On the Landau-Levich problem for a viscoplastic fluid

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Résumé :

Une étude expérimentale systématique du problème de Landau-Levich pour un fluide viscoplastique (Carbopol 980) a été réalisée. La validité de la loi asymptotique de Landau-Levich est observée grâce aux mesures, en présence ou absence de glissement, à léchelle intégrale de la largeur du film déposé dépaisseur différente suivant la concentration en Carbopol. Pour avoir une meilleure compréhension entre les origines physiques des déviations expérimentales observées par rapport au lois classiques en 2/3, une étude complète du champ de vitesse et de sa dépendance temporelle a été réalisée autour de lobjet se déplaçant. Les principales différences entre les structures de lécoulement dans le cas dun fluide viscoplastique comparé à celle(s) observée(s) dans le cas dun fluide newtonien ont été mises en évidence. Létude expérimentale a été complétée par une analyse asymptotique préliminaire en utilisant le modèle rhéologique de type Bingham.

Abstract :

A systematic investigation of the Landau-Levich problem of a viscoplastic material (Carbopol 980 is presented). The validity of the Landau-Levich scaling is assessed via integral scale measurements of the width of the coating films thickness performed for various Carbopol concentrations in the presence and in the absence of wall slip. To gain a deeper insight into the physical origins of the experimentally observed deviations from the classical 2/3 scaling a full characterisation of the time resolved flow field around the moving solid is performed and the main differences of the viscoplastic flow patterns with respect to their Newtonian counterpart are highlighted. The experimental investigation is complemented by a preliminary asymptotic analysis performed within the framework of the Bingham model.

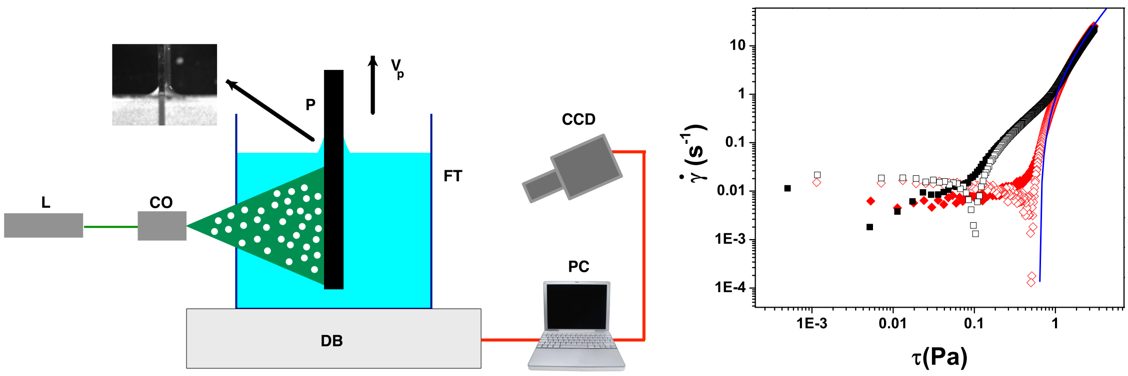
Mots clefs : **yield stress ; Landau-Levich problem ; scaling**

1 Introduction

Coating problems are of a major importance from both a theoretical and a practical perspective. In particular, the phenomenon of coating of solid bodies with non Newtonian fluids is relevant to many processes related to food industry, cosmetic industry etc. The first and probably the best known study of the scaling properties of the coating phenomenon with a Newtonian fluid in the limit of negligible inertia and small capillary numbers is due to Landau and Levich, [3]. Their central result was the scaling law of the coated fluid thickness with the capillary number : $h \propto Ca^{2/3}$. The capillary number has defined as $Ca = \frac{\eta V_p}{\sigma}$ where η, V_p and σ stand for the viscosity of the fluid, the speed of the plate and the surface tension coefficient, respectively. For the case of Newtonian fluids there exists a significant number of investigations, both theoretical and experimental which confirm the classical result of Landau and Levich and extend it, [4]. There exist, however, much fewer studies concerning with the Landau-Levich problem for non-Newtonian fluids. The main theoretical difficulty arrises in this case from the dependence of the fluid viscosity on the field of the velocity gradients and, if thixotropic effects are present, on the time. The present study concerns with an experimental and theoretical investigation of the Landau-Levich coating of a solid plate withdrawn at a constant speed from a bath filled with a yield stress fluid. The goal of the study is two-fold. First, by measurements of the width of the coating fluid layer, the prediction of Landau-Levich will be assessed. To gain a further insight, local measurements of the flow fields around the moving plate are performed. The experimental study is complemented by a preliminary theoretical analysis.

