Reciprocity and the Prediction of the Apparent Sound Reduction Index for Lightweight Structures According to EN12354

J. Mahn and J. Pearse

University of Canterbury, Department of Mechanical Engineering, Private Bag 4800, 8140 Christchurch, New Zealand
jma251@student.canterbury.ac.nz

In the derivation of the equation for the flanking sound reduction index for EN12354-1, reciprocity was used to remove the radiation efficiencies of the elements from the calculations. The exclusion of the radiation efficiencies was beneficial since they are often not known. However, the assumption of reciprocity may only be applicable to the heavy monolithic constructions for which EN12354-1 was intended. If EN12354-1 is applied to lightweight constructions which may exhibit direction dependent sound transmission, the assumption of reciprocity may contribute to an over prediction of the apparent sound reduction. A correction factor is proposed to correct for the over prediction due to the assumption of reciprocity. However, due to the inclusion of the radiation efficiencies in the correction factor, its use is limited. Alternatively, the correction factor may be used to estimate the error due to the assumption of reciprocity so that the accuracy of the EN12354-1 predictions can be assessed for lightweight constructions.

1 Introduction

In the development of the prediction method that would become the standard EN12354-1, the flanking transmission loss between two building elements was defined to be [1]:

$$\tau_{ij} = \frac{\tau_i d_{ij} \sigma_j S_j}{\sigma_i S_o} \quad (1)$$

where τ_{ij} is the calculated flanking transmission loss between elements *i* and *j*, τ_i is the *in-situ* resonant transmission loss of element *i*, d_{ij} is the *in-situ* vibration transmission factor between elements *i* and *j*, S_j is the area of element *j*, S_o is a reference area and σ_i and σ_j are the *in-situ* resonant radiation efficiencies of elements *i* and *j*, respectively. Likewise, the flanking transmission loss in the opposite direction between *j* and *i* was defined as:

$$\tau_{ji} = \frac{\tau_j d_{ji} \sigma_i S_i}{\sigma_j S_o} \quad (2)$$

Of the terms in Eq.(1) and Eq.(2), the resonant radiation efficiencies of the elements are often the least known [2]. The values are often not readily available [3] and can be determined correctly only if the velocity amplitudes and the radiation efficiencies of all participating modes are known [4]. Reciprocity between the calculated flanking transmission loss terms in each direction was used to exclude the radiation efficiency terms from the calculations such that [5]:

$$\tau_{ij,EN12354} = \sqrt{\tau_{ij}\tau_{ji}} = \sqrt{\frac{\tau_i \tau_j d_{ij} d_{ji} S_i S_j}{S_o^2}} \quad (3)$$

where $\tau_{ij,EN12354}$ is the estimation of the measurand $\hat{\tau}_{ij}$ according to EN12354-1. Eq.(3) was written under the assumption that $\tau_{ij} = \tau_{ji}$, even for lightweight building elements [6] which may have critical frequencies above the frequency range of interest. Other authors [5, 7] have suggested that the assumption of reciprocity may be acceptable for the heavy monolithic constructions for which EN12354-1 was intended, but may not be applicable to lightweight building constructions. The possible lack of reciprocity for lightweight structures is due in part to the vibration transmission factors which may differ significantly in each transmission direction for lightweight building elements [8].

2 Best Estimate of the Flanking Transmission Loss

It is assumed that τ_{ij} and τ_{ji} represent two independent observations of the possible values of $\hat{\tau}_{ij}$. The true value T_{ij} can not be known but the best estimate of the true value is an average of the observations such that [9]:

$$\tau_{ij,Proposed} = \frac{\tau_{ij}+\tau_{ji}}{2} = \frac{\tau_i d_{ij}\sigma_j^2 S_j + \tau_j d_{ji}\sigma_i^2 S_i}{2\sigma_i\sigma_j S_o} \quad (4)$$

The estimate $\tau_{ij,Proposed}$ does not assume that $\tau_{ij} = \tau_{ji}$ and therefore may be a better estimate of the measurand than $\tau_{ij,EN12354}$ when the flanking transmission loss is direction dependant as may be the case for lightweight constructions.

