

# Optimizing Instantaneous and Ramping Reserves with Different Response Speeds for Contingencies—Part I: Methodology

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**Abstract**—The need to efficiently manage different reserve types, inertia, and the largest credible contingency is critical to the continued uptake of Variable Renewable Energy (VRE) and security of a power system. This paper presents an optimization formulation for the dispatch of contingency reserves to satisfy frequency limits. Reserve options are divided into two categories: instantaneous reserve (a stepped response with a time delay) and a ramped response with both a time delay and ramp rate. The problem is to optimally select reserve capacity from a set of offers with different response speeds, i.e. different time delays, ramp rates, and prices. The optimal reserve dispatch requires the frequency transient for a contingency to be constrained against frequency limits that occur at specified times after the contingency. The first result of this paper is the demonstration of convexity of the feasible solution space, thereby retaining desirable uniqueness properties of the optimal solution, and polynomial time performance of a solver. The feasible solution space is characterized by piecewise constraints whose components are quadratic. The second result of this paper is the development of a solving methodology that utilizes the convex properties of the proposed formulation.

**Index Terms**—Reserve Markets, Primary Frequency Control Reserve, Contingency Reserve, Quadratically Constrained Programming, Convex Optimization.

## NOMENCLATURE

### Offers

IL	Interruptible Load, Instantaneous Reserve
SR	Spinning Reserve, Ramping Reserve
$p_i$	Variable for the amount of IL dispatched [MW]
$u_i$	Variable for the amount of SR dispatched [MW]
$P_i(t; p_i)$	Transient for IL output [MW]
$U_i(t; u_i)$	Transient for SR output [MW]
$p_i^{max}$	Maximum output for IL [MW]
$u_i^{max}$	Maximum output for SR [MW]
$t_{i,p}/t_{i,u}$	Time of IL/SR initiation [s]
$t_{i,e}$	Time SR stops ramping [s]
$t_{i,e}^{max}$	Maximum time SR can stop ramping [s]
$g_i$	Ramp rate of SR offer [MW/s]
$c_{i,p}/c_{i,u}$	Reserve price for IL/SR [\$/MW]

### Frequency Limits and Risk

$t_{j,l}$	Time of frequency limit step change [s]
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$f_j$	Frequency level
$R$	Risk, i.e. Largest Credible Contingency [MW]
$H$	Inertia [MWs]
$t_{min}$	First time of minimum frequency [s]
$f_{lim}$	Frequency limit function or the frequency limit at $t_{min}$

### Sets

$Q_B$	IL offers initiated before $t_{min}$
$Q_E$	IL offers initiated at $t_{min}$
$Q_{B,j}$	IL offers initiated before $t_{j,l}$
$W_B$	SR offers required to stop before $t_{min}$
$W_T$	SR offers required to stop after $t_{min}$
$W_{B,j}$	SR offers required to stop before $t_{j,l}$
$W_{T,j}$	SR offers required to stop after $t_{j,l}$

### Counts

$N_p/N_u$	Total number of IL/SR offers
$N_c$	Total number of frequency limit step changes
$N_r$	Estimated number of possible regions
$N_s$	Worst-case number of regions to solve

### Indices

$i$	Individual reserve offers
$j$	Frequency limit step changes

## I. INTRODUCTION

VARIABLE Renewable Energy (VRE) presents a challenge to power system security. VRE reduces total power system inertia, and can change the profile of the largest credible contingency on a power system. This has created a requirement for faster reserves to avoid large frequency excursions and for new methodologies for assessing the most efficient organization of resources.

As a result of potential VRE uptake, New Zealand [1], Australia [2], Texas [3], and Nordic Countries [4] are monitoring changes in inertia and noticing reductions, as well as the system operators of Ireland, Great Britain, and Quebec. The concern over reduced inertia has created limits on how much VRE can be dispatched in some systems, such as Ireland's System Non-Synchronous Penetration (SNSP) limit [5], [6], or a limit on the minimum amount of inertia on a power system, such as in Australia [7], which has resulted in plans to build synchronous condensers in South Australia [8].

In conjunction with these limits on inertia, faster reserves markets have been developed in these countries. National

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Grid in Great Britain has developed the Enhanced Frequency Response (EFR) market for providers that can respond within one second [9]. Ireland has a market for Fast Frequency Response (FFR) for reserve that can respond two seconds after a contingency [10], and Australia is also considering Fast Frequency Response arrangements [11], although it already has a Battery Energy Storage System (BESS) providing reserves [12]. These faster reserve categories can be compared to the established primary reserve categories, which define reserve capability at 5, 6, and 10 seconds for Ireland, Australia, and Great Britain respectively.

There is a clear need for holistic approaches to achieve the most efficient organization of resources. Approaches that can determine the optimal balance between minimum inertia, maximum credible contingency size, and required reserve capacity between various types of reserve with their different response characteristics. Approaches that can optimize resources for frequency stability, rather than for fixed inertia and individual reserve requirements. This paper improves the generalization of response characteristics in reserve optimization from other approaches in the literature, so that a wider range of resources can be more accurately co-optimized in Unit-Commitment and Economic Dispatch problems for greater efficiency and uptake of VRE.

