Study of bubble dynamics in gas-solid fluidized beds using ultrashort echo time (UTE) magnetic resonance imaging (MRI)

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Abstract:

A narrow column, gas-solid fluidized bed was studied using ultrashort echo time (UTE) magnetic resonance imaging (MRI) for particles of diameter 0.44 mm, 0.72 mm, and 1.5 mm. UTE enables 1D and 2D images to be acquired of an individual bubble as it rises through the sample with higher spatial resolution than has previously been possible with MRI. One-dimensional images allow calculations of bubble rise velocity and bubble frequency. They also show leading-trailing bubble coalescence. Images of bubbles in the axial plane were obtained in 2D, with the location of images adjusted to track bubbles as they rise. These measurements were used to observe the wake region of a bubble, lateral drift as the bubble rises, and in-plane bubble coalescence. Images in the vertical plane were used to study the stability of a 1D perturbation in voidage. The perturbation collapsed rapidly with particles < 1 mm in diameter, but for the 1.5 mm diameter particles the perturbation was often stable throughout the imaging region.

1. Introduction:

The behavior of fluidized beds is often challenging to predict, in part due to the difficulty in obtaining experimental data in these opaque systems. One technique to acquire data experimentally on the dynamics in a fluidized bed is magnetic resonance imaging (MRI) (Harms et al., 2006; Müller et al., 2008). However, MRI is often regarded as having relatively low spatial and temporal resolution. Recently, ultrashort echo time (UTE) MRI has been shown to permit high resolution imaging in as little as 25 ms (Fabich et al., 2016). In this paper, high resolution UTE MRI is used to observe and quantify bubble behavior in a gas-solid fluidized bed.

Gas-solid fluidized beds consist of a bed of solid particles, typically < 1 mm in size. Gas flows up from the bottom of the bed until the drag force balances the gravitational force on the particles. At this point, termed minimum fluidization, the whole system begins to behave like a fluid. As the air velocity is increased above minimum fluidization, the bed will move through several different fluidization regimes including bubbling, slugging and turbulent fluidization. If the gas velocity is increased further, the particles become entrained in the gas and are elutriated from the bed (Davidson and Harrison, 1963; Kunii and Levenspiel, 1991). Many industrial scale fluidized bed reactors are used in the bubbling phase due to efficient heat transfer characteristics and enhanced mixing caused by bubbles moving through the bed. The bubbles entrain particles in their wake, which further enhances mixing in these systems (van der Hoef et al., 2006). In narrow columns, bubbles can become large enough that they bridge the walls of the column at which point the bed is said to be in the slugging regime. Understanding bubble dynamics is imperative in quantifying fluid-solid interactions in the bed, which is used

to predict reaction rates and heat transfer properties. This study will focus on the bubbling regime and the transition to slugging near the distributor in a fluidized bed.

An understanding of bubble properties such as bubble rise velocity and bubble frequency is essential in predicting fluidized bed performance (Enwald et al., 1999; Werther and Molerus, 1973a, 1973b). Bubble rise velocity is known to be a function of the bubble size (Davidson and Harrison, 1963). The size of a bubble, and hence its rise velocity, will increase due to coalescence as it moves through the bed. Bubble frequency defines how many bubbles pass through a given plane in the sample over a set amount of time (Halow and Nicoletti, 1992). Due to bubble coalescence, the bubble frequency will vary with height in the bed with more bubbles near the distributor plate. Thus, both bubble rise velocity and frequency are strongly influenced by bubble coalescence.

Bubble coalescence is typically divided into two types: leading-trailing and in-plane (Lim et al., 1995). Leading-trailing coalescence describes the scenario where two bubbles form consecutively rather than simultaneously. As they rise, the first bubble slows due to its interaction with the trailing bubble. The trailing bubble will then 'catch' and coalesce with the leading bubble. The coalesced bubble will move up the remaining height of the bed as a single bubble, or may coalesce with other bubbles. Leading-trailing bubble coalescence is important in determining bubble size and has previously been measured using MRI (Müller et al., 2007, 2006), X-ray (Rowe, 1976), and electrical capacitance tomography (ECT) (Chandrasekera et al., 2015). In-plane bubble coalescence is where two bubbles form in the same axial plane and join at some point along their trajectory up through the fluidized bed. One of the most well established bubble size models assumes coalescence occurs exclusively in-plane and that the

average location of coalescence is related to the catchment area required to produce a bubble of a given size (Darton et al., 1977; Dennis, 2013). This type of coalescence is difficult to observe in three-dimensional fluidized beds, as they are optically opaque.

When a bubble is rising through the sample it is known that the trajectory is not always straight. The bubble will often traverse the sample horizontally in addition to its vertical motion, referred to as lateral drift (Whitehead et al., 1967). This motion has been investigated for a pseudo-two-dimensional (2D) granular system (Laverman et al., 2008). However, the bubble motion will likely be different in a three-dimensional (3D) granular system as bubble motion in pseudo-2D systems is dominated by interactions with the wall. To observe lateral drift in a 3D system, a single bubble will need to be tracked as it rises through the sample. Current three-dimensional imaging techniques are too slow to track a rising bubble. One approach recently used for an air bubble in a liquid system was to counter the rise of the bubble with a downward flow of water (Tayler et al., 2011). Such an approach is difficult to apply in a granular system as it is challenging to maintain a steady downward flow of particles. An alternative approach is to follow the bubble as it rises by changing the location of the image, this approach will be explored here.

Another key question to understanding fluidized beds considers how bubbles form. Batchelor (1993) proposed that bubbles form in a fluidized bed as a direct consequence of nonlinear processes in a growing concentration wave. In liquid-solid fluidized beds, the mechanism proposed by Batchelor has been studied and verified experimentally (Duru and Guazzelli, 2002) and numerically (Anderson et al., 1995; Glasser et al., 1997, 1996). In gas-solid systems, pictures of growing waves have been difficult to observe as

they are more unstable and disturbances grow quickly (Batchelor, 1993). However, numerical investigations and indirect experimental measurements have demonstrated the important role that frictional forces (contact stresses) play in the stabilization of fluidization and hence the formation of bubbles (Sundaresan, 2003). Here we use MRI to observe the stability of an approximately 1D concentration wave in 3D gas-solid fluidized beds.

There are few techniques that make it possible to look inside a 3D fluidized bed reactor. Some of the most commonly used are electrical capacitance tomography (ECT), X-ray, and MRI (Chandrasekera et al., 2015; Müller et al., 2008; Rautenbach et al., 2013; Rowe and Partridge, 1965). Each of these techniques have strengths and limitations. ECT can be used on relatively large fluidized beds to provide valuable information about bubble size and velocity (Makkawi and Wright, 2004). However, it is typically limited to acquiring images in a 2D plane that are averaged over a length of perhaps 50 mm axially. When expanded to electrical capacitance volume tomography (ECVT) the resolution is such that objects with a smallest dimension on the order of 5-10 mm can be resolved (Wang et al., 2010). However, this resolution is too low to resolve details of the structure of individual bubbles (Chandrasekera et al., 2012). X-ray techniques are fast, but have largely been limited to acquiring 2D projections (Yates et al., 2002). Ultrafast X-ray tomography systems now exist which can acquire up to 1000 images per second with a spatial resolution of 1 mm or better, though size and material choices may be restrictive (Verma et al., 2014). MRI can acquire 3D spatially resolved data for particle density and local or bulk velocity (Callaghan, 1991). It is traditionally a relatively slow and low-resolution technique for systems like a fluidized bed and, it is often the case that to increase the temporal resolution a decrease in spatial resolution is

required, or vice versa. However, it has the advantage that spatially resolved images can be acquired in any plane or orientation within the imaging region of the MRI system. Ultrafast MRI of fluidized beds have been acquired in 2D at a temporal resolution of 25 ms and a spatial resolution of $1.7 \text{ mm} \times 4.1 \text{ mm}$ with a 5 mm slice thickness (Müller et al., 2008). Recently it has been shown that ultrashort echo time (UTE) MRI can be used to image a bubbling fluidized bed in 25 ms with a spatial resolution of $1 \text{ mm} \times 1 \text{ mm}$ and a 2 mm slice thickness (Fabich et al., 2016). This time and spatial resolution is sufficient to capture images of bubble coalescence and bubble formation.

