

Balanced harvesting could reduce fisheries-induced evolution

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Abstract

Current fisheries management pays little attention to fisheries-induced evolution. Methods of exploitation that have benefits in the short term, while ameliorating selection in the longer term would therefore be advantageous. Balanced harvesting is a potential candidate. This tries to bring fishing more in line with natural production, and some short-term benefits for conservation of aquatic ecosystems and for biomass yield have already been documented. It is also predicted to be relatively benign as a selective force on fish stocks, because it keeps the overall distribution of mortality relatively close to natural mortality.

We test this prediction, coupling an ecological model of marine, size-spectrum dynamics to an adaptive-dynamics model of life-history evolution. The evolutionary variable is the reproductive schedule, set by the maximum body mass and the mass at maturation. The prediction is supported by our numerical analysis: directional selection under balanced harvesting is approximately an order of magnitude weaker than in a standard fishery in which fish experience a fixed rate of fishing mortality after recruitment. The benefit of balanced harvesting follows from relatively little fishing on large fish, due to the low somatic production rates the big fish have. These results therefore support the general argument for protecting big, old fish, both for ecological and for evolutionary reasons. Slot fisheries that protect large fish share some qualitative features with balanced harvesting, and show similar evolutionary benefits.

Keywords: adaptive dynamics, ecosystem dynamics, fishing-induced selection, life-history evolution, production rate, size spectrum

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1 Introduction

2 Fisheries are potentially important drivers of evolution in fish stocks, be-
3 cause fishing is often a major cause of mortality once fish reach a size at
4 which they are harvested (Heino et al., 2015). There is good evidence
5 for phenotypic change in wild populations consistent with expected effects
6 of fishing, including the much-discussed case of maturation in North East
7 Arctic cod (Eikeset et al., 2016; Enberg and Jørgensen, 2017). There is
8 also experimental evidence that such evolution can take place (Haugen
9 and Vøllestad, 2001; Conover and Munch, 2002; van Wijk et al., 2013).
10 A molecular-genetic basis for such evolution, built on change in gene fre-
11 quencies at loci linked to traits under selection in the wild, is also being
12 developed (e.g. Chebib et al., 2016).

13 The precautionary principle calls for the minimization of risks from
14 fisheries-induced evolution. We are the custodians of marine ecosystems,
15 and responsible for leaving them undamaged for the future. This is en-
16 shrined in the Malawi Principle 5 of the Convention on Biological Diver-
17 sity that motivates the ecosystem-based approach to fisheries management.
18 However, despite the case for evolutionary impact assessment (Jørgensen
19 et al., 2007), the day-to-day reality is that short-term issues of management
20 supercede longer-term issues of fisheries-induced evolution (Law, 2007).
21 An example is the plan of the European Union to eliminate discarding of
22 species subject to quota or minimum landing-size regulations in European
23 waters (Common Fisheries Policy reform EU Regulation 1380/2013). This
24 is leading to the development of technical measures that will increase the
25 selectivity of fishing, without consideration of the longer-term consequences
26 for fisheries-induced evolution. The short-term solution comes potentially
27 at the cost of exacerbating another, longer-term problem.

28 One way forward would be to develop methods of fishing that help in
29 the immediate future and, at the same time, ameliorate selection in the
30 longer term (Law, 2007). Balanced harvesting is a potential candidate for
31 this. Balanced harvesting has been proposed as a way of exploiting fish
32 stocks that would help to maintain the structure and functioning of ma-
33 rine ecosystems, by bringing fishing mortality more in line with the natural
34 production of biomass by species and body sizes (Garcia et al., 2012). For
35 clarity, we define balanced harvesting at the outset as setting fishing mor-
36 tality rate to be proportional to the rate of somatic production (dimensions:
37 $\text{mass vol.}^{-1} \text{ time}^{-1}$, or $\text{mass area}^{-1} \text{ time}^{-1}$). Perfect balanced harvesting of
38 an ecosystem is probably unachievable, but it does suggest a direction to
39 go in. The bar for improvement appears to be low: no relationship could
40 be found between fishing mortality rate and production rate of species in a
41 recent study on the West of Scotland shelf ecosystem (Heath et al., 2017).

42 Matters could be improved both by a better balance of fishing mortality
43 across species, and also by a better balance across body sizes within species.
44 These paths towards a better balance are complementary, and both could
45 bring fishing more in line with production rates. Both are the subject of
46 research, including the distribution of fishing among species or functional
47 groups (Garcia et al., 2012; Kolding et al., 2016a; Heath et al., 2017), and
48 the distribution of fishing over body sizes (Law et al., 2012; Jacobsen et al.,
49 2014; Kolding et al., 2016b; Law et al., 2016).

50 Several short-term benefits of balanced harvesting have been documented.
51 The open-access fisheries on the Zambian side of Lake Kariba, with patterns
52 of fishing mortality closer to balanced harvesting than the more regulated
53 fisheries of Zimbabwe, give greater biomass yields with less impact on com-
54 munity structure (Kolding et al., 2016b). Reducing fishing mortality in
55 species with low production rate helps to protect those that are rare and
56 vulnerable (Law et al., 2016). It also reduces ‘longevity overfishing’, aiding
57 the recovery of natural size structures, by allowing more survival of large
58 individuals (Beamish et al., 2006). In this way, it improves the resilience
59 of stocks to external perturbations (Hixon et al., 2014).

60 Here we consider a by-product of balanced harvesting, that could have
61 longer-term benefits of slowing down fisheries-induced evolution. This is
62 motivated by models that suggest balanced harvesting keeps total mortality
63 within species closer to natural mortality than do traditional size-at-entry
64 fisheries (Law et al., 2015, 2016). A better alignment between fishing mor-
65 tality and natural mortality should reduce selection on the life-histories of
66 fish stocks, and therefore reduce fisheries-induced evolution. This is pri-
67 marily a prediction about the distribution of fishing mortality over body
68 size within species, i.e. about balanced harvesting across body sizes within
69 species, rather than across species. The purpose of this paper is to test this
70 prediction, and the numerical results given here support it. In other words,
71 balanced harvesting has an incidental, longer-term advantage of reducing
72 directional selection from fishing, in addition to its short-term benefits on
73 structure and functioning of marine ecosystems.

74 To do this work, we developed a method to connect the ecological dy-
75 namics of size spectra to a simple evolutionary model of adaptive dynamics
76 (Kisdi and Geritz, 2010; Brännström et al., 2013). In technical terms, the
77 work involves analysis of a transversal eigenvalue (the invasion fitness) of
78 a high-dimensional Jacobian. The Jacobian can be resolved to a simple
79 form that will allow broader study of evolution in complex, size-structured,
80 marine ecosystems in the future.

81 2 Theory

82 The theory is built in three steps (Fig. 1). (Step 1) An ecological model
83 of the dynamics of coupled size spectra: this is needed because there is no
84 external notion of fitness in an adaptive-dynamics model—fitness of genet-
85 ically distinct phenotypes emerges directly from the ecological processes.
86 (Step 2) An evolutionary model based on adaptive dynamics, within which
87 the ecological dynamics are nested: this moves an ancestral population
88 through a sequence of mutation and selection events, driven by predation
89 in the size spectra, leading eventually to a singular point at which there is
90 no further evolution. The system is then at an evolutionarily stable state
91 (ESS), before fishing is added. Without a separation of this kind, selection
92 from fishing would be conflated with selection from predation taking place
93 inside the food web. (Step 3) Calculation of the direction and strength of
94 selection generated by a range of patterns of fishing at the ESS.

95 FIGURE 1 NEAR HERE

96 2.1 Ecological model

97 Dynamic size-spectrum models of marine ecosystems couple together an
98 arbitrary number of species through size-dependent feeding (Andersen and
99 Beyer, 2006; Hartvig et al., 2011; Blanchard et al., 2014; Jacobsen et al.,
100 2014). Like any model of a complex, real-world marine ecosystems, they
101 are a simplification. However, they are built up from realistic assumptions
102 about the frequency of predator-prey interactions between individuals of
103 a given size (Andersen et al., 2016). First, they assume that body size
104 is the primary driver of the trophic level at which marine organisms feed.
105 This property of marine trophic structure is in keeping with empirical re-
106 search on stable isotopes of nitrogen (Jennings et al., 2001). Second, they
107 deal explicitly with the growth of individuals as they eat other smaller or-
108 ganisms, so there is no external growth model, such as a von Bertalanffy
109 growth equation. Third, they assume that the most common cause of death
110 is through being eaten by larger organisms, which leaves less uncertainty
111 about rates of natural mortality. Fourth, they assume that species are
112 coupled through the body-size dependence of their prey: they are both
113 predators on other species, and cannibals on themselves. Different species
114 clearly can specialise in ways that affect their locations in food webs, and
115 size-spectrum models incorporate some species-dependent feeding param-
116 eters. Importantly, unlike most models in fisheries science, size-spectrum
117 models do the bookkeeping of biomass flowing in and out of species and
118 size categories, as individuals eat one another and grow (e.g. Law et al.,

119 2016).

