THE VULNERABILITY ASSESSMENT OF CURRENT BUILDINGS BY A MACROSEISMIC APPROACH DERIVED FROM THE EMS-98 SCALE

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SUMMARY

A hierarchical family of Damage Probability Matrices (DPM) has been derived in this paper from the ones implicitly contained in the EMS-98 Macroseismic Scale for 6 vulnerability classes. To this aim the linguistic definitions provided by the scale, and the associated fuzzy sub-sets of the percentage of buildings, have been completed according to reliable hypotheses.

A parametric representation of the corresponding cumulative probability distributions is moreover provided, through a unique parameter: a vulnerability index variable in the range from 0 to 1 and independent of the macroseismic intensity. Finally, an innovative macroseismic approach allowing the vulnerability analysis of building typologies is defined within the European Macroseismic Scale (EMS-98) and qualitatively related to the vulnerability classes. Bayes' theorem allows the upgrading of the frequencies when further data about the built-environment or specific properties of the buildings are available, allowing the identification of a different behaviours with respect to the one generally considered for the typology. Fuzzy measures of any damage function can be derived, using parametric or non-parametric damage probability matrices. For every result of the seismic analysis, the procedure allows supply to the user of the final uncertainty connected with the aforementioned fuzzy relation between the probability of the damage grade, the macroseismic intensity and the vulnerability classes.

Keywords: Ordinary buildings, Vulnerability assessment, Damage scenario

Introduction

Definitive publication in 1998 of the new European Macroseismic Scale (Grunthal 1998), stimulated the elaboration of new methodologies for the development of damage scenarios to the urban fabric (for earthquakes of predetermined intensity) or risk assessments in relation to the ascertained shakeability of the areas. In said methodologies, generally identified with the adjective "macroseismic" "(Giovinazzi and Lagomarsino 2004; Giovinazzi and Lagomarsino 2006), the conventional vulnerability measures (the 6 vulnerability classes) and the damage grade are directly assumed from the scale, together with the list of the building typologies (possibly modified taking into account the local particularities).

The applications carried out are characterised by the different territorial scale (suburban, urban, municipal or regional) and by the different catalogues used for the systematic or sampled classification of the building typologies present in the territory. In particular numerous applications are based on poor but systematic data (in particular ISTAT

1991 data) possibly checked by sampling with richer and more reliable information (Bernardini 2004).

In the definition of the damage grades that the EMS-98 Macroseismic Scale supplies, a description of the Damage Probability Matrices (DPM) of the different vulnerability classes is contained, even though in a vague and incomplete way.

The use of observed damage data, suitably processed and organised in terms of DPMs, has been introduced in Italy for the vulnerability analysis and forecast of the expected damage, starting from the Irpinia earthquake of 1980 (Braga et al. 1980). The DPM supply, for a seismic input described in terms of macroseismic intensity and for the different building classes with homogeneous behaviour (vulnerability classes), the probability of occurrence of different degrees of damage to the building (defined on the basis of the damage observed in the structural and non structural elements).

The EMS-98 supplies, in linguistic terms the percentage of occurrence of 5 different grades of global damage to buildings for six vulnerability classes correlated in a fuzzy way to building typologies.

Objectives

The objective of this work is the definition of a model, coherent with the definitions of the EMS-98, for estimation of the consequences on a building at the time of a determined seismic event. The basic idea of the method is to use the information contained in the scale to derive the DPMs for the vulnerability classes, in a numeric and complete form.The aspects relative to the uncertainties connected with the definition of the model are highlighted and represented by convex probability distribution sets corresponding to their representation through random sets (Bernardini 1999).

Furthermore, parametric representations are proposed approximating the numeric results in the form of beta discrete distributions depending on two parameters: one adimensional parameter as a measure of the vulnerability correlated, as a function of the macroseismic intensity, to the average damage value and specific values, independent of the intensity, and of a second parameter correlated to the variance.

The expected behaviour of “modified” building typologies due to the ascertained presence of specific factors (for instance the degree of maintenance) or typological features (for instance the number of storeys) were analysed, starting from the rather general building typologies defined by the scale. The definition of the DPM for building typologies was obtained, interpreting the indications of the EMS-98 table of vulnerability, in terms of frequency associated with the classes recognised as representative for each typology.

The use of Bayes’ theorem allows updating of the frequencies associated with the classes, in the case of availability of further data about the building that permit identification of modified behaviours compared to those envisaged for the typology. Numerical applications based on ISTAT 1991 data are presented.

Derivation of the Damage Probability Matrices from the definitions contained in the EMS-98 scale

In the definition of the macroseismic intensity grades of the EMS-98 (Grunthal 1998) the distribution of damage to the buildings of different classes is contained, even though in a vague and incomplete way, with the variation of intensity (Table 1). The evident vagueness of the adjectives and incompleteness of the information (for each class and intensity at most the frequency of two damage grades is characterised) does not, however, permit associating very precise numerical Damage Probability Matrices (DPM) to the scale.

Table 1. Linguistic frequencies of damage for vulnerability classes and macroseismic intensity according to the EMS-98 scale. Dk (k=0…5) represents the damage grade according to EMS-98.

I/Dk	0	1	2	3	4	5
V		Few A or B				
VI		Many A or B, Few C	Few A or B			
VII			Many B, Few C	Many A, Few B	Few A	
VIII			Many C, Few D	Many B, Few C	Many A, Few B	Few A
IX			Many D, Few E	Many C, Few D	Many B, Few C	Many A, Few B
X			Many E, Few F	Many D, Few E	Many C, Few D	Most A, Many B, Few C
XI			Many F	Many E, Few F	Most C, Many D, Few E	Most B, Many C, Few D
XII						All A or B, Nearly All C, Most D or E or F

For what concerns the first aspect, the scale suggests possible numerical values that can be associated with the three key adjectives used: *Few, Many, Most* (Figure 1). The same figure also suggests a model of possible numerical interpretation, through a "fuzzy pseudo-partition" (Klir and Yuan 1995) of the interval [0, 100] of the percentages of buildings.