In this paper we demonstrate the potential for 1D and 2D MRI to examine the bubble dynamics in a narrow gas-solid fluidized bed with three different particle types. The 1D profiles will be used to calculate the bubble rise velocity and frequency for each of the three particle types. The 2D images will be used to observe in-plane dynamics during bubble development by shifting the imaging plane to follow a rising bubble. The images can be interleaved with 1D profiles, which will provide assurance that a single bubble trajectory has been tracked rather than several different bubbles. These 2D images allow for visualization of lateral motion and in-plane bubble coalescence. In addition to the cross sectional images, images in the vertical plane are acquired to see bubbles forming at the distributor plate and rising through the imaging region. This technique is high enough spatial resolution, $1 \text{ mm} \times 1 \text{ mm} \times 2 \text{ mm}$, and temporal resolution, 40 frames per second (fps), to observe bubble formation in a narrow column 3D fluidized bed.

2. Materials and Methods:

2.1 Experimental Setup:

All experiments were performed using a Bruker, AV400 spectrometer, operating at a ^1H resonance frequency of 400.23 MHz. A three-axis, shielded gradient system with a maximum strength of 30.6 G cm^{-1} was used for gradient encoding, and a 38 mm diameter birdcage r.f. coil was used for excitation and detection of the signal.

The fluidized bed used for all MRI experiments was a 1.2 m tall glass column with an inner diameter of 35 mm. There was a porous, sintered glass, air distributor plate at the bottom of the column. The pressure drop across the distributor plate was greater than the bed pressure drop ($\sim 300 \text{ Pa}$) for all experiments, thus ensuring even gas distribution. The column was filled with sample to a height of approximately 80 mm. A pressure regulator supplies air at 1 bar(g) to a rotameter. The volumetric flowrate was measured by the rotameter, and controlled by a needle valve at the outlet. The air, at a set flowrate, was directed to the distributor at the bottom of the column. There is a filter at the top of the column to avoid elutriation of the particles, but otherwise the column was open to the atmosphere. A diagram of the setup is shown in Figure 1a with a schematic of the imaging region in Figure 1b. The system was at ambient temperature, approximately 20°C .

Figure 1. (a) Schematic of experimental apparatus. The glass column for the fluidized bed, height of 1.2 m, runs through the center of the MRI magnet, shown in grey. Pressurized air flows through a pressure regulator, needle valve, and rotameter, then through the distributor plate and into the glass column. (b) A schematic of the imaging region shows the air distributor plate (black) which distributes the air evenly across the sample (textured region). The imaging region, shown in red, is 40 mm in width and 40 mm in height. The inner diameter of the sample is 35 mm and the height of the sample is 80 mm. The top of the column has a filter to stop any particles from leaving the bed and is open to atmospheric pressure.

2.2 Materials:

Plant seeds were used for the solid phase as MRI signal may be acquired from the oil in the seeds ($T_2^* \sim 0.1$ ms, $T_2 \sim 50$ ms, $T_1 \sim 300$ ms). Three types of plant seeds were used: *Lobelia Crystal Palace*, *Nicotiana Sensation Mixed*, and *Chinese Mustard, Green in Snow* (Kings Seeds, UK). The seeds were selected for their size and relatively high sphericity. The particle sizes were measured using a Morphologi G3 Particle characterization system (Malvern Instruments Ltd.) for the *Lobelia* seeds and, for the

Nicotiana and *Mustard* seeds, a prototype Canty Sizer (JM Canty, Inc.). The Canty sizer was modified to improve the quality of the lighting and introduce a high-speed camera with a macro-zoom lens recording at 60 fps with a resolution of 1280×1024 . Both methods acquired 2D images of the particles. The images were then analyzed for average particle diameter and aspect ratio, defined as the ratio of the projected width of the particle divided by its projected length based on the images.

The diameter, aspect ratio and minimum fluidization air velocity, U_{mf} , for each particle are shown in Table 1. The U_{mf} is measured at ambient temperature, approximately 20°C, and atmospheric pressure. Experiments with *Lobelia* seeds are acquired for an air velocity of $U = 0.070 \text{ m s}^{-1}$, i.e. an excess air velocity of $U - U_{mf} = 0.035 \text{ m s}^{-1}$. *Nicotiana* seeds are fluidized with air velocities ranging from 0.087 m s^{-1} to 0.260 m s^{-1} , i.e. excess air velocities of 0 to 0.173 m s^{-1} , spaced by 0.017 m s^{-1} . Both *Lobelia* and *Nicotiana* seeds are classified as Geldart group B (Geldart, 1973). The *Mustard* seeds are fluidized with an air velocity of either 0.312 m s^{-1} or 0.364 m s^{-1} , i.e. excess air velocities of 0.035 m s^{-1} or 0.087 m s^{-1} . The *Mustard* seeds are classified as Geldart group D particles.

Table 1. Diameter, aspect ratio and minimum fluidization air velocity for each particle type used in this study.

seed variety	diameter / mm	aspect ratio	$U_{mf} / \text{m s}^{-1}$
<i>Lobelia</i>	0.44 ± 0.03	0.70 ± 0.07	0.035
<i>Nicotiana</i>	0.72 ± 0.12	0.80 ± 0.09	0.087
<i>Mustard</i>	1.51 ± 0.21	0.88 ± 0.07	0.277

2.3 Methods:

The implementation and development of UTE in this system has been described previously (Fabich et al., 2016, 2014). In this study, a combination of one-dimensional profiles and two-dimensional images are used to investigate bubble dynamics.

2.3.1 Bubble rise velocity and frequency

An average bubble rise velocity and bubble frequency can be calculated using 1D profiles of a fluidized bed. One-dimensional profiles characterize the volume of seeds at each plane along the height of the bed. The schematic in Figure 2 illustrates these measurements. Figure 2a shows a schematic of a bubble at the bottom of a fluidized bed and the corresponding profile. Signal is detected from the solid particles, thus there is a reduction in signal wherever a bubble exists as denoted by a darker shade in the profile. Figure 2b shows a bubble near the top of the fluidized bed. The reduction in signal moves with the bubble so the darker region, corresponding to a low signal magnitude in the profile, is located near the top of the profile in Figure 2b. In practice, profiles are acquired in rapid succession such that the bubble positions are tracked as they rise through the bed, as illustrated in Figure 2c. The bubble rise velocity and frequency can be calculated from these profiles.

