Determining the Rental Rate on Commercial Real Estate Leases*

Glenn Boyle

NZ Institute for the Study of Competition and Regulation
Victoria University of Wellington

Graeme Guthrie

School of Economics and Finance
Victoria University of Wellington

Neil Quigley

School of Economics and Finance
Victoria University of Wellington

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Abstract

Although the theoretical framework for determining equilibrium rental rates on commercial land is well-established, applying this framework in practice is difficult because of its dependence on unobservable parameters such as land growth and discount rates. We show how this problem can be overcome by a straightforward application of nonlinear regression methods to actual market transactions involving leased commercial land. This approach avoids the need to invoke ad-hoc parameter values, automatically incorporates any land liquidity premium into rental rates, and ensures that rental, growth, and discount rates are all estimated within a single analytically-consistent framework.

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1 Introduction

In New Zealand, the use of commercial land is often separated from its ownership: firms requiring the use of land for a business venture frequently lease the land rather than own it outright. Such leases have the following general characteristics:

1. Rental payments are due every m years in advance and are fixed in nominal terms for T years at which time they are reviewed.
2. There are n rent revisions scheduled during the life of the lease, i.e., the lease has a total length of nT years.
3. The rent set at each review is based on the current market value of the land leased in an unimproved state, with no account taken of the value of the improvements affected by the lessee.
4. All expenses associated with the use of the property are payable by the lessee.
5. If the lease is forfeited or not renewed, all buildings and other improvements on the land revert to the lessor free from any payment or compensation.
6. The lessee may assign, sub-lease or sell his interest in the land with the consent of the lessor.
7. The lessor may assign or sell his interest in the land.

A crucial aspect of these leases, at both the date of inception and at subsequent review dates, is the setting of the rental payments to be made by the lessee. Lally (2001) and Lally and Randal (2004) develop an analytical framework for determining equilibrium rental payments, but implementation of their approach requires knowledge of two unobservable parameters — the expected long-run growth rate in the value of unimproved commercial land and the expected return required by investors in such land.[1] To deal with this problem, Lally presents an example in which the two parameters are estimated independently from models that make use of historical data on land and securities prices respectively. However, the equilibrium rental rate is sensitive to both these parameters, so even a small error in the estimation of either parameter has adverse implications for the accuracy of the rental payment calculation. Moreover, allowing for a range

[1] Lally and Randal (2004) generalise the model of Lally (2001) to allow for rent to be paid at non-annual frequencies, but the basic framework is fully developed in the earlier paper, as is the suggested method for applying it to practical situations. In this paper, the 'Lally-Randal framework' thus indicates the specific model that appears in Lally and Randal (2004) together with the implementation approach outlined in Lally (2001).

of possible parameter values results in an imprecisely estimated rental rate, thereby reducing its practical utility.

In this paper, we outline a fundamentally different approach to estimating these parameters. Our method is based on the principle that unobservable parameter values can be inferred from observed market prices, thereby eliminating the need to invoke ad-hoc estimates derived from historical data. Applying nonlinear regression methods to the Lally and Randal framework, we obtain estimates of land growth and discount rates that are implied by secondary market transactions in leased commercial land. These in turn allow us to calculate very precise estimates of the equilibrium rental rate.

Although not previously applied to rental rates, such an approach has a rich tradition in finance theory and practice. For example, motivated by the evidence of Chiras and Manaster (1978) that the implicit estimates obtained from option pricing models are better forecasts of future stock price volatility than estimates based on historical data, options traders commonly use the volatility parameter implied by one contract to determine the fair price of less liquid contracts written on the same stock. Similarly, Cornell (1999), Claus and Thomas (2001), Fama and French (2002) and others argue that the discount rate implied by dividend growth models provides a better estimate of the equity premium than historical averaging methods. In both cases, the implicit approach is seen as being superior because it uses the information about expectations embedded in market prices.[2]

As well as being of practical value, our paper makes two methodological contributions. First, it shows how the equilibrium rental payment on any commercial land lease can be obtained within a single theoretical framework utilising only data directly relating to land leases. Second, because this process requires estimation of two parameters rather than one, it also shows how the implicit method described in the above paragraph can be extended to a bivariate setting.

