LP Models for Pricing Diffuse Nitrate Discharge Permits

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Abstract: Nitrate discharges from diffuse agricultural sources significantly contribute to groundwater and surface water pollution. Tradable permit programs have been proposed as a means of controlling nitrate emissions efficiently, but trading is complicated by the dispersed and delayed effects of the diffuse pollution. Hence, markets in nitrate discharge permits should be carefully designed to account for the underlying spatial and temporal interactions. Nitrate permit markets can be designed similar to the modern electricity markets which use LPs to find the equilibrium prices because the two trading problems have close analogy.

In this paper, we propose alternative LP models to find efficient permit prices for year-ahead markets. The model structure varies depending on the catchment hydro-geology and long-term goals of the community. We show how the market price structures are driven by the constraint structure under different environmental conditions. We discuss the physical and economic conditions required to assure consistent prices, the modeling of essential and optional constraints in an LP, and the problem of balancing resource allocation over time among delayed-response discharge units. We then extend the LP model to balance resource allocation over time and to improve the market performance.

Keywords: water quality markets, nitrate trading, non-point sources, tradable permits, linear programming.

1 Introduction

Nitrate is a critical water pollutant. High nitrate concentration in drinking water can cause illness including blue baby syndrome in infants (Wiederholt and Johnson 2005). Excess nitrates in surface water promote growth of algae and other nuisance weeds. Excess nitrate discharge into oceans contributes to dead zones (Alexander et al. 2008).

Agriculture is the number one source of nitrate in almost all threatened water resources (OCED 2008; Ongley 1999). Unlike most water pollutants that are carried down to water bodies via storm water runoff, nitrate loss to the environment occurs mainly due to leaching from soil layers. Intensive application of nitrogen fertilizer and livestock effluent on agricultural land leaves a surplus of nitrogen in the soil. The surplus leaches into the underlying aquifers as nitrate. Nitrate[1] is quite stable in water and flows with groundwater towards surface water sinks. In surface water, nitrate may be taken up by aquatic plants, subjected to chemical transformations, or carried down to the oceans. Tradable discharge permit programs have been proposed as a means of balancing the demand for nitrate intensive farming and the capacity of the natural water bodies to dilute nitrates (Leston 1992; Ribaudo et al. 1999; Faeth 2000). However, trading nitrate discharge permits is difficult and complicated due to the spatial and temporal impacts of diffuse agricultural sources. The physics of nitrate movement suggests four main factors need to be considered in trading these permits.

First, a trading system should account for the time lags between leaching and appearance in a water body. Nitrate, once mixed in slow-moving groundwater, can flow with groundwater for decades until being discharged into a surface water body or the ocean. The time lags vary depending on the location of loading and the hydro-geology of the flow paths.

Second, nitrate in groundwater may be transformed into other forms of nitrogen by chemical reactions such as pyrite oxidation (Conan et al. 2000) and by biological denitrification). The amount of attenuation depends on the properties of the subsurface strata and the flow paths. Hence, a trading system should consider attenuation in transport.

Third, the quantity of nitrate transported from a farm to some surface water body is not delivered in one step, but gradually over a relatively long period. Therefore, permit exchanges should take into account the protracted delivery profiles.

Fourth, a trading system must be capable of handling many distinct receptors (a water body or a point on a water body where water quality is monitored). Since monitoring nitrate everywhere is impractical, catchment planners usually select a

[1] Nitrate is the most common form of nitrogen found in water as a pollutant. But, nitrogen may found in water in other forms as ammonium.

few receptors that indicate the overall water quality in the catchment. The farms in a catchment may affect different receptors over different time scales.

Due to the above reasons, trades in nitrate discharge permits need to satisfy several spatial and temporal environmental constraints simultaneously. Other regional, political, and private constraints may add further complications. When several constraints are involved, bilateral trade in emission permits cannot achieve the efficient allocation of permits. Therefore, a centrally controlled, multilateral trading framework is required (Ermoliev et al. 2000).

Most of the existing trading programs and the conceptual market models proposed to date are simplified versions of the real problem (US EPA 2007; NIWA 2009). For example, Lock and Kerr (2008) designed a market for New Zealand's Lake Rotorua catchment, assuming that all nitrogen lost from farms would reach the lake, which is the only receptor considered, after some specific number of years. In this case, only time lags are taken into account, ignoring attenuation, protracted delivery, and other receptors. Another market model proposed by Morgan et al. (2000) for trading infinitely valid permits considers a single receptor (a selected groundwater well) and a single time period (the last year of the planning horizon). Such systems are unable to assure water quality over time. The common pitfall in all those proposed trading programs is that they ignore some important constraints to simplify the problem.

To achieve environmental feasibility, nitrate discharge permits are usually traded in year-ahead (ex-ante) markets, requiring market operators to find the equilibrium prices ex-ante. A marginal cost based pricing scheme is required to assure efficient resource allocation. Unless all the relevant constraints are imposed on the market, the prices will not reflect the true marginal costs and will lead to inefficient allocations. The problem is how to find a set of optimal prices which ration the permits relative to each farm's profit/cost function while satisfying all the environmental and other constraints. Ermoliev et al. (2000) proposed that a market coordinator who acts as a Walrasian auctioneer could lead a centrally controlled permit market towards a set of equilibrium ambient[2] and discharge prices. The auctioneer announces a set of ambient prices, and a set of discharge prices derived from the ambient prices, and lets the dischargers submit the

[2] An ambient price is a price for increasing the nitrate level at a specific receptor (McGartland 1988; Ermoliev et al. 2000).

quantities they would like to trade at the announced prices. The process can be continued by adjusting the prices, until equilibrium is achieved. However, the auction may not reach equilibrium quickly or monotonically. The authors themselves accepted that the process may take a long time.

To find the equilibrium prices, a Walrasian auction is not required if the dischargers provide the coordinator information on quantities they would trade at each possible price step, beforehand. The optimal allocation may be modelled as a mathematical program, often a linear program (LP), from which the equilibrium prices may be obtained straight off. This is the lesson that the environmental policy makers could learn from the modern electricity markets, which use LPs to clear the markets and set the prices instantaneously (Alvey et al. 1998; Hogan et al. 1999). The similarity is that both are common pool multilateral trading problems with complex interactions and constraints. If the gains from trade, and constraints on trade, can be modelled linearly, the two problems have close analogy. An added benefit is that not only the environmental constraints, but all relevant political, regional, and individual constraints could be included in the pricing models, if justified by the stake holders.

Due to the underlying spatial and temporal complexities, modelling a market in nitrate discharge permits is quite complicated. General (air and water) pollution permit market models discussed in the literature, for example, McGartland (1988), are not applicable, mainly because they are single period models. Our contribution is to present appropriate LP models for pricing nitrate discharge permits. We first discuss how a market in nitrate discharge permits could be designed and operated using an LP which models the market ex-ante, and produces the optimal allocations and prices. Then we discuss the information required in formulating an LP to price the permits and the physical and economic conditions required to justify a linear model. We present a base case LP which models the essentials of a nitrate permit market. Then we discuss different types of applicable constraints, how to model them in the market-clearing LP, and how they affect market prices. Next, we discuss the problem of balancing resource allocation over time, a problem which is specific to the delayed-response pollution permit allocations. We present an extended LP model, which have the ability to balance resource allocations over time and to reduce the risk of under-pricing resources. We demonstrate the concepts with a conceptual market model for a hypothetical lake catchment.

2 A Market in Nitrate Discharge Permits

Environmental permit markets need precise definitions of the commodities being traded. In a market for nitrate discharge permits, the maximum acceptable nitrate level (sustainable nitrate intake capacity) of each receptor in each time period is a traded resource. A "resource permit" is a right to increase the nitrate level at a specified receptor in a specified time period[3]. But nitrate discharge at a particular time and place will increase nitrate levels at one or more receptors over many periods. Thus, if only these resource permits were traded, each farm would have to assemble a whole portfolio of resource permits to match their discharge in any year. To enable farms to buy a single discharge permit rather than multiple resource permits, we define nitrate "loading permits" as rights to load a specified amount of nitrate into the aquifer underlying the farms, in a given year. A loading permit is equivalent to a bundle of resource permits. The trading system is designed so that farms can buy and sell loading permits in the same exchange market, even though those permits are not directly comparable between farms.