A comparison between $\tau_{ij,Proposed}$ and $\tau_{ij,EN12354}$ indicates that $\tau_{ij,EN12354}$ may under predict the measurand when $\tau_{ij} \neq \tau_{ji}$. This results in an over prediction of the flanking sound reduction index as shown in Fig.1.

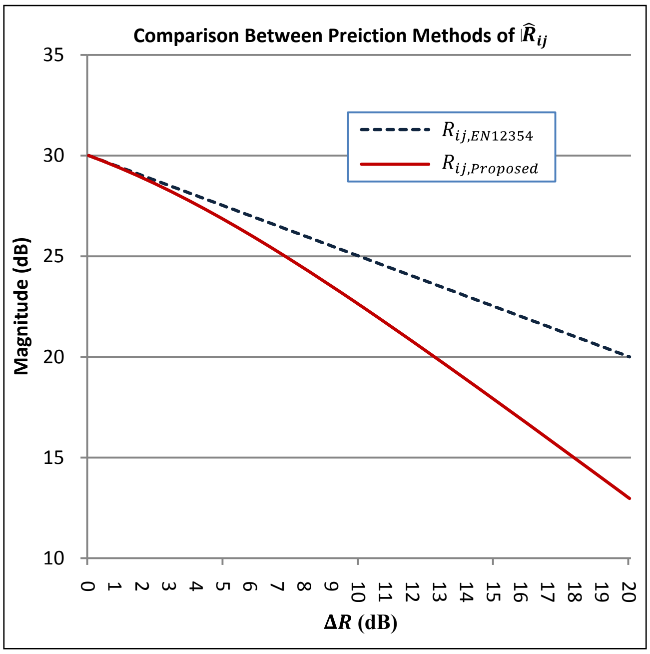

Fig.1 Comparison between $R_{ij,Proposed}$ and $R_{ij,EN12354}$ where $R_{ij} = -10\log\tau_{ij}$ and $\Delta R = 10log\left(\frac{\tau_{ji}}{\tau_{ij}}\right)$. The initial value of the calculations was set to $R_{ij} = R_{ji} = 30$dB.

The error between the predictions of the sound reduction index may be written as:

$$\varepsilon_{R_{ij}} = R_{ij,EN12354} - R_{ij,Proposed} \quad (5)$$

Fig.1 shows that $R_{ij,EN12354}$ and $R_{ij,Proposed}$ are in agreement when $\Delta R = 0$, but as ΔR increases, the magnitude of the error $\varepsilon_{R_{ij}}$ also increases. The flanking transmission loss of each of the flanking paths are summed for the calculation of the apparent sound reduction index R' [10]. For two rooms separated by a common wall, there are 12 possible first-order flanking paths [11] and the error in

R' due to the assumption of reciprocity includes contributions from each of the flanking paths.

For example, the values of R_{ij}, R_{ji}, $R_{ij,Proposed}$, and $R_{ij,EN12354}$ were calculated for two elements made of double leaf gypsum board glued and screwed to steel studs. The elements were connected by a corner junction and the vibration transmission factors were measured according to the standard, EN10848-1. The resonant sound reduction index of the elements was calculated according to the theory of Leppington [12] which was shown to give good results in one study [13]. Since the elements were identical, it was assumed that $\sigma_i = \sigma_j$ and therefore the radiation efficiency terms were excluded from the calculations. The values of R_{ij}, R_{ji}, $R_{ij,Proposed}$, and $R_{ij,EN12354}$ are compared in Fig.2.