120 The state variables of size-spectrum models are functions that describe
121 the density of organisms $\phi_i(w, t)$ as functions of body mass w and time t ,
122 where i is an index for species. The core of such a model is a system of
123 partial differential equations, one equation for each species, describing how
124 the density function $\phi_i(w, t)$ of each species unfolds over time through feed-
125 ing (and consequent growth, reproduction and death). At their simplest,
126 the partial differential equations take the form of a McKendrick—von Foer-
127 ster equation, with body mass rather than age as the independent variable
128 (Sinko and Streiffer, 1971; Silvert and Platt, 1978):

$$129 \quad \frac{\partial}{\partial t} \phi_i = - \overbrace{\frac{\partial}{\partial w} [\tilde{g}_i \phi_i]}^{(a)} - \overbrace{\tilde{\mu}_{\text{tot},i} \phi_i}^{(b)}. \quad (2.1)$$

130 To help understand the right-hand side of this equation, Fig. 2 illustrates
131 the meaning of terms (a) and (b). Term (a) describes the change in density
132 at body mass w , due to feeding on smaller fish, contained in the function
133 $\tilde{g}_i(w, t)$ the growth rate of individuals of body mass w at time t . This
134 is calculated as a function of the abundance of smaller, conspecific and
135 heterospecific individuals, of a suitable size to be prey of an individual of
136 body mass w . Term (b) describes the change in density at body mass w ,
137 due to death; $\tilde{\mu}_{\text{tot},i}(w, t)$ is the total per capita death rate for individuals of
138 body mass w . This is calculated as a function of the abundance of larger,
139 conspecific and heterospecific individuals of a suitable size to be predators
140 of an individual of body mass w at time t , plus other sources of mortality
141 including sensesence and fishing. See Appendix A for full mathematical
142 details.

143 FIGURE 2 NEAR HERE

144 In addition to species-dependent feeding, multispecies size spectra allow
145 species to have different life histories. Life-history parameters include, for
146 instance, the asymptotic body mass $w_{i,\infty}$, and body mass at 50 % mat-
147 uration $w_{i,m}$. In non-seasonal, size-spectrum models, individuals allocate
148 an increasing proportion of incoming biomass towards reproduction and
149 away from somatic growth as they mature, the proportion reaching 1 at
150 $w_{i,\infty}$ where somatic growth ends. For a given egg size, this is enough to
151 define a schedule of reproduction at the level of species. Predation mor-
152 tality and growth, which are also components of the life history, are not
153 set as externalities in size-spectrum models, as they emerge from size- and
154 density-dependent feeding in the food web. However, some additional death
155 is incorporated, recognising that predation is not the only reason why or-
156 ganisms die, and such death may include mortality from fishing. In this
157 way the life history is defined at the species level.

158 Note that the smallest organisms must have food to eat if they are to
159 grow, so size spectra have to be extended down into the spectrum of uni-
160 cellular plankton. For simplicity, we used a fixed plankton spectrum, set to
161 values that correspond approximately to those observed. This is equivalent
162 to an assumption that the plankton dynamics happen on a faster timescale
163 and cannot be exhausted by predation, and it has the effect that fish pop-
164 ulation growth is not limited by the plankton. However, the predation and
165 cannibalism among the fish are enough to hold their population growth in
166 check, as long as the upper limit of plankton body size is kept sufficiently
167 small relative to the sizes of maturation of the fish species.

168 2.2 Evolutionary model

169 We used adaptive dynamics to describe phenotypic evolution. The theory
170 of adaptive dynamics was developed in the 1990s to provide a direct link
171 between population dynamics and phenotypic evolution (Kisdi and Geritz,
172 2010). The basic dynamics and their graphical representation were given
173 in some early papers (Metz et al., 1992; Dieckmann and Law, 1996; Metz
174 et al., 1996; Geritz et al., 1998), and a review by Brännström et al. (2013)
175 gives an overview of the subject. The idea is that phenotypic traits, such
176 as asymptotic body mass $w_{i,\infty}$, although fixed in ecological time, have a
177 genetic component that is under selection driven directly by the ecological
178 processes. In the context of multispecies size spectra, adaptive dynamics
179 allows evolution of traits to emerge from natural selection generated by
180 the multispecies food web without simplifying the ecology. There is a
181 cost to this in terms of certain assumptions, the most important being a
182 time-scale separation between the ecological and evolutionary dynamics:
183 mutations to the trait have to be infrequent enough for the food web to
184 be at its asymptotic state (typically an equilibrium point) before the next
185 mutant appears. Other assumptions to make the dynamics more tractable
186 include small mutational steps, a simple asexual mutation-selection process
187 (a trait-substitution sequence), and populations that are dominated by a
188 single phenotype at each step.

189 The path of evolution is determined by the initial rate of increase (in-
190 vasion fitness) of mutants as they arise in the resident food web. An evo-
191 lutionary step starts with the ecological system running to its asymptotic
192 state with a set of resident trait values \mathbf{s} for the species. Having reached
193 this state, a function $\lambda_i(s'_i, \mathbf{s})$ defines the invasion fitness of a mutant with
194 an altered trait value s'_i in species i . Despite the complexity of the resident
195 food web, the eigenvalue corresponding to the invasion fitness is found rel-
196 atively easily from a Jacobian matrix that contains the mutant dynamics
197 Eq. (B.1) (Appendix B). Evolution of the set of traits is then given by a

198 system of canonical equations, with one equation for each evolving species:

$$199 \quad \frac{ds_i}{dt} = k_i \frac{\partial}{\partial s'_i} \lambda_i(s'_i, \mathbf{s}) \Big|_{s'_i=s_i} \quad (2.2)$$

200 (Dieckmann and Law, 1996), where k_i is an evolutionary rate constant for
201 species i . The core information about selection is carried by the partial
202 derivative of the invasion fitness in the direction of the mutant when the
203 mutant is rare (the selection gradient). What happens if the mutant in-
204 creases would seem to be left unanswered by this, but a theorem gives the
205 conditions under which invasion implies fixation of the mutant, and these
206 conditions apply quite widely (Geritz et al., 2002).

207 **2.3 Strength of directional selection from fishing**

208 To examine some basic effects of selective fishing we took just two interact-
209 ing fish species from the general framework above, and allowed evolution of
210 one trait on one of them. The evolving trait was the asymptotic body mass
211 w_∞ , and the mass at 50 % maturation w_m was assumed to be a fixed pro-
212 portion of this, so that the whole reproductive schedule would move with
213 body size as the trait evolved. This is in keeping with the similar length
214 ratios l_m/l_∞ observed (a) in similar-shaped fish species, (b) in taxonomi-
215 cally related fish species, and (c) in different populations of single species,
216 despite substantial variation in l_∞ (Beverton, 1992; Froese and Binohlan,
217 2000). (As only one species is evolving, the species index is omitted below.)

218 In an evolving system as simple as this, the invasion-fitness surface
219 $\lambda(w'_\infty, w_\infty)$ is enough to show the qualitative outcome of evolution. An
220 example is given in Fig. 3: the surface is saddle-like, and has a singular
221 point of evolution w_∞^* at which the selection gradient in Eq. (2.2) is zero.
222 The singular point can be seen by taking a section through through the
223 surface at $\lambda = 0$ known as the pairwise invasibility plot (PIP) (Fig. 3); the
224 singular point is at the intersection of the two lines (Geritz et al., 1998;
225 Brännström et al., 2013). In the system described below, the asymptotic
226 mass evolved to this point and came to rest there. Thus, in this instance,
227 the singular point is a continuously stable strategy (CSS), i.e. an evolu-
228 tionarily stable strategy (ESS) (Maynard Smith and Price, 1973), to which
229 there is convergence through evolution (Geritz et al., 1998; Brännström
230 et al., 2013). We take w_∞^* as the trait value of the evolved ancestral popu-
231 lation, prior to the introduction of fishing.

232 **FIGURE 3 NEAR HERE**

233 When fishing mortality is added, the shape of the invasion-fitness surface
234 is distorted, and the singular point at w_∞^* becomes invadable by mutants.

235 Some examples are shown in Fig. 4a. The gradient at w_∞^* clearly depends
 236 on the fishing mortality, and shows the strength of selection generated by
 237 fishing. Thus we measure the strength of directional selection S as the
 238 slope at the singular point w_∞^* :

$$239 \quad S = \frac{\partial}{\partial \log w'_\infty} \lambda(w'_\infty, w_\infty) \Big|_{w'_\infty = w_\infty^*}, \quad (2.3)$$

240 to compare the selective effects of different patterns of exploitation be-
 241 low, as shown in the inset of Fig. 4a. (Fig. 3 shows the direction along
 242 which the slope is measured.) If the slope becomes negative when fishing
 243 is introduced, mutants with smaller w_∞ can invade, and those with larger
 244 w_∞ cannot; steeper this slope, the greater the selective advantage of these
 245 mutants.

