A REVIEW OF THE HISTORY, THEORY AND OBSERVATIONS OF GRAVITATIONAL MICROLENSING UP UNTIL THE PRESENT DAY

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ABSTRACT

Gravitational microlensing has become almost a household term in the world of astronomy and astrophysics. It has derived fame from being the only technique yet developed that is capable of the detection of Earth-sized planets beyond the Solar System, the first of which was confirmed as discovered only as recently as 2004. It is, however, a concept which dates back a lot further than the modern day. Its roots may be found in the writings of Newton in the early 1700's. It was further developed by Michell, Laplace and Soldner, and reintroduced by Einstein who derived the same findings from first principles. After this period, however, the concepts of lensing due to the effect of gravity became almost forgotten, experiencing another awakening during the 1960's, but generally left undeveloped. It was not until the 1990’s that the subject began to grow. After macrolensing events were detected in 1979, it gradually became apparent that microlensing events could be detected as well. Today, various techniques of analyzing microlensing events have been introduced which can reveal, not just the existence of planets, but the shape and atmospheric conditions of individual and binary stars, brown dwarfs, and a variety of other elusive measurements.

The history of Gravitational Lensing also serves as a reminder that it is often naive to acclaim a single scientist or researcher as initiating a field of science,

but rather the progression and development of concepts is evolutionary, with each step resulting from previous works and influenced by colleagues of the day.

CHAPTER 1

The Early History of Lens Theory

Conventionally the original concept of gravitational lenses has been credited to the work of Albert Einstein in his work of 1911 when he used Newtonian mechanics to derive the deflection of light by a massive body. It is clear, however, that the roots of the theory begin at least 200 years prior to this, in the famous work ‘Optiks’ by Isaac Newton, and was developed in several forms after this date.

1.1 Newtonian Mechanics and the Deflection of Light

The first written account of an effect by mass on the path of light appeared in Isaac Newton's 3rd book of his famous work 'Optiks', published between 1704 and 1720. At the end of his third book, he poses 31 queries to the Reader as observations he had made but never pursued himself, with the intent that other scientists would consider them. Query number 1 asks:

> 'Do not Bodies act upon light at a distance, and by the action bend its Rays; and is not this action strongest at the least distance?' (Newton, 1717)

Newton's error in his queries on this topic was to invert the relationship between lightspeed and refractive ability, in that it was his presumption that light had an affinity for areas of rarer rather than denser medium, and hence light traveled at a greater velocity through more massive bodies. This led to his assertion that his ‘aether’ was displaced by gravity, rather than attracted:

> Qu 21:'Is not this medium much rarer within the dense Bodies of the Sun, Stars, Planets and Comets, than in the empty Celestial spaces between them?
>
> And in passing from them to great distances, doth it not grow denser and denser perpetually and thereby cause the gravity of those great Bodies towards one another, and of their parts toward the Bodies, every Body endeavouring to go from the denser parts of the Medium toward the rarer?…
>
> …and though this increase of density may at great distances be exceedingly slow, yet if the elastick force of this medium be exceedingly great, it may be suffice to impel Bodies from the denser parts of the medium towards the rarer, with all that power which we call gravity…' (Newton, 1717)

This is sometimes discussed in terms of Newton's Aether Model (Baird, 2000), which was an attempt to unify descriptions of light, mass and gravity. He did this by describing both the action of mass on light and the effect of gravity on light as effects of variation in lightspeed related to the density of the aether; a permeating medium throughout space. This would seem to be a three-dimensional precursor to the modern four-dimensional curved-space model of the Universe, introduced by Einstein. Newton did not attempt the calculations of the deflection of light by astronomical means, but rather left the concept in the form of a query, to be left to future researchers. He was more interested in the day-to-day nature of light able to be observed in laboratory-based experiments. His assertions were also based, understandably, on the speed of light *c* being constant relative to the emitter. Newton further proposed 'light-corpuscles'. According to his theory, particles and waves would be distorted in similar ways, and matter could therefore be described as being 'refracted' by the lightspeed gradient associated with gravity.

> Qu 29: ‘Are not the rays of light very small bodies emitted from shining substances?’ (Newton, 1717)

If Newton had revised the concept of the influence of aether density on the passage of light, then the finished theory of lensing by mass and gravity may have been proposed a long time before the currently accepted origins. The adjustment would have been a simple matter to perform, in order to align his own assertions with Huyghens’ principle. Instead of taking this course, in a manner people had come to expect from Isaac Newton, he and his supporters declared Huyghens to be in error. This led to a period of ill-feeling between theorists asserting the wave or particle natures of light, the culmination of which was reached when the speed of light measured in glass and in gas could be experimentally determined and compared. It was determined that light traveled at a greater velocity through gas than through glass. Newton's theory was found wanting, and Huygens was vindicated.

The ongoing rivalry that existed between the groups may have inhibited the progression of a modern theory of gravity and light. Inverting the relationship in Newton's theory could have led to earlier derivations of many relationships since determined using general relativity, for example the gravitational redshift and the Shapiro Effect.

From Newton's work, albeit flawed in this respect, John Michell published a paper in 1784 discussing several modern ideas (Crossley, 2000). John Michell is commonly referred to as the Father of Seismology, and was both a geologist and an astronomer. Michell determined the amount a given particle would slow in its journey from the surface of a star to the Earth, assuming all the laws of Newtonian Mechanics. He takes this concept a step further by applying the same laws to light, by treating light as being composed of ballistic particles:

> ‘Let us now suppose the particles of light to be attracted in the same manner as all other bodies with which we are acquainted; that is, by forces bearing the same proportion to their vis inertiae (or mass), of which there can be no reasonable doubt, gravitation being, as far as we know, or having any reason to believe, an universal law of nature.' (Michell, 1784)

This approach was applied, and led to the suggestion of dark stars – stars so massive that light itself is unable to travel fast enough to escape its surface. He also proposed that it would be possible to detect invisible bodies around which other stars orbit by looking for apparent irregularities in the revolving body (Will, 1987).

> 'Hence… if the semi-diameter of a sphere of the same density with the Sun were to exceed that of the Sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.' (Michell, 1784)

Henry Cavendish, to whom the above reference was addressed, was inspired to make a calculation of the deflection of light, assuming light's corpuscular nature and Newton's theories of gravity:

> 'To find the bending of a ray of light which passes near the surface of any body by the attraction of that body
> Let s be the center of a body and 'a' a point of surface
> Let the velocity of a body revolving in a circle at a distance as from the body be to the velocity of light as 1:u, then will the sine of half bending of the ray be equal to $1/(1+u^2)$'

Which was never published but was rediscovered in 1919 on an 'isolated scrap of paper' (Schneider, 1987, Will, 1987).

Laplace, apparently independently, worked on the same problem with the same starting assumptions, and again regarding light as corpuscular (Laplace, 1799). Laplace is well-known for his belief in scientific determinism. In his variations of ‘Exposition du Systeme du Monde’ he puts forward the following proof:

> ‘A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible."

Later that century Johann Georg von Soldner responded to the findings of Laplace. Soldner found Laplace’s mathematics to be sound, but could not himself agree on the constancy of the speed of light. He subsequently took the concept further and developed a theory of the deviation of light around massive bodies, to be submitted in 1801 and published in 1804 (Ueber die Ablenkung eines Lichtstrals von seiner geradlinigen Bewegung, durch die Attraktion eines Weltkörpers, an welchem er nahe vorbei geht) (Soldner and Idaczyk, 1804). These theories were based on basic Newtonian mechanics and considered light as composed of ballistic particles traveling at a speed of $3\text{x}10^5$ km/s. Soldner found that if one considers very small deflection angles in the contemplation of the orbit of a body with constant velocity passing near a spherical mass, the value for the deflection could be approximated by equation 7 in line with the findings of Cavendish. He went on to assert that light traveling close to the Sun would be deflected by 0.87 arcsec. Soldner looked quite carefully at the impact that these findings could have on practical astronomy.

$$[1]\ \Theta = \frac{2Gm}{c^2 R}$$

Of interest is Soldner’s friendship with Fraunhofer. Fraunhofer equipped the Munich Observatory with telescopes after Soldner’s appointment as director. These telescopes were of a quality high by the standards of the 1830’s in that they were able to resolve stellar parallaxes substantially smaller than 1’. They were therefore adequate to test Soldner’s predictions, but were never used for that purpose (Will, 1987).

In 1801 the concept of light as being composed of ballistic particles had been overturned by Thomas Young, who effectively demonstrated the wave nature of light. As a result, Newton and Michell's work in this arena was discredited, and it was not until the early 1900's with the introduction of quantum theory, that light particles or photons became scientifically respectable again. Additionally, American physicists Michelson and Morley disproved existence of a pervading aether in 1887.

It was interesting that these theories were developed before the introduction of General Relativity more than a century later yet remained untested, although the equipment and observational capabilities of the time were able to prove or disprove these theories.

The concept of the deflection of light was not again visited until the year 1911 by Einstein in the paper 'On the Influence of Gravity on the Propagation of Light' (Einstein, 1911). He based his working on Special Relativity, introduced in 1905, with the principle of equivalence, as General Relativity was not yet fully developed at this time. Although apparently unaware of Soldner's earlier work, he derived the same value for the deflection of light rays passing near the Sun. The formula he used was:

$$[2]\ \alpha = \frac{2GM_{sun}}{c^2 R_{sun}} = 0.87 arc\sec$$

Where M is the mass of the Sun, and R the radius, c is the speed of light, and G the gravitational constant.

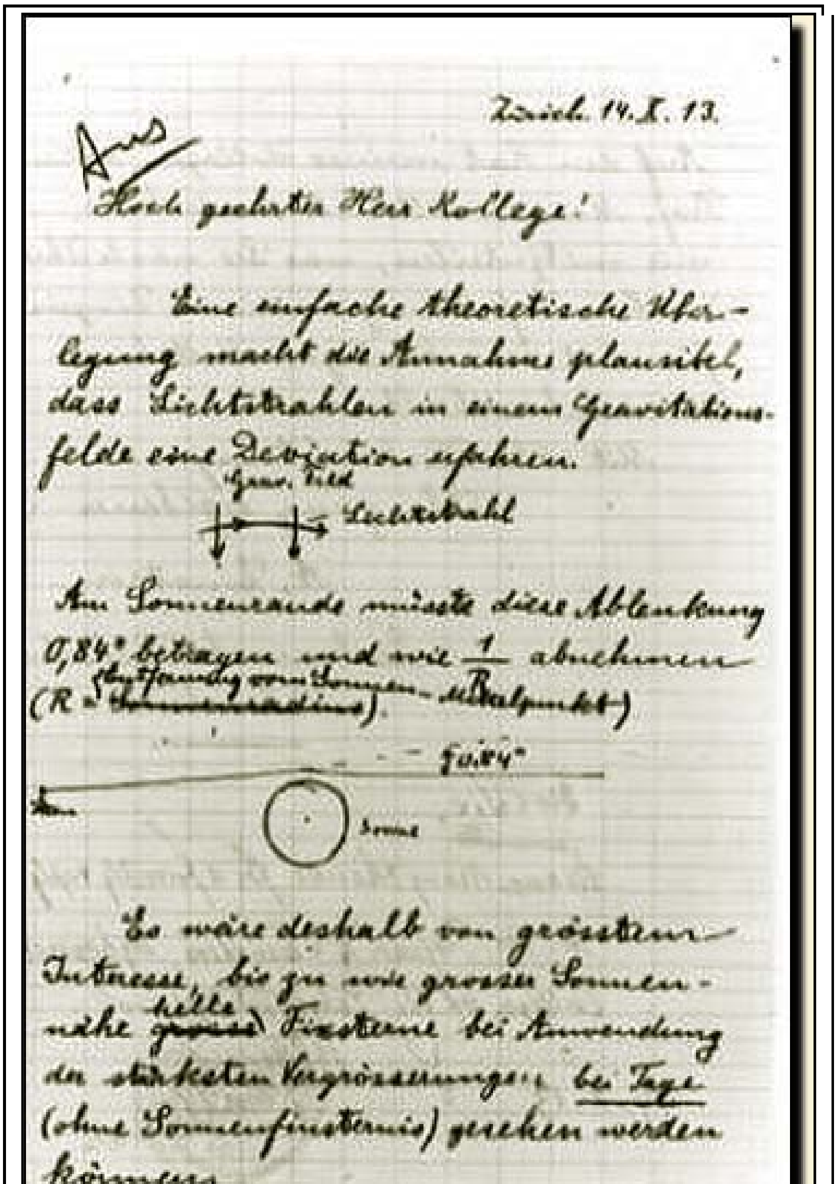
In this publication Einstein urged astronomers to think about the question raised by the theory (Renn and Sauer, 2000).

> 'It would be urgently desirable that Astronomers take up the question here, even if the considerations presented above may appear insufficiently founded or even adventurous'

Einstein spent much effort in his attempt to convince Astronomers to test the astrophysical predictions resulting from his publication, and in 1912 visited

Berlin to support his case. He met there with Freundlich, who was particularly interested in testing Einstein's theory during the eclipse of the Sun visible from the Russian Crimea Peninsula on August 21 1914. During his visit, Einstein worked on the calculations relating to gravitational lensing, as evidenced by his notebooks dating back to that time. These notes refer to the possibility of acquiring a double image or a ring because of a gravitational lens, and the resulting increase in magnitude of the images.

Einstein attempted to persuade G.E. Hale of the possibility of measuring the positions of the stars during the day in order to measure their apparent shift in position due to the Sun's mass. Hale was the director of the Mt Wilson Observatory at the time. This line of inquiry was unsuccessful.



Zürich 14. X. 13.

Ans

Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie $\frac{1}{R}$ abnehmen (R = ~~Sonnenradius~~ Entfernung vom Sonnen-Mittelpunkt).

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe ~~grosse~~ helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

Figure 1: Einstein's note to Hale, including a sketch of how he expected light to be deviated by the mass of the Sun.

Freundlich however did instigate an expedition to Russia to measure the position of the stars nearby to the Sun in the 1914 eclipse. Within a few weeks of the team of astronomers arriving in Russia, World War 1 began. The Russians captured the astronomers, and the planned observations were unfortunately not undertaken.

1.2 General Relativity

Einstein published his theory of General Relativity in 1915 (Einstein, 1915). This gave reason for him to recalculate his early findings in light of his assertion that massive objects warp nearby space-time, and hence any particle whether light or matter would be forced to travel along an altered geometry of that area. In 1916, using the same parameters as in his previous calculations, Einstein published a new value for the deflection of light grazing the edge of the Sun as below:

$$\begin{aligned} \alpha &= \frac{4GM}{c^2 R} \\ [3] \quad &= \frac{2R_{sun}}{R} \\ &= 1.74\,arc\sec \end{aligned}$$

Where *G* stands for the gravitational constant, *c* is the velocity of light; M is the mass of the Sun and *R* its radius. In this expression R_{sun} is the mass of the Sun in terms of geometry, and expressed in metres (Einstein, 1916).

This factor of two difference between the Newtonian and the general relativistic value is indicative of spatial curvature, an influence which is missed by purely considering light as particles. Given his initial calculation of the wrong value, it was perhaps fortuitous for Einstein that the Russian observing expedition of deviations in stellar positions was unsuccessful.

After the completion of WWI it was once again possible to revisit the concept of an expedition to an eclipse in order to test his theories. In 1919 such an opportunity arose: an eclipse was visible in Sobral, Brazil, and on the Island of Principe off the Western Coast of South Africa. The expedition was led by Sir Arthur Eddington, an astronomer remembered for his work on determining the workings of the interior of stars, and for ascertaining their temperatures.

The main purpose of the expedition was to conclude the truth of one of three possibilities:

1. The gravitational field of the Sun does not bend light
2. The gravitational field of the Sun influences the bending of light according to Newtonian Mechanics, with the result being a deflection of light by 0.87 arcsec

3. The gravitational field of the Sun influences the bending of light according to General Relativity, with the result being the deflection of light by 1.74 arcsec.

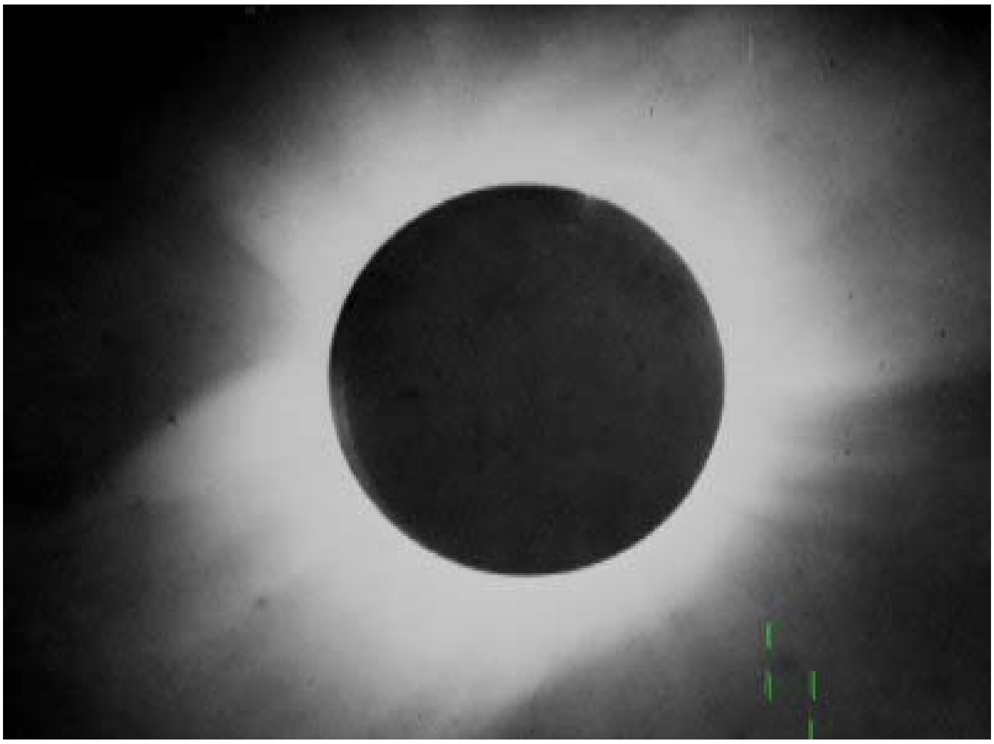


Figure 2: Image of the Solar Eclipse from the 1919 Expedition. The green lines indicate the position of the stars used for measurement.

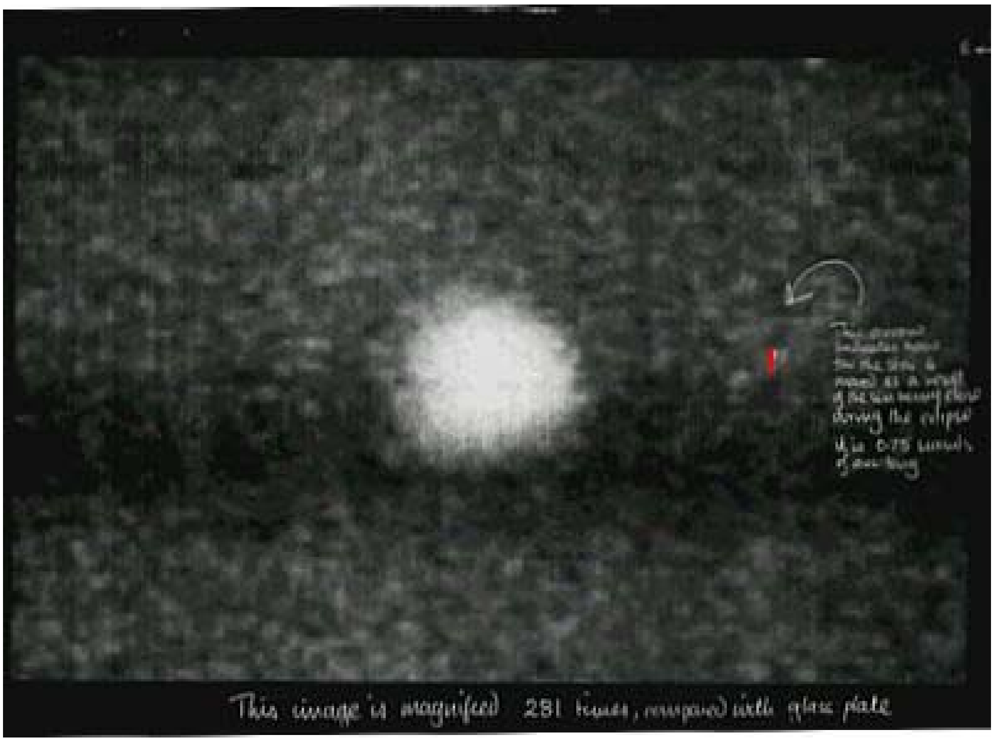


Figure 3: A star used for measurement, magnified by 281 times. The red line indicates the amount by which the image of the star had been deflected.