2.1 Market Design and Operation

To facilitate trade in nitrate loading permits, we propose a centrally mediated multilateral trading system similar to the New Zealand electricity market (Alvey et al. 1998). The dominant feature is the use of an LP which optimizes the trading of permits between participants, and determines equilibrium prices, so as to maximize the benefits of trade, as defined by the offers and bids submitted by market participants. Such a trading system is best implemented as an online trading system (although this is not a requirement) so that the farms can trade multilaterally in a virtual market place. An electronic market such as this, supported by optimization methodology, is often known as a "smart market" (McCabe et al. 1991).

[3] The resource permits are synonymous to the ambient permits discussed in the literature (McGartland 1988; Ermoliev et al. 2000) with one key difference: Conventional ambient permits are not time specific, but the resource permits discussed in this work are rights to pollute a receptor in a specified time period.

The market is designed at the catchment scale. Farms in the catchment can buy and sell loading permits. Each permit is valid for a single year, but the farms can simultaneously trade permits for a fixed number of upcoming years. Hence, this is a market in short term property leases rather than a market in long-term property "ownership" rights; the permits traded are similar to lease contracts. We assume that the government (regulator) retains "ownership" of, or at least control of and responsibility for the environment and its ability to assimilate pollutants, on behalf of the public. Any long-term discharge rights held by farms may be considered as initial holdings.

The market operates as a periodic auction. At the beginning of every trading year, the market operator calls for bids from the farms. The bids provide input data for the market clearing LP. The LP solution gives the optimal permit position and price for each farm. The market operator then calculates the net trades from the initial (pre-trade) and optimal (ex-post) permit positions. Then the operator collects money from net buyers, pays net sellers, and settles the market. For simplicity, we will assume that all payments are settled immediately. Some trial market rounds would provide a price discovery process, during which farms would adjust their bids and offers to market conditions, as in the electricity market.

2.2 Scope of the Trading System

The scope of the trading system is determined by the set of receptors considered and the planning horizon selected. The receptors and what is controlled at the receptors (whether nitrate mass or concentration) should be well defined.

A catchment, by definition, is a land area from which water drains towards a common water body. Therefore, any catchment-scale water pollution trading system should consider the common sink, towards which the catchment drains, as the main receptor. A catchment may have many sub-catchments draining to intermediate connected streams or lakes, which may also be considered as receptors to avoid excessive local pollution. The nitrate concentration in surface water may not be a good indicator of diffuse discharges, because the concentration is affected by surface water flow, which is more variable and uncertain than groundwater flow. But groundwater monitoring wells may be considered as receptors, where the nitrate concentration is to be controlled, rather than the mass nitrate discharge.

Nitrate loading in any period affects the receptors in many future periods. Hence, the market model should constrain those future impacts, ideally, for as long as the impacts extend. The planning horizon should therefore be at least the catchment's maximum nitrate travel time (residence time). Hydrologists usually estimate the maximum residence time as a tail percentile, such as 99th or 95th., of the fraction of nitrate delivered from a given discharge. Large groundwater catchments can have long residence times, for example, 200 years in the Lake Rotorua catchment of New Zealand (Lock and Kerr 2008). Therefore, the associated market models can require long planning horizons.

2.3 Information and Assumptions

2.3.1 Farm Profit Functions

Modeling the optimal distribution of nitrate loading requires knowledge of how much profit each farm can make from each unit of discharge permit allocated[4]. For both market-based and non-market based pollution management problems, this is the most difficult and vital information required. Following the convention in electricity markets, this trading system expects the farms to submit bids indicating the price they would pay (or accept) for each incremental block of quantity, starting from zero[5]. A rational farm would buy another X units of loading permits if the incremental profit from each unit is above the price, and therefore bid to buy at the marginal profit[6]. Thus, the bids indicating the additional quantity preferred at each price step correspond to a piece-wise linear profit function of the farm. We will not use time discounts in the market clearing

[4] Markets in pollution permits are usually modelled to minimise abatement cost rather than to maximise profit (Montgomery 1972; McGartland 1988; Ermoliev et al. 2000). However, allocation of diffuse nitrate discharge permits is a problem of optimising the allocation of land uses rather than the abatement responsibilities. In the case of nitrate loading permits, forgone profit from a more nitrate intensive farming option may be considered as an abatement cost. Therefore, maximising profit is the same as minimising abatement cost.

[5] A market which requires the participants to bid for quantities starting from zero is called a "gross pool" market. Alternatively, the market may operate as a "net pool" which allows the participants to submit both bids and offers. Net pool bids and offers can easily be transformed into gross pool bids and vice-versa if the initial position is known.

[6] If the market is not perfectly competitive, the buyers and sellers could game the bids above or below the marginal profit to affect market prices. However, a catchment usually has a relatively large number of farms, and a catchment scale market is expected to be workably competitive.

model, relying on the farms to discount their bids at their own rates, taking account of when settlements will be due for payment. If settlement is immediate, bids for future permits will thus indicate the discounted present value of future permits.

All the farms in a catchment are assumed to be participating in the trading system. The farms that do not actually participate in the market by submitting bids are assigned two default bid tranches: one with a price of infinity (a relatively large price) and a quantity equal to the initial position, and the other with a negative price. These two default bids indicate that the farm does not wish to either buy or sell.

To optimize the bids, the farms need to know the profits and nitrate loadings of each available land use option. A land use option is a combination of factors such as the type of crop grown, type of stock, fertilizer application rate, stocking density, irrigation method, effluent discharge system, drain layout, and other land management practices. The size of the permit required to adopt some land use option is the estimated nitrate leaching from that land use option.

To estimate the potential nitrate leaching from possible land uses, and thus the size of the permit required to cover their operations in each year, the farms can use a soil nitrogen model. Standard soil models such as SWAT (Neitsch et al. 2005), or regionalized nutrient budget models such as OVERSEER (used in New Zealand) simulate soil nitrogen dynamics (crop uptake, mineralization, immobilization, nitrification, de-nitrification) and estimate the nitrate leaching. If the farms do not have access to such models, the authorities could provide information about leaching from possible land use options and their permit requirements. The models used to estimate leaching could possibly be authorized by the market authorities, although that would raise accountability and liability issues, if the model were subsequently revised.

Farms also need to know the potential profit from alternative land uses. Farm economic models such as WFM (Beukes 2005) can estimate the potential profits from different land use and management options. Rather than independent agro-ecological and agro-economic models, integrated agro-ecological and agro-economic models which can predict both the nitrate losses and potential profits from alternative land uses (Johnson, 1991; Mohamed et al., 2000) would better help the farms in optimizing their bids. But, the assessment of potential profit is a private matter and the farms will have to be responsible for their own judgments irrespective of the model used.

2.3.2 Linkage between the Sources and Receptors

The ultimate goal of permit trading is to maintain sustainable nitrate levels at all the receptors throughout the planning horizon at least cost to society. To achieve this, the regulators divide the planning horizon into discrete time intervals, and impose water quality standards based on maximum acceptable nitrate mass or concentration in each time interval. The time intervals should be short enough to ensure that water quality is met continuously and long enough to avoid a large number of redundant constraints and computational difficulties. Taking into account the likely long planning horizons and long term resource commitments involved in commercial farming, we suggest imposing water quality constraints at yearly time intervals.

To enforce water quality standards, the relationship between nitrate loading from the farms and the nitrate level at the receptors should be known. Following previous work on optimal management of nitrate loading, such as Morgan & Everett (2005), we assume a linear relationship between the loading and the increase in nitrate level at each receptor in each time step. By assuming linearity, we can calculate the increase in nitrate mass or concentration caused by each source at each receptor, in each time step after the loading occurs, as the product of the source loading and the relevant transport coefficient. Transport coefficients measure the increase in mass nitrate discharge or concentration at each receptor after each year's delay due to one kg nitrate loading from each diffuse source (farm) during a single year.

