A Method for Robust Time-Reversal Focusing in a Fluctuating Ocean

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Abstract—In recent years, we have demonstrated time-reversal mirrors (TRM) in the ocean. A focus of up to 30 km was achieved with low frequency (445 Hz) transmissions and the focal structure could be maintained over several days at a range of 15 km. However, the stable focus was limited to less than an hour with high frequency (3.5 kHz) transmissions due to the sensitive response of high frequency sound propagation to the medium fluctuations. In this study, an approach for robust time-reversal focusing is investigated based on a method developed for matched-field processing. Instead of using a single probe source pulse, the method makes use of several probe source pulses obtained over a certain period of time where each ping represents a different propagation condition of the medium. The back-propagation from a TRM weighted by a linear combination of the dominant singular vectors obtained from the signal matrix leads to stable focusing for a longer period of time than that with a single probe pulse. The proposed method is useful in non-static propagation conditions and when frequent probe signals are not available.

I. INTRODUCTION

Time reversal [1-3] exploits the time-reversal invariance of the wave equation in a static medium such that the re-transmitted waveform from a time-reversal mirror (TRM) propagates backward to the position of probe source (PS) where the signal is generated. Since time-reversal focusing utilizes the reciprocity of a static medium, its performance can be degraded in a non-static environment where the propagation conditions change during the time of forward and backward propagation between the TRM and PS. In recent experiments the stability of the focal structure was investigated by repeatedly re-transmitting a stored probe source pulse into the time-dependent ocean medium. Measurements with high frequency (3.5 kHz) transmissions showed that the stable focus was limited to an hour or a few minutes depending on the depth of the PS and the strength of the ocean variability implying that the effect of medium fluctuation is not trivial for such high frequency transmissions.

In this study, a method for robust time-reversal focusing is investigated based on Krolik's MV-EPC processor [4] originally developed for matched-field processing. Instead of using a single probe source pulse, the method makes use of several PS pings obtained over a time interval where each ping represents a different propagation condition of the ocean

medium. The back-propagated field weighted by a linear combination of the singular vectors corresponding to the dominant singular values of the received signals produces stable focusing for a longer period of time than that with a single probe pulse. In the next section, experiment results are briefly described. In Sec. III, time-reversal focusing in a random ocean channel is addressed using adiabatic normal mode theory. In Sec. IV, the proposed method for robust time-reversal focusing is described. Simulation results are discussed in Sec. V.

II. TIME-REVERSAL EXPERIMENTS

A. Experimental setup

A series of time reversal experiments was performed in two areas, near Formiche and Elba islands, off the west coast of Italy depicted as shaded boxes in Fig. 1. Both areas are relatively flat environments with water depth of about 130 m in the Formiche area and 120 m in the Elba area, respectively. Sound-speed profiles (SSP) were measured frequently by conductivity-temperature-depth (CTD) casts. The SSPs collected over the experiment period indicate the variability of upper water column especially in the thermocline region.

Fig. 2 shows the components of a TRM experiment. In the April 1996 and May 1997 experiments, a TRM was deployed near Formiche di Grosseto. The TRM spanned 77 m of the water column with 20 receiving hydrophones and 20 contiguously located slotted cylinder sources. Each element of the TRM was operated at a nominal source level of 160 dB re 1 μ Pa with a center frequency of 445 Hz. A PS was located in the vicinity of a 48 element vertical receive array (VRA). The July 1999 experiment was performed with a center frequency of 3.5 kHz in both the Formiche and Elba areas and the May 2000 experiment in the Elba area. The experimental setup was similar to the lower frequency experiments but the TRM had 29 transducers spanning a 78-m aperture with a nominal source level of 178 dB re 1 μ Pa. The VRA covered 93 m of the water column with 32 hydrophones.

B. Measured time-reversal stability

If the ocean is static, PS pulses can be stored ahead of time to focus at specific locations. However, the temporal

Fig. 1. Locations of the time-reversal experiments. Low frequency (445 Hz) experiments were performed near the Formiche di Grosseto and high frequency (3.5 kHz) experiments were carried out both in the Elba and Formiche area

variability of the ocean is expected to limit such a procedure. In the May 1997 experiment, we found that probe pulses of 445 Hz still produced a significant focus at the original probe-source location up to 1 week later [5]. But recent experiments with 3.5 kHz transmissions showed that a stable focus was severely limited by the medium fluctuations.

