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SOME NEW APPROACHES TO FORECASTING THE PRICE OF ELECTRICITY: A STUDY OF CALIFORNIAN MARKET

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SOME NEW APPROACHES TO FORECASTING THE PRICE OF ELECTRICITY: A STUDY OF CALIFORNIAN MARKET

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ABSTRACT: In this paper we consider the forecasting performance of a range of semi- and non-parametric methods applied to high frequency electricity price data. Electricity price time-series data tend to be highly seasonal, mean reverting with price jumps/spikes and time- and price-dependent volatility. The typical approach in this area has been to use a range of tools that have proven popular in the financial econometrics literature, where volatility clustering is common. However, electricity time series tend to exhibit higher volatility on a daily basis, but within a mean reverting framework, albeit with occasional large 'spikes'. In this paper we compare the existing forecasting performance of some popular parametric methods, notably GARCH AR-MAX, with approaches that are new to this area of applied econometrics, in particular, Artificial Neural Networks (ANN); Linear Regression Trees, Local Regressions and Generalised Additive Models. Section 2 presents the properties and definitions of the models to be compared and Section 3 the characteristics of the data used which in this case are spot electricity prices from the Californian market 07/1999-12/2000. This period includes the 'crisis' months of May-August 2000 where extreme volatility was observed. Section 4 presents the results and ranking of methods on the basis of forecasting performance. Section 5 concludes.

KEYWORDS: ELECTRICITY TIME SERIES; FORECASTING PERFORMANCE; SEMI- AND NON-PARAMETRIC METHODS

JEL CLASSIFICATIONS: C14, C45, C53

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1. INTRODUCTION

Concerns over climate change, spiralling crude oil prices and security of electricity supply, have led to a resurgence of interest in energy-related issues. Electricity market modelling and forecasting has been given a particular boost following deregulation in many countries and the highly publicised Californian experiences of 2000 and more locally, the effects of two dry years in New Zealand, 2001 and 2003 and the Auckland cable failures of 1998 see, Weron (2006). Analysis of the electricity market has been facilitated by accessibility to high frequency load and price data which, in the case of New Zealand and Australia, is available at intervals as frequent as every 5 minutes.

Research into the operations and characteristics of electricity markets can be categorised into four main areas. Firstly, the modelling and forecasting of electricity load. This area has been predominately the domain of electrical or systems engineers concerned to minimise risks to supply. Modelling and forecasting here has involved both parametric, often simple multivariate regression, ARIMA time series approaches and smoothing methods see eg., and semi/non-parametric neural networks see eg., Hippert, Pedreira, and Souza (2001). Much of this literature has been published in electrical engineering outlets and has generally not entered the mainstream economics/econometrics literature. Secondly, there has been growing interest in forecasting spot and forward electricity prices. This interest has been fuelled by both the needs of a deregulated market to understand 'how the market works' and how best (most profitably) to respond to any systematic, forecastable events. Californian experiences, including widespread bankruptcy of some of the players see eg., Knittel and Roberts (2005), has added extra impetus. This area of research has typically been the realm of economists and econometricians who have used parametric time series tools from financial econometrics, and applied them to electricity data see eg., Worthington, Kay-Spratley, and Higgs (2005), Misiorek, Trueck, and Weron (2006), Conejo, Contreras, Espínola, and Plazas (2005), and Escribano, Peña, and Villaplana (2002). These methods typically comprise simple ARIMA or GARCH models whereas others have attempted to model some of the specific characteristics of electricity data. In particular, electricity price time series data tend to be highly seasonal, mean reverting with price jumps/spikes and time- and price-dependent volatility see Weron and Przybylowicz (2000), Huisman and Mathieu (2003), Goto and Karolyi (2003). Furthermore, as noted by Knittel and Roberts (2005) the data tend to exhibit large values of higher order moments relative to a Gaussian distribution which render models based on normality and log-normality of limited use. Mount, Ning, and Cai (2006) explain why price spikes are a typical feature of a deregulated market for electricity and argue in favour of a regime-switching model. Papers that have specifically considered the modelling of non-linearities and/or spikes include Huisman and Mathieu (2003) who argue that a regime jump process performs better in modelling jumps in combination with mean-reversion than a stochastic jump model. Moral-Carcedo and Vicens-Otero (2005) model the non-linearity of the response of demand to temperature using Smooth Transition (STR), Threshold Regression (TR) and Switching Regression (SR) models. They conclude that the Logistic Smooth Transition (LTSR) offers advantages over other models and is their model of choice when applied to Spanish electricity data.

The third level of interest in electricity markets has come from those interested in modelling the behaviour of firms within a newly deregulated market. Here game theory and operations control methods have been applied with or without empirical validation see eg. Batstone (2000), Wolfram (1999), Newberry (1998) and Harvey and Hogan (2000).

Finally, there has been significant discussion of the legal and political implications of the consequences of deregulation particularly related to market power, volatile prices and security of supply see eg., Barton (2003).

In this paper we will present a comparison of the forecasting performance of a range of parametric and semi/non-parametric models as applied to spot electricity prices using data from the Californian market from 07/1999 to 12/2000. Results for the parametric models are taken from Misiorek, Trueck, and Weron (2006), where they found that including a GARCH component did not improve the forecast performance of the 'best' model – an autoregressive model formulated with exogenous variables. Here we take the ARX formulation as the best parametric specification for the California CalPX market clearing prices

The motivation for the paper comes from several sources. The first is to increase our understanding of the drivers and sources of predictability in electricity markets. The second is as a response to the current perceptions regarding the applicability of non-parametric methods to the forecasting of electricity prices. Misiorek, Trueck, and Weron (2006) state that:

> "AI-based models tend to be flexible and can handle complexity and non-linearity. This makes them *promising* (emphasis added) for short term predictions."

However, they then present a somewhat sceptical view on such methods when stating:

> "We have to note, however, that the advocated models have generally been compared *only to other AI-based techniques or simple statistical methods* (emphasis added)... The results of Conejo, Contreras, Espínola, and Plazas (2005), compared three time series specifications; a wavelet multivariate regression technique, and a multilayer perceptron with one hidden layer. ... the ANN technique

> was the worst of the five tested models. . . .It would be interesting to evaluate representatives from both statistical and AI-based models. However, a comprehensive comparison of models, even from one class is a laborious task." (Misiorek, Trueck, and Weron 2006).

These same authors support regime switching models, which, "by construction should be well suited for modelling the non-linear nature of electricity prices" (Misiorek, Trueck, and Weron 2006).In this paper we seek to test whether this assumed inferiority of these techniques is supported by the data. We will compare the 'best; models for Misiorek, Trueck, and Weron (2006) with a range of semi- and non-parametric approaches discussed in section 2. however, it is worth stressing at this point that the tournament, as it stands, should favor existing approaches as the 'best' parametric models were constructed to be just that. Here we are not attempting to create the (potentially) 'best' non-parametric alternative, but to take the covariates (and lags) found optimal for the parametric approach and find the best subset for each non-parametric formulation.

The plan of the paper is as follows. In section 2 we outline the types of models that will be used to compare their ability to forecast (Californian) electricity price data. The specific models under scrutiny include a range of parametric models; ARIMA, and multiple regime (STAR)and non(semi)-parametric approaches including Artificial Neural Networks; Local regression; Linear Regression Trees and Generalised Additive Models. Section 3 describes the data, while section 4 presents the empirical results. Section 5 concludes.

2. MODELS

In this sectoin we briefly describe the parametric and the non(semi)-parametric models used in this work to forecast electricity price in the Californian Market. We present also some references on each model.

For all models we assume that y_t is the outcome and $\mathbf{x}_t = [x_{1t}, x_{2t}, \ldots, x_{pt}]'$ is the vector of regressors. Note that x_{it} can be either a exogenous variable (e.g. weather), a deterministic function (e.g. trend or dummy variables) or lags of y_t. We also define $\tilde{\mathbf{x}}_t = (1, \mathbf{x}_t')'$.

2.1. **Parametric Models.**

2.1.1. *AR(I)MA(X)-type Models.* The Autoregressive (Integrated) Moving Average (with eXogenous) variables -type models are probably the most applied type of model to forecast electricity prices. Cuaresma, Hlouskova, Kossmeier, and Obersteiner (2004) applied linear autoregressive models to short-term price forecasting in the German market, Contreras, Espínola, Nogales, and A.J. (2003) and Nogales, Contreras, Conejo, and Espinola (2002) provide a strategy to build ARIMA models to forecast next-day electricity prices and provide an application to the Spanish and Californian markets, Misiorek, Trueck, and Weron (2006) used various autoregressive schemes to forecast prices in the Californian market, and Haldrup and Nielsen (2006) studied the Nordic marked and adjusted a ARFIMA to short term price forecasting.

In the ARMA(p,q) model, the dependent variable is expressed as a linear linear combination of its past values (autoregressive part) and in terms of previous values of the noise (moving average part):

$$\Phi(B)y_t = \Theta(B)\epsilon_t, \tag{1}$$

where B is the lag operator, i.e. $B^j p_t = p_{t-j}$, $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ and $\Theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q$ are polynomials in B. A straightforward generalization is alow some coefficients be zero, in this case we do not have a complete p- or q-order polynomial. Moreover, if $q = 0$ we have a autoregressive model.

The ARMA modeling approach assumes that the time series is (weakly) stationary. If not, we can difference the time series (Box and Jenkins 1976) leading to the ARIMA model. If we also include differences of lag order larger than one, we have the Seasonal ARIMA model.