246 In due course, a new mortality regime would cause evolution to an-
 247 other phenotypic state. However, it would be inadvisable to use a simple
 248 adaptive-dynamics model to investigate this. The strong selection gener-
 249 ated by fishing would violate the time-scale separation between ecologi-
 250 cal and evolutionary dynamics assumed in the adaptive-dynamics model.
 251 Other methods avoiding this assumption would be preferred, such as quantitative-
 252 genetic and ecogenetic models (Andersen and Brander, 2009; Dunlop et al.,
 253 2009). Adaptive dynamics is used in this paper just to construct an ances-
 254 tral singular point of evolution, and to measure the strength of selection
 255 generated by patterns of fishing mortality at that singular point.

256 FIGURE 4 NEAR HERE

257 3 Numerical results

258 3.1 Ancestral singular point of evolution

259 For numerical analysis, we took an ecological system similar to that of Law
 260 et al. (2016), comprising a fixed plankton spectrum, together with two fish
 261 species, one growing to a small size, and the other to a large size (notionally
 262 Atlantic mackerel *Scomber scombrus*, Scombridae, and Atlantic cod *Gadus*
 263 *morhua*, Gadidae). The parameter values specifying the ecological system
 264 are given in Appendix C. Some effects of different fishing regimes on this
 265 and simpler systems in the absence of evolution have been shown in earlier
 266 papers (Law et al., 2015, 2016), but an evolutionary model is needed to
 267 examine the strength of selection generated by different fishing methods.

268 Cod was taken as the evolving species, and the evolving trait was w_∞
 269 with the 50 % maturation as a fixed proportion, 1/15 of w_∞ . A singular

270 point of evolution of the ancestral cod was found at $w_\infty^* \approx 85$ kg (Fig. 3),
 271 near the size of the largest cod ever recorded (Kolding personal communi-
 272 cation). Equivalently, mass at 50 % maturity w_m^* was 5.67 kg. Predation
 273 by mackerel on small cod was the main driver of late maturation in cod in
 274 our numerical model, and the strength of predation was therefore tuned to
 275 obtain the ancestral value (Appendix C). (In the absence of mackerel, evo-
 276 lution of the ancestral cod would bring cod to a singular point of evolution
 277 at $w_\infty^* = 27$ kg in our numerical analysis (results not shown).) The large
 278 asymptotic mass and longevity of ancestral cod can be interpreted as an
 279 evolutionary outcome of the escape that this gives from heavy predation
 280 early in life (Williams, 1966, p.89-91).

281 The invulnerability of the ancestral cod at $w_\infty^* \approx 85$ kg in the absence
 282 of fishing is evident from the section through the invasion-fitness surface
 283 in the direction of the mutant at w_∞^* (Fig. 4a, heavy dotted curve). This
 284 line reaches its maximum value of zero at w_∞^* : in other words, w_∞^* is
 285 an ESS, uninvadable by any mutant with another trait value w'_∞ in its
 286 neighbourhood. The point w_∞^* is taken as the state to which cod evolved
 287 prior to the introduction of fishing.

288 3.2 Patterns of fishing mortality

289 We considered three ways in which to distribute fishing mortality rate over
 290 body size (Fig. 5). (1) Balanced harvesting sets the rate to be proportional
 291 to the current rate of somatic production at each size, from some minimum
 292 size of capture onwards (see Appendix D). (2) Size-at-entry fishing has a
 293 minimum capture size above which the fishing mortality is constant irre-
 294 spective of body size. (3) Slot fishing has constant fishing mortality like
 295 size-at-entry, but has an additional maximum body size above which fish
 296 are not caught. Each fishing pattern has a parameter controlling the overall
 297 intensity of fishing. Under size-at-entry and slot fishing, this is the fishing
 298 mortality rate, F , within the exploited size range. Under balanced har-
 299 vesting, no single value of F can be used, because F is a function of body
 300 size. Instead, the constant of proportionality c (units: $\text{m}^3 \text{g}^{-1}$) between the
 301 production rate and fishing mortality, is taken as the parameter (Appendix
 302 D).

303 FIGURE 5 NEAR HERE

304 Thus the fishing patterns differ in the fishing mortality above some min-
 305 imum size of capture (assumed to be knife-edge). Notice that the fall in so-
 306 matic growth rate and biomass, which typically happens when fish become
 307 large for their species, has the effect of making the somatic production rate

308 decrease. This is therefore accompanied by a corresponding fall in fishing
309 mortality under balanced harvesting. The different fishing patterns distort
310 the invasion-fitness surface (Fig. 3) in different ways, generating different
311 selection gradients, which will be described below.

312 The key to understanding the selection on cod generated by fishing is
313 through the changing regime of mortality on cod that fishing brings about.
314 This comes in two parts. First, there is a direct effect on mortality from the
315 fish that are caught. Second, hidden beneath this, are changes in intrinsic
316 mortality, predation mortality and cannibalism inside the size-structured
317 food web, as it adjusts to the fishing. The ecological size-spectrum dynam-
318 ics automatically keep track of these internal changes, and the effects of
319 the changes are felt by non-target as well as by target species.

320 **3.3 Mortality from mackerel predation**

321 The hidden effects of predation are important. For instance, mackerel is
322 not a passive partner in the evolution of cod: predation by mackerel is
323 part of the mortality experienced by cod. If mackerel are harvested, the
324 predation by mackerel on cod is reduced, and this leaves a footprint on the
325 invasion fitness of mutants w'_∞ in cod, favouring those with lower w'_∞ (Fig.
326 4a, dash-dot line), irrespective of any fishing on cod.

327 We assumed a fixed background of fishing on mackerel, harvested as a
328 size-at-entry fishery with a fishing mortality rate 0.5 yr^{-1} starting at a body
329 mass 250 g. We did this because cod could be seriously depleted by the
330 combined effects of heavy fishing and predation from mackerel, if the latter
331 was unexploited. So fishing on mackerel here was taken as a fixed part
332 of the environment of cod, and was not balanced to match fishing on cod
333 (cf. Law et al., 2016). The selection gradients on cod under fishing should
334 therefore be taken relative to the selection gradient on cod already caused
335 by catching mackerel. However, the impact on cod of fishing mackerel at
336 this level is relatively small, as shown in Fig. 4a.

337 **3.4 Selection in balanced and size-at-entry fisheries**

338 A balanced fishery on cod leads to much less selection on the life history
339 than a size-at-entry fishery (Fig. 4a: continuous and dashed lines). This
340 can be seen from the much steeper gradient of the invasion fitness under
341 size-at-entry (continuous curve) than under balanced harvesting (dashed
342 curve), and is consistent with the prediction that balanced harvesting is
343 relatively benign in its effects on fisheries-induced evolution. Depending

344 on whether the selective effect of fishing mackerel is allowed for, the selec-
345 tion gradient in the size-at-entry fishery is from about five to twenty times
346 that in the balanced fishery at the same biomass yield. Fig. 4b extends
347 the comparison of balanced and size-at-entry fisheries to show the rela-
348 tion between selection gradient and biomass yield as cod fishing mortality
349 increases from zero (the fishing mortality rate on mackerel is fixed through-
350 out). The major benefits from balanced harvesting in reducing selection
351 are clear. Note that the selection gradient on cod is negative even when
352 there is no fishing on cod, because mackerel fishing automatically changes
353 the pattern of predation on cod.

354 Fig. 6 gives a sensitivity analysis of the effect of varying fishing pressure
355 over a range of minimum capture sizes. This confirms the much weaker se-
356 lection in balanced than in size-at-entry fisheries, as the minimum capture
357 size is varied: for a given biomass yield, the selection gradient is substan-
358 tially closer to zero in balanced than in size-at-entry fishing. Yield rises to
359 a peak as fishing increases and then falls until extinction occurs. Balanced
360 harvesting gives the greatest benefits to reducing selection with moderate
361 levels of fishing, well before the maximum yield is reached. The yield does
362 not return smoothly to zero as fishing increases; instead there is a thresh-
363 old when the combined effects of fishing, cannibalism, and predation by
364 mackerel reach a point at which cod collapses.

365 FIGURE 6 NEAR HERE

366 The main benefit of balanced harvesting comes from bringing fishing
367 in line with production rates of large (not small) fish. This is evident
368 from the fact that the minimum capture sizes in the balanced fisheries
369 have relatively little effect on the selection gradients as the biomass yield
370 is growing (Fig. 6a). In contrast, in the size-at-entry fisheries, selection
371 for earlier maturation becomes stronger (i.e., S becomes more negative),
372 as fishing becomes more concentrated on adults (Fig. 6b). In the balanced
373 fisheries, the selection gradients in fact get slightly closer to zero as the
374 minimum capture size increases (Fig. 6a), thereby countering the effect of
375 mackerel fishing.