An MRI profile was acquired every 5.1 ms. The profiles were acquired over a 70 mm field of view (FOV) and reconstructed with a resolution of 0.50 mm per pixel. The profiles are normalized to those acquired below minimum fluidization, which corresponds to a homogenous sample. The normalization is required to remove effects associated with the r.f. inhomogeneity (Fabich et al., 2016). Due to the size of the

memory buffer on the MRI spectrometer a maximum of 125 consecutive profiles can be acquired in immediate succession before the data must be written to the disk, these profiles correspond to a total time of 0.6 s. Assuming a bubble rise velocity of 0.1 m s^{-1} , a likely velocity near the distributor plate, the bubble will rise 0.5 mm or the size of a pixel over the imaging time. Near the top of the imaging region, a velocity of 0.3 m s^{-1} would mean a bubble will rise 1.5 mm, or three pixels, during the image acquisition. This will cause a blurring over the three pixels, but will not introduce significant error to the calculation of bubble rise velocity as the velocity is determined based on the velocity of the centre of mass of the bubble.

Figure 2. Schematic of how 1D profiles are used to calculate bubble rise velocity in a 3D bed. (a) A bubble just above the distributor plate, the 1D profile shows a reduction in signal at the same location as the bubble in its 1D representation. (b) The same bubble a short time later after having risen through the bed, also shown in the 1D profile. (c) Consecutively acquired 1D profiles showing the bubble trajectory over time.

Bubbles are represented by local regions of low signal intensity. The locations of the bubbles were identified by finding minima in the profiles of the signal intensity. The

locations of these minima are plotted to trace the bubble trajectory. For this study the bubble rise velocity was approximated as two linear regions, the first near the distributor plate and the second near the top of the imaging region. The bubble rise velocity in each region was calculated for five bubble trajectories and is reported as the average and standard deviation for the gradients of these trajectories.

The bubble frequency is calculated from the profiles by quantifying the number of reductions in signal across a given number of profiles. For this study, bubble frequency is calculated across 20 sets of 125 profiles. Each set of profiles was acquired over 0.6 s and a delay of 10 s was allowed between the acquisition of each set of profiles. Local minima may arise from noise or from the presence of a bubble. The signal-to-noise ratio of these measurements was approximately 50. Therefore, in order to distinguish bubbles from noise, the signal was required to drop by 3% from the maximum value, which corresponds to 1.5 standard deviations of the noise. Furthermore, the minimum had to appear in three consecutive profiles to reduce residual speckle noise. The error in the bubble frequency measurements was approximated by assuming plus or minus two bubbles from the count over 20 sets of profiles.

2.3.2 Two dimensional bubble imaging

The 1D representation of the bed does not provide any information about the motion of the bubble within a plane, or lateral drift, as shown in Figure 2a and 2b. In Figure 2a, the bubble is near the left hand side of the bed and in Figure 2b it is near the right hand side of the bed. As the profiles only have spatial resolution in the vertical direction, it is not possible to resolve this lateral motion. Furthermore, 1D profiles cannot distinguish

between a single large bubble and multiple small bubbles in a given plane. This information can be acquired using 2D images.

Two-dimensional images were acquired in both the axial, xy , and vertical, zy , planes. Images in both orientations were acquired in 25 ms and each image was reconstructed to a resolution of $1\text{ mm} \times 1\text{ mm}$ with a slice thickness of 2 mm. Images were normalized to a homogeneous image of the bed at minimum fluidization to eliminate effects arising from the sensitivity of the r.f. coil.

For the axial images, 2D images were acquired alternately with 1D profiles. Nine profiles were acquired interleaved with eight images. The images were acquired in 25 ms and the profiles in 5.1 ms, giving a total experiment time of 250 ms. The combination of images and profiles enables the axial images to be used to spatially resolve bubbles in plane, whilst simultaneously tracking the vertical position of the bubbles as they rise.

In addition to imaging a bubble moving through a single plane, the bubble rise velocities calculated from the profiles were used to adjust the location of the axial plane images to follow a single bubble as it accelerates, rising from the distributor plate through the imaging region of the fluidized bed. The slice positions were determined by assuming that at time zero the bubble is at the distributor plate and an image is therefore acquired immediately above the plate. The average bubble rise trajectory obtained from the 1D profiles was used to calculate the most probable location of the bubble in the second axial image. The second image was acquired 30.1 ms after the first image, corresponding to the time required to acquire an image and a profile. The same process

is repeated until eight image locations have been defined. Thus, an 'ideal' bubble may be tracked starting at the distributor plate and rising through the sample following the average bubble trajectory. The interleaved profiles provide certainty that a single bubble trajectory has been followed and the images give spatially resolved information about the number of bubbles and their location within a plane. As bubbles are not often 'ideal', many sets of data need to be acquired to accurately capture a single rising bubble in multiple frames. In this work, approximately 500 sets of images were acquired for each particle type and gas velocity and, from the 500 sets of images, approximately 10 bubble trajectories were captured successfully.

Images in the vertical plane provide further information by capturing bubbles forming at the distributor plate and rising through the imaging region in a set plane. The images in the vertical plane are acquired consecutively, every 25 ms, with no profiles interleaved. For vertical plane images, ten images are acquired before the memory buffer reaches capacity and the data must be written to the disk.

3. Results and Discussion:

Using a combination of profiles and images, UTE MRI was used to examine the dynamics of a 3D fluidized bed. Here we report measurements of the bubble rise velocity and bubble frequency using 1D profiles. We also characterize bubble dynamics using axial images to track rising bubbles and vertical images to study bubble stability.

3.1 Bubble rise velocity and frequency

The bubble rise velocity and frequency in a fluidized bed are calculated from the 1D profiles. Figures 3a-e show profiles for a bed of *Nicotiana* seeds at increasing air velocities from 0.087 m s^{-1} , the minimum fluidization velocity, up to 0.260 m s^{-1} . Each set of profiles in Figure 3 shows three blocks of 125 profiles. At minimum fluidization, Figure 3a, the profiles are homogeneous with no bubbles rising through the bed. Figure 3b shows the same bed, in a bubbling state, with an air velocity just above minimum fluidization. In these sets of profiles, darker lines are seen indicating the passage of bubbles rising through the bed. The signal intensity is proportional to the number of particles in the plane. Thus, the dark trajectories in Figure 3c indicate that the bubbles cover a large area in each cross-sectional plane at the high air velocity in Figure 3c compared to that in Figure 3b. The slope of the dark lines in Figure 3c is steeper on average than in Figure 3b, which indicates an increase in bubble rise velocity. Figure 3d continues the trend with darker trajectories, larger bubbles, and a higher bubble rise velocity toward the top of the trajectory. Figure 3e shows trajectories in the slugging regime. The bubble trajectories are very broad and are black meaning that there are no particles across this particular plane for several consecutive profiles.

Considering the region near the distributor in Figure 3, many small bubbles are seen. The trajectories of some of these merge indicating leading-trailing bubble coalescence. It can be seen from the profiles in Figure 3b-e that the leading bubble often decelerates before the trailing bubble accelerates, as has been shown previously (Müller et al., 2006). The merged bubbles then move up the bed as a single trajectory.