In the ARIMA model, the output only depends on its past values; however, in a number of situations we may want to add exogenous variables to the model. This new class of model is called ARMAX model. The ARMAX($p, q, r_1, \ldots, r_k$) can be compactly written as:

$$\Phi(B)y_t = \Theta(B)\epsilon_t + \sum_{i=1}^{k} \Psi_i(B)\nu_{it}, \tag{2}$$

where $r_1, \ldots, r_k$ are the orders of the exogenous factors $\nu_1, \ldots, \nu_k$ and $\Psi_i(B)$ is a r_i-th order polynomial in B. Note that like $\Theta(\cdot)$ and $\Phi(\cdot)$, $\Psi_i(\cdot)$ has not to be a complete polynomial.

Misiorek, Trueck, and Weron (2006) found that a large moving average part $\Theta(B)\epsilon_t$ typically decreases the performance of the estimator. Then, we just consider here the ARX-type model, which can be consistently estimated by least squares.

2.2. **Non(Semi)-Parametric Models.**

2.2.1. *Artificial Neural Networks.* The Artificial Neural Networks (ANN) term comprises a large number of classes of models and learning techniques. Although many people regard ANN as something mysterious they are in fact just nonlinear statistical models with the ability of approximate any Borel-measurable function under mild regularity conditions. This ability has led to their use in many fields including economics, finance and electricity load forecasting. Kuan and White (1994) surveyed the use of ANN in economics, and several financial applications in a special issue of the IEEE Transactions on Neural Networks (Abu-Mostafa, Atiya, Magdon-Ismail, and White 2001). Hippert, Pedreira, and Souza (2001) also presented a comprehensive review of ANN models applied to short-term electricity load forecasting.

The application of ANN models to forecast spot-prices in the energy market is a natural evolution of what has been done in finance and electricity load forecasting. In particular, Wang and Ramsey (1998) proposed a two-stage model to forecast weekend and holidays energy spot-price, using data from the England-Wales market. Szkuta, Sanabria, and Dillon (1999) also used a ANN model to predict half-hourly prices, presenting results from the Victorian electricity market in Australia. Finally, Conejo, Contreras, Espínola, and Plazas (2005) compared the efficiency of some models, including a heuristically built neural network, for the day ahead hourly energy prices in the Spanish and Californian markets.

The most common ANN model, usually just called a Neural Network, is the single layer perceptron network. The ANN can be interpreted as a stepwise constant function with smooth transition. This transition is given by a nonlinear function, often called an *activation function*, of the inputs.

$$y_t = \sum_{i=1}^{H} \lambda_i f(\boldsymbol{\omega}_i' \mathbf{x}_t - \beta_i) + \epsilon_t \tag{3}$$

with $f(\boldsymbol{\omega}_i' \mathbf{x}_t - \beta_i) = (1 + e^{-(\boldsymbol{\omega}_i' \mathbf{x}_t - \beta_i)})^{-1}$. These nonlinear functions together comprise the *hidden layer*, which in this case has H *hidden units*. In this example, the activation function is the logistic function; however other activation functions are also considered in the literature; e.g. see Chen, Racine, and Swanson (2001). The usual estimation algorithm is the *backpropagation* algorithm, which aims to minimize the mean square error, between the actual and the forecasted data, iteratively.

Here we consider a definition of ANN called an Autoregressive Neural Networks (AR-NN) due to Medeiros, Teräsvirta, and Rech (2006). The AR-NN is defined as

$$y_t = G(\mathbf{x}, \boldsymbol{\psi}) = \boldsymbol{\alpha}' \tilde{\mathbf{x}}_t + \sum_{i=1}^{H} \lambda_i f(\boldsymbol{\omega}_i' \mathbf{x}_t - \beta_i) + \epsilon_t \tag{4}$$

where ϵ_t is a sequence of independently normally distributed random variable with zero mean and variance σ^2.

This model is a semi-parametric nonlinear model and, under certain constraints, is similar to other nonlinear models such as the (multiple) logistic smooth transition autoregressive

((M)LSTAR) and the self-exciting threshold autoregressive (SETAR). The main difference between this model and the usual ANN models previously used to forecast spot-price of electricity is the building strategy, where in the later case a statistical approach is considered to specify and estimate the model.

Here we summarize the building strategy proposed in Medeiros, Teräsvirta, and Rech (2006). First the potential variables (and lags) are selected, then linearity is tested against a single hidden unit using a Lagrange-Multipliers type test with a significance level α. If the hypothesis is not rejected, the model with one hidden unit is estimated and tested against the model with two hidden units with a level of significance $\alpha = \rho\alpha$, $0 < \rho < 1$. This procedure continues until the first non-rejection of the null hypothesis. In this building strategy, all the hidden units contains the variables that were originally selected in the AR-NN model.

2.2.2. *Kernel Regression.* Kernel estimators are a nonparametric technique where the conditional outcome $y|\mathbf{x}$ is forecast by a weighted average of its neighbors. This technique was first proposed by Nadaraya (1964) and Watson (1964) and has been studied and applied to both cross-sectional and time series data. A clear advantage of the kernel regression is its simplicity and flexibility. A good review and selection of references can be found in Hardle (1990), Hastie, Tibshirani, and Friedman (2001) and Hastie and Tibshirani (1990).

The main idea behind kernel estimation is to evaluate the value of $y_t = f(\mathbf{x}_t)$ using a weighted average of the past values of y_t around $\mathbf{x}_t$, capturing any nonlinear relations within the data. These weights are given by a kernel $K(\cdot)$ which integrates to unity.

$$\hat{y}_t = \hat{f}(\mathbf{x}_t) = \frac{1}{\lambda k} \sum_{(y_i, \mathbf{x}_i) \in \mathbb{Q}_t} K\left(\frac{\|\mathbf{x}_t - \mathbf{x}_i\|}{\lambda}\right) y_i = \frac{1}{k}\sum_{i=1}^{k} w_i y_i, \tag{5}$$

where $\mathbb{Q}_t$ is a neighborhood of $\mathbf{x}_t$ with k elements and $w_i = K\left(\|\mathbf{x}_t - \mathbf{x}_i\|/\lambda\right)/\lambda$. The parameter λ is called the bandwidth or smoothing parameter and it controls the smoothing of the fitted curve. As data points are typically evenly spaced, we can just divide (5) by the sum of the weights. This estimator is called the *Nadaraya-Watson* estimator and provides a better approximation results, i.e. reduce the mean square error.

$$\hat{y}_t = \frac{\sum_{i=1}^{k} w_i y_i}{\sum_{i=1}^{k} w_i}. \tag{6}$$

Three key issues to consider when using kernel methods are (i) the choice of the kernel $K_{\mathbf{x}_t}(\mathbf{x}_i) = K(\|\mathbf{x}_t - \mathbf{x}_i\|/\lambda)$, (ii) the smoothing parameter λ and (iii) the Markov coefficient k. For kernel regressions, one of the most popular choices for the kernel is the Gaussian kernel defined below.

$$K(\mathbf{x}_i, \lambda; \mathbf{x}_t, \Sigma) = (2\pi)^{k/2} e^{\frac{1}{2\lambda}(\mathbf{x}_i - \mathbf{x}_t)'\Sigma^{-1}(\mathbf{x}_i - \mathbf{x}_t)}, \tag{7}$$

where Σ is the estimated covariance matrix of $\mathbf{x}$.

The choice of k is important because a large k can reduce the rate of convergence, while a small k can lead to the lose of some information about the past of the series. The choice of λ can be complicated: a large bandwidth implies a lower variance and a higher bias while a small bandwidth implies a higher variance, but smaller bias. We follow Matzner-Løber, Gannoun, and Gooijer (1998) to select the values of k and λ. It is worth mentioning that λ is usually estimated using cross-validation (see Matzner-Løber, Gannoun, and Gooijer (1998), Hastie, Tibshirani, and Friedman (2001), and Faraway (2006)). For as extensive discussion on choosing the smoothing parameter, see chapter 5 and 7 of Hardle (1990).

Some problems may arise when using locally weighted averages, as the estimator is biased on the boundaries of the domain. The same problem may occur if the the data is evenly spread. Fitting a local linear model instead of a local constant model solves this problem (Hastie, Tibshirani, and Friedman 2001). In this work, we are considering the multiple regression case.

$$\hat{f}(\mathbf{x}_t) = \tilde{\mathbf{x}}_t' \left(\mathbf{X}_k' \mathbf{W}_k(\mathbf{x}_t) \mathbf{X}_k'\right)^{-1} \mathbf{X}_k' \mathbf{W}_k(\mathbf{x}_t) \mathbf{Y}_k, \tag{8}$$

where $\tilde{\mathbf{x}}_t = (1, \mathbf{x}_t')'$, $\mathbf{X}_k = [(1, \mathbf{x}'_{(i_1)})', \ldots, (1, \mathbf{x}'_{(i_k)})']'$ is the regression matrix of all $\mathbf{x}_i \in \mathbb{Q}_t$, $\mathbf{W}_k(\mathbf{x}_t)$ is a $k \times k$ diagonal matrix of the weights with the diagonal members given by $K_{\mathbf{x}_t}(\mathbf{x}_i)$ and $\mathbf{Y}_k = [y_{i_1}, \ldots, y_{i_k}]'$ for $y_i \in \mathbb{Q}_t$. It is straightforward for the linear parameters to be estimated by weighted least squares.

2.2.3. *Linear Regression Trees.* Regression trees are now a well established non parametric modeling tool. Apart from having the advantage of being non parametric, the Regression Tree is competitive when compared to linear regression, where it often gives a better performance when faced with non linear problems, while it has a tendency to underperform in the presence of a good linear structure. The tree structure may also reveal patterns in the analysis that would not be so obvious with linear regression.

