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Direct measurement of diurnal polar motion by ring laser gyroscopes

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Abstract.

We report the first direct measurements of the very small effect of forced diurnal polar motion, successfully observed on three of our large ring lasers, which now measure the instantaneous direction of Earth's rotation axis to a precision of 1 part in 10^8 when averaged over a time interval of several hours. Ring laser gyroscopes provide a new viable technique for directly and continuously measuring the position of the instantaneous rotation axis of the Earth and the amplitudes of the Oppolzer modes. In contrast, the space geodetic techniques (VLBI, SLR, GPS, etc.) contain no information about the position of the instantaneous axis of rotation of the Earth, but are sensitive to the complete transformation matrix between the Earth-fixed and inertial reference frame. Further improvements of gyroscopes will provide a powerful new tool for studying the Earth's interior.

1. Diurnal Polar Motion

The Earth's angular velocity vector varies in time, both in direction and magnitude. Changes in magnitude correspond to a variation of the length of day (LOD) with respect to atomic clocks, amount up to a few milliseconds and consist mainly of various long period and seasonal components from solid Earth tides and the interaction with atmosphere and ocean and small diurnal and semi-diurnal terms from ocean tides.

The direction of the Earth's rotation axis also varies with respect to both space- and Earth-fixed reference systems. The principal component with respect to the Earth-fixed frame is the well-known Chandler wobble, with an amplitude of 4–6m at the poles and a period of about 432 days. This is a free mode of the Earth, i.e. it would still be present in the absence of the external gravitational forces. It is believed that the Chandler wobble would decay due to dissipative effects in the Earth's interior, were it not continually excited by seismic activity and by random noise of the atmosphere.

The Chandler wobble is overlaid by daily variations whose amplitudes are an order of magnitude smaller, some 40–60cm at the Earth's surface (c.f. figure 1) [*McClure*, 1973], [*Frede et al.*, 1999]. These Oppolzer terms arise from external torques due to the gravitational attraction of the Moon and Sun. Since the Earth is an oblate spheroid with an equatorial bulge which is inclined to the plane of the ecliptic, the net gravitational torque of the Moon and Sun on different parts of the Earth's surface does not exactly cancel out as it would if the Earth were a perfect sphere.

In an Earth-fixed reference system, such as that of a ring laser fixed to the Earth's surface, the forced retrograde diurnal polar motion is best viewed as a principal mode – the so-called "tilt-over mode" (K_1) – with the period of exactly one sidereal day (23.93447 hours), whose amplitude is modified as the angles and distances between the Earth, Moon and Sun vary over the course of their orbits. The complete spectrum of nutation modes can be understood as the beat frequencies of the tilt-over mode with frequencies corresponding to relevant orbital parameters: half a tropical month, half a tropical year, the frequency of perigee etc. The beat periods are clustered around one sidereal day. The O_1 and P_1 modes, with beat periods of 25.81934 and 24.06589 hours, have the largest amplitudes after the K_1 mode, and arise from the change in angle between the Earth's equatorial bulge and the Moon and Sun respectively. (See [*Moritz et al.*, 1987] for basic theoretical details.).

The spectrum of Oppolzer terms has the beauty that since it arises from external gravitational torques, the frequencies are known precisely but the amplitudes, which depend on the properties of the Earth such as its inertia tensor, elasticity and liquid core are not so precisely understood, and could potentially reveal much of geophysical interest. To date models of the Earth's interior have been built up which are broadly consistent with VLBI measurements of the instantaneous orientation of the Earth's rotation axis. However, direct routine measurements of the amplitude of the Oppolzer terms with ring lasers would open up a new field of geophysical studies. In this paper we report a first step in that direction.