The complete description of the distribution of damage is obtained operating a reasonable linguistic complement of the definitions supplied by the scale, making first and foremost the "fuzzy pseudo-partition" directly deducible from the EMS-98 more coherent. Indeed it does not seem logical to be able to associate membership equal to 1 to the extreme values of 0 and 100 when respectively the adjectives *Few* and *Most* are used. Furthermore, the adjectives *None*, *All* and especially *Nearly All* require a numerical interpretation that is not performed by the scale. These observations have led to the definition of five "fuzzy sets" (Zadeh 1965) associated with the adjectives *Nearly None*, *Few*, *Many*, *Most* and *Nearly All* with the condition that for each percentage the sum of the values of membership is equal to 1 (Figure 2).

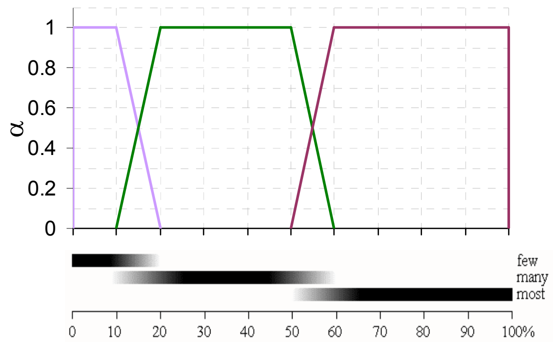


Figure 1. Percentages associated with the linguistic definitions used by the EMS-98 scale and fuzzy pseudo-partition directly deducible from the EMS-98.

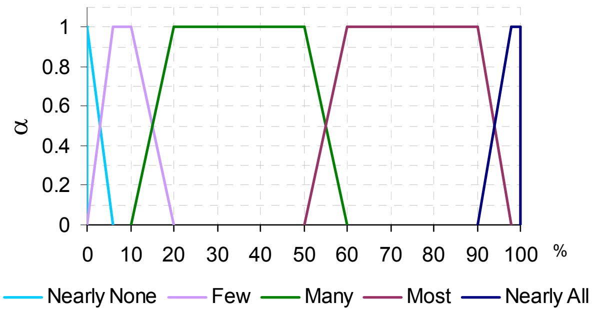


Figure 2. Proposal of a fuzzy pseudo-partition of the numerical interval [0, 100] through 5 fuzzy sets associated with *Nearly none*, *Few*, *Many*, *Most* and *Nearly All*.

Assuming the fuzzy pseudo-partition shown in Figure 2, the linguistic definitions contained in the scale were completed respecting two rules (Bernardini 2004):

– the sum of the percentages of buildings in the different damage grades (for each class and intensity) associated with the central "white" values (central value of membership by α-cut = 0.5) of the linguistic definition is equal to 100 (Bernardini 1995);

– by parity of class the increase of an intensity grade that is, by parity of intensity, the passage to the more vulnerable class produces a unitary increase of the damage grade.

The result of the linguistic completion of the EMS-98 scale is summarised in Table 2. This shows: 1) the linguistic values directly suggested by the scale (in bold) and summarised in Table 1, 2) the linguistic completions proposed. In bold on a grey background two significant modifications to the values suggested by the scale are also highlighted:

– for Class C, Intensity XI the values *Most* and *Many* associated with the damage grades 4 and 5 do not at first satisfy the rules indicated above; therefore the damage at grade 4 has been reduced from "*Many + 2Few*";
– at intesity XII it seems reasonable to differentiate the expected damage for classes D, E and F; therefore those expected for classes E and F have been reduced.

Table 2. Linguistic completion of the EMS-98 scale

Dk / I	0	1	2	3	4	5
CLASS A						
V	All-Few	**Few**	None	None	None	None
VI	Many + 7/3Few	**Many**	**Few**	None	None	None
VII	1/3Few	2Few	Many	**Many**	**Few**	None
VIII	None	1/3Few	2Few	Many	**Many**	**Few**
IX	None	None	1/3Few	3Few	Many	**Many**
X	None	None	None	5/6Few	2Few	**Most**
XI	None	None	None	None	5/6Few	Most + 2Few
XII	None	None	None	None	None	**All**
CLASS B						
V	All-Few	**Few**	None	None	None	None
VI	Many + 7/3Few	**Many**	**Few**	None	None	None
VII	7/3Few	Many	**Many**	**Few**	None	None
VIII	1/3Few	2Few	Many	**Many**	**Few**	None
IX	None	1/3Few	2Few	Many	**Many**	**Few**
X	None	None	1/3Few	2Few	Many+ Few	**Many**
XI	None	None	None	Nearly Few	8/3Few	**Most**
XII	None	None	None	None	None	**All**
CLASS C						
V	All	None	None	None	None	None
VI	All-Few	**Few**	None	None	None	None
VII	Many + 7/3Few	Many	**Few**	None	None	None
VIII	7/3Few	Many	**Many**	**Few**	None	None
IX	1/3Few	2Few	Many	**Many**	**Few**	None
X	None	1/3Few	2Few	Many	**Many**	**Few**
XI	None	None	None	4/3Few	**Many+ 2Few**	**Many**
XII	None	None	None	None	1/3Few	**Nearly All**

Dk / I	0	1	2	3	4	5
CLASS D						
V	All	None	None	None	None	None
VI	All	None	None	None	None	None
VII	All-Few	**Few**	None	None	None	None
VIII	Many+ 7/3Few	Many	**Few**	None	None	None
IX	7/3Few	Many	**Many**	**Few**	None	None
X	1/3Few	2Few	Many	**Many**	**Few**	None
XI	None	1/3Few	2Few	Many	**Many**	**Few**
XII	None	None	1/3Few	1/2Few	2Few	**Most**
CLASS E						
V	All	None	None	None	None	None
VI	All	None	None	None	None	None
VII	All	None	None	None	None	None
VIII	All-Few	Few	None	None	None	None
IX	Many+ 7/3Few	Many	**Few**	None	None	None
X	7/3Few	Many	**Many**	**Few**	None	None
XI	1/3Few	2Few	Many	**Many**	**Few**	None
XII	None	Nearly Few	2/3Few	Few	2Few	**Most-Few**
CLASS F						
V	All	None	None	None	None	None
VI	All	None	None	None	None	None
VII	All	None	None	None	None	None
VIII	All	None	None	None	None	None
IX	All-Few	**Few**	None	None	None	None
X	Many+ 7/3Few	Many	**Few**	None	None	None
XI	7/3Few	Many	**Many**	**Few**	None	None
XII	None	1/3Few	Few	Few	Many	**Many+ Few**

The numerical interpretation of the linguistic result is now expressable according to the random set theory (Bernardini 1999) and "imprecise probabilities" (Klir 2005).