An important requirement for the above formulation is that the diffuse sources have a constant impact on the groundwater flow velocities (Gorelick and Remson 1982). It is generally accepted that, in a steady state groundwater flow regime, mass nitrate discharge to a surface water body from its catchment, and nitrate concentration in groundwater discharge from a catchment have strong linear relationships with nitrate loading in the catchment (Rao et al. 2009). Transport coefficients can be estimated using a catchment nitrate transport model. Commonly available computer codes such as MT3D (Zheng 1990) are suitable to simulate nitrate transport and to estimate the transport coefficients.

2.3.3 Non-tradable sources

A catchment may have many sources of nitrate other than agriculture, for example, storm water and direct dischargers such as sewage works. Direct dischargers, usually known as point sources, may be included in the trading scheme, as discussed in Prabodanie et al. (2009). However, unmanageable sources such as nitrates loaded into the groundwater system from prior land use and currently flowing towards some surface water sink should be considered as non-tradable sources. The market authorities should calculate tradable resource capacities after providing allowances for pre-estimated non-tradable source resource commitments. As the effects of non-tradable sources change over time, the tradable capacities should be calculated again every time trading takes place, taking into account previous nitrate discharges, including the previous year's tradable discharges. The market authorities can use catchment nitrate transport models and other regional environmental models to estimate the non-tradable source contributions. Again, the tradable capacity estimates may not be perfect, and liability for the consequences of mis-estimation would be an issue to be resolved for market implementation.

2.4 Monitoring and Enforcement

Once the market is cleared for some trading year, all the participants should comply with the cleared permit positions until the next auction. The government, through a regional environmental authority, should oversee the trading system and enforce the loading limits specified by the permits. Monitoring devices may be located in the farms to measure the nitrate concentration in the leachate and thus the actual loading. However, it is not possible to measure the actual loading from the farms during a year with 100% accuracy. Since the amount of nitrate loading is determined by the type of land use, instead of monitoring the quantitative loading, the environmental authorities may restrict the farms to the land use practices allowed by the size of the loading permit held. Better monitoring may be achieved via a combination of the above two monitoring methods.

Market rules must specify whether the participants should be deemed to have purchased permits for the agreed discharge rate, or instead for the activity level that, according to the model, was expected to produce that discharge rate. Who bears liability for wrong estimates must be resolved before a trading system is implemented. Such pragmatic and political issues are beyond the scope of this paper.

3 Market Clearing LP - Optimal Loading Model

Conventionally, the pollution permit allocation problem is modelled for a single time period to allocate emission rights so that the environmental quality standards at the selected receptors are met at the least cost to society. But a market in diffuse nitrate discharge permits is complicated by the dispersed and delayed responses of diffuse discharges. In this section, we present a basic LP which models the essentials of a market in diffuse nitrate discharge permits, to serve as foundation for more general models.

Indices and Parameters:

f = farm: 1,…, F.

r = receptor: 1,…, R.

d = delay in years: 0,…, D. D = maximum nitrate residence time in the catchment.

s = permit year: 1,…, S. S = last year (also the number of years) for which permits are traded.

t = monitoring year: 1,…, S+D.

The upcoming year is given by $s = t = 1$. Rather than expressing s and t relative to the upcoming year, we may express them in terms of the absolute year, for example, as 2010,…, 2014. If the upcoming year is $\hat{S}$, s= permit year: $\hat{S}$,…, $\hat{S}+S-1$ and t = monitoring year: $\hat{S}$,…, $\hat{S}+S-1+D$.

k = bid tranche: 1,…, K.

H_{frd} = increase in nitrate level that occurs at receptor r, d years after unit (1 kg) nitrate loading in farm f during a single year (kg or mg/l).

C_{rt} = tradable nitrate intake capacity of receptor r in year t (kg or mg/l).

The quantity available for trading is calculated after providing allowances for all non-tradable source contributions. Hence, C_{rt} = the maximum acceptable nitrate mass or concentration at receptor r in year t minus the nitrate level at receptor r in year t caused by all non-tradable sources. By defining a term for each resource (commodity) traded as "resource$_{rt}$", C_{rt} can be given as the tradable capacity of resource$_{rt}$.

U_{fsk} = size (quantity) of bid tranche k submitted by farm f for year-s permits (kg).

P_{fsk}= price specified in bid tranche *k* submitted by farm *f* for year-*s* permits ($/kg).

Decision variables:

x_{fsk} = quantity accepted from bid tranche *k* submitted by farm *f* for year-*s* loading permits (kg).

q_{fs} = size of the loading permit, for year *s*, held by farm *f* after trade; this is the maximum loading allowed for farm *f* during year *s* (kg).

Optimal Loading Model (OLM):

Maximize $\sum_f \sum_s \sum_k P_{fsk} x_{fsk}$, subject to:

Upper and lower bounds on bid tranches

$$x_{fsk} \le U_{fsk} \quad \text{for all } f, s, \text{ and } k. \qquad \text{(P-1)} \quad \theta^{+}_{fsk}$$

$$-x_{fsk} \le 0 \quad \text{for all } f, s, \text{ and } k. \qquad \text{(P-2)} \quad \theta^{-}_{fsk}$$

Calculation of individual allocations

$$\sum_k x_{fsk} - q_{fs} = 0 \quad \text{for all } f \text{ and } s. \qquad \text{(P-3)}$$

$$\mu_{fs}$$

Environmental constraints

$$\sum_f \sum_{s=\max(1,t-D)}^{\min(t,S)} H_{fr(t-s)} q_{fs} \le C_{rt} \quad \text{for all } r \text{ and } t. \qquad \text{(P-4)} \quad \lambda_{rt}$$

The above LP maximises the joint total profit to farms subject to maximum acceptable nitrate level in water over time and space. The LP is formulated as a "gross pool" market, which is independent of the initial distribution of permits. The bids reflect the additional quantity preferred at each price step if there were no initial holdings. Once the LP is solved and the optimal quantities and prices are found, the net buyers and sellers are determined by the difference between the final position and the initial position. Those who have added to the initial position are net buyers and those who have reduced in the initial position are net sellers; the payments and receipts due are calculated from the net trades.

The objective function coefficients P_{fsk} indicate how much each block of nitrate loading is worth to the bidder. Hence, the objective function maximizes the total gains from trade. If the bids indicate the true economic contributions of the farms, the objective function also maximizes the true social welfare. The quantity accepted in each bid tranche cannot be negative (P-2); an upper bound on each bid tranche (P-1) ensures that the quantity cleared does not exceed the maximum specified by the bidder. The allocation constraints (P-3) specify the relationships between quantities accepted and the final permit positions. The environmental constraints (P-4) require the market to meet the maximum acceptable nitrate level at each receptor in each time period. These "spatiotemporal" environmental constraints are considered as the set of capacity constraints in the market, because they restrict the tradable resource capacities. The capacity made available to the market (the right hand side of each capacity constraint) may be set below the currently available capacity C_{rt} to reserve some resources for future allocation. The problem of balancing resource allocation between the present and the future is discussed in Section 4.5.

The variables listed to the right of the constraints are the associated shadow prices. The dual formulation of the problem provides insight into the commodity prices that match the demand and supply.

3.1 Dual Formulation and Market Prices

While the OLM models the resource allocation problem, the "dual" of the OLM models the resource valuation problem.

Minimise $\sum_f \sum_s \sum_k U_{fsk} \theta^+{}_{fsk} + \sum_r \sum_t C_{rt} \lambda_{rt}$, subject to:

$\theta^+{}_{fsk} - \theta^-{}_{fsk} + \mu_{fs} = P_{fsk}$ for all f, s, and k. (D-1) x_{fsk}

$-\mu_{fs} + \sum_r \sum_{t=s}^{s+D} H_{fr(t-s)} \lambda_{rt} = 0$ for all f, and s. (D-2) q_{fs}

μ_{fs} free for all f.

$\lambda_{rt} \geq 0$ for all r and t.

$\theta^+{}_{fsk}$ and $\theta^-{}_{fsk} \geq 0$ for all f, s, and k.