The tree-structured methods are based on recursive partitioning of the covariate space. The Regression Tree is a tree-based method where the response y is given by a local model fitted for each partition. Once the split is hard (as opposed to smooth) we can write the response y as a sum of the local models, weighted by a binary variable $I_i(\mathbf{x}) = \{1 : \mathbf{x} \in \mathbb{X}_i\}$, where $\mathbb{X}$ is the

i^{th} partition of the space.

$$y_t = \sum_{i=1}^{k} I_i(\mathbf{x}_t)\boldsymbol{\beta}'\mathbf{x}_t + \epsilon_t, \tag{9}$$

and$\boldsymbol{\beta}_i$ is the linear parameter vector for the regime i and ϵ_t is independent and identically distributed with zero mean and variance σ^2. The most relevant reference in regression tree models is the Classification and Regression Tree (CART) approach of Breiman, Friedman, Olshen, and Stone (1984). In this case, the local models are just constants.

The growing algorithm is based on recursive partitioning of the space and evaluation of the new estimated model using mean squared error (MSE)[1]. For each value of each regressor, we divide the space into two partitions, estimate the parameters using least squares and calculate the MSE of the model. Then, the smallest MSE of the tree model is compared with the MSE of the linear model, and if the difference is smaller than a threshold we keep growing the tree, otherwise the linear model is the best model. For any tree with k partitions, we estimate all possible trees with $k+1$ partitions by splitting each one into two. This model is then estimated and evaluated using MSE. The tree keeps growing in the direction of the split which minimizes de MSE. The procedure stops when the MSE stops decreasing against given a threshold.

2.2.4. *Generalized Additive Models.* The Generalized Additive Models (GAM) introduced by Hastie and Tibshirani (1990), extends the concept of linear regression, allowing the predictors to vary nonlinearly. This non/semi-parametric class of models captures the nonlinear relations between predictor and outcome with a nonparametric approach to capture the unknown nonlinear relation. The approach has been widely used in several areas including medicine, biological sciences, ecology and economics. For a comprehensive review of GAM see Hastie and Tibshirani (1990) and Hastie, Tibshirani, and Friedman (2001). Also see Faraway (2006) for a discussion of how to use GAM with the R package.

Generalized Additive Model is a non/semi-parametric and flexible class of models used to characterize and identify nonlinearity effects in regressions. The general model is given by the following:

$$y_t = \alpha + f_1(x_{1t}) + f_2(x_{2,t}) + \ldots + f_p(x_{pt}) + \epsilon_t \tag{10}$$

where x_{it} for $i = 1, \ldots, p$ are the predictors, y_t the outcome, f_i's are unspecified smooth functions and ϵ_t is an independent and identically distributed error with zero mean. Note that not all

[1] $MSE = \sum_t \left(y_t - \sum_i I_i(\mathbf{x}_t)\boldsymbol{\beta}_i'\mathbf{x}_t\right)^2$

the functions f_i must be non-parametric, in this case the GAM can be seen as a semi-parametric model.

There are two very common was to define the functional form of f_i. The first one is to evaluate the model with nonparametric functions, and if a line fits between the confidence interval of the estimated function, then $f_i = \beta_i$ is constant; otherwise, f_i is estimated non-parametrically. The second way is to use an information criteria like Akaike Information Criteria (AIC), Corrected AIC (AICc) or Generalized Cross-Validation (GCV). For a reference on these subjects see Hastie and Tibshirani (1990) and Eubank (1988).

In a more general framework, we can relate a function $g(\cdot)$ of the conditional mean ($\mu(\mathbf{x}_t)$) of y_t given $\mathbf{x}_t = [x_{1t},\, \ldots,\, x_{pt}]'$ with an additive function of the free variables:

$$g(\mu(\mathbf{x}_t)) = \alpha + \sum_{i=1}^{p} f_i(x_{it}). \tag{11}$$

The functions f_i are estimated in a flexible way using a *scatterplot smoother*[2] as a building block, more specifically a cubic spline. A simple iterative procedure to estimate the model is the *backfitting algorithm*. First we set $\hat{\alpha} = T^{-1} \sum_t y_t$, then we apply the cubic spline smoother $\mathcal{S}_k$ to the targets $\left\{ y_t - \hat{\alpha} - \sum_{i \neq k} \hat{f}_i(x_{it}) \right\}_{t=1}^{T}$, as a function of x_{kt}, to obtain a new estimate $\hat{f}_k$. This procedure is done for each $\hat{f}_i$, $i = 1, \ldots, p$, using the current estimates of the other functions until the algorithm converges, i.e. the changes in each $\hat{f}_i$ is smaller than a threshold.

3. Californian market and data

The California market was deregulated in 1998 and opened April 1st 1998. By May 1 2000 the market was in crisis which ended August 31, 2000. By that time Pacific Gas and Electric had gone bankrupt; the other two major power companies had amassed huge debts. Why did this happen? When the market was initially designed, two rules were put in place that left the utility companies unable to hedge against volatility. They were not permitted to sign long term contracts for wholesale electricity; retail rates were largely fixed and hence the companies were unable to pass-on any wholesale price increase onto customers.

Knittel and Roberts (2005) fit a range of traditional models to an hourly time series of real-time Californian electricity prices and find that the forecasting performance of traditional models is 'poor' and can be improved when they address "the unique features of electricity data in particular, volatility clustering and higher order autocorrelation". Contreras, Espínola, Nogales,

[2] A scatterplot smoother is defined to be a function of x in y whose result is a function s with the same domain as the values in $x : \mathcal{S}(y|x)$

and A.J. (2003) utilise an ARIMA model to forecast Californian next-day electricity prices for the week of April 3, 2000, being the week is prior to the beginning of the dramatic price volatility period that took place May-August 2000. Their preferred ARIMA model predicts price better before the May-August crisis and only requires the previous 2 hours of data and three differentiations. Average errors in the pre-crisis period were around 5%, whereas they jump to 11% when this volatile period is included. For more on the California market, see also Moulton (2005) and Weron (2006).

In this study we forecast the day-ahead and week-ahead hourly Californian market clearing prices from the period preceding and including the market crisis cited above. We split the dataset into estimation and evaluation sets. The estimation set comprises the period from July 5, 1999 to April 2, 2000; the day before starting the crisis. Consequently, the period from April 3, 2000 to December 3, 2000 is used for evaluation purposes. The test scheme is the same used by Misiorek, Trueck, and Weron (2006) for the linear model; however, we specify the models weekly to capture changes in the model specification, i.e. a new regime.

The variable set used to forecast the prices is: last two days log-price (p_{t-24} and p_{t-48}), last week log-price (p_{t-168}), dummy variable for Saturday (d_{Sat}), Sunday (d_{Sun}) and Monday (d_{Mon}), the logarithm transformation of the next day forecasted load (l_t) and the minimum of previous day's 24 hourly log-prices (mp_t). The logarithm transformation of price and load is used to attain more stable variances.

We forecast the clearing price in a day-based framework (24 hours of the day in a turn) and we re-estimate the models every day, re-specifying[3] the models every week. Note that we use the model estimated on Sunday to forecast the whole week to evaluate the week-ahead performance.

As noticed by Misiorek, Trueck, and Weron (2006) and Cuaresma, Hlouskova, Kossmeier, and Obersteiner (2004), modeling each hour of the day separately performs better than one specification for whole day. Then, we decide to model each hour of the day separately for all classes of models.

The variable selection procedure[4] used for the parametric models consist in select the subset of variables which minimizes the Bayesian Information Criteria (BIC). For the linear and tree-based models, the selected set contains all the variables. For the non-linear models with smooth transition (i.e. Multiple STAR and Artificial Neural Networks) we select the variables using a technique proposed by Rech, Teräsvirta, and Tschernig (2001). The idea is to approximate

[3]In re-specify the model we mean grow the model when it is needed, e.g. number of regimes in a multiple regime models.

[4]All the model/variable selection procedures were carried out using only the in-sample observations.

the non-linear model by a polynomial of sufficient high order and then apply some well-know variable selection technique to this approximation. We select all variables as they are significant for most models.

For the local regression and GAM we choose a subset of real-valued variables, which seems to present a non-linear relationship with p_t or a local behavior, to model non-parametrically. The selection of these variables is carried out using the an information criteria (Hastie, Tibshirani, and Friedman 2001, Hastie and Tibshirani 1990, Eubank 1988), where the effective number of parameters is given by the trace of the hat matrix[5]. We choose the Corrected AIC (AICc) (Hurvich, Simonoff, and Tsai 1998) which are not affected by significant problems of over-fitting (Manzan 2004). The AICc is shown below.

$$AICc = \log SSE + \frac{N + df}{N - df - 2}, \tag{12}$$

where SSE is the sum of squared errors, N is the sample size and $df = Tr(H)$ is the effective number of parameters.

For both GAM and local regression, we calculated the AICc of a number of models and select the one which minimizes the information criteria. The estimated models were the following: the dummy variables modeled linearly and all the models with 1, 2, ..., 5 non-parametric responses. In the GAM selected all five real-valued variables are modeled non-parametrically. For the local regression model the selected variables were only p_{t-24}, p_{t-168} and l_t.

Following Misiorek, Trueck, and Weron (2006) and Conejo, Contreras, Espínola, and Plazas (2005), we use the naive method as a benchmark for all models. The naive method can be described as follows: the price on hour t on Sundays, Mondays and Saturdays are equal to the same hour of the previous week; the price on hour t on Tuesdays to Fridays are equal to the same hour of the previous day. For the week-ahead forecast, the price is the same as last week. The naive test is passed if the errors for the model are smaller than the errors obtained for the naive method.