2. The Effect of Polar Motion on Ring Laser Gyroscopes

Ring lasers measure absolute rotation [*Stedman*, 1997]. In the experimental set up two laser beams propagate in opposite directions around a closed path. If the instrument rotates the effective path-length is slightly shorter for the counter-rotating beam. In an active laser cavity, as is the case for our instruments, lasing is achieved when an integral number of wavelengths circumscribe the ring perimeter. Since the path length is slightly different for the co-rotating and the counter-rotating beams the lasing frequencies are

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also slightly different in each case and the beat frequency of the two laser beams, the Sagnac frequency is readily measurable. For an Earth-fixed rectangular cavity of perimeter length P, and area A, with normal $\mathbf{n}$ the Sagnac frequency is given by

$$\delta f = \frac{4A}{\lambda P}\,\mathbf{n}\cdot\mathbf{\Omega}, \tag{1}$$

where $\mathbf{\Omega}$ is the instantaneous angular velocity vector (referred to an inertial frame), and λ is the wavelength of the laser beams in the absence of rotation. The Sagnac frequency will vary with: (i) changes in the instantaneous position of the Earth's rotation axis and its angular velocity, which alter $\mathbf{\Omega}$; (ii) tilts from solid Earth tides and tidally induced ocean loading changes, which alter the local normal, $\mathbf{n}$; (iii) any changes to the scaling factor A/P, which could result from local thermal or pressure induced variations in the dimensions of the ring laser; and (iv) any changes in the refractive index seen by each beam in the laser cavity, as might result from changes in gas composition.

Although different mechanisms contribute to changes in δf, their individual effects can be separated. Firstly, unwanted effects from thermal expansion and the like can be minimised by suitably isolating the ring lasers and by building ring lasers with the largest possible ratio A/P to increase their sensitivity. Likewise variation in refractive index is an engineering design question and while problems such as tiny cavity leaks may give slow drifts in δf they would not have strongly periodic signatures. Finally, tilts from solid Earth and ocean tides can be readily distinguished from diurnal polar motion, by tiltmeter measurements on the ring laser.

Table 1 summarizes the performance of some of the largest existing ring lasers. The fourth column represents the experimentally obtained instrumental resolution (averaging time of 3 hours), while the last column shows the maximum amplitude of diurnal polar motion (DPM) as measured at each ring laser site. While the longitude of a ring laser location determines the phase of the polar motion signal, the latitude defines the projection of the rotation vector onto the ring laser normal and therefore determines the amount of the amplitude of the polar motion signal that is mapped into the ring laser measurements by equation 1.

Table 1. The sensitivity of some of our ring lasers and the theoretical maximum amplitude of the diurnal polar motion effect at these instruments.

Ring laser	Area m^2	f_{Sagnac} Hz	Gyroscope Sensitivity $\delta\Omega/\Omega$	DPM_{max} $\delta\Omega/\Omega$
C-II	1	79.4	$1\cdot 10^{-7}$	$1\cdot 10^{-7}$
G	16	348.6	$1\cdot 10^{-8}$	$9.8\cdot 10^{-8}$
UG1	367	1512.8	$3\cdot 10^{-8}$	$1\cdot 10^{-7}$

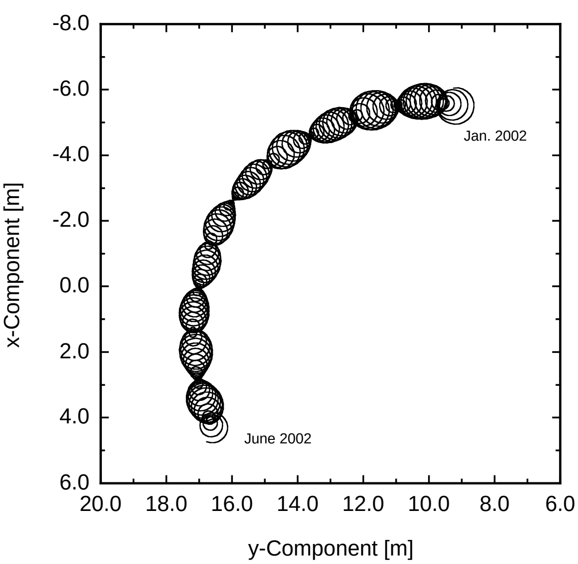

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Figure 1. The behaviour of the Earth's instantaneous axis in an Earth-fixed frame (w.r.t. the Conventional International Origin) in the period of January to June 2002. The large circle is the Chandler wobble. Superimposed are the smaller circles of the Oppolzer terms: diurnal variations whose dominant amplitude varies over half a month.