Fig. 3 shows the measured time-reversed pulses received on the VRA by re-transmitting the received probe pulse after a short time (16 min), 1 day, and 1 week later, respectively. The range between the TRM and PS was 15 km with the PS depth at 81 m. The PS signal was a 50-ms pure tone pulse at 445 Hz. During the period, the focus moved a few meters downward resulting from the changes of modal shapes due to the increased sound speed. Although the focus is significantly degraded after a week along with the appearance of a side-lobe in the upper water column, the TRM clearly retained a focus.

Fig. 2. Experimental setup for the high frequency (3.5 kHz) time-reversal experiment. The time-reversal mirror (vertical source-receiver array) consisted of 29 transducers with inter-element spacing 2.786 m spanning 78 m of the water column.

Fig. 3. Measured time-reversal foci with 50-ms low frequency (445 Hz) transmissions. A 50-ms pulse received by a TRM from a PS 15.2 km away at a depth of 81 m was re-transmitted (a) 16 min, (b) 1 day, and (c) 1 week later.

Fig. 4 shows the measured high frequency time-reversal foci obtained by re-transmitting the probe-source pulse every minute over 45 min. The pulse was 10-ms long. The probe source was 8.1 km away from the TRM at the depth of 68 m. Although the focal size does not change very much, the peak level of the focus decreases about 5 dB over the 45 min. The focal structure shows slight upward movement between 15 and 30 min and the corresponding strong side-lobes appear at a depth around 75 m.

Fig. 5 shows the results of a stability test made at different source depths. The TRM transmitted 100-ms, 3-4 kHz chirp signals every minute for 35 min by exciting the transducers one by one at a 333-ms interval. The signals received by one element of the VRA are data sets of dimension 29×35 , corresponding to the number of elements in the TRM and the number of transmission sequences. The first 29 data are correlated with successive data sets to examine the time variation of the signal correlation. This is a passive time-reversal (phase-conjugation) process [6] performed using one-way transmission data. The result with all the receiver depths shows the vertical structure of the time-reversal stability. As we see the focus exists longer at deeper depths below the thermocline where the sound-speed fluctuations are expected to be small. The stability decreases with the probe source moving to shallow depths and shows minimum stability at a depth of 50 m which corresponds to the bottom of the thermocline.

III. TIME-REVERSAL FOCUSING IN A RANDOM OCEAN

In this section, we examine time-reversal focusing in a time-dependent ocean waveguide using adiabatic normal mode theory and a first-order perturbation approximation.

The time-reversal process consists of exciting a TRM by the phase conjugated received signals. Let the acoustic field at frequency ω at the j th element of the TRM due to a probe source located at a horizontal distance R and depth z_s be

Fig. 4. Measured time-reversal foci with high frequency (3.5 kHz) transmissions. A stored 10-ms probe pulse received by the TRM from a PS at a range of 8.1 km was re-transmitted repeatedly for 45 min.

$G(\mathbf{r}_j, \mathbf{r}_s)$, where $\mathbf{r}_j = (0, z_j)$ and $\mathbf{r}_s = (R, z_s)$. Then, the pressure field observed at a field point $\mathbf{r} = (r, z)$ reduces to a sum of time-reversal processes over the TRM transducer positions,

$$p(\mathbf{r}) = \sum_j G_{t2}(\mathbf{r}, \mathbf{r}_j) G_{t1}^*(\mathbf{r}_j, \mathbf{r}_s) \quad (1)$$

where $[]^*$ is complex conjugation. Specification of $t1$ and $t2$ allows for a time delay in the two-way propagation between the forward probe signal propagation and the back-propagation from the TRM. Here we assume that environmental changes during the short time interval of the one-way propagation between the PS and TRM are negligible. Using adiabatic normal mode theory, the acoustic field at a field point is

$$p(\mathbf{r}, \Delta t) \approx \sum_n \sum_m \sum_j \frac{u_m(\mathbf{r}) u_m(\mathbf{r}_j) u_n(\mathbf{r}_j) u_n(\mathbf{r}_s)}{\rho(z_s) \rho(z) \sqrt{k_n^* k_m} r R} \times \exp i \left(\int_0^r k_m dr' - \int_0^R k_n^* dr \right) \quad (2)$$

where $\Delta t = t2 - t1$ is the elapsed time of the time-reversal process and the subscript n and m are associated with the variables at time $t1$ and $t2$, respectively. If a TRM adequately samples most of the modes and the modal functions do not change very much during Δt , according to the orthogonality condition of modes, Eq. (2) becomes

$$p(\mathbf{r}, \Delta t) \approx \sum_n \frac{u_n(\mathbf{r}) u_n(\mathbf{r}_s)}{\rho(z_s) k_n \sqrt{r R}} \exp i [k_n^{(0)} (r - R) + \Delta \theta_n] \quad (3)$$