4. RESULTS

To assess the forecasting performance of each model, we use different statistical measures. This performance can be evaluated once the true market prices are available. For every day and all the weeks three types of average prediction errors (typically used in the electricity price forecasting literature, see e.g. Weron (2006)) were computed: one corresponding to the 24 hours of each day and two to the 168 hours of each week.

[5]The hat matrix H is defined as $\hat{y} = Hy$, where $\hat{y}$ is the forecasted outcome and y the actual outcome.

The Mean Daily Error (MDE) is computed as

$$MDE = \frac{1}{24}\sum_{h=1}^{24}\frac{|p_h - \hat{p}_h|}{\bar{p}_{24}}, \tag{13}$$

where p_h and $\hat{p}_h$ are respectively the actual price and the forecasted price for hour h and $\bar{p}_{24}$ is the mean hourly price for a given day. The use of $\bar{p}_{24}$ avoid the adverse effect of prices close to zero.

Analogous to the MDE, the Mean Weekly Error (MWE) is computed as:

$$MWE = \frac{1}{168}\sum_{h=1}^{168}\frac{|p_h - \hat{p}_h|}{\bar{p}_{168}}, \tag{14}$$

where p_h and $\hat{p}_h$ are respectively the actual price and the forecasted price for hour h in the week and $\bar{p}_{168}$ is the mean hourly price for a given week. Additionally, we compute the Weekly Root Mean Square error (WRMSE). The WRMSE is calculated as the square root of the 168 square differences between the actual and forecasted price:

$$MDE = \sqrt{\frac{1}{168}\sum_{h=1}^{168}(p_h - \hat{p}_h)^2}. \tag{15}$$

The WRMSE puts more weight to differences in the high-price range than MDE and MWE. Such measures are important because price spikes may lead to financial losses in electricity trading. However, both measures are not robust against outliers.

4.1. **Forecast Results.** Tables 4, 5, 6 (Appendix) and table 1 below, refer to the daily forecasts presented as a weekly measure. The entry "*Linear*" refers to the preferred model from Misiorek, Trueck, and Weron (2006) and entries for this model in Tables 4, 5 and 6 replicate his results for this approach and likewise for "*Naïve*". The other entries (GAM; Local Regression; ANN and Tree) presented below and in the Appendix are new. Table 1, below, summarises the results and demonstrate the following; for the MWE both GAM and Local Regression dominate Linear with Naïve fourth. For WRMSE GAM followed by Local Regression with Linear and Naïve joint third. Looking at the 'calm' (weeks 1-10) versus 'volatile periods' (weeks 11-35); Linear seems to forecast better in the early periods, less so in the more volatile episodes.

Tables 7, 8, 9 (Appendix) and table 2 below, refer to week ahead forecasts. These are new including the columns headed "*Linear*" and "*Naïve*". Table 2 below summarise these results and shows that Local Regression and GAM dominate all other approaches for both MWE and WRMSE. Also new are Tables 10, 11 – 17 and Table 3 which relate to day-ahead forecasts

TABLE 1. WEEKLY BEST MODEL SUMMARY - DAY AHEAD FORECAST

This table contains the weekly 'best model' (model with smallest error) summary in a Day-Ahead forecasting framework. For each model we show how many times it was be best model in each error measure (MWE and WRMSE), where 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Model	MWE	WRMSE
Local Regression	8	10
GAM	9	7
ANN	1	2
Naive	6	6
Tree	4	4
Linear	7	6

where the day of the week effect is reported explicitly. Table 3 shows the models with smaller MDE sorted by day of week. The naive method is the best "forecasting method" in a day ahead forecast, followed by GAM and Local Regression. The linear, ANN and linear regression tree models have the worst performance.

5. CONCLUSION

Interest in modelling and in particular, forecasting, electricity prices is growing globally. Much interest has been focussed on modelling a small number of key markets including CalPX and NordPool, with British, Spanish and Australasian markets being included as high frequency data becomes available.

In this study we have analysed the CalPX data as a precursor to a more wide-ranging testing programme. In particular, we have sought to formally investigate the potential for using a range of non- (semi-) parametric methods that have proven useful in other areas of applied statistics. The particular nature of the electricity data eg., highly seasonal, mean reverting with occasional jumps/spikes and time- and price-dependant volatility, appears on the face of it to be a prima facie case for using a range of parametric methods developed for financial data. Weron et. al. (various) have demonstrated with the CalPX data the apparent dominance of linear ARX and 'Naïve' forecasting methods. Incorporating GARCH-type effects apparently does not enhance

TABLE 2. WEEKLY BEST MODEL SUMMARY - WEEK AHEAD FORECAST

This table contains the weekly 'best model' (model with smallest error) model summary in a Week-Ahead forecasting framework. For each model we show how many times it was be best model in each error measure (MWE and WRMSE), where 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Model	MWE	WRMSE
Local Regression	14	16
GAM	19	16
ANN	2	2
Naive	0	1
Tree	0	0
Linear	0	0

TABLE 3. DAILY BEST MODEL - DAY-AHEAD FORECAST

This table contains summary of the models with smallest MDE in each day of week. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

WEEK	**Monday**[6]	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**	**Total**
Local Regression	4	6	11	6	9	8	7	20.8%
GAM	10	2	6	10	7	9	9	21.6%
ANN	10	3	3	1	3	5	3	11.4%
NAIVE	8	18	11	11	9	5	9	29.0%
TREE	3	3	3	3	3	4	2	8.6%
LINEAR	0	3	1	4	4	4	5	8.6%

the performance of these simple methods (see Weron (2006)) although these results relate to a small range of cases and would appear to contrast with those of **?** who used both CalPX and Spanish data.

In this study we have contributed to the literature by formally testing the proposition on page 4 that casts doubt on the assumed poor performance of AI-based techniques. Our results fall into two groups. For the experiments undertaken by Weron; daily forecasts - weekly measure - the Linear (ARX) model performs well, but is dominated by Local Regression and GAM. ANN does not perform well, as postulated by Misiorek, Trueck, and Weron (2006), nor do Trees. However, 'Naïve' works very well - the simplest and often 'best' way to forecast electricity prices is to assume your forecast tomorrow is simply informed by the same hour of the previous day (or week for Saturday, Sunday and Monday)! New results presented here, however, are somewhat more encouraging for the benefits of using non-parametric methods. Week ahead forecasts are dominated by Local Regression and GAM formulations and day ahead forecasts show a strong role for these two approaches and also the ANN. Linear models have somewhat less success. In addition it must be stressed that these comparisons were made within the constraints of the best linear model formulation where the covariates were chosen to maximise the performance of that formulation.

Future work in this area will involve; application of the parametric and non- (semi-) parametric methods to a range of alternate data sets and to include a number of other co-variates, i.e., NordPool and New Zealand data sets and the inclusion of weather and hydrological data.

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APPENDIX A. TABLES

TABLE 4. MEAN WEEKLY ERROR - DAY-AHEAD FORECAST

This table contains the Mean Weekly Error (MDWE) for each model. We also show the median of the MWE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	3.756%	5.625%	4.574%	3.777%	4.470%	5.004%
2	5.190%	5.409%	6.941%	5.324%	5.510%	8.619%
3	8.550%	9.298%	8.924%	8.635%	9.380%	9.736%
4	13.374%	13.193%	11.842%	13.782%	14.770%	17.139%
5	17.311%	17.631%	17.837%	17.074%	17.070%	19.307%
6	8.193%	8.050%	7.628%	8.248%	9.200%	14.698%
7	9.811%	9.449%	9.379%	9.834%	9.900%	12.558%
8	47.290%	41.551%	1544900.000%	42.837%	48.270%	62.970%
9	13.772%	14.224%	15.754%	13.975%	13.650%	33.220%
10	7.633%	8.428%	7.185%	7.611%	7.980%	16.228%
11	44.227%	38.256%	546.460%	45.853%	41.580%	35.587%
12	19.789%	19.984%	33.079%	20.009%	20.000%	19.407%
13	44.880%	38.517%	33.513%	47.244%	41.550%	23.306%
14	27.588%	25.208%	31.844%	28.431%	24.440%	49.471%
15	11.881%	15.152%	13.758%	11.932%	15.210%	22.374%
16	24.567%	18.823%	19.998%	24.727%	21.430%	32.347%
17	23.605%	19.240%	20.483%	25.008%	20.620%	27.742%
18	12.948%	25.264%	16.182%	13.156%	12.370%	15.004%
19	13.849%	13.174%	13.883%	13.896%	13.930%	30.420%
20	8.638%	16.497%	14.584%	8.642%	11.390%	8.602%
21	16.062%	20.597%	19.645%	16.652%	17.280%	18.223%
22	19.991%	18.745%	23.554%	20.115%	21.430%	50.328%
23	22.799%	23.438%	21.327%	22.593%	22.440%	44.169%
24	14.259%	15.362%	15.198%	14.187%	15.320%	22.861%
25	16.999%	22.400%	21.162%	17.027%	19.380%	27.895%
26	15.044%	14.320%	14.846%	14.999%	15.960%	24.696%
27	12.991%	12.146%	11.635%	12.996%	14.270%	16.660%
28	11.717%	10.361%	9.794%	11.794%	13.120%	20.226%
29	11.515%	9.882%	10.947%	11.646%	12.410%	14.817%
30	14.008%	11.467%	10.331%	14.103%	14.450%	12.407%
31	13.853%	11.173%	11.276%	13.895%	14.140%	12.200%
32	13.690%	11.882%	12.753%	13.871%	14.620%	16.534%
33	16.280%	16.992%	15.253%	16.804%	16.740%	26.987%
34	15.465%	14.816%	11.033%	15.981%	15.710%	7.631%
35	9.720%	9.279%	6.245%	9.986%	10.910%	6.177%
Calm	9.181%	9.373%	9.152%	9.235%	9.640%	15.463%
Volatile	15.044%	16.497%	15.198%	14.999%	15.710%	22.374%