2 Experimental section

The experimental setup is schematically illustrated in Fig. 1(a). A solid plate **P** is withdrawn at a constant speed V_p from a fluid contained in a acrylic made rectangular tank **FT**. The flow around the moving plate is illuminated laterally by a laser sheet obtained by passing a laser beam emitted by the solid-state laser **L** through a cylindrical optics block **CO**.



(a) (b)

FIGURE 1 – (a) Schematic view of the experimental setup : **L** - laser, **CO** - cylindrical optics bloc, **P** - moving plate, **FT** - fluid tank, **DB** - digital balance, **CCD** - digital camera, **PC** - personal computer. The insert illustrates the free meniscus formed around the moving plate. (b) Rheological measurements performed with a polished geometry (□, ■) and with a rough geometry (◇, ♦). The full/empty symbols refer to the increasing/decreasing branch of the flow curves. The full line is a Herschel-Bulkley fit that gives $\tau_y = 0.64 \pm 0.003(Pa)$, $K = 0.4 \pm 0.002(Pas^n)$, $n = 0.54 \pm 0.001$. A 0.1% Carbopol solution has been used.

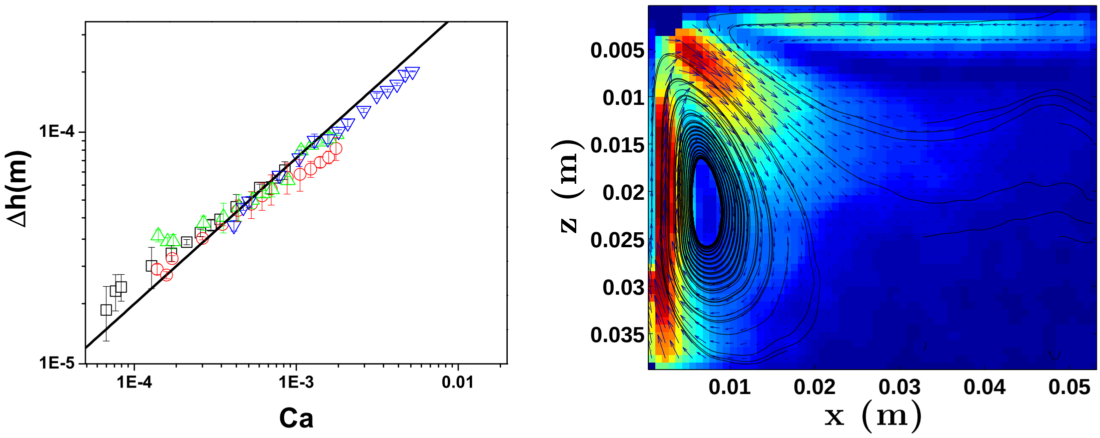
The working fluids are seeded with a minute amount of polyamide particles and the flow is visualised frontally with a digital camera **CCD**. A long sequence of images is acquired and individual image pairs are passed to a multi-grid Particle Image Velocimetry (**PIV**) algorithm.

As working fluids we have used aqueous solutions of Carbopol 980 at three distinct (weight) concentrations : $c = 0.1\%$, $c = 0.2\%$ and $c = 0.3\%$. The rheological behaviour of each solution has been investigated by performing controlled stress linear ramps for both increasing and decreasing values of the applied stress. To test the role of the wall slip, such measurements have been performed with both smooth and rough surfaces as illustrated in Fig. 1(b). A more comprehensive of the yielding behaviour of the Carbopol gels has been reported in Ref. [5]. In addition to the local measurements of the flow fields around the moving plate, integral measurements of the width of the fluid film have been performed. For this purpose the fluid tank has been placed on a digital balance and the mass of withdrawn fluid was measured in real time which. Using the assumption of the homogeneity of coating, the measurements of the withdrawn fluid mass provide an indirect mean of estimating the thickness of the coating fluid layer. The surface tension coefficient σ of Carbopol solutions decreases with the polymer concentration, [1].