The literature assesses three constraints on frequency stability within an optimization framework: maximum absolute Rate of Change of Frequency (RoCoF) for transient stability, the minimum frequency constraint to avoid activation of under frequency load shedding schemes, and a steady-state frequency limit to bring the power system back to a normal state of operation. The minimum frequency constraint presents the greatest challenge for optimization formulations, as it requires evaluating a solution of a differential equation. Depending on how the constraints are formulated, the minimum frequency constraint can significantly limit how generally the reserve offers can be defined. [13] considers a single total reserve quantity in its minimum frequency constraints, and develops linear constraints from dynamic simulations. [14] and [15] also consider the total reserve quantity in evaluating frequency dynamics. In contrast to [13], they develop their minimum frequency constraints by Mixed Integer LP (MILP) reformulations of analytical equations.

[16], [17] consider the total individual reserve quantity for two different reserve types in their constraints, which is an incremental improvement on [13]–[15]. [16], [17] follow [13] forming minimum frequency constraints by applying linearizations to dynamic simulation results. Similarly [18] applies a two reserve type model to obtain an analytical value for the minimum frequency. The constraint is non-linear and multiple approximation techniques are applied to keep it solvable using MILP.

The next set of approaches provides further improvement in generality. [19] specifies a different ramp rate for each reserve provider, and a shared time delay. However in the development of the minimum frequency constraint it makes the worst case assumption that all reserve takes the same amount of time to respond, i.e. all reserve finishes ramping at the same time. [20] extends [19] to include fast acting battery response at

the time of the contingency. [21] has a two stage optimization while considering reserves of different ramp rates, but does not include a delay after the contingency. [22]’s approach is the most generalized, by discretizing time so that reserve response is uniquely defined at each discrete time step. To retain linear constraints, [22] assumes that the whole reserve response is scaled in proportion to how much reserve capacity is available.

This paper extends the generality of reserve offers in [19], [20] by letting each reserve provider be uniquely defined by a time delay and constant ramp rate. If the response is sufficiently fast, an instantaneous change in output is allowed. This is completed while avoiding the linearizations of [20], and allowing for inertia and the size of the largest credible contingency to be variables in the optimization. If multiple reserve offers are combined to approximate more complicated reserve types, then the proposed approach can quickly approximate the level of generality found in [22], while avoiding the expansion of variables that [22] requires, and also avoiding the assumption that reserve responses are scaled in proportion to the reserve dispatched.

In addition to the above methods, [23] [24] presents a formulation that manages the minimum frequency constraint by iterating between the market optimization and dynamic simulations of a power system, where the results from each are used to update each others parameters in each step. This is not dissimilar to reserve procurement in New Zealand, which is done in a real-time co-optimized energy and reserve market. In New Zealand, the market solver (Scheduling, Pricing, and Dispatch [25]) iterates against the dynamic simulation tool (Reserve Management Tool [26]) to find the optimal requirement for reserves. However, these approaches do not represent an improvement in the generality of reserve offer definitions within the optimization.

This paper presents a tool that can minimize the costs of reserve by selecting among reserve providers with different response speed and price. The constraints of the formulation require that the frequency during a contingency has to be above limits. Reserve capacity constraints are expressed as well. These constraints could be applied in energy and reserve co-optimizations that form the basis of many real-time electricity markets around the world. The main contributions of this work are:

- A new reserve optimization formulation that can optimize a number of reserve types.
- A proof of convexity in the feasible solution space.
- A new solving methodology that utilizes the convex properties of the formulation, even though some of the constraints are discontinuous, non-smooth, and piecewise.

This paper is organized as follows: Section II presents the optimization problem, Section III demonstrates the convexity of the feasible space, Section IV explains why the feasible space is divided into regions, Section V describes the searching algorithm to find the global minimum.

## II. PROBLEM FORMULATION

This section describes the optimization problem, defines the generalized reserve options, defines the frequency limits,

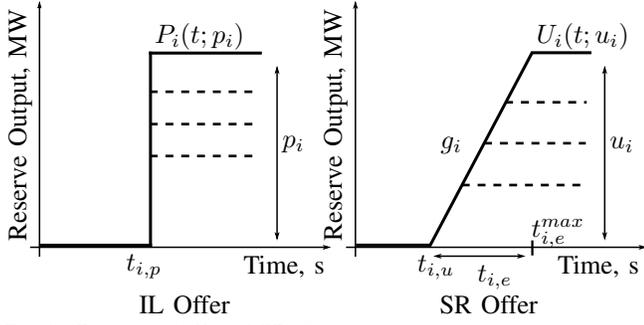


Fig. 1. Dynamics of IL and SR offers.

and gives the first step in the derivation of the constraints. A reserve offer is defined by a delay in initiation, and ramp rate, where each offer can have its own unique characteristics. The ramp rate can be instantaneous, as shown on the left in Fig. 1. The instantaneous reserve,  $P_i(t; p_i)$  in (1), is called IL reserve after Interruptible Load. Potential sources of IL include industrial refrigeration, large water pumps, and industrial furnaces. The finite ramping reserve,  $U_i(t; u_i)$  in (2), is called SR after Spinning Reserve. This naming convention does not exclude new types of reserve, as new reserve can be approximated by IL and SR reserve types when appropriate.