In the next section, we briefly describe the Lally and Randal (2004) framework, note its key implications, and outline their suggested approach to practical implementation. Section 3 explains our alternative approach in detail, and provides an example that illustrates its principal features. Section 4 offers some concluding remarks.

2 The Lally-Randal model

Lally and Randal (2004) consider a traditional lease (i.e., where the rental payment is expressed as a proportion r of land value L) with the characteristics described in (i) and (ii) of the Introduction. Using standard finance valuation techniques, they show that

$$r = \frac{1}{(1-t_c)m}\left(1-\left(\frac{1+g}{1+k}\right)^T\right)\left(\frac{1-(1+k_R)^{-m}}{1-(1+k_R)^{-T}}\right). \tag{1}$$

[2]Campbell et al. (1997) note that this approach sometimes involves strong assumptions about the specific pricing model used to back out the parameters being estimated.

where t_c is the corporate tax rate, g is the expected long-run growth rate in the value of commercial land, k is the expected return required by investors in such land, and k_R is the after-tax riskless interest rate.

The intuition underlying equation (**??**) is straightforward. In equilibrium, the current value of the land must equal the present discounted value of all future rent payments (plus any residual land value at the end of the lease); if this were not the case, then either the lessee or the lessor would prefer to transact in land rather than enter the lease. Equation (**??**) gives the rental rate that ensures this equality holds. Indeed, (**??**) is simply a mathematically-correct version of the so-called "total return" model sometimes used by land valuers. This can be seen by linearising (**??**) to obtain

$$k = r(1 - t_c) + g \tag{2}$$

which states that the total expected return is equal to the after-tax rental rate plus the expected capital gain. Equation (**??**) thus differs from the total return model only in that it (correctly) takes account of the fact that rent is fixed for T years and is not reset at each rental payment date.

The equilibrium rental rate given by (**??**) depends on six parameters: t_c, m, g, k, T and k_R. Of these, the rent payment frequency m and the time between rent revisions T are observable characteristics of the lease, while the tax rate t_c and the riskless interest rate k_R are readily available from public sources. However, g and k are not directly observable and hence must be estimated.

Note, however, that the rental rate r given by (**??**) does not depend on the individual values of g and k, but only on their relative value, as indicated by the term $\phi = (1+g)/(1+k)$. If ϕ is high, then the proportion of total return offered by expected capital gain is high, and hence the required rental rate is low; if ϕ is low, then the proportion of total return offered by expected capital gain is low and the required rental rate is high. As a result, alternative combinations of g and k that yield the same value of ϕ have no effect on r.

Lally (2001) suggests a three-step procedure for applying equation (**??**) to practical situations:

1. Use 40 years of data on rural land prices to obtain an estimate $\hat{g}$ of the long-run expected rate of growth in commercial land value.

2. Use historical financial data and the Capital Asset Pricing Model (CAPM) to obtain an estimate $\hat{k}$ of the expected return on land as follows:

$$\hat{k} = k_R + \beta\Phi,$$

   where β is the systematic risk of land (its 'beta') and Φ is the market risk premium.

3. Substitute $\hat{g}$ and $\hat{k}$, along with t_c, m, T and k_R into (1) in order to arrive at the appropriate value of r.

This procedure estimates g and k independently — the former as an average over past realisations and the latter via the theoretical CAPM relationship between risk and return. Unfortunately, two problems arise. First, g and k are endogenous price variables that are determined jointly, so any attempt to estimate them independently necessarily ignores this property. This error will be particularly problematical if the data series used to estimate these parameters are short or ill-matched. Second, using the CAPM to estimate the expected return on land investment is fraught with problems. In particular, the CAPM assumes that all assets are freely marketable, perfectly divisible, and highly liquid. Although such assumptions are perfectly reasonable for most financial markets, they are not a very good description of land markets. Sales of land are often time consuming, while marketing of the land and transfer of title are both costly. Land is also a lumpy asset: it is not divisible into small units to facilitate individual title to multiple owners. As a result, the CAPM is likely to significantly understate k because it makes no allowance for the risks associated with poor liquidity.[3]