For each α-cut of the fuzzy sets associated with the linguistic definitions, the frequencies of the damage grades (j from 0 to 5) are measured by "interval probablities" [lj, uj], to which is associated a convex set of possible damage probability distributions. In Figure 3 for instance the interval probabilities are represented for class A and intensity VI and VIII; the precise distributions corresponding to the average values of the "white" percentages are also shown and, for comparison, the binomial distributions elaborated from the damage from the Irpinia earthquake for the same Class A.

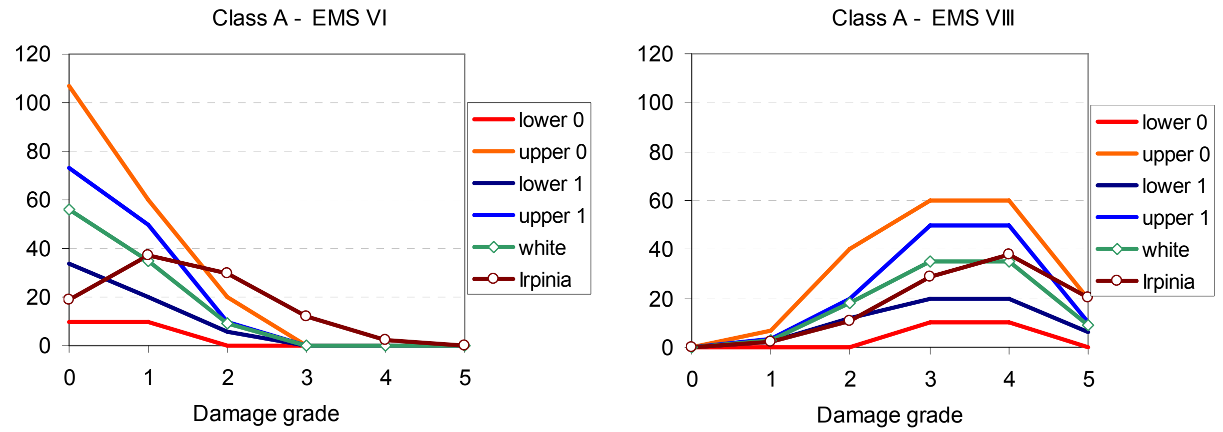


Figure 3. Class A, Intensity EMS98 VI and VIII: Interval probabilities and "white" values for alpha = 0 and 1 and comparison with the DPM from Irpinia (Braga et al., 1980) for the same Class and Intensity (MSK).

The Damage Probability Matrices may be represented in terms of vulnerability curves, showing the value taken on by the averages of the damage distributions (μ_D) with variation of macroseismic intensity.

In Figure 4 the curves I - μ_D obtained from the linguistic and numeric completion of the EMS-98 scale are shown for the 6 vulnerability classes and for three different values of the α –cut of the fuzzy sets associated with the linguistic definitions supplied by the scale.

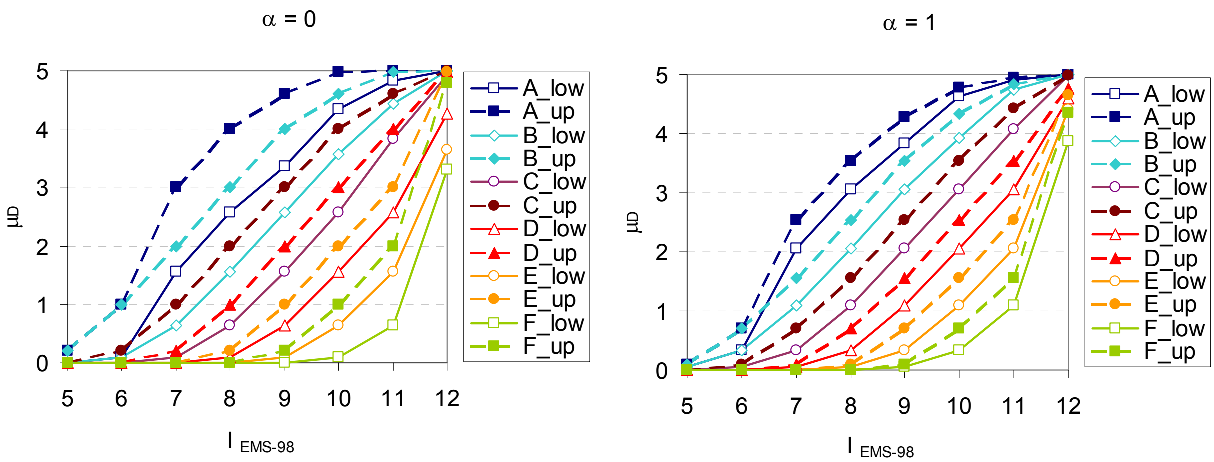


Figure 4. Curves I-μ_D obtained from completion of the EMS-98 matrices for the 6 vulnerability classes, for two different values of the α-cut: α = 0 and α = 1

A first rapid, but somewhat significant validation of the model of vulnerability deduced from the EMS-98, may be obtained from a comparison with the Damage Probability Matrices relative to the Irpinia earthquake (Figure 5), it also being represented in terms of vulnerability curves. To this end it should be pointed out that: 1) the DPM for Irpinia refer to the MSK-76 macroseismic scale which may reasonably be assumed analogous to the EMS-98 (Grunthal 1998); 2) the maximum intensity recorded with the Irpinia earthquake is equal to I_{MSK}=X, therefore the information relative to grades XI and XII is missing.