The average value for the displacement of the stellar images from the Sobral observations was 1.98 arcsec ±0.12, and the Principe observations yielded a value of 1.61 arcsec ±0.3. These measurements, although imprecise, could clearly show that two of the initial possibilities could be rejected, leaving the predictions of General Relativity as the most accurate. It is now common knowledge that this expedition determined Einstein's findings to be correct to within 20%; in 1995 it was determined by radio-interferometric (VLBI) methods that Einstein was correct to within 0.02% (Lebach, 1995).

The results from the 1919 expedition caused a world-wide media frenzy, and helped elevate the status of such research in the arena of professional science.

Eddington published a report on this expedition in his book ‘Space, Time and Gravitation’. In chapter 7 he writes

> ‘In a superstitious age a natural philosopher wishing to perform an important experiment would consult an astrologer to ascertain an auspicious moment for the trial. With better reason, an astronomer to-day consulting the stars would announce that the most favourable day of the year for weighing light is May 29. The reason is that the sun in its annual journey round the ecliptic goes through fields of stars of varying richness, but on May 29 it is in the midst of a quite exceptional patch of bright stars---part of the Hyades---by far the best star-field encountered. ... by strange good fortune an eclipse did happen on May 29, 1919.’ (Eddington, 1920)

In the same chapter he describes how some plates were unusable as no stars could be seen through the clouds until near the end of the eclipse. They performed some measurement with a micrometric measuring machine and comparing with images of the star field taken at night. Further plates were not developed until their return to Britain, as they were of a brand that could not stand development in hot climates. One of these plates had sufficient stars to render measurement possible, and confirmed the initial findings. Although measurements were difficult and rough, they were sufficient to disprove the Newtonian deflection. It was not until the mid-60’s that more accurate observations were possible.

The person first credited with the published use of the word ‘lens’ in the context of the deviation of light be massive objects was O.J. Lodge (Schneider et al, 1992) in a half-page note published in 1919 where he tackled the concept of a gravitational lens, although he asserted that

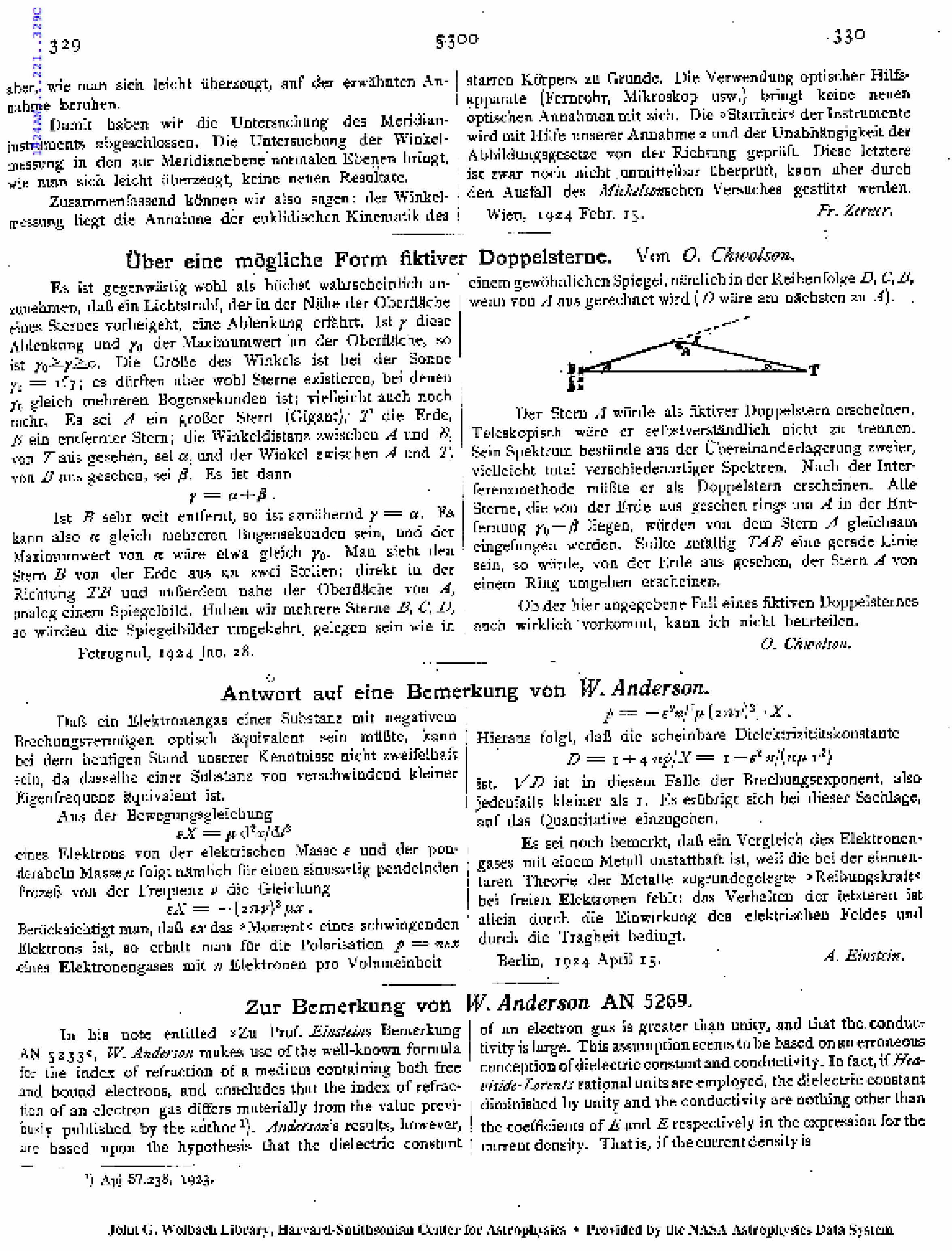
> ‘it is not permissible to say that the Solar gravitational field acts as a lens, for it has not a focal length’ (Lodge, 1919)

In 1920 Eddington published a suggestion that multiple images may be a result of a lensing event, given that the two stars were of great enough difference in distance from the observer, and given that the two objects were aligned to within 1 arcsec (Eddington, 1920). These circumstances, he suggested, would mean that an observer would detect the primary image of the more distant star as well as an illusory image on the other side of the nearer star. As the alignment improved, he asserted, the primary image should fade. This was included in a section on observational tests of General Relativity. He went on to investigate the expected intensity of the deflected light, and stated

> ‘it is easily calculated that the increased divergence would so weaken the light as to make it impossible to detect when it reaches us.’

However the first published suggestion of the ‘Einstein Ring’ was by Orest Chwolson in 1924 (Chwolson, 1924). Chwolson considered the circumstance of two stars of considerable distance between them, but rather precisely aligned from the viewpoint of an observer. This would lead to the formation of a 'fictitious double star' being formed with the foreground star apart from the primary. He stated that the secondary image would be visually inseparable from the foreground star from an observer's viewpoint, except perhaps by spectral analysis. The foreground star would exhibit a fictitious Doppler effect. The spectra from the stars would accumulate together and, applying the interference method the foreground star with images will appear as a Doppler star. Additionally he predicted that if there were multiple stars, the secondary images of these stars would appear reversed as in a mirror. If the observer, foreground star and background star were in perfect alignment, there would not be a secondary image, but a ring of light from the background star centered on the foreground star.

This paper was published in a journal of great note, and was in fact on the same page as a paper by Einstein, yet it appeared to have gone unnoticed in the eyes of the scientific community. Einstein himself appears to have not been aware of the article, although he also had an article printed immediately below the article by Chwolsen. Whether or not Einstein had in fact been aware of the article and simply ignored it as not important, or didn't notice it at all, we can never be sure.



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aber, wie man sich leicht überzeugt, auf der erwähnten Annahme beruhen.

Damit haben wir die Untersuchung des Meridianinstruments abgeschlossen. Die Untersuchung der Winkelmessung in den zur Meridianebene normalen Ebenen bringt, wie man sich leicht überzeugt, keine neuen Resultate.

Zusammenfassend können wir also sagen: der Winkelmessung liegt die Annahme der euklidischen Kinematik des starren Körpers zu Grunde. Die Verwendung optischer Hilfsapparate (Fernrohr, Mikroskop usw.) bringt keine neuen optischen Annahmen mit sich. Die »Starrheit« der Instrumente wird mit Hilfe unserer Annahme a und der Unabhängigkeit der Abbildungsgesetze von der Richtung geprüft. Diese letztere ist zwar noch nicht unmittelbar überprüft, kann aber durch den Ausfall des *Michelson*schen Versuches gestützt werden.

Wien, 1924 Febr. 15. *Fr. Zerner.*

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).

Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestünde aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

Petrograd, 1924 Jan. 28. *O. Chwolson.*

Antwort auf eine Bemerkung von *W. Anderson.*

Daß ein Elektronengas einer Substanz mit negativem Brechungsvermögen optisch äquivalent sein müßte, kann bei dem heutigen Stand unserer Kenntnisse nicht zweifelhaft sein, da dasselbe einer Substanz von verschwindend kleiner Eigenfrequenz äquivalent ist.

Aus der Bewegungsgleichung

$$\varepsilon X = \mu\, d^2x/dt^2$$

eines Elektrons von der elektrischen Masse ε und der ponderabeln Masse μ folgt nämlich für einen sinusartig pendelnden Prozeß von der Frequenz ν die Gleichung

$$\varepsilon X = -(2\pi\nu)^2 \mu x .$$

Berücksichtigt man, daß εx das »Moment« eines schwingenden Elektrons ist, so erhält man für die Polarisation $p = n\varepsilon x$ eines Elektronengases mit n Elektronen pro Volumeinheit

$$p = -\varepsilon^2 n/\mu(2\pi\nu)^2 \cdot X .$$

Hieraus folgt, daß die scheinbare Dielektrizitätskonstante

$$D = 1 + 4\pi p/X = 1 - \varepsilon^2 n/(\pi\mu\nu^2)$$

ist. $\sqrt{D}$ ist in diesem Falle der Brechungsexponent, also jedenfalls kleiner als 1. Es erübrigt sich bei dieser Sachlage, auf das Quantitative einzugehen.

Es sei noch bemerkt, daß ein Vergleich des Elektronengases mit einem Metall unstatthaft ist, weil die bei der elementaren Theorie der Metalle zugrundegelegte »Reibungskraft« bei freien Elektronen fehlt; das Verhalten der letzteren ist allein durch die Einwirkung des elektrischen Feldes und durch die Trägheit bedingt.

Berlin, 1924 April 15. *A. Einstein.*

Zur Bemerkung von *W. Anderson* AN 5269.

In his note entitled »Zu Prof. *Einsteins* Bemerkung AN 5233«, *W. Anderson* makes use of the well-known formula for the index of refraction of a medium containing both free and bound electrons, and concludes that the index of refraction of an electron gas differs materially from the value previously published by the author[1]. *Anderson*'s results, however, are based upon the hypothesis that the dielectric constant of an electron gas is greater than unity, and that the conductivity is large. This assumption seems to be based on an erroneous conception of dielectric constant and conductivity. In fact, if *Heaviside-Lorentz* rational units are employed, the dielectric constant diminished by unity and the conductivity are nothing other than the coefficients of $\dot{E}$ and E respectively in the expression for the current density. That is, if the current density is

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[1] Ap J 57.238, 1923.

Figure 4: Chwolson's paper describing the deviation of light, directly beneath on of Einstein's own papers.

Some modern writers suggest that 'Einstein Rings' should be more accurately described as 'Chwolson Rings', others defend the name by pointing out that Chwolson did not pay due attention to the flux or radius of the resulting ring. Chwolson appears to have been unconvinced that the effect would be at all observable.

In 1925, H. Uhler of Yale published the workings of finding the deflection of light in both Newtonian and relativistic mechanics, apparently in frustration at not finding the complete working in any other publication (Uhler, 1925).

Eleven years later, in 1936, Einstein was approached by an amateur scientist in order to present his theories on another test of General Relativity by the deflection of light from a background star around a foreground star. He further theorized that the resulting increase in intensity of light from the background star may be responsible for a variety of Earth-based phenomena including sudden evolutionary changes, the detection of cosmic rays, and the extinction of dinosaurs (Renn and Sauer, 2000). Rudi Mandl had attempted to have his theories published by a variety of means and had been rebutted by a number of distinguished scientists already. Einstein agreed to meet with him, and at the time sketched out some calculations about the effects on light from a background star by a foreground mass. Einstein at some stage of their meeting or following correspondence committed to publishing these publications.

Initially it appears that Einstein attempted to renege on this commitment, as indicated by his letter to Mandl on April 18 1936:

> ‘I have come to the conclusion that the phenomenon in question will, after all, not be observable so that I am no longer in favor of publishing it’

In a cross-over of correspondence, a letter sent to Einstein from Mandl dated April 23 of the same year contests that view:

> ‘Meanwhile I have found a method to measure the intensity increase in the domain of the focal line of a star and to confirm it experimentally. It would be, according to my view, in the interest of science to begin with these experiments as soon as possible’

Although Einstein was still apparently reluctant to have any further association with Mandl, this statement was reminiscent of Einstein’s own assertion in 1912. On May 12 1936, Einstein wrote back to Mandl.

> ‘Your fantastic speculations associated with the phenomenon would only make you the laughing stock of reasonable Astronomers. I warn you in your own interest against such a publication. On the other hand, one cannot object against a modest publication of a derivation of the two characteristic formulae for the ‘halo effect’ and the ‘intensification effect’.

Referring to the calculations behind the Einstein Ring and the apparent intensification of the light from the background star. Mandl remained unsuccessful in his attempts to have his findings published, and beseeched Einstein again to publish the calculations himself, calling again on his early commitment to publish. A combination of this coupled with Mandl's unrelenting enthusiasm may have been the cause for the eventual publication of Einstein's 1936 paper. The first paragraph however alludes to his reluctance to do so, and appears to act as a disclaimer to the contents of the work:

> 'Some time ago, R.W. Mandl paid me a visit and asked me to publish the results of a little calculation which I made at his request. This note complies with this wish.'

This view is reinforced by a letter to the Editor J. McKeen Cattell penned on the 18th of December later the same year.

> Let me also thank you for your cooperation with the little publication, which Mr Mandl squeezed out of me. It is of little value, but it makes the poor guy happy.'

The paper made some interesting conclusions. One is Einstein's conviction that these effects would never be observed (Einstein, 1936).

> 'There is no chance of observing this phenomenon, even if dazzling by the light of the much nearer star is disregarded'

And the analytical result.

> This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming indefinite, with x vanishing, but since with increasing distance D of the Observer not only does it not decrease, but event increases proportionally to $\sqrt{D}$'

With D referring to the distance between the observer and the background star, star A, and x referring to the distance between the observer and the foreground star, star B.

Regardless of Einstein's apathy toward his own work, this paper caused a flurry of work by other scientists of note in the same topic, many of whom claimed to have performed the same calculations and made the same predictions prior to Einstein (Tikhov, 1938). It may be that other scientists were more comfortable asserting their findings after the concept had been deemed appropriate as a line of inquiry and had been published by a scientist of note, or it may have been that the same scientists were not convinced of the importance of their assertions until after a journal of note had published something on the same topic.

One such scientist was the Russian researcher Tikhov. In his publication of June 1938 he claims that he had started working on the concept in the Summer of 1935 and sent a communication to that effect in January 1936 to the Poulkovo Observatory. He also indicated the work of Chwolson. In the same paper, Tikhov deduces the lensing formulae in both the relativistic and the Newtonian cases.

H.N. Russell in February of 1937 published a paper after investigating Einstein's calculations, which was entitled 'A Relativistic Eclipse' (Russell, 1937). He was focusing more on the idea of observability, and agreed with the assertion that the lensing effect would not be verifiable by Earth-bound observers, but pursued the idea anyway. He considered the idea of using the Sirius system as a test case. Using this system, Russell discusses the orders of magnitude resultant from the case of having a white dwarf lens. He goes on to consider the case of Sirius' companion being a lens star to Sirius, and how this would be observed if the observer were on an imaginary planet in this system. He published in the paper sketches of what the effect might be like for such an observer, comparing them to a solar eclipse. He included the situation of the two stars being in perfect alignment with the subsequent 'Halo' or Einstein Ring. Although based on a purely imaginary planet, this paper was arguably important in keeping the interest in lensing to the forefront, and in giving a more realistic and tangible side to an essentially abstract notion.

The publication was dated Dec 2, 1936 and thanks Einstein for his help in allowing Russell to view Einstein's manuscript before it was published. He examined each stage of a lensing event by the pup of Sirius, and concludes

> 'Our hypothetical space-tourist, therefore, could settle down with his planet in such a place that general relativity would no longer be a matter of the utmost refinement of theory and observation. It

> would instead be a needed to account for the most bizarre and spectacular phenomena of the heavens, as he saw them.'

In 1937, the Swiss Astronomer Fritz Zwicky replied to Einstein's paper with a note on possible gravitational lensing of galaxies, or extragalactic nebulae, in a paper titled 'Nebulae as Gravitational Lenses' (Zwicky, 1937). He indicates in the publication that his first encounter with the problem had arisen indirectly as a result of Mandl's approaches to the scientific community.

> 'Last summer Dr. V.K. Zworykin (to whom the same idea had been suggested by Mr. Mandl) mentioned to me the possibility of an image formation through the action of gravitational fields. As a consequence I made some calculations which show that extragalactic nebulae offer a much better chance than stars for the observation of gravitational lens effects.'

In this paper, Zwicky concludes that 'extragalactic nebulae', due to their apparent diameter and their mass, were more probable candidates for the observation of gravitational lensing events. He suggests that their observation be important, not merely as another test for the validity of General Relativity, but that the lens effect acts as a form of natural telescope, extending our view of the Universe beyond what was hitherto possible.

> 'The discovery of images of nebulae that are formed through the gravitational fields of nearby nebulae would be of considerable interest for a number of reasons. First, it would furnish an additional test for the general theory of relativity; secondly, it would enable us to see nebulae at distances greater than those ordinarily reached by even the greatest telescopes. Any such extension on the known parts of the Universe promises to throw

> very welcome new light on a number of cosmological problems. Thirdly, the problem of determining nebular masses at present has arrived at a stalemate… Observations on the deflection of light around nebulae may prove the most direct determination of nebular masses.'

Only two months later, Zwicky published a further letter, entitled 'On the Probability of Detecting Nebulae Which Act as Gravitational Lenses' (Zwicky, 1937). In this he made the revolutionary claim that detecting the gravitational lensing effect by extragalactic nebulae as 'practically a certainty'. He based this claim on a number of assumptions: Firstly that the mass of these nebulae were approximately $4 \times 10^{11} M_{sun}$ (at the time, it was estimated at only 10^9). Using the Coma and Virgo clusters of galaxies, Zwicky estimated that approximately 1/400 the area of photographic plates were covered in nebular images. If one included gravitational focusing, he found that

> 'around one in about one hundred nebulae the ring-like image of a distant nebula should be expected'
>
> 'The probability that nebulae which act as gravitational lenses will be found becomes practically a certainty.'
>
> 'Present estimates of masses and diameters of cluster nebulae are such that the observability of gravitational lens effects among the nebulae would seem ensured.'

Zwicky, although enthusiastic about the possibility of discovering these effects, demonstrated a sense of realism on the laborious nature of searching for them.

