phi = \Phi_0 \) and \( \Phi(0, \tau) = 1 \). Closer examination of equation (214) shows that no true maxima is obtained during this period and that the rate increases as \( \Phi \) decreases. The maximum during this period will be obtained at the end of this period when \( \Phi = 1 \). Hence the maximum rate is:

\[
\frac{\partial \Phi}{\partial t}_{\text{max}} = -\left( \frac{\Phi}{\Phi_0} - 1 \right)
\]

As the value of \( [\Phi_0 - \Phi(0, \tau)] \) is increasing as the critical point moves through the dryer, this maximum rate is decreasing.

The drying rate during the third period of drying is described by the equation

i.e.,

\[
\frac{\partial \Phi}{\partial t} = -\left( \frac{\Phi}{\Phi_0} - \Phi \right)f
\]

\[
\frac{\partial \Phi}{\partial t} \left[ \Phi_0 - \Phi(0, \tau) \right]
\]
The maximum drying rate is given by the solution of equation (218).

\[
\frac{\partial^2 \Phi}{\partial t^2} = - (\Phi - \Phi_0) \frac{df}{d\Phi} \left[ (\Phi - \Phi_0) \frac{df}{d\Phi} + f(0, \tau) \right]
\]  

(218)

The position of the maximum drying rate will occur for the value of \( \Phi \) given by the solution of the nonlinear equation (219),

\[ (\Phi - \Phi_0) \frac{df}{d\Phi} = f \cdot f(0, \tau) \]  

(219)

and the drying rate maximum will take the following value,

\[ \frac{\partial \Phi}{\partial t_{\text{max}}} = - \frac{(f \cdot f(0, \tau))}{[\Phi - \Phi(0, \tau)] \frac{df}{d\Phi}} \]

(220)

The two equations (219) and (220) are conditional upon \( \Phi \leq 1 \).

At the limit when \( \Phi = 1 \) then;

\[ (\Phi - 1) \frac{df}{d\Phi} \bigg|_{\Phi = 1} = 1 - f(0, \tau) \]  

(221)

so for the maximum drying rate to occur when the critical moisture content reaches that position in the bed the following inequality must hold.

\[ f(0, \tau) \geq 1 + b_C \Phi_0 \]

(222)

where \( b_C = \frac{df}{d\Phi} \bigg|_{\Phi = 1} \)

When the equality given by equation (222) is satisfied a true maximum is observed at the critical point. For the other case, when \( f(0, \tau) > 1 + b_C \Phi_0 \), an artificial maximum will be observed at the critical point discontinuity. This can be shown by equations (215) and (218).
as these derivatives will have opposite signs about this point. When the drying rate maximum coincides with the critical point the maximum drying rate will be given by equation (216).

Equation (222) shows the range for which the assumption made by Krischer and Jaeschke\(^5\), Key\(^7\) and Mykelstad\(^10\) will hold that the maximum drying rate will occur at the critical point. To illustrate the error that arises from this assumption, the above equations are evaluated for the case of the linearised characteristic curve, i.e., \( f = b_c \phi + 1 - b_c \).

From equation (219) the value of \( \phi \) when the maximum is reached \( \Phi_{\text{max}} \) is:

\[
\Phi_{\text{max}} = \phi(0, T) + \frac{\phi_0}{2}
\]  
(223)

and the maximum drying-rate is:

\[
\frac{\partial \Phi}{\partial T}_{\text{max}} = -\frac{f_{\text{max}}}{2}
\]  
(224)

From equations (99) and (219) it can be shown that

\[
f_{\text{max}} = 1 - b_c + b_c \Phi_{\text{max}} = \frac{f_0}{2(1 - e^{-f_0 T})}
\]  
(225)

and

\[
\frac{\partial \Phi}{\partial T}_{\text{max}} = -\frac{f_0}{4(1 - e^{-f_0 T})}
\]  
(226)

An absolute criterion for the maximum to occur always at the critical moisture content can be derived from equation (222) by putting \( f(0, \psi) = 0 \).
Fig. 204 to 206  Moisture content $\Phi_{\text{max}}$ at drying-rate maxima for positions within batch-dryer for range of characteristic drying-rate curves ($f(\Phi) = 1 - b_c + b_c \Phi$) as function of initial moisture content (1.25 to 3.0).