3 Results

3.1 Experimental results

To validate the experimental setup and the measurement techniques, we have studied the coating of plate with Newtonian fluids (solutions of sucrose at various concentrations). Measurements of the dependence of width of the coating film on the capillary number Ca performed with several sugar solutions are presented in Fig. 2(a). The capillary number Ca has been varied over nearly two decades by



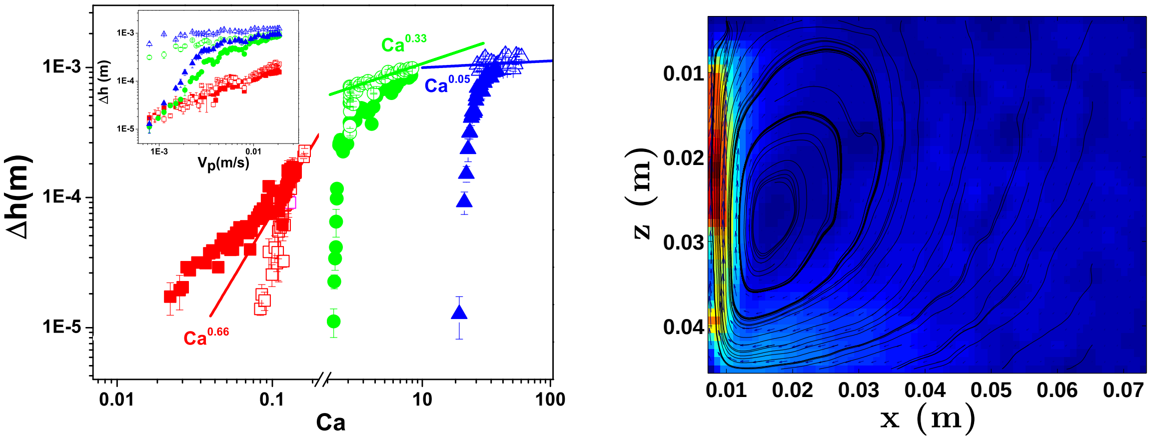
(a) (b)

FIGURE 2 – (a) Dependence of the film thickness on the capillary number for sugar solutions with various concentrations : (□) - $c = 50\%$, (○) - $c = 55\%$, (△) - $c = 60\%$, (▽) - $c = 65\%$. The full line is a guide for the eye, $Ca^{2/3}$. (b) Flow field measured for a 65% wt sugar solution. The edge of the vertically moving plate corresponds to $x = 0$ and the free surface of the fluid to $z = 0$.

varying the speed V_p of the plate, the viscosity and surface tension coefficient of the fluid (by modifying the concentration of sucrose). The error bars have been calculated by repeating each measurement three times and computing the standard deviation. The data acquired for the Newtonian fluids follows fairly well the classical Landau - Levich scaling $\Delta h \propto Ca^{2/3}$ which is an indicator for the reliability of our measurement technique. Slight deviations from this scaling might be attributed to both the finite size of the withdrawn plate and the limited accuracy of the digital balance. A typical flow pattern around the plate withdrawn from a 65% sugar solution is illustrated in Fig. 2(b). The flow is organised in two counter-rotating vortices, one near the moving plate and the other in the vicinity of the free fluid surface. This flow organisation was reproduced for various Newtonian solutions and for various withdrawing speeds and is consistent with previous experimental observations, [4].

Similar measurements of the dependence of the width of the coated fluid layer performed with several Carbopol solutions on the capillary number Ca within a wide range of capillary numbers are presented in Fig. 3(a). The raw measurements of the width of the film versus the speed of the withdrawn plate V_p are presented in the inset.

As the rheological behaviour of the Carbopol solutions is strongly nonlinear (see Fig. 1(b)) the calculation of the capillary number is not straight forward and deserves a separate discussion. Measurements of the flow patterns around the withdrawn plate such as the one presented in Fig. 3(b) allowed the determination of the velocity gradients in its direct proximity. By connecting the measured velocity gradients with the rheological measurements (such as the one illustrated in Fig. 1(b)) one can assess a local (near the moving plate) viscosity which is ultimately used for the calculation of the capillary number Ca. For each of the solutions we have investigating the data acquired in the presence of slip with a smooth plate (the full symbols) scales differently with Ca than the data acquired in the absence of slip with a rough plate (the empty symbols). This can be understood in terms of significant differences in both the stress magnitude and its distribution near the moving plate induced by the presence of wall



(a) (b)