$$P_i(t; p_i) = \begin{cases} 0 & t < t_{i,p} \\ p_i & t \geq t_{i,p} \end{cases} \quad (1)$$

$$U_i(t; u_i) = \begin{cases} 0 & t < t_{i,u} \\ g_i(t - t_{i,u}) & t_{i,u} \leq t < t_{i,u} + u_i/g_i \\ u_i & t_{i,u} + u_i/g_i = t_{i,e} \leq t \end{cases} \quad (2)$$

The index  $i$  identifies the offer provider. To determine the impact on frequency the swing equation is used:

$$2H \frac{df}{dt} = -R + \sum_i P_i(t; p_i) + \sum_i U_i(t; u_i) \quad (3)$$

where a contingency of size  $R$  in MW occurs at time zero.  $R$  is also called the risk.  $H$  is the inertial constant in MWs and  $f$  is the frequency in per unit deviation from nominal. The frequency transient is found by integrating (3) with respect to time:

$$2Hf(t) = -Rt + \sum_{t \geq t_{i,p}} p_i(t - t_{i,p}) + \sum_{t \geq t_{i,u}} \left( U_i(t; u_i)(t - t_{i,u}) - \frac{U_i(t; u_i)^2}{2g_i} \right) \quad (4)$$

it is assumed that  $f(0) = 0$ . The frequency is kept above a limit  $f(t) \geq f_{lim}(t)$ . The limit is shown in Fig. 2 and (5).

$$f_{lim}(t) = \begin{cases} f_0 & 0 \leq t < t_{1,l} \\ f_1 & t_{1,l} \leq t < t_{2,l} \\ \vdots & \\ f_j & t_{j,l} \leq t < t_{j+1,l} \\ f_{N_c} & t_{N_c,l} \leq t \end{cases} \quad (5)$$

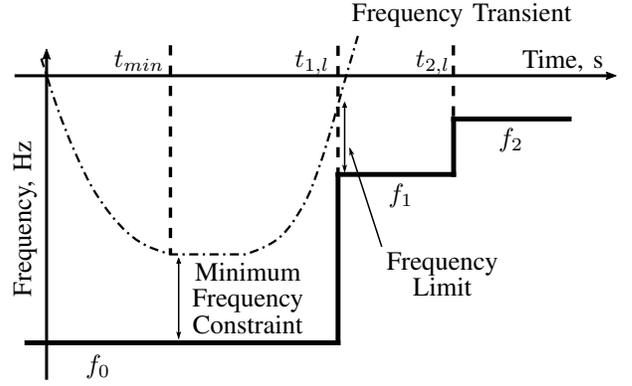


Fig. 2. The frequency limit for which the frequency transient has to be above.

The frequency limit has steps occurring at  $t_{j,l}$  where  $j$  is an index of the time the frequency limit transitions from  $f_{j-1}$  to  $f_j$ . A problem will have a finite number of frequency steps,  $N_c$ , including possibly none. A condition is placed on the steps that they are increasing,  $f_{j-1} < f_j$ .

The form of the constraint  $f(t) \geq f_{lim}(t)$  is difficult, as it has to be evaluated for every time  $t > 0$ , i.e. potentially an infinite number of constraints. The rest of this section converts this constraint into a finite number of practical constraints.

#### A. Reserve Requirement

The first constraint is that the total amount of reserve dispatched has to be greater than the risk,  $R$ . This ensures that the frequency transient reaches a minimum, and does not keep declining. This constraint is:

$$-R + \sum_i p_i + \sum_i u_i \geq 0 \quad (6)$$

Graphically, this is seen in Fig. 2 as a positive or zero gradient in the frequency transient after all reserve has been dispatched.

#### B. Frequency Limits

The second set of constraints ensure at each step in frequency limit,  $t_{j,l}$ , that the frequency transient is above the limit,  $f_j$ . Fig. 2 shows a gap between the frequency transient and limit at the time of the step. These constraints are binding when these two curves touch. This constraint is not required when there are no step changes in frequency limit. Substituting (4) into  $f(t) \geq f_{lim}(t)$  and evaluating at  $t = t_{j,l}$  gives the constraint:

$$2Hf_j \leq -Rt_{j,l} + \sum_{t_{i,p} \leq t_{j,l}} p_i(t_{j,l} - t_{i,p}) + \sum_{t_{i,u} \leq t_{j,l}} \left( U_i(t_{j,l}; u_i)(t_{j,l} - t_{i,u}) - \frac{U_i(t_{j,l}; u_i)^2}{2g_i} \right) \quad (7)$$

### C. Minimum Frequency Constraint

The third constraint requires that at the time of the minimum frequency, the frequency has to be above the limit. The first step is to define the time of the minimum frequency,  $t_{min}$ , as the first time the total reserve is equal to the risk:

$$-R + \sum_i P_i(t_{min}; p_i) + \sum_i U_i(t_{min}; u_i) \geq 0 \quad (8)$$

$$\forall t < t_{min} \quad -R + \sum_i P_i(t; p_i) + \sum_i U_i(t; u_i) < 0 \quad (9)$$

The reason why  $t_{min}$  is defined through two equations, (8) and (9), is because of IL offers. If before an IL offer,  $t_{i,p}$ , risk is greater than reserve and after  $t_{i,p}$  reserve is greater, then there is no time at which total reserve equals risk, yet  $t_{min}$  should equal  $t_{i,p}$ . Secondly, (9) removes a problem when the frequency transient at the minimum is flat, which is seen in Fig. 2, and uniquely defines  $t_{min}$  as the first time on that flat section.

