

Teaching and learning about place value at the Year 4 level

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ABSTRACT: Teaching and learning about place value at the Year 4 level

This project aimed to find out more about ways that Year 4 children who had participated in the Early Numeracy Project (ENP) understood place value concepts and to develop a conceptual model to assist teaching and learning about place value.

Data were gathered firstly by a review of the literature about place value concepts. Two groups of Year 4 children were then observed working on place value activities: one that had not participated in the ENP and one that had. The first group was selected from National Educational Monitoring Project (NEMP) data gathered in 2001 and the activities used by the teacher / administrators with this group were then replicated with a small group of Year 4 children from my own school.

Two themes emerged from this study. The first was concerned with children's ownership of learning and how this is an important aspect for teachers to consider when they are setting up learning environments and activities for children. The challenge for teachers is to develop realistic activities with which children can engage and to encourage children to explore flexible approaches to problem-solving. The second theme raised concerns about ability grouping children and the potentially limiting consequences that this method of classroom organisation might have on children's learning experiences. The stages theory assumptions behind the ENP are challenged and an alternative pedagogical model is proposed.

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Chapter 1: Introduction and literature review

During my time as a teacher, I have noticed how some Year 4 children struggle with place value concepts. Place value is an important concept in the teaching and learning of multi-digit numbers at the Year 4 level. It is important because a secure knowledge of place value equips children to solve problems with number operations (Faire, 1990 and Jones et al, 1996). Given the central place that place value has in number operations and our number system in general, it is important for us as teachers to also have a secure knowledge of how children learn about it. We seek this knowledge so as to gain insights into best practice with which to help children learn about place value.

At the Year 4 level, many New Zealand children, by 2003, will have been introduced to the ideas for learning about place value contained in the Early Numeracy Project (ENP). The Number Framework of the ENP (Ministry of Education, 2003) is described by Thomas and Ward (2002, p.ii) as,

“... providing teachers with a knowledge of how students acquire number concepts, an increased understanding of how they can assist students make progress and an effective means to assess students’ levels of thinking in number.”

I have participated in the ENP professional development this year and am interested in looking closely at how children who have been taught in the ENP show their understandings of place value. As a teacher I want to improve my own practice. As a critical practitioner, however, I am not content to merely take on a new pedagogy, such as the ENP, without giving it careful consideration. In this project I survey the current literature in order to find out how it fits with the assumptions of the ENP before gathering my own data about how children show their understanding of place value. I intended to look closely at two small groups of children - one group to consist of children who had participated in the ENP and the other of some who had not. I hoped that the insights gained during this process would enable me to construct a model of how best to teach place value that would inform my own teaching practice as well as that of other teachers.

Research Objective

To develop a conceptual model to support teaching and learning about place value for Year 4 children.

Research Question

How do Year 4 children, who participated in the ENP in Year 3, show their understanding of place value?

Supplementary Questions

1. What are some of the ideas about place value in current educational theory, research and practice?
2. How do Year 4 children who have not participated in the ENP show their understanding of place value?

Defining place value

As briefly defined in Mathematics in the New Zealand Curriculum (1992), place value means,

“The value of the place a digit occupies, for example, in 57 the 5 occupies the tens place.” (p.214)

The concept of place value enables us to express an infinite range of numbers with only ten different digits. It is characterised by the following four mathematical properties:

1. **Additive:** the quantity represented by the whole numeral is the sum of the values represented by the individual digits.
2. **Positional:** the quantities represented by the individual digits are determined by the positions that they hold in the whole numeral. The value given to a digit is according to its position in a number.
3. **Base-Ten:** the values of the positions increase in powers of ten from right to left.
4. **Multiplicative:** the value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position. Ross (1989)

Literature review

The literature about place value examines a number of teaching and learning issues that I will describe in this chapter. It informed me of reasons why some children may find learning about place value difficult. It also described various models of how children learn about place value, approaches to the teaching of place value and an examination and critique of the model mentioned above, the Early Numeracy Project (ENP), that is currently used in New Zealand primary schools.

Difficulties with understanding place value at the Year 4 level

Researchers have found that children at the Year 4 stage often have difficulty understanding place value concepts. Three suggested reasons for these confusions and gaps in knowledge are the English language (Cotter, 2000; Ross, 1995), children being taught place value too early (Kamii, 1986; Thompson, 2000 and Ross, 2002) and teachers being unsure about how best to teach place value (Young-Loveridge, 1998).

The English language does not provide consistent patterns with all of its numbers. The words that young children have to learn can make learning about numbers and place value more difficult than it might be for children who have other first languages, such as Japanese or te reo Māori. In English, the concept of ten, for example, has three names: ten, -teen, and -ty. In te reo Māori, the number concepts are integrated into the word for multi-digit numbers. For example, *tekau ma rua* (ten and two) is much more revealing of the part-whole nature of this number than is shown by the English word, twelve.

Likewise, *rua tekau ma rua* (two tens and two) tells the young child more about the part-whole character of this number than is revealed by the word *twenty two*. According to Cotter (2000), counting to 100 in an Asian language requires knowledge of just eleven words, whereas an English-speaking child needs to know a total of twenty-eight words.

Kamii (1986), Thompson (2000) and Ross (2002) claim that when children are taught place value too early they may become confused by it if they are also engaged in counting activities. While their knowledge is situated within a counting-based model of number, children are not ready to move to working with collections-based, or part-whole,

concepts. This focus on counting by ones is thought to interfere with the development of place value understanding. It can be contrasted with the approach of the Japanese school system where children are discouraged from using only one-by-one counting (Cotter, 2000) and are encouraged to see multi-digit numbers as part-whole concepts from an early stage.

Young-Loveridge (1998) argues that the best age to introduce place value is after the concept of ones has been established and the child has built up a network of relationships that will lead to the concept of ten. This approach is disputed by Cotter (2000) who sees children becoming confused by an early unitary, or counting-by-ones focus. Clearly there is a range of opinion within the literature about how best to teach place value. Kamii (1986), Cobb and Wheatley (1988), Fuson et al. (1997), Beishuizen and Anghileri (1998) and Thompson (2000) support introducing place value gradually in the context of developing mental strategies for solving multi-digit addition and subtraction problems. These research findings support the approach taken by the ENP with its focus on building concepts and mental strategies. Within the literature there appears to be a variety of approaches to the teaching of place value, which can lead to confusion about best practice methods.

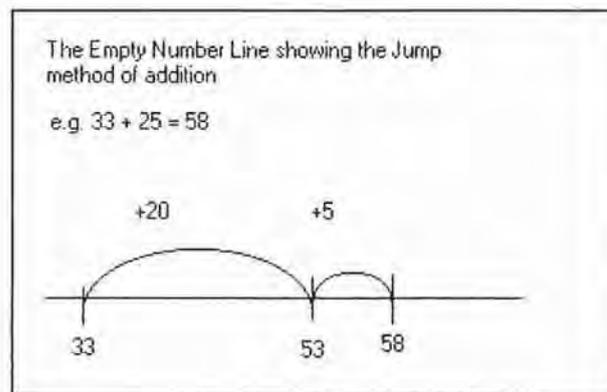
Approaches to teaching place value

Effective teachers, according to Askew, Brown, Rhodes, Johnson and Wiliam (1997) have a clear “mental map” of how children develop concepts. Research has found, however, that many teachers are not as effective as they could be because they have only fragmented and vague mental models of children’s development of place value knowledge (Jones et al., 1996). Higgins (2001) recommended, after studying the teaching and learning of place value in ten Wellington classrooms, that enhancing teachers’ pedagogical content knowledge was the most effective way of improving learning outcomes about place value. Apart from teachers perhaps lacking confidence and competence in the teaching of mathematics, one of the reasons many teachers lack a clear mental model for the teaching of place value is that researchers have not agreed upon a best practice approach (Young-Loveridge, 1998).

Several models show what is thought to be the conceptual development of place value knowledge and strategies in children. Knowledge, in these models, shows a recognition and understanding of an increasing range of numbers and also a move from concrete to abstract concepts about number. Young-Loveridge (2001) notes that there are two broad concepts of number that are the basis of children's understandings when adding or subtracting multi-digit numbers. They are counting-based and collections-based models.