Figure 5 shows a comparison between the different vulnerability curves, from which it clearly emerges how the trend of the two curve families is analogous and how there is a discrete correspondence between the three classes of DPM in Irpinia and the first three EMS-98 vulnerability classes. However, one notices for class B and above all for class A a net increase of the expected damage in grade VI intensity, as confirmed (for Class A) by the comparison shown in Figure 3. Furthermore, for class B and especially for class C the typologies of Irpinia show behaviour systematically more vulnerable at all levels of intensity. One could presuppose that such a deviation is connected with the typological composition of the building population in Irpinia and with the choice of proceeding in any case with their classification in three vulnerability classes.

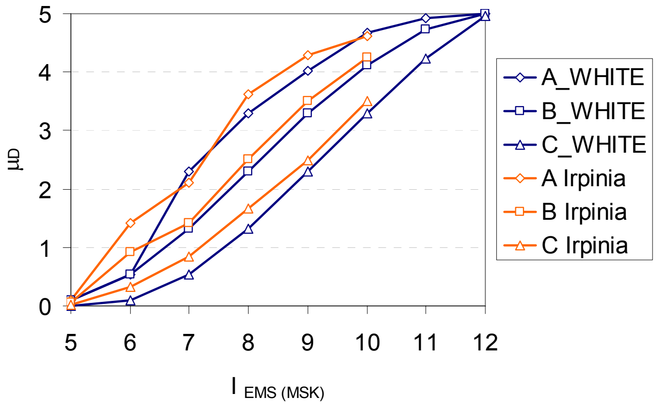


Figure 5. Comparison for the three classes ABC of the average values of “white” DPM with those of the DPM of Irpinia (Braga et al., 1980).

Parametric representation of the DPM

Following a more operative representation of the method of vulnerability obtained, the Damage Probability Matrices were parameterised with respect to a single parameter V∈ [0, 1], independent of the intensity and measured by a fuzzy set associated with each vulnerability class, according to that already proposed by Giovinazzi and Lagomarsino (2001). Such a representation is shown in the following as a “parametric representation”.

Having fixed a value of V and intensity I, the average value (μ_D) of a given damage distribution is determined by means of a very precise analytical function:

$$\mu_D = 2.5 + 3\tanh\left(\frac{I + 6.25V - 12.7}{3}\right)\cdot f\left(V,I\right) \quad \Big| \quad 0 \le \mu_D \le 5 \qquad (1)$$

where f(V, I) is a function depending of the vulnerability index and intensity, introduced to understand the trend of the numerical vulnerability curves taken from the EMS-98 even for the lower extremes of the intensity grades (I=V and VI).

$$f\left(V,I\right) = \begin{cases} e^{\frac{V}{2}\cdot(I-7)} & I \le 7 \\ 1 & I > 7 \end{cases} \qquad (2)$$

Figure 6 shows the comparison between the vulnerability curves relative to the central “white” values of the definitions of the EMS-98 scale and the parametric representation of the same (Eq. (1)) obtained for values of the vulnerability index shown in Table 3.

Table 3. Values of the index V corresponding to the six vulnerability classes.

Class	A	B	C	D	E	F
V	0.88	0.72	0.56	0.40	0.24	0.08

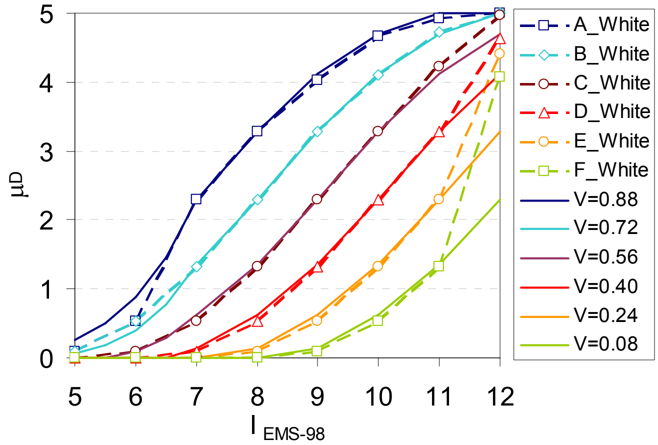


Figure 6. Curves I-μ_D corresponding to the central "white" values of the definitions of the scale for the 6 vulnerability classes and corresponding parametric vulnerability curves.

It is interesting to note the perfect coherence between the two representations especially for the central grades of intensity, from I=VII to I = XI.

Inverting the function that defines the trend of the parametric vulnerability curves (Eq. (1)) it is possible to find a fuzzy set in the interval of V for each vulnerability class. The modalities chosen for completion of the EMS-98 matrices and the relation defined between the quantities μ_D – V – I (Eq. (1)) has led to the definition of fuzzy sets more or less linear with the variation of α and sufficiently regular. The fuzzy sets shown in Figure 7 were obtained averaging the values obtained for different grades of intensity separately for the different values of α (0, 0.2, 0.4, 0.6, 0.8, 1). The trend found results regular for the central values of intensity (from VII to XI) for most of the vulnerability classes with the exception of classes E and F for which the values of the parameter V obtained resulted stable only for intensity values equal to XI and XII. With this in mind only the values found for these intensity grades were considered in the averaging operation.

In application of the parametric method, in order to succeed in having a completely regular fuzzy partition in the interval of V $\in$ [0, 1], the fuzzy sets obtained from the EMS-98 were linearized as shown in Figure 7 (continuous lines of greater thickness).

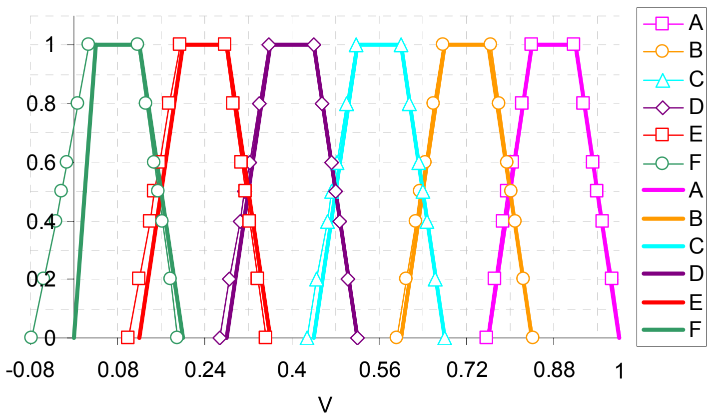


Figure 7. Fuzzy sets of the EMS-98 vulnerability classes obtained by means of the values of V found inverting Eq. (1) and corresponding linearized fuzzy sets assumed in the parametric methodology

(continuous lines of greater thickness).