> In searching through actual photographs, a number of nebular objects arouse our suspicion. It will, however, be necessary to investigate certain composite objects spectroscopically, since differences in the redshift of the different components will immediately betray the presence of gravitational effects. Until such tests have been made, further discussion of the problem in question may be postponed.'

A further note in the same publication refers to the dated notion of Einstein's ideas.

> 'Dr. G. Stroemberg of the Mt Wilson Observatory kindly informs me that the idea of stars as gravitational lenses is really an old one. Among others, E.B. Frost, late Director of the Yerkes Observatory, as early as 1923 outlined a program for the search of such lens effects among stars.'

Although there appears to be no record of published material by either.

It appears that Zwicky's suggestion of waiting was heeded – there was very little in the way of further publication until the early 1960's.

1.3 1960 Developments

After this initial burst of scientific activity following Einstein's 1936 publication, gravitational lensing as a topic for research appears to have been left aside for a couple of decades. After the advent of radio astronomy, however, lensing again became a topic of great interest. Radio astronomers detected in 1963 'quasi-stellar radio sources', or quasars. These interesting objects had very strong spectral features, high redshift, great distances, were intensely luminous and appeared point-like on optical plates. They defied cosmological models of the time, and appeared to be ideal candidates for gravitational lens events.

After the discovery of these quasars, it was Barnothy who was the first to connect these strange objects to the gravitational lens phenomenon. In his 1965 paper, Barnothy suggested that quasars

> ‘are nuclei of Seyfert I galaxies intensified through the gravitational lens action of foreground galaxies’ (Barnothy, 1965).

This suggestion was not warmly received by the scientific community, partly due to Barnothy’s favouring of an unusual cosmological model.

Gravitational lens theory was once again a hot topic, and was explored independently by a number of notable scientists.

Yu G. Klimov explored the concept of lensing galaxies by galaxies. He concluded this line of investigation by asserting that when galaxies were aligned sufficiently precisely, the resultant ring would be visible, and easily distinguishable from a field of galaxies. A less perfect alignment though would lead to a visual effect much harder to differentiate from other double or multiple galaxies.

S. Liebes investigated other occasions where lensing may occur (Liebes, 1969), concentrating in particular on globular star clusters. He considered stars lensing stars within the Milky Way, stars in the Milky Way lensing globular star clusters, and stars lensing stars where both stars lie within the same globular cluster. Interestingly, he also estimated the probability of observing such events.

In 1969 Liebes devised a plexiglass simulation of the effects of microlensing (Liebes, 1969). That is, he was able to calculate the shape and design of an actual physical lens which would replicate the effect of gravitational lenses in space. This has proven useful in understanding the possible effects of lenses and multiple lenses, and for demonstrating the effects of lensing to pupils and others.

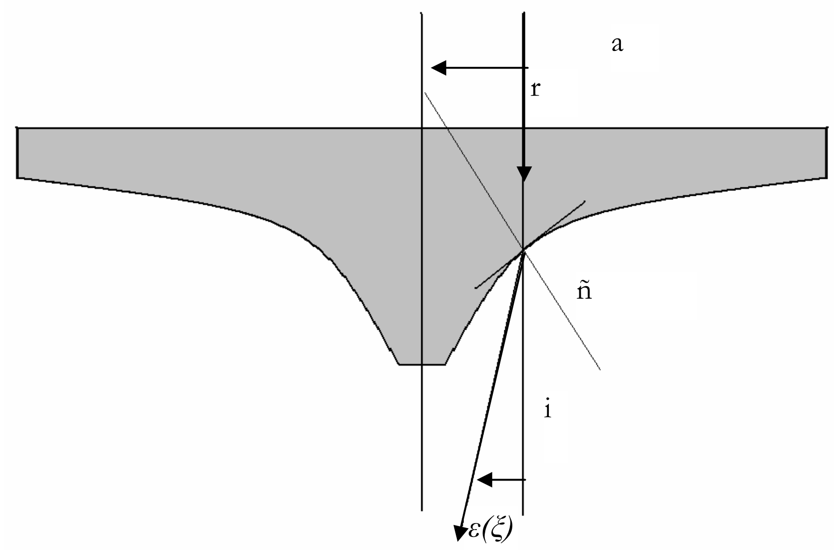

Figure 6: A plexiglass replica of a gravitational microlens. The light ray a is deflected by an amount $\varepsilon(\xi)$***, resulting from Descrates' law. This was first applied by Liebes in 1969.***

Liebes also referenced and acknowledged all the references he could locate in literature about gravitational lens phenomena, 'apologizing for those oversights which had undoubtedly been made'.

S. Refsdal also considered star-star lensing, but he also paid particular attention to the time delay of the two images resulting from an imperfect alignment of stars (Refsdal, 1966). He also asserted that geometric optics could confidently be used in considering gravitational lensing effects.

In a paper published in 1964 (Refsdal, 1964), Refsdal speculated on the possible uses for gravitational lensing, beyond simply providing a definitive test of General Relativity.

> The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the Observer, and the difference Δt in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble's parameter and the mass of the galaxy can be expressed by Δt the redshifts of the supernova and the galaxy, the luminosity of the supernova 'images' and the angle between them. The possibility of observing the phenomenon is discussed.

There was a definite shift in attitude from not considering gravitational lensing to be observable at all, to there being a certain probability of detecting one. Refsdal also went on to assert that lensed background quasi-stellar objects may be used in the determination of the mass of lens masses. Armed with that, the Hubble constant may be determined using equation 11.

$$[4]\ t = \frac{1}{H_0}$$

Other topics explored by scientists of the day included examining the statistical effects of local inhomogeneities on the propagation of light, examining the concept of lensing in relation to quasars and galactic clusters, and the effect of Universal inhomogeneity affecting distance/redshift relations, and the development of a formalism for transparent lenses.

Despite the definite prediction and the calculations of the possibility of observing these predictions, observational aastronomers showed very little interest in searching for these effects, let alone instigating a systematic search for them, whether lensing by and of extragalactic nebulae (now referred to as macrolensing, in which generally the secondary image or ring is observable) or by stellar sources and lenses (referred to as microlensing where in which in general the secondary image or ring is not observable).

1.4 Detection

After another decline in the development of lensing theory, there was a sudden and remarkable success: the first confirmed gravitational macrolensing event was detected.

In 1979, Walsh, Carswell and Weymann asserted that the unusual double-quasar 0957-561 was in fact a single quasar undergoing lensing (Walsh, 1979). This conclusion was brought about because of the discovery that the two images had similar spectra, and had similar ratios of optical to radio fluxes. The two images were only 6 arcsecs in separation, and they were both redshifted at about z~1.4. Subsequently, a galaxy amidst a galaxy cluster was discovered to be the lens, at only z~0.39. The large separation of the two images (about 6 arcsec) was due to the lens being a cluster and not just a single galaxy.

In 1979 as well, Chang and Refsdal (Chang, Refsdal, 1979) published a paper in which it was detailed how stars lying between the observer and observed quasars could affect the image brightness of the quasar. This could be considered the first real investigation into the effects of microlensing. It was said at the time that Refsdal, rather than thinking of the proposition as a new line of discovery, was instead concerned that, due to this effect, it may have been impossible to determine time delays in quasar lensing events, and hence render impossible the calculation of the Hubble constant from those events.

A year later, in 1980, Young et al reported a quadruple event, a single quasar split into four images (PG115 + 080). The first Einstein Ring was discovered in a radio survey in 1986, in which the ring was clearly visible in radio, and the lens visible in optical and IR wavelengths. Also in 1986, the first example of galaxies being lensed by clusters of galaxies was discovered in Abell 370.

These had been cases of strong lensing, where there were clear separate images due to the intervening lens. Around this time, however, it was realized that a weak lensing effect should be detectable, as a result of the large-scale distribution of matter throughout the Universe. This effect is also described as ‘Cosmic Shear’ and may only be detected by statistical analysis from surveys of 10^6 galaxies or more. It is described as ‘weak’, as the lensing produces only single, slightly distorted images.

It has been demonstrated more recently that, if observations are made to the limit of detectability in magnifications, tangentially sheared faint images will be found in most galaxy clusters. This effect can assist astronomers ascertain the mass distribution in galaxy clusters, determine sizes of galactic halos, and cosmological parameters by observing these ‘weak’ events.

In 1986 Paczynski suggested that it would be possible, by looking for lensing events in stars of galaxies in the Local Cluster, to detect dark matter, objects estimated to be planet-sized and EM dark (Paczynski, 1986). Again it would be required to compile observations of approximately 10^6 objects.

Several groups took up the initiative, and in 1993, over a decade after the first ‘macrolensing’ event, the first microlensing event was announced. In fact, two were announced that year, one by OGLE (Udalski, Szymanski et al, 1993) and one by MACHO (Alcock, Akerlof et al, 1993).

MACHO detected its first microlensing event toward the Large Magellanic Cloud.

OGLE had 85 allocated observing nights between Apr 3 and Sep 6 of 1993 at the Las Campanas Observatory, operated by the Carnegie Institute of Washington, using the 1-m Swope telescope. This time was subdivided into 7 ‘sub-runs’, each lasting between 7 to 20 consecutive nights. Over the course of the observing run 1192 usable images of the galactic bulge were obtained.

The strategy for observing and detecting events was to construct a database of non-variable stars derived from their I-band photometry of every bulge field acquired in their initial observing run in 1992. The total number of non-variable stars compiled in this manner was 1.1 million.

Each object in the 1993 run from which photometric results were gathered was compared against the average of the 1992 run. If any object exhibited a variation of more than $3\sigma_{1992}$ from their average 1992 value the object would be flagged in a ‘warning system’ database. This system was designed to run online, although in 1993 the capability was not available, so all measurements were performed after the observing season was over.

Any object flagged more than 4 times in the warning system database was selected and run through a variety of filtering systems, a step which removed any object which decreased in brightness, or exhibited random fluctuations. Any object passing all tests would undergo analysis as potential lensing events. 650,000 stars were investigated in this manner, of which it was expected many would turn out to be long-period intrinsic variables.

One object was found to exhibit behavior similar to theoretical predictions, object number I 117281 in the BW7 field of the Baade’s Window. Baade’s Window refers to the small area in the constellation Sagittarius which is relatively free of dust, through which one can observe the central region of the Milky Way.

The group ascertained that, given that the event observed was in fact a gravitational lensing effect, the mass of the source star could be estimated at approximately 0.3 solar masses based on its period of 24 days, indicating a late-type M dwarf star.

It is of interest to note that the first observations of lensing events coincide with the advent of CCDs in the 1980s. It is the increasing availability of good imaging equipment that has made the task of searching for these events possible and, with every year, CCD cameras are becoming better and more accessible. The positive effect of this is the increasing involvement of amateur astronomers in the search for microlensing events, and the rapid and effective imaging of the events when they are detected.

For the remainder of this thesis, particular attention will be paid to gravitational microlensing as separate and distinct from macrolensing.

CHAPTER 2

Lens Theory and Lensing phenomena

In this chapter the general theory of the deflection of light by gravity, of gravitational microlensing, and various phenomena are discussed in some depth.

2.1 The Deflection of Light by Massive Objects

As the original workings of Einstein in his calculations of the 'Halo Effect' and the "Intensification Effect' were not published, much of the work here is based on the work of Uhler. These examples are based on the case of the displacement of the image of a star due to the effect of the Sun, as this has been the base argument for each calculation performed since the 1700s.

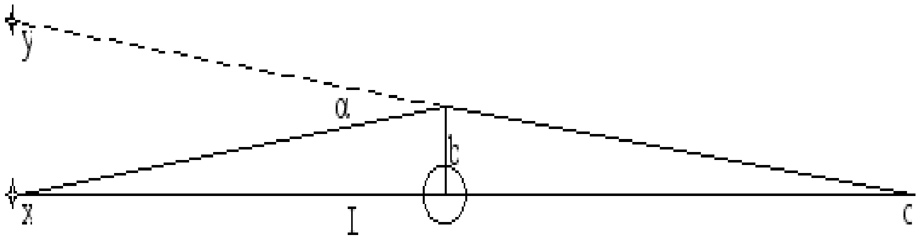

Figure 7: A simple demonstration of the deflection of light by a massive body. The Observer rests at C, and the source at X, and the image of X lies at Y.

The general concept is demonstrated in the above diagram. An observer, o, is viewing the background object. Without an intervening mass, the observer will see the background star in the position x, its proper position. With an intervening mass, the light from that object will be bent around the massive object by a certain amount, and will appear to be in the position y from the observer's perspective.

2.2 Newtonian Mechanics

Although it has been confirmed that in the case of massive bodies Newtonian mechanics cannot apply, the workings are reproduced here for interest. These were performed by Michell, Laplace, Cavendish, Soldner, Uhler and Tikhov at various times with results varying only slightly.

Start with the three Newtonian equations for the path, acceleration and radial velocity of an infinitesimal particle in orbit around the Sun:

$$[5]\, r = \frac{p}{(1 + e\cos\theta}; \alpha = \frac{-h^2}{pr^2}; h = \frac{r^2 d\theta}{dt}$$

Consider the situation of the Sun, with a ray of light grazing it. You will have then *R* as the radius of the Sun, *G* the acceleration of the particle due to gravity, and the speed of light at this point V_0. We then find

$$[6]\, r = R; \theta = 0; \frac{d\theta}{dt} = \frac{V_0}{R}; \alpha = -G$$

Substituting into the initial equations we have

$$[7] \quad \begin{aligned} h &= \frac{R^2 V_0}{R} = RV_0 \\ G &= \frac{R^2 {V_0}^2}{pR^2} \\ R &= \frac{{V_0}^2}{G(1+e)} \\ e &= \frac{{V_0}^2}{RG} - 1 \\ p &= \frac{{V_0}^2}{G} \\ (1+e) &= \frac{{V_0}^2}{RG} \end{aligned}$$

Given the circumstance, it can be shown that *e* has a value of the order of 10^6, and hence we can rewrite the final equation as

$$[8]\, \frac{1}{e} = \frac{RG}{{V_0}^2}$$

It can be seen that the path is hyperbolic because of the size of *e*. The asymptotes to the hyperbola make an angle

$$[9]\, \alpha = \tan^{-1} \sqrt{e^2 - 1}$$

with the axis of symmetry. For the same reason as above, e^2-1 may be substituted with e, giving

[10] $\alpha = \tan^{-1} e$

The deflection we are aiming to deduce is the amount that light is deflected, *D*. Half this value, it can be seen, is 90°-α. Therefore

[11] $\frac{D_0}{2} = \cot^{-1} e = \tan^{-1}(\frac{1}{e}) = \tan^{-1}(\frac{RG}{V_0^{\ 2}})$

As D_0 is very small, it can be expressed in seconds of arc:

[12]
$$\frac{D''_0}{2} = \frac{RG}{(V_0^{\ 2} \tan 1'')}$$
$$D''_0 = \frac{2RG}{(V_0^{\ 2} \tan 1'')}$$

If one were to use the premise in Special Relativity that light travels at a constant speed *c,* then V_0 may be substituted for *c*. The difference between the Newtonian and relativistic values V_0 and *c* is

[13] $V_0 = c + \sqrt{2RG}$

Substituting in the mass of the Sun as a unit of length

[14] $$R_S = \frac{2\,GM_S}{c^2}$$
$$1475\,m$$

and the radius of the Sun, we find that:

[15] $$\begin{aligned} D''_{newt} &= 0.8702 arcec \\ D''_{SR} &= 0.8737 arc\sec \end{aligned}$$

Where D”$_{\text{Newt}}$ and D”$_{\text{SR}}$ refer to the Newtonian and Special Relativistic conditions.

2.3 General Relativistic Mechanics

These were the final set of predictions, which have since been proven to be correct to within 0.02%, and were based on Einstein’s 1915 publication of General Relativity (Lebach, 1995).

In order to find the general relativistic value for the deflection of a light path around the Sun, it is possible to evaluate four geodesic equations. Another efficient method is to consider it a 2-dimensional problem with the plane of the path running through the centre of mass of the system.

Start with the Schwarzschild metric in polar coordinates:

[16] $$(d\tau)^2 = (\frac{r-2m}{r})(dt)^2 - (\frac{r}{r-2m})(dr)^2 - r^2(d\theta)^2 - r^2\sin(\theta)^2(d\phi)^2$$

Then restrict attention to the two-dimensional plane

$$[17]\,(dt)^2 = (\frac{r}{r-2m})^2 (dr)^2 + \frac{r^3}{r-2m}(d\theta)^2$$

which can be regarded as the positive-definite line element of a two-dimensional surface with *t* serving as the metric distance. The problem reduces as it is only necessary to determine the geodesic paths on this surface.

The covariant and contravariant metric tensors are

$$[18]\;\; g_{\mu\nu} = \begin{bmatrix} (\frac{r}{r-2m})^2 & 0 \\ 0 & \frac{r^3}{r-2m} \end{bmatrix} \text{and}\, g^{\mu\nu} = \begin{bmatrix} (\frac{r-2m}{r})^2 & 0 \\ 0 & \frac{r-2m}{r^3} \end{bmatrix}$$

The non-zero partial derivatives of *g* are

$$[19]\;\; \frac{\partial g_{rr}}{\partial r} - \frac{-4mr}{(r-2m)^3} \text{and}\; \frac{\partial g_{\theta\theta}}{\partial r} = 2r^2 \frac{r-3m}{(r-2m)^2}$$

giving the nonzero Cristoffel symbols

$$[20]\;\; \Gamma^{r}{}_{rr} = \frac{-2m}{r(r-2m)}, \quad \Gamma^{r}{}_{\theta\theta} = -(r-3), \quad \Gamma^{\theta}{}_{r\theta} = \Gamma^{\theta}{}_{\theta r} = \frac{1-3m}{r(r-2m)}$$

So, taking *t* as the path parameter, the two equations for geodesic paths on the surface are

$$[21]\quad \begin{aligned} \frac{d^2r}{dt^2} &= \frac{2m}{r(r-2m)}\left(\frac{dr}{dt}\right)^2 + (r-3m)\left(\frac{d\theta}{dt}\right)^2 \\ \frac{d^2\theta}{dt^2} &= -2\frac{(r-3m)}{r(r-2m)}\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) \end{aligned}.$$

The second equation is integrated into

$$[22]\quad \frac{d\theta}{dt} = K\left(\frac{r-2m}{r^3}\right)$$

To determine *K*, the metric equation may be divided by $(dt)^2$, then consider the situation where $dr/dt=0$ and $r=r_0$:

$$[23]\quad 1 = \frac{r_0{}^3}{r_0 - 2\mathrm{m}}\left(\frac{dq}{dt}\right)^2$$

This expression may be substituted into the equation above above, yielding

$$[24]\quad K^2 = \frac{r_0{}^3}{r_0 - 2\mathrm{m}}$$

from which we find

$$[25]\quad \frac{dq}{dt} = \sqrt{\frac{r_0{}^3}{r_0 - 2\mathrm{m}}}\left(\frac{r-2\mathrm{m}}{r^3}\right)$$

This may be substituted back into the metric equation, and then solved for $\frac{dr}{dt}$:

[26] $$\frac{dr}{dt}=\frac{r-2\mathrm{m}}{r}\sqrt{1-\left(\frac{r_0}{r}\right)^3\frac{r-2\mathrm{m}}{r_0-2\mathrm{m}}}$$

Divide $\frac{dq}{dt}$ by $\frac{dr}{dt}$ to give

[27] $$\frac{dq}{dr}=\frac{\sqrt{\frac{r_0^3}{r_0-2\mathrm{m}}}}{\sqrt{1-\left(\frac{r_0}{r}\right)\frac{r-2\mathrm{m}}{r_0-2\mathrm{m}}}}\frac{1}{r^2}$$

This may be integrated from r=r_0 to Y to give the mass-centered angle swept out by the photon as it moves from the perihelion to an infinite distance. Defining r=r_0/r, the equation becomes

[28] $$dq=\int_{\rho=0}^{1}\frac{1}{\sqrt{1-\rho^2}}\frac{1}{\sqrt{1-2\left(\frac{1-\rho^3}{1-\rho^2}\right)\frac{m}{r_0}}}d\rho\,.$$

The second term in the integral will always have a magnitude<1 for all cases where $r_0>3m$, which is the radius of light-like orbits. Hence the square root my be expanded into a power series:

$$[29]\ dq = \int_{\rho=0}^{1} \frac{1}{\sqrt{1-\rho^2}} (1 + \frac{1}{2}\left[2\frac{1-\rho^3}{1-\rho^2}\frac{m}{r_0}\right] + \frac{3}{8}\left[2\frac{1-\rho^3}{1-\rho^2}\frac{m}{r_0}\right]^2 + \ldots) d\rho$$

The integral of the first term here is just *p*/2, which would be expected as a photon with a mass of zero would travel in a straight line and sweep out a right angle as it traveled from perihelion to infinity. The rest of the terms account for the deflection angel. If m/r_0 is small, then only the second term in the power series is significant. It must be remembered as well that the path of light is symmetric about the perihelion, which means that the amount the path is deviated as measured as the difference between the asymptotes of the incoming and outgoing paths is twice that indicated in the terms of the power series past the first term.