FIGURE 3 – (a) Dependence of the film thickness on the capillary number for Carbopol solutions with various concentrations : (□) - $c = 0.1\%$, (○) - $c = 0.2\%$, (△) - $c = 0.3\%$. The full lines are guides for the eye, $\Delta h \propto Ca^{\alpha}$ with the scaling exponents α indicated in the inserts. The raw dependencies of the film thickness on the plate's speed are presented in the upper inset. The full/empty symbols refer to smooth/rough plates. (b) Flow field measured for a 0.3% *wt* Carbopol solution. The edge of the vertically moving plate corresponds to $x = 0$ and the free surface of the fluid to $z = 0$.

slip which originate from different rheological responses of the material in the presence/absence of wall slip and in a range of small rates of shear, see Fig. 1(b). The data acquired with and without slip merge only a critical capillary number Ca_c which increases monotonically with the Carbopol concentration. Beyond this critical value of the capillary number a power law scaling of the width of the coated fluid film is observed, $\Delta h \propto Ca^{\alpha}$, with the exponent α monotonically decreasing with the concentration of Carbopol (the yield stress), Fig. 3(a). This rather unexpected fact is, most probably, related to a significant re-organisation of the flow pattern due to the plasticity of the material. Indeed, preliminary observations of the flow field indicate the presence of a single large scale vortex located in the vicinity of the moving plate, Fig. 3(b), as opposed to the two-vortices flow pattern observed with a Newtonian fluid, Fig. 2(b).

3.2 Dimensional analysis

Focusing on flow regimes close to the yield point we model the fluid as a Bingham fluid such that the fluid viscosity is given by $\mu = \eta + \tau_y/\dot{\gamma}$. Then for the dynamic meniscus thickness δh we seek a functional relationship

$$\Delta h = f(V_p, \sigma, g, \tau_y, \eta, \rho) \ , \tag{1}$$

where g is the acceleration due to gravity.

Taking V_p, σ, g as a set of dimensional parameters with independent units in the length-mass-time class, we can rewrite (1) in nondimensional terms as

$$H = F(\mathrm{Yi}, \mathrm{Ca}, \mathrm{Bo}) \ , \tag{2}$$

where

$$H = \frac{h_0 g}{V_p^2} \ , \ \mathrm{Yi} = \frac{\tau_y V_p^2}{g\sigma} \ , \ \mathrm{Ca} = \frac{\eta V_p}{\sigma} \ , \ \mathrm{Bo} = \frac{\rho V_p^4}{g\sigma} \tag{3}$$

are the nondimensional thickness, yield number, capillary number, and Bond number, respectively, and the function F is unknown at this stage.

3.2.1 Limit as $\mathrm{Yi} \to 0$

We approach a Newtonian fluid in the limit as $\mathrm{Yi} \to 0$ with all other nondimensional parameters fixed and of $O(1)$. A Newtonian fluid certainly has a dynamic meniscus of finite non-zero thickness δh under the conditions of order unity capillary and Bond number, thus following [2]

$$\Delta h = \frac{V_p^2}{g} F_1(\mathrm{Ca}, \mathrm{Bo}) \text{ when } \mathrm{Yi} \ll 1 \tag{4}$$

for some function F_1.

In this Newtonian limit we expect the dynamic meniscus thickness to tend to zero as both η and ρ tend to 0 (independently), and so we conclude that for some positive and real α, β,

$$\Delta h \sim \frac{V_p^2}{g} \mathrm{Ca}^{\alpha} \mathrm{Bo}^{\beta} \text{ when } \mathrm{Yi} \ll 1,\ \mathrm{Ca} \ll 1,\ \mathrm{Bo} \ll 1. \tag{5}$$

The exponents α, β cannot be derived from a dimensional analysis, even in principle. However, we know from the classic result of Landau and Levich [3] that for Newtonian fluids in the small capillary number regime

$$\Delta h \sim \left(\frac{\sigma}{\rho g}\right)^{\frac{1}{2}} \left(\frac{\eta V_p}{\sigma}\right)^{\frac{2}{3}} , \tag{6}$$

which is consistent with (5) when $\alpha = 2/3$, $\beta = -1/2$, and thus

$$\Delta h \sim \frac{V_p^2}{g} \frac{\mathrm{Ca}^{\frac{2}{3}}}{\sqrt{\mathrm{Bo}}} . \tag{7}$$

which is also confirmed for the case of a viscoplatic fluid with low yield stress ($c = 0.1\%$ in our case) as can be seen in Figure 3.