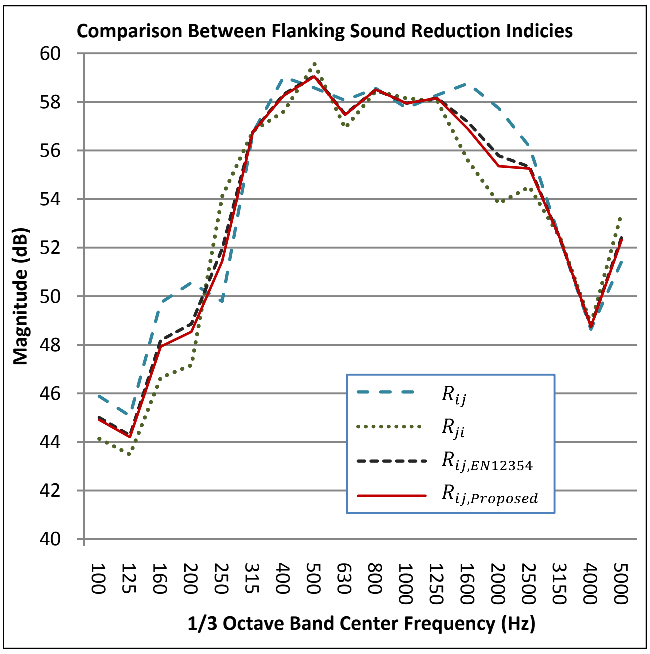

Fig.2 Comparison between R_{ij}, R_{ji}, $R_{ij,Proposed}$, and $R_{ij,EN12354}$ for a lightweight, double-leaf constructions of gypsum board on metal studs. The elements both had critical frequencies in the 4000 Hz 1/3 octave band.

The figure shows that the values of R_{ij} and R_{ji} differed by as much as 4.3 dB in the 2000 Hz 1/3 octave band resulting in an error of $\varepsilon_{R_{ij}}$ = 0.5 dB. If each of the 12 possible flanking paths showed similar errors, the over estimation of R' due to the assumption of reciprocity would be 0.5 dB.

3 Proposed Correction Factor

For convenience, Eq.(4) may be rewritten in terms of a correction factor which may be applied to $R_{ij,EN12354}$ such that:

$$R_{ij,Proposed} = R_{ij,EN12354} - 5\log C_R \qquad (6)$$

The correction factor C_R is defined as:

$$C_R = \frac{1}{4}\left[Q + \frac{1}{Q} + 2\right] \qquad (7)$$

where:

$$Q = 10^{\left(\frac{R_{j,situ} - R_{i,situ}}{10}\right)} 10^{\left(\frac{D_{v,ji,situ} - D_{v,ij,situ}}{10}\right)} \left[\frac{\sigma_{j,situ}^2}{\sigma_{i,situ}^2}\right]\left[\frac{S_j}{S_i}\right] \qquad (8)$$

The correction factor includes the radiation efficiencies and therefore may be difficult to apply in practice. If the radiation efficiencies of the elements are predicted theoretically using the equations in Annex B of EN12354-1, for example, the corresponding uncertainty of the predictions may exceed the uncertainty due to the assumption of reciprocity. Therefore, the correction factor is best applied in cases where the radiation efficiencies are known, in cases where the radiation efficiencies of the elements along the flanking path are believed to be similar (as could be the case if the walls of the rooms were made of identical constructions), or in cases where the frequency is above the critical frequency and therefore the radiation efficiencies tend to a value of 1 [4]. The correction factor may also be used to estimate the error, $\varepsilon_{R_{ij}}$ in cases where the difference between d_{ij} and d_{ji} is large compared to the difference between the predicted radiation efficiencies and transmission losses of the elements as may be the case with lightweight constructions.

4 Evaluation of Assumptions

The assumption according to EN12354-1 that τ_{ij} and τ_{ji} represent observations of the same distribution may be evaluated using a one-way analysis of variance (ANOVA) on the data [14]. The one-way ANOVA requires sets of data for each observation, but in practice, τ_{ij} and τ_{ji} each represent single, calculated values which are insufficient for the analysis. However, if the uncertainty and the probability density function (PDF) of τ_{ij} and τ_{ji} are known, repeat observations of each value may be produced by sampling from the PDF describing the calculated values using a pseudo-random number generator that is appropriate for the PDF [15].