To approximate the damage distributions observed following Italian earthquakes the binomial distribution depending only on the binomial coefficient p has often been used; this is proportional to the average value ($p = \mu_D/5$), from which the standard deviation $\sigma_D^2 = p(1-p)/5$ directly derives. Completion of the damage matrices taken from the EMS-98 scale has, however, pointed out how the value of the standard deviation associated with the damage distributions is lower. For this reason it was decided to adopt a probabilistic distribution derived from the discretisation of a beta distribution defined in the interval [0, 5]:

$$\text{PDF}: p_\beta(x) = \frac{\Gamma(t)}{\Gamma(r)\Gamma(t-r)} x^{r-1}(5-x)^{t-r-1} \tag{3}$$

where t and r are the parameters of the distribution, defined as a function of the average value μ_x and the variance σ_x^2 from Eq.(4), and Γ the gamma function.

$$t = \frac{\mu_x(5-\mu_x)}{\sigma_x^2} - 1 \qquad r = t \cdot \frac{\mu_x}{5} \tag{4}$$

A discrete distribution also dependent on two parameters t and r may therefore be defined in the following form:

$$\begin{aligned} p(0) &= P_\beta(0.5) \\ p(k) &= P_\beta(k+0.5) - P_\beta(k-0.5) \\ p(5) &= 1 - P_\beta(4.5) \end{aligned} \tag{5}$$

where

$$P_\beta(x) = \int_0^x \frac{\Gamma(t)}{\Gamma(r)\Gamma(t-r)} x^{r-1}(5-x)^{t-r-1} dx \tag{6}$$

The limited variation found in the values assumed by the parameter t for the numerical damage distributions taken from the EMS-98 allows one to assume a single value for t (equal to 8) as representative of the variance of all the possible damage distributions (Bernardini et al. 2007a).

Defining such parameter a priori, it is thus possible to define the damage distributions exclusively through knowledge of the average value, but characterised by a variance coherent with that found from completion of the EMS-98 matrices (Figure 8).

In Figure 8 the comparison between the standard deviations of the numerical distributions, obtained for class A for three values of the α-cut, of the binomial distribution and the beta discrete distribution with t =8 is shown.

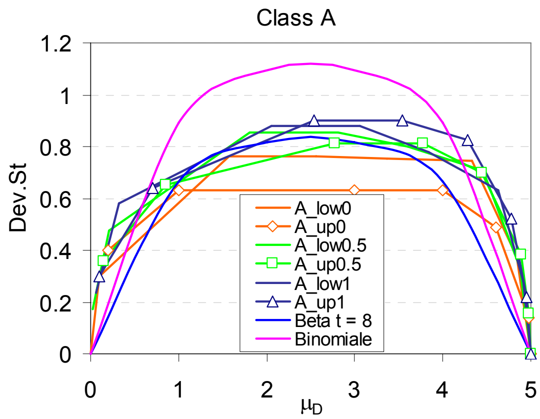


Figure 8. Standard deviation as a function of the average damage for class A compared with the correlations deriving from the binomial distributions and beta discrete distributions with t = 8.

Vulnerability typologies and classes in the EMS-98 scale

With the aim of defining the DPM by building typologies the indications of the EMS-98 table of vulnerability were interpreted in terms of frequencies associated with the classes recognised as representative for each typology.

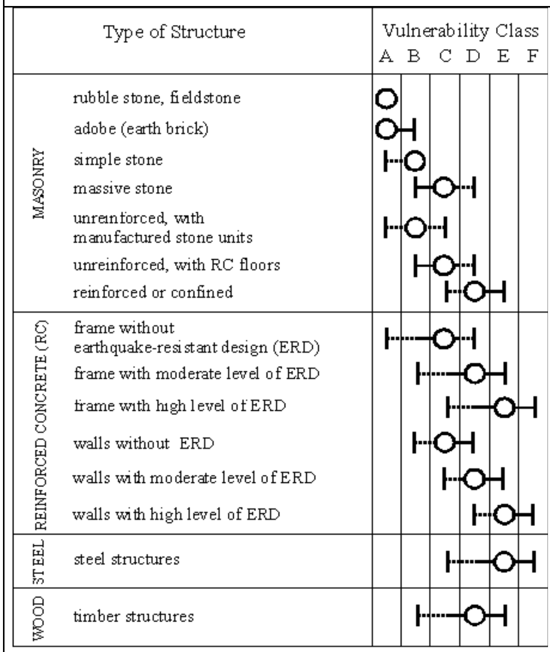
The correlation between the 6 vulnerability classes and the 15 typologies (of which 7 relative to masonry buildings and 6 to buildings in r.c.) are summarised in Table 4.

In almost all cases one is not dealing with a deterministic relation, but with an implicit probabilistic relation of which a "modal" class is explicit, the "most likely vulnerability class" alongside two groups of classes judged "probable" and "less probable" or "exceptional". An explicit reasonable hypothesis for interpretation of the above mentioned probabilistic relation is shown in Table 5: assuming for "less probable" the average "white" value (Bernardini et al. 2007a) of FEW (9%) and for "probable" the analogous value of 2.5*FEW (22.5 %), the modal frequency may be calculated for a difference at 100%. Furthermore, one assumes that in any case, the probability distribution takes on positive values in at least 3 classes, assigning to such a purpose a percentage equal to 4.5 (corresponding to the white value of FEW/2) to contiguous classes not envisaged by the EMS-98 and adding a class Y of buildings with greater vulnerability than that of class A.

The percentages shown in Table 5 may be interpreted as probabilities of the classes Cj (j from 1 to 7) conditioned by typology Mi (i from 1 to 15):

$$m_j^i = \Pr(C_j \mid M_i), \qquad \forall i : \sum_j m_j^i = 1 \tag{7}$$

Table 4. Correlation between vulnerability classes and typologies according to the EMS-98.