Counting-based models of number involve keeping one number intact when adding or subtracting. This model is reflected in Stage 4 (Advanced Counting) of the ENP where the child, adding two numbers together, can count on from one of the numbers and does not have to count both numbers individually. The Jump method for addition also reflects the counting-based model of number. This is shown in the Empty Number Line (in Diagram 1 below) that was developed by Beishuizen and Anghileri (quoted in Wright et al, 2002). It can be used as a visual prompt when adding or subtracting. Yackel (quoted in Young-Loveridge, 2001) argues that even when there is no obvious counting, solutions to problems such as these are still counting-based.

Diagram 1: The Empty Number Line



Collections-based models of number involve the partitioning of numbers (e.g. tens and ones) so that, when adding or subtracting, numbers of the same value can be combined separately. Resnick (quoted by Young-Loveridge, 2001) emphasised that developing an understanding of the part-whole properties of multi-digit numbers is perhaps the most important mathematical achievement of the early years at school.

Examples of collections-based models of number are described by Fuson et al (1997) and are also included in the part-whole stages of the ENP Number Framework. Other researchers (Ross, 1989, Cobb and Wheatley, 1988, Baroody, 1990 and Young-Loveridge, 1998) have developed similar models of place value understanding. They all follow a “stages theory” approach showing progression from unitary to part-whole, and from concrete to abstract concepts. As children become more sophisticated in their number knowledge they can recognise and problem-solve with an increasing range of numbers. Yackel (quoted in Young-Loveridge, 2001) argues that it is important for children to have access to both counting-based and collections-based models of number if they are to become flexible in their approach to problem-solving.

A model for teaching place value: the Early Numeracy Project (ENP)

The ENP provides a model of number and place value learning for teachers of children at the Year 0 – 4 levels. It is an example of cognitively guided instruction (CGI), an approach that assumes that if teachers understand children’s thinking about specific content domains, such as mathematics, then they can shape their lessons to better meet the learning needs of the children (Steffe and Cobb, 1988; Carpenter et al., 1996). As such it answers some of the concerns voiced by researchers (Jones et al., 1996) about teachers lacking the necessary knowledge to inform best practice. The Number Framework is divided into two sections: knowledge and strategy. The knowledge section describes the key items of knowledge that children need to learn, of which grouping / place value is one. The strategy section is divided into eight stages that describe the use of increasingly sophisticated strategies when working with the five areas of knowledge about number. These strategies are grouped in two parts: counting and part-whole relationships. They are:

1. Counting

Stage 0 - Emergent: children are not yet able to consistently count a group of objects.

Stage 1 – One-to-one counting: can count and form a set of objects up to ten but cannot join or separate two sets.

Stage 2 – Counting from one (materials): when joining or separating sets, the objects are all counted from one and the child relies on using physical objects.

Stage 3 – Counting from one (imaging): as with the previous stage but the child can work out the problem mentally.

Stage 4 – Advanced counting: the child counts on or back from one set when joining or separating sets.

2. Part-whole

Stage 5 - Early additive: can see numbers as abstract units that can be treated simultaneously as wholes or as parts of that whole. Answers can be derived from other known facts. Strategies used at this stage may include the Jump or Split method in addition or subtraction.

Stage 6 - Advanced additive: able to choose from a range of strategies and derive multiplication facts from known facts. Strategies used may include compensation, reversing numbers and doubling.

Stage 7 – Advanced multiplicative: able to choose from a range of strategies to solve multiplication and division problems. Strategies used may include halving, doubling, place value partitioning, reversing and multiplying within.

Stage 8 – Advanced proportional: able to choose from a range of strategies to solve problems that involve fractions, ratios and proportions.

In their survey of approximately 15 000 children, Thomas and Ward (2002, p.13-14) found that, over the course of six months, the average gains that children made in their ability to use place value strategies (0.8 of a stage) were lower than in the other aspects of number knowledge. This is shown in Table 3.1.

Table 1: Children who made no gains in stages over six months

<i>Knowledge areas of the Number Framework</i>	<i>%</i>
Base Ten System (BTS or place value)	44
Stages of Arithmetic Learning	30
Numerical Identification	18
Forward Number Word Sequence	15
Backward Number Word Sequence	15

Thomas and Ward also found that the advances that children made in place value strategies were greater at the lower stages of the Number Framework. It seemed that children moved through the counting-based levels more rapidly but that they took longer to become secure in their understandings of collections-based or part-whole concepts. The transition between early additive and advanced additive was, in fact, beyond the understanding of some children. This may mean that there might be larger gaps between the higher stages of part-whole understanding than there are between the early stages of counting. Children working at the Early Additive stage are operating with 2-digit numbers and then at the Advanced Additive stage are working with numbers that contain up to six digits. Clearly, coming to grips with these kinds of numbers will take time. Examples of the kind of problem-solving that children are doing at these stages are:

Example 1 (Early Additive): addition of two- or three-digit numbers. The groupings and strategies that could be used,

$$\begin{aligned} \text{e.g. } 43 + 25 &= (40 + 20) + (2 + 5) \text{ (Split method)} \\ &= 43 + 20 + 5 \text{ (Jump method)} \\ &= 40 + 28 \text{ (Tidy numbers)} \end{aligned}$$

Example 2 (Advanced Additive): using place value strategies with larger numbers,

$$\begin{aligned} \text{e.g. } 273 - 106 &= (273 - 100) - 6, \\ &\text{forward and backward counting in 10s, 100s, 1000s} \\ &\text{ordering numbers up to a million.} \end{aligned}$$

Thomas and Ward (2002) found that older children who started later made greater gains than younger children thus suggesting that there may be a link between age and number concept development. Older children have more background information with which to understand new concepts. This suggestion is in agreement with the cautions made by Kamii (1986) about the dangers of attempting to teach place value concepts too soon.

A critique of the Numeracy Project

The literature suggests a number of aspects relevant to the ENP model that may hinder children's understandings of place value. These are:

- a focus on unitary (counting by ones) concepts,

- children not having ownership of place value concepts,
- an emphasis on conscious thinking skills at the expense of other forms of thinking,
- too much stress on strategies and procedures and
- the stages model of learning upon which the ENP is based.

Counting strategies take up the first four stages of the Number Framework before children engage in part-whole thinking. As Cotter (2000) has suggested, this emphasis on counting by ones may interfere with the development of place value concepts. When a child uses a counting-based strategy to count a collection of objects by ones, he or she may lose the idea of the whole. By focusing on counting first it may be difficult to then integrate ideas about the part-whole nature of multi-digit numbers. Because of this, as well as because of the irregularities of the English language, children may remain stuck in thinking of multi-digit numbers as collections of ones rather than moving on to a concept of parts and wholes.

The children do not own the concepts that are contained in the Number Framework. The model that is embedded within it comes from the levels of conceptual development derived from the work of researchers, not the children themselves. As such, it is not a constructivist approach to learning. Fuson et al (1997) reported that after participating in classes where the learning of multi-digit concepts and procedures were treated as problem-solving activities, children did show significant gains in understanding about place value. The key element in the lessons that they observed was that the children were encouraged to come up with their own solutions, rather than relying on instructions on how to use procedures and rules.

Conscious thinking skills, which can be articulated by the child, are emphasised in many schools (Claxton, 1997). Claxton differentiates between conscious and unconscious thinking: in the former, reason and logic are used and articulated whereas in the latter, unconscious thoughts can lead to intuitive knowledge. He argues that intuition as a valid way of thinking has been neglected in schools. Slower types of thinking or contemplation, while not infallible, are better suited to understanding complex or ill-defined situations. As a result of repeated observations, the child gradually uncovers patterns that are embedded in, or distributed across, a wide variety of experiences. Perhaps the development of deep place value knowledge may be fostered by an intuitive,

unconscious process as well as by an explicit conscious way of thinking. Encouraging children to explain their thinking, according to Claxton, may hinder concepts that are slowly developing in the unconscious mind.

The Number Framework sets out stages of learning about the number system. Within these stages, children are taught strategies to help them think about and solve number problems. Haskell (2001) claims that teaching strategies is insufficient because learning by this method is too often restricted to the context within which it was experienced. It makes little use of the prior learning of the child. He says that teaching that promotes transfer of learning to problems outside of the initial context involves looking at an idea or procedure many times in different ways, on different levels and contexts and in different examples. This supports the thinking of Claxton (1997) about the gradual building up of intuitive knowledge through the observation of patterns and examples. The ENP encourages teachers to make links for the children when they are discussing concepts. Haskell warns that although providing examples is important, it may increase procedural efficiency at the expense of a deep conceptual knowledge base. The examples may make the children more expert in “doing” problems without having to really understand the concepts that lie behind them. Children become proficient at problem-solving within the confines of the examples given but may not develop a broader understanding of the concepts as they present themselves in other settings. Greeno (1991) argues that mastery is more than the ability to convert a form of skill: it is characterised as global, transferable, flexible and owned by the learner.

