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OPINION PAPER

A critical examination of seismic response uncertainty analysis in earthquake engineering

Brendon A. Bradley*,†

Department of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch, New Zealand

ABSTRACT

The last decade of performance-based earthquake engineering (PBEE) research has seen a rapidly increasing emphasis placed on the explicit quantification of uncertainties. This paper examines uncertainty consideration in input ground-motion and numerical seismic response analyses as part of PBEE, with particular attention given to the physical consistency and completeness of uncertainty consideration. It is argued that the use of the commonly adopted incremental dynamic analysis leads to a biased representation of the seismic intensity and that when considering the number of ground motions to be used in seismic response analyses, attention should be given to both reducing parameter estimation uncertainty and also limiting ground-motion selection bias. Research into uncertainties in system-specific numerical seismic response analysis models to date has been largely restricted to the consideration of 'low-level' constitutive model parameter uncertainties. However, 'high-level' constitutive model and model methodology uncertainties are likely significant and therefore represent a key research area in the coming years. It is also argued that the common omission of high-level seismic response analysis modelling uncertainties leads to a fallacy that ground-motion uncertainty is more significant than numerical modelling uncertainty. The author's opinion of the role of uncertainty analysis in PBEE is also presented. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Performance-based earthquake engineering (PBEE) has been under continual development for several decades. In addition to continual improvement in the characterization of seismic hazard and the seismic response of engineered systems, the last decade has seen a major swing in emphasis towards the explicit inclusion of uncertainties in such performance assessments. Such a swing in emphasis was cultivated within the Pacific Earthquake Engineering Research Centre on the backbone of the so-called PEER framework formula [1].

The most prominent seismic hazard generally results from earthquake-induced ground motions and will therefore be the focus of attention herein. The consideration of ground-motion uncertainty is most often accounted for via a significant number of ground motions in the seismic response history analysis of a numerical model of the system of interest. Although the rigorous consideration of uncertainties in the seismic response analysis of engineered systems, for a given input ground motion, is arguably in its infancy (compared with the attention devoted to ground-motion uncertainty), it ultimately results in

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*Correspondence to: Brendon A. Bradley, Department Of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch, New Zealand.

†E-mail: brendon.bradley@canterbury.ac.nz

various realizations of the numerical model of the system, leading to a further increase in the number of analyses performed.

The increased focus on uncertainties in research on ground-motion and seismic response modelling is, on face value, a significant step forward in being able to improve the reliability of seismic performance assessments. However, in numerous instances, as elaborated herein, it appears that there is a propensity to consider a large number of analyses (i.e. large set of ground motions or large number of seismic response model uncertainties) at the detriment of consistently capturing the underlying physics of the problems considered and the breadth of uncertainties present.

This paper is presented in three sections. The first section examines the consideration of uncertainty in input ground motions. The second section examines the consideration of uncertainties in system-specific numerical seismic response analysis modelling. Finally, the third section discusses the role of uncertainty analysis within the overarching goals of PBEE.

2. CONSIDERATION OF UNCERTAINTIES IN INPUT GROUND MOTION

2.1. *Seismic response history analysis*

Naturally, seismic response history analyses require the specification of an input ground-motion time series. It remains almost universally common for seismic response analyses within PBEE assessments to utilize amplitude-scaled ground motions. While time-domain or frequency-domain modifications to ground-motion time series are common in some situations for design code verification [e.g. 2], they are generally unreasonable for PBEE calculations, which seek to faithfully represent ground-motion uncertainty, because, by definition, such methods attempt to minimize the uncertainty in the seismic response for an ensemble of spectrum-compatible motions.

The particular manner in which ground motions are selected should be a function of the type of seismic performance assessment considered, namely, (i) a particular future scenario earthquake (scenario assessment); (ii) a single ground-motion intensity level which could result from different earthquake sources (intensity-based assessment); or (iii) multiple ground-motion intensity levels (multiple intensity-based assessments). Attention here will be given to the multiple intensity-based assessment case, because it is where the issues to be discussed are most apparent, although the comments made also apply to some extent for the other two assessment types.

One aim of performing seismic response analyses is to assess the relationship between the ground-motion intensity (quantified by an intensity measure, IM) and numerous seismic response parameters (or engineering demand parameters, EDPs), which collectively describe the seismic response of the system considered. That is, for each EDP considered, one seeks the relationship $EDP \sim f(IM)$. Only scalar IMs are considered here for reasons elaborated upon by Bradley [3, p.1420]. As a single IM is a highly simplified description of the ground-motion severity, it follows that the value of EDP from different ground-motion time series, which have $IM = im$, will be different, and hence, the relationship $EDP \sim f(IM)$ will be probabilistic in nature. The N_{gm} values of EDP obtained by subjecting the system to N_{gm} ground motions, which have $IM = im$, can be used to quantify the distribution of $EDP|IM = im$ on the basis of the assumption that each of the considered ground motions are equally likely to occur so that the EDP values can be considered as independent and identically distributed observations.