376 3.5 Selection in slot fisheries

377 A detailed balancing of fishing to production rate by species and body size
378 would be hard to achieve in practice. Evidently, low fishing mortality on
379 the big fish that have low production rates is the key to reducing fisheries-
380 induced selection on the reproduction schedule. We therefore examined the
381 sensitivity of selection to a range of slot fisheries, as a first approximation to

382 balanced harvesting (Fig. 7), using two fixed ratios of maximum/minimum
383 capture size of 5 and 10. Like balanced and size-at-entry fisheries, the yield
384 rises to a peak as fishing increases. But unlike balanced harvesting, the
385 extinction point under fishing can be close to the peak unless the minimum
386 capture size is large. Since collapse could occur with little warning, slot
387 fisheries on small fish would need to be implemented with care.

388 FIGURE 7 NEAR HERE

389 The effect of sliding the slot fisheries across the life history of cod is
390 consistent with a basic notion of life-history theory, that organisms evolve
391 to avoid states where they are at their most vulnerable (Williams, 1966).
392 When small cod are caught (minimum capture sizes: 30, 100, 300 g), the
393 ancestral advantage in large body size as an escape from predation weakens
394 and, as in balanced harvesting, relatively weak selection for earlier maturation
395 occurs. Such fishing is undoing part of the ancestral selection pressure
396 for late maturation. When intermediate-sized cod are caught (minimum
397 capture size: 1 kg), delayed maturation allows faster growth through the
398 vulnerable size range, pushing the selection gradient a little in the opposite
399 direction, even to the point of reversing the direction of selection (Fig. 7b).
400 When large cod are caught (minimum capture size: 3 kg), delaying maturation
401 carries the heavy cost of potentially not reproducing at all, and, as
402 in size-at-entry fishing, there is strong selection for early maturation.

403 The reversal of fisheries-induced selection is remarkable (Fig. 7b, fishing
404 from 1 to 10 kg). We interpret it in part as an interaction with the mackerel
405 fishery, since this slot size range would include cod that would otherwise
406 be eating the exploited size range of mackerel to a major degree. Catching
407 these cod thus allows more of these mackerel to escape predation, despite
408 the fishery on them (and also more escape from predation by cod of a
409 similar size). The outcome is heavier predation on cod still earlier in life,
410 and overall selection for later maturation.

411 4 Discussion

412 Our results support the prediction that balanced harvesting is a good deal
413 more benign than traditional size-at-entry fisheries, as a selective pressure
414 on the life histories of fish. This is contingent on fishing mortality being set
415 at a moderate level. Although the ecological context of multispecies size
416 spectra is different from previous work, the basic feature, that organisms
417 evolve not to linger in vulnerable states, is congruent with earlier work on
418 life-history evolution (Williams, 1966; Edley and Law, 1988), suggesting
419 a robustness of the results that goes beyond particular model structures.

420 The simple message is that, to keep fishing-induced selection small, it helps
421 to protect big fish with low production rates.

422 Importantly, balanced harvesting is as much about *reducing* fishing on
423 components of ecosystems that have low production, as it is about fishing
424 on those that have high production. Fish that are big for their species typ-
425 ically have relatively low somatic production rates, (a) because they have
426 low mass-specific somatic growth rates, and (b) because a history of fishing
427 tends to truncate size structures, leaving the remaining big fish with low
428 biomass densities. The somatic production rate is simply the product (a) \times
429 (b), and balanced harvesting therefore calls for correspondingly little fish-
430 ing on these big fish. Balanced harvesting thus aligns with a major stream
431 of thinking that big, old fish need protection both for ecological and for
432 evolutionary reasons (Beamish et al., 2006; Hsieh et al., 2006, 2010; Hixon
433 et al., 2014). Balanced harvesting contributes to this literature in sug-
434 gesting somatic production rate as a quantitative guide for setting relative
435 levels of fishing mortality.

436 A precise balancing of fishing mortality to production rate by body size
437 would be hard to achieve in practice. Slot fisheries that select an interme-
438 diate range of body size resemble balanced harvesting at a qualitative level,
439 as they create a refuge for large fish. Our results on fisheries-induced selec-
440 tion caused by slot fishing are consistent with those of a recent study on the
441 use of gillnets in NE Arctic cod (Zimmermann and Jørgensen, 2017). Slot
442 fishing deserves attention in the drive for increased selectivity to reduce
443 discarding (Common Fisheries Policy reform EU Regulation 1380/2013).
444 Selectivity *per se* is not the issue—it is what is being selected that mat-
445 ters. To get the evolutionary benefits from slot fisheries, their upper limits
446 should not extend too far into adult life, as that would generate a strong
447 selective advantage for early reproduction. Slot fisheries involving juve-
448 niles have to be implemented with caution because of the clear danger that
449 stocks could collapse from over-exploitation.

450 Taking a multispecies, size-spectrum model as the ecological input into a
451 model of adaptive dynamics, provides a new route into life-history evolution
452 in general, and fisheries-induced evolution in particular. It deals internally
453 with all the density-dependent growth and mortality generated by preda-
454 tion and cannibalism in the size-structured, food-web model. In this way, it
455 removes an artificial separation of natural mortality from fishing mortality.
456 This has some interesting consequences. For instance, it shows how fishing
457 on one species generates selection on another (unexploited) species, as the
458 food web adjusts to the fishing. It also shows that a fishing regime, ap-
459 propriately chosen, could change the predation mortality generated within
460 the food-web, reversing the direction of selection caused by fishing. This

461 would be system specific, and would require a detailed understanding of
462 how the food web works. The framework we have developed offers a route
463 to exploring the selection pressures generated by fishing on multiple species
464 within a marine ecosystem.

465 Quite apart from the context of fisheries-induced evolution, coupling
466 size-spectrum dynamics to adaptive dynamics should facilitate research
467 into broader issues about evolution in aquatic food webs. Current models
468 of size spectrum dynamics contain a number of parameters that could be
469 evolutionary variables, such as how far down the food web predators are
470 feeding, how broad their diets are, and how active they are. Further ecolog-
471 ical parameters are likely to become part of the language of size-spectrum
472 models as the research field develops, and adaptive dynamics provides a
473 flexible framework for studying their evolution.

474 One general evolutionary issue is whether there is a simple maximisation
475 principle at work. Such a principle, that species evolve to reproduce at the
476 body size at which cohort biomass is greatest, has been suggested
477 by Froese et al. (2016) as an argument for the implausibility of peaks in
478 biomass at small body size (Law et al., 2016). Our evolutionary analysis
479 does not support this maximum-biomass principle: irrespective of biomass
480 peaks, predation by mackerel on small cod generates an advantage for late
481 maturation in cod. Peaks and troughs in cohort biomass (and equivalently
482 somatic production rate) occur at body sizes where mass-specific growth
483 rate and death rate intersect (Law et al., 2016, Appendix E). These rate
484 functions are nonlinear and rather labile as they are strongly affected by
485 the prevailing predation in the food web. We would therefore expect the
486 peaks of cohort biomass to move around during the course of evolution.
487 Until more is known about such evolution, it is probably sensible to keep
488 an open mind about where peaks in cohort biomass are located with respect
489 to body size, and to try to understand more about the location of peaks
490 from empirical work.

491 Among the caveats about this study is the reduction of the life history
492 to a single scalar measure of reproduction, to allow the whole reproduction
493 schedule to shift to smaller or larger body sizes. This allows some basic
494 calculations, but it simplifies the multidimensional, phenotypic structure
495 of the life history. For instance, there is special interest in probabilistic
496 maturation reaction norms (PMRNs) as sensitive indicators of fisheries-
497 induced evolution (Heino et al., 2002; Heino and Dieckmann, 2008). The
498 ecological, size-spectrum dynamics do carry dependence of growth on food,
499 so there is an implied PMRN, which would be seen as prey densities change;
500 this PMRN would depend on age (not body size) with the size-spectrum
501 model as implemented here.

502 A second caveat is that we have not dealt with the rate at which fisheries-
503 induced evolution takes place. This is because it would be hard to justify
504 adaptive-dynamics' time-scale separation between ecological and evolution-
505 ary dynamics in contemporary fisheries. Our results say only that, for a
506 given biomass yield, the strength of selection could be brought down by
507 roughly an order of magnitude by moving from size-at-entry fishing to
508 balanced and appropriate slot fisheries. The rate of evolutionary change
509 caused by fishing is widely discussed (e.g. Jørgensen et al., 2007; Ander-
510 sen and Brander, 2009; Audzijonyte et al., 2013a; Heino et al., 2015), but
511 has not gained traction in the practical management of fisheries. This
512 is unfortunate because the longer, decadal time-scale of fisheries-induced
513 evolution does not absolve managers of marine ecosystems from responsi-
514 bility for such changes. One reason for linking fisheries-induced evolution
515 to balanced harvesting is that, as well as helping to resolve some short-term
516 issues, it can evidently also assist conservation of fish stocks in the longer
517 term.