3.2.2 Limit as $\mathrm{Yi} \to \infty$

The experimental data shows that when τ_y, and hence Yi, take on large values while the remaining nondimensional groups remain fixed and of order unity, the dynamic meniscus thickness approaches a finite non-zero limit, suggesting that

$$\Delta h \sim \frac{V_p^2}{g} F_2(\mathrm{Ca}, \mathrm{Bo}) \text{ when } \mathrm{Yi} \gg 1 \tag{8}$$

for some function F_2. However, the experimental data also shows that in this large-Yi regime the thickness h_0 is independent of plate speed V_p, hence we must consider a power-law monomial of the form

$$\Delta h \sim \frac{V_p^2}{g\sqrt{\mathrm{Bo}}} \left(\frac{\mathrm{Ca}}{\mathrm{Bo}^{\frac{1}{4}}}\right)^{\delta} , \tag{9}$$

which can be rewritten as

$$\Delta h \sim a \overline{\mathrm{Ca}}^{\delta} , \tag{10}$$

where $a = V_p^2/g\sqrt{\mathrm{Bo}} = \sqrt{\sigma/\rho g}$ is the capillary length, and $\overline{\mathrm{Ca}} = \mathrm{Ca}/\mathrm{Bo}^{\frac{1}{4}}$ is a renormalized capillary number.

Once again, the value of δ in (10) cannot be determined by dimensional analysis. But for any given experiment, the following should be a constant :

$$\delta = \frac{\log\left(\frac{\Delta h}{a}\right)}{\log \overline{\mathrm{Ca}}} . \tag{11}$$

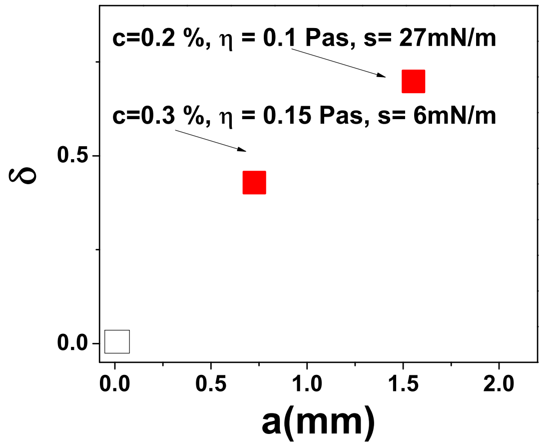


Fig. 4 :Dependence of the estimated scaling exponent δ on the capillary length a.

$\mathbf{c(\%, wt)}$	σ**(mN/m)**	η**(Pas)**	$\overline{\text{Ca}}$	δ
0.2	27	0.1	0.45	0.69
0.3	6	0.150	2.1	0.42

Table 1 :Physical properties of Carbopol solutions.

Using the physical parameters given in Table 1, we can estimate the values of the scaling exponent δ at various capillary lengths obtained in our experiments, Fig. 4. Bearing in mind that the origin should naturally be part of this dependence (the added empty symbol in Fig. 4), the linear dependence of scaling exponents is likely to be valid in the limit of large yields numbers. However, to fully validate this result, some additional experiments performed for larger concentrations of Carbopol are needed.

4 Conclusions, outlook

An experimental investigation of the coating of a solid surface with a viscoplastic fluid (Carbopol 980) by combined measurements of the thickness of the coating film and of the flow patterns around the solid object is presented. The experimental setup and the measuring techniques have been validated by studying the coating of a solid object withdrawn at a constant speed from various Newtonian fluids (sucrose solutions with various concentrations). For each of the Newtonian solutions we have investigated the classical Landau-Levich scaling of the width of the coated layer $\Delta h \propto Ca^{2/3}$ has been found, Fig. 2(a). Measurements of the film's thickness performed in a wide range of capillary numbers Ca with several Carbopol solutions reveal a clear departure from the classical Landau-Levich scaling : an increase of the polymer concentration (yield stress) practically suppresses the dependence of the film's width on Ca. Below a critical value of the capillary number, the presence of wall slip strongly affects this dependence. An increase of the Carbopol concentration (yield stress) results in a gradual suppression of the scaling of the film's width with the capillary number.

Through careful dimensional analysis we have shown that the Newtonian scaling with Ca is recovered if $\tau_y \ll 1$. Following our experimental results, when $\tau_y \gg 1$ we have shown that the witdh of film will be proportional to the capillary length with no dependence on the speed of the plate V_p. This linear dependence derived via scaling arguments is fairly well supported by our experimental results.

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