2.2. *Incremental dynamic analysis (IDA) and 'optimal' intensity measures*

Incremental dynamic analysis [4] is the most common approach to compute the $EDP|IM$ relationship in a probabilistic manner [e.g. 5, 6 and 7]. In a conventional IDA, N_{gm} ground motions are amplitude scaled to have $IM = im$, seismic response analyses are performed and the N_{gm} results for EDP are used to determine the distribution of $EDP|IM = im$. The process is then repeated by varying the amplitude scale factors so that numerous values of IM are considered.

The problem with the use of IDA stems from the fundamental questions: how representative, of the seismic hazard at the site considered, is a single ensemble of ground motions amplitude-scaled to a range of intensities, and how does this impact on the seismic response analysis results? Some have

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argued that the aforementioned criticism of the incremental ground-motion scaling in IDA can be overcome by the use of an *IM*, which is 'sufficient' with respect to the *EDP* considered [4, 8], that is, *EDP* is a function of the conditioning *IM* alone. Noting further that an *IM* which leads to a smaller standard deviation in *EDP* will allow estimation of statistical distribution parameters (i.e. μ *and* σ) with lower uncertainty, for a given number of ground motions, so-called *IM* 'efficiency' is also deemed important [8]. As a result, a vast amount of research has been devoted to the investigation of efficient and sufficient *IM*s for various seismic response problems (see Bradley [3] and references therein). The results of some such research suggests that the choice of conditioning *IM* is fundamental in the estimation of the demand hazard, when in fact it should be independent of the conditioning *IM* [3].

A critical limitation in the majority of the aforementioned 'optimal *IM*' studies has been the focus on a single *EDP* (often the peak inter-storey drift in a structural system). Bradley *et al*. [9], however, clearly illustrated that no *IM*s were sufficient when considering 21 different *EDP*s, comprising peak drifts and peak accelerations, within a 10-storey structure, and hence that the results of seismic response analyses will be impacted by the manner in which ground motions are selected. That is, if the ground motions utilized for a given intensity level are biased with respect to the seismic hazard at the site, then the seismic response analysis results will also likely be biased [3, 10]. It is also worth emphasizing that as the level of complexity in a numerical seismic response model increases to realistically capture salient nonlinear deformation mechanisms, the seismic response becomes naturally more complex and, thus, less and less like that of a simple elastic single-degree-of-freedom system upon which the most common *IM*s are based. It is therefore critical that procedures for considering ground-motion uncertainty within seismic performance assessment do not break down when one attempts to consider a realistic numerical model of a system, or such research is sure to have a short shelf life as today's 'advanced analysis' quickly becomes the norm in the future.

2.3. Seismic hazard-consistent ground-motion selection

Acknowledging that a single simplistic ground-motion *IM* will always be insufficient in representing the multitude of *EDP*s which characterize the system response, then the distribution of *EDP|IM* will be dependent on the ground-motion time series which are utilized. Thus, the connection between the seismic hazard at the site and the ground-motion ensemble that is utilized is a critical causal link which is often under-appreciated.

The analytical framework of contemporary seismic hazard analysis accounts for the fact that: (i) seismic hazard is posed by numerous seismic sources; (ii) there is uncertainty in the likelihood of a specific source rupture in a given time interval; and (iii) there is uncertainty in the ground-motion characteristics at a site from a specific source rupture. It is well recognized that the marginal distribution of ground-motion *IM*s is a function of source and site characteristics such as rupture magnitude, source-to-site distance etc. and that the joint distribution of ground-motion *IM*s is also dependent on correlations between these *IM*s [11]. As a corollary, an ensemble of seismic hazard-consistent ground motions must be a function of the ground-motion intensity level considered because: (i) as the value of the conditioning *IM* changes, different seismic sources tend to have a different contribution to the seismic hazard at the site; and (ii) even for a single seismic source, as the conditioning *IM* changes the distribution of other *IM*s will change at a different rate [e.g. 12, 13].

Thus, with reference to the comments in the previous section regarding the use of IDA, it must be stressed that the use of amplitude-scaled ground motions is not the principal issue. The principal issue is that a single ensemble of ground motions scaled over a wide range of *IM* does not account for the fact that ground-motion characteristics change as a function of the ground-motion intensity, and therefore will almost certainly result in a ground-motion ensemble which is not consistent with the considered seismic hazard at the site for one or more *IM* levels.

It is important to note that the initial five objectives of an IDA, as noted by Vamvatsikos and Cornell [4, p.492], can still be obtained by using different ground motions at different intensity levels for forming the distribution *EDP|IM,* but with a physically consistent seismic intensity representation. Goulet *et al*. [14] and Bradley *et al*. [15] are two examples, among others, where different sets of ground motions at different intensity levels were utilized. However, in both of these studies, ground motions were heuristically selected using implicit causal parameters (e.g. M_w, R_{rup}, ϵ *and* SF),

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rather than explicit ground-motion (i.e. *IM*) parameters. The robust selection of different ground motions for different intensity levels requires that the selection of ground-motion records is consistent with the site-specific seismic hazard in terms of explicit ground-motion *IM*s [12]. That is, ground-motion selection must be performed within a theoretically consistent framework, rather than simply requiring ad-hoc decisions, something which the majority of proposed ‘ground-motion selection methods’ ignore.