3. Details of a Diurnal Polar Motion Model

McClure carried out a detailed investigation of diurnal polar motion, using a purely elastic deformable Earth as the basic theoretical model [*McClure*, 1973]. The additional correction for liquid-core effects has been shown to produce no significant differences for the motion of the rotation axis [*Wahr*, 1981; *Brzeziński*, 1986]. Adopting body–fixed coordinates, the instantaneous position of the rotation axis is determined as a linear superposition of sinusoidal Oppolzer modes. By convention, the modes are expressed in terms of fundamental arguments corresponding to particular orbital parameters. Specifically, the longitude diurnal variation, $\Delta\varphi$, and the obliquity diurnal variation, $\Delta\epsilon$, are given respectively by

$$\Delta\varphi = \sum_i -A_i \sin(\phi_M + \sum_j N_{ij} F_j), \tag{2}$$

$$\Delta\epsilon = \sum_i A_i \cos(\phi_M + \sum_j N_{ij} F_i) \tag{3}$$

where ϕ_M is the Greenwich mean sidereal hour angle, A_i are amplitudes and N_{ij} are integer coefficients (as given in [*Brzeziński*, 1986]) which multiply the fundamental arguments like for example

$$\begin{aligned} F_1 \equiv l &= 134.^\circ 96340251 + 171791592.''2178t \\ &\quad +31.''8792t^2 + 0.''051635t^3, \end{aligned} \tag{4}$$

the mean anomaly of the Moon. Similar expressions are obtained for the mean anomaly of the Sun, the difference between the mean longitude of the Moon and the mean longitude of the ascending node of the lunar orbit, the mean elongation of the Moon from the Sun and finally the mean longitude of the ascending node of the Moon. In all these expressions, the time t is measured in Julian Centuries of 36525 days of 86400 seconds since J2000. Furthermore, the Greenwich mean sidereal hour angle ϕ_M relates to the Greenwich Mean Sidereal Time (GMST) as follows:

$$\phi_M = \mathrm{GMST}\frac{2\pi}{86400} + \pi + 2\pi d_u, \tag{5}$$

$$\begin{aligned} \mathrm{GMST} &= 24110^s.54841 + 8640184^s.812866T_u \\ &\quad +0^s.093104T_u^2 - 6^s.2\cdot 10^{-6}T_u^3, \end{aligned} \tag{6}$$

where $T_u = d_u/36525$, with d_u the number of days elapsed since 12h UT1 January 1, 2000. Finally, the longitude λ of each ring laser must be added to the Greenwich mean sidereal angle (5) to synchronise observations.

The tables given in [*McClure*, 1973],[*Brzeziński*, 1986] provide up to 160 values of diurnal polar motion amplitudes and coefficients. We will keep only the largest 6 coefficients, as all the remaining coefficients together contribute less than 1 milliarcsecond (mas) to the amplitude of polar motion and are not yet within the resolution obtained with a gyroscope. Calculations with these assumptions superimposed on the official International Earth Rotation Service (IERS) C04 polar motion series [*Dick and Richter*, 2002] were used to compute figure 1. This C04 series refers to the Celestial Intermediate Pole (CIP) [*Capitaine et al.*, 2002] defined to be free of diurnal polar motion terms, as opposed to the instantaneous axis of rotation. One can clearly see the diurnal circles of the rotation axis as it progresses through a larger cycle of the Chandler wobble. The resulting "latitude oscillations" have two main components with a periodicity of nearly 24 hours and approximately 14 days. The maximum amplitude represents 0.02 seconds of arc at the pole. We wish to point out that besides the here discussed forced diurnal polar motion there are many more effects that have subdaily or daily contributions to Earth rotation (e.g. from atmosphere, ocean). However at the current level of performance of our ring laser gyroscope they are not yet observable. This also applies for length of day variations.

4. Direct Observation of Polar Motion by Large Ring Lasers

The effect of Earth tides and ocean loading as small periodic variations has already been successfully identified in the time series of the Sagnac frequency of the small C-II ring laser [*Schreiber et al.*, 2003] located in an underground cavern in Christchurch (New Zealand). An unambiguous detection of polar motion has now been provided by our other ring lasers, G and UG1, the results from G being the most accurate and definitive. G is a semi-monolithic square HeNe ring laser of 16 m^2 area constructed from Zerodur and located in a thermally stable underground laboratory at the Fundamentalstation Wettzell (Germany) [*Schreiber et al.*, 2001]. The G ring laser was operated for most of the time in the year 2002. The most stable conditions ever obtained from any of our large ring lasers were met between July 24 and October 27, corresponding to the days 205 and 300 of the year 2002. The overall drift of the raw data over the entire time is as low as 2 mHz or expressed as an relative error $\Delta f/f < 6 \cdot 10^{-6}$ over 95 days, to our knowledge a world record for gyroscope stability.