518 A third caveat is that fishing gear obviously has many selective effects
519 other than changing the mortality rate, for instance on behaviour or repro-
520 ductive phenology (Heino et al., 2015; Andersen et al., 2018; Tillotson and
521 Quinn, 2018). Such selective effects of fishing gear can be quite different
522 from those generated by natural predators. The prediction in this paper is
523 simply about the distribution of mortality on the evolution of life histories
524 under different schemes of fishing.

525 Our main result, that fisheries-induced selection would be reduced by
526 lowering fishing mortality on fish that are big for their species, should be
527 robust. However the fine details of feedbacks within food webs are bound
528 to be context dependent. Feedbacks in multispecies, size-structured food
529 webs are intricate, and the challenge as fisheries science moves towards an
530 ecosystem approach is to see what, if any, broad robust patterns emerge
531 from the fine details (Audzijonyte et al., 2013b).

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Table 1: Model parameters and values.

Parameter	Mackerel	Cod	Unit	Comments
<i>Fish life histories:</i>				
$w_0 e^{x_{i,0}}$	0.001	0.001	g	mass of fish egg
$w_0 e^{x_{i,m}}$	200	evolving	g	mass at 50% maturity
$w_0 e^{x_{i,\infty}}$	650	evolving	g	asymptotic mass
$\rho_{i,m}$	15	8	–	controls the body-size range over which maturation occurs
ρ	0.2	0.2	–	exponent for approach to asymptotic body size in reproduction function
<i>Dynamic size spectra of fish species:</i>				
K	0.2	0.2	–	food conversion efficiency
α_i	0.8	0.8	–	search rate scaling exponent
A_i	750	700	$\text{m}^3 \text{yr}^{-1} \text{g}^{-\alpha}$	feeding rate constant
β_i	6	4.5	–	natural log of mean predator prey mass ratio
σ_i	2.5	1.9	–	diet breadth
$\mu_{o,i}^{(0)}$	0.1	0.1	yr^{-1}	intrinsic mortality rate at birth
ξ	-0.15	-0.15	–	exponent for intrinsic mortality
<i>Fixed plankton size spectrum:</i>				
$w_0 e^{x_{0,min}}$	4.8×10^{-11}		g	lowest body mass of plankton
$w_0 e^{x_{0,max}}$	0.03		g	greatest body mass of plankton
$u_{0,0}$	100		m^{-3}	plankton density at 1 mg
γ	2		–	exponent of plankton spectrum

718 **Figure legends**

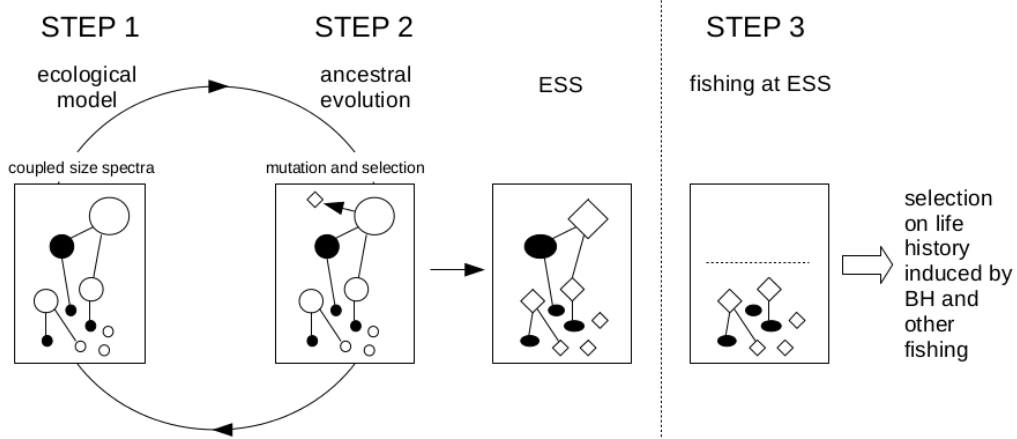


Figure 1: Road map of modelling steps. Boxes are cartoons of size spectra, with two species (filled and empty), and shapes depicting different phenotypes. The ecological model is run to determine the equilibrium state of the two species (STEP 1). New phenotypes are generated by mutation (STEP 2). The fate of a new phenotype is decided by the ecological dynamics (STEP 1). These steps are iterated as shown by the arrows until eventually the system reaches a state at which no further mutant can invade, an evolutionarily stable state (ESS). Contemporary fishing at this ancestral ESS generates new selection on the life history (STEP 3). The paper contrasts the strength of selection generated by balanced harvesting with size-at-entry and slot fishing.

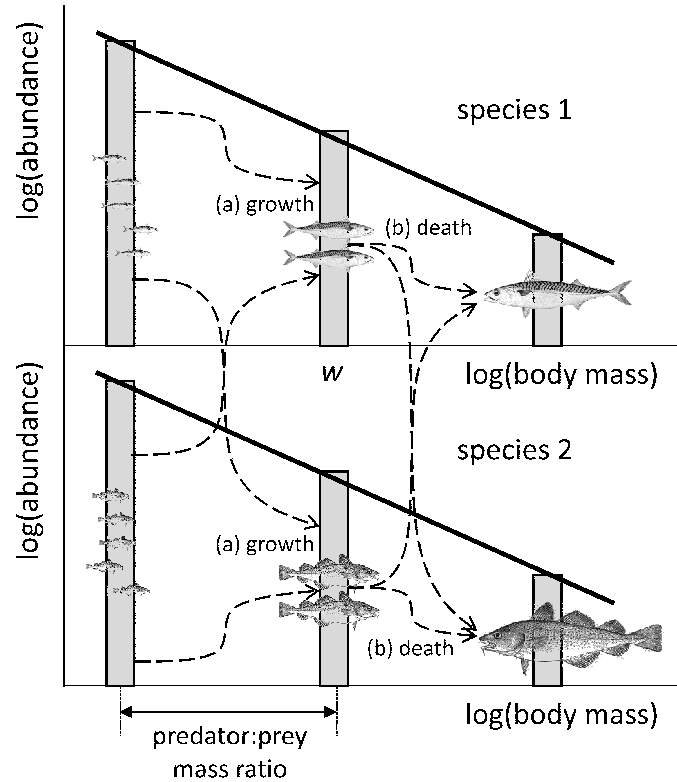


Figure 2: Processes acting on fish of body mass w . Growth comes from feeding on smaller fish of the same and other species, given by rate term (a) in Eq. (2.1). The main cause of death is predation and cannibalism by larger fish, a component of the rate term (b) in Eq. (2.1). Feeding is set by a preference function for prey relative to size w , determined by a species-specific predator:prey mass ratio. Heavy lines are examples of size spectra on log-log axes; these lines can change in shape over the course of time, as fish grow and die. Dashed lines show biomass flows from prey to predator.

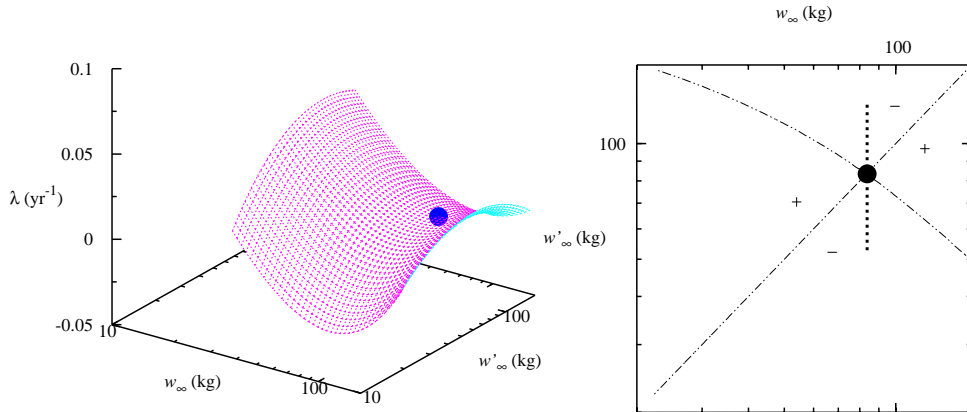


Figure 3: An example of an invasion fitness surface λ for a mutant with trait value w'_∞ as it enters a resident population with trait value w_∞ , and its corresponding pairwise invasibility plot (PIP), the section through the surface at $\lambda = 0$. Filled circles mark the singular point of evolution, w_∞^* . Signs show the sectors of the PIP in which the invasion fitness of mutants is positive and negative, with boundaries given by the dash-dot line. The dotted line shows the direction in which selection gradient, Eq. (2.3), is measured.