The minimum frequency constraint is now similar to the other frequency limits, except  $t_{j,l}$  is replaced with  $t_{min}$ :

$$2Hf_{lim}(t_{min}) \leq -Rt_{min} + \sum_{t_{i,p} \leq t_{min}} p_i(t_{min} - t_{i,p}) + \sum_{t_{i,u} \leq t_{min}} \left( U_i(t_{min}; u_i)(t_{min} - t_{i,u}) - \frac{U_i(t_{min}; u_i)^2}{2g_i} \right) \quad (10)$$

It is with these three constraints and the definition of  $t_{min}$  that  $f(t) \geq f_{lim}(t)$  is implemented. The reserve requirement ensures the existences of  $t_{min}$  and the minimum frequency constraint.

### D. Reserve Constraints

To bound this problem it is necessary to add minimum and maximum limits to the reserve offers:

$$0 \leq p_i \leq p_i^{max} \quad (11)$$

$$0 \leq u_i \leq u_i^{max} \quad (12)$$

### E. Objective Function

Lastly an objective function is required: minimize

$$\sum_i c_{i,p} p_i + \sum_i c_{i,u} u_i \quad (13)$$

The prices on IL and SR offers are  $c_{i,p}$  and  $c_{i,u}$  respectively in \$/MW. They can either be negative, positive, or zero.

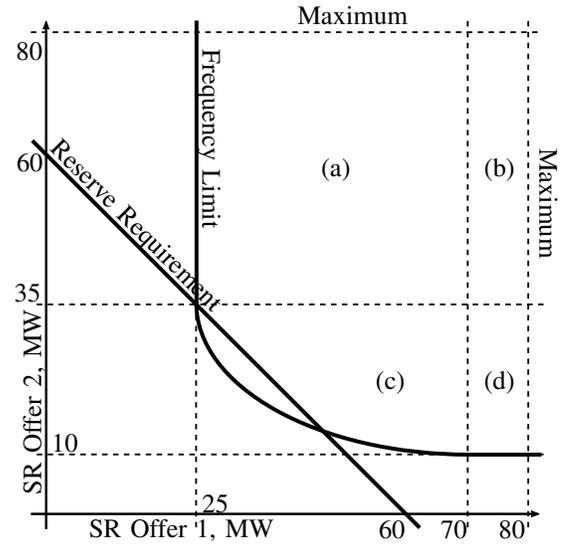


Fig. 3. Two SR offer problem with constraints.

### III. CONVEXITY

The current form of the constraints, (6)-(12), cannot be applied in an optimization formulation yet. Commonly optimizations are posed to have one set of equations without functions in the constraints. The presence of  $U_i(t, u_i)$ , a function rather than a variable, as well as incorporating  $t_{min}$  poses a difficulty. To avoid this, the feasible solution space is divided into regions defined by a set of constraint equations with only variables. Before this, it is important to demonstrate the convexity of the feasible solution space.

A convex space which is optimized with a linear objective function has useful properties: the optimal solution is usually unique, unless the feasible space has a optimal boundary parallel with the objective function; the optimal solution is on the boundary of the feasible space as long as the space is bounded; and a solving methodology can efficiently traverse the feasible space in the direction of minimal cost. A full proof of convexity is not given here, but an example is shown. The full proof can be found in [27]. For this problem the space is defined by the vector,  $\mathbf{v} = [H, R, p_1, \dots, p_{N_p}, u_1, \dots, u_{N_u}]$ , constrained by (6)-(12). This allows for each of these components to be variables in the optimization.  $N_p$  and  $N_u$  are the number of IL and SR offers respectively.

The example starts with two SR offers: the first is slow to initiate,  $t_{1,u} = 2.91$  s, but ramps quickly,  $g_1 = 22.68$  MW/s; the second is faster to start,  $t_{2,u} = 1.0$  s, but ramps at 7 MW/s. Both offers have a maximum capacity of 80 MW. To limit the number of dimensions,  $u_1$  and  $u_2$  will remain the only variables, therefore inertia is set at 5228 MWs and 60 MW of risk is to be covered. Initially frequency is limited to 48 Hz ( $f_0 = -0.04$ ), but after six seconds the limit becomes 49 Hz ( $f_1 = -0.02$ ).