As m/r_0 is expected to be small, one can safely focus on the second term. The deflection may then be found as

$$[30]\ d = 2\frac{m}{r_0} \int_{\rho=0}^{1} \frac{1-\rho^3}{(1-\rho^2)^{\frac{3}{2}}} d\rho$$

and hence

[31] $\int \frac{1-\rho^3}{(1-\rho^2)^{\frac{3}{2}}} d\rho = -\sqrt{\frac{1-\rho}{1+\rho}} - \sqrt{(1-\rho)(1+\rho)}$

Evaluating the integral between ρ ==0,1, one achieves a constant factor of 2. Hence the first-order value for the deflection is

[32] $\delta = \frac{4m}{r_0} = 1.75\text{arcsec}$.

Which is just over double the value achieved with Newtonian mechanics.

2.4 Gravitational Lensing

When a massive object lines up almost directly with the line of sight between an observer and the source, the gravity of that massive object may act as a lens, bending the light of the source around it. Gravitational macrolensing refers to the effect seen when objects such as galaxies and quasars act as the lenses or sources, and one may resolve individual and distinct images caused by the lensing event. Microlensing refers to those situations, such as those commonly found in our own Galaxy, where the individual images cannot be optically resolved, and the event may only be detected by analysis of the resultant change in apparent magnitude of the source.

2.5 Lens Equation

For most practical purposes, the lens is considered to be a single matter inhomogeneity between the source and Observer. This situation is called the 'Thin Lens Approximation'.

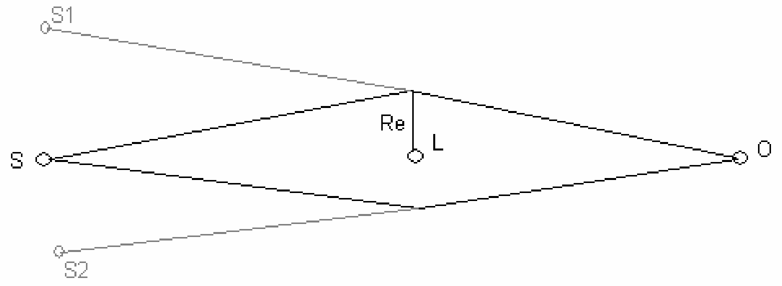

Figure 8: A simplified Lensing Scenario

A simplified microlensing scenario is illustrated in figure 8. Here, the source is S, the lens L, and the observer O. The light, as indicated in the figure, is emitted from the source, and subsequently bent around the lens before reaching the observer.

When there is essentially a point-like lens, at least two images of the source will be seen. These are shown in the figure as S_1 and S_2. If external sheer is also taken into consideration, an effect caused by the tidal effect of objects in close proximity to the light bundles, additional images may also be seen.

The position of these images will be observed to be along the directions corresponding to the real incoming light paths. The images will have identical observed spectra and red shift.

Figure illustrates the comparative angles and angular diameter distances referred to in the Lens Equation. For the thin-lens approximation, hyperbolic paths are approximated by their asymptotes. Hence, in the circular-symmetric case, we find that:

[33] $$\tilde{\alpha}(x) = \frac{4GM(x)}{c^2}\frac{1}{x}$$

where $\tilde{\alpha}(x)$ is the deflection angle and $M(x)$ is the mass inside radius x. Here the origin is chosen to be in the position of the observer for simplicity. From figure (8) it can also be seen that:

[34] $$qD_S = \beta D_S + \tilde{\alpha} D_{LS} \, .$$

where q, β and $\tilde{\alpha}$ <<1. This is true for almost all astrophysical situations relative to gravitational lensing.

The reduced deflection angle is defined as:

[35] $\alpha(q)=(\frac{D_{LS}}{D_S})\tilde{\alpha}(q)$.

or

[36] $\beta = q - \alpha(q)$.

This relation may also be readily derived for non-symmetric situations where all the angles are vector-valued. Hence the 2D lens equation becomes:

[37] $\beta = q - \alpha(q)$

The observable features of something lensed following the lens equation are that the brightness varies with time dependent on the relative motion of the lens and source compared to the observer with durations of between hours and years, the event will be non-repeated (or the chances of the event occurring again precisely are so slim as to be negligible), for a simple event, the light curve will be symmetrical about the peak, with variations for parallax, binary sources/lenses etc, and finally the variation in the magnitude of the event will not be dependent on the wavelength of light but be independent of it. Following a lensing event, the source object would be expected to return to its normal magnification.

2.6 Einstein Rings

The Einstein Radius relates to the unique scenario where the source and the lens are exactly aligned in relation to the Observer. In this case, the two or more images are elongated, distorted and stretched enough that the the symmetry results in a ring-like image being observed, the angular radius of which is referred to as the 'Einstein Radius'. This is an important concept, as it defines the angular scale for a lens situation q_E .

To find the Einstein radius, use the equation for the deflection angle, the lens equation and $x = D_L q$, it can be seen that:

[38] $$\beta(q) = q - \frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 q_E} \ .$$

When the source and lens are exactly aligned, as in the definition, $\beta = 0$, and hence the angular radius is given by:

[39] $$\begin{aligned} q_E &= \frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 q_E} \\ q_E &= \sqrt{\frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 q_E}} \end{aligned} \ .$$

For macrolensing events (the lensing of large objects at large distances), q_E may be of the order of arcsecs and hence the Einstein ring may be easily resolved. In microlensing events however (the lensing of small objects at small distances) this is of the order of milliarcsecs, and cannot be resolved with modern equipment (except possibly using long baseline interferometry, see Chapter 8). The event would only be directly observable by measuring the difference in the apparent magnitude of the source due to the increase in the unresolvable number of source images.

2.7 Gravitational Microlensing

Gravitational microlensing occurs when multiple images are formed with very small angular separations, usually of the order of 10^{-6} to 10^{-3} arcsec. These images are usually unresolvable with the optical telescopes of today. What can be measured, however, is the changing apparent magnitude of the source star over the course of the event.

The brightness of the source star appears to increase and decrease with a very specific lightcurve. This lightcurve is symmetric if the lens star is a point mass (not a binary or without significantly detectable planetary systems), and is achromatic, as all the light from the star is lensed regardless of its wavelength.

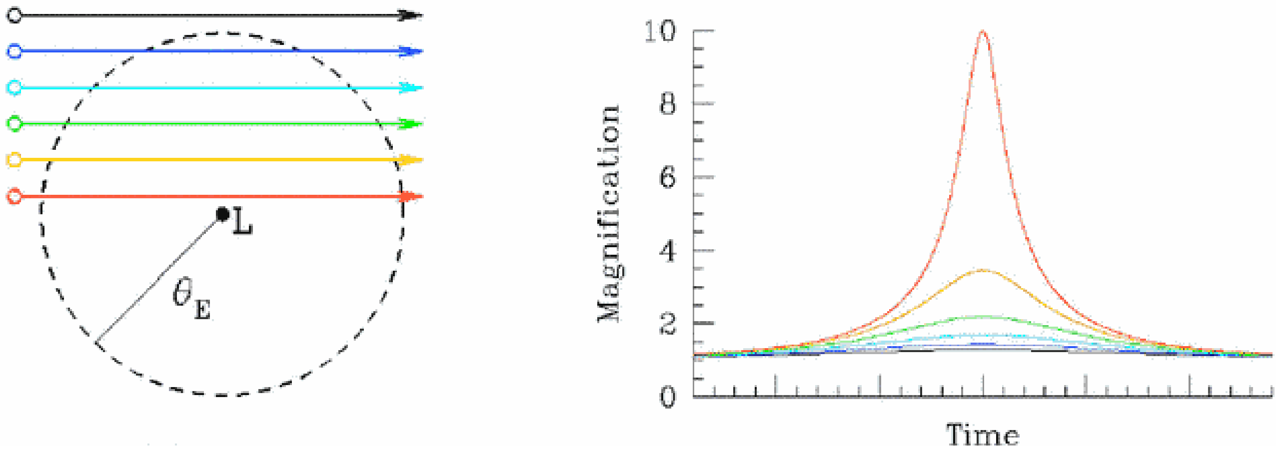

Figure 8: The light curves resulting from microlensing events, dependent on how precisely aligned the source and lens are to the line of sight.

The reason for the change in magnitude of the source is due to the multiple images (or ring). The surface brightness of the source star remains unaltered, but the effective surface area increases with the number of images, and depending on how perfectly the source and lens are aligned with the observer. Hence, although the individual images remain unresolved, the flux detected from the source increases and decreases over the course of the event.

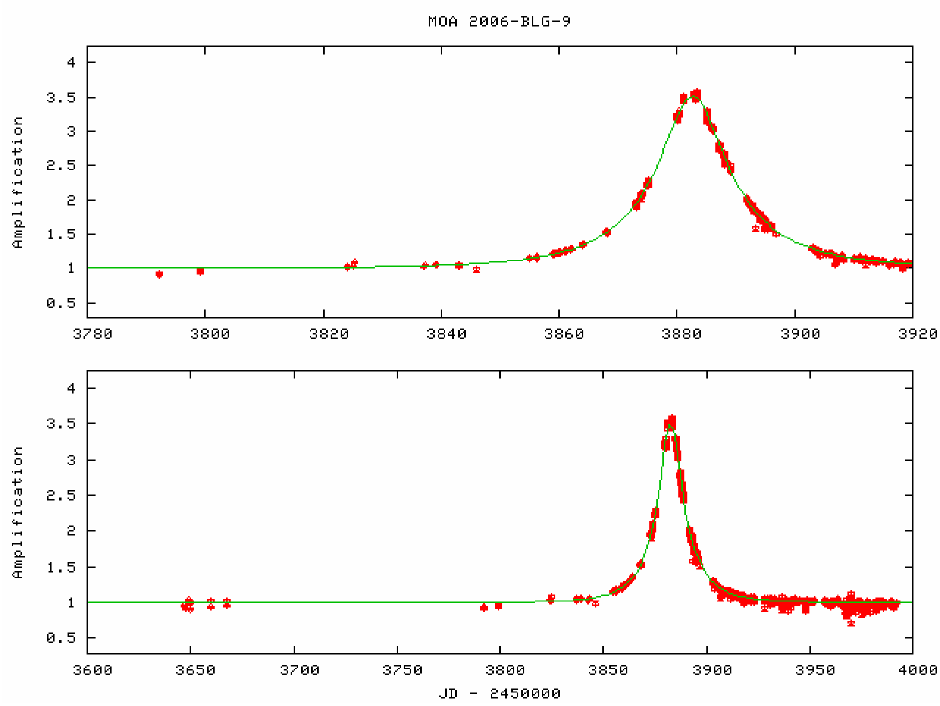

Figure 9:A very typical microlensing event. This is MOA-2006-BLG-9, id gb9-6-63, an event which lasted 19.50 days and reached a maximum amplitude of 3.51.

The effectiveness of gravitational microlensing as a tool for exploring the local area is profound. An estimate of the mass of the lensing object may be obtained if one applies certain assumptions and models. This is currently constrained as a result of the limited reliability of these assumptions without further data on the individual objects. A further advantage of microlensing, however, is the ability to search for multiple systems (binary systems) and extrasolar planets. The extra bodies involved in these lensing events also exhibit specific lightcurves, from which details may be discovered.

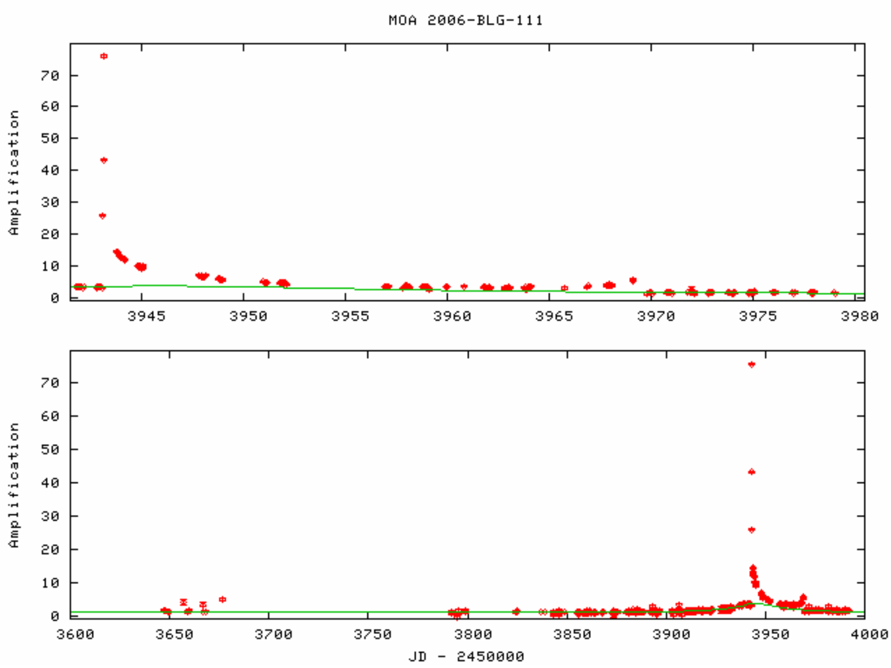

Figure 10: A typical lightcurve for a binary lens system. The green curve shows the best model fit for a single lens, and the red points indicate observations made. This event is MOA-2006-BLG111, id gb3—8-1275, had a time period of 36.66 days, and reached a maximum amplification of 3.62.

Microlensing events do not exhibit a change in the total number of photons emitted from an event, rather it is a redistribution of the photons dependent on the lens. In any given location during the event, the observer will experience either amplification or deamplification in comparison with the lens-less situation (Kubas, 2005). The surface brightness is also unchanged during the event, and hence the only manner in which amplification may be detected is by changing the solid angle under which the source is observed. The magnification of an event is then proportional to the image solid angle, and inversely proportional to the source solid angle. The amplification factor would be given by $1/\det J$, where J is the Jacobian of the lens equation.

Long baseline interferometry (see chapter 8) may soon provide scientists with the ability to observe astrometrically the centroid in microlensing events, a feat which has hitherto been impossible. The prospect of obtaining these measurements is exciting, as it provides and opportunity to reduce the amount of assumption involved in the data analysis of these events, and will remove some of the inherent degeneracy.

On one occasion, in 2000, a lensing event occurred where both the lens and the source were visible. This was particularly advantageous as it meant that further details could be acquired about the mass and intensity of these objects. As time progresses and the two objects have a wide enough angular separation, the individual details will be determined to a much greater degree of certainty, which will also help determine and pin down some of the assumptions made in the processing of other events.

2.8 Quasar Microlensing

Soon after the discovery of the first multiply-imaged quasar in 1979, Chang and Refsdal made the suggestion that the images detected from Earth of quasars may be affected by the presence of stars lying close to the line of site.

Later, in 1981, Gott stated that for the hypothesis of a heavy halo consisting of low mass stars, it ‘should produce fluctuations of order unity in the intensities of the QSO images on time scales of 1-14 years'.

Between 1981 and 1987 various teams used a variety of different techniques in considering potential light curves and magnification distributions. There were several different scientific successes that could be achieved by quasar microlensing, namely that determination of the effect of compact bodies between the observer and the source should be achieved, that the size of quasars could be determined, that the two-dimensional brightness profile of quasars should be discernible, and finally that the mass and mass distribution of the lens objects should be determined.

The first real detection of this event however was to come in 1989, in the quadruple quasar Q2237+0305, as published by Irwin et al (Irwin, 1989). The fluctuations that they detected and analysed could only be explained by the presence of intervening ordinary stars in our own Galaxy, lensing the images of the quasar as a source. These fluctuations were beneficial in observations of the quasar, as published by Wambsganss, Paczynski and Schneider in 1990 (Wambsganss, 1990), as they can put a limit on the possible size of the source quasar.

Based on work done by Grieger et al. in 1988 and 1991, and on the work of Agol and Krolic in 1999, new techniques in determining the one-dimensional brightness profile of a quasar were developed. In 1999, Agol and Krolic (1999) published how, by frequent monitoring of a caustic crossing event in a range of wave bands, it is possible to construct a map of the frequency-dependent brightness distribution of a quasar.

At this stage, some of the potential exhibited by quasar microlensing has been fulfilled, in that it is now known what the effect of an intervening body has on detection, that some limits have been placed on the size of quasars, and that the results of analysis are consistent with other findings on the mass ranges published using other techniques. There is however still a lot more to be achieved in this field.

2.9 Light Curve Variations and Implications

There is a diverse range of light curves resulting from lensing events, arising from the different properties evident in the lensing system. The size and the shape curve depends on the mass of the lens object, the relative transverse velocity of the lens and source, the presence of a body orbiting the lens, the ratio of that body's mass to the lens object's mass, its orbital radius projected onto the lens game, the relative orientation of the source star motion compared to the axis of the lens-body axis, the geometry of the system, the impact parameter of the event, and a host of other variables . It is in the unknitting of the detected light curves that the greatest challenges lie, some of the techniques of which are described in chapter 3.

2.9.1 Parallax

One variation of the light curve that lends itself to breaking the degeneracy often present in microlensing events is the influence of parallax. The effect of the Earth's motion around the Sun skews the light curve in a way that breaks the usually symmetric nature of the event, i.ee it will give rise to small perturbations in the detected light curve (Kaiser, 1986). The other conditions of the light curve remain the same however, ie the light curve is achromatic and non-periodic. This distortion is detectable and useful if the period of the event is slightly less than a year, and if the projected velocity of the lens is not much more than the orbital velocity of the Earth around the Sun.

This effect was first detected in 1995 (Alcock, 1995), where an event of 220 days was found to exhibit a light curve with characteristics consistent with a parallax impact. The team determined from this distortion that the projected velocity of the lens was 75±5 km s^{-1} at an angle of 28°±4° from the direction of increasing galactic longitude. Degeneracy in this event could be broken as a result of the influence of the motion of the Earth. As the orbital motion of the Earth is well known, the light curve may be fit under the reasonable assumption that the only real deviation from the constant velocities of the bodies in the system is the motion of the Earth. If a good fit is reached using the parameters of the Earth's orbit around the Sun, then it may be reasonably asserted that the Earth's motion is the only such deviation, as the probability of there being another deviation with the same characteristics is insignificant.

2.9.2 Caustic Curves

For a binary or planetary lens system, there are some positions where the radiation from the background star is strongly amplified by gravitational potential of the foreground system. These positions take the form of closed curves known as ‘caustics’. When the source star reaches a position on a caustic curve, the lensing system image becomes split into two, or converges into one, depending on the direction of the relative motion of the source star. A direction progressing towards the enclosed region will produce a double image, and progressing out will produce one image. This is certainly a very interesting and useful effect able to be detected in microlensing event light curves. The caustic curves detected in the light curves are useful in that they can, for well-sampled data, provide information on the proper motion of the lens relative to the line of site. This in turn can assist to determine the nature of the lens object, as different populations of object tend to exhibit different proper motions.