Figure 3. 1D profiles of *Nicotiana* seeds in a fluidized bed at different air velocities. The air velocities were (a) minimum fluidization at an air velocity of 0.087 m s^{-1} , (b) bubbling at 0.122 m s^{-1} , (c) 0.156 m s^{-1} , (d) 0.208 m s^{-1} , (e) and 0.260 m s^{-1} . Each of the three sets of 125 profiles were acquired over approximately 0.6 s.

Table 2. Bubble rise velocity and corresponding calculated bubble diameter for each different fluidization air velocity in Figure 3.

Fluidization Velocity (m s^{-1})	0.122	0.156	0.208	0.260
<i>Rise velocity - distributor plate (m s^{-1})</i>	0.11 ± 0.02			
<i>Predicted bubble diameter (mm)</i>	2.5			
<i>Rise velocity - top of imaging region (m s^{-1})</i>	0.29 ± 0.04	0.33 ± 0.11	0.43 ± 0.08	0.56 ± 0.15
<i>Predicted bubble diameter (mm)</i>	17	23	38	65

The bubble rise velocities obtained from the profiles in Figure 3 near the distributor plate and near the top of the imaging region are given in table 2. The rise velocity near the distributor plate is similar for all flow rates. The average bubble rise velocity near the top of the imaging region increases with an increase in air velocity. As the bubbles grow in size, they accelerate. At higher air velocities this acceleration happens closer to the distributor plate, presumably because of the increased rate of coalescence at higher air velocities. In Figure 3e, which shows profiles in the slugging regime, the rise velocity for the thick, dark trajectories is approximately $0.24 \pm 0.08 \text{ m s}^{-1}$ near the top of the bed. These slugs rise at a much slower velocity than the bubbles rising between the slugs. When a slug forms, it encompasses the full cross section of the sample, as can be seen by the very dark, or black, sections of the profile decreasing the rise velocity due to the wall effects (Davidson and Harrison, 1963). The bubbles rising between the slugs have velocities up to 1.3 m s^{-1} . It is likely that these bubbles will coalesce with a slug higher in the bed, outside of the visible imaging region.

The bubble rise velocity is typically described as a function of the bubble diameter (Dennis, 2013). Using the bubble rise velocities calculated from the 1D profiles, the average bubble diameter can therefore be estimated. One of the most widely used correlations is that of Davies and Taylor (1950):

$$U_{b,\infty} = 0.71\sqrt{gD_e} \quad (1)$$

where $U_{b,\infty}$ is the bubble rise velocity, g is the acceleration due to gravity, and D_e is the volume-equivalent diameter of the bubble, i.e. the equivalent diameter of a sphere of the same volume as the bubble. The bubble rise velocities calculated from Figure 3 are used to determine an average volume-equivalent bubble diameter. The bubbles rising

at 0.11 m s^{-1} , near the bottom of the bed, are predicted to have an average diameter of 2.5 mm. The predicted bubble diameters near the top of the imaging region increase with increasing superficial air velocity and are approximately 17 mm, 23 mm and 38 mm for the trajectories shown in Figures 3b, c, and d, respectively. As the diameter of the column is only 35 mm, the bubble rise velocity calculated for Figure 3d is clearly overestimated. Slugging is clearly visible in Figure 3e, however, due to the narrow column diameter and significant wall effects, the bed may have already begun slugging behavior in Figure 3d.

There are three types of slugs to consider in the system discussed here, axisymmetric slugs, wall slugs and square nosed slugs. Axisymmetric slugs span the sample symmetrically. Wall slugs travel along the wall and span the sample asymmetrically with particles only flowing around the slug on one side. Square-nosed slugs are symmetric but particles fall down through the slug rather than around the outside, thus square-nosed slugs tend to rise more slowly than either axisymmetric or wall slugs. Axisymmetric and wall slugs in aerated particles typically rise with the same velocity as a slug of gas in an inviscid liquid (Stewart and Davidson, 1967). The correlation for the rise velocity of an axisymmetric slug in a fluidized bed is therefore:

$$U_{s,\infty} = 0.35\sqrt{gD} + (U - U_{mf}), \quad (2)$$

Where D is the column diameter and other symbols are as defined previously. When calculating the rise velocity for a wall slug, the first term in Eq. 2 must be multiplied by a factor of $\sqrt{2}$. Assuming wall slugs for Figure 3d, the predicted rise velocity is 0.41 m s^{-1} in fair agreement with the measured rise velocity of 0.43 m s^{-1} . However, the rise velocity measured in Figure 3e is too high for either wall slugs or axisymmetric slugs.

The rise velocity for square-nosed slugs can be estimated from (Rudolph and Judd, 1985):

$$U_s = 0.18\sqrt{gD} + (U - U_{mf}) \quad (3)$$

Using Eq (3), square-nosed slugs are expected to rise at a velocity of only 0.28 m s^{-1} in this system, within one standard deviation of the measured velocity. Thus, the measured bubble rise velocities are consistent with the classical theory of slugging fluidized beds.

Due to bubble coalescence the bubble frequency will vary relative to the height in the fluidized bed. Figure 3 shows that there are many small bubbles forming just above the distributor plate. As the bubbles rise they coalesce which means there will be a lower bubble frequency observed near the top of the imaging region. Figure 4 plots bubble frequency at five different heights above the distributor plate for superficial gas velocities ranging from 0.1 m s^{-1} to 0.26 m s^{-1} . The bubble frequencies are averaged over 20 sets, each of 125 profiles. The bubble frequency at 5 mm and 10 mm above the distributor plate decreases slightly as the air velocity increases. In general, bubble frequency near the distributor is expected to rise with gas velocity. As the 1D profiles do not provide information about the number of bubbles at a given vertical height, the decrease in frequency seen here may suggest the presence of multiple bubbles in plane for higher air velocities. At 15 mm above the distributor the bubble frequency remains approximately constant across all air velocities and is significantly lower than at the distributor. This height corresponds approximately to the average point for leading-trailing coalescence seen in Figure 3. At heights greater than 15 mm above the distributor, the bubble frequency continues to decrease with height. The continuing decrease in bubble frequency with height indicates that leading-trailing coalescence

continues to occur throughout the bed, even though coalescence does tend to be concentrated around a height of 11 mm above the distributor in this bed.

Figure 4. Bubble frequency plotted against superficial air velocity for *Nicotiana* seeds in a bubbling fluidized bed for five positions in the bed measured as height above the distributor. The bubble frequency decreases with increasing position in the bed due to leading-trailing bubble coalescence.