$b_c = 0.4, 0.6, 0.8, 1.0, 1.5$ and 2.0.
Fig. 207 to 212. Rate enhancement factor $\left(1 + \frac{\Delta \theta}{\Delta \theta_{\text{max}}} \right)$ and maximum drying rate within batch-dryer for initial moisture content $\theta_0=2$ for dryers of various length ($\lambda=0.5$ to 4) and various characteristic drying-rate curves as functions of outlet moisture content.

- Fig. 207 and 208: $f(\theta) = \theta$.
- Fig. 209 and 210: $f(\theta) = (1, 1.5, 0.8, 0.1, 0.4)$. 
- Fig. 211: $f(\theta) = (1, 1.5, 0.4, 0.6, 0.4, 0.2)$.
For $1 < \phi_0 \leq 1 + \frac{b_c}{b_G}$, the rate maxima will occur at the critical moisture content for the first part of the dryer.

The limiting position $\zeta_k$ for $\phi_{\text{max}} = 1$ is given by:

$$
\zeta_k = \frac{1}{1-b_c^2-b_c \phi_0} \ln \left( \frac{2}{1+b_c^2-b_c \phi_0} \right)
$$

For the remainder of the dryer, $\zeta > \zeta_k$, the drying-rate maximum will correspond to a moisture content less than the critical moisture content as given by equation (225). These equations are illustrated by Figs. 204 to 206 where $\phi_{\text{max}}$ is plotted against $\zeta$ for values of $b_c = 0.4, 0.6, 0.8, 1.0, 1.5$ and 2.0.

A suggestion has been made that a knowledge of these drying-rate maxima could provide information for enhancing the inlet conditions once the constant-rate period has ended. The maximum drying rate is plotted in Figs. 207, 209 and 211, for three types of characteristic curve (linear, Quasilinear ($1, 1; 0.5, 0.8; 0.1, 0.4$), and Quasilinear ($1, 1; 0.6, 0.4; 0.4, 0.2$)) for dryers of NTU = 0.5, 1, 2 and 4 when $\phi_0 = 2$.

These curves have been plotted as a function of the outlet moisture content. When the moisture content is small ($\phi < 0.5$) the rate function is practically independent of the dryer NTU. Figs. 208, 210 and 212 show the possible rate enhancement factor $\frac{\partial \phi_{\text{max}}}{\partial \zeta}$ which would restore the maximum rate to the initial inlet rate. These results show that for long dryers (NTU $> 2$) it is necessary to monitor both the inlet and outlet moisture content so as to know when the rate can be safely boosted. This is demonstrated in Fig. 207 where it can be seen for NTU = 2 that the inlet moisture content gives good control for $\frac{\partial \phi}{\partial \zeta} > 0.5$ and the outlet moisture content will provide control sensitivity for the remainder of the drying.
i.e., when $\frac{\partial \delta}{\partial t} \leq 0.5$.

For one-way circulation a sophisticated control scheme is necessary. When flow-reversals are used it will be sufficient to monitor the moisture content at either end of the dryer. For this latter case the drying rate maxima will vary in a different manner from that shown in Figs. 207, 209 and 211.

If a variable inlet potential policy is superimposed upon the flow-reversal policy it has already been observed that unsatisfactory rates are obtained for changes made together, Hence care will be needed to ensure that an increasing inlet potential scheme and a flow-reversal policy complement and enhance each other.

B. Theoretical Thermal Efficiency of a Batch-Dryer

We are interested in finding the energy requirements for batch-drying. It was mentioned earlier that energy is used for vaporisation of the moisture, for heating the makeup fresh air which replaces the energy lost through the ventilators, and some energy is lost through the walls of the dryer.

The recirculation ratio, $r$, of the humid air necessary to be mixed with the cool fresh air to obtain the desired inlet humidity potential $\pi_M^I$ is given by the following expression.

$$\pi_M^I = r \pi_M^{out} + (1 - r) \pi_M^o$$  \hspace{1cm} (229)

In equation (229), $\pi_M^o$ is the fresh-air humidity potential and $\pi_M^{out}$ is the outlet humidity potential. This equation also implies that the residence time of the air in the dryer is negligible compared with the drying time (for timber-drying the residence time is 1 sec compared to a drying time of 100 hours).