The Diagnostic Interview is an assessment tool provided for teachers in the ENP. It enables them to group children within the stages of the Number Framework. Children are interviewed on a one-to-one basis with a bank of activities designed to show their level of number knowledge and skill. The measurement of knowledge using a stages approach is problematic, however, as it is difficult to measure the extent of knowledge that may lie between the stages. By defining a set of skills and knowledge in a teaching model and then assessing against them the teacher is at once limiting the possibility that the child can demonstrate a breadth of learning that is outside of the model. Children may have progressed laterally and this may not feature in the assessment.

The difficulty that children appear to have in moving from the Early Additive to the Advanced Additive level as reported by Thomas and Ward (2002) could perhaps be a consequence of the focus on learning strategies and getting to the next stage, rather than exploring concepts from a broader base. Perhaps the children need more time to construct their own ways of knowing before they can progress to the more expert levels of the Number Framework. Social constructivists, such as Greeno (1991) argue that learning is more than strategies: it is encouraged by exploring, relating and creating understandings.

Greeno (1991) argues that little has been proven about the way that learners progress from a level of learned competence to that of automatic expertise and suggests that lesson sequences other than those based on stages models may need to be considered. In order to understand more complex ideas Ertmer and Newby (quoted in Mergel) assert that a constructivist perspective is more effective. Clay (2001), commenting about reading, describes how children begin to use more than one strategy or resource simultaneously to problem-solve, using different types of information and showing alternative ways of using information. This model of learning is relevant to the development of place value concepts and describes an increasingly unconscious or automatic expertise. As a dynamic, exponential process, the idea of the sequential learning that is assumed by a stages approach becomes untenable.

Conclusion

Looking towards the interviews I was interested to see whether aspects of the literature review and the critique of the ENP model might or might not be borne out. It was possible that the two groups of children would reveal completely different knowledge and skills. These might give insight into the initial unitary focus of the ENP (where children start with counting-based knowledge before they move on to a part-whole way of thinking) and how this might hinder their place value concept development. The activities that I planned to give to the children might also provide data to suggest that the children owned their own concepts of place value rather than merely copying procedures. This could be revealed by the amount of flexibility that I observed in their attempts to problem-solve. The data might also reveal that children showed intuitive ways of thinking and also how the stages model, upon which the ENP is founded, fits with the conceptual development that I was able to observe.

Chapter 2: Methodology, research design and ethical issues

Methodology

The purpose of this investigation was to develop a conceptual model to support teaching and learning about place value. My research was informed firstly by a literature review and then by looking at how children showed their understandings of place value. A tentative conceptual model emerged from the literature and was refined by the insights gained from observations of these children. The research, aiming for descriptions of their knowledge and ways of knowing, is situated within a qualitative research methodology. The qualitative research methodology shifts the focus away from knowledge as objective and recognises it as contextual and negotiated. Therefore I used observations and interviews as my data gathering methods. These methods have been described as the tools with which researchers attempt to capture rich, thickly layered data that provide descriptions and understandings about individuals or groups (Kvale, 1996). My role as the researcher was to try to interpret and make sense of the data by looking for the patterns and themes that emerged.

Research design

In this study I analysed interviews with two groups of children. The first group was made up of children who had not participated in the ENP. The second group was made up of children from my school who had participated in the ENP. I wanted the two groups to be as similar as possible except for whether or not they had taken part in the ENP.

For the first group I approached the National Educational Monitoring Project (NEMP) to see if they could provide me with suitable video recordings of Year 4 children working on place value activities from their annual surveys. I hoped that from these I would be able to find a group of Year 4 children who had not yet participated in the ENP. NEMP provided me with a list of schools that had been surveyed in 2001 that matched my ENP group as closely as possible. I refer to these children as the NEMP group and my criteria for choosing them were:

- Year 4 children

- similar decile to my school
- situated in the Christchurch area
- had not participated in the ENP.

NEMP did not have information on this last criterion. Following a number of telephone calls, however, I was able to find a school that had not participated in the ENP professional development. After explaining my project to the principal, I received permission to use the NEMP data (see Appendix 1, p.40) collected from his school in 2001. This group was made up of twelve children who completed several of the activities in smaller groups of four. These children were identified by NEMP as A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3 and C4. One reason for looking at video data rather than just checking the NEMP report was that I wanted to observe the children closely and see cues, such as body language, that were not included in the report. In choosing this school I assumed that the children in the videos had not recently come from another school that had participated in the ENP.

For the second group of children, I decided that five would be a manageable number of children to interview. They were in their second year of participation in the ENP at my school and were selected using a purposive sampling strategy. This meant that, rather than choosing them randomly, I chose them according to pre-determined criteria. These were:

- self-confidence in an interview situation
- representative of a range of mathematical knowledge (as identified by the children's teacher).
- Year 4 level
- had participated in the ENP
- ability to articulate their thinking.

This last criterion contrasts with views about intuitive learning and presents a problem for assessment tools, such as the ones that I used, that rely on children being able to articulate and explain their thinking (Claxton, 1997). I called this group the ENP group. In order to keep their names confidential I refer to these children in this report as A, B, C, D and E.

Data gathering

Data gathering involved, firstly, watching the video recordings provided for me by NEMP. This allowed me to become familiar with the types of questions and activities that the children had been asked about place value. The NEMP (2001) survey consisted of a series of interviews conducted by a teacher / administrator with individual Year 4 children from a group of twelve. Each child completed some of the eight place value activities. As a group they covered them all. I made notes of most of the conversations and actions of the individual children where they seemed relevant to the research questions.

My next step was to set up structured, face-to-face interviews with the ENP children from my school. I recorded these on video. I wanted to make the groups similar in as many areas as possible and this included the structure of the interview. Therefore I decided to replicate the NEMP (2001) activities relating to place value when I interviewed the ENP group at my school. The following activities were used:

- **Girls and Boys:** showing 26 and 32 with place value blocks; dividing 26 and 32 by two using place value blocks.
- **Population:** reading 4-digit numbers and explaining what the digits represent.
- **Calculator ordering:** putting 4010, 140, 41, 4100, 104 and 14 in order of size and reading these numbers.
- **Number A:** multiplying by 100
- **Number C:** writing words for 3,4 and 6 digit numbers in figures; multiplying by 10
- **Motorway:** estimating and then multiplying by 100 and describing a strategy.
- **36 + 29:** mental calculation
- **Speedo:** adding and subtracting 1,10, 100, 1000 to and from a 4-digit number.

(for more detail on these activities see Appendix 1, p40.)

It is possible to question the validity of some of the activities by building on observations from the literature. Kamii (1986), for example, suggests that numbers such as those listed in *Calculator ordering* (see above) could be arranged in order according to cues other than the place values of the digits. The child could know that larger numbers have more digits without recognising specifically that place values are ordered according to ones,

tens, hundreds and thousands. Likewise the 14 and 41 could be ordered according to the face value rather than the place value of the digit in the tens place. In *Girls & Boys*, the second part of this activity involved division with regrouping. One strategy for this would have been to exchange one of the tens blocks for ten ones. The choices for regrouping were limited by the provision of only tens and ones blocks for the children. Cotter (2000) would suggest the availability of fives as well as tens blocks. This would probably fit in better with the ENP teaching that focuses on fives from early stages of the Number Framework.

Claxton (1997) reminds us that schools tend to focus on the cognitive modes of thinking where being able to articulate and explain are valued thinking skills. He suggests that, as a result, intuitive thinking can receive less emphasis because assessment activities tend to focus on explaining and articulating thinking. This could be a criticism of the sorts of activities that I used in this project, as they mostly required the child to explain their thinking. When we ask a child to explain, we are making an assumption that thought processes are in the conscious domain. The inability to articulate reasoning may reflect the intuitive mode of thinking described by Claxton, and needs to be taken into account when designing assessment activities.

One of the assumptions of any qualitative methodology is that during a research interview, data is actively created that would not exist apart from the interaction of the participants (Silverman, 2001). It is important to allow the voice of the children to emerge and shape the interview and indeed the research itself. Therefore I tried to remain open to unanticipated answers that the children might give to my questions and to keep the interview process as informal, encouraging and comfortable as possible.