Figure 5 shows bubble trajectories for all three types of seeds acquired at the same excess gas velocity of 0.035 m s^{-1} . This excess air velocity was selected to show the transition from bubbling to slugging fluidisation arising from the change in particle size. Figure 5a shows bubble trajectories for *Lobelia* seeds in a fluidized bed at an air velocity of 0.070 m s^{-1} . *Nicotiana* seeds are shown in Figure 5b with an air velocity of 0.122 m s^{-1} and *Mustard* seeds are shown in Figure 5c with an air velocity of 0.312 m s^{-1} . The bubble frequency 5 mm above the distributor plate is $14.5 \pm 0.2 \text{ s}^{-1}$ for both *Lobelia* and *Nicotiana* seeds. The frequency decreases toward the top of the imaging region to $12.3 \pm 0.2 \text{ s}^{-1}$ for the *Lobelia* seeds and $11.7 \pm 0.2 \text{ s}^{-1}$ for *Nicotiana* seeds. In both cases, the bubble frequency decreases near the top of the bed as a result of leading-trailing coalescence; in-plane coalescence may also occur but is not detectable with the 1D measurements. The bubble rise trajectories for the *Mustard* seeds (Figure 5c) show

very different behavior. The calculated bubble frequency is $8.5 \pm 0.2 \text{ s}^{-1}$ near the distributor plate and $4.8 \pm 0.2 \text{ s}^{-1}$ near the top of the imaging region. This reduction in frequency and the thicker, darker trajectories indicate the formation of larger bubbles with the *Mustard* seeds than the *Lobelia* and *Nicotiana* seeds. The change in behavior for the *Mustard* seeds is attributed to the change in particle size (and hence Geldart's classification) for the particles, with the *Mustard* seeds being $1.5 \pm 0.2 \text{ mm}$ in diameter, whilst the *Lobelia* and *Nicotiana* are $0.44 \pm 0.03 \text{ mm}$ and $0.7 \pm 0.1 \text{ mm}$, respectively.

Figure 5. 1D profiles of a bubbling fluidized bed for (a) *Lobelia*, (b) *Nicotiana* and (c) *Mustard* seeds. Each set of 125 profiles was acquired over approximately 0.6 s. The excess air velocity ($U - U_{mf}$) of 0.035 m s^{-1} is constant for (a)-(c) with U_{mf} , and seed diameter, increasing from (a)-(c).

Figure 6 shows the bubble rise velocity as a function of particle size for the trajectories in Figure 5. The rise velocities show a decreasing trend with increasing particle diameter. Theoretically, bubble size and bubble rise velocity should not depend heavily on particle diameter (Davidson and Harrison, 1963). The result in Figure 6 shows the bubble rise velocity has a high dependence on particle diameter, specifically near the top of the imaging region. In this narrow column, the large particle size of the *Mustard* seeds, $d_p = 1.5 \text{ mm}$, means that it is relatively easy for the particles to bridge the walls of

the column. This bridging stabilizes the bubbles and encourages the onset of slugging, which, in turn, reduces the bubble frequency and rise velocity. Previous investigations of slugging also found strong dependence on particle size in narrow columns (Baeyens and Geldart, 1974). At an excess gas velocity of 0.035 m s^{-1} , the expected rise velocity for square-nosed slugs in this system is 0.14 m s^{-1} (Rudolph and Judd, 1985). This velocity is in agreement with the rise velocity of bubbles measured using MRI at the top of the imaging region for the mustard seeds. Thus, it is likely square-nosed slugs (i.e. horizontal voids) formed with the large particles.

Figure 6. Bubble rise velocity plotted against average seed diameter for three different seed types at the same excess gas velocity or 0.035 m s^{-1} . The bubble rise velocity near the distributor is less dependent on seed diameter than that at the top of the imaging region though both locations show a decreasing velocity with an increasing seed diameter.

3.2 Spatially resolved bubble tracking

Bubbles are imaged as they rise by using the rise velocity calculated from the 1D profiles to approximate the locations for an ‘average’ bubble as they rise through the bed. The offset for each subsequent image is increased according to the local average bubble velocity. Acquisitions of profiles and images are alternated to simultaneously

track the bubble rise velocity and to observe the bubble in a 2D horizontal plane. In some images, individual bubbles could not be resolved near the distributor, but rather the signal intensity over the entire cross section was reduced. In this situation it was difficult to be certain that the same bubble was tracked as it rose. We will consider these cases in more detail in Section 3.4 using vertical 2D images. Here we restrict ourselves to considering bubbles that had already formed in the first frame. In this context, three types of bubble events are investigated: a bubble wake, lateral drift, and in-plane bubble coalescence.

Figures 7-9 show 1D profiles and 2D images of bubbles in an air-fluidized bed of *Nicotiana* seeds with an air velocity of 0.156 m s^{-1} , excess air velocity of 0.069 m s^{-1} . The acquisition of each profile takes 5.1 ms and the acquisition time of each image is 25 ms. A profile is acquired first, then an image is acquired immediately after the profile. Profile and image acquisitions are then alternated as indicated by the relative time, recorded as the time at the midpoint of the acquisition. Increasing the slice offset makes it possible to acquire several images of the same bubble as it develops while rising through a fluidized bed. To describe the images, a Cartesian coordinate system, as shown in Fig. 7, will be used assuming the center of the image to be the origin.

The red markers between each profile, as in Figure 7a, depict the height above the distributor for the images as in 7b. The markers indicate the location, slice thickness, and relative timing of the axial images. The first image, at a relative time of 15 ms, is acquired just above the distributor plate, as shown between the first two profiles (acquired at relative times of 0 ms and 30 ms).

The profiles in Figure 7a show a bubble rising through the sample. The images acquired at 15 ms, 45 ms and 75 ms show the formation of an annular ring that contains few particles. It is known that bubbles typically form a spherical cap shape with a wake region that follows the bubble. The wake region of a bubble is often such that if an image slice were taken through the wake it would show an annulus, as captured in Figure 7b. The profiles indicate that the bubble is rising at a velocity of 0.1 m s^{-1} , as was commonly observed for the bubbles shown in Figure 3.

Figure 7. Images of a fluidized bed of *Nicotiana* seeds with air velocity of 0.156 m s^{-1} . Profiles and images are acquired alternately as indicated by the relative timing, or the time at which each acquisition started assuming the first profile to be acquired at time 0 ms. (a) Profiles along the z -axis showing a bubble rising through the fluidized bed with a spatial resolution of 0.5 mm per pixel and a temporal resolution of 5.1 ms with 25 ms in between each acquired profile. The red markers indicate xy slice location, slice thickness and relative timing. (b) Axial images acquired between each profile showing the wake region of a bubble as it moves up through the fluidized bed as indicated on the profiles in (a). Images are $1 \text{ mm} \times 1 \text{ mm}$ with a slice thickness of 2 mm and are acquired over 25 ms with 5.1 ms between each acquisition.

The profiles in Figure 8a confirm that a single trajectory has been followed with an initial rise velocity of 0.1 m s^{-1} , accelerating to 0.25 m s^{-1} at the top of the imaging region. In the first frame in Figure 8b, there is a bubble in the $+x, +y$ quadrant. Over the next 120 ms, the bubble drifts toward the $-x$ quadrant showing an example of the lateral motion of the bubble, or lateral drift. Lateral drift is known to occur in 2D systems (Laverman et al., 2008) and in simulations (Busciglio et al., 2008). It has also been inferred in 3D systems using x-ray measurements, but is difficult to observe owing to the use of projection measurements (Yates et al., 2002) or the fixed location in the vertical plane of the images (Verma et al., 2014). The lateral drift here may have arisen from in-plane coalescence, as may be indicated in the images at 75 ms and 105 ms.

Figure 8. Profiles and images acquired of a fluidized bed under same conditions as in Figure 7, Nicotiana seeds with air velocity of 0.156 m s^{-1} (a) Profiles along the z -axis showing a bubble rising through the fluidized bed with a spatial resolution of 0.5 mm per pixel and a temporal resolution of 5.1 . The red markers indicate xy slice location, slice thickness and relative timing. (b) Axial images acquired between each profile showing a single bubble as it moves up the fluidized bed as indicated in (a). Images are $1 \text{ mm} \times 1 \text{ mm}$ with a slice thickness of 2 mm and are acquired over 25 ms .