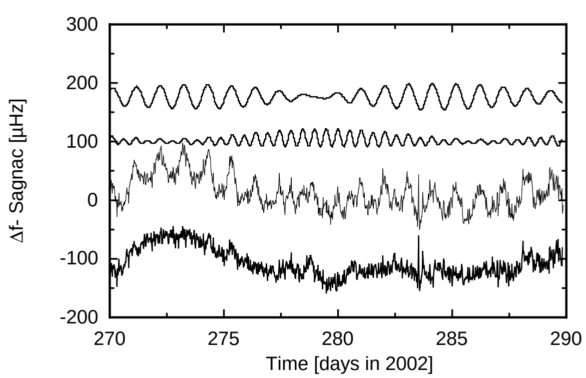

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Figure 2. Time series of a section of the Sagnac frequency taken by the G ring laser. The computed contributions of the Earth body tides and diurnal polar motion to the Sagnac frequency are shown offset by 100 μHz and 180 μHz respectively at the top. The resultant time series with both corrections applied is shown offset at the bottom.

A section of the observed time series – data taken between days 270 and 290 – is shown in figure 2. A constant value of 348.635 Hz has been subtracted from each measurement. In addition to the raw ring laser data one can find the corresponding tilt signatures from the body tides [*Agnew*, 1997] and the diurnal polar motion [*McClure*, 1973; *Brzeziński*, 1986], both converted to the respective variations of the Sagnac frequency according to equation 1 and offset by 100 and 180 μHz for better illustration in the diagram. The residual data set is displayed offset by -100 μHz after the corrections were applied and shows a good agreement between the measurements and the models adopted. What

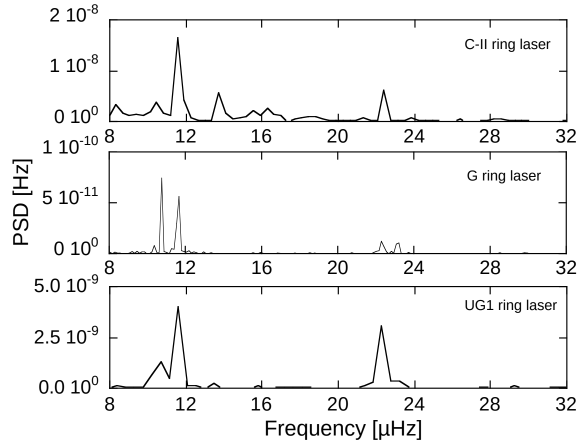

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Figure 3. Power spectra of ring laser time series from C-II, G and UG1. The expected contributions from solid Earth tides and contributions to polar motion from the K_1 and O_1 mode are clearly present in all measurements. The spectrum of G reveals the most details because the data set is much longer and the instrument is mostly free from backscatter and drift.

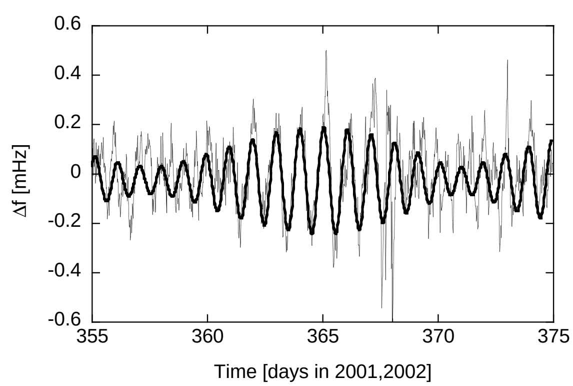

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Figure 4. An example of a time series of a measurement of diurnal polar motion carried out on the UG1 ring laser. Because of slow instrumental drift the data were band-pass filtered. The computed theoretical polar motion signal was treated similarly and is superimposed on the Sagnac frequency measurements. The data set was also corrected for Earth tides and ocean loading.

remains is the signature from instrumental drift and some transient pertubations of still unknown geophysical origin.