O most likely vulnerability class; — probable range;
······range of less probable, exceptional cases

Table 5. Correlation between vulnerability classes and typologies according to the EMS-98 in terms of probability of the classes conditioned by typology.

Ti	Vulnerability classes Cj						
	Y	A	B	C	D	E	F
M1	4.5	91	4.5				
M2	4.5	73	22.5				
M3		9	86.5	4.5			
M4			22.5	68.5	9		
M5		9	82	9			
M6			22.5	68.5	9		
M7				9	68.5	22.5	
RC1		9	22.5	59.5	9		
RC2			9	22.5	46	22.5	
RC3				9	22.5	46	22.5
RC4			9	68.5	22.5		
RC5				9	68.5	22.5	
RC6					9	68.5	22.5
S				9	22.5	46	22.5
T			9	22.5	46	22.5	

Influence of behaviour modifiers on the vulnerability of typologies

One supposes that additional information present in the catalogue of buildings allow one to specify the typology better, in that they belong to disjointed sub-groups of the M_i typology. The sub-groups are characterised by the homogeneity of certain typological characteristics considered as vulnerability modifiers (for instance, with reference to the catalogue of ISTAT 1991 data, “age of building”, “height” “state of maintenance”,“aggregation to other buildings”); in general the k-th modifier (k from 1 to m) is a variable of state that may take on r_k = 1 to n_k values (for instance for the “height” modifier the 3 values “low”, “medium”, “high”, corresponding to three specified intervals of the number of storeys). Each sub-group of the M_i typology, defined in the index s (1 to ss), is characterised by the states $S_s = (S_{r1}, S_{r2}, \ldots S_{rm})$ respectively assumed by the modifiers.

Were the matrix of the cumulative probabilities $Pr(C_j, S_s)$ known, each sub-group could be characterised by the relative frequencies $m_j^{i,s}$ in the different vulnerability classes and by the corresponding absolute frequencies if the count of the buildings in different states is known (in an assigned territory). In Table 6 the structure of the matrix and the relative marginals is indicated.

Table 6. Matrix of the cumulative probabilities of Classes and Modified States and relative marginals.

	S_1		S_s		S_{ss}	Total
C_1 = Y	$Pr(C_1, S_1)$		$Pr(C_1, S_s)$		$Pr(C_1, S_{ss})$	m^I_1
	…		…		…	…
C_j	$Pr(C_j, S_1)$		$Pr(C_j, S_s) = m_j^{I,S}\, Pr(S_s)$		$Pr(C_j, S_{ss})$	m^I_j
	….ì		…		…	…
C_7 = F	$Pr(C_6, S_1)$		$Pr(C_6, S_s)$		$Pr(C_6, S_{ss})$	m^I_6
Total	$Pr(S_1)$		$Pr(S_s)$		$Pr(S_{ss})$	1

The cumulative matrix completely describes (in probabilistic terms) the influence of the different modifiers on the seismic vulnerability, including their correlation. In the reality of this matrix one knows (rather one presupposes to know from the definitions of the EMS-98 and hypothesis indicated assumed) only the m_j^i marginals.

There is not just one way to try to reconstruct the matrix of cumulative probabilities, lacking the corresponding statistical information in cumulative form. One proposes the use of a Bayesian procedure (Eq. (9)) of progressive, separated in consideration of the effect of each modifier k, to which the possible states correspond.

Naturally the $Pr(S_{rk} / C_l)$ are not known: they may however be supposed monotonically increasing or decreasing with the index l, depending on the expected effect of the modifier on vulnerability.

In the assigning one must however take into account the possible correlations between the modifiers. The influence on vulnerability in the application of a certain modifier is that which also summarises the effects of other modifiers which are related to this one. For instance, in the archive of the reports on damage to masonry buildings relative to class A, there might be many “low” buildings not in as much as low buildings are more vulnerable than high buildings (with parity of structural quality the opposite should be true, especially if there is no anti-seismic project), but because low buildings have a dreadful state of maintenance. Symmetrically then, when one analyses the effect of the state of maintenance and this appears very poor, one must take into account that we are reasoning about buildings that are prevalently low and therefore not particularly vulnerable.

$$
\begin{aligned}
&\forall s=(r_1,r_2,...,r_m),\ \text{posto } S_s=(S_{r_1},S_{r_2},...,S_{r_k}):\\
&\forall j:\quad m_j^{i,0}=m_j^i\\
&m_j^{i,1}=\frac{Pr(C_j\cap S_{r_1})}{Pr(S_{r_1})}=\frac{Pr(S_{r_1}/C_j)}{\sum_l Pr\left(S_{r_1}/C_l\right)\cdot m_l^{i,0}}m_j^{i,0}\\
&m_j^{i,k}=\frac{Pr(C_j\cap S_{r_k})}{Pr(S_{r_k})}=\frac{Pr(C_j\cap(S_{r_{k-1}},S_{r_k}))}{Pr(S_{r_{k-1}},S_{r_k})}=\frac{Pr\left(S_{r_k}/C_j\right)}{\sum_l Pr\left(S_{r_k}/C_l\right)\cdot m_l^{i,k-1}}m_j^{i,k-1}\\
&m_j^{i,s}=m_j^{i,m}
\end{aligned}
\tag{8}
$$

The application of more modifiers that systematically operate in the sense of increasing (or decreasing) vulnerability will progressively move the probability of the modal class to that more (or less) vulnerable, reducing or at most almost annulling the variance of the distribution. Viceversa the application of non-homogeneous modifiers from this point of view may substantially leave the modal class unchanged, but increase the variance of the distribution.

Expected damage value or consequently associated to it

If one considers a generic real damage function f, measured by the 6 grades of the conventional EMS-98 scale, from grade 0 (no damage) to damage 5 (structural collapse), it is possible to assess the expected value, for a fixed value of macroseismic intensity I, both starting from the DPM directly taken from the EMS-98 scale and from analogous matrices parameterised by the vulnerability index V.