Such a bias can have major implications for estimates of the equilibrium rental rate r because, as Figure 1 illustrates, the value of r given by equation (1) is highly sensitive to variations in k. Thus, even relatively small under-estimates of k can have an economically significant impact on estimates of r. For example, suppose that the true k for the lease depicted in Figure 1 is 10%; then the equilibrium r from equation (**??**) is 9.1%. But suppose k is wrongly estimated to be 9%; then the rental rate given by (1) is only 8.0%, more than one percentage point below its true value.

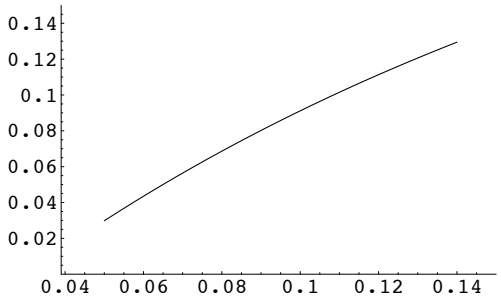

Figure 1: The rental rate (r) as a function of the Land Discount Rate (k). The figure plots the rental rate given by equation (**??**) for various values of k. Parameter values are $m = 1$, $t_c = 0.33$, $g = 0.03$, $T = 5$, $k_R = 0.045$.

This source of error is potentially magnified by the independence of the k and g estimates. Consider again the lease depicted in Figure 1 and suppose that, in addition to the underestimate of k, the growth rate g is wrongly estimated to be 4% when it is in fact 3%. Then the rental rate given by equation (**??**) is only 6.8% — 2.3 percentage points below its true value.

Clearly, even quite small errors in the estimates of g and k can result in very misleading

[3]Many studies in finance (e.g., Silber, 1991; Longstaff, 1995; Acharya and Pedersen, 2005) suggest that investors rationally require a higher rate of return to invest in illiquid assets such as land, and that this premium can be large.

estimates of r. This suggests the need for an alternative method that either minimises these errors, or minimises their impact on the rental rate calculation. In the next section, we describe such a method.

3 Inferring k and g from market transactions

Using the CAPM to estimate k represents a theoretical attempt to determine the expected return investors would require on commercial land *if* claims to such assets were re-packaged as securities and traded in financial markets. Similarly, using historical data on rural land prices to estimate the future expected growth in commercial land value assumes that past realisations from one price series provide a useful guide to future realisations of another price series. Clearly, it would be more desirable to estimate these parameters using information contained in commercial land data. In this section, we describe one method for doing so, based on actual market transactions for leased commercial land.

A relatively common transaction in commercial land is the so-called sale of lessor's interest (SOLI), in which the lessor sells his interest in a land lease to a third party (or sometimes the lessee). The asset traded in such transactions offers (i) the right to receive rental payments during the remaining term of the lease and (ii) the right to the land at the termination of the lease. Thus, the equilibrium price P_S of a SOLI that occurs S years into a lease must be equal to the present value of the remaining rental payments plus the residual land value at the end of the lease. In the Appendix, we show that P_S is given by

$$\begin{aligned} P_S &= (1-t_c)rmL_0\left(\frac{1-(1+k_R)^{-(T-S)}}{1-(1+k_R)^{-m}}\right) \\ &\quad + (1-t_c)rmL_S\left(\frac{1+g}{1+k}\right)^{T-S}\left(\frac{1-(1+k_R)^{-T}}{1-(1+k_R)^{-m}}\right)\frac{1-\left(\frac{1+g}{1+k}\right)^{(n-1)T}}{1-\left(\frac{1+g}{1+k}\right)^{T}} \\ &\quad + L_S\left(\frac{1+g}{1+k}\right)^{nT-S} \end{aligned} \tag{3}$$

where L_0 is the value of the land at the last rent-setting date. Letting $rL_0 = C$, we can rewrite this as