There are arguably two ground-motion selection methods which are exceptions to the aforementioned comment: (i) the generalized conditional intensity measure (GCIM) approach [10, 12]; and (ii) the conditional mean spectrum/conditional spectrum (CS) approach [13, 16], the former developed as a generalization of the latter. The GCIM approach, in particular, is fully consistent with the results of seismic hazard analysis, whereas the CS approach contains only a few inconsistencies for simplicity of use [10]. As elaborated upon by Bradley [10, 12], the particular manner in which the GCIM method is employed for ground-motion selection is a function of the application considered. If the analyst feels that response spectral ordinates alone provide a sufficient representation of a ground-motion time series, then ground motions can be selected on the basis of numerous spectral ordinates, and this resembles the CS approach of Baker [13]. In the more general case, where seismic response is a function of features in the acceleration time series not well represented via spectral ordinates alone, ground motions can be selected using the GCIM approach with various other ground-motion *IM*s (peak ground velocity, *PGV*; cumulative absolute velocity, *CAV*; and significant duration, D_s etc.).

2.4. How many ground motions to use? The bias–variance trade-offs

Because the distribution of seismic response for a given ground-motion intensity level, $f_{EDP|IM}$, is estimated on the basis of a finite number of seismic response history analyses, it follows that the statistical moments of this distribution contain uncertainty. As the assumption of lognormality is usually reasonable, then this uncertainty is that in the parameters of the distribution. For example, the standard error in the estimate of the lognormal mean of the distribution, $\mu_{\ln EDP|IM}$, can be given by

$$\sigma_{\mu_{\ln EDP|IM}} = \sigma_{\ln EDP|IM} / \sqrt{N_{\mathrm{gm}}} \tag{1}$$

where $\sigma_{\ln EDP|IM}$ is the lognormal standard deviation of the *EDP*|*IM* distribution, and N_{gm} is the number of ground motions which have been used to obtain the estimates of the mean and standard deviation. Hence, the error in the statistical moments of the distribution can be reduced by finding *IM*s which correlate strongly with the *EDP* (i.e. smaller $\sigma_{\ln EDP|IM}$), or by increasing the number of ground motions considered (i.e. larger N_{gm}). As previously noted, because no *IM* will provide a strong correlation with all ***EDPs*** considered, then an increase in N_{gm} is the only way to systematically reduce the parameter estimation error for all *EDP*s considered.

The use of a larger ground-motion ensemble clearly reduces the variance (i.e. uncertainty) in seismic response parameter estimation, and therefore is beneficial, but may lead to potential bias in the estimated seismic responses as a result of a biased ensemble of ground motions. This represents an example of the classical bias–variance trade-off within statistics. The use of a large ground-motion ensemble, but a lack of regard to seismic hazard-consistent ground-motion selection, thus results in a response prediction with lower variance but potentially high bias, which is obviously a poor trade. Clearly, the optimal solution would be to utilize a large ground-motion ensemble, which is also seismic-hazard consistent; however, as noted later, ensuring ground-motion ensembles are seismic hazard consistent becomes increasingly difficult as the ensemble size increases.

In making rational decisions around the number of ground motions to be used in seismic performance assessment in the face of this bias–variance trade-off, it is necessary to approximately estimate both the bias and variance in the estimated seismic response distribution parameters. The variance in parameter estimates can be easily assessed (e.g. Equation (1)). In contrast, the estimation of bias in seismic response resulting from biased ground-motion selection first requires a quantitative estimate of ground-motion selection bias and second the assessment of seismic response bias resulting from ground-motion bias. Such estimation of seismic response bias is not possible without a ground-motion selection method with a theoretically rigorous framework as elaborated upon by Bradley [3, 10, 12].

Figure 1(a) and (b) illustrate the distribution of seismic demand obtained from seismic response analyses using 7 and 15 ground motions, respectively. For illustrative purposes, in both cases, it has been assumed that the uncertainty in the distribution of *EDP*|*IM* based on the seismic responses is $\sigma_{\ln EDP|IM} = 0.4$. Thus, using 7 and 15 ground motions leads to an estimate of the lognormal mean seismic demand with an uncertainty of 0.15 and 0.10, respectively, clearly illustrating the variance reduction from increasing the number of ground motions. To obtain these seismic responses, the author selected ground motions and scaled them on the basis of a specific value of a conditioning *IM*. Figure 1(c) and (d) illustrates the distribution of some other *IM* of interest to assess ground-motion selection bias. In Figure 1(c) and (d), a solid red line is used to illustrate the 'target' distribution of this other *IM* (based on the GCIM theory), whereas the dashed grey line represents the empirical distribution of the selected ensemble of N_{gm} ground motions. To assess the representativeness of the ensemble as compared with the theoretical distribution, the Kolmorogov–Smirnov goodness of fit test is used, which is graphically illustrated as two acceptance bounds. If the empirical distribution of the ground-motion ensemble lies 'within' these acceptance bounds, as is the case in Figure 1(c), then the ensemble is not biased with respect to the theoretical distribution, and therefore, no (statistically significant) bias will result in the seismic response as a result of this *IM*. Using a larger ensemble of ground motions to reduce the variance in the estimated seismic response parameters, for example, $N_{gm} = 15$ as in Figure 1(d), results in the 'width' of the acceptance region between the two Kolmorogov–Smirnov acceptance bounds reducing. As a result, it becomes more difficult to find a ground-motion ensemble which is unbiased with respect to the theoretical 'target' distribution because: (i) there are a finite number of ground motions in empirical databases from which to select from; and (ii) an unbiased ensemble is desired with respect to multiple *IM*s of interest.