The first image in Figure 9 shows two bubbles just above the distributor plate. The bubbles approach each other and coalesce over the next 90 ms. The images show a clear instance of in-plane bubble coalescence. These bubbles are seen to join at the side. This is evidence that, in addition to the leading-trailing coalescence visible in the 1D profiles, lateral coalescence can also be observed by MRI. Further work is required to determine the relative frequency of these coalescence mechanisms and how this varies with height in the column.

Figure 9. Profiles and images acquired of a fluidized bed under same conditions as in Figure 7 and 8, *Nicotiana* seeds with air velocity of 0.156 m s^{-1} (a) Profiles along the z -axis showing a bubble rising through the fluidized bed with a spatial resolution of 0.5 mm per pixel and a temporal resolution of 5.1 . The red markers indicate xy slice location, slice thickness and relative timing. (b) Axial images acquired between each profile showing a two bubbles starting just above the distributor plate coalescing into a single bubble as it moves up through the fluidized bed as indicated in (a). Images are $1 \text{ mm} \times 1 \text{ mm}$ with a slice thickness of 2 mm and are acquired over 25 ms .

In summary, the axial images in combination with profiles have been used to track rising bubbles in the fluidized bed. These measurements were used to observe a wake region of a bubble, show lateral drift, and capture images showing in-plane bubble coalescence.

3.4 Bubble stability

Images in the vertical plane are acquired of the bed with a $1\text{ mm} \times 1\text{ mm}$ resolution and a 2 mm slice thickness. The images are acquired with a time resolution of 25 ms. Depending on the rise velocity and trajectory of an individual bubble, multiple images can be acquired of a single bubble rising.

Figure 10a captures sequential images of a fluidized bed of *Nicotiana* seeds with an air velocity of 0.156 m s^{-1} , excess air velocity of 0.069 m s^{-1} . The first image shows a thin dark band spanning the sample. A dark band such as this was seen in perhaps half of the series of images that were acquired, with the remainder of the images showing conventional bubbles. This band corresponds to a predominantly one-dimensional (1D) variation in voidage across the bed, similar to the perturbations introduced to study bubble stability using computational fluid dynamics (CFD) (Glasser et al., 1997, 1996). The diagonal lines in the images are artifacts arising from the small amount of data used to produce these images and should be ignored. In Figure 10a, the central region of the band is slightly thicker than the edges in the first frame. The second image shows the thin, 1D perturbation spanning the sample at approximately the same height; however, the central region is now larger than in the first image. Over the next 50 ms, the perturbation collapses to form a bubble above where the 1D perturbation had existed.

Figure 10b is acquired at the same experimental conditions as 10a. This time series also starts with a 1D perturbation, however, in Figure 10b the perturbation develops into two bubbles. Though it was more common to see the development of a 1D perturbation into a single bubble, Figure 10b shows it is possible for two bubbles to form in the same plane in this system. The break down of the 1D perturbation is analogous to what has been seen in liquid-solid fluidized beds (Duru and Guazzelli, 2002) and in numerical simulations (Glasser et al., 1996).

Figure 10. Images are acquired in the zy -plane of an air fluidized bed of *Nicotiana* seeds showing a 1D wave, or perturbation, starting just above the distributor plate and collapsing into (a) a single bubble and (b) two bubbles as they move up through the fluidized bed. The air velocity for both data sets is 0.156 m s^{-1} .

When the perturbation collapses into a bubble in the second frame, the diameter is on the order of 5 mm. From the velocity profiles the bubble rise velocity was calculated to be 0.11 m s^{-1} near the bottom of the bed. At this velocity, Equation 1 predicts a spherically equivalent bubble diameter of approximately 2.5 mm. The discrepancy in the bubble size imaged and that estimated from the rise velocity may be due to the formation of the perturbation or to rapid coalescence of small bubbles near the distributor, both of which may slow the rise velocity of the bubbles.

Vertical images of *Lobelia* seeds with an air velocity of 0.070 m s^{-1} , that is an excess air velocity of 0.035 m s^{-1} , are shown in Figure 11. The first image shows a large bubble at the top of the imaging region and three, much smaller, bubbles near the bottom of the image. The three bubbles are along a single plane, similar to the two bubbles formed in Figure 10b. The following frames show the bubbles moving up through the imaging region. In the second image the bubble on the left hand side appears larger and darker than in the previous image. The change in appearance could be due to the bubble increasing in size, but is more likely due to the bubble moving further into the imaging plane. The bubble in the middle is much smaller, though still visible and the bubble at the right is difficult to resolve. The bubble on the right hand side is likely to have moved away from the imaging plane during the second acquisition, as it is easily resolved again in the third and fourth images. The fourth image shows another bubble below the original bubbles. In the fifth image, this new bubble appears to be coalescing with the right hand bubble from the first image. The final image shows a circular bubble on the right hand side of the image, which is much larger than in the first four images, confirming the bubbles have coalesced. The bubble on the right hand side is larger and accelerating faster than that on the left hand side of the image. The sequence of images shown in Figure 11 is representative of all images acquired of the *Lobelia* seeds. These images indicate that in the bed of *Lobelia* seeds, the smallest particles studied here, no 1D perturbation was observed. It is therefore likely that bubbles form immediately above the distributor and proceed to rise through the bed.

Figure 11. Images acquired in the zy -plane of an air fluidized bed of *Lobelia* seeds at 0.70 m s^{-1} showing three bubbles forming along a plane near the distributor plate. Two of these bubbles then rise through the bed in the imaging region.

The vertical images for the largest particles used in this study, *Mustard* seeds, with an air velocity of 0.364 m s^{-1} , excess air velocity of 0.087 m s^{-1} , are shown in Figure 12.

Figure 12a shows a thick, 1D perturbation along the bottom of the fluidized bed. This perturbation collapses into a bubble over approximately 100 ms. This behavior is similar to the *Nicotiana* seeds, but it was not consistently observed. The more common progression is shown in Figure 12b. All seven images in Figure 12b show the trajectory of a single perturbation as it moves from the distributor plate up through the imaging region. A second perturbation can be seen in the first two images. This upper perturbation started at the distributor plate before the start of the acquisition for this data set so it is not considered further. Considering the lower perturbation, the perturbation is seen to have traversed the imaging region without collapsing. This set of data show that, for the *Mustard* seeds, a 1D perturbation can be stable through the imaging region and we never observed bubbles right at the distributor.

Figure 12. Images acquired in the zy -plane of an air fluidized bed of *Mustard* seeds, 0.364 m s^{-1} , showing a 1D wave, or perturbation starting just above the distributor plate (a) collapsing into a single bubble and (b) moving up through the imaging region as a wave without collapsing.