Analysing the data

As I viewed each tape, I recorded my observations in a table so as to make a written record of key points (see Appendix 3, pp.43 - 52). Reading through this table and my notes I looked for similarities and differences, groupings, patterns and ideas of particular significance. Some came up during the interview and before I had completed writing up the notes. I initially read once through what I had recorded so as to gain an overall picture

and then start looking for data that related to the research questions themselves while keeping on open mind for unexpected responses. I found that I needed to re-examine my notes and the video recordings several times during the analysis process to check on some of the interpretations that I was making.

For example, in an activity that required the children to explain the place value of each digit in a four-digit number (see Appendix 3, Population, Child E, p.51) one of the ENP group seemed confused by the question and did not identify the hundreds and tens digits correctly. She did, however, at the end of the activity say that the 2 in 2495 was in the thousands column. I interpreted this answer to mean that she might have understood more about place value than the question had initially revealed. In hindsight, it would have been interesting to ask this child to go to the earlier parts of this activity. She may simply have needed more time than I gave her to think about the question. In the NEMP group, B2 and B4 also seemed confused by the same question and were not able to furnish an answer. The children appeared confused by the teacher prompt ("What does this mean?") which was ambiguous as it could imply face, place or total value. It is not clear whether the NEMP children had a lesser degree of place value knowledge or whether they were simply confused by the question.

Kvale (1996) likens the interview process to a snapshot, where the interviewer records data at a given time. During the interviews I chose to reword questions where I felt that they were ambiguous. My thinking was influenced by the concept of reflexivity (Denzin and Lincoln, 1994) that describes the interview as a dialogue that is created by both of the participants in the interview. I would allow my own thoughts about the questions and activities to be expressed where appropriate in the interview rather than attempting to adhere rigidly to the format followed by the NEMP teacher / administrators.

Ethical issues

In a project such as this, researchers are guided by the following ethical principles: no harm is done, participation is voluntary, consent is informed and the interviewee retains the right to withdraw data at any stage (Tolich and Davidson, 1999). It is essential that anything that the interviewee shares is kept confidential to the researcher (myself), the

interviewees (the individual teachers) and the Christchurch College of Education supervisors and examiners.

I initially contacted the principal of my school to explain the research that I had in mind and gained his verbal and written consent. The next step was to approach the teacher of the Year 4 children. After gaining her verbal and written consent for the research we discussed which children would be suitable for the interviews. The reason for this was that I wanted children who would be confident enough when faced by a video camera to provide data and attempt the place value activities. I also wanted to have a range of mathematical ability represented in the group. Once this was completed I introduced myself to the group of children, briefly explained my research and gave them an information letter and consent form for their parents to read and sign. All of the children returned their forms with parental approval to take part in the interviews.

I was aware of the implications of conducting this research in my own workplace. It was possible that a child might divulge personally sensitive information or information that compromised another teacher or child. I conducted the interviews at school in classrooms that were temporarily vacated by classes who had gone to Manual training. I tried to keep the room informal and comfortable for the children, keeping doors open to encourage the normal interactions with other children as far as possible and to make the interview environment as close to “normal” as possible.

Chapter 3: Data analysis and discussion

The two groups had some similarities and some differences in the knowledge and skills that they showed about place value. Making a decision about whether a child had showed knowledge in a particular area was a challenge in some of the activities and I had to rely on my own knowledge and experience as a teacher as well as by referring to other sources. In some of the activities, I used the criteria provided by the NEMP (2001) report. For example, in the Motorway activity (see Appendix 3, p. 49), a reasonable estimate for 98×9 was considered, by NEMP, to be any number between 850 and 900. In some cases, during the interview, where the children's answers made it hard to tell what knowledge they were showing, I rephrased the question. This opportunity was only possible with the ENP children, however, not the NEMP group. In the case of Child E (NEMP activity: Population, see Appendix 3, p. 51) her initial answers suggested confusion between face and place values for the digits in 2495. Her last answer, however, revealed that she did in fact understand that the 2 was in the thousands place. It seemed that she had misunderstood the question rather than that she had lacked the knowledge necessary to answer it. Table 3.1 gives an overview of the broad similarities and differences that the children seemed to have in their knowledge of place value. It shows six areas of similarity and four areas of difference between the two groups.

Table 3.1 Place value knowledge shown by the children.

Item	Area of knowledge or skill	ENP group	NEMP group
1.	Recognising multi-digit numbers	✓	✓
2.	Reading and writing numbers	✓	✓
3.	Ordering numbers according to size	✓	✓
4.	Using place value blocks to show two-digit numbers	✓	✓
5.	Adding or subtracting 1 or 1000 from a four-digit number.	✓	✓
6.	Not being able to suggest more than one strategy.	✓	✓
7.	Explaining the meanings of the different digits in a four-digit number	✓	

8.	Using the split method of addition when adding two-digit numbers	✓	
9.	Adding or subtracting 10 or 100 from a four-digit number	✓	
10.	Using the algorithm method when adding two-digit numbers		✓

In Items 4 and 6, although the outcome was the same, the methods used by the children differed. Item 4 is examined in more detail later in this chapter. Appendix 3 provides more reported data from the interviews and mention is made of a number of points that may be worthy of further study by other researchers.

I have chosen to report in this chapter on two themes that emerged as I analysed the data: ownership of learning and the potentially limiting effect of ability grouping on children's learning.

Ownership of learning

The theme of ownership of learning encompasses two main subthemes. They are concerned, firstly, with the problem for teachers of developing realistic activities and contexts for learning and, secondly, with encouraging flexible approaches to both teaching and learning.

Ownership of learning extends beyond the ownership of strategies and concept development to ownership of the activities and types of materials used as well. Haskell (2001) and Greeno (1991) suggest that mathematical problems should be situated in realistic contexts that are recognised by the child. This is to allow children to make use of their prior knowledge and to be better able to engage with and think of the problem as a whole. For example, when asked whether it was possible for there to be an equal number of girls and boys in a class of 26 children, Child C2 (NEMP activity: Girls and Boys, Appendix 3, pp. 46-47), explained that, in his opinion, there were normally more girls than boys in any given classroom. For this child the idea of equal numbers of boys and girls in a class was unrealistic and initially prevented him from engaging with the division operation that the teacher was trying to assess. Only when the question was

rephrased did this child attempt to work out the answer. Setting up realistic activities without negotiating their possible meanings with the children presents a problem for the teacher.

Some of the activities used in the interviews showed how children can and do take ownership of their learning. The ways that the children used materials highlighted this. When asked to use place value blocks to make up a number and show a division operation (see NEMP activity: Girls & Boys, Appendix 3, pp. 46-47), the children demonstrated a variety of strategies. For example, Child C (in the ENP group) used her fingers to work out the problem and then made up the correct number of blocks afterwards. She seemed to have decided to use her own method as an intermediary step to solving the problem in the manner that I had requested. Child B and Child C4 worked out the answer mentally before using the blocks. These children seemed to have moved beyond the need to use concrete strategies in solving a question like this. It appeared that the question was unduly focused on getting them to use a concrete strategy and had lost sight of conceptual flexibility as an important skill worth encouraging. Teachers need to be aware of the limiting nature of questions such as these where the use of any type of materials or strategies could be mistaken for the end point rather than just one of many potential pathways of learning.

Claxton (1997) argues that, in order to develop a multi-pathway, flexible approach to problem-solving, children must be helped to develop a greater sense of ownership of knowledge and the knowledge-making process. There is a big difference between working with a rule that has been given by the teacher and one that is based on "owned" images and connections (Greeno, 1991). This was illustrated when the children were asked to mentally add 36 and 29 (NEMP activity: $36 + 29$, see Appendix 3, p. 44). The ENP children all successfully used either the split or the jump method, while, of the three NEMP children who solved the problem, two of them used a mental algorithm. Despite using different strategies to add $36 + 29$, there was not much difference in the end result between the two groups of children. The relative uniformity of the strategies used, together with the fact that none of the children in either group were able to suggest any alternative ways of solving this problem, suggest that the children may have been applying a strategy that had been taught rather than one that they had thought of themselves. This highlights a lack of breadth in the children's learning and raises the

question of whether the children's learning had become too narrowly focused on learning set procedures rather than developing a more flexible approach to problem-solving. The findings from this activity made me curious to see how the children performed when working with larger numbers that might make these strategies less effective.