$$P_S - (1-t_c)mC\left(\frac{1-(1+k_R)^{-(T-S)}}{1-(1+k_R)^{-m}}\right) = \left(\frac{L_S}{L_0}\right)A,$$

where, using equation (**??**) and simplifying

$$\begin{aligned} A &= (1-t_c)mC\left(\frac{1+g}{1+k}\right)^{T-S}\left(\frac{1-(1+k_R)^{-T}}{1-(1+k_R)^{-m}}\right)\frac{1-\left(\frac{1+g}{1+k}\right)^{(n-1)T}}{1-\left(\frac{1+g}{1+k}\right)^{T}} + L_0\left(\frac{1+g}{1+k}\right)^{nT-S} \\ &= \left(\frac{1-(1+k_R)^{-T}}{1-(1+k_R)^{-m}}\right)\left(\frac{(1-t_c)mC}{1-\left(\frac{1+g}{1+k}\right)^{T}}\right)\left(\frac{1+g}{1+k}\right)^{T-S}. \end{aligned}$$

Therefore

$$Q_S = \left(\frac{\left(\frac{1+g}{1+k}\right)^{T-S}}{1-\left(\frac{1+g}{1+k}\right)^{T}} \right) \left(\frac{L_S}{L_0} \right), \tag{4}$$

where

$$Q_S = \frac{P_S}{(1-t_c)mC} \left(\frac{1-(1+k_R)^{-m}}{1-(1+k_R)^{-T}} \right) - \left(\frac{1-(1+k_R)^{-(T-S)}}{1-(1+k_R)^{-T}} \right)$$

is a linear transformation of the lease's price-earnings ratio.[4] Note that all components of Q_S are observable, so for any given realisation of $\frac{L_S}{L_0}$ there is a unique value of ϕ that satisfies equation (**??**).

Now suppose that the value of unimproved land, L_S follows a geometric Brownian motion with volatility σ and drift equal to $\log(1+g)$.[5] Then

$$L_S = L_0 \exp\left(\left(\log(1+g) - \frac{\sigma^2}{2} \right) S + \varepsilon_S \right), \quad \varepsilon_S \sim N(0, \sigma^2 S),$$

Taking the natural logarithm of each side of (**??**), using the above process for L_S, and splitting k into a riskless component (k_R) and a time-invariant risk premium (λ) yields

$$\log Q_S = S\left(\log\left(1 + k_{R,i} + \lambda\right) - \frac{\sigma^2}{2} \right) - \log\left(\left(\frac{1 + k_{R,i} + \lambda}{1+g} \right)^{T} - 1 \right) + \varepsilon_S, \tag{5}$$

which is readily recognisable as a standard regression model with heteroskedastic disturbances and unknown parameters g, λ, and σ. Although this model is nonlinear, it is straightforward to obtain maximum likelihood estimates of these parameters from SOLI data, which can then be used in equation (**??**) to arrive at a value for r.[6]

This approach has several attractive features. First, because it is based on actual market transactions involving land under lease, the estimates we obtain necessarily include any relevant allowance for liquidity risk. Second, because it involves estimating the 'best' combination of g and λ, values for these parameters are obtained jointly rather than independently. Third, all unknown parameters (the rental, growth and discount rates) are calculated within a single theoretical framework that relates market values to the present value of future rent payments, thereby dispensing with the need to introduce other models such as the CAPM. Fourth, if the lease under consideration has a so-called ratchet clause (whereby rents can never fall from one period to the next) that necessitates the use of option pricing methods, our model's estimate of σ eliminates the need to undertake a separate estimation of land value volatility.

[4] For a lease with a one-year term ($T = 1$) and annual rent payments ($m = 1$) that is sold on a rent-setting date ($S = 0$), $1 + Q = \frac{P_S}{(1-t_c)C}$.

[5] Note that this assumption is consistent with the requirement that $E_0[L_S] = (1+g)^S L_0$.