This recirculation of air will also produce a change in the temperature potential at the dryer inlet.
and from eqn. (229)

\[ \eta_H = \frac{\eta_H^{\text{out}} (1 - \eta_H^{\text{in}})}{\eta_H^{\text{out}} - \eta_H^{\text{in}}} + \eta_H^{\text{in}} \]

As the fresh air is colder than that at the inlet position this temperature potential \( \eta_H \) will be less than or equal to the desired inlet temperature potential \( \eta_H^0 \). In this case energy \( Q \) will be required to restore the inlet potential to the desired level.

i.e. \[ Q + C_G (T_G - T_0) + Y_G [\Delta H_v + C_v (T_G - T_0)] = C_G (T_G^0 - T_0) \]

\[ + Y_G [\Delta H_v + C_v (T_G^0 - T_G)] \]

where \( T_G \) is the temperature of the resulting air mixture and \( T_G^0 \) is the desired inlet temperature. Rearrangement of eqn. (232) gives

\[ Q = C_G (T_G^0 - T_G) + Y_G C_v (T_G^0 - T_G) \]

The temperature \( T_G \) can be obtained from the temperature potential \( \eta_H \) \( \eta_H = \frac{T_G - T_W}{T_G^0 - T_W} \) so, equation (233) becomes:

\[ Q = (C_G + Y_G C_v) (1 - \eta_H) (T_G^0 - T_W) \]

If the inlet conditions to the dryer are held constant, then \( \eta_M^0 = 1 \) and \( \eta_H^{\text{out}} = \eta_H^{\text{in}} \). Combining equations (234) and (231) then gives

\[ Q = \frac{C_H (\eta_H^{\text{in}} - T_W) (\eta_M^{\text{in}} - \eta_H^{\text{in}}) (1 - \pi(\lambda, \tau))}{\{\eta_M^{\text{in}} - \pi(\lambda, \tau)\}} \]

i.e. \[ \eta_H = r \eta_H^{\text{out}} + (1-r) \eta_H^{\text{in}} \] (230)
Q is the specific energy required per unit mass of dry air flowing through the dryer. The complete energy requirements for drying, ignoring losses through the walls, will be given by the summation of Q over the total time of drying for the volume of air circulated through the dryer.

\[ Q_{\text{TOT}} = C_H G H A (T_G^0 - T_H^0) \int_0^\infty \frac{(H^0 - H^0)}{H^0 - H(\lambda, \tau)} d\tau \]

\[ = C H (1-c) G H (\overline{\chi_G} - \overline{\chi_E}) A \int_0^\infty \frac{(H^0 - H^0)}{H^0 - H(\lambda, \tau)} d\tau \]

where \( A \) is the cross-sectional area of the dryer at right angles to the airflow direction. Hence by monitoring the outlet potential the energy requirements for drying can be estimated.

The thermal efficiency of the drying process can be gauged by comparing \( Q_{\text{TOT}} \) with the energy required for vaporisation of the moisture, \( Q_{\text{VAP}} \).

\[ Q_{\text{VAP}} = \rho S (1-c) V \Delta H_{\text{VE}} (\overline{\chi_G} - \overline{\chi_E}) \]

where \( V \) is the volume of the dryer. By combining equations (236) and (237) the theoretical efficiency \( E_T \) is given by:

\[ E_T = \frac{Q_{\text{VAP}}}{Q_{\text{TOT}}} \]

\[ = \frac{\rho S V}{C H (1-c) G H (\overline{\chi_G} - \overline{\chi_E})} \int_0^\infty \frac{(H^0 - H^0)}{H^0 - H(\lambda, \tau)} d\tau \]  

The integral in equation (238) is difficult to solve analytically. When the initial moisture is greater than 1, it can only be solved for the case of a linear characteristic drying rate curve when \( \Phi_0 = 2 \). To evaluate the integral, it is convenient to split it into three parts, each part corr-
Fig. 2.13 and 2.14. Fresh-air humidity and temperature potentials as a function of wet-bulb temperature and temperature difference $v^o$. Fresh-air state, $T_e = 0^oC$ and $Y_e = 0.0$ kg/kg$^{-1}$. 
Fig. 215 to 217
Theoretical thermal efficiency
Comparator of batch-dryer
as a function of wet-bulb
temperature (30°C to 90°C)
and temperature difference
$\Delta T$ for dryers of length
$\lambda = 0.1, 1.0, 10$. (eqn. 24.0)
$\frac{f(\Delta T)}{\Delta T} = \Phi$, $\Phi_0 = 2$. 
responding to a particular drying period,