In the methodology proposed, the parameterisation using the vulnerability index is, indeed, an operation not strictly necessary. On the other hand the introduction of a numerical parameter representative of the propensity of buildings to be damaged by an earthquake, may be useful in order to simplify the methodology and the preservation of the symbolic concept of vulnerability index, adopted in many methodologies currrently used in Italy.

$$f:\left\{D_0,D_1,D_2,D_3,D_4,D_5\right\}\rightarrow Y=\mathbb{R} \tag{9}$$

In reality taking into account the uncertainty with which the EMS-98 scale defines the implicit damage matrices, such a value may only be described by means of a fuzzy sub-set, which will be determined operating systematically on a discrete number of α-cuts of the fuzzy sets that measure the linguistic frequencies of damage for the different classes and for the different levels of macroseismic intensity.

Each vulnerability class and each macroseismic intensity is also associated with a specific central DPM called “white expected”, which might be useful for a rapid determination of the most reliable expected value.

Wishing instead to highlight the effective uncertainty of the estimates, it is opportune to keep the representation fuzzy. For each vulnerability class (Cj) the frequencies associated with the damage grades, for each value of α, are measured by “interval probabilities”:

$${}^{\alpha,j}IP=\left\{\left[{}^{\alpha,j}l_i,{}^{\alpha,j}u_i\right],i=0,1,...5\right\} \tag{10}$$

This means therefore that in fact the DPM is not univocally determined even if one fixes the value of α: a convex set of DPM are possible and a corresponding interval ${}^{\alpha,j}Y$ of the expected value of the function f may be determined from the extreme values.

The theory of “interval probabilities” (Klir 2005) permits one to solve the problem quite easily and in an exact way by means of Choquet’s integral: this means carrying out a permutation of the indices of the damage space in such way as to make the function f of which one wishes to assess the expected value monotonically decreasing, calculating the lower and upper limits of the probability cumulated on the reordered space and finally calculating the corresponding lower and upper limits of the expected value with two ordinary Lebesque integrals.

If one considers the matrices parameterised with the index $V \in [0, 1]$, each vulnerability class (C_j) results associated with a fuzzy sub-set of the interval [0, 1] and thus, having a certain discrete number of α, ordinary intervals of said index V:

$$ {}^{\alpha,j}V = \left\{ \left[{}^{\alpha,j}l_V, {}^{\alpha,j}u_V \right] \right\} \tag{11} $$

Fixing the value of the macroseismic intensity, the two extreme DPM corresponding to the interval of variation of V and the consequent interval of the expected value of the function f of damage ${}^{\alpha,j}Y$ therefore turn out to be determined.

If the function f of damage is monotonic (or quasi monotonic) compared with the damage grades ordered in the scale, the result may be deemed exact, in the limits of the approximation introduced with the parameterisation and the representation by means of beta discrete distributions. If, instead, the damage function results highly non-monotonic the error with regard to the non-parametric procedure described above may be much more sensitive.

For each typology (index i) modified (index s) one now considers the random set (Dubois and Prade 1991; Bernardini 1999), this also dependent on α, obtained by attributing the intervals ${}^{\alpha,j}Y$ with the probabilities $m_j^{i,s}$, independent of α.

One is dealing with a random set of the non consonant type, with focal elements ${}^{\alpha,j}Y$ which might be non disjointed and probabilistic assignments given by the $m_j^{i,s}$.

Thus it is possible to calculate the cumulative extreme functions of the random set and the interval of their expected values of the function y = f of the damage considered, ${}^{\alpha}Y_{i,s}$. It is also possible to calculate a specific value of “white expected” ${}^{\alpha}y_{i,s}^{WHITE}$.

If the calculation is repeated for different values of α, one generates (under certain conditions of continuity of the analytical functions used) a fuzzy set that measures the expected value of y = f for the modified typology, conditioned by the macroseismic intensity assumed. The interval of variation with α of the white expected ${}^{\alpha}y_{i,s}^{WHITE}$ values and the barycentre of the fuzzy set remain determined, usable as the central “defuzzified” value for a central independent measure of the effective uncertainty of the DPM.

One observes that a central defuzzified value may also be calculated substituting the iterative procedure concerning α with a direct calculation that uses the “white expected” DPM of each vulnerability class. The parametric class (with respect to α) of the random sets (${}^{\alpha}Y_j$, $m_j^{i,s}$), is reduced to a single discrete probability distribution (y_j, $m_j^{i,s}$), for which calculating the average value is immediate.

Numerical applications

In the following the results obtained from the application of the methodology to a group of 19 municipalities chosen in the provinces of Belluno and Pordenone (Italy) are shown, with reference to the information deducible from the ISTAT 1991 census for masonry buildings alone (Bernardini et al. 2007b).

The ISTAT 1991 catalogue allows one to identify the building in terms of structural typology, age of building, height, state of maintenance and aggregation conditions. Structural typology and age class are useful in terms of characterisation of the distribution in the EMS-98 building typologies in the territory. Later, for each EMS-98 typology, it is possible to identify the groups of buildings homogeneous by height, state of maintenance and aggregation conditions.

Applying the procedure described previously, for each modified typology it is possible to define the interval of variation of the expected values of the damage function chosen and the "white expected" value, having fixed the intensity and for each value of alpha.

One considers, for instance, the two functions of damage (both monotonically increasing with the damage grade ordered in the EMS-98 scale):

- y1 = f (0,0,0,0.4,1,1) which defines the percentage of unusable buildings,
- y2 = f (0,0,0,0,0,1) which defines the percentage of collapsed buildings,

and a value of intensity equal to 8.

The fuzzy sets are shown for the expected values of the expected percentage of unusable buildings, respectively obtained with the parametric procedure (Figure 9) and non-parametric one (Figure 10). One also calculates the corresponding defuzzified "white expected" values (coinciding with the central value of the α -cut for α = 0.5).

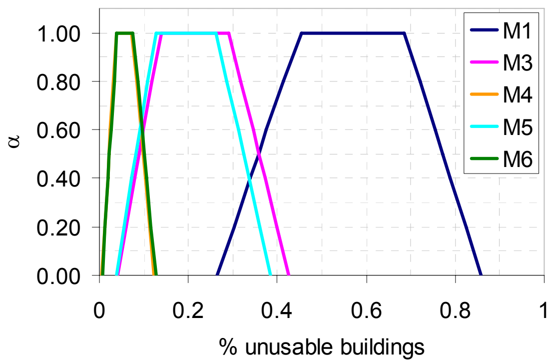


Figure 9. Fuzzy sets of the percentage of unusable buildings for the typologies, obtained using the non-parametric procedure.