The uncertainty of the τ_{ij} term may be calculated from the uncertainty of the inputs of Eq.(1) following the guidelines of GUM [16] such that:

$$u^2(\tau_{ij}) = \tau_{ij}^2\left[\frac{u^2(\tau_i)}{\tau_i^2} + \frac{u^2(d_{ij})}{d_{ij}^2} + \frac{u^2(\sigma_i)}{\sigma_i^2} + \frac{u^2(\sigma_j)}{\sigma_j^2} + \frac{u^2(S_j)}{S_j^2} + \frac{u^2(S_o)}{S_o^2}\right] \qquad (9)$$

where $u^2(x)$ represents the squared uncertainty of term x. A similar equation may be written for $u^2(\tau_{ji})$. Eq.(9) includes the uncertainty of the resonant transmission loss and the resonant radiation efficiencies, both of which may be calculated theoretically rather than experimentally. The uncertainty of the theoretical calculations is dependent upon the uncertainty of the inputs values which may not be known and upon the bias from the true value of the measurand which can not be known.

Alternatively, GUM allows for the estimation of the Type B uncertainty from previous measurement data, experience or general knowledge [16]. If it is assumed that the theoretical predictions of the resonant components of the transmission losses and the radiation efficiencies have similar uncertainty to the measured values, then the uncertainty may be estimated from experience or from standards such as ISO140 [17] which provides a standard deviation of reproducibility for the measurement of the sound reduction index.

The Central Limit Theorem suggests and Monte Carlo simulations have confirmed that the flanking transmission loss has a Gaussian PDF. The Hill-Wichmann algorithm

[18, 19], for example may be used to generate random observations from the Gaussian distribution based on the uncertainty and the mean of τ_{ij} and τ_{ji}. Since the simulated observations involve random values, the results are subject to statistical fluctuations [20]. Therefore, the simulation requires an adequate number of observations which may be estimated by comparing the mean of the calculated value to that of the estimated mean of the simulated values. A maximum allowable error is set and the number of trials is then increased until the difference between the calculated and the simulated mean is within the allowable error.

For example, the distribution of τ_{ij} and τ_{ji} of the double-leaf constructions of gypsum board on metal studs shown in Fig.2 were simulated using up to 500,000 observations to achieve a maximum of 1% error in the mean. The hypothesis that τ_{ij} and τ_{ji} are observations from the same normal population was rejected at the 95% confidence level in thirteen of the eighteen 1/3 octave bands evaluated by EN12354-1. The 1/3 octave bands where the hypothesis was accepted included the 315, 800, 1250, 3150 and 4000 Hz 1/3 octave bands which can be seen in Fig.2 to be bands where the values of R_{ij} and R_{ji} were almost identical and where the error $\varepsilon_{R_{ij}}$ was negligible. The rejection of the hypothesis in the remainder of the 1/3 octave bands indicates that the assumption of reciprocity did not hold and therefore suggests that $\tau_{ij,Proposed}$ may be a better estimator of the measurand than $\tau_{ij,EN12354}$.

5 Uncertainty of Estimates

The uncertainty of $\tau_{ij,EN12354}$ is calculated using the procedure outlined in GUM to be:

$$u^2(\tau_{ij,EN12354}) = \frac{\tau_{ji}^2 u^2(\tau_{ij}) + \tau_{ij}^2 u^2(\tau_{ji})}{4\tau_{ij,EN12354}^2} \quad (10)$$

and the combined uncertainty of $\tau_{ij,Proposed}$ is:

$$u^2(\tau_{ij,Proposed}) = \frac{(\tau_{ij} - \tau_{ji})^2 + u^2(\tau_{ij}) + u^2(\tau_{ji})}{4} \quad (11)$$

Histograms of the estimates are compared for the gypsum board on metal studs construction in Fig.3.