Thus, when attention is given to the number of ground motions for use in seismic response analyses, due consideration should be given to the impact of variance reduction (from using a larger ensemble)

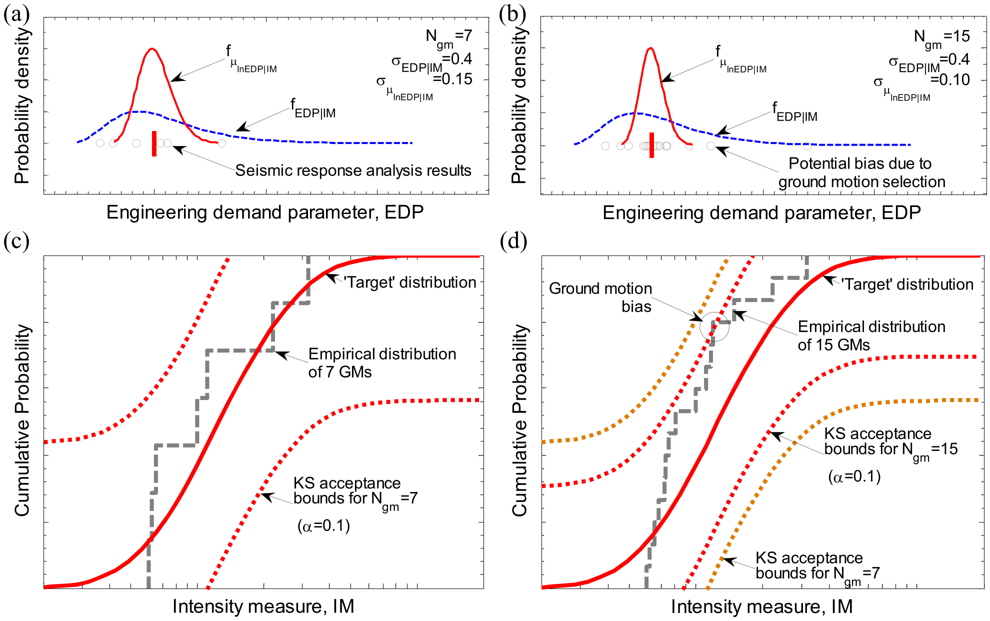

Figure 1. Schematic illustration of the bias–variance trade-off resulting from an increase in ensemble size from 7 (left side) to 15 (right side) ground motions. Increasing the ensemble size results in a reduction in parameter uncertainty (variance) in the distribution of seismic response, but a greater potential for ground-motion selection bias (and consequent seismic response bias) as a result of a reduction in the 'width' of the acceptance region for differences between the target intensity measure distribution and the empirical distribution of the ground-motion ensemble (modified after Bradley [10]). KS, Kolmorogov–Smirnov.

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and also increasing potential bias (due to improper ground-motion selection). Furthermore, bias and variance due to ground-motion uncertainty should be considered with a consistent treatment of numerical seismic response model uncertainty as elaborated upon in the next section.

3. CONSIDERATION OF UNCERTAINTIES IN THE NUMERICAL MODEL OF THE SYSTEM

The previous section focused on the uncertainty in the *EDP|IM* relationship as a result of uncertainty in the incident ground motion, that is, uncertainty in the seismic demand. Naturally, there is also uncertainty in the seismic capacity/resistance of the system considered, as represented by the seismic response analysis model employed. Often, this uncertainty is referred to as 'seismic response modelling uncertainty' or simply 'modelling uncertainty'.

3.1. Classification of uncertainties in numerical seismic response modelling

Numerical seismic response modelling uncertainty results from the inability of an idealized numerical seismic response model to predict the actual response of an engineered system to ground motions. Because the source of these uncertainties is manifold, it is useful to differentiate them, with one such classification being (Figure 2) [17]: (i) uncertainties in the measurement of physical quantities; (ii) uncertainty in the correlation between measurable physical quantities and constitutive model parameters; (iii) uncertainty in selecting an appropriate constitutive model; and finally, (iv) uncertainty in the overall idealized model methodology. Examples of type (i) uncertainties include the estimation of soil shear stiffness or reinforcing bar yield strength that can be directly measured. As not all constitutive models utilize directly measurable physical parameters, type (ii) uncertainties account for the inexactness of correlations between measurable quantities and constitutive model parameters. Type (iii) uncertainties result from the specific choice of constitutive model in a numerical seismic response model, because of its underlying assumptions. Finally, type (iv) uncertainties account for the fact that the numerical model domain always represents a gross simplification of reality (e.g. one-dimensional/two-dimensional analyses, neglect of the interaction of specific components, assumed boundary conditions and formulation of damping).