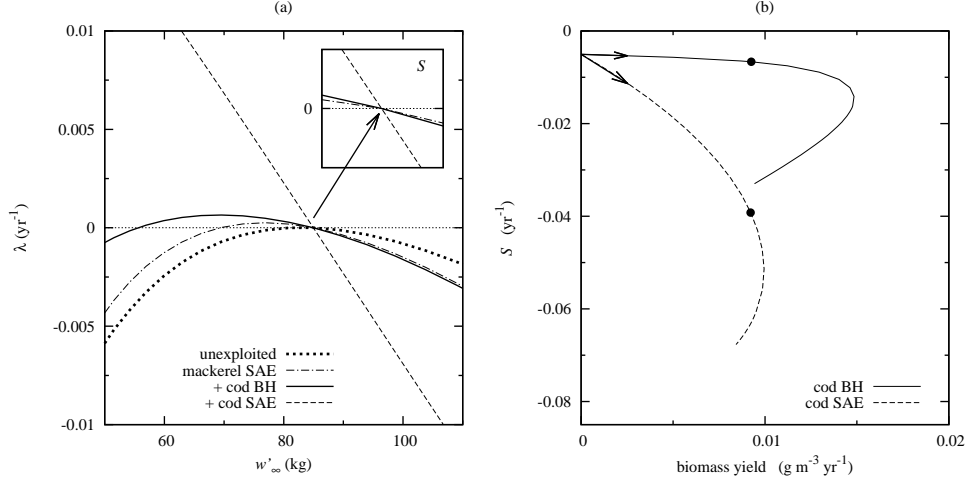


Figure 4: Invasion fitnesses and selection gradients S of cod mutants, under fishing schemes defined in the text. (a) Sections through invasion-fitness surfaces λ in the mutant direction w'_∞ at w_∞^* (the direction of the dotted line in Fig. 3). The ancestral, unexploited system has a singular point of evolution w_∞^* at 85 kg ($S = 0$). The selection gradient on cod from fishing is measured by the gradient S at w_∞^* , Eq. (2.3), as shown in the inset. A size-at-entry fishery (SAE) on mackerel, and no fishing on cod, leads to some selection on cod: $S = -0.0051 \text{ yr}^{-1}$. Adding balanced harvesting (BH) on cod to the background fishing of mackerel, slightly increases selection on cod: $S = -0.0067 \text{ yr}^{-1}$ (cod minimum capture size 100 g, $c = 11.0 \text{ m}^3 \text{ g}^{-1}$). Adding size-at-entry (SAE) fishing on cod to the background fishing of mackerel, gives much stronger selection on cod: $S = -0.0392 \text{ yr}^{-1}$ (cod minimum capture size 1 kg, $F = 0.2 \text{ yr}^{-1}$). (b) Effects of increasing cod fishing on selection gradients S and biomass yields (minimum capture sizes remain as in (a)). Arrows show the direction of increasing fishing on cod, starting from 0 and ending close to extinction of cod (near $c = 70 \text{ m}^3 \text{ kg}^{-1}$ in the case of balanced harvesting, BH, and $F = 0.32 \text{ yr}^{-1}$ in the case of size-at-entry, SAE). Filled circles mark the selection gradients of the balanced and size-at-entry fisheries on cod, shown in panel (a).

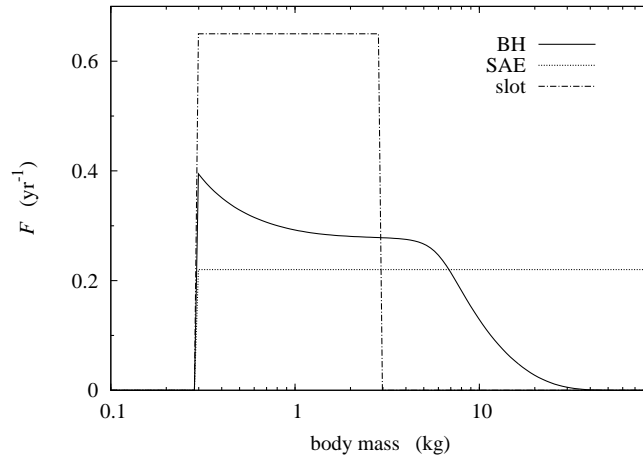


Figure 5: Three kinds of fishing mortality F : balanced harvesting (BH), size-at-entry (SAE), and slot. Each fishing pattern has a parameter controlling the overall fishing intensity, which moves the fishing mortality rates up or down; here their values are: BH $c = 30 \text{ m}^3 \text{ g}^{-1}$, SAE $F = 0.22 \text{ yr}^{-1}$, slot $F = 0.65 \text{ yr}^{-1}$. These parameter values were chosen to generate biomass yields near to $0.01 \text{ g m}^{-3} \text{ yr}^{-1}$ at steady state. They give selection gradients $S \text{ (yr}^{-1}\text{)}$: BH -0.008 , SAE -0.045 , slot -0.014 .

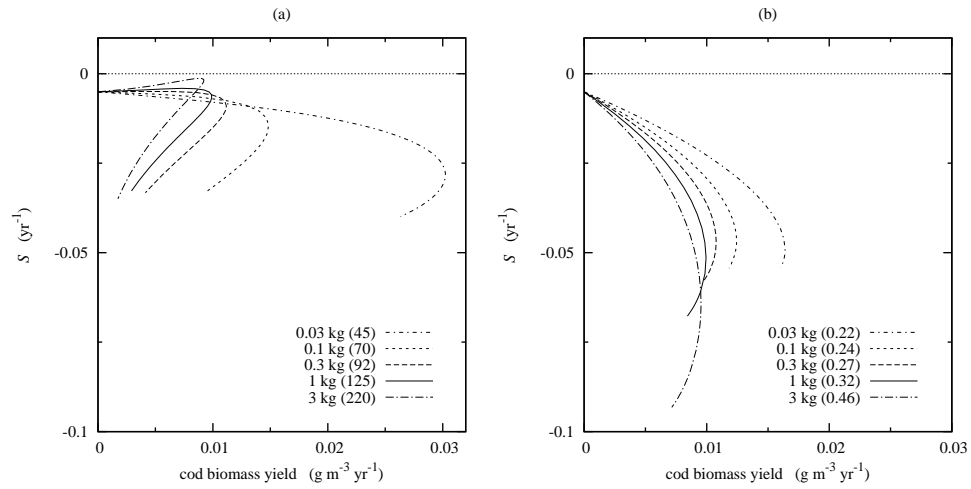


Figure 6: Selection gradients S and biomass yields of cod as fishing mortality on cod increases in: (a) balanced harvesting, and (b) size-at-entry fisheries. Each line describes a different minimum capture size, as given in the keys. Lines end where fishing mortality rate causes extinction of cod; this is close to the value given in brackets, c (m³ kg⁻¹) in the case of balanced harvesting, and F (yr⁻¹) in the case of size-at-entry fishing.

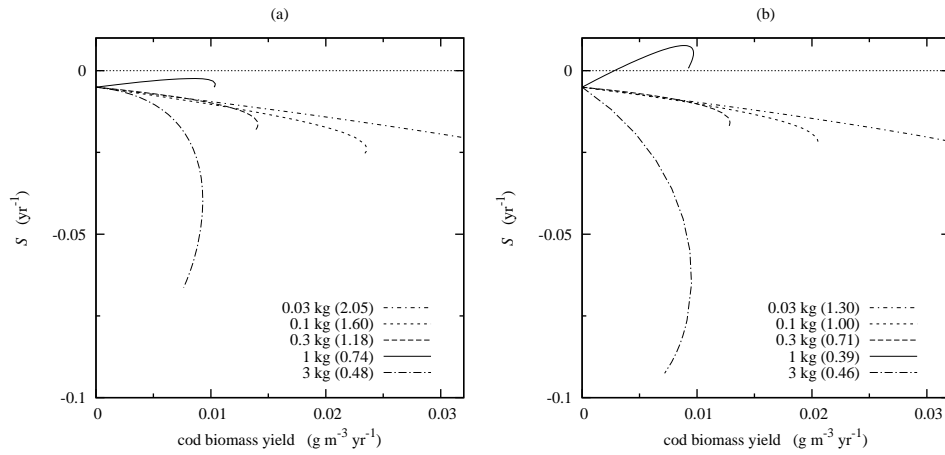


Figure 7: Selection gradients S and biomass yields of cod obtained as fishing mortality on cod increases in slot fisheries: (a) maximum capture size at 5 times the minimum; (b) maximum capture size at 10 times the minimum. Each line describes a different minimum capture size, as given in the keys. Lines end where fishing mortality rate F causes extinction of cod close to the value (yr^{-1}) given in brackets, except for minimum capture size 30 g, which is off the scale.