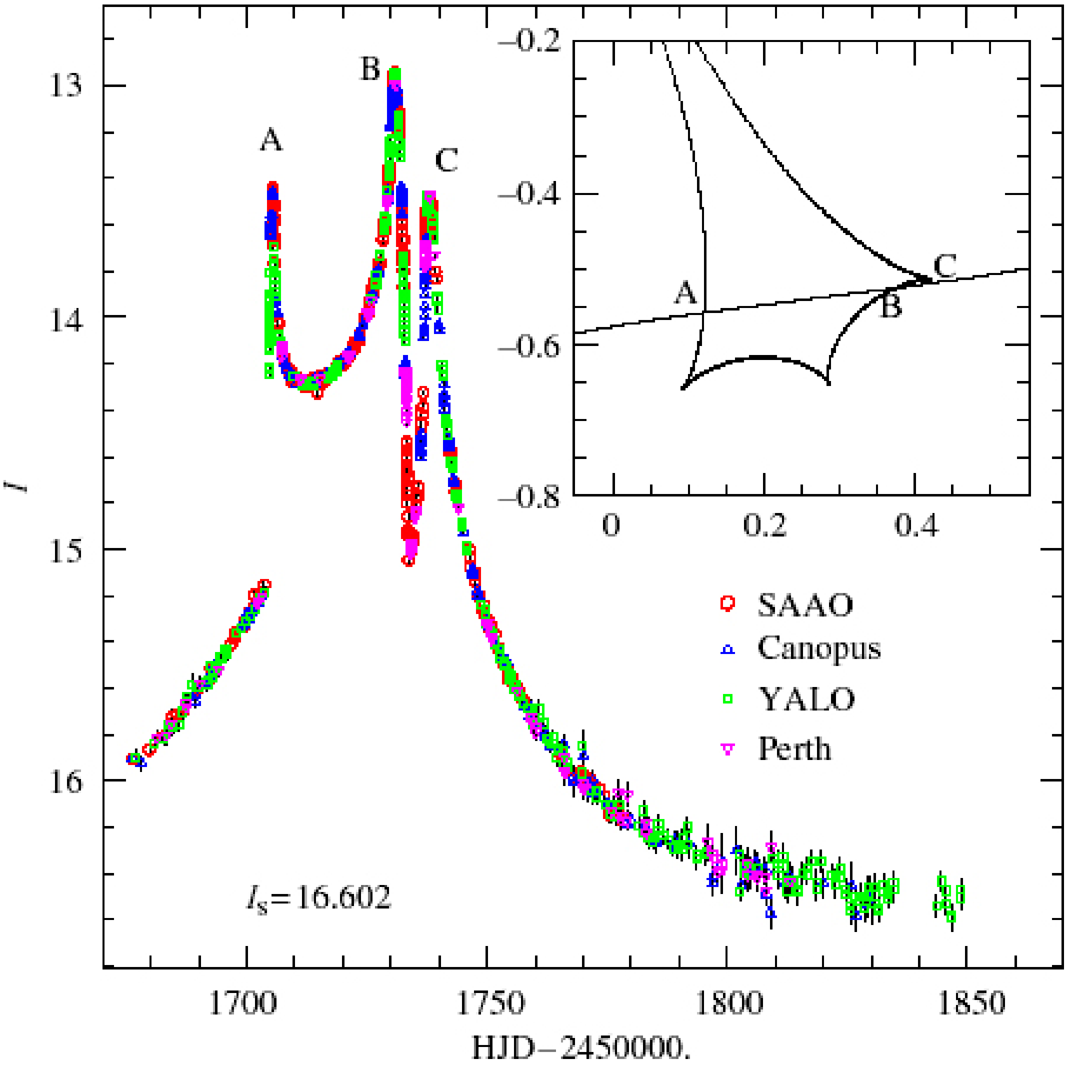

Figure 11: A microlensing event (EROS BLG-2000-5) showing the lightcurve produced when considering the caustic produced with a binary lens. Peaks A and B relate to the ingress egress caustic crossings and C shows the passage close to a caustic cusp.

As caustic curves are mathematical entities rather than physical objects, the curves are precise and infinitely sharp (An, 2002). In binary or planet systems, there is ways a small central caustic lying near to the observer's line of sight, and one or two planetary caustics. The properties of these caustics – their size, position and orientation – are determined by the planet/star mass ratio as well as the projected orbital radius. These caustics can be very prominent in a light curve, as there is typically an abrupt and large change in the apparent magnitude of the source on crossing the edge of a caustic. The deviation from the standard light curve caused by caustic curves is also extremely distinctive, allowing for ready determination of the cause of a deviation in a typical event.

Another 'class' of microlensing events occurs when the source star passes close to the central caustic. In this case, with the source approaching the cusp of two caustic lines, another type of perturbation will occur. This kind of perturbation is much smaller than discussed above but, because of the much smaller impact parameter, the source would be undergoing intense magnification. This class of events is then referred to as high-magnification events.

PLANET have used this technique extensively, which is well illustrated by the event MOA 2002-BLG-33 (Rattenbury, 2005). In this event the source star was determined to be solar-like, and the lens determined to be a close binary. By analyzing the light curve resultant from this event, researchers were able to examine the source's stellar atmosphere, for which the limb darkening was found to be consistent with current stellar atmosphere theory to within 4%.

This was an important proof of the use of microlensing in determining the parameters of a stellar atmosphere.

For more information on this event see chapter 7 below.

CHAPTER 3

Modern techniques and their applications

In the present day there is a variety of different techniques available for analyzing and for detecting lensing events. These are discussed in some depth in this chapter.

3.1 Inverse Ray Technique

Using the Inverse Ray Technique, the reverse of the typical situation is considered. In this scenario, the observer shoots rays toward the lens, which are deflected according to the lens equation, and then collected on the source plane (Wambsgass, 1997). According to the modeled density of light rays at the source plane, the amplification of the source at that position may be determined (Kubas, 2005).

The lens and source planes are both modeled as uniform pixel grids, with the pixel size dictating the lower limit for the source size. The size of the considered grids is important in order to reduce the number of rays that become 'lost'.

Using the Inverse Ray Technique, it is possible to determine the magnitude of the source star as well as a number of its atmospheric details – colour, temperature, etc.

3.2 Inverse Polygon Mapping

A relatively new technique of calculating microlensing magnification patterns has been developed which has been formulated by looking at a ‘backward gravitational lens’, ie a lensing event analyzed in reverse. A lattice of polygons is mapped at the image plane to a first order approximation, from which the local linearity allows the computation of each cell’s contribution to the magnification. This is done by apportioning the area of the inverse image of the cell among the source-plane pixels covered by it.

The second order approximation has been considered in order to control departures from linearity. These deviations may cause variations in the magnification within the boundaries of a transformed cell. The second order approximation can hence be used in identifying and controlling the cells surrounding critical curves.

This technique has been shown to be more effective than the Inverse-Ray-Shooting (IRS) technique in the range $\kappa=0-0.8$ range of mass surface densities. Considering an image-plane lattice of one ray per unlensed pixel gives magnification patterns with theoretical relative errors of only $\sim 5\text{x}10^{-4}$. This level of accuracy would be very difficult to achieve using the IRS technique, as more than 10,000 rays per pixel would be required to produce a similar result. Additionally, the IPM technique needs less than 1% of the computing time of the IRS technique to achieve the same theoretical accuracies.

3.3 Difference Imaging

Difference imaging is a method by which stars of varying brightness may be efficiently detected in a field of millions of stars. For a given star field the average brightness of the stars as they have been ascertained over extensive observations may be subtracted from the field, leaving behind only those objects which have varied. Over the periods of observing star field for a number of years, a comprehensive database of variable stars has been compiled – these can also be subtracted out of the star field, leaving only those stars which display very long-term variability or those stars which are anomalously luminous. These stars can then undergo analysis as potential lensing candidates.

3.4 Astrometry

Typically, the angular separation of the images of a source star in a lensing event within the Milky Way or toward the Magellanic Clouds is approximately 1mas (Paczynski, 1996), and the movement of the centroid of the combined image is also approximately 1 mas. There are currently no telescopes capable of detecting this motion, although the Space Interferometry Mission (see chapter X) will be able to once it is operational.

By considering the case of a lensing event with a point lens forming an Einstein Ring, the angular radius of the ring is:

[40]
$$j_E = [(\frac{4GM}{c^2})(\frac{D_S - D_d}{D_S D_d})]^{1/2}$$
$$0.902\text{mas}(\frac{M}{M_{()}})]10kpc(\frac{1}{D_d} - \frac{1}{D_S})]^{1/2}$$

Where D_s is the distance to the source, D_d is the distance to the lens, and M is the mass of the lens.

The timescale is then defined as:

[41]
$$t_0 = \frac{j_E}{j}$$

where j is the proper motion of the lens relative to the source.

CHAPTER 4

Applications

Gravitational Microlensing has proven to be more than just an interesting observable effect. There are a variety of different ways that microlensing may be used to assist our greater understanding and exploration of the Universe in ways that have hitherto not been possible.

4.1 Dark Matter

It has become evident in recent years that there exists far more mass in the Universe than is directly visible or detectable. The problem was first postulated some time ago (Oort 1932, Zwicky 1933), but only recognised as important in the mid-1970's (Ostriker, 1974; Faber and Gallagher, 1979). This has become particularly apparent by the observation of spiral galaxies such as the Milky Way, although the problem exists for stellar systems through to superclusters of galaxies. In spiral galaxies, observed stars, dust and gas can account for at most 20% of the matter contained within the Milky Way (Griest,1991), and in M31 at least 20% of the matter consists of MACHOs if their average mass is between 0.5-1 Earth masses (Calchi Novati, 2008). The rest is composed of matter that does not emit any detectable form of radiation (it is dark), and interacts very little with the visible matter in the Universe. The rotational velocity distribution of the visible matter in spiral galaxies, however, points to the existence of this dark matter. It is understood that the dark matter component of a given galaxy (spiral or otherwise) and globular clusters form a roughly spherical halo which extends significantly beyond the visible halo. Using gravitational microlensing, it has become possible to test some of the different theories that have been proposed to account for this matter.

One such theory is that dark matter comprises of MACHOS, which may be brown dwarf stars and Jupiter-sized planets, both of which are types of objects emitting very little radiation, and are virtually undetectable if not interacting with other bodies. Paczynski, in 1986, suggested that in order to detect dark matter, one could continuously observe the stars in the Large Magellanic Cloud (LMC) for fluctuations due to gravitational microlensing of the background stars by dark matter within the Milky Way.

The main disadvantage, he noted, was the low probability of detection on a given night. He demonstrated that, if the halo of the Milky Way consists mostly of brown dwarf stars, then one could expect approximately one star in one million to undergo a lensing effect lasting about 40 days. It was shown in 1993 (Alcock et al, 1993) that gravitational microlensing may be applied to test this hypothesis. The premise was that, if dark matter were comprised of these small, faint objects, then they should be detectable by gravitational microlensing. If it were not, then a microlensing survey would be able to establish this fact.

In the situation of light from a background source star being lensed around a foreground object, it is conceivable that the lens object is not a star but is dark matter. The observations of such events can lead to a better understanding of the nature of dark matter – its distribution, size, mass and proper motion. Hence with the application of techniques used in the observation of gravitational microlensing events, a more comprehensive theory around dark matter may be developed.

The MACHO Project, among others, was actively looking for such objects as those suggested by theory. Simulations done prior to the study suggested that the survey undertook would be sensitive to objects between 10^{-7} - 100 M_{Sun}, and that if theory were correct, approximately 10 events over 4 years of survey would be detected.

4.2 Binary Systems

In 1986, Scheinder and Weiss considered the case of a two-point lens in a lensing event. This line of inquiry was due to the knowledge of the time being that most stars in the galaxy were binary rather than single – which has since been disproven.

Schneider and Weiss determined that the resulting light-curve of the event would be different from the standard for single-point lenses.

By analyzing the light curves of a number of microlensing events, it can be seen that there are often anomalies – double peaks, deviations from theoretical curves, and more. These deviations can indicate the presence of other bodies in the system. A short peak close to the main peak (as in figure 10) can indicate the presence of a planet for example, or another star. Disentangling the different possibilities for a system is a complex matter, with a number of degeneracies.

At that stage only binary stars were considered, and the possibility of searching for planetary systems was not yet explored. Howeve,r observable effects of orbiting planetary bodies would have been a natural extrapolation of these theories.

In 2002, Albrow et al. proposed that with a particular observed event, it was possible to show that the lens in the system was a double star of extreme separation, and not a planet. This was an event with a very high signal-to-noise ratio with a short-period deviation near the peak. Normally such an anomaly would be attributed to either a planetary body being present, or an extreme binary system. With this event however it could be shown by the light-curve fit, the extreme value of the event duration, and the blending fraction required for planetary events, that it was probably an extreme binary system.

Additionally, if the two components are separated by an order of Einstein radii, a more complex effect must be accounted for. There will be a certain amount of gravitational interference between them, causing a more complex pattern of magnification than just the superposition of two or more masses (An, 2002).

4.3 Extrasolar Planets

In 1991, Mao and Paczynski proposed a search for extrasolar planets. This was a new concept, although the case of binary stars had been considered long before.

In 1996, three years after the first published detection of a microlensing event, it was proposed that Earth-sized planets may be discovered by monitoring lensed stars in the Galactic Bulge. Bennett and Rhie (2002a) developed the theory behind a binary lens and the resulting light curve to adapt to the situation of a planet orbiting a star. They found that, for planets orbiting in the 'Lensing Zone' (centered between 1 and 4 AU from the host star and extending around a factor of 2 in distance) around the kind of host star reasonably expected in the Galactic Bulge, that the resultant light curve would exhibit a distinct deviation from the curve expected of a typical single lens. It was also proposed that approximately 2% of all M_{Earth} planets may be detected using gravitational microlensing.

Although the use of Gravitational Microlensing to detect Earth-mass planets had been proposed earlier (Tyler, 1995), and theoretical deviations had also been determined earlier (Mao, 1991; Gould, 1992; Bolatto, 1994), this was the first time that this analysisdid not use a point-source approximation and considered the planetary-binary lensing events with realistic finite sized source stars.

In 1998 further investigation was performed into the possibility of detecting extrasolar planets. It was proposed that high-magnification events where the lens star is host to a planet will always have a perturbation in the resulting light curve near the peak. The reason for this is due to the small caustic caused by the planet. This effect is detectable for planets of the same order of mass as Jupiter, and may be detectable for smaller planets.

In 2004, the first planet unequivocally discovered via gravitational microlensing was discovered.

4.4 Black Holes

The search for the population of black holes in our Galaxy has been complicated by their minimal radiation, and the lack of certainty in the various parameters exhibited by them. Most standard methods of detection are inadequate for the purpose of detecting black holes, or even the estimation of their population in the Milky Way. Microlensing, however, is an effect well suited to the detection of black holes, as it is dependent only on the gravity of the lensing mass.

The parameter which best characterizes a microlensing event is the event timescale

[42] $\tau = \frac{2\mathrm{R}_E}{v}$

The Earth's motion causes fluctuations in the detected magnification (the microlensing parallax) in events with long enough time scales, which allows the reduced velocity to be measured.

The result is that, for given D_S and the equation above, the lens mass may be determined solely by x, where if x=1 then the mass goes to 0, and where x=0, the mass reaches infinity. If an event is observed where x approaches 0, then it may be interpreted that the lens for that event is a black hole.

A single black hole may be considered as the simplest example of a gravitational lens, and its lensing behavior can be well mapped using the solution of the Einstein equation of a point mass in a vacuum – the Schwarschild metric. It has been proposed (Bennett et al 2002, Mao et al 2002) that the three longest-period microlensing events detected toward the galactic bulge were due to a black hole microlensing system. From the Schwarschild metric, the solution to the Einstein Equation of a point mass in a vacuum, the lensing behaviour of a black hole may readily be determined.

In 2002 (Bennett, 2002; Mao, 2002), it was claimed that three black holes had likely been discovered. There had been three events detected with particularly long durations and, assuming the lenses were of the stellar population for which for which the velocity and spacial distributions were characteristic of the disk and bulge, the μ-π measurements would indicate that those lenses were medium-weight black holes, typical of the type expected in this area of the Galaxy.

4.5 Stellar Mass

In 2002, An and others (An, 2002) published what they asserted to be the first microlens mass measurement. The microlensing event labeled EROS-2000-BLG-5 was a caustic-crossing binary lens event, and it was found that in modeling this event it was necessary to allow for the microlens parallax and the orbital motion of the binary system. From the measurement of the microlens parallax, the projected Einstein Radius could be determined. The finite source effect on the light curve combined with the estimated angular size of the source, derived from the source position in a colour-magnitude diagram, allowed the team to infer an angular size for the source. From the combination of the angular size of the source star and the projected Einstein Radius, the mass of the lens object was determined to be 0.612+-0.57 M_{Sun}. Given these factors, it was then determined that the lens was relatively close to the Sun at only about 2.6 kpc distant, and the radial velocity of the source was such that it was determined to be a Galactic Bulge giant.

This was a momentous announcement, given the problems with degeneracy faced when analyzing light curves. The determination of the mass of the lens in this event was the first time reported when the degeneracy had been completely broken by photometric means alone.

4.6 Stellar Atmospheres

In 1999, microlensing was used to investigate a stellar atmosphere. The lens in the particular case was a binary system. The light curve showed peaks when the source star crossed a caustic, or a linear region where the amplification by the lens is considered effectively infinite, as predicted in Einstein's 1936 paper. This event was ‘self-lensing’ in the sense that both the source and the lens were in the Small Magellanic Cloud (SMC).

In 1998, Lennon demonstrated that investigating stellar spectra using microlensing was feasible by Lennon, and the following year by various members of MACHO (Lennon, 1998).

Briefly, the concept was that the lensing effect would effectively increase the magnification over what would be achievable by the telescope alone, making the observation of faint objects more achievable. Minniti and others from MACHO used this technique to measure the lithium abundance in a main sequence bulge star during such an event (Minniti et al). The star at its peak intensity was magnified by approximately 1 magnitude.

This technique of enhancing the observable light for the purpose of measuring spectra is particularly effective because of the achromatic impact on the light – light is intensified by the same degree independent of the wavelength of that light.

It is also possible to measure the limb darkening of the background star by the observation of transiting microlensing events. In essence, for events with a small impact parameter, the amplification of the flux of the background star allows for greater scope of measurement, as the amplified flux is sensitive to the brightness distribution of the source. It may then be possible to measure the limb darkening, stellar spectra and stellar spots of the source star, and possibly other variations in the stellar atmosphere.

4.7 Quasar abnormalities

In 1979, and subsequently in greater detail in 1997, it was suggested that the variation of light curves of quasars may be explained by gravitational microlensing of the quasars (Hawkins and Taylor, 1997). This external reason for the anomalies was investigated resulting from the fact that certain detected variations were independent of variations intrinsic in their redshift.

4.8 Understanding Galactic Structure

In 1996 investigations began into detected microlensing effects in M31. Six potential events were detected, of which only one was demonstrably a possible candidate for a lensing event, subject to follow-up observations. The reason for investigating possible events in M31 was to help scientists better understand our own.

M31 is investigated by Pixel Lensing due to the constraints experienced by observing such a densely packed, distant object.

The earliest use of pixel lensing in the search for microlensing events has been in the search for events located towards the bulge of the Andromeda Galaxy (M31). M31 presents an interesting candidate for studying using pixel lensing, because the galaxy is an earlier type than our own Milky Way, and the stellar crowding towards the bulge is far more severe (Kerins, 2006). This issue is compounded by the distance between astronomers and M31, meaning that sounrces in the galactic bulge of M31 are not individually resolvable, let alone the light curve from individual stars.

This limitation is overcome in a fashion by pixel lensing. In this technique, stars are not tracked individually, but instead the flux per pixel is measured. It is postulated that although one pixel may be the result of the light from a number of stars, just one of those stars undergoing a lensing event will cause an overall change in the flux of the entire pixel. AGAPE, POINT-AGAPE and GEST have all either used this premise, or will be using it in their observations.