Using first-order perturbation theory, the perturbed horizontal

Fig. 5. High frequency stability test with 100-ms, 3-4 kHz chirp signals. A TRM transmitted the pulse element by element for 35 min. The received signals at the VRA were correlated with the initial arrivals. The result shows different stability with different probe-source depths.

wavenumber due to small changes in sound speed $\delta c(r, z)$ from the background sound field $c_0(z)$ is approximated as $k_n(r, t) \approx k_n^{(0)} + \delta k_n(r, t)$. The modal phase perturbation from the two-way time-reversal propagation is $\Delta \theta_n = \theta_n(t2) - \theta_n(t1)$ where $\theta_n = \int \delta k_n dr$. At the range of the probe source position, $r = R$, the closure property of the modes, $\sum_n u_n(z) u_n(z_s) / \rho(z) \approx \delta(z - z_s)$, implies the focus is at the depth $z = z_s$. However, modal attenuation and the modal phase perturbation degrade the focus as

$$p(R, z) \approx \sum_n \frac{u_n(z) u_n(z_s)}{\rho(z_s) k_n R} \exp(-2\alpha_n R) \exp(i \Delta \theta_n) \quad (4)$$

where α_n is the imaginary part of $k_n^{(0)}$. Modal attenuation increases the focal size with increasing range due to the mode stripping of the higher order modes. Although different modes would have different $\Delta \theta_n$, these values can be highly correlated with each other due to the fact that the number of effective modes of the ocean fluctuations is normally much smaller than that of acoustic modes. This correlated $\Delta \theta_n$ between modes results in an environment-driven focal shift which is observed in the experimental data as the in-and-out of focal structure and is similar to the focal shift with a frequency change [7].

IV. ROBUST FOCUSING WITH SINGULAR VECTORS

Given the model of Eq. (3), a direct approach for reducing the sensitivity of the time-reversal process to sound-speed variations is to minimize the effect of $\exp(i \Delta \theta_n)$ by modifying the received signal to a form suitable for the environment of back-propagation. Without having exact *a priori* environmental information along the propagation path,

Fig. 6. Simulated time-reversal foci at 4 different probe-source depths. The pulse is a 100 ms pure tone at 3.5 kHz and the range between the TRM and PS is 8 km.

one possible method is using weight vectors that span the range of possible wave-front perturbations. By back-propagating multiple wave fronts we can minimize the phase perturbation and avoid the case of extreme mismatch between forward and backward propagation. This method [4] was originally developed for matched-field processing to improve the robustness to environmental mismatch. In matched-field processing, the possible wave-front perturbations for a particular geographical region are calculated from historical databases of sound-speed profiles. Similarly, in time-reversal processing we use multiple probe-source pings obtained over a period of time where each ping represents a different propagation condition of the ocean medium.

If we measure successive PS pings, the number of data sets spanning all possible wave-front perturbations can be large. But this does not mean the effective dimension of the perturbation space is also large. The modal phase perturbation can be highly correlated among the modes as well as signal pings. The design of an efficient constraint space for the weight vector consists of selecting minimum vectors that can best approximate the phase perturbation space. This order reduction can be achieved using a singular value decomposition of the signal matrix $\mathbf{P}$ with rank K approximation,

$$\mathbf{P} = \mathbf{U}\mathbf{D}\mathbf{V}^+ \quad (5)$$

where $[\cdot]^+$ is the Hermitian transpose, $\mathbf{U}$ is $J \times K$ matrix whose columns are left singular vectors, $\mathbf{D}$ is $K \times K$ matrix whose diagonal elements are singular values of $\mathbf{P}$ and $\mathbf{V}$ is $N \times K$ matrix whose columns are right singular vectors. Here J is the number of receivers in TRM and N is the