Classification of uncertainties in the aforementioned manner is important when reflecting on research into seismic response uncertainty quantification published in literature to date, which principally has focused on only the lower level uncertainties (types (i) and (ii)). It is also fundamentally important to understand that the contribution of the various types of uncertainties above to the total numerical seismic response modelling uncertainty is not constant. For example, type (i) uncertainty is solely dependent on the quality of the physical characterization of the system considered, but independent of the particular numerical model considered. If a simplistic numerical model, with simple constitutive model(s) is used, it will likely have small type (ii) uncertainties,

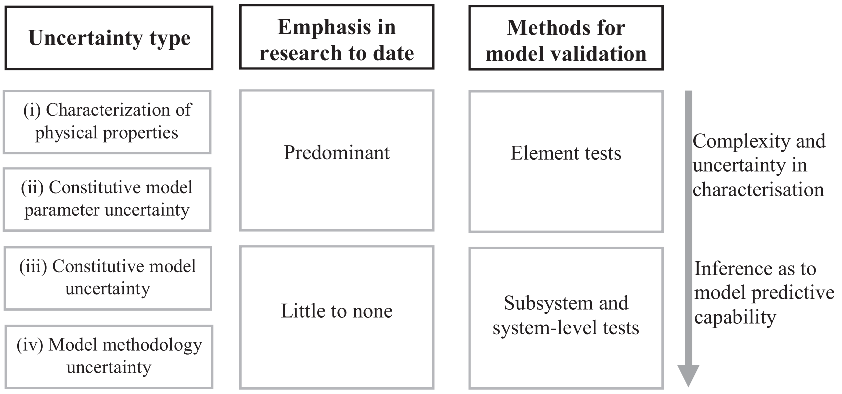

Figure 2. Uncertainties in seismic response modelling (adapted from Bradley [17]).

because simple models naturally use few parameters whose physical basis is often well established, but potentially large types (iii) and (iv) uncertainties because of the inability of the simplified model to capture salient deformation mechanisms. In contrast, if a complex numerical model with complex constitutive relationships is utilized, it will likely have larger type (ii) uncertainty (as a result of additional constitutive model parameters required, as well as less established methods to determine such parameters), but potentially smaller type (iii) and (iv) uncertainties if the model is a more faithful representation of reality.

3.2. *Consideration of numerical seismic response model uncertainties to date*

The consideration of numerical modelling uncertainty in PBEE research is in its infancy in comparison with the attention that has been devoted to the consideration of ground-motion uncertainty. Such a status is understandable for two reasons. First, from a historical standpoint, seismic response analyses have long been considered in a deterministic manner [18], whereas uncertainties in ground-motion *IM*s have been considered within seismic hazard analyses for over 40 years [19] (and with various levels of rigor [20]), and the consideration of uncertainty in ground-motion time series is a natural extension of seismic hazard analysis. Second, and more importantly, ground-motion uncertainty is treated in a highly simplified manner through the use of statistically-based empirical ground-motion predictions, which directly provide uncertainties, while seismic response analyses generally utilize system-specific simulation methods. As a result, numerical modelling uncertainty is significantly harder to quantify than ground-motion uncertainty, and this is compounded by the vast array of different structural systems and materials which engineering systems comprise.

Historically, the explicit consideration of uncertainties in seismic response was considered via the use of factor-of-safety-based system-level fragility functions obtained principally via expert judgement [e.g. 21]. More recently, for system-specific numerical seismic response models, the rigorous consideration of numerical seismic response modelling uncertainty in PBEE research has been almost exclusively focused on the consideration of uncertainties in constitutive model parameters (that is, both type (i) and type (ii) uncertainties as depicted in Figure 2). In the seismic response of structural systems—Chryssanthopoulos *et al*. [22], Ibarra and Krawinkler [23], Lee and Mosalam [24], Haselton [25, Chapter 5], Dolsek [6], Vamvatsikos and Fragiadakis [7], Liel *et al*. [26]; in seismic site response analysis—Rathje *et al*. [27], and Bazzurro and Cornell [28]; in seismic slope stability—Rathje and Saygili [29] and in seismic soil–foundation–structure interaction—Shin [30], among others, are examples of studies which have systematically evaluated the uncertainty in numerical seismic response analyses but restricted their attention to only uncertainties in constitutive model parameters.