In the Motorway activity (Appendix 3, p.49) the children had to think of a way of estimating the product of 98 and 9. The familiarity that both groups had shown with the split method or mental algorithms did not appear to serve them well in this problem. Only one child (A4 from the NEMP group) managed to answer this question correctly with most of the children unable to demonstrate ways of using estimation as a strategy. One way of solving this problem involved rounding the 98 up to 100 and then multiplying 100 times 9, which I suspected was well within the children's capability as all of the ENP children were able to multiply 10 times 12 (see NEMP activity: Independent, Appendix 3, p. 48). Child D managed to work out 316 times 10 and yet could not solve the Motorway problem. I decided to test my theory that the children were capable of solving this problem but simply lacked the knowledge of rounding up to estimate. In the next interview I provided Child C (who was unsure of how to start) with a prompt to think of a number close to 98 that might make the problem easier for her and she was able to round 98 up to 100. When I asked her what 100 times 9 might be, she gave the correct answer. Without similar prompts the other children attempted to use an addition or multiplication algorithm. Although the algorithm method that they used had been successful with smaller numbers, the children were unable to work with this larger number and either gave up or arrived at an inaccurate estimate. This could confirm the concerns of researchers (Pirie & Kieran, 1994, Fuson et al, 1997 and Greeno, 1991) that too much emphasis on learning how to follow the procedures of number operations can narrow the focus and prevent children from thinking about alternative ways of solving a problem.

The problems that the children had suggest the need to encourage greater breadth of learning by creating realistic contexts for mathematical activity and for teachers to be open to the different ways that children choose to go about their learning. The focus in the ENP on getting children to think about numbers in different ways is helpful in this regard and needs to be further emphasised if children are to build a broader concept of number and place value. By building up a sense of their own ideas rather than just ones

that have been taught to them, children may become more secure and better equipped to think of ways to solve problems of this type.

The potentially limiting effect of ability grouping on children's learning

Three issues related to ability grouping are worth discussing in greater detail. In this section I show, firstly, that trying to fit children's learning to the curriculum is problematic for teachers. Secondly I discuss the challenge that teachers have in deciding when a child is ready to learn a concept and, thirdly, I question the assumption that there is a pre-set order for learning. In all of these examples, I am raising issues concerned with the validity and consequences of grouping children according to criteria which are based on a pre-set, stages model of learning.

A problem for teachers who are working within the stages paradigm is knowing how children's conceptual development fits in with the different stages described in the Mathematics curriculum or the Number Framework of the Early Numeracy Project. The ENP answers this need by supplying teachers with the Diagnostic Interview assessment tool. This enables teachers to sort children into ability groups according to the different stages of the Number Framework. Being placed in an ability group, however, may hinder the concept development of some children. I found an example of this in the $36 + 29$ activity (see Appendix 3, p. 44). Child E had been placed by her teacher at the Advanced Counting level (Stage 4) of the Number Framework and appeared to have the weakest mathematical knowledge of the group yet her choice of method suggested otherwise. She was able to perform subtraction operations (see NEMP activity: Speedo, Appendix 3, p. 52) as well or better than her peers who had been placed in a group that was working at the higher Early Additive level (Stage 5). In the case of Child E, her learning did not seem to fit with a linear, stages perspective.

Bowers and Nickerson (2000) suggest that forms of assessment, such as the ENP Diagnostic Interview, focus on the individual at the expense of shared learning. By isolating children from their more or less able peers, teachers could be limiting children's learning and discussion as well as threatening their self-esteem as mathematicians.

Bowers (1999) described a shift to a view that mathematical activity is inherently social and cultural in nature. All children can learn from each other and although some learning

is individual, Greeno (1991) suggests that social learning is the more effective way to learn.

Deciding when a child is ready to learn a concept is a problem that all teachers face. At the Advanced Additive stage of the Number Framework, children are described as being able to choose from a range of strategies to solve problems. As none of the children that I interviewed were working at this level in their classroom it was perhaps to be expected that none of them managed to provide more than one way of adding two 2-digit numbers together (see NEMP activity: $36 + 29$, Appendix 3, p. 44). The lack of breadth in their problem-solving ability, although perhaps "acceptable" within a stages approach, may have hindered their potential to solve problems that were within their grasp. The implication of the stages approach is that children are not expected to be learning about, or showing understandings of, concepts that are "above their level". Although this appears to have been the case with showing alternative strategies, the children showed on a number of occasions that they were operating at levels above their designated classroom groupings. An example of this, from the ENP group, was Child E who showed higher ability in some areas of knowledge and lower ability in others (see NEMP activities, Appendix 3: Speedo, p.52 and $36 + 29$, p. 44). Her learning would seem to be at a variety of levels simultaneously and did not fit into a neat category or stage. The consequences of consigning her to a lower ability group could be boredom, reduced self-esteem and fewer learning opportunities.

Another example of children showing knowledge "above their level", concerns four-digit numbers. In the ENP, children are not taught about groupings within 1000 until they reach the Advanced Additive stage. It would, therefore, be reasonable to expect that the ENP children, who were working at stages below this (Advanced Counting and Early Additive) would solve problems with smaller numbers more successfully than they could with larger numbers. Contrary to this however, Fuson et al (1997) suggest that teachers should focus on using larger (four-digit) numbers before smaller (two-digit) ones because of the irregularities of English language words. This allows children to construct multi-digit concepts with the more regular hundreds and thousands. An example of this would be practising multi-digit addition with $2300 + 6800$ instead of $23 + 68$. I was interested to explore this assertion further with the Speedo activity and find out whether the children

were more successful when adding and taking away hundreds and thousands rather than two-digit numbers.

In this activity (NEMP activity: Speedo, see Appendix 3, p. 52), in the context of a car speedometer, the children were asked to add and subtract 1,10,100 and 1000 from a four-digit number. If the stages theory was correct for these children, they could be expected to be more successful with the smaller numbers and less so as the size of the numbers increased. The ENP group showed slightly more success overall and both groups were more successful when adding or subtracting 1000 from a four-digit number, as opposed to 10 or 100. The NEMP (2001) report also found that children were more successful in using ones and thousands than in using tens and hundreds. This activity suggested to me to be wary of using a lock-step, stages approach in my teaching. For some children, learning appears to follow different pathways that are not as predictable and ordered as a stages approach might have us believe.

The themes of ownership of learning and the limitations of ability grouping described so far highlight the importance of teachers encouraging children to develop a wide, multi-pathway approach to problem-solving that makes use of their own prior knowledge and understandings. In the next section of this chapter I discuss possible ways that teachers might address the problems of when to teach concepts and how to cater for children of all abilities. Firstly, I suggest how teachers might develop the learning culture of the whole class as a social group and, secondly, discuss an approach that seeks to address the problem of when to teach a concept and how to group children.

An alternative “emergent” model of learning: the example of the Candy Factory

Children with rich prior knowledge are able to learn more quickly and effectively (Nash, 2002 and Mergel, 1998). They have more information with which to make their own links to new ideas and experiences, to understand taught procedures and to create their own strategies. If a child has not had a rich background of experience, he or she may not have the necessary knowledge upon which to securely base the strategies that are being taught. To assist teachers with this problem, Cobb and McClain (2002) propose what Haskell (2001) describes as a systematic approach to constructivism using real life scenarios rather than decontextualised artificial situations. By building up shared

imagery, all of the children in the classroom gain similar knowledge upon which to base their ideas about place value. The children who already know a great deal can share their knowledge with their less informed peers and may increase their own understanding by having to explain their ideas to the class as a whole. Rather than having the more able children isolated and working in their own group, their ideas become available for the whole class and can enrich everyone in it.

In helping children develop from merely knowing how to follow procedures to having the kind of automatic knowledge and skill that Greeno (1991) defines, McClain and Cobb (2001) suggest that the development of the classroom learning expectations and culture should be a focus for the teacher. An example of this might be a shared understanding of what is meant by a “satisfactory” explanation to an answer given by a child. During a study of a first-grade classroom, they observed a teacher negotiating with the children the criteria for a “different” solution to a problem after it had become apparent that the children were often repeating what their peers had said. McClain and Cobb (2001, p. 252) noted that over time this gave more rigour to discussions and that the children engaged more actively in their learning. Developing shared knowledge and expectations as a whole class adds to the base of knowledge that the children need so as to become flexible problem-solvers.