number of probe-source pings observed. Now the weighting vector $\mathbf{H}$ for back-propagation is obtained by the linear combination of left singular vectors,

$$\mathbf{H} = \mathbf{U}\mathbf{q} \quad (6)$$

where $\mathbf{q}$ is a $K \times 1$ vector representing the coefficient used for the linear combination of the singular vectors. Since the singular values decrease rapidly with increasing index number, in many cases the first singular vector is sufficient for stable focusing, which gives $\mathbf{q} = [1, 0, \dots, 0]^T$. Substituting G_{i1} in Eq. (1) with $\mathbf{H}$ in Eq. (6), the focused acoustic field becomes

$$p(\mathbf{r}) = \sum_j G_{i2}(\mathbf{r}, \mathbf{r}_j) H^*(\mathbf{r}_j, \mathbf{r}_s). \quad (7)$$

Here $p(\mathbf{r})$ should show a stable focal structure since the weight vector $H(\mathbf{r}_j, \mathbf{r}_s)$ maintains high correlation with G_{i2} for all possible wave-front perturbations. The singular value decomposition of the received signal vectors makes the focal size larger resulting in a stable focus under a slight shifting of the focal center caused by the perturbed modal phases.

V. NUMERICAL SIMULATIONS

In this section, numerical simulation results for the proposed method are described. The first step is to generate the time series of sound-speed perturbations. A major source

Fig. 7. Simulated time-reversal foci with 2 different probe source depths (a) and (b) at 42 m and (c) and (d) at 72 m. Upper panels (a) and (c) correspond to using a single probe signal and lower panels (b) and (d) correspond to using the dominant singular vector.

of sound-speed fluctuations for ranges on the order of 10-km is internal waves. The internal wave generation parameters were chosen to simulate the environmental conditions of the May 2000 experiment performed near Elba Island. The representative buoyancy frequency profile is determined by the average value of the CTD casts obtained during the experiment. We used the Garret-Munk internal wave spectrum with modified parameters suitable for shallow water (see Yang and Yoo, 1999 [8] for details).

A parabolic equation model was used to calculate the acoustic field between the TRM and the VRA for each time frame (1-min interval) of the sound-speed evolution. The TRM consists of 29 elements spanning water column from 30 to 107 m with 2.75-m inter-element spacing. The VRA has 32 elements covering 93 m of the water column from 15 to 108 m. Model parameters are selected from the waveguide condition of the Elba area.

Fig. 6 shows the simulated time-reversal foci for 4 different probe-source depths. The probe pulse is 100 ms long at 3.5 kHz. The calculated TRM signal was re-transmitted into the simulated time-dependent medium every min for 45 min. As expected the result shows increased stability at deeper depth below the thermocline zone. For the probe source at 39 m, it appears that high frequency components of internal waves limit the stable focus to within 10 min. At the 51 m and 63 m depths, the focus exists for about 20 min. Even for the focus at a depth of 75 m, which is in the relatively stable portion of the water column, we can identify a slow upward motion of the focus due to internal waves. The result matches well with the general tendency of the stability observed from the experiment.

In order to test the proposed method, the singular value decomposition approach was applied to the simulated probe-source signals. We used 30 pings received for an hour and the analysis was performed at a center frequency 3.5 kHz. Fig. 7 shows the results from using the proposed method. The upper panel shows the time-reversal foci with a single probe signal transmitted repeatedly for 45 min. The lower panel shows the results from transmitting the signal using the first dominant singular vector as data for the TRM. At 42 m, the back-propagation with a single probe signal maintains a good focus for the first few minutes, then the focus begins fluctuating after 10 min and the focus is lost two times between 20 to 40 min. In contrast, the back-propagation using the proposed method maintains the focus for the entire time interval. For the 72-m case, the focus is separated into two parts after 20 min for 15 min but the proposed method maintains the focus at a single spot. Although the focal size becomes larger, the proposed method maintains the focus longer showing the improved stability.

VI. SUMMARY

The results from recent time-reversal experiments show that for high frequency transmissions (3.5 kHz) stable focusing is severely limited due to the time-dependent sound-speed structures. A method for robust time-reversal focusing is proposed and tested through numerical simulations. The method utilizes multiple probe signals to extract singular vectors. Focusing using the dominant singular vector shows improved robustness of time-reversal focusing in a time-dependent ocean.

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