[6] In applying this procedure, g, λ, and σ are chosen to minimize the weighted sum of squared errors, where the weight allocated to each data point is the reciprocal of the number of years between the sale and the most recent revision. It follows that sales that occur a long time into a lease cycle have a relatively low influence on our estimates.

Finally, one potential problem with implementing this approach is the presence of "fire sales" in the data — leases that are sold for an artificially low price due to financial distress on the part of the original lessor. If such transactions were prevalent in the data used to estimate (**??**), then the estimated value of k — and hence r — would tend to be biased upwards. However, (**??**) is only defined when $Q_S > 0$; that is, when

$$\frac{P_S}{(1-t_c)C} > \frac{m(1-(1+k_R)^{S-T})}{1-(1+k_R)^{-m}},$$

where the right-hand term is approximately equal to $T - S$. Consequently, equation (**??**) can only be estimated when the SOLI sale price exceeds annual rent payments by some minimum threshold, thereby automatically ruling out the inclusion of fire sales observations.

Application

To illustrate the approach described in Section **??**, we utilise data from 30 SOLIs that occurred in the Wellington region between April 1993 and March 2007.[7] The details of these transactions are listed in Table **??**. As a proxy for k_R, we use the 5-year government bond rate prevailing on the date that the SOLI occurred.

Estimating equation (**??**) using the Table **??** data yields the results appearing in Table **??**.[8] The first three columns show the maximum likelihood estimates of the long-run expected growth rate in the value of unimproved land ($\hat{g}$), the risk premium component of the expected return on land ($\hat{\lambda}$), and the volatility in the growth of land value ($\hat{\sigma}$); asymptotic standard errors for these estimates are given in parentheses. The fourth column shows the resulting estimate of $(1+g)/(1+k'_R+\lambda)$ ($\hat{\phi}$), where k'_R is the 'current' 5-year government bond rate, i.e., the rate prevailing on the rent-setting day for the lease whose equilibrium rental rate we wish to determine. To facilitate comparison with the approach suggested by Lally (2001), we set $k'_R = 0.045$ and $m = 1.0$. The final three columns then use equation (**??**) to estimate the rental rate $\hat{r}$ for this lease, assuming lease terms of 5, 10 and 21 years respectively. We use the delta method (see Greene, 2003; and Xu and Long, 2005) to obtain standard errors for $\hat{\phi}$ and $\hat{r}$.

We first check the model specification: if correctly specified, then the standardized residuals (i.e., the residual for each SOLI divided by $\sqrt{S_i}$) are normally distributed. The Bera-Jarque test statistic for normality of the adjusted residuals equals 0.125. Since the test statistic is asymptotically distributed according to $\chi^2_{[2]}$, implying a p-value of 0.939, we cannot reject the null hypothesis that the residuals are normally distributed at conventional levels of significance.

Turning to the Table **??** parameter estimates, we see that $\hat{g} = 0.082$, $\hat{\lambda} = 0.118$, and $\hat{\sigma} = 0.264$. These estimates seem sensible, insofar as they broadly correspond to those obtained

[7] We are grateful to Wareham Cameron Ltd for providing this information. One further transaction was deleted from the sample because it failed to pass the fire sale test described above.

[8] Because information on the frequency of rent payments is unavailable, we assume that all leases in Table **??** make annual payments. We also assume $t_c = 0.33$.

Table 1: Data on Sales of Lessor's Interest

This table summarises data from 30 Sales of Lessor's Interest (SOLIs) that occurred in the Wellington region between April 1993 and March 2007. T is the term of the lease in years, S is the number of years between the date on which the SOLI occurred and the previous rent-setting date, C is the annual rent payment on the lease as of the date on which the SOLI occurred, P is the price at which the lease was sold, and k_R is the after-tax 5-year government bond rate prevailing on the date that the SOLI occurred.