Drawing the feasible solution space begins with the minimum and maximum limits of (12). This is seen by the outside box of Fig. 3. The minimum reserve requirement is added by the 45 degree line,  $u_1 + u_2 \geq 60$ . The next step is to add the single frequency limit at  $t_{1,l} = 6$  s. The key simplification is that any reserve offered after 6 seconds has

no influence on the limit, e.g. consider SR offer one, by six seconds it can produce 70 MW of reserve, if it is dispatched to 75 MW then the extra 5 MW has no benefit for satisfying the frequency limit. Therefore the same constraint used for  $u_1 = 70$  MW can be applied for  $u_1 = 75$  MW. For the subspace where  $u_1 \leq 70$  and  $u_2 \leq 35$ , (7) is simplified to:

$$2Hf_1 \leq u_1(t_{1,l} - t_{1,u}) - \frac{u_1^2}{2g_1} + u_2(t_{1,l} - t_{1,u}) - \frac{u_2^2}{2g_2} \quad (14)$$

which results in an ellipse as seen with its lower left quadrant in region (c) of Fig. 3. For  $u_1$  and  $u_2$  outside this subspace their value can be reduced until it is on the border  $u_1 = 70$  or  $u_2 = 35$ . This results in the constraint  $u_2 \geq 10$  in region (d), and  $u_1 \geq 25$  in region (a).

The minimum frequency constraint can be found in a similar manner to the frequency limits, as all reserve offered after  $t_{min}$  has no influence on the constraint. The time of minimum frequency is found when total reserve is equal to risk, i.e. on the reserve requirement line. The feasibility of all points to the minimum frequency constraint above the reserve requirement line is determined by the feasibility of the same point if all reserve offered after  $t_{min}$  is removed. Therefore only points on the reserve requirement line have to be directly checked for feasibility. The feasibility of all points above the line can be inferred by their equivalent points on the reserve requirement line.

This has a significant implication for simplifying (10);  $t_{min}$  can be removed from the right hand side of the equation:

$$2Hf_{lim}(t_{min}) \leq -u_1t_{1,u} - \frac{u_1^2}{2g_1} - u_2t_{2,u} - \frac{u_2^2}{2g_2} \quad (15)$$

when only considering points on the reserve requirement line, i.e.  $-Rt_{min} + u_1t_{min} + u_2t_{min} = 0$ .

The next step is to determine how  $t_{min}$  progresses along the reserve requirement line so that the left hand side of (15) can be simplified. The earliest  $t_{min}$  can be is if all the earliest reserve is dispatched:  $t_{min} = 4.48$  s,  $u_1 = 35.61$  MW, and  $u_2 = 24.39$  MW. This point is seen by the dot in Fig. 4, shifting in either direction away from this dot,  $t_{min}$  increases as later reserve is required to meet the requirement. Eventually after moving upwards along the reserve requirement line  $t_{min}$  surpasses six seconds and the frequency limit changes from 48 to 49 Hz. Therefore two equations are required to describe the minimum frequency constraint:

$$2Hf_0 \leq -u_1t_{1,u} - \frac{u_1^2}{2g_1} - u_2t_{2,u} - \frac{u_2^2}{2g_2} \quad (16)$$

when  $t_{min} < 6$ , and for when  $t_{min} \geq 6$ :

$$2Hf_1 \leq -u_1t_{1,u} - \frac{u_1^2}{2g_1} - u_2t_{2,u} - \frac{u_2^2}{2g_2} \quad (17)$$

These two curves are ellipses as shown in Fig. 4, but only small arcs are within view. The feasible space is the interior of the ellipse, so that the reserve requirement line inside the ellipse is feasible, and all other points after removing reserve offered after  $t_{min}$  result in a feasible point on the reserve

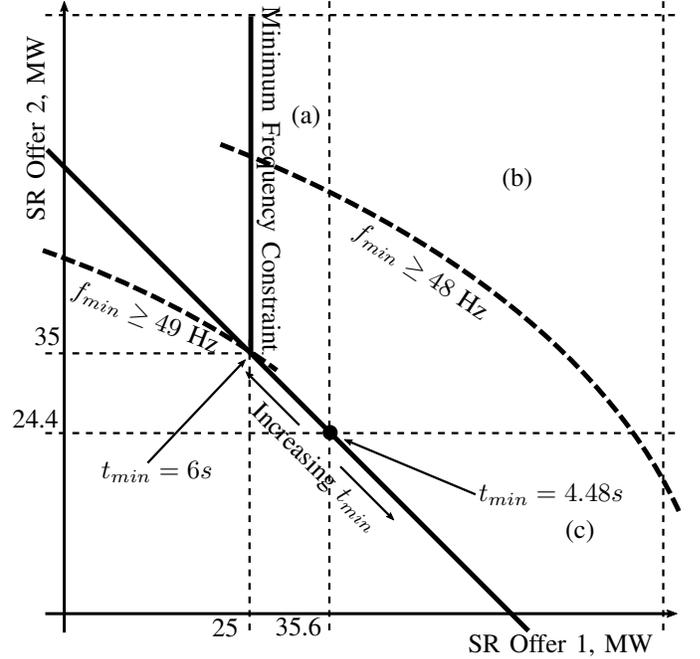


Fig. 4. Feasible space from the minimum frequency constraint for the two SR offer problem. Note that the elliptic lines do not divide the regions.

requirement line. The first ellipse, (16), does not cross the reserve requirement line at any meaningful point, but as soon as  $t_{min} = 6$  s the second ellipse intersects the line and forms the minimum frequency constraint. This constraint is a vertical line in Fig. 4, as all points along this line have their equivalent point as the intersection point between the ellipse and the reserve requirement line.