Greeno (1991) calls for teachers to design activities that involve the use of numbers and quantities in ways that make children think about the properties and relationships between them rather than just by learning about the rules and procedures. Haskell (2001) and Pirie and Kieran (1994) support the use of analogies and metaphors that enable children to see these properties and relationships in new or unfamiliar experiences. Encouraging questioning and revising, and modelling by teachers that knowledge is contestable, all help children become more secure and better equipped to problem-solve in unfamiliar contexts. The ENP supports the use of linking and the making of connections in the discussions that are encouraged between the teacher and children. The sharing and discussion that ensues before, during and after engagement with an activity may help children form their own ideas about the concepts that are being worked on by the class.

Both the Mathematics Curriculum document and the Number Framework are based on a stages approach which assumes that learning proceeds in a linear fashion towards increasingly abstract levels (Wright et al., 2002). As an alternative to a stages approach, Bowers and Nickerson (2000) describe an emergent model where the values, practices and social motivations of the classroom play important roles in children's conceptual development. In this model the evolution of each child's learning can be seen as reflexively related to the emerging practices in which he or she participates. Learning is a by-product of activities that are done in a social environment rather than a result of direct cognitive instruction of knowledge and skills in a sequential manner. In her research of children creating their own conceptions of place value and number, Bowers (1999) reported on an instructional sequence called the Candy Factory. The learning took place over a nine-week period in a class of 23 third grade children and centred on an imaginary Candy Factory that the children were to control (see Appendix 2, p. 41 for more detailed description). The instructional sequence included setting the scene, exploring and interpreting numbers, creating and regrouping numbers, recording and problem-solving with number.

A key part of the lessons was the use of a software programme that enabled the children to explore numbers and change groupings in the context of packing and unpacking cartons of candies. The programme freed the children from having to manually group and regroup using materials and allowed them to think at a higher level about the relationships between the numbers. Place value relationships were revealed in the course of "playing around" with the different options for packing and unpacking candies and the discussions that the children engaged in with their teacher and peers. The programme showed a focus on the communal process – an alternative to the idea that mathematics is only accessible to children via the curriculum.

The Candy Factory lesson sequence aimed at promoting an understanding of increasingly sophisticated number concepts and place value. It also addressed ownership of concepts by the emphasis of sharing and discussion, intuitive thinking as reflection on patterns and analogies situated within a realistic activity and social learning as a result of the children being encouraged to work and discuss problems together. This shows an important shift towards seeing mathematics as a more socio-cultural, situated, human activity. The implications for teachers of this shift are towards seeing learning as best situated within

the whole class rather than ability groups and to embrace the development of a classroom culture with shared knowledge and expectations that arise from engagement with realistic activities.

Chapter 4: Conclusion - a conceptual model to support teaching and learning about place value for Year 4 children

During the course of this research project I have found no reason to dispute the understandings about place value that inform the Early Numeracy Project. Children develop their ideas about place value from early attempts to count objects through more advanced counting to an appreciation of the part-whole nature of multi-digit numbers. As their understanding develops they leave behind a reliance on using materials and become more abstract in their knowledge. Concepts that had been previously uncertain become accepted as givens and form the basis of an increasingly complex conceptual structure. When they encounter challenging problems that are unmanageable this structure allows children to fold back to earlier understandings that then enable the problem to be reconstructed in a more simple form. Increasing knowledge and skill allow for greater flexibility in problem solving and also eventually enable the children to create their own understandings. Whether these understandings develop in a linear, hierarchical manner however, is open to question.

The Number Framework of the ENP sets out stages of concept development about place value. As such, it supplies teachers with a useful model for understanding children's place value concept development. An alternative pedagogical model for teachers could include a number of other areas, however, which may be useful to consider using in addition to the current model of teaching and learning. These are allowing for social rather than ability groupings within a class, encouraging ownership of learning and aiming for a balance between instruction about strategies and concepts and allowing place value to emerge from realistic activities with which the children engage.

Ability grouping is an outcome of a stages approach to learning which makes assumptions about the nature of knowledge. Knowledge is seen as developing in a relatively ordered fashion which does not seem to allow for children who may have quite advanced knowledge in one area and less in another. By placing them in groups (stages) at the start of a lesson sequence, the teacher may limit the kinds of experiences that a child has access to. A more flexible arrangement of groupings, alternatively, may allow

the child to have a wider variety of experiences with which to consolidate his or her learning. As has been shown in this project, children's levels of knowledge and skill are different and may go unrecognised by assessment tools such as the ENP Diagnostic Interview. Different levels of understanding are evident among children across a range of place value concepts. Intuitive modes of thinking have been discussed as a type of thinking that has, to an extent been left out of assessment practices. By relying on an assessment tool such as the Diagnostic Interview, teachers may inaccurately categorise ability and place a child in a group that limits the amount of learning that he or she is capable of. This research raises questions about the kinds of teaching strategies that teachers might use as alternatives to ability grouping.

The Number Framework identifies the knowledge and strategies that the children are to learn and shows teachers activities with which the children can practice. A focus of the lessons is the shared discussions that enable the children and teacher to establish meanings as a platform for communal understanding. The key aspect of the ENP in emphasising the importance of discussion and shared understandings is to be encouraged but the problem of ownership remains. The stages aspect of the ENP model assumes that children progress in a relatively linear manner and that teaching is best focused on defined skills and strategies. This approach may restrict the learning that the children are capable of.

The importance of encouraging a social learning culture in the classroom has already been examined and this is an aspect that seems to be a strength of the ENP programme. Teachers are encouraged to focus their discussions with children on understanding the previously established skills and strategies of The Number Framework and to make links between them. It is possible however that they may not go far enough in allowing children to create their own skills and strategies. In their model of recursive development, Pirie and Kieran (1994) suggest that the creative stage is the highest level of learning. It is implied that it is beyond the level of children operating at lower levels. A constructivist approach, however, would suggest that children can formulate owned concepts at any level, which may help them to develop a more secure understanding of place value.

Developing an understanding of place value that is based on the children's own concepts is a key aspect of an emergent approach. It sits within a constructivist paradigm and

acknowledges that children develop their concepts in a variety of ways. It allows for the child who may learn in a linear fashion and also for those who may follow less obvious paths to understanding. The strength of this approach is that the child's understanding is likely to be better equipped to problem-solve in areas that are unfamiliar.

When I started this research project I had in mind the idea of constructing a model of learning to illustrate children's understandings about place value at the Year 4 level. As the research progressed this idea evolved into that of developing a model to support teaching and learning. It recognises that learning is a complex process for both children and for teachers.

For children, the learning process encompasses constructing their own meanings from realistic experiences and discussing emerging meanings within the context of a classroom community. This also includes using a variety of approaches, being able to explain ideas in different ways and owning the concepts that are developed. It includes elements of an emergent model (as shown in the Candy Factory) and the ENP Number Framework.

For teachers, the learning process involves both the ability to conceptualise the children's process (noted in the previous paragraph) and to develop strategies to support them in becoming inquiring learners. Rather than teaching skills and strategies in isolation they could be taught as part of an environment created by the teacher and children together. The challenge for teachers is to find suitable activities in consultation with children that are sufficiently realistic to encourage them to engage with them. The instructional sequence could then follow much the same path as identified in the Candy Factory - setting the scene, exploring, interpreting, creating, regrouping, recording and problem-solving with number. The Number Framework provides the teacher with support in understanding the aspects of place value that the children work through. As such it goes some way towards answering the concerns expressed by researchers in the introduction to this project about teachers lacking a mental map of how children progress in the development of place value concepts. The important insight for teachers, however, is to recognise that children may not progress through the identified knowledge and skills in a linear fashion but that they are likely to emerge as they are needed in problem-solving activities.

The model is focused on teaching and learning. Both teachers and children have their own understandings to develop and their own work to do. Teachers, through their own professional development, and children, within the context of their classroom activities, need to construct their own understandings. They need to work together to articulate their emerging confidence and knowledge and share their different ways of understanding and explaining the different aspects of place value. The conceptual model to support teaching and learning of place value at the Year 4 level, which has emerged from this study, is a pedagogical model. It recognises that the different ways of understanding that emerge from engagement with realistic classroom activities need to be conceptualised individually and collectively by both teachers and children.