The shadow price λ_{rt} of the capacity constraint (P-4) for some r and t, indicates how much the objective function would increase if the nitrate intake capacity of receptor r in time period t were increased by one unit. Farms would make an incremental profit of λ_{rt} if they were allowed to violate the capacity constraint by one unit. To stop the farms exceeding the limit, they should be charged at λ_{rt} per unit increase in nitrate mass or concentration at receptor r in period t. If the bids indicate the true marginal profit functions, λ_{rt} is a true marginal cost based price which results in efficient allocation of the resources. We call λ_{rt} a "resource price," the market price which matches the demand and supply for resource$_{rt}$. Since a loading permit is equivalent to a bundle of resource permits, the price of a loading permit for any farm can be derived from the prices of the resources in the bundle as $\sum_r \sum_{t=s}^{s+D} H_{fr(t-s)} \lambda_{rt}$.

Demand for the resources varies depending on farm locations and characteristics. If most farms are far upstream with long delay times, their discharge may imply a heavy burden on constrained resources in later years, particularly if the farms have already been operating in an unconstrained way for some time. Hence the (undiscounted) price for the constrained resource capacity of later years may be greater than for earlier years.

Even if the ability of a receptor to accept nitrate is constant over time (e.g., 400 tonnes per year), the tradable capacities and thus the associated resource prices may vary over time due to the temporal variations in non-tradable sources. For example, a huge plume of nitrate already in the aquifer may be flowing towards the receptor and expected to reach the receptor after another 20 years. Then the tradable resource capacities of the years after the 20th (C_{r21}, C_{r22}, …) will be lower and the associated resource prices are likely to be higher.

The shadow price μ_{fs} of the allocation constraint (P-3) indicates how much the objective function would increase if farm *f* were given another kg of year-*s* permits. Unlike the receptor prices (λ_{rt}) which describe the market equilibrium in receptor permits, μ_{fs} is indexed to a particular farm, and may be called the "participant price". Since each farm has a specific location in the catchment, the participant prices are similar to the "nodal" or locational prices used in electricity markets. For this simple formulation, these prices could be described as "market clearing prices". But they have not been derived from strictly locational demand and supply balance constraints and, as discussed in Section 4.3, they may be affected by private constraints.

Thus, the OLM formulation above does not always provide *locational* prices which can be charged to the farms directly. A true locational price for nitrate loading should describe the social marginal cost of nitrate loading from the particular location, irrespective of which entities are operating in that location. And they should be consistent for all farms in a similar location. To obtain such market clearing prices, we have to separate the locational effects of nitrate loading and the effects of participants' private characteristics. A simplified market model could be developed by dividing the catchment area into a set of locations or zones, in which all farms are assumed to have the same transport coefficients.

Let $z = 1,\ldots, Z$ index the zone and let H_{zrd} be the zonal transport coefficient which measures the increase in nitrate level at receptor *r*, *d* years after 1 kg nitrate loading in zone z during a single year (kg or mg/l). Let q^{zone}_{zs} be the total loading from zone z (kg). The set of zonal loading variables would add another set of constraints to the OLM and change the environmental constraints as follows.

$\sum_{f\in z} q_{fs} - q^{zone}_{zs} = 0$ for all z and s. (P-5) β_{zs}

$\sum_z \sum_{s=\max(1,t-D)}^{\min(t,S)} H_{zr(t-s)} q^{zone}_{zs} \le C_{rt}$ for all r and t. (P-6) λ_{rt}

The new primal variables create a new set of dual constraints (D-4). The new primal constraint creates a new dual variable, β_{zs}, and change the dual constraint associated with primal variable q_{fs} (D-2) to (D-3).

$-\mu_{fs} + \beta_{zs} = 0$ for all f and s. (D-3) q_{fs}

$-\beta_{zs} + \sum_r \sum_{t=s}^{s+D} H_{zr(t-s)} \lambda_{rt} = 0$ for all z and s. (D-4) q^{zone}_{zs}

The shadow price β_{zs} of the constraint (P-5), which defines q^{zone}_{zs}, for some z and s, indicates how much the objective function would increase if another 1 kg of year-s loading permits were assigned to zone z. This is the social marginal cost of nitrate loading from zone z in year s. We call β_{zs} the “zonal loading price”. Unlike the participant price, the zonal loading price is a locational price which is not indexed to a particular participant, and it does not vary among individual farms in the same zone based on any private constraint (or other side constraint) which applies to q_{fs}. Since the zonal loading price is a market price determined by the zonal demand and supply, it can be directly charged to each farm in zone z. The dual constraint associated with q^{zone}_{zs} (D-4) indicates that the zonal loading price equals the value of the bundle of resource permits equivalent to a unit (1 kg) loading permit allocated to zone z. The advantage of such a simplification is that all farms in a zone can readily verify that they have been treated fairly by the market and, in fact could trade permits between themselves without reference back to the wider market.

3.4 Settlement

Let Q^*_{fs} be the initial year-s permit position of farm f, kg. Then, the payment due from (to) farm f for buying (selling) year-s permits is $\beta_{z(f),s} \times (q_{fs} - Q^*_{fs})$, where $z(f)$ is the zone to which farm f belongs.

Let $\Omega = \sum_f \sum_s \beta_{z(f),s} \times (q_{fs} - Q^*_{fs})$ be the market operator’s net revenue after clearing the payments for the farms. If the initial distribution of permits is feasible, Ω is non-negative. If the farms do not possess any previously purchased permits, Ω is the total payment made by the farms for buying resources (or leasing contracts)

from the regulator. Then, Ω is the total lease payment due to the regulator. If the farms currently possess loading permits, and if the initial distribution of loading permits fully allocates all the resources (i.e., if the initial allocation binds all the capacity constraints), Ω is zero. The market re-allocation being bound by a capacity constraint which was not binding for the initial allocation means that the farms have bought resources from the regulator, and Ω will be non-zero. If the initial allocation was infeasible, Ω may be negative, as the regulator has to buy back the over-allocated resources. Such payments will offset one another if some resources were previously over-allocated, and some under-allocated.
The regulator may choose to re-distribute the total revenue or a portion of the revenue among the market participants, based on some free initial allocation criteria. It may be better to redistribute the total revenue among the farms by making the free initial allocation in terms of resource permits rather than loading permits (because it is difficult to find a fair allocation of initial loading permits which also binds all the ex-post binding capacity constraints).

3.5 Gains from Trade

The shadow price $\theta^{+}{}_{fsk}$ of the bid upper bound constraint (P-1) indicates the increase in economic benefit if farm *f* were able to utilize another 1 kg at a marginal value of P_{fsk}. If the k^{th} bid of farm *f* corresponds to some land use option, then $\theta^{+}{}_{fsk}$ is the net gain from expanding that land use (in terms of fertilizer application rate, area cultivated, stocking density, etc.) by enough to cause another 1 kg of nitrate loading. Hence, the bid will be fully accepted if $\theta^{+}{}_{fsk}=P_{fsk}\ -\mu_{fs} > 0$. The shadow price $\theta^{-}{}_{fsk}$ of bid lower bound constraint (P-2) indicates the loss in economic benefit if one unit were to be accepted from that bid. Hence, the bid will not accepted if $\theta^{-}{}_{fsk} = \mu_{fs}.\text{-}\ P_{fsk} > 0$. Bids will be partially accepted if $P_{fsk} = \mu_{fs}$, in which case $\theta^{-}{}_{fsk} = \theta^{+}{}_{fsk} = 0$.Figure 1 highlights how these dual variables relate to the gains from trade. For example, if the initial position of farm *f* is zero, then *f* is a buyer of year-*s* permits, and the buyer surplus is $\sum_k \sum x_{fsk}\ \theta^{+}{}_{fsk}$. If the initial position of farm *f* is $\sum_k U_{fsk}$, then *f* is a seller of year-*s* permits, and the seller surplus is $\sum_k \sum x_{fsk}\ \theta^{-}{}_{fsk}$ Hence the market allocation always satisfies the participant preferences indicated in the bids.