Comparing Fig. 3 and 4, the minimum frequency constraint does not further restrict the feasible space of Fig. 3 and so the whole feasible space can be seen in that figure. In Fig. 3, the feasible space of (a)-(d) bound by the constraints is seen to be convex.

#### IV. REGION FORMULATION

From Fig. 3, it appears that forming the set of equations for the optimization would be matter of finding all the linear and quadratic equations, and solving through a Quadratically Constrained Programming (QCP) solver. However this direct method does not work for one important reason: if the piecewise components of both the frequency limits (7) and minimum frequency constraint (10) are allowed to extend past their domain, then a space smaller than the full feasible space is being optimized. This section explains how the frequency limits confine the solution space. Then the general equation set is given for each region.

In Fig. 3, the constraints of the feasible space appear to be the following: maximum limits  $u_1 \leq 80$  and  $u_2 \leq 80$ , reserve requirement  $u_1 + u_2 \geq 60$ , and the frequency limit (14),  $u_1 \geq 25$ , and  $u_2 \geq 10$ . If these were the constraints for a QCP solver then the optimal solution will be searched for inside the ellipse, (14), and above the reserve requirement line as seen in Fig. 5 marked by the words ‘Reduced Feasible

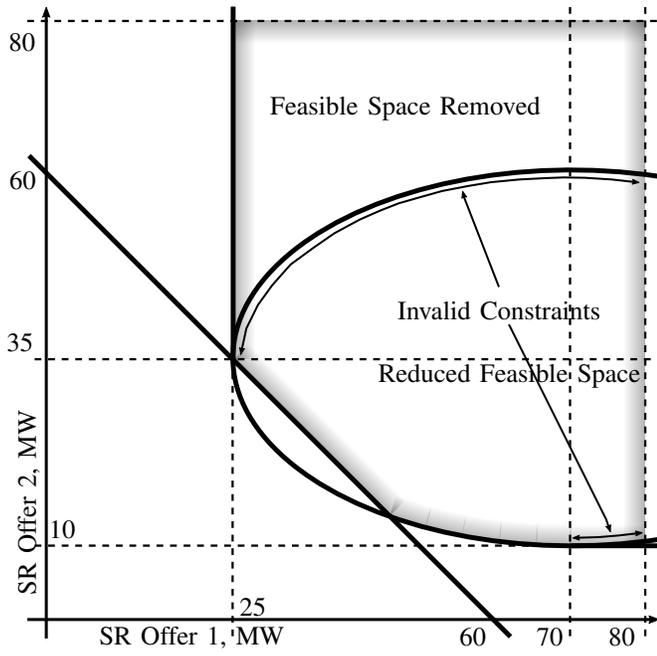


Fig. 5. Feasible space of the two SR offer problem, marking the space that is reduced, and the feasible space removed. The full feasible space is marked by the shading.

Space'. This is not the full feasible space, as seen by the large area above the ellipse that is removed, and the small area below and right of the ellipse. Therefore portions of the ellipse (14) are invalid constraints when  $u_1 \geq 70$  and  $u_2 \geq 35$ . Therefore to solve this problem the feasible solution space is divided into regions.

The boundary between regions occurs whenever the form of the frequency limit or the minimum frequency constraint changes. There are two types of region: the IL type and the SR type. The difference is dependent on which type of reserve is last to satisfy the risk. This is important because it determines how  $t_{min}$  changes. The generalized optimization problem for each region is (18) to (25). The following remarks are made:

- For an IL type region  $p_m$  is a variable,  $t_{min}$  is constant, and (23) is removed.
- For a SR type region  $t_{min}$  is a variable,  $p_m = 0$ , and (22) is removed.
- $p_m$  is the IL reserve offered at the minimum time,  $t_{min}$ , in order to satisfy the reserve requirement, (19). This value cannot be greater than the amount dispatched at the minimum time, (22), where the set  $Q_E$  is all the IL offers,  $t_{i,p} = t_{min}$ .
- $Q_B$ , set of IL offers initiated before the minimum time,  $t_{i,p} < t_{min}$ .
- $W_B$ , set of SR offers that finishes ramping before the minimum time,  $t_{i,u} < t_{min}$  and  $t_{i,u} + u_i/g_i \leq t_{min}$ . The latter of these conditions is expressed in (25) as  $w_i^{max} = g_i(t_{min} - t_{i,u})$ .
- $W_T$ , the set of SR offers that continue ramping past the minimum time,  $t_{i,u} < t_{min}$  and  $t_{min} \leq t_{i,u} + u_i/g_i$ . In (25),  $w_i^{min} = g_i(t_{min} - t_{i,u})$ . For a SR type region,  $W_T$  is required to have at least one element. This is to ensure the definition of  $t_{min}$  by (19) so that the condition of (8)

and (9) is satisfied.