In addition to the problems inherent in obsserving stars which are individually unable to be resolved, there are other problems that must be overcome (Paulin-Henriksson, 2008). A degeneracy is present between the Einstein crossing time, the impact parameter and the source flux (Gould, 1996). Additionally, it is much more difficult to distinguish between variable stars and genuine microlensing events. The detection efficiency is necessarily more limited as, for example, if a microlensing event occurs very close to a variable star, the microlensing event may be rejected in the search. To overcome the variable tendencies, the event must be of a magnification large enough to distinguish it from the other nearby objects. This would mean that most detected microlensing events would be expected from bright source stars. This again causes a problem, as it is the brighter stars that are most likely to be intrinsically variable themselves.

An event would be detected when a lens passes sufficiently close to the line of sight of a background source star, this distance expressed as a fraction of the lens star's Einstein Radius. When this happens, the source is magnified by a factor A:

[43] $$A = \frac{(u^2+2)}{[u^2(u^2+4)]^{1/2}}$$

$$u^{-1}$$

for high magnification events.

An event would actually be detected when the resultant difference in photon count is detectable over the local noise, so giving:

[44] Lensing event photon count: $(A-1)N$

Galaxy photon count: N_{gal}

Resultant noise (Poisson distribution): $(A-1)N \approx u^{-1}N > \alpha N_{gal}^{1/2}$

Where α=3 would be the typical assumption.

Defining u_t as the threshold impact parameter such that impacts for which $u_0<u_t$ satisfy the above, then the visibility timescale may be determined by

[45] $t_v = 2(u_E^2 - u_0^2)^{1/2}\frac{\theta_E}{\mu}$

$= 2(u_E^2 - u_0^2)^{1/2} t_E$

where

$$\theta_E = [\frac{4GM}{c^2 d_s}(\ell^{-1} - 1)]^{1/2} .$$

So to characterize a pixel lensing event accurately, the event must be sampled at a rate much higher than t_v^{-1}

CHAPTER 5

Systematic Observational Networks

In the early 1990's, technology had developed to the point whereby simultaneous observations of large numbers of stars could be made. This essentially meant that systematic searches for microlensing events became possible. These systematic searches would look for objects which exhibited a change of apparent magnitude in a very distinctive pattern. The conditions imposed on detecting a lensing event of single mass objects (ie not binary systems or systems with planets) were that its light curve was achromatic, that the event was not repeated (the chances of a single object being lensed twice from the same vantage point is practically nil), and that the light curve was symmetric in time around its maximum intensity.

Systematic searches are required in order to detect microlensing events because of the small probability of observing an event at any given time. The more sophisticated the search, the faster the results can be analysed, and the more stars observed at any given time, contributes to the higher probability of observing events.

Most systematic searches concentrate on continuous monitoring of dense star fields towards the galactic bulge of the Milky Way, and towards the

Magellanic Clouds, restricting the observing 'seasons' to between May and September when the Galactic Bulge is visible. For those networks observing other objects such as M31 of course, the observing season differs. Lensing events due to disk objects may be expected when observing toward the galactic bulge due to the long line of sight to most stars through the galactic disk. In both circumstances, the probability of observing an event at a given time is very small. Towards the Galactic Bulge, the probability of a star being lensed at a given moment is just 10^{-6}.

Since 1991 a number of different networks have been established in order to detect and observe these events. These networks become more adept at detecting and following-up these events, with 488 alerts put out by the MOA network alone for the 1997 observing season.

5.1 AGAPE

AGAPE (the Andromeda Gravitational Amplification Pixel Experiment) was operational between 1994 and 1996, observing from the 2-m TBL Telescope with a 1024x1024 CCD in the Pic du Midi (Le Du, 2001). AGAPE observed the bulge of M31 with Pixel Lensing, dividing the area into six individual frames A-F and focusing on red filter images for areas A-D (due primarily to time restrictions), as well as an additional field Z taken toward the nucleus as a reference. AGAPE only observed when M31 was higher than 35 degrees above the horizon. Exposure times were 20 minutes using the red filters, and 30 minutes using the blue (Ansari et al, 2008).

Preprocessing was performed at Pic du Midi using MIDAS.

5.2 Angstrom Project

The Angstrom (ANdromeda Galaxy STellar RObotic Microlensing) project is a network of telescopes from the Canary Island (2-m robotic Liverpool Telescope at La Palma), Hawaii (2-m robotic Faulkes Telescope North, Maui), Arizona (2.4-m Hiltner Telescope), Korea (1.8-m Bohyunsan Observatory) and Uzbekistan (1.5-m Maidanak Telescope), and a collaboration of Astronomers from the UK, the USA, Korea and Uzbekistan. The spread of telescopes allows for 24-hr coverage of M31 between August and February, and the collaboration aims to achieve at least three observations per 24 hr, which will provide enough data to characterize events induced by low mass M dwarf stars properly (Darnley, 2008).

Currently the collaboration primarily takes data from the Liverpool and Faulkes telescopes. Both telescopes use identical optical cameras of 2kx2k CCD arrays with a 4.6 arcsec field of view. The images from the telescopes are automatically preprocessed and made available online within approximately 15 minutes of observations, which are normally conducted in the I band. A sequence of exposures is taken, totaling approximately 30 min, although each individual exposure is a maximum of 200 seconds in duration to prevent saturation.

The Angstrom Collaboration has developed the Angstrom Project Alert System (APAS) which allows them to identify potential events in real time (Darnley, 2008). This is the first alert system that has been seriously attempted outside the Milky Way System, and once it is in regular operation it will increase the detection of lensing events in the same way that real-time detection has assisted in the discovery of events in our own Galaxy. The APAS uses reprocessed data from Point-AGAPE (see 5.14 below) to provide an extended baseline for the events which occur in the overlapping region of where both Point-AGAPE and the Angstrom Project have imaged – approximately 65% of the region observed by Angstrom.

The APAS is not designed to provide high quality automated reduction of data in real time, but to perform basic processing from which rough lightcurves may be derived, which are then scanned by an astronomer, from which an alert may be raised.

Angstrom uses stellar microlensing events in order to study in detail the structure and composition of the inner regions of the Andromeda Galaxy

(M31) (Kerins, 2006), and uses pixel lensing to obtain the data. Many of these events will only have a duration on the scale of days, which is why the APAS will be extremely valuable to this line of study. Such an alert system would allow for more intensely studied events, leading to a better characterization of the events detected, and which could lead to the discovery of gas giant planets and binary star systems in the bulge of M31.

5.3 DUO

DUO (the Dark Unseen Objects Project) uses the ESO Schmidt Telescope to investigate stellar populations between the Sun and the Galactic Centre. With each photographic plate covering approximately 30 square degrees, they are able to cover a relatively large are of sky in any observing run (Alard and Guibert, 1997). Their objectives include seeking the detection of weak (if not dark) objects through the microlensing amplification of background stars.

5.4 EROS

EROS (Expérience pour la Recherche d'Objets Sombres) is a collaboration between astronomers and particle physicists from DAPNIA, IN2P3 and INSU, and has as its objective the search for and study of baryonic dark matter, brown dwarfs or MACHOs in the Galactic halo by looking for microlensing events toward the Magellanic Clouds (Ansari, 2004)). It was initiated in 1990 under inspiration of J. Rich and M. Spiro, following the suggestion of Paczynski in 1986. The first phase (EROS I) opened two complementary programs operating from the ESO (La Silla Observatory, Chile). The first used a 40-cm telescope with a 3.5-million pixel wide-field CCD camera, and searched specifically for short timescale events on the order of hours to days. This was unsuccessful. The second program used the 1-m Schmidt telescope and a dedicated CCD-T40 camera using alternatively a blue and red filter, and was sensitive to longer timescale events of a duration of days. This generated 380 photographic plates, which were subsequently digitized using MAMA. Analysis of these plates revealed two candidate events detected in 1993, out of approximately 8 million stars. One of these candidates was found by EROS II to vary after the event, thus essentially ruling it out as a microlensing candidate.

EROS II was initiated to economize on EROS I, and to remove the inherent problems faced with the use of photographic plates. With an extended collaboration including Danish and Chilean participation, It uses the MARLY Telescope (a 1.5-m telescope recuperated from a French observatory), and is ultimately more sensitive than its forebears, and able to differentiate the optical depth between the effects caused by objects in the Galactic disk, the Galactic bar and the halo. A new wide-CCD camera has been acquired, and the system is guided using a smaller CCD camera.

MARLY was specifically refurbished and automated for EROS, and is able to image simultaneously in two pass-bands over a one-square-degree field of view. There is a CCD camera for each focal plane, between them covering a total field of RA0.7 and Dec 1.4. The global seeing detected is typically less than 2 arcsec. The two cameras each have more the 32-million pixels of 0.6 arcsec. EROS II finally started making observations of July 1996 and continued to 2003. Over the seven years, more than $2x10^6$ image frames were taken of about 80 fields toward the Large Magellanic Cloud, and 10 toward the Small Magellanic Cloud. Additionally, about 150 fields were monitored towards the Galactic bulge, and 29 in the Galactic plane away from the bulge.

The facilities and work done by EROS naturally lend itself not only to the detection of microlensing events but, like other observational networks, the study of variable stars and supernovae. The EROS automated SuperNova search was able to detect about 1 supernova for every two hours of observing time.

A small amount of observing time was given over to the search for white dwarfs and red dwarfs in the solar neighborhood using proper motion measurements.

Using the results from the EROS survey, a constraint was able to be placed on the distribution of compact dark bodies in the Galactic halo. It was found that massive compact objects with masses in the range $2 \times 10^{-7} M_{\mathrm{Sun}}$ and $1 M_{\mathrm{Su}}$ cannot represent more than 25% of the halo mass, in the case of a standard spherical, isothermal Galactic halo, encompassing $4 \times 10^{11} M_{\mathrm{Su}}$ out to 50 kpc.

5.5 GMAN

The Global Microlensing Alert Network was designed to assist in following up MACHO alerts on a nightly basis (MEGA, 2007). It communicates with a global array of 1-m class telescopes, including the UTSO 0.6-m telescope, the CTIO 0.9-m telescope, the CTIO 4.0-m telescope, the MSO 0.8-m telescope, the Wise Observatory 1.0-m telescope, and the MJUO observatory 0.6-m telescope. GMAN observes events towards the Galactic Bulge and the Magellanic Clouds.

5.6 MACHO

The MACHO Project was a collaboration between scientists at the Mt Stromlo & Siding Spring Observatories, the Center for Particle Astrophysics at the

Santa Barbara, San Diego, & Berkeley campuses of the University of California, and the Lawrence Livermore National Laboratory. It was led by Charles Alcock, of the Lawrence Livermore National Laboratory, California (MACHO, 2007).

The ambition for the MACHO project was to test the MACHO hypothesis that the dark matter in the Galactic halo is composed of dense objects such as planets or brown dwarfs. MACHO is an acronym for Massive Compact Halo Object, and the MACHO Project derives its name from this hypothesis.

The MACHO Project utilized a two-channel system including 8 2048x208CCD cameras mounted on the Mt Stromlo 50-inch telescope. This combination leads to an extremely large data rate which is accommodated by custom electronics and online data reduction.

About 27,000 images were taken by the MACHO project in its decade of use, after which analysis has yielded databases containing the light curves in two colours for 8 million stars in the Large Magellanic Cloud, and 10 million in the Galactic Bulge. Further inspection has revealed 4 microlensing candidate events toward the LMC, and 45 toward the Galactic Bulge.

5.7 MEGA

The ‘Microlensing Exploration of the Galaxy and Andromeda’ is a UN/NC collaboration which surveys objects primarily toward the Andromeda Galaxy (MEGA, 2007). Their objective is to determine the MACHO content in the extended halo of M31.

It is anticipated that this will give researchers further data from which conclusions may be drawn about our own Galaxy.

MEGA utilizes a network of telescopes including the 2.2-m Isaac Newton Telescope, the 1.3-m McGraw-Hill Telescope, the 2.4-m Hiltner Telescope and the 4.0-m KPNO Telescope, each observing 1 hr per night in 2002. Using difference image photometry, they monitor apparently varying but unresolvable objects in M31.

5.8 MicroFUN

MicroFUN is a collaboration of Astronomers spanning five continents dedicated to taking follow-up observations of interesting, high-magnitude events. Its purpose is to assist in the detection of extrasolar planets, and focus in particular on events occurring in the galactic bulge. MicroFUN is supported by the NSF and NASA, and is affiliated with GMAN. It selects targets from OGLE-III and MOA.

This follow-up network consists of professionals and amateurs from around the globe, particularly below 30 degrees northern latitude in order to obtain optimal imagery of the Galactic Bulge. Observers come from the USA, Korea, New Zealand, Australia, French Polynesia, South Africa and Israel. Equipment ranges from 0.25-m telescopes with basic CCDs, to the 2.4-m telescope at Kitt Peak. Its wide range of instrumentation and locations gives this network a corresponding range of advantages and disadvantages over the more conventional networks. However, perhaps its most appealing feature is the accessibility it provides for dedicated amateur astronomers to participate and contribute to the field. Each observatory will typically have a small to medium aperture telescope with an automated equatorial mount on a permanent pier, and be carefully polar-aligned. A scientific-grade CCD camera will be used with a control computer with a high-speed internet connection used for operating the CCD, telescope, and do basic data analysis. Many of the observers are also members of the Center for Backyard Astrophysics (CBA).

MicroFUN has provided some critical data in events determined to be of interest, such as in OGLE-2005-BLG-169 (see chapter 7).

Once an alert has been issued, the observers will image the event target every couple of minutes, when conditions are favourable, and then upload the data to the MicroFUN Headquarters at OSU for photometric processing using relative

PSF-fitting photometry. The amplitude of the event is measured in comparison to five nonvariable 'reference' stars, which allows fairly precise photometric results even in non-photometric conditions. The purpose of processing all the data at the Headquarters, or at least by the same software as used by Headquarters, rather than observers processing the data individually, is to ensure the results are obtained in a uniform fashion, and hence some of the disadvantages in the wide spread of telescope and observer types are reduced. Because of the spread of locations, MicroFUN can provide almost 24-hr. continuous data on a given event towards the Galactic Bulge. The imaging and photometric time-series data become the property of the MicroFUN consortium, and the raw uncalibrated photometry is made available online to other networks and interested parties to assist in the tracking of these events.

Dependent on the results of processing, MicroFUN will either cease observing the event, or observe intensively. For events which exhibit particularly interesting features, a more rigorous photometric time-series analysis will be performed on the data using difference-imaging. This provides the maximum detail possible, and using this detail a paper will be drawn up and published describing the scientific results, with all the contributors listed as co-authors.

5.9 MOA

MOA is an acronym for Microlensing Observations in Astrophysics, and is collaboration of astrophysicists from New Zealand and Japan, headed by Prof Yusashi Muraki of Nagoya University (Hearnshaw et al, 2005). The main research centre for this group is at Mt John, Tekapo, in New Zealand, primarily using the recently-installed 1.8-m telescope dedicated to the project, which routinely monitors approximately 10 million stars toward he galactic bulge and the Magellanic Clouds. They use a customised microlensing detection system, and share alerts and data with other member institutions. Beyond this, software was developed in order to reduce digital images automatically with the aim to be used in large astronomical projects where the task of keeping up with data reduction would be beyond a person or a group of people. The data are stored in an object database implemented in C++, from which it is possible to access hundreds of observations of million-star star fields without unnecessary delay. Under testing, it was found that the reductions made possible by this system were such that for a task requiring 6 months of analysis by a group of people, the system could complete the data within 4 months. Although the software may be used with nearly any telescope/detector combination, it is specifically designed for use in larger projects with mosaic CCD detectors which typically produce too much data to be processed manually.

MOA has a new task scheduler in development that is designed to coordinate parallel reduction on a cluster of workstations. The improvement in reduction throughput will increase in a linear fashion dependent on the number of workstations involved. The design used should allow for heterogeneous computers to be added to increase throughput so, for example, computers normally used during the day may be included in the cluster at night.

MOA concentrates on the detection and observation of high-magnification gravitational microlensing events, events which yield a much higher sensitivity to the detection of extrasolar planets. As of the current date, every planet discovered using gravitational microlensing has had observations contributed by MOA. In addition to finding planets, an objective of the group is to detecting variations in stellar atmospheres and the nature of dark matter.

5.10OGLE

OGLE is an acronym for Optical Gravitational Lensing Experiment, established in 1992 and based at Warsaw University. It is led by Prof Andrzeg Udalski. The collaboration consists of the Observatory at Las Campanas Observatory in Chile, as well as Princeton University and the Carnegie Institution.

The primary focus for OGLE is the search for dark matter by observing events toward the Galactic Bulge and the Magellanic Clouds, although it has been involved in the detection of a number of extrasolar planets.

OGLE’s history may be separated into three distinct phases, with OGLE-I the initial pilot period when the collaboration was established. It began in 1992 and continued throughout four consecutive observing sessions using the 1-m Swope Telescope at the Las Campanas Observatory. This initial phase was extremely successful, but did suffer from some limitations, most importantly the limited amount of telescope time available. It was clear that a dedicated telescope was required.

In 1996, construction had been completed on the 1.3-m Warsaw Telescope and accommodation, and OGLE-II could begin in January the following year.

In June 2001, OGLE-III commenced regular observations at the Las Campanas Observatory. The new second generation CCD mosaic camera used consists of eight SITe 2048x4096 CCD chips acquiring a total field of about 35’x35’, and focusing on transit events toward the Galactic Bulge and the constellation of Carina. It mow regularly monitors 130 million stars in the galactic bulge, and 33 million toward the Magellanic Clouds. At this stage, over 500 events are detected each observing season. It has been an extremely successful project, leading to the detection of interesting events followed up by other observers the world over.

5.11PLANET

The aim of PLANET is to 'perform precise and frequent multi-band observations of ongoing microlensing events in order to study departures from a light curve that are due to lensing of a point source by a single point-like lens' (Dominik, 2003), . The intent is to provide valuable contributions to the fields of Galactic Structure and dynamics, binary stars, extra-solar planets, stellar atmospheres and variable stars. PLANET is a follow-up Network, reacting to alerts published by the detection networks OGLE, MACHO, EROS and MOA.

Used by PLANET are four 1-m-class telescopes spread around the southern hemisphere - in Australia, Chile, Tasmania and South Africa. These telescopes are all distributed in such a manner as to allow for continuous observation of the Galactic Bulge when necessary between April and September.

PLANET observes predominantly in the I-band, although it does image in the V-band approximately half the number of times as the I.

Since 2005, PLANET have worked with Robo-Net.

5.12 POINT-AGAPE

This survey has now ceased, however the archived data obtained between 1999 and 2002 provide the Angstrom Project with valuable historic material with which it may compare recent images to search for fluctuations in magnitude. POINT-AGAPE performed a dark matter search via microlensing of M31 using the wide-field camera of La Palma's 2.5-m Isaac Newton Telescope, and was initially the result of a joint effort from MEGA and AGAPE. It surveyed a 0.6° area of the disk and bulge of M31 (http://star.pst.qub.ac.uk/~sjs/m31micro/m31micro.html).

Most exposure times were limited to between 5 and 10 minutes per night and, as the telescope time allocated was normally of an hour or less, not all filters were used each night (Paulin-Henriksson, 2008). Another limitation encountered was that the observations tended to be clustered together, as the Wide Field Camera was not permanently mounted on the telescope.

Four microlensing candidates were discovered by POINT-AGAPE in the years 1999-2001, of which one is most likely caused by a stellar lens in the bulge of M31, whereas the other three may be explained by MACHOs or stellar lensing.

In 2004 (Paulin-Henriksson, 2004) POINT-AGAPE published the results of seven microlensing events from three years of observation with pixel lensing, which implied a constraint on MACHOS towards M31. The results indicated that less than 60% of standard halos may be composed of objects between 0.1 and 1 Earth masses.