We have computed power spectra of ring laser measurements from various available time series of different lengths of 3 different ring lasers, C-II, G and UG1. Figure 3 shows the result. There are 2 groups of signals evident in the spectrum of the G ring, one at a period around 12 hours 25 minutes and one at a period of around one day. In the latter group we can identify the K_1 and O_1 mode contributions to diurnal polar motion. Their periods of 23.9345 and 25.8195 hours [*Frede et al.*, 1999] correspond to frequencies of 11.606 and 10.758 μHz. Our measurements yield frequencies of 11.665 and 10.754 μHz, which agree well with theory within the spectral measurement resolution of 129 nHz. The P_1 mode also has an expected amplitude well within current resolution, but since its period is very close to that of K_1 longer time series are required to separate out its peak in the spectrum.

The group of higher frequencies is due to variations of the orientation of the ring laser on the Earth surface induced by solid Earth tides. Again the frequencies of 23.204 and 22.293 μHz are within the resolution of the expected S_2 and M_2 tides signals. The Earth tides contribute little to the diurnal terms, because north directed diurnal tidal tilts are close to zero in mid-latitudes.

The best available time series of UG1 and C-II are much shorter. Therefore the resolution of their spectra is not so high. In addition, C-II shows a significant masking of the diurnal polar motion signal, probably due to backscatter induced frequency pulling as an instrumental artefact [*Schreiber et al.*, 1998]. Furthermore, the power spectral densities obtained for daily polar motion and solid Earth tides as measured with the C-II ring laser are about one order of magnitude larger than expected. By contrast, for UG1 and G this is not the case. We believe that this is related to the backscatter induced pulling of the Sagnac frequency which is strong in C-II but negligible for UG1 and G. Due to long timescale instrumental drift frequencies below 5 μHz (G) or 8 μHz (UG1) were fully cut off by the application of an 8th degree Butterworth high pass-filter. However, one can clearly identify diurnal polar motion signals and solid Earth tides in all three instruments. The tidal signal is stronger at Christchurch due to local ocean loading on Banks Peninsula.

The larger a ring laser in a given location is, the higher is its sensitivity to such small signals. Figure 4 shows a time series from the 367 m^2 UG1 which we believe is the largest working ring laser gyroscope in the world [*Dunn et al.*, 2002]. It dramatically shows one of the advantages of ring lasers, namely the very high time resolution of the measurements, making this technique very promising for the study of subdaily variations in Earth rotation. In addition continuous observations are possible over long time intervals. In contrast gyroscopes are local sensors and the data may contain contributions from local effects which are not yet understood. Indications for this are apparent in figure 4 as well.

Since the raw data of the UG1 ring laser shows far more drift than the G ring laser, the data were band-pass filtered with cut-off frequencies at 5 and 40 μHz. The computed polar motion signal was treated in the same way in order to avoid artefacts from phase shifts caused by the filter process, and is superimposed on the measurement data in figure 4. The data set has been corrected for Earth tides and ocean loading [*Agnew*, 1997]. Local disruptions of the ring laser setup have not been reduced from the data set. This plot shows the impressive sensitivity of modern large ring laser gyroscopes applied to the field of geophysics.

5. Summary

The sensitivity and performance of large ring lasers has improved significantly in recent years with the construction of the C-II, G0, G and UG1 gyroscopes. Today it is possible to measure variations in the location of the rotational pole of the Earth to within a few centimetres. As our instruments progressively improve in stability we expect evidence of further geophysical signals with periods of well over a day in the future. Since both our ring laser laboratories in Germany and New Zealand are nearly at antipodal points we unfortunately cannot yet distinguish the different effects of the components $\Delta\varphi$ and $\Delta\epsilon$ of the polar motion. This will require at least one additional high quality ring laser site displaced 90 degrees in longitude from our current laboratories.

It is important to note that over the past 30 years the theoretical model of forced diurnal polar motion has been developed and is used to reduce these contributions from VLBI measurements. There are still some uncertainties remaining, since the theoretical models use some simplifications to account for a deformable Earth. Large ring laser gyroscopes provide a new and independent technology which can measure the amplitudes and frequencies of the forced modes of Earth rotation directly and monitor polar motion to unprecedented high temporal resolution. This effectively provides a direct probe of the Earth's moment of inertia. Therefore we expect that future improvements in ring laser technology will lead to quantitative improvements in the nutation models themselves, and provide a new tool for studying aspects of the Earth's interior.

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