TABLE 5. WEEKLY ROOT MEAN ERROR - DAILY FORECAST

This table contains the Weekly Root Mean Square Error (WRMSE) for each model. We also show the median of the WRMSE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	1.474	2.001	1.771	1.482	1.721	2.930
2	1.704	1.719	2.197	1.740	1.776	3.196
3	2.534	2.704	2.853	2.543	2.812	5.588
4	4.427	4.565	4.052	4.521	4.822	8.551
5	7.296	8.157	7.454	7.288	7.399	6.150
6	3.434	3.463	3.145	3.462	3.815	6.410
7	4.808	4.743	4.669	4.805	4.891	97.979
8	84.138	79.527	17360000.000	82.174	84.992	30.346
9	10.049	10.168	11.544	10.139	9.977	12.953
10	5.383	5.653	5.187	5.386	5.743	99.880
11	121.360	103.688	10041.000	127.490	113.715	27.660
12	25.511	24.981	70.172	25.532	25.321	93.168
13	151.210	131.456	144.680	161.500	139.370	37.344
14	20.247	19.216	24.676	20.401	16.915	18.582
15	8.990	12.555	10.888	8.992	12.210	69.832
16	63.201	47.435	52.007	63.680	55.849	96.727
17	76.115	58.926	64.318	79.421	65.353	61.971
18	42.497	115.285	63.640	43.438	41.270	70.626
19	25.722	26.829	26.416	25.661	25.918	16.703
20	18.027	33.695	28.558	18.025	21.534	45.096
21	36.080	45.047	44.611	37.146	38.901	77.402
22	30.735	27.650	34.805	30.769	31.481	60.336
23	29.832	30.198	29.884	29.710	29.333	41.541
24	28.209	28.061	28.150	28.153	28.004	50.209
25	31.353	44.218	39.889	31.262	34.295	36.646
26	23.110	21.884	23.567	23.056	25.153	24.138
27	18.327	17.881	17.463	18.335	19.972	26.058
28	13.760	12.429	11.934	13.811	15.235	18.684
29	14.786	12.374	13.967	14.908	16.129	15.609
30	17.103	13.887	13.001	17.201	17.406	14.583
31	18.146	14.753	14.018	18.199	18.733	26.558
32	23.378	20.498	19.809	23.613	24.567	64.918
33	37.499	39.101	34.470	38.297	39.494	21.522
34	40.260	38.889	31.061	41.585	40.076	24.890
35	27.226	25.464	21.669	27.855	31.299	26.841
Calm	4.618	4.654	4.361	4.663	4.857	7.481
Volatile	27.226	27.650	28.558	27.855	28.004	36.646

TABLE 6. WEEKLY BEST MODEL - DAY AHEAD FORECAST

This table contains the weekly 'best model' (model with smallest error) in a Day-Ahead forecasting framework. For each week, we show the best model for each error measure (MWE and WRMSE) where 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	MWE	WRMSE
1	'Linear'	'Linear'
2	'Tree'	'Tree'
3	'Linear'	'Linear'
4	'Local Regression'	'Local Regression'
5	'Tree'	'Tree'
6	'Local Regression'	'Local Regression'
7	'Local Regression'	'Local Regression'
8	'GAM'	'Naive'
9	'Linear'	'Linear'
10	'Local Regression'	'Local Regression'
11	'Naive'	'Naive'
12	'Naive'	'GAM'
13	'Naive'	'Naive'
14	'Tree'	'Tree'
15	'Linear'	'Linear'
16	'GAM'	'GAM'
17	'GAM'	'GAM'
18	'Linear'	'Linear'
19	'GAM'	'Naive'
20	'Naive'	'ANN'
21	'Linear'	'Linear'
22	'GAM'	'GAM'
23	'Tree'	'Tree'
24	'ANN'	'GAM'
25	'Linear'	'ANN'
26	'GAM'	'GAM'
27	'GAM'	'Local Regression'
28	'Local Regression'	'Local Regression'
29	'Local Regression'	'GAM'
30	'Local Regression'	'Local Regression'
31	'GAM'	'Local Regression'
32	'GAM'	'Local Regression'
33	'Local Regression'	'Naive'
34	'Naive'	'Naive'
35	'Naive'	'Local Regression'

TABLE 7. MEAN WEEKLY ERROR - WEEK-AHEAD FORECAST

This table contains the Mean Weekly Error (MWE) for each model. We also show the median of the MWE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	7.220%	4.290%	5.370%	6.590%	7.830%	7.010%
2	19.610%	5.680%	7.440%	17.220%	22.300%	15.690%
3	21.880%	9.540%	10.090%	20.190%	23.410%	11.010%
4	12.890%	13.090%	12.770%	12.080%	13.330%	16.500%
5	21.220%	17.940%	18.550%	21.080%	Inf	22.170%
6	14.020%	9.150%	7.220%	12.470%	15.500%	24.060%
7	17.400%	9.940%	9.540%	16.590%	17.960%	14.030%
8	57.880%	39.530%	969.770%	57.120%	58.370%	58.220%
9	24.040%	15.850%	17.340%	24.020%	23.960%	90.640%
10	11.200%	7.490%	8.090%	11.660%	11.840%	20.620%
11	73.090%	41.200%	132.720%	75.520%	72.890%	68.380%
12	29.720%	17.690%	83.330%	33.920%	32.350%	132.720%
13	75.890%	42.030%	80.980%	80.630%	74.750%	67.670%
14	91.460%	22.920%	58.430%	130.940%	75.610%	360.160%
15	29.740%	15.410%	12.920%	31.320%	24.200%	32.900%
16	51.260%	18.200%	30.860%	55.080%	39.280%	49.150%
17	57.040%	21.100%	25.090%	59.630%	50.300%	44.290%
18	33.140%	25.230%	18.530%	35.450%	29.710%	30.020%
19	20.960%	20.860%	18.730%	30.460%	69.550%	86.740%
20	25.760%	16.780%	17.120%	29.920%	21.140%	16.860%
21	37.870%	24.920%	15.590%	41.340%	30.710%	22.790%
22	49.010%	20.030%	24.770%	59.070%	104.300%	74.090%
23	32.770%	22.320%	22.680%	29.720%	37.350%	66.140%
24	32.740%	14.280%	17.930%	33.280%	26.190%	33.700%
25	25.190%	22.000%	21.890%	23.030%	36.590%	39.470%
26	19.930%	14.600%	14.710%	21.830%	32.810%	45.070%
27	19.940%	12.350%	13.950%	19.780%	22.790%	21.420%
28	15.130%	10.690%	9.150%	14.240%	20.620%	27.180%
29	17.040%	10.630%	10.030%	19.320%	16.270%	11.920%
30	16.150%	11.670%	9.320%	14.680%	16.130%	11.920%
31	10.860%	11.960%	11.090%	10.700%	13.460%	11.930%
32	20.090%	13.250%	9.930%	21.720%	17.040%	19.160%
33	36.670%	24.890%	13.900%	40.500%	34.170%	36.810%
34	22.790%	28.960%	10.090%	25.810%	18.450%	16.040%
35	15.260%	16.230%	7.180%	18.320%	12.120%	10.690%
Calm	18.505%	9.740%	9.815%	16.905%	17.960%	18.560%
Volatile	29.720%	18.200%	17.120%	30.460%	30.710%	33.700%

TABLE 8. WEEKLY ROOT MEAN ERROR - WEEKLY FORECAST

This table contains the Weekly Root Mean Square Error (WRMSE) for each model. We also show the median of the WRMSE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	3.022	1.601	2.099	2.610	3.171	2.544
2	6.035	1.862	2.403	5.085	6.692	4.502
3	6.855	2.779	3.185	6.292	7.262	3.374
4	4.551	4.700	4.370	4.115	4.375	5.618
5	8.756	8.162	7.611	8.613	Inf	9.995
6	6.140	3.688	3.049	5.500	6.603	9.965
7	7.538	4.871	4.703	7.276	7.748	6.549
8	102.420	78.867	6511.948	102.509	103.214	105.560
9	17.133	11.133	14.857	17.319	17.531	96.302
10	7.605	5.471	5.275	7.873	8.144	14.479
11	207.039	116.364	600.404	211.359	207.039	197.744
12	38.689	20.958	273.017	42.425	44.399	167.634
13	281.364	151.745	405.989	291.956	276.734	251.960
14	55.359	16.744	73.578	89.766	42.992	286.497
15	20.187	11.655	10.433	21.878	17.129	22.699
16	112.116	44.963	71.019	118.130	86.082	108.487
17	157.512	63.862	84.135	162.811	137.402	138.460
18	108.362	105.821	69.834	115.294	96.649	103.757
19	40.224	43.708	36.727	64.748	165.084	160.564
20	44.804	34.806	34.290	52.286	37.412	34.019
21	76.271	53.359	35.366	85.905	65.515	52.304
22	68.285	28.928	38.752	85.351	177.265	102.240
23	40.791	29.624	30.604	38.131	47.877	81.267
24	59.800	26.465	34.246	62.390	46.561	55.935
25	41.932	43.067	38.361	39.289	70.980	63.820
26	26.946	22.277	20.520	28.711	48.064	58.566
27	27.919	17.973	23.334	28.408	32.764	29.610
28	17.697	12.486	10.858	16.968	26.873	32.271
29	21.870	13.041	13.348	24.502	21.308	15.941
30	18.398	14.082	11.857	16.989	19.422	14.538
31	12.789	15.609	13.054	12.664	16.306	14.424
32	30.534	22.643	15.874	34.133	26.883	28.418
33	78.023	52.698	31.884	85.296	73.751	75.385
34	55.537	69.473	28.783	64.333	46.986	42.380
35	42.255	43.280	23.105	50.962	35.738	34.754
Calm	7.196	4.785	4.536	6.784	7.262	8.257
Volatile	42.255	29.624	34.246	52.286	46.986	58.566

TABLE 9. WEEKLY BEST MODEL - WEEK AHEAD FORECAST

This table contains the weekly 'best model' (model with smallest error) in a Week-Ahead forecasting framework. For each week, we show the best model for each error measure where 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast and 'Local Regression' for Local Regression Model.