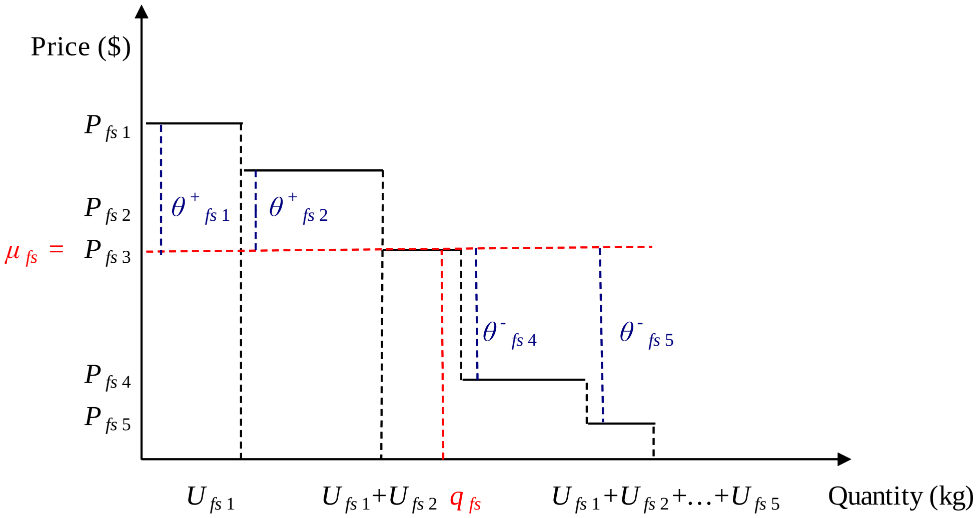

Fig 1: Buyer and seller surplus relative to the participant price.

4 Model Generalization

In the above discussion, the only commodity traded is the ability of the receptors to accept pollution. Hence, the receptor capacity constraints are the only supply-side constraints. The “private” constraints faced by particular participants are assumed to be internalized into the bids, and no other demand-side constraints are imposed. If other constraints are required, the price structure becomes a little more complex, and we see particular participants facing new prices. As a result, the price which rations the final permit allocation is the participant price. If the resource capacities were the only limiting factor in the market (as in the above basic model), the participant price equals the zonal price.

The capacity constraints (P-4) alone would sufficiently model the supply-side restrictions if water quality limits for the entire catchment can be stated entirely as a set of annual targets for the selected receptors, with no reference to any other limit, or allowance for trade-offs between receptors, or periods. Large and hydro-geologically complex groundwater catchments do not necessarily satisfy all those conditions, so the model may need additional environmental constraints. Commercial or private constraints such as minimum operating levels may also directly affect the demand for permits. We recognize two major types of applicable side constraints: receptor-based side constraints; and source-based side constraints.

4.1 Receptor-Based Side Constraints

Nitrate may take many years to travel through large receptors such as lakes. For example, it takes 11 years for water to flow through New Zealand's Lake Taupo (Morgenstern 2008). Due to uncertainties in both mixing and residence of nitrate in slow moving water bodies such as lakes, and due to the fluctuations in water quantity, the environmental authorities cannot rely solely on either a restriction on cumulative discharge over many periods, or independent restrictions on the discharges in each period. Hence, a workable solution is to impose relatively relaxed limits on the effects in each time period, together with a stringent limit on cumulative effects (for example, if the nitrate intake capacity and maximum nitrate residence time in a lake are roughly given as 40-50 tonnes/year and 5 years, a restriction of 50 kg nitrate load in each year together with a restriction of 200=40×5 tonnes aggregate over any 5 consecutive years). Such multi-period constraints give more flexibility to the permit users in scheduling their operations. The environmental authorities of large catchments may select several connected receptors to maintain water quality in the sub-catchments. For example, in a river catchment with many sub-catchments draining to different river segments, the catchment authority may wish to control nitrate discharge into each segment (considered as one receptor) as well as the total discharge into the river (total discharge into all receptors). Such cases require multi-receptor constraints to control the aggregate effect over several receptors in a single time period.
We can model all the receptor-end constraints using a receptor-end variable y_{rt} defined as the total allocation of resource$_{rt}$ (kg or mg/l). Let i = 1, 2,…, I index the receptor-end side constraints (excluding capacity constraints of individual receptors); V_i is a restriction on the aggregate effect over multiple receptors or multiple time periods as specified by constraint i (kg or mg/l); and A_{rti} is a unitless constraint coefficient. Then the set of receptor-end environmental constraints can be written as follows.

$\sum_f \sum_{s=\max(1,t-T)}^{\min(t,S)} H_{zr(t-s)} q^{zone}{}_{zs} - y_{rt} = 0$ for all r and t. (P-7) γ_{rt}

$y_{rt} \le C_{rt}$ for all r and t. (P-8) λ_{rt}

$\sum_r \sum_t A_{rti} y_{rt} \le V_i$ for all i. (P-9)

δ_i

Adding a new variable y_{rt} to the primal would add a new constraint to the dual and change the dual constraint associated with primal variable $q^{zone}{}_{zs}$ (D-4) as follows.

$$-\beta_{zs} + \sum_r \sum_{t=s}^{s+T} H_{zr(t-s)} \gamma_{rt} = 0 \qquad \text{for all } z \text{ and } s. \qquad \text{(D-5)} \quad q^{zone}{}_{zs}$$

$$-\gamma_{rt} + \lambda_{rt} + \sum_i A_{rti}\delta_i = 0 \qquad \text{for all } r \text{ and } t. \qquad \text{(D-6)}$$

y_{rt}

The dual variable γ_{rt} associated with the primal constraint which defines y_{rt} (P-7) indicates that the farms would make an incremental profit of γ_{rt} if they were allowed to increase the nitrate level at receptor *r* in year *t* by another one unit. Thus they should be charged at γ_{rt} per unit increase in nitrate level at receptor *r* in year *t*. Once more, λ_{rt} is the marginal value of the nitrate intake capacity of receptor *r* in year *t* and hence the market price of resource_{rt}. The capacity owner (the regulator if the resource capacities are not fully owned by the farms) should be paid at rate λ_{rt}. The dual constraint associated with primal variable y_{rt} (D-6) describes the relationship between γ_{rt} and λ_{rt} as $\gamma_{rt} = \lambda_{rt} + \sum_i A_{rti}\delta_i$.

The relationship shows that if the receptor-end side constraints are binding, farms may have to pay the resource price plus a price for the side constraint. When the buyers (farms) are charged at γ_{rt} and the capacity owners (other than farms) are paid at λ_{rt}, the market would clear with surplus revenue, because the same final commodity is traded at different prices. The surplus is explained by the amount paid by the farms for the binding side constraints. Who should collect the money paid for the side constraints? The market requires a mechanism for handling the possible surpluses. This is similar to the problem of handling the rental surpluses associated with transmission line capacity constraints in electricity markets. The issues and alternatives are discussed in Hogan (1992) and Oren et al. (1995).

One way to handle multi-receptor constraints is to define tradable rights for each side constraint and let them be traded separately in a combined market. That would be analogous to the "flow gate right" approach to electricity market design dicussed by Chao et al. (2000). Another option is to define tradable rights in terms of locational price differences, as is common in electricity markets, following Hogan (1992), and/or inter-temporal price differences, as discussed for gas markets by Read et al (2011). In other cases it may be appropriate to treat the constraint as a restriction on the capacity of a service provided to the market by someone outside the market. The service need not be explicitly traded among the market participants, but their utilisation of that service is implicit in their trades, creating rent implicitly collected via price differences. Then, while the capacity owner does not participate in the market, the surplus revenue associated with the constraint could be paid to them.[7] Otherwise, the surpluses may be taken by the regional environmental authority to fund remediation projects to clean the threatened water bodies, or re-distributed amongst the market participants, e.g., in proportion to permit holdings (Raffensperger 2011).

In the case multi-period constraints, though, defining tradable rights may cause confusion, since the rights involved (ie rights to pollute a receptor in each year vs rights to pollute a receptor any time within a period of several consecutive years) are actually close substitutes for each other. It would seem better to trade only the former annual type of rights for early years of the planning horizon, and only the latter multi-annual type for later years, where uncertainty is greater.