There are few exceptions to the above statement which have examined constitutive model parameter uncertainties as well as also comparing the differences between the predictions obtained using different constitutive models. Aslani [5] compared different numerical models of the Van Nuys Hotel on the basis of design code prescriptions. Browning *et al*. [31] compared three different numerical models of the Van Nuys Hotel response to the Northridge earthquake instrumental responses, but did not consider constitutive model parameter uncertainties. Andrade and Borja [32] and Kwok *et al*. [33] considered the uncertainty in constitutive model parameters for seismic site response analysis at vertical array sites and also the differences among using different constitutive models. In both cases, with the exception of the difference between time-domain and frequency-domain equivalent linear analysis [34], the various different predictions in Andrade and Borja [32] and Kwok *et al*. [33] all utilize the same overall modelling methodology assumptions (i.e. one-dimensional wave propagation).

The consideration of spatial correlations between constitutive model parameter uncertainties is also important in correctly capturing this aspect of prediction uncertainty. In the seismic response of structural systems, it has been common to assume the extreme cases of zero or perfect correlation [e.g. 6, 7, 23, 26]. In seismic site response, it still remains common to assume perfect correlation of uncertainties in, for example, shear wave velocity, V_s, with depth [28]. Although geo-statistics research findings often enable the realistic spatial uncertainty consideration of a single parameter, such as shear-wave velocity [27, 30], general results are not yet available for the joint spatial distribution of multiple material properties, which are needed in constitutive models considering nonlinear seismic response.

3.3. Why is the rigorous consideration of high-level numerical seismic response modelling uncertainties not well developed?

Quantifying the impact of uncertainties is not simply a matter of having a method to propagate the uncertainties to the performance metric of interest; the nature of the uncertainty must itself be known. For example, to consider the effect of uncertainty in the yield stress of steel reinforcing in structural elements, it is necessary to know the distribution of possible yield stresses. The type (i) uncertainty distribution in such 'basic' parameters is often easily quantified, because such properties are directly measurable. Similarly, uncertainties in the correlation between a physically measurable quantity and a constitutive model parameter (type (ii)), can also be developed by the use of a simple probabilistic model with paired physical and constitutive model parameter data [e.g. 35, 36].

It is fundamentally important to understand that type (i) and (ii) uncertainties, which as previously discussed have essentially been the focus of numerical model uncertainty in PBEE to date, require only element tests to obtain the necessary experimental data for parameter uncertainty validation (Figure 2). In contrast, characterization of type (iii) (constitutive model) and type (iv) (overall model methodology) uncertainties requires the use of sub-system or system-level experimental testing [17, 37] (Figure 2). This difference is considered as one of the primary reasons for the lack of research into rigorously quantifying these higher-level uncertainties in numerical seismic response modelling. Another important reason is that type (i) and (ii) uncertainties are also more generalizable, whereas type (iii) and (iv) uncertainties tend to be more problem- and system-specific and, hence, less amenable to generalized studies. Nonetheless, it is the author's opinion that these high-level uncertainties are likely significantly larger than those low-level uncertainties related to constitutive model parameters, and should therefore be seen as major research focus within uncertainty quantification in PBEE over the coming years. Because of the difficulty in directly quantifying system-specific numerical seismic response model uncertainty, guidance documents tend to provide default uncertainty values (e.g. Tables 5.1–5.3 of ATC-58 [38] or Tables 3.1–3.3 of FEMA-P695 [39]) for use in the absence of direct analysis.

The most appropriate manner to assess constitutive model and model methodology uncertainties is via the systematic validation of numerical seismic response models with observational data [37]. Although such validation data can take several forms, seismic instrumentation coupled with post-event field documentation provides the most reliable means to obtain system-level validation data at full scale and with the correct boundary conditions. Vertical downhole arrays in site response analysis (e.g. Bradley [17] and references therein) and instrumented structures [e.g. 31] have been utilized to provide significant insight into the efficacy of common numerical seismic response models and assumptions. From the view point of uncertainty quantification however, these aforementioned comparisons with seismic instrumentation observations can essentially be regarded as deterministic in that no uncertainties in the constitutive model parameters of the seismic response models were considered, despite such uncertainties often being significant because of the complexity of such 'real-world specimens' compared with laboratory equivalents [17]. A consequence of the failure to account for such uncertainties is that it cannot be determined if a good agreement between a single model prediction and an instrumental observation is due to a capable numerical model or possibly from 'cancellation' of errors due to the neglect of uncertainties. Therefore, to assess the uncertainties in numerical seismic response modelling arising from constitutive model and model methodology uncertainties, it is necessary to compare deterministic observations from seismic instrumentation with the probabilistic prediction of a particular numerical seismic response model resulting from constitutive model parameter uncertainties. One possible framework within which such high-level modelling uncertainties can be assessed is outlined in Bradley [17].

3.4. The potential for simplified numerical model predictions with low variance but high bias

The current use of simplified numerical models in uncertainty quantification studies presents several potential problems, which stem from the fact that, as previously noted, it has been conventional to

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only consider numerical modelling uncertainties resulting from constitutive model parameter uncertainties and neglect high-level uncertainties associated with the adopted constitutive models and model methodology. No claim is made that simplified models will systematically produce poorer predictions than the use of more advanced models. Clearly, the adoption of a complicated model, without adequate system characterization data to calibrate constitutive model parameters, and an analyst with inadequate experience, can lead to the possibility of erroneous results, but such cases are not discussed further herein.