$SOLI$	T	S	C	P	k_R
1	21.083	0.764	35438	390000	0.043
2	14.075	0.833	149688	1855000	0.050
3	5.000	0.833	60000	640000	0.040
4	13.997	0.842	87857	850000	0.048
5	5.000	0.917	6977	110000	0.050
6	7.000	1.303	100000	920000	0.059
7	14.000	1.314	59500	740000	0.040
8	5.000	1.750	24464	330000	0.049
9	2.964	1.881	90000	800000	0.049
10	5.000	2.167	39000	420000	0.060
11	5.000	2.250	5450	62500	0.054
12	12.000	2.333	315000	3302500	0.059
13	21.000	2.400	24625	352000	0.045
14	21.000	2.917	172500	2350000	0.038
15	7.000	3.333	28125	354000	0.059
16	21.000	3.583	12378	165000	0.049
17	5.000	4.167	516250	9831375	0.044
18	5.000	4.167	225060	4083750	0.044
19	5.083	4.917	13500	235000	0.057
20	7.000	4.919	15148	168000	0.041
21	7.000	5.667	47303	525583	0.047
22	21.000	8.658	41915	450000	0.059
23	12.000	8.917	122000	2000000	0.040
24	21.000	9.333	550400	5181500	0.048
25	21.000	9.917	76000	875000	0.039
26	20.000	10.333	6175	65000	0.040
27	21.000	12.000	514528	4100000	0.040
28	13.997	12.342	87857	6700000	0.041
29	21.000	15.083	51800	550000	0.045
30	21.083	18.667	30987	750000	0.041

from different settings. For example, Quotable Value report that the median growth in industrial land value over the 25 years to 2006 was 10.74%, while Lally and Randal (2004) estimate the standard deviation of rural land price growth to be 30%. Moreover, 11.8% is consistent with risk premium estimates for illiquid assets – see, for example, Kerins et al (2004) and Acharya and Pedersen (2005). Together, the estimates of g and k imply $\hat{\phi} = 0.93$ and, for a 10-year lease, $\hat{r}$ equals 9.3%.

An interesting feature of these results is that ϕ is very precisely estimated even though g and λ, are not. This can be seen more clearly in Figure 2 which plots the 'confidence ellipse' for the latter two parameter estimates. For any combination of g and λ inside the region bounded by the solid curve, we cannot reject the null hypothesis (at a 5% level) that the parameters take these values. However, we can reject this hypothesis for any combination outside the region bounded

Table 2: Maximum likelihood estimates of equation (**??**) using SOLI data

Assuming annual rent payments ($m = 1.0$), this table uses the data in Table **??** to obtain maximum-likelihood estimates of g, λ, and σ from equation (**??**). These are in turn used to obtain estimates of, firstly, $\phi = (1+g)/(1+k_R+\lambda)$, and then, from equation (**??**), the rental rate r for leases of varying length. For the latter, we set $k_R = 0.045$. Asymptotic standard errors are in parentheses; for $\hat{\phi}$ and $\hat{r}$, these are calculated using the delta method. The second and third panels repeat this exercise for SOLI transactions where the purchaser is an outside investor and the lessee respectively.

Sample	$\hat{g}$	$\hat{\lambda}$	$\hat{\sigma}$	$\frac{1+\hat{g}}{1+k_R+\hat{\lambda}}$	$\hat{r}$		
					$T=5$	$T=10$	$T=21$
Full ($n = 30$)	0.082	0.118	0.264	0.930	0.099	0.093	0.083
	(0.029)	(0.035)	(0.034)	(0.005)	(0.006)	(0.004)	(0.002)
Sales to outside investors ($n = 12$)	0.077	0.114	0.233	0.929	0.100	0.094	0.084
	(0.042)	(0.049)	(0.048)	(0.006)	(0.007)	(0.005)	(0.003)
Sales to the lessee ($n = 18$)	0.085	0.120	0.283	0.931	0.098	0.092	0.083
	(0.040)	(0.048)	(0.047)	(0.007)	(0.008)	(0.006)	(0.004)