Multi-period or multi-receptor constraints can significantly affect the resource prices charged to the farms. If the nitrate level at receptor r in year t has critical impacts on the receptor-end side constraints, farms may have to pay a higher price for the resource, even if the associated resource capacity constraint itself is non-binding. For example, relatively clean sub-catchments performing well below the maximum acceptable nitrate discharge into local streams or lakes may still incur non-zero prices, because major downstream rivers or lakes are at a critical state, with binding environmental constraints. This is a proper reflection of the impact that their discharges will eventually have on those downstream receptors. Conversely, relatively polluted sub-catchments will have to pay a higher price, reflecting local pollution impacts, even though downstream rivers or lakes may not be in a critical state.

4.2 Source-based Side Constraints

Water quality standards at a few receptors alone may not guarantee local groundwater quality. The environmental authorities may impose source-end constraints (loading caps) on individual loading rates, regional totals, or catchment totals. Zonal loading caps can be used to deal with local hot-spots, because managing a loading cap is simpler and easier than having a large number

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[7] This is similar to the concept of paying the surpluses generated from binding transmission line capacity constraints in electricity markets to the transmission network operator.

of groundwater receptors. Non-zonal restrictions might also be imposed on total permit allocation to farms having some specific characteristic (for example, dairy farms or effluent irrigation farms. Hence, side constraints may be imposed on zonal loading $q^{zone}{}_{zs}$; or directly on combinations of individual allocations q_{fs}. We use two separate indices $m = 1,\ldots, M$ and $n = 1, \ldots, N$, for these two types of source-based constraints. Let $W^{Z}{}_{m}$ and $W^{F}{}_{n}$ be two restrictions on zonal loading and permit allocations respectively (kg), and $\underline{B}^{Z}{}_{zsm}$ and $\underline{B}^{F}{}_{fsn}$ be unitless constraint coefficients. Then the source-based side constraints can be expressed as below.

$\sum_z\sum_s \underline{B}^{Z}{}_{zsm}q^{zone}{}_{zs} \leq W^{Z}{}_{m}$ for all m. (P-10) $\pi^{Z}{}_{m}$

$\sum_f\sum_s \underline{B}^{F}{}_{fsn}q_{fs} \leq W^{F}{}_{n}$ for all n. (P-11) $\pi^{F}{}_{n}$

In the dual formulation, the prices attached to the (P-10) constraints expressed in terms of zonal loading appear as a component of the zonal loading prices, as in $\beta_{zs} = \sum_r \sum_{t=s}^{s+T} H_{zr(t-s)}\gamma_{rt} + \underline{B}^{Z}{}_{zsm}\pi^{Z}{}_{m}$, and hence the zonal loading prices reflect the market values attached to the restrictions on zonal loading. The prices attached to the (P-11) constraints expressed in terms of individual allocations appear as a component of the participant prices rather than of the zonal loading prices, as $\mu_{fs} = \beta_{zs} + \underline{B}^{F}{}_{fsn}\pi^{F}{}_{n}$. So, if a constraint n, not directly related to zonal loading, stops a farmer from buying more permits up to the point where her marginal profit falls to equal the zonal price β_{zs}, while the participant price (the farm's marginal value of the next 1 kg) is still above the zonal price. In this case, the zonal price alone would not justify the final allocation, in terms of the bid prices. But the zonal price, in combination with the shadow price(s) on binding non-zonal constraint(s), forms a participant price at which the final allocation is optimal, in terms of the bid prices.

If the farms who affect a binding (zonal) source-based constraint face a price that includes a price component associated with that constraint, while other farms face a price that does not include the price associated with that constraint, the market will clear with surplus revenue. These surpluses can be handled by variations on the methods discussed in Section 4.1 for receptor-end side constraints.

4.3 Private Constraints

Besides environmental constraints imposed by regulators, farms may have private constraints which they cannot easily internalize into their bids. For example, a farm may want to have a constant annual quantity for the next ten years; or a farm may be prepared to trade off quantities across years, provided an average is met, because some of the fertility from fertilizer applied in one year remains for the next. Such inter-temporal constraints are difficult to internalise into bids submitted for each year separately. Trial market rounds would help the farms to iterate towards a set of bids which meet the additional constraints, but this may need a large number of trials.

Alternatively, these "private" inter-temporal constraints can be included in the model as source-based non-zonal constraints on individual allocations as in (P-11) above. Such private constraints may provide space for strategic price manipulations (Oren and Ross 2004), but such manipulation is equally possible, in principle, by manipulating simple offers over multiple market rounds. Conversely, private constraints can provide more flexibility to the market, in choosing solutions that are feasible and acceptable to permit users, thus avoiding the need for many market rounds. Provided the private constraints form a convex LP feasible region, the model can still generate efficient market prices for the bids submitted. "Participant prices" will then differ by participant, though.

Farms may also have conditional permit requirements such as "I need at least 50 kg or none otherwise", because particular activities will not be economical below a certain scale. Participants could express this kind of requirement in their bids. But that would create a non-convex (integer) optimization problem, thus complicating price interpretation considerably (Bjørndala and Jörnsten 2008). Some electricity markets do allow participants to include conditions, such as non-divisible quantities and start-up costs or restrictions, in their quantity/price bids (Contreras et al. 2001). But here we exclude private constraints that require integer variables, assuming that farms can internalize those constraints into their bids based on past learning, or by iterating through trial market rounds.

5 Trading Both Resource and Loading Permits

The LP discussed above is intended to clear a market in which the total available capacity C_{rt} for any year and any receptor is in the market. Apart from the forecasted availability, the proposed model has no capacity restrictions such as reserve requirements. The participants collectively decide whether they allocate the total capacity now, or leave some for the future. The prices are determined solely by competitive interaction and trading amongst the farms. The total capacity, for the entire planning horizon, is in the market, but the regulator who is presumed to "own" any unsold capacity does not actively participate in the market. Thus this capacity may be sold for price zero, because the regulator is assumed to always have a reserve price of zero.

Thus, regardless of whether the resource capacities are fully allocated before market start, any of the resources may be fully allocated by the market. If a resource capacity constraint for some year t becomes binding in the solution, the capacity of year t is fully allocated, in terms of year 1 to year S loading permits traded in the current auction. Consequently, in the future, farms will not be able to buy any loading permit that affects resource utilisation in year t, unless someone who has bought a loading permit that affects resource utilisation in year t, offers to sell. Another disadvantage of fully allocating future capacities is that the estimated capacity C_{rt} may not be accurate. If the actual capacity turns out to below the estimate, or transport coefficients turn out to be mis-estimated, the environmental goals will not be achieved. Therefore, it is always risky to fully allocate the estimated capacity.

Resource allocation can be balanced over time in several different ways. The trivial method is to reserve some specified amount for the future, but authorities would have to decide how much. The regulator could set a penalty per unit resource allocated beyond some limit, but would have to choose the penalty and the limit. More generally, the regulator could set reserve prices on unallocated resource capacities, creating what would effectively be a stepped offer function. We would hope that the regulator would set reserve prices to price out over-allocation, not to price out the farms entirely and monopolize the market. Even so, such mechanisms push the market further away from free trading. As far as possible, we would like to facilitate the market itself to collectively decide how much resource capacity to allocate in each year.

The market discussed above traded only farm loading permits. If resource permits could also be traded, some entities might buy future capacity in the hope of profiting by saving it for the future. The regulator could also trade resource permits, as a way of balancing resource use over time. But there are other reasons why resource banks would be desirable. Our goal here is to create a market environment in which participants can trade interactively in a way which improves economic efficiency. Bilateral trading cannot readily achieve this, because discharges at different times and locations are not directly comparable. The previously described market in loading permits effectively overcomes this by providing a precise mathematical description of the pattern of impacts from each source. But it might still turn out that real opportunities for trade are quite limited. The ability of the farms to trade future capacities in the market is limited by the catchment hydro-geology which determines the composition each farm’s loading permit, in terms of ultimate resource capacity impacts. The catchment hydro-geology is encapsulated in the transport coefficients. Hence, the extent to which the farms can really interact via the market partly depends on the extent to which their transport coefficient matrices overlap (Prabodanie and Raffensperger 2009). Also, the “loading permits” traded by farms each imply a bundle of resource permits, in fixed proportions that may differ for each farm. These bundles are thus not interchangeable, and it may be quite difficult for the LP to find matching patterns that would allow trading to occur. And the delayed impacts of diffuse discharges mean that future receptor capacities could easily become fully allocated in later years, due to spatio-temporal interactions which farms may not readily understand, or be able to trade around.