Week	MWE	WRMSE
1	'GAM'	'GAM'
2	'GAM'	'GAM'
3	'GAM'	'GAM'
4	'ANN'	'ANN'
5	'GAM'	'Local Regression'
6	'Local Regression'	'Local Regression'
7	'Local Regression'	'Local Regression'
8	'GAM'	'GAM'
9	'GAM'	'GAM'
10	'GAM'	'Local Regression'
11	'GAM'	'GAM'
12	'GAM'	'GAM'
13	'GAM'	'GAM'
14	'GAM'	'GAM'
15	'Local Regression'	'Local Regression'
16	'GAM'	'GAM'
17	'GAM'	'GAM'
18	'Local Regression'	'Local Regression'
19	'Local Regression'	'Local Regression'
20	'GAM'	'Naive'
21	'Local Regression'	'Local Regression'
22	'GAM'	'GAM'
23	'GAM'	'GAM'
24	'GAM'	'GAM'
25	'Local Regression'	'Local Regression'
26	'GAM'	'Local Regression'
27	'GAM'	'GAM'
28	'Local Regression'	'Local Regression'
29	'Local Regression'	'GAM'
30	'Local Regression'	'Local Regression'
31	'ANN'	'ANN'
32	'Local Regression'	'Local Regression'
33	'Local Regression'	'Local Regression'
34	'Local Regression'	'Local Regression'
35	'Local Regression'	'Local Regression'

TABLE 10. DAILY BEST MODEL - DAY-AHEAD FORECAST

This table contains the model with smallest MDE each day. ’GAM’ aims for the Generalised Additive Model, ’ANN’ for the Artificial Neural Networks, ’Naive’ the Naive method for week ahead forecast, ’Local Reg’ for Local Regression Model, ’Tree’ the Linear Regression Tree model and ’Linear’ the ARX model.

WEEK	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
1	ANN’/’Linear’	’Linear’	’Local Reg’	’Linear’	’Local Reg’	’ANN’	’Local Reg’
2	’GAM’	’Naive’	’GAM’	’Naive’	’Naive’	’Linear’	’GAM’
3	ANN’/’Linear’	’Local Reg’	’Local Reg’	’Naive’	’GAM’	’ANN’	’GAM’
4	’GAM’	’Local Reg’	’Local Reg’	’Naive’	’GAM’	’GAM’	’Linear’
5	’Tree’	’Naive’	’Naive’	’GAM’	’GAM’	’Naive’	’ANN’
6	ANN’/’Linear’	’Naive’	’Local Reg’	’Naive’	’Naive’	’GAM’	’GAM’
7	’GAM’	’Naive’	’Naive’	’GAM’	’Naive’	’GAM’	’GAM’
8	’GAM’	’Naive’	’ANN’	’GAM’	’Local Reg’	’ANN’	’Naive’
9	ANN’/’Linear’	’Naive’	’Tree’	’ANN’	’Local Reg’	’Local Reg’	’GAM’
10	ANN’/’Linear’	’ANN’	’Naive’	’Naive’	’Tree’	’Local Reg’	’Linear’
11	’Naive’	Tree’	’GAM’	’GAM’	’Linear’	’Naive’	’ANN’
12	’Tree’	’Naive’	’Local Reg’	’GAM’	’Naive’	’Local Reg’	’Local Reg’
13	’GAM’	’Local Reg’	’Local Reg’	’Naive’	’Naive’	’Naive’	’Local Reg’
14	ANN’/’Linear’	’Tree’	’Tree’	’Naive’	’ANN’	’Tree’	’Local Reg’
15	ANN’/’Linear’	’Tree’	’Naive’	’Naive’	’Naive’	’Local Reg’	’Linear’
16	’Tree’	’ANN’	’GAM’	’GAM’	’GAM’	’GAM’	’Local Reg’
17	’GAM’	’GAM’	’Tree’	’Tree’	’Naive’	’Local Reg’	’Local Reg’
18	’Local Reg’	’Naive’	’Naive’	’Naive’	’Local Reg’	’Tree’	’ANN’
19	ANN’/’Linear’	’Naive’	’GAM’	’Naive’	’GAM’	’Tree’	’GAM’
20	’Naive’	’Naive’	’Naive’	’Linear’	’ANN’	’ANN’	’Tree’
21	’Naive’	’Naive’	’Naive’	’Local Reg’	’Naive’	’Linear’	’Linear’
22	ANN’/’Linear’	’Naive’	’GAM’	’GAM’	’Local Reg’	’GAM’	’GAM’
23	’Local Reg’	’Linear’	’ANN’	’Tree’	’Local Reg’	’Local Reg’	’Tree’
24	’GAM’	’Linear’	’GAM’	’Local Reg’	’ANN’	’ANN’	’Linear’
25	ANN’/’Linear’	’Naive’	’Naive’	’GAM’	’GAM’	’Linear’	’GAM’
26	’Local Reg’	’ANN’	’ANN’	’GAM’	’Tree’	’Local Reg’	’Naive’
27	’Naive’	’Local Reg’	’Local Reg’	’Tree’	’Local Reg’	’GAM’	’GAM’
28	’GAM’	’GAM’	’Local Reg’	’Local Reg’	’Local Reg’	’GAM’	’Naive’
29	’GAM’	’Naive’	’Naive’	’GAM’	’Tree’	’Linear’	’Naive’
30	’GAM’	’Naive’	’Local Reg’	’Local Reg’	’Local Reg’	’GAM’	’Naive’
31	’Local Reg’	’Local Reg’	’Naive’	’Local Reg’	’GAM’	’GAM’	’Local Reg’
32	’Naive’	’Local Reg’	’Naive’	’Linear’	’Linear’	’Local Reg’	’Naive’
33	’Naive’	’Naive’	’Local Reg’	’Linear’	’Linear’	’Tree’	’Naive’
34	’Naive’	’Naive’	’Linear’	’Naive’	’Linear’	’Naive’	’Naive’
35	’Naive’	’Naive’	’Local Reg’	’Local Reg’	’Naive’	’Naive’	’Naive’
Best	GAM’/’ANN’/’Linear’	’Naive’	Local Reg’/’Naive’	’Naive’	’Local Reg’	’GAM’	GAM’/’Naive’

TABLE 11. MEAN DAILY ERROR - MONDAY

This table contains the Mean Daily Error (MDE) on all Mondays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	5.130%	8.430%	8.890%	5.130%	6.260%	5.680%
2	6.480%	3.270%	3.800%	6.480%	6.010%	16.910%
3	8.280%	9.570%	8.890%	8.280%	11.240%	9.800%
4	10.640%	9.880%	9.980%	10.640%	12.180%	10.910%
5	23.890%	25.640%	26.210%	23.890%	22.550%	34.810%
6	5.820%	5.940%	6.300%	5.820%	7.300%	25.720%
7	10.050%	7.810%	9.850%	10.050%	9.580%	8.250%
8	57.500%	56.790%	58.680%	57.500%	56.940%	69.900%
9	9.740%	17.340%	20.420%	9.740%	11.090%	120.050%
10	6.850%	10.260%	6.940%	6.850%	8.670%	20.660%
11	31.920%	27.210%	36.740%	31.920%	30.510%	25.480%
12	12.200%	11.510%	14.710%	12.200%	10.240%	20.990%
13	50.170%	42.700%	48.090%	50.170%	47.210%	48.520%
14	21.600%	25.480%	26.310%	21.600%	24.370%	70.280%
15	12.380%	12.630%	15.700%	12.380%	15.210%	47.770%
16	15.300%	16.620%	20.050%	15.300%	12.020%	36.700%
17	47.060%	27.020%	28.550%	47.060%	38.420%	61.680%
18	17.450%	28.690%	13.230%	17.450%	15.490%	22.400%
19	10.740%	21.980%	11.050%	10.740%	16.210%	78.360%
20	13.200%	16.110%	13.020%	13.200%	13.400%	5.430%
21	21.840%	23.300%	28.440%	21.840%	19.340%	9.030%
22	10.430%	10.710%	15.090%	10.430%	12.790%	18.480%
23	29.520%	20.270%	11.980%	29.520%	24.210%	194.710%
24	13.140%	10.140%	15.760%	13.140%	17.630%	47.270%
25	11.320%	20.190%	12.450%	11.320%	13.540%	27.820%
26	14.590%	8.410%	6.810%	14.590%	15.140%	73.000%
27	19.140%	21.120%	20.860%	19.140%	19.780%	16.640%
28	13.010%	11.110%	15.730%	13.010%	14.600%	11.190%
29	16.540%	12.960%	17.560%	16.540%	17.870%	14.390%
30	10.400%	7.860%	10.960%	10.400%	11.510%	8.650%
31	6.010%	5.390%	4.930%	6.010%	5.630%	8.240%
32	11.430%	8.630%	12.550%	11.430%	12.530%	6.380%
33	21.040%	21.510%	20.630%	21.040%	22.260%	15.030%
34	13.490%	11.170%	13.740%	13.490%	14.310%	9.710%
35	12.080%	10.990%	6.940%	12.080%	13.780%	4.250%
Calm	14.438%	15.493%	15.996%	14.438%	15.182%	32.269%
Volatile	18.240%	17.348%	17.675%	18.240%	18.320%	35.296%