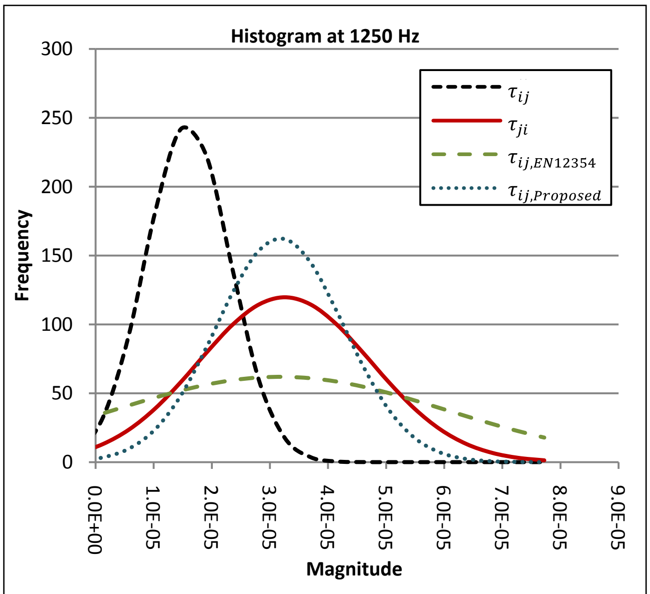

Fig.3 Comparison between the histograms of τ_{ij}, τ_{ji}, $\tau_{ij,Proposed}$ and $\tau_{ij,EN12354}$.

The histogram of $\tau_{ij,EN12354}$ is shown to be broader and shorter than that of $\tau_{ij,Proposed}$, indicating that in this case, $\tau_{ij,EN12354}$ had greater uncertainty than $\tau_{ij,Proposed}$. However, since Eq.(11) also includes the Type A uncertainty due to averaging the τ_{ij} and τ_{ji} terms, the uncertainty of $\tau_{ij,Proposed}$ may exceed that of $\tau_{ij,EN12354}$ if the difference between τ_{ij} and τ_{ji} becomes large. The uncertainty of $\tau_{ij,EN12354}$ depends only on the uncertainty of τ_{ij} and τ_{ji} and remains the same regardless of the magnitude of the difference between τ_{ij} and τ_{ji}. Therefore Eq.(11) may give a better estimation of the uncertainty.

6 Discussion

While the proposed correction factor can be used to correct for the underestimation of $\tau_{ij,EN12354}$ when $\tau_{ij} \neq \tau_{ji}$, the correction does not address the validity of the method when the assumption of reciprocity does not hold. It has been shown [11, 21, 22] that the equations of EN12354-1 may be equated to a first order Statistical Energy Analysis (SEA) model where there is only one junction and where only bending waves are considered. Therefore, the method of EN12354-1 is subject to the same restrictions as SEA [23]. For SEA models, if reciprocity does not hold then the coupling loss factors can be negative and power can flow from a subsystem with low energy to one with high energy which is against the spirit of SEA [24]. Therefore, the validity of using EN12354-1 may be in question for systems where reciprocity does not hold.

The proposed correction to the flanking sound reduction index is itself only an estimate since the true value of the flanking sound reduction index is unknown. The proposed estimate $\tau_{ij,Proposed}$ lacks the elegance of $\tau_{ij,EN12354}$ since it includes the radiation efficiencies of the elements, the magnitude and uncertainties of which may not be known. However, even if the correction factor is not applied to the EN12354-1 predictions, it may still be used to estimate the error $\varepsilon_{R_{ij}}$ in the predictions due to the assumption of reciprocity. If the error is large, then the accuracy of applying the EN12354-1 prediction method may be questionable.

7 Conclusions

The EN12354-1 method for predicting the flanking sound reduction index of each flanking path may over estimate the measurand if the assumption of reciprocity does not hold. The contributions from each of the flanking paths are summed to predict the apparent sound reduction index, resulting in a possible overestimation in the predictions made according to EN12354-1. A correction factor for the flanking sound reduction index of EN12354-1 has been proposed to correct for the deviation from reciprocity, especially in the case of predictions for lightweight constructions. However, the correction factor includes the radiation efficiencies, the values of which may be unknown. If the values of the radiation efficiencies are predicted theoretically, the uncertainty of the estimated values may be greater than the uncertainty due to the assumption of reciprocity. Therefore, the correction factor is suggested for use in cases when the radiation efficiencies are known, when the radiation efficiencies of the elements may be assumed to be identical or when the difference between the ratio of the radiation efficiencies and the transmission loss is small when compared to the ratio of the vibration transmission factors.

Even if the value of the radiation efficiencies are estimated, the error between the proposed best estimate of the flanking sound reduction index and the estimate according to EN12354-1 may be used to estimate the error due to the assumption of reciprocity. However, the correction does not address the validity of applying the EN12354-1 method when the assumption of reciprocity does not hold.

Acknowledgments

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