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Appendix 1

The range of NEMP activities

The NEMP activities (in **bold**) covered the following features of place value:

Girls and Boys

- showing 26 and 32 with place value blocks.
Showing links between multi-digit numbers and the objects that they represent. A child can show that the tens digit refers to groups of ten objects.
- dividing 26 by two and dividing 32 by two using place value blocks.
Conservation of number. The value of a number is not affected by changes in the grouping.

Population

- explaining the meaning of individual digits within a four-digit numeral. This extends the concept development of showing to being able to express it verbally.

Calculator Ordering

- put 4010, 140, 41, 4100, 104 and 14 in order of size - comparing relative magnitude.
- reading and writing numerals.

Number A

- multiplying by 100

Number C

- writing words for 3,4 and 6 digit numbers in figures
- multiplying by 10

Algorithms

- adding and subtracting one and two-digit numbers.

Motorway

- estimating and then multiplying by 100 (knowledge of one strategy).

36 + 29

- mental calculation of addition

Speedo

- adding and subtracting 1,10, 100, 1000 to and from a 4-digit number.

There do not appear to be any activities that show a child's ability to count groups of ten.

Appendix 2: The Candy Factory: an example of an “emergent” instructional sequence

Bowers (1999) reported on an instructional sequence aimed at promoting an understanding of increasingly sophisticated number concepts and place value. The learning took place over a nine-week period in a class of 23 third grade children and centred on an imaginary Candy Factory that the children were to control. The instructional sequence developed as follows:

1. **Setting the scene** - the children were given a description of the Candy Factory and were asked how the workers might pack and keep track of candies. After a variety of suggestions had been made by the children, the teacher told them that the workers had decided to pack the individual candies in rolls of ten and to pack ten rolls into a box.
2. **Exploring number** - exploring ways of packing and unpacking the candies (using Unifix cubes) and working on activities that required them to make up set amounts and to record them in picture form, showing boxes, rolls and pieces.
3. **Interpreting number** - children were asked to interpret how many candies there might be in different drawings.
4. **Creating / regrouping** - the children created different ways of packing (regrouping) specified numbers of candies. They were introduced to a software programme, which enabled them to do this more quickly and easily than by using the Unifix cubes. This developed the idea of the conservation of quantity and led to the understanding that a number can be rearranged in many ways without changing its value. As the children's concepts developed, old concepts that previously had had to be justified came to be accepted as givens and no longer required justification.
5. **Recording** - moving towards symbolic representations of number. The children started to make use of digits instead of pictures.
6. **Problem-solving with number** - keeping track of the candies required the children to develop ways of solving addition and subtraction problems.

When confusion occurred the teacher made use of the “folding back” strategy (Pirie and Kieran, 1994). A key part of the lessons was the use of a software programme that enabled the children to explore the packing and unpacking problems without having to go through the relatively time consuming task of physically changing pieces for rolls and rolls for boxes using the Unifix cubes. Using the computer the children could see, at a glance, the effect of rearranging the candies when they changed groupings. Place value relationships were revealed in the course of just “playing around” with the different options for packing and unpacking candies. Bowers (1999) maintains that the software, therefore, enabled the children to operate at a higher level of thinking about place value. Regrouping and use of symbols were significant ideas at the core of the developing understanding of place value.

During the discussions that followed these activities the children's ideas were shared with those of their peers and, together, these contributed to developing communal understandings about the Candy Factory and the number system. The development of sociomathematical norms was seen as an important part of the developing learning culture. These included an awareness of what counted as a different mathematical solution, what counted as a clear explanation and what would be the most efficient solution. Because of the conventions established at the outset the children viewed the boxes and rolls as composite units and the children's understanding of this emerged from

the activities and discussion. As the classroom mathematical practices evolved, a reflexive relationship was established with individual children contributing to the class understanding and, in turn, learning from the classroom culture. Children who thought in original and different ways and did not “follow the herd” had to justify their ideas and sometimes they were the ones who prompted the emergence of new practices within the classroom culture. This analysis shows the focus on the communal process – an alternative to idea that mathematics is only accessible to children via the curriculum.

Appendix 3: Overview showing the number of children who performed successfully in the place value activities.

Name of activity	Place value knowledge or skill	ENP group (5 children)	NEMP group (4 children)
Girls and Boys	Showing multi-digit numbers with place value blocks	all	all
Population	Reading a 4-digit number (2495)	all	3
Number C	Writing a 4-digit number	all	all
Number C	Writing a 6-digit number	2	0
Population	Knowledge of all positional values in 2495	4	2
Calculator Ordering	Ordering numbers	all	all
Girls and Boys	Dividing an even number into two groups with place value blocks	4	3
Girls and Boys	Dividing a number using regrouping with place value blocks	3	1
Speedo	$1996 + 1$	4	3
Speedo	$1996 + 10$	2	1
Speedo	$1996 + 100$	2	1
Speedo	$1996 + 1000$	2	2
Speedo	$3402 - 1$	4	2
Speedo	$3402 - 10$	2	1
Speedo	$3402 - 100$	3	1
Speedo	$3402 - 1000$	3	2
Number order	Arrange four digits to make smallest possible number	all of 3 recorded	3
Number order	Arrange four digits to make largest possible number	2 of 3 recorded	3
Number A	$700 + 100$	all	all
Number A	$15\,735 + 100$	all	3
Number B	$400 - 100$	all	2
Number B	$643 - 100$	all	2
Number C	6×10	all	3
Number A	12×10	all	2
Number C	78×10	1	1
Number A	316×10	1	2
$36 + 29$	Adding $36 + 29$	all	3
Motorway	Problem-solving (estimating 9×98)	1	1

NEMP activity	ENP children					NEMP children			
	A	B	C	D	E	A1	A2	A3	A4
36 + 29									
Method used by the children to solve this problem	✓ <i>I'd probably do 30 + 20 = 50 and then 9 + 6 = 15 and ten onto fifty is sixty and five onto sixty is 65.</i> SPLIT.	✓ <i>I'd take away the 9 and 6 and add 20+30=50 and put on the 9+6 then it would be 65.</i> SPLIT	✓ Gave 65. Explained that she added 30+20=50 and 9+6=15 so 50+15=65 SPLIT	✓ Gave 65, smiled and said that... <i>9+6=15 and 2+3=5 (adding in tens) and that equals 65.</i> SPLIT	✓ After a long think she gave 65 and explained... <i>36+10=46 plus ten equals 56 plus 9 is 65.</i> JUMP	x 3+2=5 (adding tens) and 9+6=15 so... 515.	✓ <i>30+20=50 add on... repeats 9+6 =15 ... add on... the answer's 65</i> SPLIT	✓ <i>it's 65, put 29 under 36, add 6+9=15 put down 5 carry one, 1+3 is 4 and 2 is 6</i> Algorithm method	✓ <i>3+2 = 5 and 6+9 = 15 put it in working form, 9+6=15 put 5 down, carry 1 to 3, 4+2=6... 65</i> Algorithm method
Other strategies	Another version of the above method where 50+15=65	No	No	No	No	No	No	No	No

Notes: ENP children

1. Split method dominant
2. Confident use of this mental strategy

Notes: NEMP children

1. Algorithms well established as a mental strategy
2. No other strategies

NEMP activity	ENP children				
	A	B	C	D	E
Calculator Ordering					
Arranged in order correctly	y	y	y	y	y
Read the numbers 14 41 104 140 4010 4100	y	y But read 4100 as 400 and 401 then as 4001 finally as 4100	y	y	y but read 4100 as 4400 initially then correctly

NEMP children			
B1	B2	B3	B4
y	y	y	Omitted from tape
T/A forgot to get her to read the answers	Read 4100 as 400 then asked for if that was right.	y	Omitted from tape

Notes:

ENP children

1. The question was unclear to some children; C needed some prompting to move the cards around before she established the right answer.
2. B needed some time to process the answer

NEMP children

3. B2 could read 4010 but not 4100. She knew 2 and 3-digit numbers and recognised the increasing size of digits.

NEMP activity	ENP children				
	A	B	C	D	E
Girls & Boys					
Make 26 with blocks	✓	✓	✓	✓ Tried to use all of the blocks as ones initially	✓
Is it possible to have equal girls and boys in this class?	✓	✓	✓	✓	not sure
Show what two equal sets from 26 would be	✓	✓ Picks one rod and takes away three. <i>There would be 16 boys and 16 girls.</i> Changed this to 13 because 16 was "too big"	✓ Says she figured it out with her fingers.	✓ Says he just "knew" that 13 was half of 26 and could not explain further.	Not sure
Make 32 with blocks	✓	✓	✓	✓	✓
Is it possible to have equal girls and boys in this class?	✓	✓ Explains... 15+15=30 and 1+1=2 so 16 is the answer. Initially could not see how to use blocks to make 16+16. Prompted, managed to make 2 groups of 16.	✓	✓	not sure