Figure 3 provides a schematic illustration of the problems that can result from the neglect of high-level modelling uncertainties, which illustrates the true system response in comparison to the: (i) model prediction with ground motion (GM) uncertainty only; (ii) model prediction with the addition of low-level model parameter uncertainties; and (iii) model prediction with both low-level and high-level constitutive model and modelling methodology uncertainties. It is illustrated schematically that the addition of model parameter uncertainties leads to only a small increase in the prediction uncertainty compared with that from ground motion uncertainty alone. This small increase due to model parameter uncertainty is likely to be particularly the case when simplified numerical models are used compared with more complex models, because simplified models often utilize pre-defined failure mechanisms (e.g. Newmark sliding block assumes a specific failure plane location with a constant failure stress), and therefore, the consideration of model parameter uncertainties serves only to alter the capacity of such failure mechanisms (e.g. k_y in the Newmark block case). In contrast, more complex models do not require a priori assumptions for the location and nature of failure mechanisms (e.g. slope stability using finite element methods) or the post-elastic path dependency of the corresponding constitutive models, and therefore, constitutive model parameter uncertainties are likely to result in notably greater uncertainty in the response prediction. If the high-level modelling uncertainties (i.e. types (iii) and (iv) [17]) associated with the use of simplified models were accounted for explicitly, then this would compensate for the lack of sensitivity to model parameter uncertainties and result in a prediction with significantly greater uncertainty, as shown in Figure 3. However, the fact that such uncertainties are almost always neglected results in a prediction with relatively small variance, but potentially large bias, in comparison with the true seismic performance of the system as shown in Figure 3.

Given that the aforementioned discussion pushes the point that the greatest modelling uncertainty in a simplified model is the uncertainty in the model methodology itself, then if such modelling uncertainty is neglected, it poses the question: is the use of simplified models, which neglect the majority of the salient physics of the problem, compatible with the idea of rigorous uncertainty analysis?

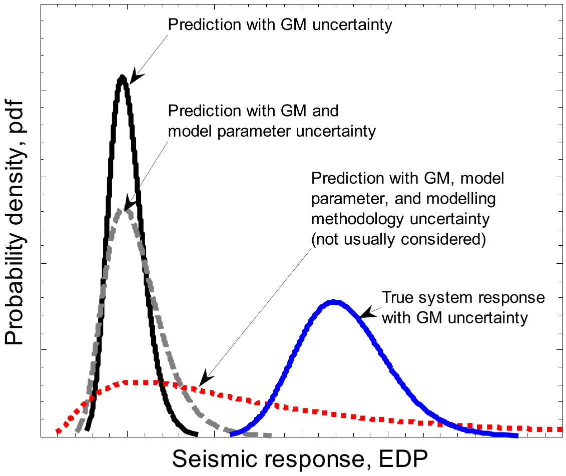

Figure 3. Illustration of the potential for a numerical model prediction with low variance but high bias. GM, ground motion.

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3.5. Comparison of modelling uncertainties and ground-motion uncertainty

Previous studies have often made the comment that the uncertainty in seismic response as a result of ground-motion uncertainty is generally significantly larger than that due to numerical seismic response modelling uncertainty [6, 7, 24, 26, 27]. There are two significant biases in such statements. The first is that such statements have been made for cases in which seismic response analyses have been conducted with ground-motion ensembles which are not seismic hazard consistent, and hence, the ground-motion uncertainty is misrepresented. Second and more importantly, such statements have been made on the basis that only constitutive model parameter uncertainties have been considered but not uncertainties in the constitutive models themselves or the overall modelling methodology (i.e. type (iii) and (iv) uncertainties as per Figure 2).

4. HOW AND WHERE DOES UNCERTAINTY ANALYSIS FIT WITHIN PERFORMANCE-BASED EARTHQUAKE ENGINEERING?

The question ‘what does performance-based earthquake engineering mean?’ may be obvious to an earthquake engineering researcher. However, one merely needs to view research literature in which the word ‘performance-based earthquake engineering’ is used to see clearly the particulars of what is implied by the term vary widely, particularly geographically. On the basis of the author’s experience, it is clear that US and Japan research views represent two extreme cases of what would be embodied in a PBEE assessment, with New Zealand, Europe and others lying somewhere in between. In the USA, PBEE is almost synonymous with the ‘PEER framework formula’ [1] and thus is largely focused on probabilistic quantification of seismic performance. In Japan, by contrast, PBEE is still largely taken to mean simply anything that is not ‘factor-of-safety based’, that is, assessment and/or design in which deformations are considered. Which of these emphasized views can be considered as better? Arguably, the answer is neither, because PBEE should provide a continuum of methods, each of which provides complementary information for the design and assessment of engineered systems [40]. Which particular methods from this continuum are utilized should be specific to the system considered and the objectives of the design/assessment. What is common to all methods within such a continuum however is that they provide information to enable additional insight into ‘understanding the system’ so that rational decisions can be made.