by the solid curve. The narrow shape of the ellipse indicates that the estimates of g and λ are highly positively correlated, the source of which is apparent from equation (**??**). Other than via the L_S term, P_S depends only on ϕ and not on the individual values of g and λ, so the regression model — equation (**??**) — is close to being under-identified. As a result of this property, any estimation error in $\hat{g}$ (which affects the numerator of ϕ) will tend to be substantially offset by an estimation error in $\hat{\lambda}$ (which affects the denominator of ϕ). Consequently, although the components of ϕ have substantial estimation errors, ϕ itself does not. And since the rental rate is a function of ϕ only (and not g and λ separately), the precision in estimating ϕ feeds through into the estimated rental rates. For example, the lease with ten-yearly rent reviews ($T = 10$) has an estimated rental rate of 9.3% with a 95% confidence interval of $[0.085, 0.101]$.

It is instructive to compare these estimates with those of Lally (2001). Using historical financial and land price data to obtain a range of estimates for g and k, he arrives at a mid-point rental rate estimate of 0.064 for the 10-year lease, within a possible range of $[0.044, 0.096]$. This is somewhat lower than the estimates we obtain for the same lease, as would be expected given the inability of Lally's approach to account for a liquidity premium in land discount rates (see the discussion in Section **??**). Moreover, the possible range of rental rates of 4.4% to 9.6% is very wide, and thus of questionable value for practical purposes. Finally, because Lally's approach is not based on any statistical inference, it is impossible to construct confidence intervals for these estimates. Overall, relative to our rental rate estimates, those generated by Lally's approach are low, imprecise, and lack any statistical reference point. Clearly, the question of how rental rates might best be estimated is not an economically-trivial matter.

Our SOLI data sample contains two types of transactions: those where the lease is sold to an outside investor and those where it is sold to the lessee. In case the dynamics underlying these two cases differ, we repeat our estimation exercise for each group separately. Although, as

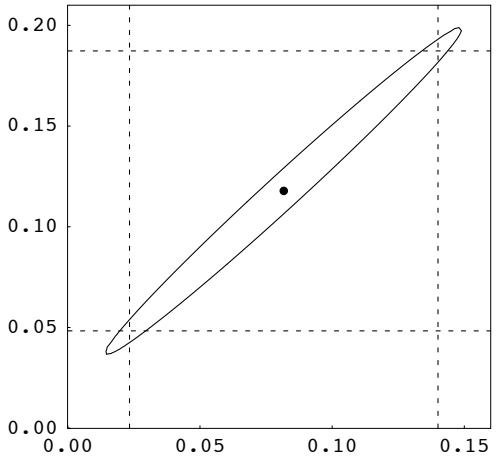

Figure 2: Confidence ellipse for average growth rate and risk premium. The point in the middle of the graph shows the point estimate of g and λ, while the two sets of dashed lines show the 95% confidence intervals for each parameter separately. For any combination of g and λ inside the region bounded by the solid curve, we cannot reject the null hypothesis (at a 5% level) that the parameters take these values. However, we can reject this hypothesis for any combination outside the region bounded by the solid curve.

can be seen in the second and third panels of Table **??**, this results in slightly higher standard errors due to the smaller number of observations, it has no meaningful effect on the rental rate point estimates: regardless of the type of transaction, the estimated rental rate is essentially identical to that obtained for the full sample.

4 Concluding Remarks

Disputes over commercial land rentals are not infrequent. Although the analytical framework appropriate for resolving such disputes is now largely agreed, application of this framework to real-world leases is made difficult by our inability to directly observe certain key parameters.

One approach advocated in prior literature is to, first, use additional analytical models in order to estimate these unobservable parameters from historical data and, second, substitute these estimates back into the original framework in order to calculate the appropriate rental rate. However, the problems associated with this approach can lead to significant errors in the estimated rental rate. In this paper, we suggest an alternative approach based on maximum-likelihood estimation that involves only the application of the original analytical framework to actual market transactions in leased commercial land. By using data from such transactions, this alternative approach avoids the above-mentioned problems and yields more precise rental rate estimates.

This approach also follows a long-standing finance tradition of using the information in market prices to infer the values of underlying pricing parameters. The application of this method to rental rates as we have described it here is undoubtedly capable of further refinement. We hope this paper will encourage further research along such lines.