NEMP children			
C1	C2	C3	C4
✓	✓	✓	✓
✓	✓	✓	✓
✓	x Did not use the 26 that he had made up at first. Said 14 was the answer as $14+12=26$; then gave 28 as the sum of 14 and 14.	✓	✓
✓	Made shape of 32 first. ✓	✓	✓
✓	✓ At first said no because there are " <i>normally more girls than boys in a class</i> ".	✓	x Because 3 is not an even number

<p>Girls & Boys (continued)</p> <p>Show what two equal sets from 32 would be</p>	<p>✓ Did not regroup with blocks. Said $16+16=32$ because $2 \times 10=20$ and $2 \times 6=12$ which makes 32.</p>	<p>✓</p>	<p>not sure</p>						
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Notes:

Early Numeracy Project children

1. A and B both used the split method to work out half of 32. $10+10=20$ and $6+6=12$ which makes 32 therefore the answer is 16; or $15+15=30$ and $1+1=2$, therefore $15+1=16$.
B couldn't work it out initially with blocks which may support the suggestion in Note 1.
- D used the blocks as single units initially and then, once prompted, remembered and used them in the correct manner but then over-ruled his own correct answer.

NEMP children

1. Used place value blocks confidently to make up 26 and 32; C1 attempted a split method and came close to the correct answer.
2. C2 tried a split method but confused division with multiplication; C3 exchanged a ten block for a set of 10 ones and showed the correct answer.
3. C4 got stuck on 3 tens being an odd number and therefore not divisible by 2.

NEMP activity	Early Numeracy Project children				
	A	B	C	D	E
Independent					
Number A					
700 + 100	✓	✓	✓	✓	✓
15 735 + 100	✓	✓	✓	✓	✓
12 x 100	✓	✓	✓	✓	✓
316 x 100	4060	no attempt	4060	✓	4060
Number B					
400-100	✓	✓	✓	✓	✓
643 – 100	✓	✓	✓	✓	✓
Number C					
6 x 10	✓	✓	✓	✓	✓
78 x 10	150	700	no attempt	✓	156
write in figures					
five hundred and eighty	✓	✓	✓	✓	✓
two thousand five hundred and eighteen	✓	✓	✓	✓	✓
two hundred thousand and forty three	✓	2340	no attempt	✓	2243

NEMP children			
A1	A2	A3	A4
✓	✓	✓	✓
n/a	✓	✓	✓
n/a	112	✓	✓
n/a	416	✓	✓
✓	100	✓	100
✓	364	✓	143
0.6	✓	✓	✓
1.91	70	✓	150
✓	✓	✓	✓
✓	✓	✓	✓
n/a	2000043	2.140	2043

Notes:

ENP children

1. Adding and subtracting 100 well established
2. Multiplication of a 3-digit number by 10 was difficult for most; curiously same incorrect result for three children.
3. Could all write 3 and 4 digit numbers correctly.
4. Starting to establish multiplying of 10 and 100 correctly.

NEMP children

1. Adding 100 was established; most were successful with adding 100 to a 4 digit number.
2. Subtraction less secure than adding 100; subtracting century similar results.
3. 3 and 4 digit numbers well known; thousands concept and how it is written is not established yet.

NEMP activity	ENP children				
Motorway	A	B	C	D	E
estimate 9×98	✓ 966...I just thought that if $98 + 98$ is... then used split method to work out 196 ... and I thought it would be quite big so got about 900.	x 188...I started with 98 and went all the way up to 188	x Not sure. When I gave the prompt that 98 was close to a number that might make it easier she got 100 when asked what 9×100 was she got 900 then 995.	x long thinking... 166 because he used 9×8 and $9 \times 9 = (72 + 81)$	x 20...why? ... I guessed

NEMP children			
A1	A2	A3	A4
x Looking at T/A ... smiles and says 98×9 . T/A asks for an estimate. Says 500. T/A asks How did you get that? I guessed... I did 9×95 it was too big so I just guessed	x T/A repeats question. Says 1000 and ... looks away... 100+26 or 24... T/A asks how did you get that? I just worked out that there would be 80... 98... child did not explain any more.	x 8172... T/A asks for an explanation I multiplied 8×9 then 9×9	✓ T/A repeats question Whispering to herself... it's 882... I timesed 9×98

Notes:

ENP children

1. This seemed a difficult one for most of the children.
2. Evidence of trying to use the split method to work it out by B; D tried to use the algorithm method.

NEMP children

1. Mental algorithms attempted by all children, only A4 was successful. A3 multiplied correctly but could not rename.

NEMP activity	ENP children				
	A	B	C	D	E
Number Order					
Arrange the digits 3, 5, 8, 1 to make the smallest possible number	✓	✓	lost this section of tape	lost this section of tape	✓ 1835 read it correctly
Read 1358	✓	✓	lost	lost	x
Arrange the digits 3, 5, 8, 1 to make the biggest possible number	✓	✓			8351 read it correctly
Read 8531	✓	✓	lost	lost	x

NEMP children			
A1	A2	A3	A4
x Adds $5 + 8 = 13$ Unsure T/A prompt "Is that the biggest number you can make using all of the cards?" Still unsure	✓	✓	✓ Initially unsure and starts to add $3 + 5$ then multiplies $3 \times 5 = 15$.. = 8 is $23 + 1 = 24$
	✓	✓	✓
Not sure of the question	✓	✓	✓ Gets it and also changes answer to previous question
	✓	✓	✓

Notes:

ENP children

1. This seemed to be an easy activity for the children

NEMP children

1. The question caused some confusion with A1 and A4. They both thought that they were being asked to add the digits together.

NEMP activity	ENP children				
	A	B	C	D	E
Population					
Read 2495	✓	✓	✓	✓	✓
place value of 4	✓	✓	✓ Needed the question clarified first.	✓	40
place value of 5	✓	✓	✓	✓	✓
place value of 9	✓	✓	✓	✓	9
place value of 2	✓	✓	✓	✓	2

NEMP children			
B1	B2	B3	B4
✓	249...no... 24095	✓	✓
✓ 100s um... 400	x It's the second number.	✓	Do you mean like what column is it in? T / A repeats question... 4 tens
✓	x It's the last number so that's what everything has.	✓	x one... I don't know
✓ tens	x not sure	✓	x one... not sure
✓	x not sure	✓	✓ 2000

Notes:

ENP children

1. "What does (the number) stand for?" was not initially understood by children who later answered the question correctly.
2. Is the prompt in the NEMP manual unclear? E read "2" for the value of the 2 in 2495 and when asked how she got that explained that it was in the thousands column which would suggest that she understands more about place value than the question reveals.

NEMP children

1. Two children had difficulty understanding the question; B4 knew that the 2 was 2000 so she may have understood the other questions if she had been asked in another way.

NEMP activity	ENP children				
	A	B	C	D	E
Speedo					
1996 + 1	✓	3906	✓	✓	✓
1996 + 10	✓	12906	1916	✓	2060
1996 + 100	✓	no attempt	no attempt	2996	✓
1996 + 1000	✓	no attempt	no attempt	✓	no attempt
3402 - 1	✓	4804	✓	✓	✓
3402 - 10	✓	13804	no attempt	unclear	✓
3402 - 100	✓	no attempt	no attempt	✓	✓
3402 - 1000	✓	no attempt	no attempt	✓	✓

NEMP children			
A1	A2	A3	A4
✓	3224	✓	✓
1916	9999	1906	✓
1600	99999	2006	✓
6000	400999	✓	✓
3403	4402	✓	✓
3412	12402	✓	not attempted
3100	100402	✓	3300
2000	100402	✓	✓

Notes:

ENP children

1. Why use 1996 for the addition question? It involves two steps: adding and renaming as well for adding 10 and 100.
2. Why use 3402 for the same reason? Renaming with the take away ten question.
3. Suggest using 1546 and 3547 as questions that have only the place value component and do not rely on renaming as well.

NEMP children

1. Achieved slightly above the NEMP (2001) report results.
2. A wide range of answers given, with some six-figure numbers that showed a lack of understanding of larger numbers.