Although an overemphasis on uncertainty consideration at the detriment of a physically faithful representation of the problem will lead to the potential for significant performance estimation bias, a neglect of uncertainties will lead to false confidence in performance estimation. The explicit consideration of uncertainties in seismic performance estimation is a recent trend, and further research on understanding and quantifying such uncertainties is clearly needed to obtain robust estimates of seismic performance. However, this can only be expediently obtained if empirical studies utilize physically consistent representations of seismic intensity (i.e. ground-motion selection) and seismic demand (i.e. seismic response analysis).

Just as the general goal of PBEE should be to improve the understanding of the system considered, a focus on developing a further understanding of the system should be forthright when embarking on uncertainty analysis. The aim of an uncertainty analysis for quantifying seismic performance is not simply to derive numerical values, such as the probability that the peak inter-storey drift ratio for a given ground-motion intensity level is below some code-prescribed level (a statistical viewpoint). Performance quantification utilizing uncertainty analyses should provide further information to understand the system so that performance can be potentially improved, whether that be through a greater understanding of which aspects of the seismic response result in system vulnerability, or which particular uncertainties are most prominent and therefore targeted to effectively reduce overall performance uncertainty (an engineering viewpoint).

Rigorous uncertainty consideration requires numerous analyses to quantify the uncertainty in system response. As a result, for a given time available, the level of interrogation that can be afforded for each of these numerous analyses must be inversely proportional to the number of analyses considered. For example, when considering the seismic response of a system with

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ground-motion and numerical modelling uncertainties, it is conventional to simply express the relationship between cause and effect in the form of *EDP* versus *IM* plots for the various *EDP*s considered. While in certain cases attempts have been made to infer mechanisms of seismic response from the trends in the *EDP* versus *IM* plots [e.g. 4], clearly the examination of such trends provides a very superficial understanding of the seismic response of the system. Performing an uncertainty analysis can therefore provide information which is complementary to, but certainly not a substitutive for, a rigorous interrogation of the seismic response of the system at global, element, section and material levels under a limited subset of analysis cases in which emphasis is placed on a detailed understanding of the response focusing on deformation mechanisms. Such a limited subset of analyses may utilize one (or a few at most) ground motion(s) at various intensity levels and sensitivity analyses of numerical seismic response model uncertainties (e.g. 'swing analyses' or 'tornado diagrams'). Furthermore, in nontrivial seismic response problems, for which nonlinear dynamic analyses are most useful, the seismic response of the system may not be well understood a priori, and therefore such a rigorous interrogation is first necessary to determine which *EDP*s can be adequately used to characterize the system response for use in comprehensive uncertainty analyses [15]. The aforementioned statements are particularly aimed at the use of simplified numerical models (e.g. single-degree-of-freedom representations of structures and Newmark sliding block representation of slopes), which can be utilized within uncertainty analyses to attempt to quantify seismic performance, but with which often little information is provided for an understanding of the system considered and therefore little information as to how seismic performance can be improved if found to be undesirable.

5. CONCLUSIONS

This opinion paper has discussed uncertainty consideration in ground-motion selection and seismic response analysis within the context of PBEE. The key conclusions related to the topics addressed are:

- The use of incremental dynamic analysis [IDA] and a single ensemble of ground motions scaled over a wide range does not account for the fact that ground-motion characteristics change as a function of the ground-motion intensity, and therefore will almost certainly result in a ground-motion ensemble which is not consistent with the considered seismic hazard at the site for one or more *IM* levels.
- The use of a large ground-motion ensemble, but with a lack of regard to seismic hazard-consistent ground-motion selection, results in a response prediction with lower variance but potentially high bias, which is obviously a poor trade.
- While there are a multitude of uncertainties in numerical seismic response analyses, constitutive model parameter uncertainties have been the focus of research to date, with high-level uncertainties in constitutive models and overall modelling methodology generally neglected. These high-level uncertainties are, in the author's opinion, likely significantly larger than those low-level uncertainties related to constitutive model parameters and thus should be a key research area in the coming years.
- Previous studies, which have concluded that ground-motion uncertainty is generally significantly larger than numerical seismic response modelling uncertainty, are typically biased because ground-motion uncertainty is often overestimated when ground-motion ensembles are not rigorously selected and because numerical modelling uncertainty is generally significantly underestimated when high-level modelling uncertainties are neglected.
- Despite the presented shortcomings in current research, it is obvious that uncertainty assessment plays an important role in PBEE. However, it is critical to consider uncertainty analysis within PBEE as providing more than just a numerical quantification of seismic performance, but providing additional insight into understanding the system to aid in decision-making. In particular, the results of an uncertainty analysis provide information that is complementary to, but not a substitute for, a rigorous interrogation of the seismic response of the system at global, element, section and material levels under a limited subset of analysis cases in which emphasis is placed on a detailed understand of the response.

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