While farms alone cannot create an active market, resource banks could create the liquidity to make a market work. The LP may not be able to directly match the bundles of resource permits implicit in the trades farms want to undertake, but it should be able to find resource banks willing to buy the bundle of resource permits implicitly offered by one farm and willing to sell the bundle of resource permits implicitly sought by another. Put another way, resource banks should be in a position to buy/sell the net discrepancies in year-by-year resource permit requirements arising out of the interaction between all farm trades.

Private parties may buy permits hoping to sell them later, at a profit, or simply arbitrage between farm buy/sell bids in the same period, thus improving market liquidity, and hence competitiveness, by acting as “market makers”. The regulator, or other public spirited parties, may also act as “banks”, to improve liquidity, to hold capacity off the market in order to create headroom for future trading, or even to retire capacity permanently for the sake of the environment. Parties able to clean the receptor water bodies, would also be natural “resource bank” participants, selling the tradable capacity they create in the form of year by year resource permits. Hence, the expanded market could incentivise active cleanup technologies and methods.

5.2 Optimal Resource Allocation Model (ORAM)

This section presents a combined market model trading both "loading permits", and trading resource permits. For simplicity, we assume that the farms trade only loading permits because the resource permits purchased from the market cannot be directly utilized to cover nitrate loading. A farm who wants to trade both loading permits and resource permits is considered as participating in the market as two independent players: a farm and a bank. We present a model to facilitate any number of banks participating in the market. Farms bid for loading permits, as above. Banks bid for each resource separately, also using bid steps for quantities starting from zero (offers can be converted to gross pool bids as discussed above). For this model, we assume that tradable rights (resources) are defined for all receptor end constraints in all time periods. We use a common index $\acute{r}$ = 1, 2, …, $\acute{R}$ for all the receptor end constraints including receptor capacity constraints, multi-receptor constraints, and multi-period constraints. The term resource $\acute{r}$ refers to a tradable right (resource) defined relative to receptor end constraint $\acute{r}$. A set of additional indices, parameters, and variables are required.

Additional Indices and Parameters:

b = bank: 1,…, B.
l = bank bid tranche: 1,…, L.
$U^{Bank}_{b\acute{r}l}$ = size of bid tranche l submitted by bank b for resource $\acute{r}$ (kg or mg/l).
$P^{Bank}_{b\acute{r}l}$= price for bid tranche l submitted by bank b for resource $\acute{r}$ ($/kg or $/mg/l).
$C_{\acute{r}}$ = tradable capacity of receptor end constraint $\acute{r}$ (kg or mg/l). $C_{\acute{r}} = C_{rt}$ or Z_i.
$G_{rt\acute{r}}$ = unitless constraint coefficient. $G_{rt\acute{r}}$ = 1, 0, or B_{rti}.

Additional Decision variables:

$x^{bank}_{b\acute{r}l}$ = quantity accepted from bid tranche l submitted by bank b for resource $_{\acute{r}}$ (kg or mg/l).
$q^{bank}_{b\acute{r}}$ = aggregate position of resource $_{\acute{r}}$ for bank b; this is the size of the resource permit held by bank b after trade (kg or mg/l).

Model: ORAM

Maximize $\sum_f\sum_s\sum_k P_{fsk}x_{fsk}$, + $\sum_b\sum_{\acute{r}}\sum_l P^{Bank}{}_{b\acute{r}l}x_{b\acute{r}l}$, subject to:

Upper and lower bounds on bid tranches

$x_{fsk} \le U_{fsk}$ for all f, s, and k. (P-1) $\theta^+{}_{fsk}$

$x_{fsk} \ge 0$ for all f, s, and k. (P-2) $\theta^-{}_{fsk}$

$x^{bank}{}_{b\acute{r}l} \le U^{Bank}{}_{b\acute{r}l}$ for all b, $\acute{r}$, and l. (P-12) $\alpha^+{}_{b\acute{r}l}$

$x^{bank}{}_{b\acute{r}l} \ge 0$ for all b, $\acute{r}$, and l. (P-13) $\alpha^-{}_{b\acute{r}l}$

Calculation of final permit positions

$\sum_k x_{fsk} - q_{fs} = 0$ for all f and s (P-3)

μ_{fs}

$\sum_l x^{bank}{}_{b\acute{r}l} - q^{bank}{}_{b\acute{r}} = 0$ for all b and $\acute{r}$. (P-14)

$v_{b\acute{r}t}$

$\sum_{f\in z} q_{fs} - q^{zone}{}_{zs} = 0$ for all z and s. (P-5) β_{zs}

Receptor end environmental constraints

$\sum_f \sum_{s=\max(1,t-T)}^{\min(t,S)} H_{zr(t-s)} q_{zs} - y_{rt} = 0$ for all r and t. (P-7) $\gamma_{\acute{r}}$

$\sum_r\sum_t G_{rt\acute{r}}y_{rt} + \sum_b q^{bank}{}_{b\acute{r}} = C_{\acute{r}}$ for all $\acute{r}$. (P-15)

$\lambda_{\acute{r}}$

Source based side constraints

$\sum_z\sum_s \underline{B}^Z{}_{zsm}q^{zone}{}_{zs} \le W^Z{}_m$ for all m. (P-10) $\pi^Z{}_m$

$\sum_f\sum_s \underline{B}^F{}_{fsn}q_{fs} \le W^F{}_n$ for all n. (P-11) $\pi^F{}_n$

The dual of the above formulation would show that the market price of resource$_{\acute{r}}$ $\lambda_{\acute{r}}$ is determined by competition among and between the farms and the banks. The dual constraints associated with primal variables $x^{bank}{}_{b\acute{r}l}$ and $q^{bank}{}_{b\acute{r}}$ produce the price relationship $\lambda_{rt} = v_{b\acute{r}t} = P^{Bank}{}_{b\acute{r}l} + \alpha^-{}_{b\acute{r}l} - \alpha^+{}_{b\acute{r}l}$. The relationships indicate that a bank can be the marginal trader who determines the market price ($\lambda_{rt} = P^{Bank}{}_{b\acute{r}l}$). The market price of resource$_{\acute{r}}$ cannot be zero unless the banks all bid at price zero, or not enough bid for the resource. We could allow the regulator, who offers to sell resource permits, to also act as a “Bank”, thus effectively refusing to sell unless the market price is above its bid (reservation) price. Hence the farms bidding for loading permits have to offer to pay permit prices that imply resource prices above the regulator’s reservation price ($H_{zr(t-s)}P_{fsk} \ge \lambda_{rt} \ge P^{Bank}{}_{b\acute{r}l}$), otherwise the farms cannot buy the resource (i.e., the farms who affect the particular resource constraint cannot buy loading permits). Thus, even if only one farm's year-1 to year-S permits affect some year t, the farm's ability to manipulate the price is limited.

6 A Conceptual LP Model for a Lake Catchment

This section presents a market model for a hypothetical lake catchment inspired by the nitrate pollution problem in the Lake Taupo catchment[8]. The main purpose is to show how the complex market interactions in a large catchment can be modeled as an LP, sufficiently addressing the underlying physical transport systems while maintaining the simplicity of the model. We do not present a numerical simulation of the model[9], but discuss how the price structure is driven by the constraint structure and the ability of the model to generate theoretically efficient prices. The model does not correspond to any particular catchment.