TABLE 12. MEAN DAILY ERROR - TUESDAY

This table contains the Mean Daily Error (MDE) on all Tuesdays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	2.670%	5.090%	2.870%	2.680%	3.480%	3.770%
2	4.790%	7.770%	4.660%	4.800%	5.080%	4.180%
3	7.550%	8.300%	2.670%	7.620%	8.570%	4.390%
4	12.580%	11.300%	6.020%	12.770%	13.990%	9.180%
5	18.390%	22.270%	12.960%	18.880%	18.740%	8.990%
6	10.120%	9.080%	9.100%	10.140%	10.960%	8.670%
7	22.750%	23.200%	18.610%	22.720%	22.650%	18.440%
8	66.470%	64.460%	70.970%	67.160%	65.990%	55.490%
9	14.150%	13.950%	23.010%	14.520%	14.040%	13.030%
10	4.220%	7.260%	6.690%	4.170%	4.660%	7.040%
11	21.590%	31.880%	31.020%	21.950%	20.350%	24.770%
12	29.000%	48.050%	41.420%	28.710%	37.410%	20.580%
13	50.560%	40.510%	29.590%	50.580%	46.250%	46.170%
14	85.080%	75.470%	71.970%	83.120%	62.770%	92.700%
15	16.530%	16.840%	16.380%	16.530%	14.710%	19.760%
16	10.470%	11.620%	14.120%	10.360%	16.480%	11.720%
17	20.580%	8.110%	22.500%	21.410%	13.360%	8.780%
18	9.310%	47.160%	27.950%	10.070%	14.030%	5.270%
19	5.710%	7.300%	7.640%	5.660%	4.970%	3.390%
20	7.070%	23.240%	20.450%	6.870%	12.830%	5.700%
21	16.950%	24.360%	19.540%	17.210%	17.610%	12.590%
22	7.390%	10.330%	12.440%	7.510%	7.580%	3.880%
23	7.660%	8.480%	9.470%	7.860%	7.950%	12.090%
24	8.550%	12.660%	12.740%	8.590%	12.540%	11.000%
25	9.000%	29.710%	23.870%	9.090%	14.520%	7.830%
26	7.100%	11.920%	10.650%	7.060%	8.060%	8.850%
27	8.250%	5.770%	4.930%	8.190%	6.870%	14.790%
28	6.530%	5.840%	7.540%	6.680%	9.320%	6.950%
29	7.640%	7.040%	8.280%	7.830%	8.380%	5.300%
30	13.860%	11.100%	12.830%	13.960%	14.820%	9.750%
31	4.840%	5.270%	4.730%	4.820%	6.080%	7.170%
32	5.140%	6.270%	5.070%	5.180%	6.670%	8.970%
33	11.970%	15.110%	11.760%	12.270%	12.880%	5.800%
34	13.750%	13.040%	9.210%	13.950%	14.940%	7.850%
35	8.390%	9.890%	7.140%	8.450%	10.040%	4.650%
Calm	16.369%	17.268%	15.756%	16.546%	16.816%	13.318%
Volatile	15.717%	19.479%	17.730%	15.756%	16.057%	14.652%

TABLE 13. MEAN DAILY ERROR - WEDNESDAY

This table contains the Mean Daily Error (MDE) on all Wednesdays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	2.710%	4.570%	1.880%	2.740%	3.330%	2.190%
2	7.540%	5.440%	11.540%	7.700%	6.220%	5.990%
3	4.960%	7.000%	3.210%	5.300%	6.870%	3.230%
4	16.520%	17.900%	12.210%	17.020%	17.260%	15.080%
5	23.910%	28.030%	19.740%	24.660%	24.620%	16.560%
6	12.540%	14.730%	8.920%	12.710%	13.560%	14.630%
7	6.450%	6.470%	4.150%	6.430%	6.260%	3.430%
8	44.190%	27.340%	9170640.770%	17.630%	51.250%	119.850%
9	9.160%	9.670%	8.980%	10.090%	7.450%	9.710%
10	9.680%	7.920%	7.210%	9.600%	9.290%	5.930%
11	68.770%	59.860%	63.780%	69.170%	67.270%	66.920%
12	18.170%	17.900%	15.720%	18.650%	16.230%	16.980%
13	46.040%	34.790%	17.560%	48.930%	42.130%	29.910%
14	17.550%	16.730%	39.260%	18.380%	16.340%	19.540%
15	15.270%	28.130%	21.890%	15.150%	27.210%	9.640%
16	13.430%	6.730%	10.990%	13.420%	10.080%	13.650%
17	13.460%	11.930%	20.500%	14.150%	11.840%	21.210%
18	13.500%	29.510%	12.910%	14.520%	14.540%	5.770%
19	16.680%	7.480%	14.130%	16.930%	14.490%	16.940%
20	12.440%	22.550%	18.120%	12.560%	15.660%	9.910%
21	15.850%	22.700%	17.150%	16.540%	15.310%	9.630%
22	30.050%	27.760%	34.320%	29.480%	30.240%	38.980%
23	26.230%	28.770%	30.940%	25.810%	26.530%	29.830%
24	30.100%	21.660%	24.520%	30.110%	25.950%	30.880%
25	7.310%	15.840%	9.780%	7.540%	10.450%	5.330%
26	10.220%	14.070%	14.360%	10.080%	11.630%	11.900%
27	15.360%	14.680%	10.780%	15.480%	17.930%	11.290%
28	6.930%	5.800%	4.260%	6.990%	11.030%	26.910%
29	13.510%	10.990%	8.950%	13.690%	16.290%	8.950%
30	9.790%	8.470%	7.380%	9.870%	12.250%	8.880%
31	15.760%	11.890%	11.960%	15.750%	17.850%	9.180%
32	22.700%	19.200%	17.500%	22.740%	24.790%	16.510%
33	30.120%	31.610%	23.140%	30.350%	32.010%	38.500%
34	4.440%	6.180%	6.420%	4.700%	7.060%	12.840%
35	14.060%	11.680%	5.230%	14.190%	16.840%	17.150%
Calm	13.766%	12.907%	917071.861%	11.388%	14.611%	19.660%
Volatile	19.510%	19.476%	18.462%	19.807%	20.478%	19.489%

TABLE 14. MEAN DAILY ERROR - THURSDAY

This table contains the Mean Daily Error (MDE) on all Thursdays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	1.770%	3.340%	2.260%	1.770%	1.830%	2.970%
2	3.490%	5.370%	5.020%	3.750%	3.990%	1.480%
3	5.770%	6.790%	5.500%	6.020%	5.670%	4.650%
4	18.330%	20.460%	13.680%	19.480%	18.680%	12.390%
5	22.920%	14.570%	30.610%	21.260%	22.640%	40.700%
6	6.550%	7.160%	6.560%	6.650%	7.420%	4.610%
7	5.140%	4.130%	5.410%	5.110%	5.430%	6.410%
8	42.910%	15.900%	48.760%	45.240%	39.490%	96.840%
9	4.050%	6.230%	8.720%	4.020%	5.670%	6.060%
10	11.840%	7.960%	9.580%	11.900%	11.720%	3.400%
11	42.800%	20.830%	1748.880%	46.980%	37.230%	21.880%
12	32.400%	21.520%	30.560%	34.320%	31.630%	36.700%
13	36.090%	31.370%	44.210%	42.500%	33.340%	11.090%
14	41.090%	17.480%	46.540%	47.220%	35.970%	10.860%
15	4.950%	10.460%	8.670%	5.110%	14.410%	2.940%
16	55.690%	40.710%	45.530%	55.790%	49.920%	57.720%
17	12.050%	10.840%	15.350%	13.350%	9.040%	9.580%
18	9.820%	10.370%	13.450%	10.270%	9.030%	7.190%
19	31.580%	30.910%	28.840%	32.680%	32.600%	27.630%
20	6.210%	13.240%	15.760%	6.300%	9.580%	6.250%
21	10.940%	14.610%	9.900%	10.730%	14.040%	10.390%
22	62.320%	51.220%	55.610%	62.700%	60.350%	73.500%
23	26.550%	26.320%	26.510%	26.560%	24.090%	24.420%
24	12.710%	11.420%	9.120%	13.070%	11.300%	13.240%
25	41.820%	34.550%	41.750%	41.340%	41.940%	48.020%
26	13.970%	9.390%	13.300%	13.960%	11.450%	22.540%
27	8.740%	9.860%	13.180%	9.010%	8.250%	29.800%
28	16.220%	17.230%	8.220%	16.070%	14.380%	48.420%
29	6.600%	5.620%	11.710%	6.880%	5.990%	21.760%
30	5.970%	7.880%	3.770%	6.030%	6.130%	14.680%
31	20.520%	22.550%	14.370%	20.390%	19.430%	17.830%
32	7.820%	8.370%	15.730%	8.020%	8.310%	30.750%
33	21.180%	22.120%	22.810%	21.860%	21.980%	47.000%
34	25.330%	23.860%	17.900%	25.740%	24.330%	6.100%
35	4.460%	5.310%	3.750%	4.670%	6.120%	5.170%
Calm	12.277%	9.191%	13.610%	12.520%	12.254%	17.951%
Volatile	22.313%	19.122%	90.617%	23.262%	21.634%	24.218%