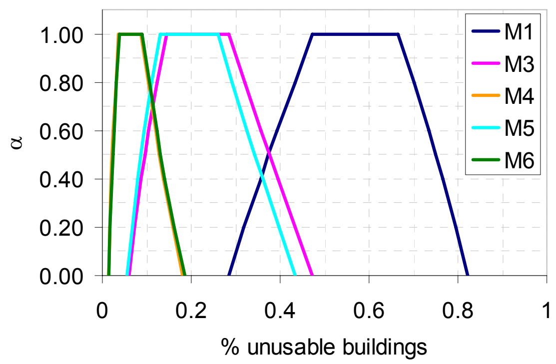


Figure 10. Fuzzy sets of the percentage of unusable buildings for the typologies, obtained using the parametric procedure.

Analogous comparisons relative to the percentage of collapsed buildings are shown in Figure 11 and Figure 12.

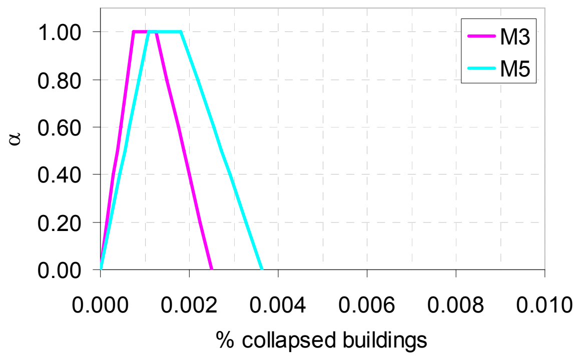


Figure 11. Fuzzy sets of the percentage of collapsed buildings for typologies M3 and M5, obtained from the non parametric procedure.

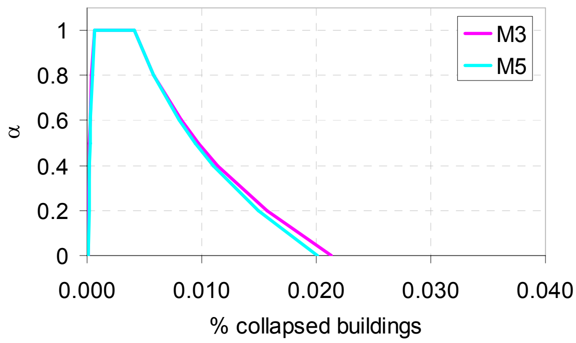


Figure 12. Fuzzy sets of the percentages of collapsed buildings for typologies M3 and M5, obtained from the parametric procedure.

One observes how the approximation of the parametric procedure seems totally acceptable, at least for the calculation of the expected values of monotonic functions of damage. This conclusion is further confirmed by the summarised comparison of the "white expected" values of the percentages of unusable buildings (Table 7) and of the collapsed buildings (Table 8).

Table 7. Comparison of the white values of the percentage of unusable buildings obtained with the two methods.

% unusable buildings	M1	M3	M4	M5	M6
non-parametric	56.47	22.41	5.99	20.26	6.18
parametric	56.24	23.54	7.57	21.47	7.73

Table 8. Comparison of the white values of the percentage of collapsed buildings obtained with the two methods.

% collapsed buildings	M1	M3	M4	M5	M6
non-parametric	8.63	0.11	0.00	0.16	0.00
parametric	7.18	0.50	0.07	0.48	0.07

Conclusions

The methodology described confirms how the information contained in the EMS-98 scale, suitably interpretated, completed and re-elaborated may determine Damage Probability Matrices, even if in an imprecise form. These matrices substantially make up an effective conventional definition of the Vulnerability Classes, usable therefore for a classification coherent with the EMS-98 of buildings.

The idea of taking Damage Probability Matrices (DPM) from the definition supplied by the EMS-98 macroseismic scale had already been proposed by the authors in previous works which however showed parametric distributions of binomial type (Bernardini 2004) or precise type (Giovinazzi and Lagomarsino, 2001).

Both hypotheses turn out to be irrealistic: on the one hand binomial parametric distributions overevaluating the variance of the distributions implicit in the EMS-98 scale; on the other the uneliminable uncertainty connected with the qualitative nature of the definitions cannot be ignored.

For the distributions of damage probability a parametric representation has been proposed here obtained by means of the discretisation of beta calibrated distributions for the DPM obtained from the EMS-98, parameterised by introducing a single parameter V independent of the macroseismic intensity, which takes on the analogous meaning of that of the vulnerability index currently used in many Italian methodologies. This representation approximates in a completely satisfactory way to the Damage Probability Matrices directly deduced from the definitions, apart from inessential shifts at the extreme intensities (VI and XII).

The macroseismic methodology described here allows one to calculate in a manner coherent with the conventional definitions of damage grade and macroseismic intensity supplied by the EMS-98 scale expected values of any functions of seismic damage to populations of ordinary buildings, starting from systematic information, even very approximative, relative to buildings. In particular applications have been carried out on populations of buildings described by ISTAT 1991 data.

The results of the comparison between the two procedures proposed (one of parametric type, the other direct and non-parametric) for the description of the Damage Probability Matrices implicit in the scale, show very small differneces, at least when the damage function to be assessed results monotonic compared to the ordered damage grades. These differences concern both the estimate of the expected central values of expectation and the fuzzy representation of the entire uncertainty with which they are determined.

However, the non-parametric procedure supplies values that are computationally “exact” even for damage functions that are highly non-monotonic.

In order to improve the assessments of the consequences, further statistical information will have to be sought to support the Bayesian procedure proposed for recognition of the distributions of probability of the EMS-98 vulnerability classes for each modified typology, possibly defining modifiers of national or regional nature when the average definitions at the level of the European scale are too far from the reality of the built-up environment.

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Acknowledgments

This research was financed by the executive project 2005-2008 of the ReLUIS Consortium (in implementation of a programme agreement with the Italian Department of Civil Defence), under the scope of the Line of Research no.10.