719 Appendices

720 A Multispecies dynamics

721 It is convenient to work in terms of the logarithmic body mass variable,
 722 $x = \ln(w/w_0)$, where w_0 is an arbitrary body mass. This gives a state
 723 variable $u_i(x, t)dx = \phi_i(w, t)dw$, with dimensions L^{-3} , which corresponds
 724 to the density of individuals of type i with log body mass in the range
 725 $[x, x + dx]$ at time t . ‘Type’ may be a species or a mutant within a species.
 726 The dynamics of $u_i(x)$ are given by the partial differential equation (Law
 727 et al., 2016):

$$728 \quad \frac{\partial u_i}{\partial t} = \overbrace{-\frac{\partial}{\partial x} [\epsilon_i g_i u_i]}^{\text{growth}} + \overbrace{\frac{1}{2} \frac{\partial}{\partial x} \left[e^{-x} \frac{\partial}{\partial x} [\epsilon_i G_i u_i] \right]}^{\text{diffusion}} + \overbrace{\frac{b_i R_i}{2} e^{-x}}^{\text{reproduction}} - \overbrace{\mu_{\text{tot},i} u_i}^{\text{mortality}}, \quad (\text{A.1})$$

729 where the arguments x and t have been omitted from each function. The
 730 functions $g_i(x, t)$, $G_i(x, t)$ and $\mu_{\text{tot},i}(x, t)$ respectively represent the rates of
 731 mass-specific prey biomass assimilation, diffusion and mortality for type i
 732 at log body mass x and time t . The function $R_i(t)$ is the reproduction rate
 733 (number of eggs produced per unit volume per unit time) of type i at time
 734 t . The function $\epsilon_i(x)$ is the proportion of assimilated prey biomass that is
 735 used for somatic growth by individuals of type i and log body mass x . Each
 736 of these functions will be defined below. The function $b_i(x)$ represents the
 737 mass distribution of eggs of type i . This is assumed to be a Dirac-delta
 738 function, corresponding to a unique log mass $x_{i,0}$ for type i . Eq. (A.1) is an
 739 extension of the size-based McKendrick–von Foerster equation to include
 740 a second-order diffusion-like term. This allows for demographic variability
 741 in size-at-age trajectories (Datta et al., 2010, 2011), although in practice
 742 this is small.

743 The model assumes that a predator of type i and log body mass x
 744 searches a volume of water $A_i e^{\alpha_i x}$ per unit time, and has a relative pref-
 745 erence for prey that is given by a function $s_i(r)$ of the predator:prey mass
 746 ratio r . The relative encounter rate between individuals of type i and indi-
 747 viduals of type j is denoted θ_{ij} . The mass-specific prey biomass assimilation
 748 rate $g_i(x)$ is calculated as an integral over the abundance of potential prey:

$$749 \quad g_i(x) = A_i K e^{(\alpha_i - 1)x} \sum_{j=0}^n \theta_{ij} \int e^{x'} s_i(e^{x-x'}) u_j(x') dx'. \quad (\text{A.2})$$

750 Similarly, the rate function for the second-order diffusion term $G_i(x)$ is

751 given by (Law et al., 2016)

$$752 \quad G_i(x) = A_i K^2 e^{(\alpha_i - 1)x} \sum_{j=0}^n \theta_{ij} \int e^{2x'} s_i(e^{x-x'}) u_j(x') dx'. \quad (\text{A.3})$$

753 Three sources of mortality are included: predation mortality, natural
754 non-predation mortality (referred to as intrinsic mortality) and fishing mor-
755 tality

$$756 \quad \mu_{\text{tot},i}(x) = \mu_i(x) + \mu_{o,i}(x) + \mu_{F,i}(x).$$

757 The predation mortality rate $\mu_i(x)$ is calculated as an integral over the
758 abundance of potential predators:

$$759 \quad \mu_i(x) = \sum_{j=1}^n A_j \theta_{ji} \int e^{\alpha_j x'} s_j(e^{x'-x}) u_j(x') dx'. \quad (\text{A.4})$$

760 The intrinsic mortality rate $\mu_{o,i}(x)$ accounts for sources of mortality other
761 than predation and fishing. We assume that this is proportional to the
762 mass-specific needs for metabolism, relative to the mass-specific rate at
763 which food becomes available at size x . These rates are set relative to their
764 values at egg size, so $\mu_{o,i}(x_{i,0}) = \mu_{o,i}^{(0)}$ is a fixed baseline intrinsic mortality at
765 birth for type i . The metabolic need should scale with body mass, and we
766 write this as $\exp(-\xi(x - x_{i,0}))$, using the same exponent for all types. The
767 mass-specific prey intake rate at size x relative to size $x_{i,0}$ is $g_i(x)/g_i(x_{i,0})$.
768 Thus

$$769 \quad \mu_{o,i}(x) = \mu_{o,i}^{(0)} \exp(-\xi(x - x_{i,0})) g_i(x_{i,0})/g_i(x), \quad (\text{A.5})$$

770 which is also a function of time because it depends on the mass-specific
771 prey intake rate $g_i(x)$.

772 The feeding kernel for type i is a Gaussian function of log predator-to-
773 prey mass ratio r , with mean β_i and standard deviation σ_i . The feeding
774 kernel is assumed to be 0 when $r < 1$ so that predators are always larger
775 than their prey:

$$776 \quad s_i(r) = \left\{ \begin{array}{ll} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(r)-\beta_i)^2}{2\sigma_i^2}\right) & r \geq 1 \\ 0 & r < 1 \end{array} \right\}. \quad (\text{A.6})$$

777 The function $\epsilon_i(x)$ the proportion of incoming prey biomass that is al-
778 located to reproduction, using a form suggested by Hartvig et al. (2011):

$$779 \quad 1 - \epsilon_i(x) = [1 + \exp(-\rho_{i,m}(x - x_{i,m}))]^{-1} \exp(\rho(x - x_{i,\infty})). \quad (\text{A.7})$$

780 Here $w_0 e^{x_{i,m}}$ is the body mass at which 50 % of the fish of type i are mature,
781 and $\rho_{i,m}$ defines the body-mass range over which fish are maturing. The

782 asymptotic body mass $w_0 e^{x_{i,\infty}}$ is the size at which all incoming mass is
 783 allocated to reproduction and no further somatic growth is possible, the
 784 approach to this size being scaled by a parameter ρ common to all types.

785 The egg size $x_{i,0}$ and asymptotic size $x_{i,\infty}$ together give boundary condi-
 786 tions for Eq. (A.1), over which there is no flux of individuals. For simplicity,
 787 we do not deal with the dynamics of the plankton. This can be thought
 788 of as an assumption that the plankton operate on a short timescale rel-
 789 ative to the fish community. The fixed plankton spectrum was taken as
 790 $u_0(x) = u_{0,0} \exp^{(1-\gamma)x}$, where $u_{0,0}$ is the abundance of plankton of mass 1
 791 mg, giving a power-law relationship between body mass and abundance.
 792 Parameter values are given in Table 1.

793 B Invasion fitness

794 We consider a resident community consisting of two species coexisting at
 795 a stable equilibrium (though the following easily generalises to more than
 796 two species). The discretised version of the size-spectrum model consists of
 797 the abundance u_i of each species in size classes x_k ($k = 1, \dots, N$) with step
 798 size Δx . The Jacobian matrix of the two-species system takes the form

$$799 \quad \mathbf{J}_{\text{res}} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}$$

800 where \mathbf{J}_{ij} is the $N \times N$ matrix describing the dependence of species i on
 801 species j . We require that this two-species system has a stable equilibrium
 802 in which both species are non-zero, so that all eigenvalues of the Jacobian
 803 evaluated at this equilibrium, $\mathbf{J}_{\text{res}}^*$, have negative real part.

804 Now suppose the community is augmented by a mutant of species 2
 805 indexed $2'$. The expanded system has a Jacobian matrix of the form

$$806 \quad \mathbf{J}_{\text{aug}} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} & \mathbf{J}_{12'} \\ \mathbf{J}_{21} & \mathbf{J}_{22} & \mathbf{J}_{22'} \\ \mathbf{J}_{2'1} & \mathbf{J}_{2'2} & \mathbf{J}_{2'2'} \end{bmatrix}.$$

807 The state at which the resident species 1 and 2 are at the two-species equi-
 808 librium and the mutant $2'$ is absent is also an equilibrium of the augmented
 809 system. When the Jacobian matrix \mathbf{J}_{aug} is evaluated at this equilibrium,
 810 the submatrices $\mathbf{J}_{2'1}$ and $\mathbf{J}_{2'2}$ are zero. Hence the Jacobian is

$$811 \quad \mathbf{J}_{\text{aug}}^* = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} & \mathbf{J}_{12'} \\ \mathbf{J}_{21} & \mathbf{J}_{22} & \mathbf{J}_{22'} \\ 0 & 0 & \mathbf{J}_{2'2'}^* \end{bmatrix} \quad (\text{B.1})$$

812 The eigenvalues of this matrix consist of the eigenvalues of $\mathbf{J}_{\text{res}}^*$, which all
 813 have negative real part, together with the eigenvalues of $\mathbf{J}_{2'2'}^*$, which is $\mathbf{J}_{2'2'}$
 814 evaluated at the coexistence equilibrium of 1 and 2, with $2'$ at zero.