Comparing the images from the three different seed diameters, the particle size can be seen to have an effect on the bubble formation. The use of a narrow column and a porous distributor plate means that the excess air is introduced to the bed as a nearly uniform perturbation of the voidage. The smallest seeds, *Lobelia*, do not show this 1D perturbation. The perturbation may never form, or it may collapse almost immediately to produce the bubbles. The *Nicotiana* seeds frequently form a 1D perturbation, but this perturbation was found to collapse into one or two bubbles soon after formation. The largest particles, *Mustard* seeds, form a much larger, thicker 1D perturbation, which can either collapse into a bubble or traverse the imaging region as a stable structure. As the seeds have similar properties, other than a variation in size, this suggests that it is particle size that is influencing bubble formation. The change in behavior could be attributed to either the transition from group B to group D particles, or the increased effect of wall friction. No 1D perturbation is seen for the *Lobelia* seeds (i.e. the system is very unstable), and the *Nicotiana* seeds are moderately stable. As both of these particles belong to group B, this observation suggests that wall friction is more important here than the transition from group B to group D. Friction is known to enhance the stability

of fluidized beds (Sundaresan, 2003). In this case, the confining walls provide an additional source of friction that further stabilizes perturbations in the voidage. With smaller particles, these additional frictional forces will need to span a larger number of particles to have an effect on dynamics, which may mean that the frictional forces arising from the wall are of less importance for small particles relative to larger particles.

4. Conclusion:

UTE MRI was used to acquire images of individual bubbles in fluidized beds. Images of bubbles forming and coalescing in a three-dimensional system have been acquired at higher spatial resolution than was previously possible for an acquisition rate of 40 fps. Profiles were used to characterize the average bubble rise velocity and frequency in a fluidized bed. The profiles show clear examples of leading-trailing bubble coalescence at all air velocities. Images in combination with profiles were used to simultaneously track bubble rise velocity and visualize in-plane bubble behavior. Using an increasing image offset to follow a single bubble trajectory, this technique was able to record the wake region of a rising bubble, in-plane bubble coalescence, and lateral drift, showing that the bubble did not move exclusively vertically, but rather migrates around the bed as it rises up through the imaging region.

Vertical images show the bubble behavior near the distributor plate in a fluidized bed for three different particle sizes. The narrow column and porous distributor plate mean

that the excess air is introduced to the bed as a nearly uniform perturbation of the voidage. For *Lobelia* seeds, at small particle sizes, the formation of a 1D perturbation was not visible with the current technique, though multiple small bubbles were observed near the distributor plate. The *Nicotiana* seeds, or medium sized particles showed the formation of a 1D perturbation near the distributor plate, which would then collapse into a bubble (or bubbles). For large particles, *Mustard* seeds, the perturbation was often found to be stable and persisted throughout the imaging region. It is hypothesized that this difference in behavior is due to the greater transmission of frictional forces from the walls to the center of the bed when larger particles are used.

From these images the unpredictability of bubble behavior is observed. Not only does the timing of bubble formation appear to be inconsistent within a set sample, the formation and collapse of a perturbation is also inconsistent. A perturbation is shown collapsing into one bubble while the next is shown to collapse into two, and in other cases is not observed at all. When modeling a fluidized bed it is important to quantify parameters such as bubble rise velocity, bubble frequency, location of coalescence, and bubble paths as they rise. UTE MRI provides a method of investigating these phenomena in detail in small-scale systems.

5. Acknowledgements

The authors would like to thank Matthew Bryan and Professor Ian Wilson for helping to characterize the particles with the Canty Sizer they had modified. Thank you to Zlatko Saracevic for his help with sizing the *Lobelia* seeds. We are grateful to Kings Seeds for helping us to identify a variety of sizes of relatively “spherical” seeds. HTF would like to acknowledge the financial support of the Gates-Cambridge Trust. All authors would like

to acknowledge the financial support of the EPSRC (EP/K008218/1, EP/F047991/1 and EP/K039318/1).

Data shown in this publication and further data in support of the paper will be made available at the University of Cambridge Open Access repository as soon as possible.

6. References

- Anderson, K., Sundaresan, S., Jackson, R., 1995. Instabilities and the Formation of Bubbles in Fluidized Beds. *J. Fluid Mech.* 303, 327–366. doi:10.1017/S0022112095004290
- Baeyens, J., Geldart, D., 1974. An Investigation into Slugging Fluidized Beds. *Chem. Eng. Sci.* 29, 255–265. doi:10.1016/0009-2509(74)85051-7
- Batchelor, G.K., 1993. Secondary Instability of a Gas-Fluidized Bed. *J. Fluid Mech.* 257, 359–371. doi:10.1017/S0022112093003118
- Busciglio, A., Vella, G., Micale, G., Rizzuti, L., 2008. Analysis of the Bubbling Behaviour of 2D Gas Solid Fluidized Beds Part I. Digital Image Analysis Technique. *Chem. Eng. J.* 140, 398–413. doi:10.1016/j.cej.2007.11.015
- Callaghan, P.T., 1991. *Principles of Nuclear Magnetic Resonance Microscopy*. Oxford University Press, Oxford.
- Chandrasekera, T.C., Li, Y., Moody, D., Schnellmann, M.A., Dennis, J.S., Holland, D.J., 2015. Measurement of Bubble Sizes in Fluidised Beds Using Electrical Capacitance Tomography. *Chem. Eng. Sci.* 126, 679–687. doi:10.1016/j.ces.2015.01.011
- Chandrasekera, T.C., Wang, A., Holland, D.J., Marashdeh, Q., Pore, M., Wang, F., Sederman, A.J., Fan, L.S., Gladden, L.F., Dennis, J.S., 2012. A Comparison of Magnetic Resonance Imaging and Electrical Capacitance Tomography: An Air Jet Through a Bed of Particles. *Powder Technol.* 227, 86–95. doi:10.1016/j.powtec.2012.03.005
- Darton, R.C., LaNauze, R.D., Davidson, J.F., Harrison, D., 1977. Bubble Growth Due to Coalescence in Fluidised Beds. *Trans. Inst. Chem. Eng.* 55, 274.
- Davidson, J.F., Harrison, D., 1963. *Fluidised Particles*. Cambridge University press, Cambridge.
- Davies, R.M., Taylor, G.I., 1950. The Mechanics of Large Bubbles Rising Through Extended Liquids and Through Liquids in Tubes. *Proc. R. Soc. A Math. Phys. Eng. Sci.* 200, 375–390. doi:10.1098/rspa.1983.0054
- Dennis, J.S., 2013. Properties of Stationary (Bubbling) Fluidised Beds Relevant to Combustion and Gasification Systems, in: Scala, F. (Ed.), *Fluidized Bed Technologies for Near-Zero Emission Combustion and Gasification*. Woodhead Publishing, Sawston, Cambridge, pp. 77–146. doi:http://dx.doi.org/10.1533/9780857098801.1.77
- Duru, P., Guazzelli, É., 2002. Experimental Investigation on the Secondary Instability of