5.13Robo-NET

Robo-NET 1.0 is a collaboration of ten UK Universities led by Liverpool JMU and originally funded by PPARC (now defunct, the funding has been replaced by STFC), and operates global network of 2-m-class telescopes. Observations are based on the observing time obtained at the Liverpool Observatory for given projects, and time on the North and South Faulkes Telescopes. The Liverpool and Faulkes telescopes are identical 2.0-m telescopes with similar instrumentation. There is an off-axis CCD autoguider and a science fold mirror able to direct the science beam to one of four side ports or to be retracted to allow the beam to pass through the straight-through port, allowing different instrumentation to be placed at the focus of the telescopes in less than 20 seconds. These alternative instruments at the Liverpool Observatory include a 2048x2048 CCD camera with a 4.6 arcsec field of view, an infra-red imaging camera with a 1.7 arcsec field of view, a prototype fibre-fed spectrograph and a double-beam fibre-fed spectrograph. The Faulkes Observatories instrumentation also include identical copies of a RATCAM camera with eight filters and a low-resolution spectrograph.

Its principal aims are to determine the origin and nature of Gamma Ray Bursts, to providing rapid response and optimized robotic monitoring using a global telescope network able to react automatically to alerts from the Swift Spacecraft, to integrate a global network of telescopes to act effectively as a single instrument, and to apply developments in e-science to maximize returns. It also has as a primary goal the search for cool, Earth-sized extrasolar planets. Their initial strategy is to discover microlensing events, to detect in these events planet-like anomalies, and subsequently to characterize them.

The discovery of microlensing events is primarily through the OGLE-III Early Warning System, after which an effective follow-up observation strategy has been developed and coupled with an over-ride to the characterization once an appropriate anomaly is detected.

5.14SuperMACHO

SuperMACHO is the successor to the highly successful MACHO Project. The primary purpose of SuperMACHO is to search for microlensing events in the Large Magellanic Cloud in order to determine the reason for the number of microlensing events detected in this part of the sky.

CHAPTER 6

MAJOR CONTRIBUTORS IN MICROLENSING (POST 1960)

This field has been influenced by the work of a great number of people over the last few hundred years. There are some people which, however, have influenced it more than others.

The people who heavily influenced the pre-1960 era have already been covered in Chapter 1. Below are just some of the people who have helped develop gravitational microlensing into the well-known phenomenon it is today.

6.1 Alcock

Charles Alcock is a New Zealander, and currently director at the Harvard-Smithsonian Institute for Astrophysics. He was awarded the Beatrice Tinseley award in 2000. Alcock credited Pacynski's 1986 paper with inspiring him to pursue the search for MACHOs via microlensing, and from there the MACHO evolved (Harvard University Gazette, 2004).

Alcock graduated with a BSc from Auckland University in 1973, qnereceived his PhD in Astronomy and Physics from the California Institute of Technology in 1977. From there, he became a Member of the Institute for Advanced Technology in Princeton and, in 1979, he held a visiting Professorship at the Niels Bohr Institute in Copenhagen and in 1983, a visiting Fellowship at the Australian National university. He has also held the Reese W. Flower of Astronomy and Astrophysics at the University of Pennsylvania. (CFA Press Release, 2004)

In 1986, Alcock became the Head of the Astrophyics Centre of the Lawrence Livermore National Laboratory, and from 1994 became the heald of the laboratories' Institute of Geophysics and Planetary Physics.

Alcock's primary interests lie in the detection of massive compact halo objects by gravitational microlensing, and in comets and asteroids.

6.2 Bennett

David Bennett was a founder of MACHO, and very active in the field of microlensing. He was born in America in 1959.

Bennett graduated with a Bachelors Degree in Mathematical Physics from the Case Western Reserve University in 1981, and received in PhD in Physics from Stanford University in 1986. He has held academic positions in a diverse range of Universities – The University of Notre Dame, the University of California, the Lawrence Livermore National Laboratory, Princeton University and the University of Chicago. He currently holds the position of Research Assistant Professor at the University of Notre Dame.

In his academic career, Bennett has published over 120 papers and 35 Conference proceedings, and (with Rhie) demonstrated the theoretical possoibility of detecting Neptune-mass planets in Jupiter-like orbits around distant stars within the Milky Way. His areas of interest lie in Gravitational Microlensing (near and far) (Bennett CV).

Bennett is involved with the proposal for GEST.

6.3 Gaudi

Scott Gaudi is an Assistant Professor in the Department of Astronomy at the Ohio State Universiity. His interests lie in extrasolar planets, Kuiper Belt objects, and gravitational lensing.

Gaudi studied Astrophysics at the Ohio State University as a graduate student, where one of his Supervisors was Prof Andrew Gould. He started his Postdoctoral career as a Hubble Fellow and Member of the Institute for Advanced Study in Princeton. Gaudi was a Menzel Postdoctoral Fellow in the Theoretical Astrophysocs Division at the Harvard-Smithsonian Center for Astrophysics in Cambridge (Gaudi homepage).

6.4 Gould

Andrew Gould was one of the Supervisors of Scott Gaudi on his graduation. He is currently a Professor at the Ohio State University.

Gould has a particular interest in Gravitational Microlensing, Dark Matter, Planetary Searches, Galactic Structure, and the Cosmic Distance Scale. He is very active in PLANET, and has been involved in discovery of most of the planets as yet detected using microlensing (Gaudi homepage)

6.5 Paczynski

Bohdan Paczynski was a prolific Polish astrophysicist who specialised in theory surrounding stellar evolution, accretion disks, gamma ray bursts and gravitational microlensing. His first paper was published when he was 18, in Acta Astronomica (Paczynski, 1958). He studied astronomy at Warsaw University between 1959 and 1962, receiving his doctorship in 1964 under the tiutelage of Stefan Piotrowski and Wlodzimierz Zonn.

From 1962 Paczynski joined the Center of Astronomy of the Polish Academy of Sciences, where he worked for around 20 years. At the Academy, he achieved Habilation in 1974, and Professorship in 1979. As a result of his work in theoretical Astronomy, he became the youngest member of the Polish Academy of Sciences at 36.

Paczynski is well known in the field of microlensing as the initiator of OGLE (see chapter 5) and the ASAS (All Sky Automated Survey) (Paczynski CV), and for first using the term 'microlensing' to describe the effect. He has received the title of Honoris Causa by Wroclur University and the Nicolaus Copernicus University in Poland, and awarded the Henry Norris Russell Lectureship of the American Astronomical Society, amongst a range of other prestigious awards.

Paczynski authored and co-authored over 258 papers before he died in 2007 of brain cancer.

6.6 Udalski

Andrzej Udalski is one of the founders of OGLE, and currently holds the position of Professor in the Astronomical Observatory of Warsaw University in Poland. In 2001, he developed an advanced camera to improve the sensitivity of the Polish telescope in use in Chile (OGLE Homepage)

6.7 Wambsganss

Joachim Wambsganss is a German Astrophysicist born in 1961 in Landau. He studied from 1981 to 1987 in Heidelberg and Munich in Astronomy and Physics, graduating with a thesis written on Gravitational Microlensing under Luneburg and kegel. Following this he attended Princeton University in 1992, and then the Max Planck Institute for Astrophysics in Garching. In subsequent years, Wambsganss attended the Astrophysical Institute in Potsdam, and is now a Professor at the Astronomical Institute at Heidelberg.

Wambsganss is a member of the American and German Astronomical Societies, and assists with the Children's University. His specialty subjects are Cosmology, X-Ray Astronomy and Physics. His particular passion iis the search for extrasolar planets, and the possibility of life throughout the Universe. He has authored or co-authored a large number of papers relating to gravitational lensing, including some recognized as keeystone papers in the field (Living Reviews, German Wiki)

CHAPTER 7

Successes and their Ramifications

Gravitational microlensing, whilst for a long time being an effect that was described only in theoretical terms, has now been detected many times throughout our own galaxy and neighbouring ones. Events are being detected almost nightly now, and the rate of detection is only increasing. With the advent of space-based surveys, this rate will climb to the point where the detection of events will be commonplace.

Some detected events however have been particularly significant. These are described below. The first detection of a microlensing event is not described here, as it has already been covered in some detail prior to this chapter.

7.1 OGLE-2005-BLG-071

This was a high-amplification event where the source passed very near to two caustic cusps in the lens system (Rattenbury, 2006). The light curve exhibited a dramatic double-peaked feature at the event maximum. This event was extremely well sampled, and the accuracy was high, with observations from MicroFUN, Robo-NET, MOA, OGLE and PLANET. The best fit model found had the properties:

[46] $$\begin{aligned} q &= (7.1 \pm 0.3) \times 10^{-3} \\ d &= 1.294 \pm 0.002 R_E \end{aligned}$$

This model was found from preliminary analysis of finite source star effects during the event, and the parallax effects seen during the event wings. The mass estimate for the lens was very low, at

[47] $$0.08 \leq \frac{M_L}{M_{(*)}} \leq 0.5,$$

and the lens distance was found to be most likely

[48] $$1.5kpc \leq D_1 \leq 5kpc.$$

This means that the it was determined to have a planet, and absolute mass of the planet was found to be around

[49] $$0.05 \leq \frac{M_P}{M_J} \leq 4.$$

Hence if

[50] $$D_S = 8kpc, D_L = 4kpc,$$

then the planetary orbit radius is

[51] $$1.14 \leq a \leq 2.9AU.$$

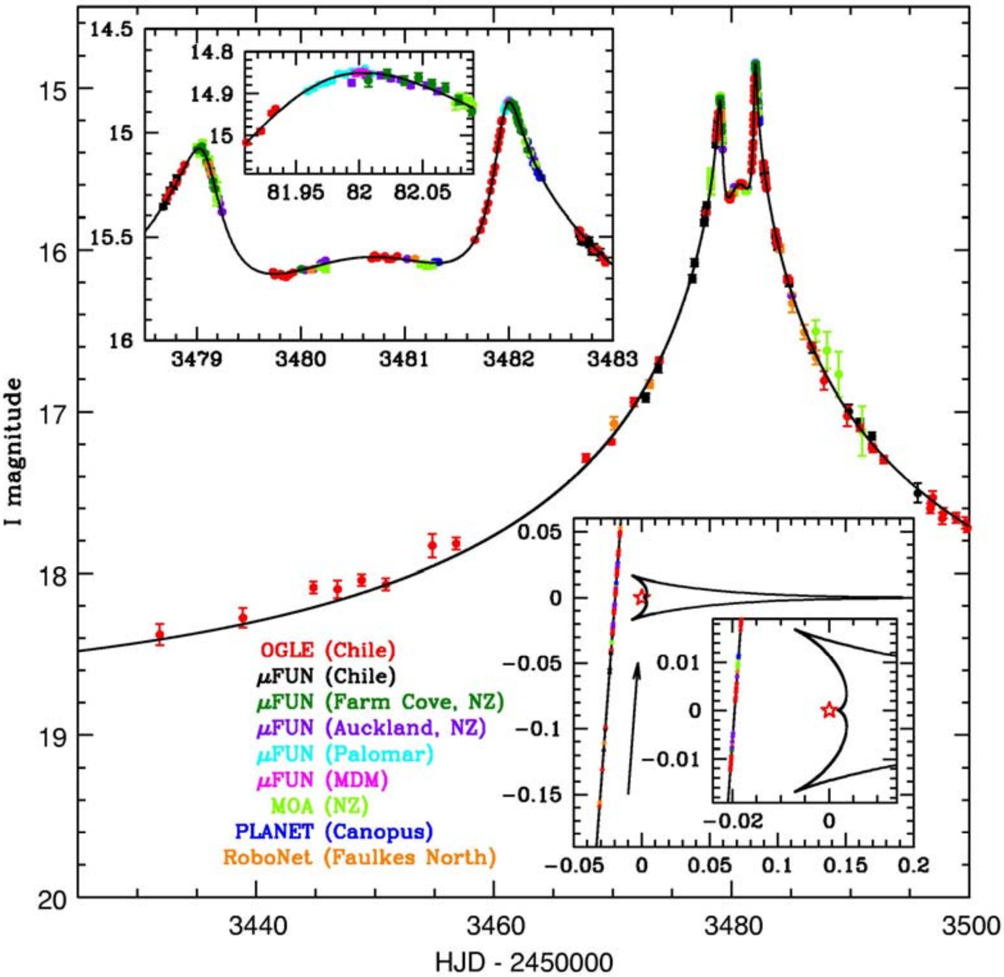

Figure 12: The light curve resulting from OGLE-2005-BLG-071

This was the second clear detection of a planet using Gravitational microlensing, and helped fuel public interest in the stars – as seen in figure 13 below.

Figure 13: An illustration of the enthusiasm shown by the public for the discovery

7.2 OGLE-2005-BLG-390

This event had a very low maximum amplification, with

[52] $A_{\max} \approx 2.8$

(OGLE, 2006a; OGLE, 2006b). The planetary deviation was found only in the wings of the event, as seen in figure 14 below.

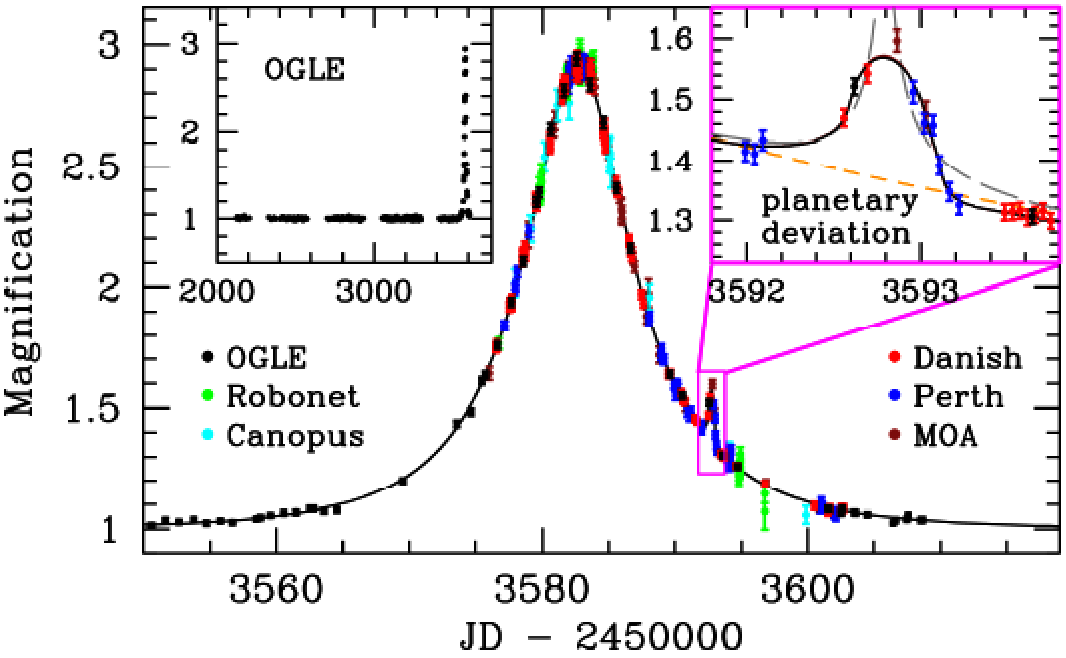

Figure 14: The best fit ligyhtcurve for OGLE-2005-BLG-390

The best fit model for this event had

[53] $$q = (7.6 \pm 0.7) \times 10^{-5}$$ $$d = 1.610 \pm 0.008 R_E$$

With the lens star mass

[54] $$M_L = 0.22^{+0.21}_{-0.11} M_{(*)}$$

and estimated at a distance of

[55] $D_L = 6.6 \pm 10 kpc$

with 68% confidence. This suggests a planet mass of only

[56] $5.5^{+5.5}_{-2.7} M_{\oplus}$,

which is extremely low, and orbiting at about

[57] $2.6^{+1.5}_{-0.6} AU$,

with a period of 10 years. It is expected that the lens star was an M-class dwarf of 0.2 solar masses. The mechanics of the system would have resulted at some time in three images of the source star.

The planet detected in this event would not be big enough to have accreted a large amount of gas such as Saturn or Jupiter, and its surface temperature is estimated to be between 50 and 70 K. This makes it of a similar nature to a large Pluto, or to the cores of Neptune or Uranus. Of the 170 planets detected before this event, 3 of which were found by microlensing, this was the smallest and most similar to the Earth. It provided evidence of gravitational microlensing's effectiveness in the search for Earth-sized extrasolar planets, and helped intensify microlensing planet searches, as well as assisting the public in seeing the possibilities in such observations.

7.3 O GLE-2005-BLG-169

OGLE-2005-BLG-169 was a very high magnification event, with

[58] $A_{\max} \approx 800$.

It exhibited a brief and small, but significant deviation at the peak. The best fit model indicates

[59] $$\begin{aligned} q &= 8^{+2}_{-3} \times 10^{-5} \\ d &= 1.00 \pm 0.02(3\gamma) \end{aligned}$$

Further analysis reveals that

[60] $$\begin{aligned} M_L &= 0.49^{+0.23}_{-0.25} M_{(*)} \\ D_L &= 2.7^{+1.6}_{-1.3} kpc (90\% \text{confidence}) \end{aligned}$$

The most probable mass of the host planet is about $13 M_{Earth}$ orbiting at approximately 2.7 AU. This event highlights the usefulness of these high-magnification events in the search for low-mass planets. This was the fourth planet detected by microlensing, and the second Neptune-mass planet. It also helped to develop the assertion that these 'Cool Neptunes' are relatively common in the Galaxy, with a frequency of 16%, with 90% confidence.

7.4 MACHO-98-BLG-35

This was also a high-magnification event, at

[61] $A_{\max} \approx 96$

(Rattenbury, 2006). There was a perturbation detected near the peak as detected by MOA and MPS, consistent with a low-mass planet. It was first analysed by Rhie et al (1999), then reanalysed using difference imaging photometry on an expanded dataset. The subsequent best-fit model for this reanalysed data corresponds to a star/planet system with properties

[62] $q = 1.3^{+0.2}_{-0.9} \times 10^{-5}$,

orbiting at

[63] $1.225 R_E$.

This leads to a planet mass of

[64] $M_L = (0.4 - 1.5) M_{Earth}$

at 2.3 AU. The assumptions inherent in this system are

[65] $M_L = 0.3 M_{(*)}, D_L = 6kpc, D_S = 8kpc, \rho = 2 \times 2 \times 10^{-3}, R_S = R_{(*)}$.