References

Acharya, V. and L. Pedersen, 2005. Asset pricing with lliquidity risk. *Journal of Financial Economics* 77, 375–410.

Campbell, J., A. Lo and C. MacKinlay, 1997. *The Econometrics of Financial Markets.* Princeton University Press: Princeton, NJ.

Chiras, D. and S. Manaster, 1978. The information content of options prices and a test of market efficiency. *Journal of Financial Economics* 6, 213–234.

Claus, J. and J. Thomas, 2001. Equity premia as low as three percent? Evidence from analysts' earnings forecasts for domestic and international stock markets. *Journal of Finance* 56, 1629–1666.

Cornell, B., 1999. *The Equity Risk Premium.* John Wiley and Sons: New York.

Fama, E. and K. French, 2002. The equity premium. *Journal of Finance* 57, 637–659.

Greene, W. 2003. Econometric Analysis, 5th Ed. New York: Prentice-Hall.

Kerins, F., J. Smith and R. Smith, 2004. Opportunity cost of capital for venture capital investors and entrepreneurs. *Journal of Financial and Quantitative Analysis* 39, 385–405.

Lally, M., 2001. The rental rate on land, revision frequency and inflation. *Pacific Accounting Review* 13, 17–34.

Lally, M. and J. Randal, 2004. Ground rental rates and ratchet clauses. *Accounting and Finance* 44, 187–202.

Longstaff, F., 1995. How much can marketability affect security values? *Journal of Finance* 50, 1767–1774.

Muth, J., 1961. Rational expectations and the theory of price movements, *Econometrica* 29, 315–335.

Silber, W., 1991. Discounts on restricted stock: The impact of illiquidity on stock prices. *Financial Analysts Journal* 47, 60–64.

Xu, J. and J. Long, 2005. Confidence intervals for predicted outcomes in regression models for categorical outcomes. *Stata Journal* 5, 537–559.

Appendix A: Proof of equation (??)

Consider a commercial land lease that, at its most recent review date, was determined to have a land value L_0 and a rental rate r. If the lessor's interest in this lease is sold S years after this

revision date, the equilibrium price is given by

$$\begin{aligned}
P_S &= (1-t_c)rmL_0\left(1+\frac{1}{(1+k_R)^m}+\ldots+\frac{1}{(1+k_R)^{T-S-m}}\right) \\
&\quad+\frac{(1-t_c)rmL_S(1+g)^{T-S}}{(1+k)^{T-S}}\left(1+\frac{1}{(1+k_R)^m}+\ldots+\frac{1}{(1+k_R)^{T-m}}\right) \\
&\quad+\frac{(1-t_c)rmL_S(1+g)^{2T-S}}{(1+k)^{2T-S}}\left(1+\frac{1}{(1+k_R)^m}+\ldots+\frac{1}{(1+k_R)^{T-m}}\right) \\
&\quad+\ldots+\frac{(1-t_c)rmL_S(1+g)^{(n-1)T-S}}{(1+k)^{(n-1)T-S}}\left(1+\frac{1}{(1+k_R)^m}+\ldots+\frac{1}{(1+k_R)^{T-m}}\right) \\
&\quad+L_S\left(\frac{1+g}{1+k}\right)^{nT-S}.
\end{aligned}$$

Simplifying the various geometric series appearing in this equation yields

$$\begin{aligned}
P_S &= (1-t_c)rmL_0\left(\frac{1-(1+k_R)^{-(T-S)}}{1-(1+k_R)^{-m}}\right) \\
&\quad+(1-t_c)rmL_S\left(\frac{1+g}{1+k}\right)^{T-S}\left(\frac{1-(1+k_R)^{-T}}{1-(1+k_R)^{-m}}\right)\frac{1-\left(\frac{1+g}{1+k}\right)^{(n-1)T}}{1-\left(\frac{1+g}{1+k}\right)^{T}} \\
&\quad+L_S\left(\frac{1+g}{1+k}\right)^{nT-S}.
\end{aligned}$$