- In the minimum frequency constraint, (20), the frequency limit is found,  $f_{lim} = f_{lim}(t_{min})$ . The frequency level,  $f_k = f_{lim}$ , if  $k > 1$  then the frequency limit constraints for  $j < k$  in (21) are not required. These constraints will be satisfied regardless.
- The term  $p_m t_{min}$  would generally imply (20) not to be a convex constraint. For IL or SR type regions either  $t_{min}$  or  $p_m$  is constant, therefore (20) is convex.
- $Q_{B,j}$ , set of IL offers initiated before the step change in frequency limit,  $t_{i,p} < t_{j,l}$ .
- $W_{B,j}$ , set of SR offers that stop ramping before the step change,  $t_{i,u} < t_{j,l}$  and  $t_{i,u} + u_i/g_i \leq t_{j,l}$ . In (25),  $w_i^{max} = g_i(t_{j,l} - t_{i,u})$ .
- $W_{T,j}$  set of SR offers that continue ramping past the step change,  $t_{i,u} < t_{j,l}$  and  $t_{j,l} \leq t_{i,u} + u_i/g_i$ . In (25),  $w_i^{min} = g_i(t_{j,l} - t_{i,u})$ .
- The constants  $t_{min}^{min}$  and  $t_{min}^{max}$  can be either  $t_{j,l}$ ,  $t_{i,p}$ ,  $t_{i,u}$ , or  $t_{i,e}^{max}$  depending on the region chosen.

An upper limit on the total number of regions in a problem is given in [27, eq. (5.98)]:

$$N_r = (2N_p + N_c + 2)(N_c + 2)^{N_u} \quad (26)$$

The relationship is exponential because of the number of possible partitionings of  $W_{B,j}$  with  $W_{T,j}$ , and  $W_B$  with  $W_T$ . Clearly it is not advised to solve every region to find the global minimum.

## V. SOLVING METHODOLOGY

Reformulating constraints (7) and (10) as functions  $F_j(\mathbf{v}) \leq 0$  and  $F_{min}(\mathbf{v}) \leq 0$ , it has been demonstrated that they are piecewise functions whose components are at most quadratic in form. It has been shown in the previous section that the optimization problem cannot be reformulated into a QCP problem without restricting the feasible solution space. The form of  $F_j$  and  $F_{min}$  classifies this problem under Nonsmooth Convex Optimization. [28] provides an introduction for these types of solvers. However, to the best knowledge of the authors, efficient algorithms, which utilize properties of the constraints, cannot be applied for the proposed formulation because  $F_{min}$  is discontinuous. Therefore a new solving methodology is developed to take advantage of the convex properties.

The solving methodology consists of a starting point, i.e. a region with a known feasible point, a QCP solver to solve each region, and a set of rules that determine the next region to be solved. These rules guide the algorithm to finding the region with the global minimum. Within these rules are terminating conditions when the global minimum is found. This methodology is first demonstrated using the previous example, and is then generalized with an algorithm.

Consider the two SR offer problem of the previous sections, the individual regions are shown in Fig 6, marked by (a) to (h). Each region is of SR type as there are no IL offers. The details of each region are shown in Table I. The second column of Table I specifies the range of  $t_{min}$  in (23). If there is a single value for  $t_{min}$ , instead of a range,  $t_{min}$  is held constant in this region. Other equations can be removed when they no longer

$$\text{minimize } \sum_i c_{i,p} p_i + \sum_i c_{i,u} u_i \quad (18)$$

$$\text{subject to } p_m + \sum_{i \in Q_B} p_i + \sum_{i \in W_B} u_i + \sum_{i \in W_T} g_i (t_{min} - t_{i,u}) = R \quad (19)$$

$$p_m t_{min} + \sum_{i \in Q_B} p_i t_{i,p} + \sum_{i \in W_B} \left( u_i t_{i,u} + \frac{u_i^2}{2g_i} \right) + \sum_{i \in W_T} \frac{g_i}{2} (t_{min}^2 - t_{i,u}^2) \leq -2H f_{lim} \quad (20)$$

$$\forall j, \quad -R t_{j,l} + \sum_{i \in Q_{B,j}} p_i (t_{j,l} - t_{i,p}) + \sum_{i \in W_{B,j}} \left( u_i (t_{j,l} - t_{i,u}) - \frac{u_i^2}{2g_i} \right) + \sum_{i \in W_{T,j}} \frac{g_i}{2} (t_{j,l} - t_{i,u})^2 \geq 2H f_j \quad (21)$$

$$0 \leq p_m \leq \sum_{i \in Q_E} p_i \quad (22)$$

$$t_{min}^{min} \leq t_{min} \leq t_{min}^{max} \quad (23)$$

$$\forall i, \quad 0 \leq p_i \leq p_i^{max} \quad (24)$$

$$\forall i, \quad 0 \leq u_i \leq u_i^{max} \quad \text{or} \quad w_i^{min} \leq u_i \leq w_i^{max} \quad (25)$$

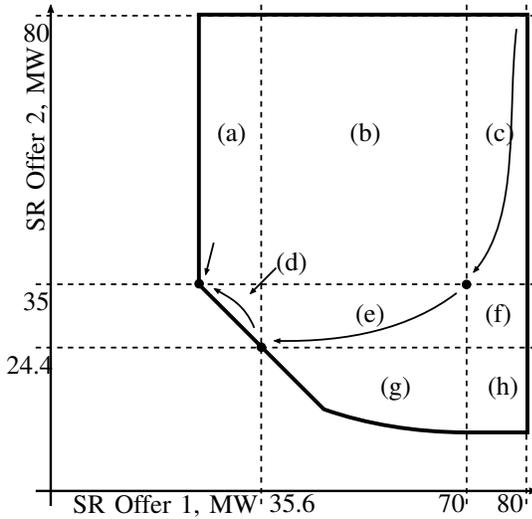


Fig. 6. Individual regions of the two SR offer problem, showing the progress towards the global minimum.

contain variables, these are identified in the last column of Table I.