- Liquid-Fluidized Beds and the Formation of Bubbles. *J. Fluid Mech.* 470, 359–382. doi:10.1017/S0022112002002100
- Enwald, H., Peirano, E., Almstedt, A.E., Leckner, B., 1999. Simulation of the Fluid Dynamics of a Bubbling Fluidized Bed. Experimental Validation of the Two-Fluid Model and Evaluation of a Parallel Multiblock Solver. *Chem. Eng. Sci.* 54, 311–328.
- Fabich, H.T., Benning, M., Sederman, A.J., Holland, D.J., 2014. Ultrashort Echo Time (UTE) Imaging Using Gradient Pre-equalization and Compressed Sensing. *J. Magn. Reson.* 245, 116–124. doi:10.1016/j.jmr.2014.06.015
- Fabich, H.T., Sederman, A.J., Holland, D.J., 2016. Development of Ultrafast UTE Imaging for Granular Systems. *J. Magn. Reson.* 273, 113–123. doi:10.1016/j.jmr.2016.10.016
- Geldart, D., 1973. Types of Gas Fluidization. *Powder Technol.* 7, 285–292. doi:10.1016/0032-5910(73)80037-3
- Glasser, B.J., Keverekidis, I.G., Sundaresan, S., 1997. Fully Developed Travelling Wave Solutions and Bubble Formation in Fluidized Beds. *J. Fluid Mech.* 334, 157–188.
- Glasser, B.J., Keverekidis, I.G., Sundaresan, S., 1996. One- and Two-Dimensional Travelling Wave Solutions in Gas-Fluidized Beds. *J. Fluid Mech.* 306, 183–221.
- Halow, J.S., Nicoletti, P., 1992. Observations of Fluidized Bed Coalescence Using Capacitance Imaging. *Powder Technol.* 69, 255–277.
- Harms, S., Stapf, S., Blümich, B., 2006. Application of k- and q-space Encoding NMR Techniques on Granular Media in a 3D Model Fluidized Bed Reactor. *J. Magn. Reson.* 178, 308–317. doi:10.1016/j.jmr.2005.10.009
- Kunii, D., Levenspiel, O., 1991. Fluidization Engineering, Fluidization Engineering. Butterworth-Heinemann, Stoneham. doi:10.1016/B978-0-08-050664-7.50003-2
- Laverman, J.A., Roghair, I., van Sint Annaland, M., Kuipers, H., 2008. Investigation into the Hydrodynamics of Gas-Solid Fluidized Beds Using Particle Image Velocimetry Coupled with Digital Image Analysis. *Can. J. Chem. Eng.* 86, 523–535. doi:10.1002/cjce.20054
- Lim, K.S., Zhu, J.X., Grace, J.R., 1995. Hydrodynamics of Gas-Solid Fluidization. *Int. J. Multiph. Flow* 21, 141–193.
- Makkawi, Y.T., Wright, P.C., 2004. Electrical Capacitance Tomography for Conventional Fluidized Bed Measurements - Remarks on the Measuring Technique. *Powder Technol.* 148, 142–157. doi:10.1016/j.powtec.2004.09.006
- Müller, C.R., Davidson, J.F., Dennis, J.S., Fennell, P.S., Gladden, L.F., Hayhurst, A.N., Mantle, M.D., Rees, A.C., Sederman, A.J., 2007. Rise Velocities of Bubbles and Slugs in Gas-Fluidised Beds: Ultra-Fast Magnetic Resonance Imaging. *Chem. Eng. Sci.* 62, 82–93. doi:10.1016/j.ces.2006.08.019
- Müller, C.R., Davidson, J.F., Dennis, J.S., Fennell, P.S., Gladden, L.F., Hayhurst, A.N., Mantle, M.D., Rees, A.C., Sederman, A.J., 2006. Real-Time Measurement of Bubbling Phenomena in a Three-Dimensional Gas-Fluidized Bed Using Ultrafast Magnetic Resonance Imaging. *Phys. Rev. Lett.* 96, 154504. doi:10.1103/PhysRevLett.96.154504
- Müller, C.R., Holland, D.J., Sederman, A.J., Mantle, M.D., Gladden, L.F., Davidson, J.F., 2008. Magnetic Resonance Imaging of Fluidized Beds. *Powder Technol.* 183, 53–62. doi:10.1016/j.powtec.2007.11.029

- Pauly, J.M., 2012. Selective Excitation for Ultrashort Echo Time Imaging. *Encycl. Magn. Reson.* 1, 381–388. doi:10.1002/9780470034590.emrstm1271
- Rautenbach, C., Mudde, R.F., Yang, X., Melaaen, M.C., Halvorsen, B.M., 2013. A Comparative Study Between Electrical Capacitance Tomography and Time-Resolved X-Ray Tomography. *Flow Meas. Instrum.* 30, 34–44. doi:10.1016/j.flowmeasinst.2012.11.005
- Rowe, P.N., 1976. Prediction of Bubble Size in a Gas Fluidised Bed. *Chem. Eng. Sci.* 31, 285–288. doi:10.1016/0009-2509(76)85073-7
- Rowe, P.N., Partridge, B.A., 1965. An X-Ray Study of Bubbles in Fluidised Beds. *Trans. Inst. Chem. Eng.* 43, S116–S134. doi:10.1016/S0263-8762(97)80009-3
- Rudolph, V., Judd, M.R., 1985. Circulation and Slugging in a Fluid Bed Gasifier Fitted with a Draft Tube, in: *Circulation Fluidized Bed Technology - Proceedings of the First International Conference on Circulating Fluidized Beds*. Pergamon Press, New York, pp. 437–442.
- Stewart, P.S.B., Davidson, J.F., 1967. Slug Flow in Fluidised Beds. *Powder Technol.* 1, 61–80.
- Sundaresan, S., 2003. Instabilities in Fluidized Beds. *Annu. Rev. Fluid Mech.* 35, 63–88. doi:10.1146/annurev.fluid.35.101101.161151
- Taylor, A.B., Holland, D.J., Sederman, A.J., Gladden, L.F., 2011. Time Resolved Velocity Measurements of Unsteady Systems Using Spiral Imaging. *J. Magn. Reson.* 211, 1–10. doi:10.1016/j.jmr.2011.03.017
- van der Hoef, M.A., Ye, M., van Sint Annaland, M., Andrews IV, A.T., Sundaresan, S., Kuipers, J.A.M., 2006. Multi-Scale Modeling of Gas-Fluidized Beds. *Adv. Chem. Eng.* 31, 65–149.
- Verma, V., Padding, J.T., Deen, N.G., Kuipers, J.A.M., Barthel, F., Bieberle, M., Wagner, M., Hampel, U., 2014. Bubble Dynamics in a 3-D Gas-Solid Fluidized Bed Using Ultrafast Electron Beam X-Ray Tomography and Two-Fluid Model. *AIChE J.* 60, 1632–1644. doi:10.1002/aic
- Wang, F., Marashdeh, Q., Fan, L.-S., Warsito, W., 2010. Electrical Capacitance Volume Tomography: Design and Applications. *Sensors* 10, 1890–1917. doi:10.3390/s100301890
- Werther, J., Molerus, O., 1973a. The Local Structure of Gas Fluidized Beds - I. A Statistically Based Measuring System. *Int. J. Multiph. Flow* 1, 103–122. doi:10.1016/0301-9322(73)90007-4
- Werther, J., Molerus, O., 1973b. The Local Structure of Gas Fluidized Beds - II. The Spatial Distribution of Bubbles. *Int. J. Multiph. Flow* 1, 123–138.
- Whitehead, A.B., Dent, D.C., Young, A.D., 1967. Fluidisation Studies in Large Gas-Solid Systems Part II: The Effect of Distributor Design and Solid Properties on Fluidisation Quality. *Powder Technol.* 1, 149–156. doi:http://dx.doi.org.ezproxy.unal.edu.co/10.1016/0032-5910(67)80026-3
- Yates, J.G., Cheesman, D.J., Lettieri, P., Newton, D., 2002. X-Ray Analysis of Fluidized Beds and Other Multiphase Systems. *KONA Powder Part. J.* 20, 133–143. doi:10.14356/kona.2002016