TABLE 15. MEAN DAILY ERROR - FRIDAY

This table contains the Mean Daily Error (MDE) on all Fridays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	3.290%	5.320%	2.660%	3.330%	4.110%	2.890%
2	2.810%	4.960%	4.520%	2.990%	5.440%	1.270%
3	4.290%	3.430%	8.280%	4.410%	3.610%	7.190%
4	13.340%	9.250%	16.880%	12.400%	12.330%	25.560%
5	7.290%	5.520%	7.980%	6.410%	5.610%	11.290%
6	9.350%	8.060%	9.290%	9.450%	10.470%	4.390%
7	6.600%	7.600%	6.110%	6.700%	6.170%	6.070%
8	8.460%	11.760%	3.130%	3.340%	10.630%	10.960%
9	23.530%	22.720%	20.500%	23.290%	22.470%	20.580%
10	5.170%	9.260%	6.870%	4.960%	4.600%	13.310%
11	24.110%	42.610%	42.410%	32.990%	25.100%	30.480%
12	17.270%	23.560%	90.850%	17.000%	18.670%	11.200%
13	28.030%	36.170%	29.060%	37.200%	28.130%	9.660%
14	13.080%	17.770%	18.190%	12.410%	15.030%	21.960%
15	5.170%	5.890%	5.860%	5.270%	6.330%	3.990%
16	17.320%	12.290%	13.010%	18.530%	14.930%	13.400%
17	17.090%	14.360%	11.450%	19.480%	14.830%	5.890%
18	14.160%	24.540%	6.740%	14.560%	11.620%	14.730%
19	20.790%	13.200%	19.940%	19.520%	16.790%	27.810%
20	4.280%	11.820%	10.780%	4.260%	9.660%	5.670%
21	16.150%	18.740%	14.010%	17.050%	16.230%	10.190%
22	21.400%	19.550%	13.820%	23.570%	20.500%	19.340%
23	26.980%	26.720%	20.630%	26.780%	29.630%	32.140%
24	10.080%	20.910%	13.880%	9.330%	15.960%	14.670%
25	27.420%	16.950%	23.330%	27.810%	25.370%	36.660%
26	7.740%	8.160%	8.050%	7.730%	7.500%	9.590%
27	14.040%	12.090%	6.090%	13.620%	13.930%	18.540%
28	7.990%	14.440%	7.000%	7.930%	7.740%	31.960%
29	6.820%	8.110%	7.980%	6.600%	6.370%	19.470%
30	13.400%	14.260%	7.320%	13.260%	12.520%	16.480%
31	13.350%	10.310%	17.650%	13.390%	13.820%	15.590%
32	5.090%	6.380%	10.030%	5.250%	6.450%	20.570%
33	6.920%	7.850%	8.520%	7.230%	7.140%	44.070%
34	6.400%	6.580%	14.120%	6.480%	8.390%	8.830%
35	9.560%	11.220%	4.280%	10.000%	11.680%	3.000%
Calm	8.413%	8.788%	8.622%	7.728%	8.544%	10.351%
Volatile	14.186%	16.179%	17.000%	15.090%	14.573%	17.836%

TABLE 16. MEAN DAILY ERROR - SATURDAY

This table contains the Mean Daily Error (MDE) on all Saturdays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	4.140%	4.900%	9.340%	4.140%	5.380%	8.720%
2	5.800%	5.860%	12.180%	6.090%	6.180%	14.410%
3	21.010%	22.510%	28.730%	20.610%	20.780%	27.770%
4	6.900%	6.300%	8.030%	7.720%	12.250%	22.650%
5	8.740%	7.810%	13.680%	7.950%	7.500%	5.380%
6	6.250%	4.520%	6.180%	6.200%	7.630%	29.420%
7	5.980%	5.560%	6.730%	5.960%	6.360%	14.930%
8	5.190%	5.770%	9.550%	4.960%	6.730%	9.640%
9	20.650%	18.310%	17.170%	20.670%	20.130%	37.220%
10	8.670%	5.920%	5.480%	8.770%	9.160%	32.840%
11	99.050%	60.960%	109.050%	72.650%	85.560%	39.250%
12	17.170%	11.070%	5.600%	15.710%	13.850%	14.410%
13	157.180%	105.800%	78.450%	120.890%	133.240%	13.770%
14	18.830%	17.440%	23.250%	21.710%	11.050%	84.470%
15	12.400%	13.040%	9.480%	12.470%	12.470%	35.650%
16	17.410%	8.960%	9.120%	15.540%	12.170%	41.830%
17	17.070%	24.870%	10.040%	19.650%	22.630%	33.240%
18	17.380%	12.190%	30.250%	15.960%	9.650%	20.260%
19	7.930%	8.620%	14.180%	8.170%	7.690%	30.900%
20	8.570%	13.260%	12.230%	8.430%	8.840%	14.690%
21	22.980%	26.770%	28.130%	24.250%	27.700%	40.440%
22	10.300%	7.840%	16.520%	11.190%	18.890%	229.290%
23	13.270%	19.810%	10.000%	12.280%	13.850%	37.000%
24	10.580%	12.700%	13.370%	10.240%	10.850%	31.590%
25	7.960%	11.650%	10.760%	8.300%	9.190%	67.300%
26	13.410%	11.910%	10.570%	13.390%	14.820%	16.610%
27	11.800%	8.340%	9.650%	11.780%	16.360%	11.150%
28	9.280%	6.570%	7.880%	9.420%	11.120%	8.580%
29	6.580%	10.830%	7.230%	6.780%	9.110%	23.570%
30	16.420%	10.180%	11.130%	16.520%	16.370%	11.500%
31	13.410%	8.070%	11.540%	13.490%	14.120%	11.740%
32	10.970%	7.870%	7.010%	11.310%	11.560%	13.340%
33	6.350%	9.610%	9.760%	7.200%	5.750%	18.900%
34	22.870%	26.090%	9.420%	23.800%	22.250%	6.880%
35	6.610%	5.790%	9.630%	7.030%	8.730%	5.140%
Calm	9.333%	8.746%	11.707%	9.307%	10.210%	20.298%
Volatile	22.231%	18.410%	18.970%	19.926%	21.113%	34.460%

TABLE 17. MEAN DAILY ERROR - SUNDAY

This table contains the Mean Daily Error (MDE) on all Sundays for each model. We also show the mean of the MDE for the calm (weeks 1-10) and volatile (weeks 11-35) periods. 'GAM' aims for the Generalised Additive Model, 'ANN' for the Artificial Neural Networks, 'Naive' the Naive method for week ahead forecast, 'Local Regression' for Local Regression Model, 'Tree' the Linear Regression Tree model and 'Linear' the ARX model.

Week	Linear	GAM	Local Regression	ANN	Tree	Naive
1	7.110%	7.850%	4.650%	7.180%	7.390%	10.110%
2	5.570%	5.190%	8.020%	5.620%	5.790%	18.420%
3	12.640%	11.800%	12.300%	12.700%	13.050%	19.230%
4	12.580%	13.640%	14.750%	13.530%	14.760%	25.310%
5	5.930%	6.800%	8.070%	5.600%	7.480%	12.190%
6	5.790%	5.780%	6.610%	5.790%	6.060%	19.030%
7	12.170%	11.410%	15.010%	12.310%	13.160%	29.030%
8	15.980%	15.750%	16.930%	15.650%	17.180%	13.700%
9	11.120%	9.560%	11.450%	11.540%	11.200%	34.180%
10	6.410%	10.460%	7.200%	6.420%	7.190%	35.550%
11	7.500%	14.120%	6.940%	6.850%	10.660%	17.010%
12	5.860%	6.890%	5.360%	6.260%	7.180%	7.460%
13	14.660%	15.670%	8.670%	12.210%	20.570%	9.650%
14	7.930%	10.290%	6.560%	7.730%	10.220%	40.680%
15	15.750%	18.460%	18.200%	15.890%	16.390%	38.140%
16	13.450%	16.630%	7.710%	14.300%	9.740%	36.310%
17	32.950%	37.550%	30.310%	35.380%	32.000%	50.630%
18	9.520%	11.020%	10.810%	8.230%	9.370%	51.360%
19	8.000%	4.870%	4.910%	8.480%	8.940%	21.700%
20	8.530%	13.650%	9.710%	8.700%	8.240%	15.550%
21	7.370%	13.150%	21.800%	8.440%	9.760%	34.180%
22	20.520%	18.520%	34.290%	19.540%	24.610%	123.950%
23	24.140%	25.600%	29.480%	24.140%	23.770%	35.940%
24	11.400%	15.880%	17.320%	11.550%	11.560%	16.430%
25	25.370%	23.030%	32.870%	25.010%	28.970%	44.760%
26	35.660%	32.880%	36.380%	35.580%	39.670%	30.750%
27	13.070%	11.480%	14.670%	13.190%	17.510%	11.600%
28	22.740%	12.470%	17.350%	23.130%	23.730%	10.900%
29	23.930%	14.680%	14.760%	24.250%	24.130%	11.010%
30	27.760%	20.150%	18.320%	28.190%	27.020%	17.040%
31	22.560%	15.590%	13.560%	22.840%	21.500%	15.870%
32	29.300%	23.790%	19.280%	29.730%	28.890%	17.550%
33	16.050%	10.750%	9.100%	17.290%	14.750%	6.180%
34	18.990%	14.060%	6.030%	20.530%	16.420%	2.650%
35	12.770%	9.850%	6.820%	13.400%	8.950%	4.080%
Calm	9.530%	9.824%	10.499%	9.634%	10.326%	21.675%
Volatile	17.431%	16.441%	16.048%	17.634%	18.182%	26.855%