A price is added to SR offer 1 of \$20/MW, and for SR offer 2 \$10/MW. The first step is to find a region that has feasible points. The easiest way to find a region is to check whether  $(u_1^{max}, u_2^{max})$  is feasible. Since  $(80,80)$  is feasible, region (c) is the first to be solved.

The local minimum for region (c) is the point  $(70,35)$ , which is highlighted by a dot in Fig. 6. There are three options to choose for the next region to be optimized: (b), (e), or (f). Region (e) is chosen because it is opposite to (c).

The local minimum of (e) is  $(35.6,24.4)$  which is on the reserve requirement line, i.e. an external boundary. It is possible for this to be global minimum, but since it is also on

TABLE I  
LIST OF REGIONS. FOR THE  $W$  SETS THE NUMBER REFERS TO WHICH SR OFFER.

Label	$t_{min}$	$W_B$	$W_T$	$W_{B,1}$	$W_{T,1}$	Omitted Eqs.
(a)	4.48 - 6	1	2	1	2	(22,24)
(b)	4.48	-	1,2	1	2	(19,20,22-24)
(c)	4.48	-	1,2	-	1,2	(19-24)
(d)	4.48 - 6	1	2	1,2	-	(22,24)
(e)	4.48	-	1,2	1,2	-	(19,20,22-24)
(f)	4.48	-	1,2	2	1	(19,20,22-24)
(g)	4.48 - 6.44	2	1	1,2	-	(22,24)
(h)	4.48 - 6.44	2	1	2	1	(22,24)

two internal boundaries it is better to move to another region, than to check whether it is the global minimum. There are no regions directly opposite, so a decision is required between (d) and (g). Region (d) is chosen because SR offer 1 is more expensive than SR offer 2.

After finding the local minimum in (d), it directs to region (a) to be optimized, and its local minimum is the same as region (d)'s,  $(25,35)$ . Therefore this point is the global minimum. In Fig. 6, it can be easily checked that this is the global minimum, as it is on the vertex of two external boundaries.

The general form of the solution algorithm is shown in Fig. 7. The full version can be found in [27], which further explains issues around finding the local minimum, isolating boundaries, and what to do if non-unique local minima are found.

Lastly, it is useful to have an worst case estimate for the number regions to be solved [27, eq. (5.128)],  $N_s$ :

$$N_s = 2(N_u + N_p) + N_c - 1 \quad (27)$$

In practice, the actual number of regions solved is much lower depending on the parameters of the problem. (27) requires further proof to guarantee worst-case performance.

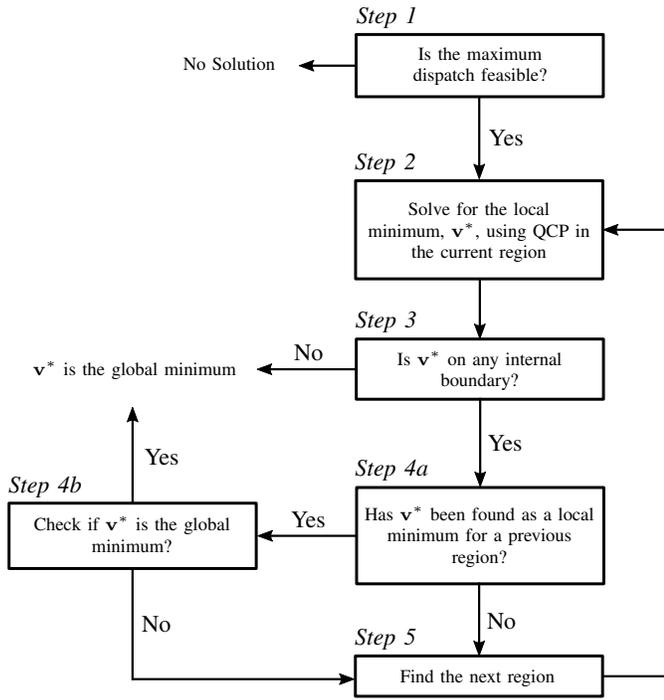


Fig. 7. Flow chart of the solving methodology to find the global minimum of the optimization problem.

## VI. CONCLUSION

This paper develops a reserves optimization with the ability to simultaneously optimize an indefinite number of different reserve responses, while ensuring the frequency transient for a credible contingency remains above minimum limits. This paper demonstrates by example that the formulation is convex, but leaves the proof in [27]. A new solving methodology is developed. The feasible solution space is divided into regions, which a QCP solver finds the local minimum, and a heuristic directs it to the region with the global minimum. The number of regions optimized to find the global minimum is estimated to be at most linearly dependent on the problem size. The companion paper [29] provides more examples of the methodology while highlighting the implications of this formulation on reserve market design.

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