815 The elements of the Jacobian $\mathbf{J}_{2'2'}$ can be obtained from the partial
 816 differential equation Eq. (A.1) for the mutant, after discretization of log
 817 mass x . For brevity, we drop the mutant index by using u_k to denote
 818 $u_{2'}(x_k)$, and similarly g_k , G_k , ϵ_k , $\mu_{\text{tot},k}$. The discretised version of the
 819 partial differential equation is then:

$$\begin{aligned}
 820 \quad \frac{du_k}{dt} &= \frac{\epsilon_{k-1}g_{k-1}u_{k-1} - \epsilon_k g_k u_k}{\Delta x} \\
 821 &+ e^{-x_k} \frac{\epsilon_{k-1}G_{k-1}u_{k-1} + \epsilon_{k+1}G_{k+1}u_{k+1} - 2\epsilon_k G_k u_k}{2\Delta x^2} \\
 822 &+ e^{-x_k} \frac{\epsilon_{k-1}G_{k-1}u_{k-1} - \epsilon_k G_k u_k}{2\Delta x} \\
 823 &- \mu_{\text{tot},k} u_k + \frac{\delta_{k1} R e^{-x_0}}{\Delta x}, \tag{B.2}
 \end{aligned}$$

824 where δ_{kl} is the Kronecker-delta symbol.

825 From Eq. (B.2), the elements of $\mathbf{J}_{2'2'}^*$ are:

$$\begin{aligned}
 826 \quad a_{kk} &= -\frac{\epsilon_k g_k}{\Delta x} - e^{-x_k} \epsilon_k G_k \left(\frac{1}{\Delta x^2} + \frac{1}{2\Delta x} \right) - \mu_{\text{tot},k}, \\
 827 \quad a_{k,k-1} &= \frac{\epsilon_{k-1}g_{k-1}}{\Delta x} + e^{-x_k} \epsilon_{k-1} G_{k-1} \left(\frac{1}{2\Delta x^2} + \frac{1}{2\Delta x} \right), \\
 828 \quad a_{k,k+1} &= \frac{e^{-x_k} \epsilon_{k+1} G_{k+1}}{2\Delta x^2}, \\
 829 \quad a_{1k} &= \frac{e^{x_k - x_0} (1 - \epsilon_k) g_k}{2}. \tag{B.3}
 \end{aligned}$$

830 All other elements of $\mathbf{J}_{2'2'}^*$ are zero because terms of the form $\partial/\partial u_l(g_k u_k)$
 831 are all zero when evaluated at the equilibrium $u_k = 0$. The functions g_k ,
 832 G_k and $\mu_{\text{tot},k}$ depend on the resident abundances via Eqs. (A.2)–(A.4).
 833 In the special case considered in this model, where the only difference
 834 between the mutant $2'$ and the resident 2 is in its reproduction schedule
 835 (ϵ_k), the functions g_k , G_k and $\mu_{\text{tot},k}$ will be identical to those for the resident
 836 2. In other words, the mutant experiences the same size-dependent food
 837 intake and mortality rates as the resident, but differs in the proportion of
 838 incoming biomass that is allocated to reproduction. In the simpler case

839 of the McKendrick—von Foerster equation without diffusion, the Jacobian
 840 elements above omit all terms containing G_k .

841 The two-species coexistence equilibrium is stable to introduction of the
 842 mutant (i.e. a rare mutant will die out) if all eigenvalues of $\mathbf{J}_{2'2'}^*$ have
 843 negative real part. If $\mathbf{J}_{2'2'}^*$ has an eigenvalue with positive real part, the
 844 two-species equilibrium is unstable to the introduction of the mutant (i.e.
 845 a rare mutant will increase in abundance). The eigenvalue with largest real
 846 part λ is the rate of increase of the mutant population when the mutant is
 847 rare, i.e. the invasion fitness.

848 C Numerical methods

849 We took a community of two fish species, one growing to a small size, and
 850 the other to a large size, together with a fixed plankton spectrum. This
 851 was based on Law et al. (2016), the two species having parameter values
 852 motivated by mackerel and cod (Table 1) as described in Law et al. (2016).
 853 The dynamics were described by Eqns (A.1), with mackerel indexed $i = 1$,
 854 and cod $i = 2$. The asymptotic body mass of cod, $x_\infty = \ln(w_\infty/w_0)$, was
 855 set to evolve, and the mass at 50 % maturation, $x_m = \ln(w_m/w_0)$, evolved
 856 with it as fixed proportion $\ln(1/15)$ of this. The matrix of preferences θ_{ij}
 857 of predators of type i for prey of type j was:

$$858 \quad \boldsymbol{\theta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0.2 & 0.2 \\ 1 & 0.2 & 1 & 1 \\ 1 & 0.2 & 1 & 1 \end{pmatrix}. \quad (\text{C.1})$$

859 The first three rows of $\boldsymbol{\theta}$, indexed $i = 0, 1, 2$, refer respectively to: (0)
 860 plankton, (1) mackerel, and (2) cod with resident trait value x_∞ . The final
 861 row refers to predation by mutant cod x'_∞ , with predation preferences set
 862 to be the same as resident cod. The cross-species predation parameters,
 863 $\theta_{ij} = 0.2$, were chosen to take cod's x_∞ to a singular point of ancestral
 864 evolution x_∞^* near that of the largest recorded cod.

865 TABLE 1 NEAR HERE

866 For numerical analysis, the continuous equations were discretized to
 867 a system of ordinary differential equations using as small a step size as
 868 practicable ($\Delta x = 0.05$). For given parameter values, we obtained a close
 869 approximation to the steady state from a numerical integration over a 100
 870 yr time period, using a time step $\Delta t = 0.0005$, based on the Euler method.
 871 The Gaussian feeding kernel $s_i(r)$ Eq. (A.6) was truncated at $\pm 3\sigma$, and
 872 normalised to sum to 1. Fast Fourier transforms were used to compute

873 the convolution integrals. In cases where convergence to the steady state
 874 was slow, the time period of integration was extended. We terminated
 875 sequences of increasing fishing mortality at extinction of the cod.

876 Having reached the steady state of an arbitrary resident community
 877 (with cod's trait value at x_∞), we constructed the life history of a rare mu-
 878 tant with an altered trait value x'_∞ . The Jacobian matrix of the resident
 879 community, augmented by the rare mutant, could then be built, with ele-
 880 ments as given in Eqs (B.3). The invasion fitness, $\lambda(x'_\infty, x_\infty)$, of the mutant
 881 cod in the resident community is the real part of the leading eigenvalue of
 882 this matrix.

883 A singular point of evolution x_∞^* occurs at

$$884 \quad 0 = \frac{\partial}{\partial x'_\infty} \lambda(x'_\infty, x_\infty) \Big|_{x'_\infty = x_\infty}, \quad (\text{C.2})$$

885 obtained numerically from a pairwise invasibility plot, using a grid of values
 886 (x'_∞, x_∞) of invasion fitness (Fig. 3). The strength of directional selection
 887 generated by fishing on cod at the singular point x_∞^* , was measured as

$$888 \quad S = \frac{\lambda(x'_\infty + \delta x, x_\infty^*) - \lambda(x'_\infty - \delta x, x_\infty^*)}{2\Delta x}. \quad (\text{C.3})$$

889 We checked the integrations by running two independently constructed
 890 versions of the code. We also checked the eigenvalue measure of invasion
 891 fitness by direct measurement of the rate of increase of rare mutants.

892 **D Fishing mortality under balanced harvest-** 893 **ing**

894 Balanced harvesting, as defined in this paper, sets the fishing mortality
 895 rate on species i at time t in proportion to the current rate of somatic
 896 production at each body mass x , from some minimum capture size x_{min}
 897 onwards. Production rate is measured as

$$898 \quad p_i(x, t) = \epsilon_i(x) g_i(x, t) u_i(x, t) w_0 e^x, \quad (\text{D.1})$$

899 where $g_i(x, t)$ is the mass-specific assimilation rate of prey biomass Eq.
 900 (A.2), $\epsilon_i(x)$ is the proportion of this prey biomass allocated to somatic
 901 growth, $u_i(x, t)$ is the the density of individuals with log body mass in
 902 the range $[x, x + dx]$, and $w_0 e^x$ is the predator mass. This gives a fishing
 903 mortality rate $F_i(x, t)$

$$904 \quad F_i(x, t) = \begin{cases} 0 & \text{if } x < x_{min} \\ cp_i(x, t) & \text{if } x \geq x_{min} \end{cases}. \quad (\text{D.2})$$

905 Here c is a constant of proportionality with dimensions vol. mass⁻¹ or
906 area mass⁻¹, and can be thought of as a mass-specific exploitation ratio.
907 Production rate changes over time as the density functions $u_i(x, t)$ change.
908 Balanced harvesting tracks the changing production rate until the ecosys-
909 tem reaches its ecological steady state. The calculations in this paper use
910 the fishing mortalities at this steady state.