6.1 Hypothetical Lake Catchment

We consider a slow-moving lake with many rivers flowing in and one outlet. We assume that nitrogen loss from farms in the catchment occurs as nitrate leaching (runoff losses are negligible). Nitrate loaded by the farms may be carried down to the lake via streams or direct groundwater seepage. Therefore, the streams and the lake are considered as receptors where the mass of nitrate discharge into each receptor is to be controlled. We assume that in-stream nitrate residence time is less than a year, and the proportions of in-stream nitrate attenuation are known. We assume a simple nitrate mass balance model, in which any nitrate ion in the lake has a known probability of being lost (due to de-nitrification or other processes) and of being drained to the outlet.

[8] Since the mid 1970s, increased nitrate levels have been observed in Lake Taupo due to intensive farming and urbanization in the catchment. Catchment hydro-geology is complex with groundwater nitrate residence times ranging from 20 to 180 years. The lake waters can take 11 years to travel through the lake to the Waikato River. Even without further intensification of land use, the total nitrogen load into the lake is expected to increase in the future (Morgenstern 2008). To maintain current water quality, the nitrogen load into the lake has to be reduced by at least 20% (Petch et al. 2003). Tradable nitrogen discharge permits have been proposed as a means of achieving the target.

[9] Detailed numerical illustrations of the proposed models are available in Prabodanie (2011).

All the farms in the catchment are assumed to be participating in the trading program as discussed in Section 2.4.1. An independent entity called the “market operator” operates the market. A regional environmental authority called the “Bank” participates in the market as a resource bank which buys and sells resource permits. Trading takes place once every year. At the beginning of every trading year, the operator calculates the tradable resource capacities, taking into account all non-tradable sources, including the previous year’s discharges. We assume no previously allocated permanent discharge rights, and only pre-purchased permits are considered as initial holdings. A previous market allocation cannot cause infeasibility unless the previous capacity estimates were inaccurate or some unexpected event occurred (e.g., in this region, a volcanic eruption leading to large nitrogen inflows). All initially free (unallocated) resource capacity is considered to be owned by the Bank.

6.2 Market Modeling

A market model for the catchment requires two types of environmental constraints specifying the ability of the receptors to accept nitrates (restrictions on total nitrate discharge into the receptors in each year) and the ability of the lake to store nitrates (restrictions on annual nitrate storage in the lake) while maintaining its health[10]. The trading program would be simpler if tradable resource permits are defined only relative to the constraints on annual discharge limits. Surplus revenue generated by binding storage constraints can be handled by methods discussed in Section 4.1. A cap on loading rate per hectare may be imposed (outside the market) to secure local groundwater quality.

Model ORAM presented in Section 5.3 above is the best structure to model a market for this hypothetical catchment. We present only the receptor-end environmental constraints, because the other constraints and the objective are similar to those of ORAM.

[10] If the nitrate fate and transport in the lake is well understood with certainty, either type of constraints alone may be sufficient to maintain lake water quality. However, under a high level of uncertainty, having both types of constraints is safer. Another option is having only the discharge based constraints in early years and only the storage based constraints in later years of the planning horizon.

Additional indices, parameters, and variables:

Receptor r = 1 is the lake, and r = 2, 3, …, R are the streams.

y^{lake}_t = total mass nitrate discharge into the lake during year $t = \sum_r E_r y_{rt}$ where $1-E_r$ = proportion of in-stream nitrate attenuation. $E_1 = 1$.

y^{store}_t = mass nitrate storage in the lake at the end of year t.

G^W= proportion of mass nitrate in the lake that flows into the downstream river annually.

G^L = proportion of annual nitrate attenuation in the lake.

$y^{store}_t = G\,(y^{store}_{t-1} + y^{lake}_t)$ where $G = (1-G^W-G^L)$: lake nitrate balance equation, assuming both nitrate losses and outflow take place at the end of the year.

C^0 = maximum acceptable mass nitrate storage in the lake.

Model: LakeModel (Receptor-end environmental constraints)

$\sum_f \sum_s H_{fr(t-s)} q_{fs} - y_{rt} = 0$ for all r and t. (P\`-1) γ_{rt}

Maximum annual nitrate load into the streams

$y_{rt} + q^{bank}_{rt} = C_{rt}$ for all r and t. (P\`-2) λ_{rt}

Maximum annual nitrate load into the lake

$\sum_r E_r y_{rt} - y^{lake}_t = 0$ for all t. (P\`-3) ε_t

$y^{lake}_t + q^{bank}_{1t} = C_{1t}$ for all t. (P\`-4) λ_{1t}

Maximum in-lake nitrate storage

$Gy^{lake}_t/2 + Gy^{store}_{t-1} - y^{store}_t = 0$ for all t. (P\`-5) δ_t

$Y^{store}_t \le S_0$ for all t. (P\`-6) λ^0_t

6.3 Pricing Nitrate Discharges into a Lake

The above model contains both multi-period and multi-receptor environmental constraints. Hence, the prices should be spatially and temporally dependent. The constraint structure of the model provides the following price relationships.

$v_{rt} = \lambda_{rt}$ for all r and t.

$\gamma_{rt} = E_r\varepsilon_t + \lambda_{rt}$ for r = 2,3, …, R and all t.

$\gamma_{1t} = \varepsilon_t$ for all t.

$\varepsilon_t = \lambda_{1t} + G\delta_t$ for all t.

$\delta_t = G\delta_{t+1} + \lambda^0_t$ for all t.

Price λ_{rt} for some $r \in \{2,3, \dots, R\}$ is the market price per unit nitrate discharge into stream r in year t. The market price per unit nitrate discharge into the lake in year t is λ_{1t}. The regulator which participates in the market as a bank (possibly as a resource owner) will be selling at λ_{rt}. However, the prices charged to the farms, γ_{rt} may be higher than λ_{rt} because discharges into the streams have effects on the lake also. For the farms, the stream price equals the stream's market price plus the lake price adjusted by the proportion of in-stream nitrate carried down to the lake ($\gamma_{rt} = E_r\varepsilon_t + \lambda_{rt}$). Thus, even if the farms in a sub-catchment perform well below the maximum nitrate intake capacity of the local streams, they may face a high price because a relatively large proportion of their discharges flow down to the lake. The farms may face an even higher price if the local constraints also bind. The storage cost (cost of carrying a kg of in-lake nitrate into the next year) is δ_t. For discharging nitrate into the lake, the farms should pay the expected cost of carrying forward in addition to the market price ($\varepsilon_t = \lambda_{1t} + G\delta_t$). The storage cost of each year is determined by current and future storage capacity constraints ($\delta_t = G\delta_{t+1} + \lambda^0{}_t$). Even if the total discharge into the lake in some year t is well below the maximum acceptable level, the price of discharge in year t may be increased by the capacity shortages expected in the future. In large agricultural catchments, due to unmanaged nitrate discharges in the recent past, capacity shortages are more likely to occur in the future than in the present. For example, in the Lake Taupo catchment, a large amount of nitrate leached from current and previous land use is currently flowing towards the lake via groundwater, and nitrate discharge into the lake from this non-tradable source is likely to increase in the future (Morgenstern 2008). The above price structure has the capability of pricing the current discharges to reflect the value of future capacity.

7 Discussion and Conclusions

We have presented alternative LP models to price diffuse nitrate discharge permits. The basic model simulates a market in loading permits where the diffuse dischargers, mainly agricultural dischargers, trade rights to load nitrates into groundwater aquifers. But we note that restricting the market to only trade discharge permits creates rigidities which may make trading difficult. Thus we propose an expanded model which allows third parties to trade (unbundled) resource permits, together with the loading permits required by farms, in the same combined market. We suggest that participation of third parties, including the environmental authorities who represent the public interest, as "resource banks" could improve the performance of the market by balancing resource allocation over time, establishing competitive market prices, and aiding liquidity.

This latter model provides a potentially workable approach to the inherently difficult problem of creating an environment in which farms can trade interactively and adjust nitrate loadings now, while recognizing that the physical implications of those adjustments may not be seen for decades hence. This very long time scale remains a challenging issue, though, and success probably depends on the intervention of far sighted regulators to establish, monitor, and probably participate in such markets. Those parties will also have to address important practical issues such as the monitoring and enforcement of compliance with permit limits allocated by market trading.

8 References

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