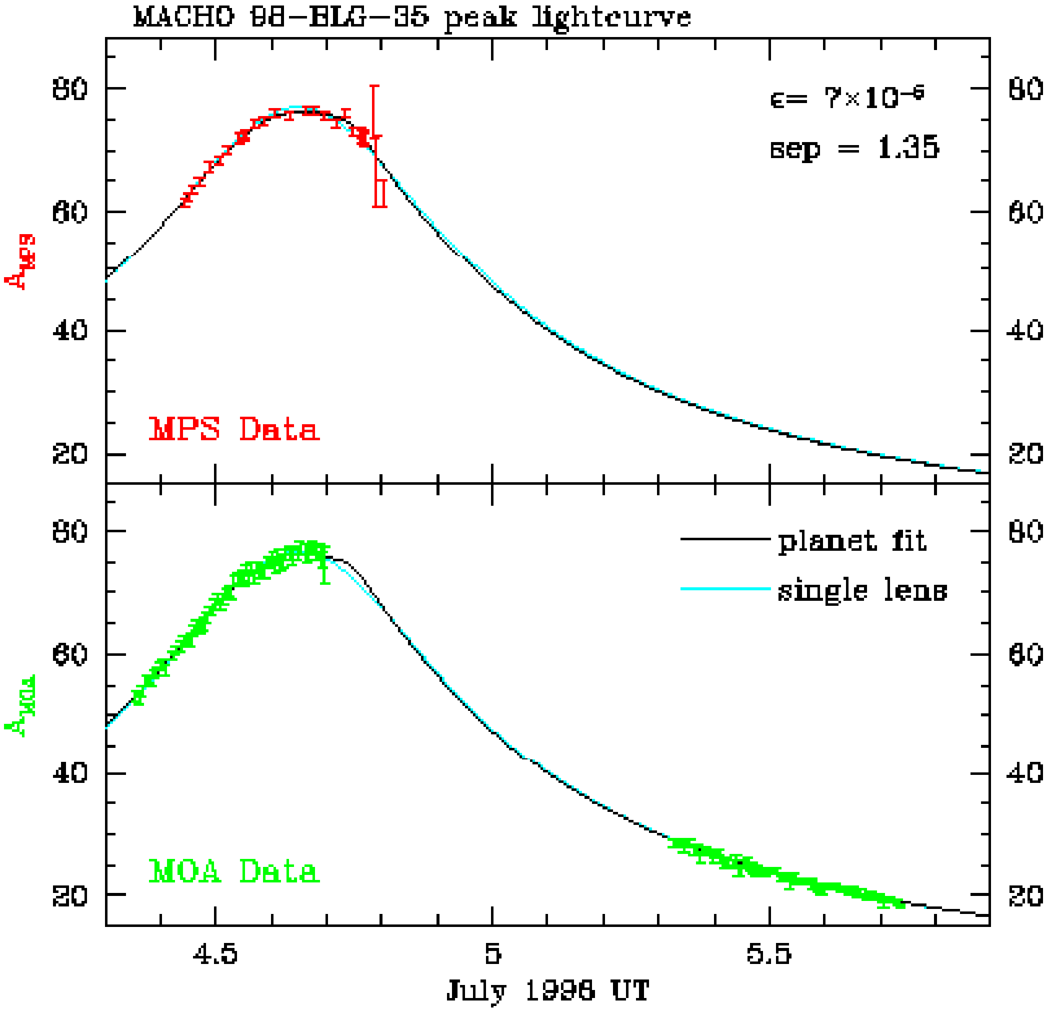

Figure 15: The best fit curve for MACHO-98-BLG-35

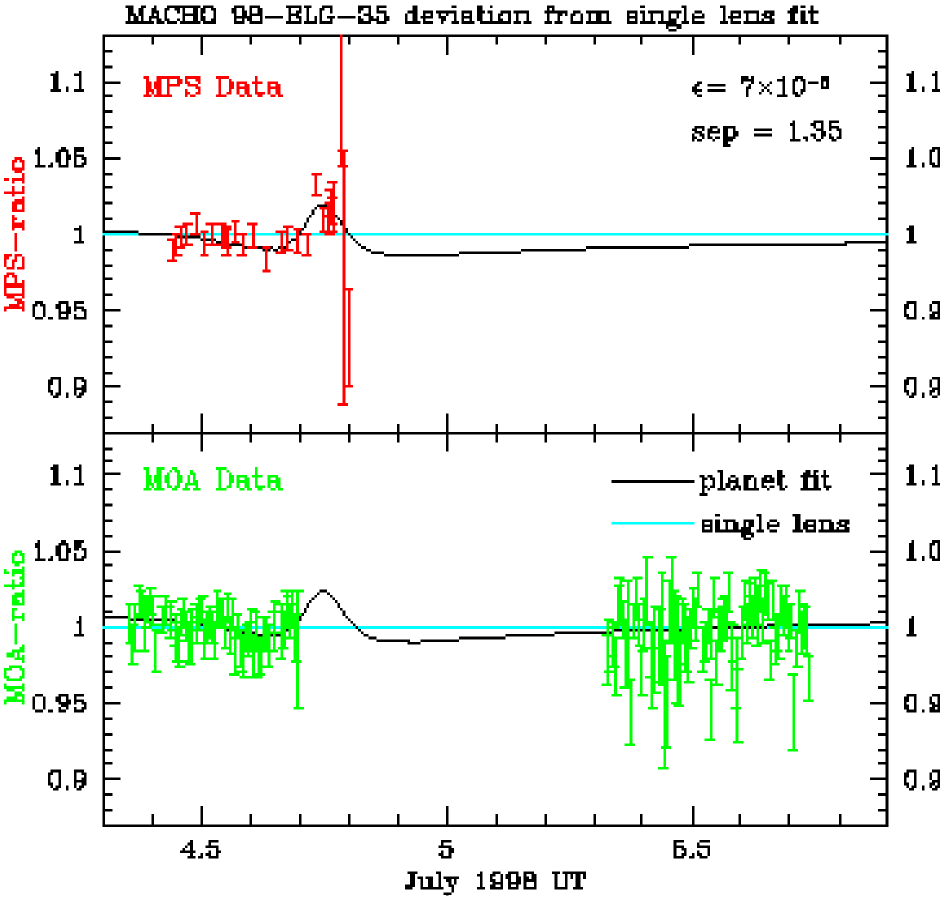

Figure 16: The deviation exhibited by MACHO-98-BLG-35

One of the difficulties with this event was that some of the photometry was not analysed using state-of-the-art difference imaging analysis techniques, and until the images are analysed using difference imaging methods the results for this event remain inconclusive. PLANET subsequently released its own data on this particular event, which evidenced no perturbation near the peak, and which supported the hypothesis that this was a single lens system with no companion such as that proposed (PLANET, 1999).

Despite the uncertainty of this discovery, this event sparked a lot of public interest, especially when it was later published that, given the assumptions, this planet could be capable of hosting life. As would be expected, speculations on extraterrestrial life abounded. This could only be an advantage in that it has kept microlensing high in general esteem as the only method for detecting Earth-sized planets.

7.5 MOA 2002-BLG-33

This particular event was unusual in that the size of the caustic for this event was similar to the size of the source star, which allowed the shape of the source star to be determined. With a resolution of 0.04 arcsec, the shape of the source star was seen to vary from circularity by less than a few percent. The source star in this event traveled through the caustic in 15.6 hours. The star itself is very similar to the Sun, however 7±2 kpc distant. The resultant image of this star was extraordinarily crisp, the sharpest image of any star other than our own Sun.

The limb darkening of the source star was able to be measured as a result. The baseline flux was determined from the observations of the Wise and MDM telescopes at high magnification. The absorption and reddening of the source star due to galactic dust was assumed and led to an absolute magnitude of 2.6±0.6 with a colour index of 0.74±0.1. The source star was hence an F8-G2 main-sequence turn-off star, with an age of approximately 3-10 Gy, and a temperature in the range of 6200-5800 K, placing the star as very similar to our own Sun.

MOA 2002-BLG-33 was first detected approximately 48 hr before peak magnification by MOA on June 18 2002 (Abe, 2002). Using the data gathered from MOA and others (including EROS), the inverse ray technique was used along with MOA's Cluster Computing.

This result is important, as it demonstrates the successful application of gravitational microlensing in the characterisation of stars within the Galaxy – their mass, temperature, age, atmosphere and more.

7.6 GSC3656-1328 - The Tago Event

In October 2006, the gravitational microlensing event in a sparse stellar field was detected. It was found by amateur Astronomer and variable star hunter A. Tago (subsequently independenelty discovered by Y Sakurai), and involved the brightest and closest source star to that date (Fukui et al, 2008), at just magnitude 11.4 and about 1kpc distant in the constellation of Casseiopea. The increase is magniude was extremely large, at 4.5 magnitudes.

This has been the only event detected toward a sparse stellar field since observations have been taken, which is roughly consistent with the frewuency of events expected of stars down to 12th magnitude. What is unusual is the extraordinarily large amplification of this event, which is much higher than would be normally expected. Other data from the event taken from other variable star observers confirmed that it was a microlensing event and not a variable star.

This unusual event may indicate that events toward the sparsely populated regions of the galaxy may be more frequent than theorized. It is reasonable to expect approximately 0.05-0.2 events per year over the whole sky for source stars brighter than 12th magnitude, however the expected frequency decreases by 50 times for an event this bright. It could be pure luck that this event was rare, or it may indicate that the true event rate is higher than expected. Detection of events with smaller amplifications may have been overlooked by variable star and nova searches due to efficiency concerns.

The detection of this event gives reason to encourage further survey of the entire sky, in order to discover the true rate of events and their amplification.

7.7 OGLE-2006-BLG-109

On February 14 2008, it was announced that a multiple planet system had been detected by gravitational microlensing, with observers from OGLE, MicroFUN, Robo-NET, MOA and PLANET, and the analysis lead by Gaudi (Gaudi, 2008). The event was detected over two weeks in late March and early April in 2006, with the background star lying 26,000 ly (85 kparsecs) from the Sun. The foreground star lies at a distance of approximately 5,000 ly (Gaudi, 2008)

The light curve from the event indicated two planets orbiting the foreground star, with masses of approximately 71% and 27% that of Jupiter. The orbits of these planets are 4.6 and 2.3 times that of the Earth, which means the ratios of size to orbit are very similar to the sizes and orbits of Jupiter and Saturn in the Solar System, orbiting a red dwarf star.

The detection of this system suggests that multiple planet systems may be common throughout the Universe, giving weight to current planetary formation theory.

CHAPTER 8

Future of Gravitational Microlensing

Even considering its long history, gravitational microlensing as a means of probing the Universe is still a young technique. It is only in the past 15 years that lensing events have been detected and investigated, hence it is reasonable to expect that theory and observations will continue to develop considerably in coming years. In the following sections various aspects of the future of gravitational microlensing are explored, from theory and techniques to proposed missions and networks.

8.1 Space-Based missions

There are a number of space-based mission in the planning and development stages, which have the potential to provide a statistical analysis of planets with masses down to 0.1 Earth masses, and with orbital separations exceeding 0.5 AU. These would be capable of detecting every planet in the Solar System except for Mercury. This also includes most planets predicted by modern planetary formation theory, although does not include planets of stars with shorter life spans.

Space-based missions such as SIM will also enhance the capability of Earth-based observatories in the collection of data from detected events, with events detected from Earth able to be followed up from space.

It also includes most types of planets predicted by various planet formation theories. It does not, however, include planets of short-life-span host stars. This is the only method of finding comprehensive statistics on mass and semi-major axis distribution of extrasolar planets.

Terrestrial observations are limited to detecting planets within the vicinity of the Einstein Radius (~2-3 AU), hence space-based imaging is required to identify and determine the mass of planetary host stars for most of the planets discovered by microlensing. Space-based microlensing surveys may then be the only way to gain a comprehensive understanding of the nature of planetary systems, which is a requirement of achieving a comprehensive understanding of planetary formation and habitability. One such survey would be the Microlensing Planet Finder (MPF), another the Galactic Exoplanet Survey Telescope (GEST).

8.2 Long Base Line Interferometry

Using Earth- and Space-based interferometers, the two individual images in a microlensing event will be able to be resolved. This will help to lift the degeneracy present in each system between physical parameters (Delplancke, 2001).

This degeneracy is a well-known problem in microlensing. The only parameter that can reliably be determined from a lensing light curve is the Einstein Radius crossing time, a value dependent on the lens mass, the transverse velocity, and the distance to the lens and source. In general, the mass of a lensing object cannot be uniquely determined from a single light curve. Statistical analysis must therefore be used to derive information on the lens population.

In order to break this degeneracy completely, a measurement must be made of both the lens-source relative parallax π_E, and the angular size of the Einstein Radius. To determine the relative parallax, the microlensing event must be observed from at least two non-collinear locations, for example from one Earth-bound observatory and one space-based observatory such as the planned Space Interferometry Mission. Alternatively, for events with timescales in excess of months, the lens-source parallax may be determined by observing the event from two different points in the Earth's orbit around the Sun, given that the binary-source motion is distinguishable from the parallax (Dalal, Lane, 2003).

The Einstein Radius is not normally resolvable by Earth-bound single-aperture telescopes, except in some rare events where the caustic crossing allows it. In order to resolve the Einstein Radius in normal circumstances, Long Base Line Interferometry must be used.

8.3 Potential Direct Single-Star measurement

In 2004 (Ghosh, 2004) it was proposed that the data obtained from the event OGLE-2003-BLG-175/MOA-20030BLG-45, in conjunction with future astrometric observations, may be able to yield a measurement for the mass of the lens star involved. This is a particularly exciting possibility, as it would remove some of the uncertainty surrounding the geometry of the event. If the mass of the lens star were known, that would mean that the relative distances of the two stars could be determined. A lot of the uncertainty surrounding the observations and analysis of the microlensing events would be removed. It would be more exciting to find a method by which the mass could be determined on every occasion.

The method by which the team Ghosh et al. proposed to determine the mass was by using the unique properties displayed by this particular event. The first property was that the light curve of the event showed distortion from the Earth’s accelerated motion during the period of the event. From this comes a measurement of the projected Einstein Radius. The second property was found by taking precise astrometric observations. It was found that the blended light in the event was coincident with the lensed source light to within 15 mas. This implies that the blended light is the lens star, and hence the lens-source proper motion could potentially be measured with future high-precision astrometry. With the combination of these details, the mass of the lens star could be determined. Although there is a large amount of degeneracy for a given Einstein Radius, with the relative proper motion factored in, the degeneracy falls away.

8.4 Galactic Exoplanet Survey Telescope

The Galactic Exoplanet Survey Telescope (GEST) is a planned mission designed to detect planets toward the Galactic Bulge down to the mass of Mars, at a separation of greater than 0.5 AU using microlensing – the first mission that would be capable of discovering every planet except Mercury in our own Solar System if it were observed from space (Bennett, 2000).

The GEST baseline mission has been set to run for 3.7 years, including four 8-month continual observing sessions of a particularly dense star field near the Galactic Centre. Because of the large number of stars observed, a very large format detector will be used, which implies a very high data rate, meaning a nearly circular geosynchronous orbit must be maintained. The orbit must be highly inclined at over 47 degrees with respect to the ecliptic plane so that

GEST can maintain both constant observation of the Galactic Centre and a ground station.

The instrumentation will include a 1.0-1.5-m telescope of a 3-mirror anastigmatic design with a 1.2x2.4 degree field of view, and a focal plane assembly of 32 backside thinned CCDs. The line of sight pointing stability will be accomplished by a closed-looped fine attitude control system, and low frequency drift errors will be corrected by imagery obtained by GEST acting as a high-precision star tracker. There will also be four additional CCDs which will be read out ten times per second used for closed-loop instrument and spacecraft pointing.

GEST will utilize a CCD focal plane array of 6.0×10^8 pixels which, at 14 bits per pixel, will provide images of 8.5 Gbits each. Images will be taken every two minutes, and the images will be read out in ten seconds, using a narrow shutter covering one row of the CCD at a time. Exposures will be added together to provide ten minute exposures, and cosmic rays removed by median filtering. The resultant image will be compressed using the rice algorithm which will reduce the image so that it may be transmitted at 10Mbits/s to the ground station. At the ground station, the images will be immediately processed to photometric measurements using a difference images algorithm and a dedicated parallel-computing data processing system. Analysis will automatically run on the data which will provide real-time detection of lensing events, and quicker discovery of planetary events, leading to a greater efficiency in coordinating follow-up observations, and hence more detailed data for each detected event.

The goal of GEST is to measure the planetary mass function and separation of extra-solar planets, the abundance of free-floating planets and the ratio of the number of free-floating planets to bound planets, planets which have become independent of their host's gravitational bonds at some stage in their system's history. In this way theories of planetary and stellar evolution may be tested. Combined with Earth-based IR observations, the host star for at least half of the extrasolar planets may be detected and observed, particularly those in the F, G and K spectral classes.

GEST will be measuring the abundance of planets around stars in the inner galaxy for planets down to 0.1 Earth masses, and the planetary mass function

[66] $f(\varepsilon = \frac{M_p}{M_{Earth}})$

When the Galactic Bulge is not visible, a wide-angle deep lensing survey, a high redshift supernova survey, a Kuiper Belt object search sensitive to about 100,000 objects, and a Guest Observer program will be carried out.

8.5 Pixel lensing toward the Galactic Bulge

A new method has been developed in order to search for gravitational microlensing events towards the Galactic Bulge which makes use of a small camera rather than a conventional telescope. In this manner, new regions of parameter space may be explored.

The small aperture (normally ~65 mm) of the camera used allows the detection of stellar flux variations between magnitudes 7 and 16, an advantage when current techniques are typically limited to less than 15 m due to the effects of saturation.

Due to the large pixel sizes, conventionally ~10arcsec and covering $(6^{\circ})^2$, observation of the entire bulge may be accomplished with minimal adjustments. The large pixel size may be perceived as a problem as, especially when incorporating a point spread function of approximately 30 arcsec, most bulge stars remain unresolved. Instead, microlensing events are detected by pixel lensing, where lensing and other variable events are detected by the difference in pixel counts determined in successive images.

There are three readily apparent applications of this method. There is the obvious application whereby the method used is similar to that used in pixel lensing observations towards M31 (see below), with the difference being that detected events may subsequently be monitored in greater detail. This particular application also allows for a check on the methods used to acquire observations towards the bulge of M31.

The second application is in the compilation of a complete catalogue of bright bulge variables, also of significant benefit to the wider astronomical community.

Finally, extreme microlensing events, with a maximum of A ~200, may be detected and identified in real time. Capturing these events and taking continuous observations of them could yield the mass, distance and speed of the gravitational lenses, providing another potentially invaluable set of data to the astronomical community.

A significant advantage to this method is its cost, providing an opportunity for observers on a restricted budget to break into the field. The benefits of that are self explanatory.

This new method may exist as a complement to search and follow-up networks.

The data reduction is simplified in comparison with standard pixel lensing techniques as the PSF may be fixed by the optics, and will not vary with the atmospheric conditions.

8.6 Space Interferometry Mission

The Space Interferometry Mission (SIM) is an ambitious project led by NASA and managed by JPL. Primarily because of budgetary constraints, however, the launch date has been pushed out from the intended 2005 at least seven times to, at the earliest, 2015 [NASA].

The ambition of SIM is to perform astrometry to a positional accuracy of 1 microarcsec with an angular resolution of 10 mas for stars down to a resolution of mag 20. It should therefore be capable of measuring the displacement of the light centroid in microlensing events. An exciting prospect, this will mean that events detected photometrically from Earth may be subsequently measured astrometrically from Earth orbit. The advantages of this are immediately apparent. Combining the photometric with the astrometric data for an event involving a MACHO massive enough to induce lensing, the mass, distance, and proper motion of the MACHO may be determined, and the angular stellar radii and temperature of the source may be determined.

The SIM Planet-Finder had eight specific technological milestones to achieve prior to launch. These were

(http://planetquest.jpl.nasa.gov/SIM/sim_milestones.cfm)

1. To create a 'ruler' able to measure increments less than the diameter of a hydrogen atom
2. To develop a mechanism to suppress the subtle vibrations caused by the machinery of the SIM.
3. To demonstrate the instruments are able to measure angles consistent with those expected from stellar and planetary interactions
4. To demonstrate that the laser metrology gauges (the 'rulers' developed in milestone 1) can work together in a network and produce consistent results
5. To demonstrate a Microarcsecond Metrology (MAM) testbed performance of 3200 picometres over its wide-angle field of regard is achieved, in line with the minimum global astrometry requirement
6. Ensure the scientific goal of 1 microarcsecond narror-angle sensitivity is achievable by the MAM
7. Ensure the scientific goal of 4 microarcsecond wide-angle accuracy is achievable
8. Build a composite picture of the SIM instrument performance based on all previous milestones

These milestones were completed to satisfaction in November 2006. At the present date, the only issue preventing the launch in a timely fashion, is the budgetary constraints of NASA, and what priority they place on this mission.

CHAPTER 9

Conclusion and Discussion

Gravitational Microlensing is an effect of which our understanding is still in its infancy. It has been proven to be effective in the detection of objects otherwise undetectable – extrasolar planets, MACHOs, binary systems and others.

The Astronomical Community has embraced the evolution of the theory involved with enthusiasm, its potential being more than apparent.

Six planets have been discovered through the detection of microlensing events, planets which would have otherwise not been visible directly or by other means. These planets, and the countless planets still to be found, have assisted in the development and understanding of stellar evolution and the prevalence of systems like the Solar System in the Universe. The hundreds of other events detected have assisted our understanding of galactic structure and the nature of dark matter.

The future of gravitational microlensing is promising, with established networks detecting more events each year, and space-based missions in progress. The planned missions will improve the amount of detail able to be derived from each event. In the near future, the observation of microlensing events will reveal far more detail about the Universe hitherto uncovered.

In the more distant future, the effects of gravitational microlensing will need to be taken into account when detailed observations are taken in any direction. With the amount of matter in the Universe, in particular dark matter, it may be expected that light from distant objects will, for the most part, fluctuate depending on the intervening matter. It may even be suggested that the effects of gravitational microlensing will cause frustration to future Astronomers when observing these distant objects.

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