\[ Q_{\text{TOT}} = \int_{0}^{\beta_{0}} \frac{A_{I}(1-e^{-\lambda})}{B_{I}e^{-\lambda}} \, d\tau + \int_{0}^{\lambda} \frac{A_{I}(1-\pi(\lambda, \chi_{CR}))}{B_{I}e^{-\lambda}} \, d\tau \cdot d\chi_{CR} \]

\[ + \int_{0}^{\infty} \frac{A_{I}(1-\pi(\lambda, \chi_{CR}))}{B_{I}e^{-\lambda}} \, d\tau \cdot d\chi_{CR} \quad (239) \]

where \( A_{I} = \pi_{H}^{0} - \pi_{H}^{0} \) and \( B_{I} = \pi_{M}^{0} \). The functions \( \pi(\lambda, \chi_{CR}) \)
are given by equations (68) and (69). Expansion of the integrals leads to the following expression for \( Q_{\text{TOT}} \):

\[ Q_{\text{TOT}} = \frac{A_{I}}{B_{I}} \left\{ \ln \left( \frac{1+e^{2\lambda}}{2} \right) - (1-B_{I}) \ln \left[ \frac{2B_{I} e^{\lambda} + e^{-\lambda} - e^{\lambda}}{2B_{I} e^{2\lambda}} \right] \right\} \]

\[ + \frac{(1-B_{I})B_{I}}{\sqrt{B_{I} e^{-2\lambda}}} \ln \left[ \frac{B_{I} - 1}{B_{I} - 1 + e^{-2\lambda}} \right] + \frac{B_{I} - 1}{B_{I} - 1 + e^{-2\lambda}} \ln \left( B_{I} e^{\lambda} A_{I} \right) \]

\[ + \ln \left[ \frac{2B_{I} - 1 - e^{-2\lambda}}{(B_{I} - 1)(1 + e^{-2\lambda})} \right] \quad (240) \]

This function was evaluated for dryers of various lengths and for various drying conditions over the range of \( T_{W} = 30^\circ \) to \( 90^\circ C \) and \( \Delta T = 2 \) to \( 50^\circ C \). The state of the fresh air is
assumed to be given by \( T_{G} = 0^\circ C \) at 0 per cent relative humidity. The parameters \( -\pi_{H}^{0} \) and \( \pi_{M}^{0} \) are plotted in Figs. 213 and 214 as a function of the wet- and dry- bulb temperatures. Similarly the efficiency \( E_{I} \) is plotted in Figs. 215 to 217 for \( \lambda = 0.1, 1, \) and 10 as a function of the same variables. It can be seen that the efficiency is higher for the longer dryers. However the most significant factor is the
inlet temperatures of the airstream. The efficiency decreases as the driving force is increased and the efficiency increases
Fig. 218 and 219  

Significance of initial moisture content and shape of characteristic drying-rate curve upon theoretical thermal efficiency of a batch-dryer as a function of dryer length ($\lambda = 0.5$ to 8).
as the dry-bulb temperature increases. For drying at low temperatures the efficiency is poor. The significance of the shape of the characteristic drying-rate curve and the initial moisture content upon the efficiency is shown to be small by Figs. 218 and 219.

This efficiency has been termed the theoretical energy efficiency of the dryer. The actual thermal efficiency of a dryer will be less than the values presented in Figs. 215 to 217. Energy will be lost through the walls of the dryer and further energy will be lost when the warm dry solids are taken from the dryer. Both these losses will be roughly proportional to the difference between the dry-bulb temperature and the surroundings. Further losses will result if the dryer doors are leaky and if the ventilators are not operated in a fully effective manner. This latter influence may greatly reduce the efficiency predicted above, particularly when the wet-bulb temperature depression is small. Under these conditions only a small proportion of the air will be exhausted and so exact control of the ventilators will be difficult.

The results presented above also show that the enhancement of drying schedules by increasing the inlet potential will decrease the efficiency. It is also expected that flow reversal policies will decrease the efficiency. This can be deduced as flow reversals effectively decrease the extensiveness of the dryer. Hence any operating policies that improve the drying time and smooth the moisture-content distribution will require higher energy usage.

This Chapter, overall, suggests methods of examining the operating costs of a batch-dryer. The lumped parameter model is adequate to allow accurate estimations of optimal operation procedures and to calculate energy requirements. The additional costs of labour and the cost of operating the air-circulation fans can be simply deduced from knowing the drying times. To examine the costs of degradation, consideration of the internal moisture movement is necessary. This aspect of batch-drying is studied in the following chapters.
REFERENCES


